Entanglement and Bell non locality in high-energy collisions

Wien
19 November 2024

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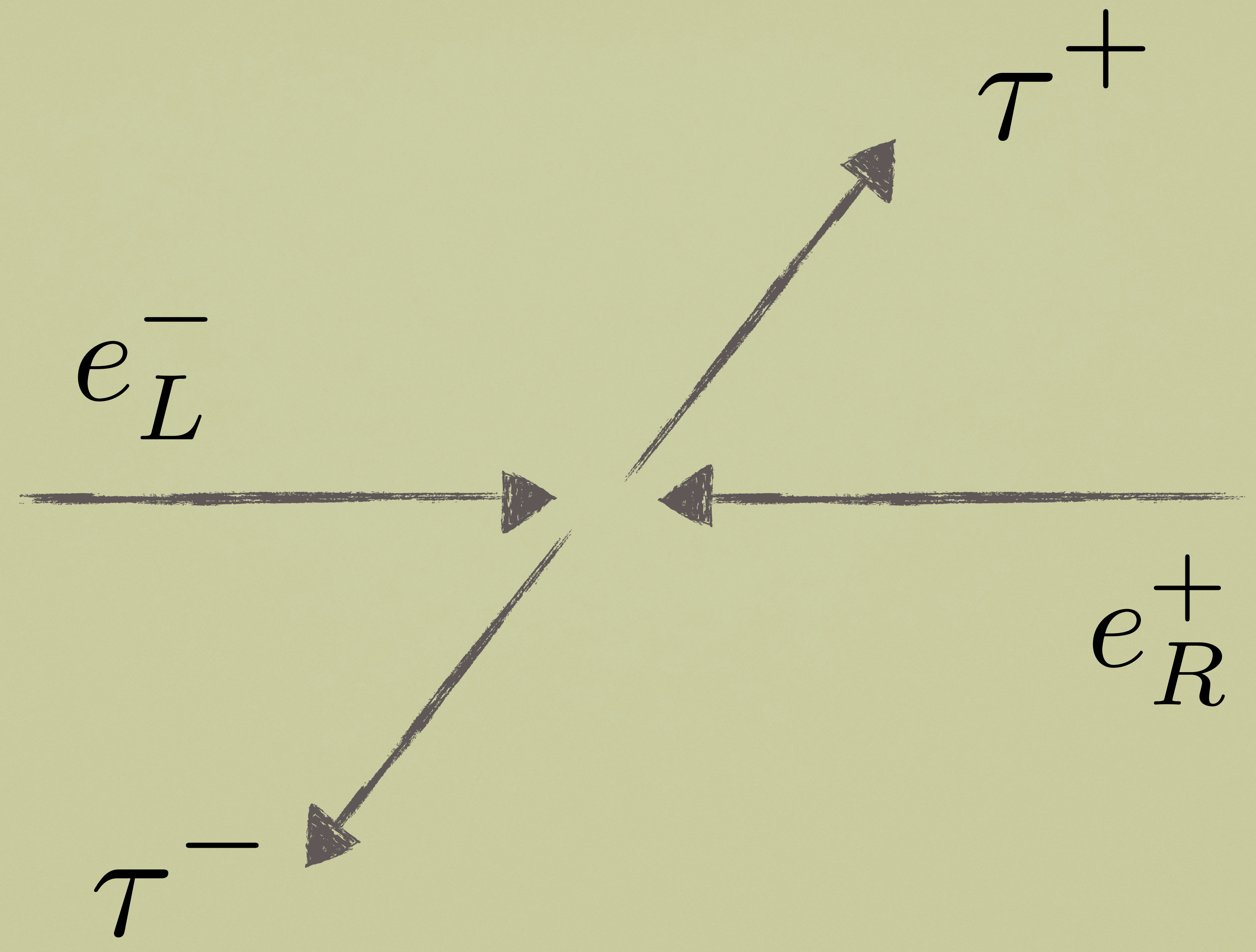
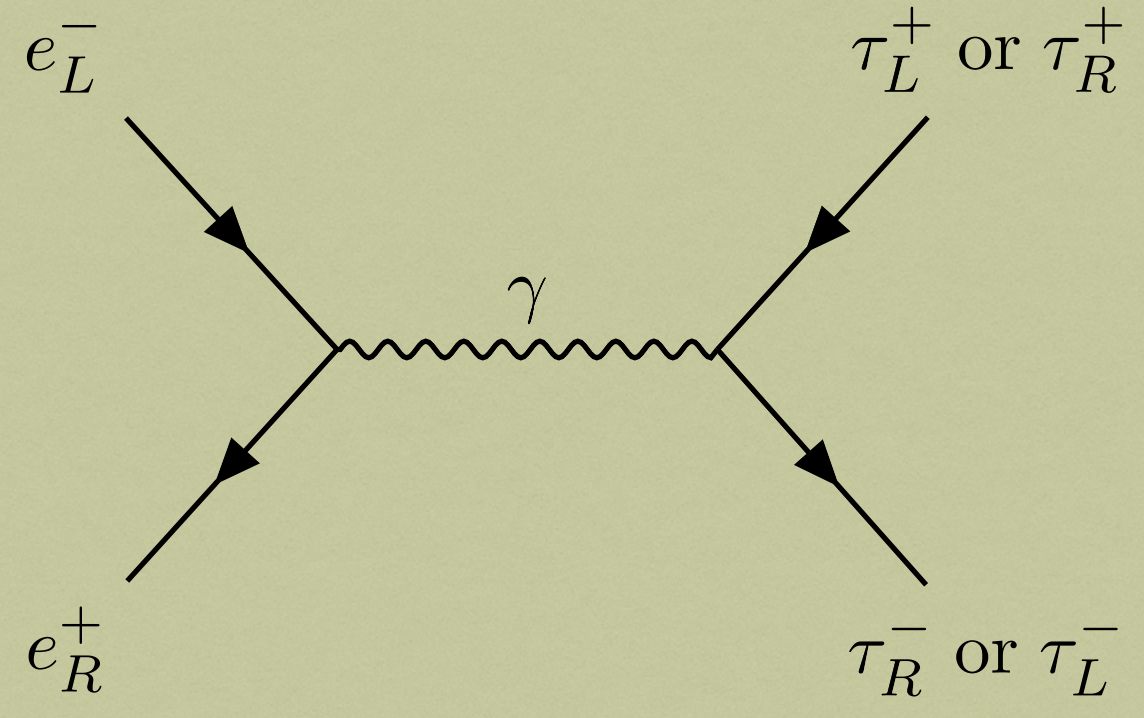


$$\zeta_1 |\tau_L^-\rangle |\tau_L^+\rangle + \zeta_2 |\tau_R^-\rangle |\tau_L^+\rangle + \zeta_3 |\tau_L^-\rangle |\tau_R^+\rangle + \zeta_4 |\tau_R^-\rangle |\tau_R^+\rangle \quad \left(\sum_i |\zeta_i|^2 = 1 \right)$$



$$\underbrace{\left(1 + \cos\Theta\right)}_{\zeta_2 = D^{(1)}_{1,1}(\Theta)} |\tau_R^-\rangle\, |\tau_L^+\rangle + \underbrace{\left(1 - \cos\Theta\right)}_{\zeta_3 = D^{(1)}_{1,-1}(\Theta)} |\tau_L^-\rangle\, |\tau_R^+\rangle$$

$J = \pm 1 \quad J_z = \pm 1 \quad (\Theta = 0)$	$\|\tau_R^-\rangle\, \|\tau_L^+\rangle$	separable
$J = \pm 1 \quad J_z = 0 \quad (\Theta = \pi/2)$	$\frac{1}{\sqrt{2}}\left(\|\tau_R^-\rangle\, \|\tau_L^+\rangle + \|\tau_L^-\rangle\, \|\tau_R^+\rangle\right)$	entangled (Bell state)

The quantum in quantum field theory

Entanglement

Bell inequality violation

Entanglement

$$\Psi = \frac{1}{\sqrt{2}}\left(|\tau_R^-\rangle\,|\tau_L^+\rangle + |\tau_L^-\rangle\,|\tau_R^+\rangle\right)$$

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$$\Psi = |\tau_R^-\rangle\,|\tau_L^+\rangle$$

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Bell inequality violation

J. Bell, *On the Einstein Podolsky Rosen paradox*, *Physics Physique Fizika* **1** (1964) 195.

probabilities

$\mathcal{P}(\uparrow_{\hat{n}_i};-)$ spin of one tau-lepton up in the direction n_j

$\mathcal{P}(\uparrow_{\hat{n}_i};\downarrow_{\hat{n}_j})$ spin of one tau-lepton up in the direction n_i other tau-lepton spin down in the direction n_j

Bell inequality violation

stocastic variables

$$\mathcal{P}(\uparrow_{\hat{n}_1}; -) = \int d\lambda\, \eta(\lambda)\, p_\lambda(\uparrow_{\hat{n}_1}; -)$$

$$\int d\lambda\, \eta(\lambda) = 1$$

$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) = \int d\lambda\, \eta(\lambda)\, p_\lambda(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}})$$

$$p_\lambda(\uparrow_{\hat{n}}; \downarrow_{\hat{m}}) = p_\lambda(\uparrow_{\hat{n}}; -)\, p_\lambda(-; \downarrow_{\hat{m}})$$

Bell locality assumption

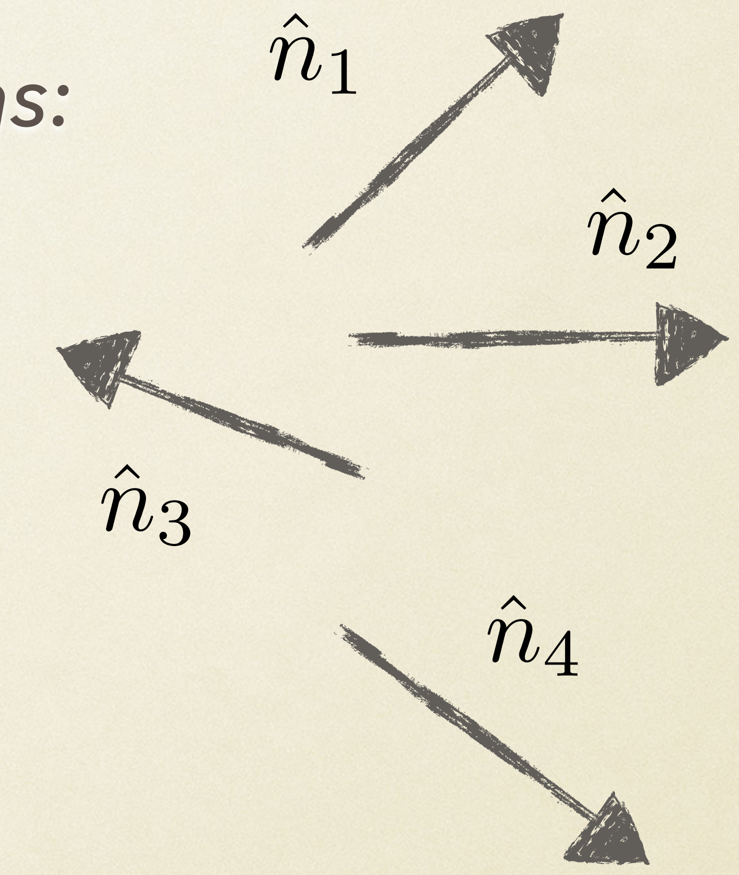
probability independence

Bell inequality violation

any four non-negative numbers

$$x_1 x_2 - x_1 x_4 + x_3 x_2 + x_3 x_4 \leq x_3 + x_2$$

four directions:



$$\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_4}) \leq \mathcal{P}(\uparrow_{\hat{n}_3};-) + \mathcal{P}(-;\uparrow_{\hat{n}_2})$$

$$\Psi = \; |\tau_R^-\rangle\,|\tau_L^+\rangle \longrightarrow \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\mathcal{P}(\uparrow_{\hat{n}_i};\uparrow_{\hat{n}_j}) = \frac{1}{4}\,\langle\Psi|(1_{2\times2}+\hat{n}_i\cdot\vec{\sigma})\otimes(1_{2\times2}+\hat{n}_j\cdot\vec{\sigma})|\Psi\rangle = \frac{1}{4}\,(1-\hat{n}_i^z+\hat{n}_j^z-\hat{n}_i^z\hat{n}_j^z)$$

$$\hat{n}_1=\hat{z},\quad \hat{n}_2=\frac{-1}{\sqrt{2}}(\hat{z}+\hat{x}),\quad \hat{n}_3=-\hat{x},\quad \hat{n}_4=\frac{1}{\sqrt{2}}(\hat{z}-\hat{x})$$

$$\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_2})-\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_4})+\mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_2})+\mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_4}) = \frac{1}{2}$$

$$\leq \; \mathcal{P}(\uparrow_{\hat{n}_3};-)+\mathcal{P}(-;\uparrow_{\hat{n}_2}) = 1-\frac{\sqrt{2}}{4}$$

✔

$$\Psi = \frac{1}{\sqrt{2}}\Big(|\tau_R^-\rangle\,|\tau_L^+\rangle + |\tau_L^-\rangle\,|\tau_R^+\rangle\Big) \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$

$$\mathcal{P}(\uparrow_{\hat{n}_i};\uparrow_{\hat{n}_j}) = \frac{1}{4}\,\langle\Psi|(1_{2\times2}+\hat{n}_i\cdot\vec{\sigma})\otimes(1_{2\times2}+\hat{n}_j\cdot\vec{\sigma})|\Psi\rangle = \frac{1}{4}\,(1+\hat{n}_i^x\hat{n}_j^x+\hat{n}_i^y\hat{n}_j^y-\hat{n}_i^z\hat{n}_j^z)$$

$$\hat{n}_1 = \hat{z}, \quad \hat{n}_2 = \frac{-1}{\sqrt{2}}(\hat{z}+\hat{x}), \quad \hat{n}_3 = -\hat{x}, \quad \hat{n}_4 = \frac{1}{\sqrt{2}}(\hat{z}-\hat{x})$$

$$\mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1};\uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3};\uparrow_{\hat{n}_4}) = \frac{1}{2}+\frac{\sqrt{2}}{2}$$

$$\not\leq \quad \mathcal{P}(\uparrow_{\hat{n}_3};-) + \mathcal{P}(-;\uparrow_{\hat{n}_2}) = 1$$

the toolbox

Qubits

$$\rho = \frac{1}{4}\left[\mathbb{1}_2 \otimes \mathbb{1}_2 + \sum_{i=1}^{3} \mathrm{B}_i^{+}(\sigma_i \otimes \mathbb{1}_2) + \sum_{i=1}^{3} \mathrm{B}_j^{-}(\mathbb{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^{3} \mathrm{C}_{ij}(\sigma_i \otimes \sigma_j)\right]$$

$$R = \rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$$

Concurrence $\mathscr{C}[\rho] = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$

CC^T $[m_1, m_2, m_3]$

Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

Qutrits

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3\right] + \sum_{a=1}^{8} f_a \left[T^a \otimes \mathbb{1}_3\right] + \sum_{a=1}^{8} g_a \left[\mathbb{1}_3 \otimes T^a\right] + \sum_{a,b=1}^{8} h_{ab}\left[T^a \otimes T^b\right]$$

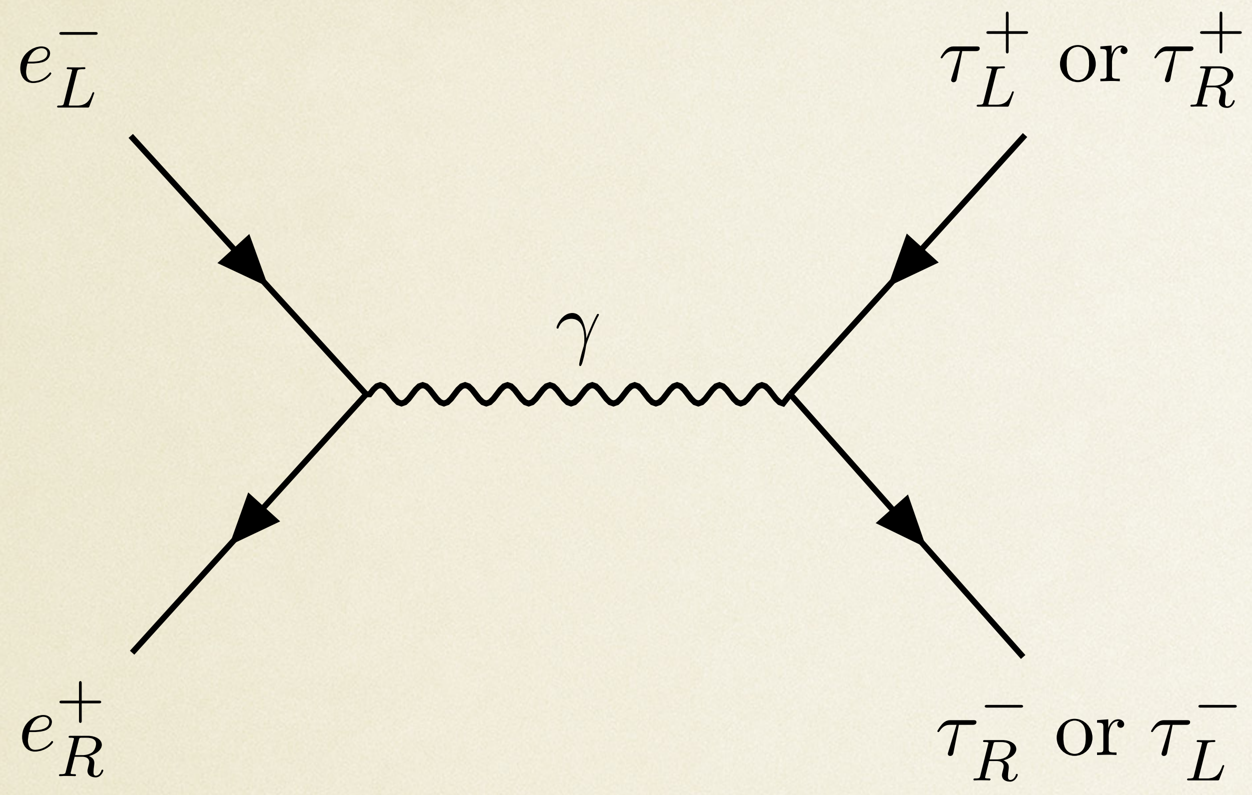
$$\mathscr{C}_2 = 2\max\Big[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,;\; -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2,\; 0\Big]$$

Entropy $\mathscr{E}[\rho] \equiv -\mathrm{Tr}[\rho_A \ln \rho_A] = -\mathrm{Tr}[\rho_B \ln \rho_B]$

Negativity $\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2}$

Bell operator $\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathscr{B}_3\right]$

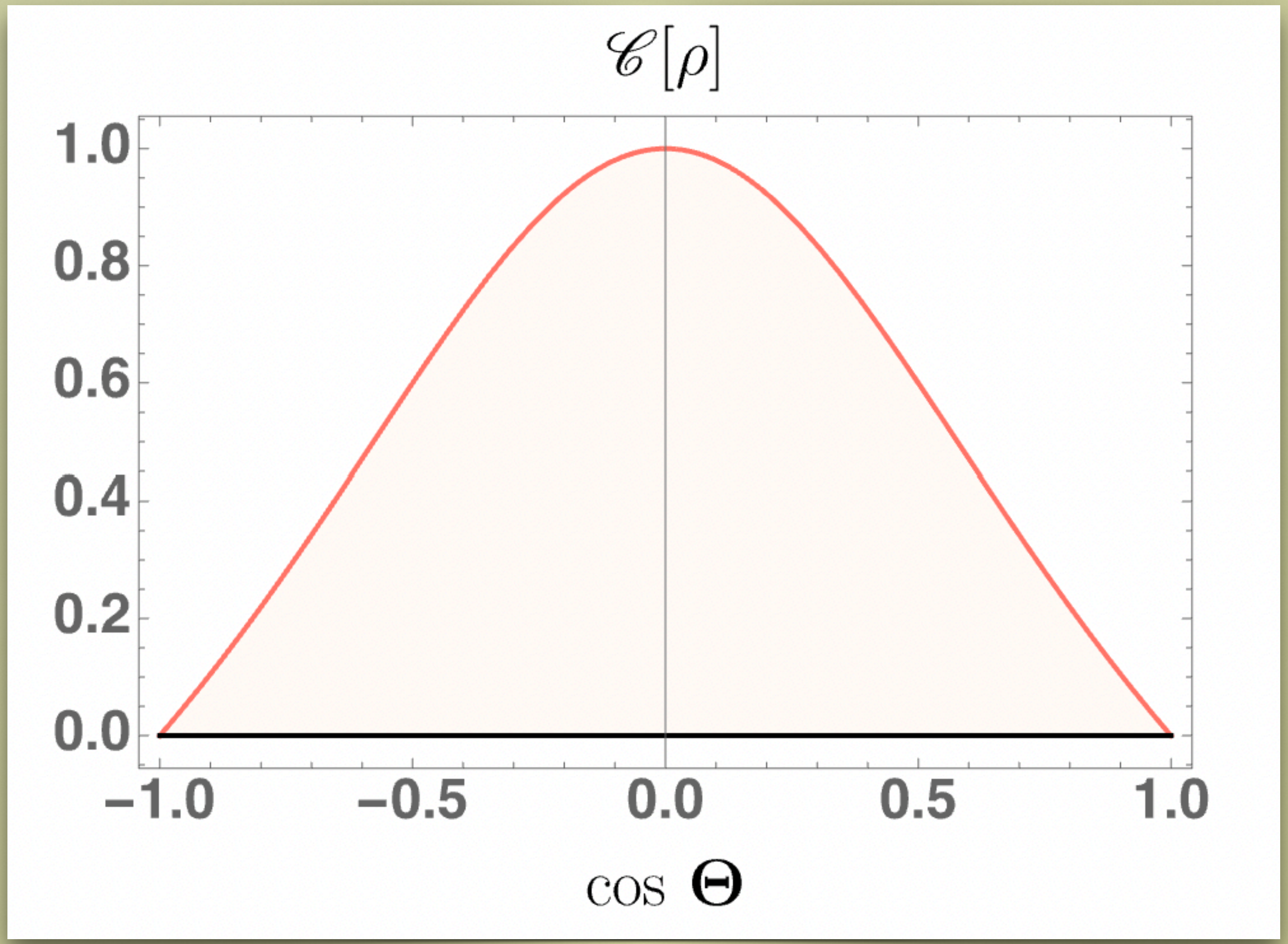
A. J. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, and Marzola, Quantum entanglement and Bell inequality vi lation at colliders, Prog. Part. Nucl. Phys. **139**, 1041: (2024).

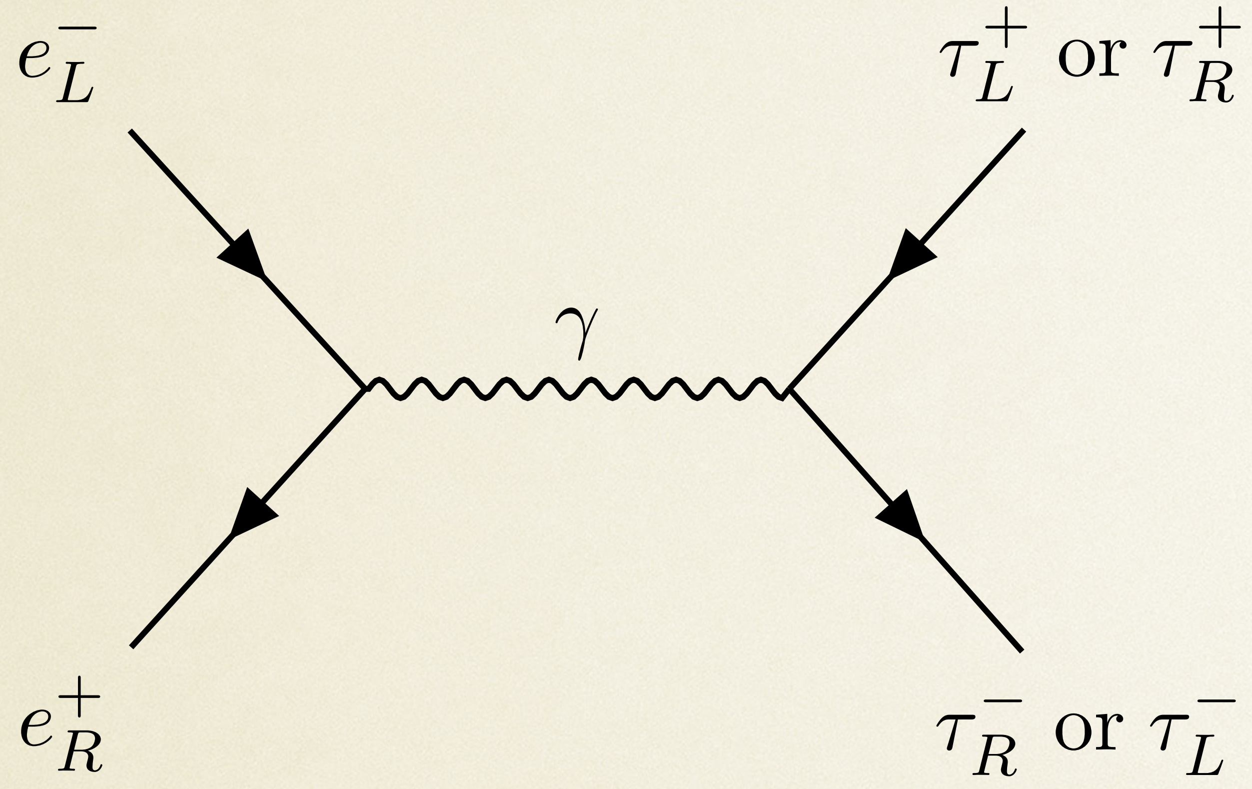


$$\Big(1 + \cos\Theta\Big)|\tau_R^-\rangle\,|\tau_L^+\rangle + \Big(1 - \cos\Theta\Big)|\tau_L^-\rangle\,|\tau_R^+\rangle$$

Concurrence

$$\mathcal{C}[\rho] = 2|\zeta_1\zeta_4 - \zeta_2\zeta_3| = \frac{\sin^2\Theta}{1+\cos^2\Theta}$$

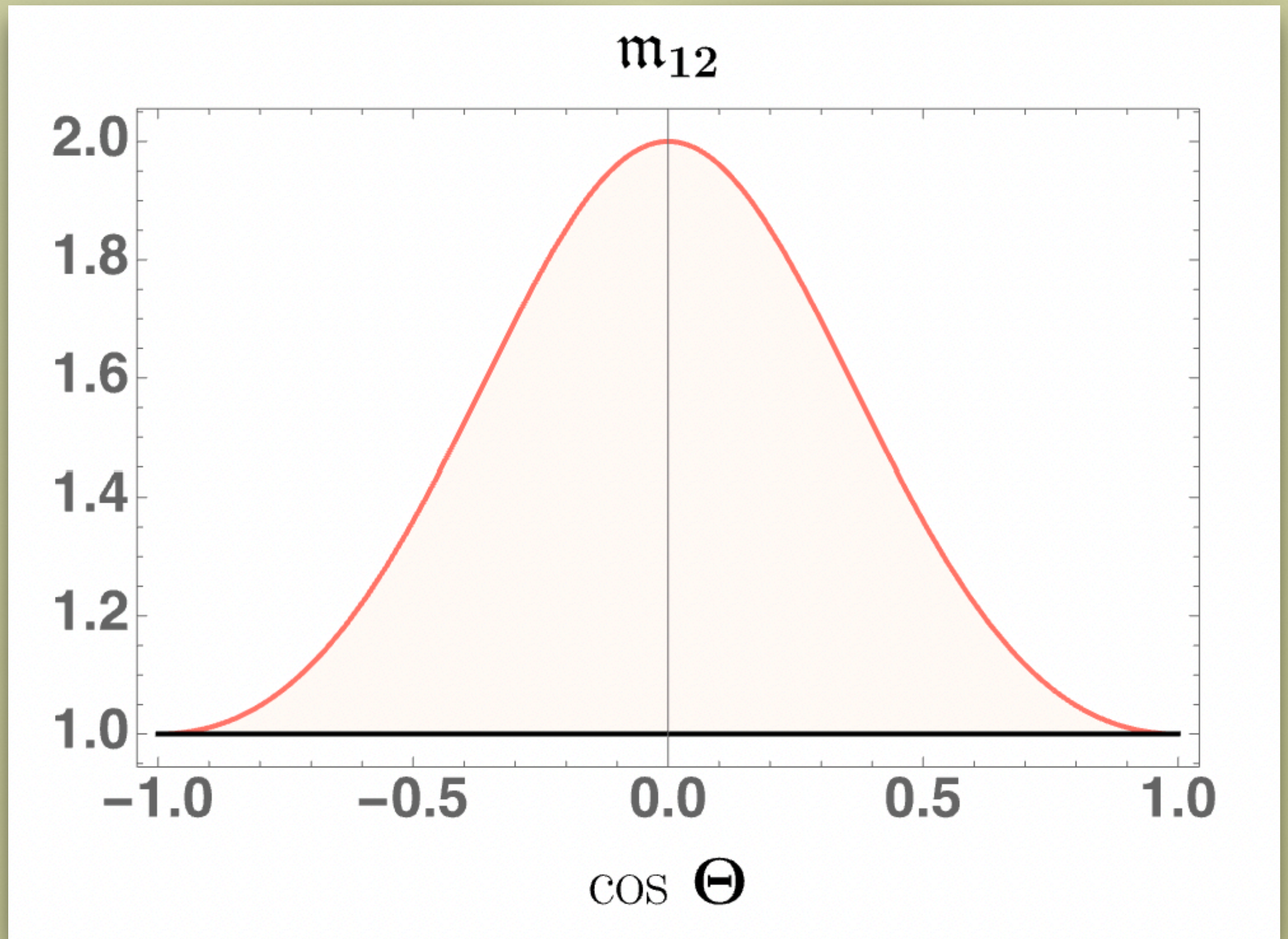




$$\Big(1+\cos\Theta\Big)|\tau_R^-\rangle\,|\tau_L^+\rangle + \Big(1-\cos\Theta\Big)|\tau_L^-\rangle\,|\tau_R^+\rangle$$

Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

$$\mathfrak{m}_{12} = 1 + \frac{\sin^4\Theta}{(1+\cos^2\Theta)^2}$$



Low-energy tests with photons and solid-state devices

A. Aspect, J. Dalibard and G. Rogers, Phys. Rev. Lett. 49 (1982) 5039

J.F. Clauser, M.A. Horne, Phys. Rev. D 10 (1974) 526

J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett. 23 (1969) 880

G. Weihs, T. Jennewein, C. Simon, H. Weinfurther and A. Zeilinger, Phys. Rev. Lett. 81 (1998) 5039

W. Tittel, J. Brendel, H. Zbinden, N. Gisin, Phys. Rev. Lett. 81 (1998) 3563

M. Ansmann et al, Nature 461 (2009) 504

PHYSICAL REVIEW LETTERS

VOLUME 81 — 7 DECEMBER 1998 — NUMBER 23

Violation of Bell's Inequality under Strict Einstein Locality Conditions

Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
(Received 6 August 1998)

We observe strong violation of Bell's inequality in an Einstein-Podolsky-Rosen-type experiment with independent observers. Our experiment definitely implements the ideas behind the well-known work by Aspect *et al*. We for the first time ... the condition of locality, a central assumption in the derivation of ... like separation of the observations is achieved ... ations, by ultrafast and random setting of the ... [S0031-9007(98)07901-0]

VOLUME 81, NUMBER 17 — PHYSICAL REVIEW LETTERS — 26 OCTOBER 1998

Violation of Bell Inequalities by Photons More Than 10 km Apart

W. Tittel,* J. Brendel, H. Zbinden, and N. Gisin
Group of Applied Physics, University of Geneva, 20, Rue de l'Ecole de Médecine, CH-1211 Geneva 4, Switzerland
(Received 10 June 1998)

A Franson-type test of Bell inequalities by photons 10.9 km apart is presented. Energy-time entangled photon pairs are measured using two-channel analyzers, leading to a violation of the inequalities by 16 standard deviations without subtracting accidental coincidences. Subtracting them, a two-photon interference visibility of 95.5% is observed, demonstrating that distances up to 10 km have no significant effect on entanglement. This sets quantum cryptography with photon pairs as a practical competitor to the schemes based on weak pulses. [S0031-9007(98)07478-X]

... 47, NUMBER 7 — PHYSICAL REVIEW LETTERS — 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

... ear polarization correlation of the photons emi... m. A high-efficiency source provided an i... orm new tests. Our results, in excellent ... tions, **strongly violate the generalized Be... realistic local theories. No significant c... rizer separations of up to 6.5 m.**

Article | Open access | Published: 10 May 2023

Loophole-free Bell inequality violation with superconducting circuits

Simon Storz, Josua Schär, Anatoly Kulikov, Paul Magnard, Philipp Kurpiers, Janis Lütolf, Theo Walter, Adrian Copetudo, Kevin Reuer, Abdulkadir Akin, Jean-Claude Besse, Mihai Gabureac, Graham J. Norris, Andrés Rosario, Ferran Martin, José Martinez, Waldimar Amaya, Morgan W. Mitchell, Carlos Abellan, Jean-Daniel Bancal, Nicolas Sangouard, Baptiste Royer, Alexandre Blais & Andreas Wallraff

Nature **617**, 265–270 (2023) | Cite this article

Selected for a Viewpoint in Physics
PRL 115, 250402 (2015) — PHYSICAL REVIEW LETTERS — week ending 18 DECEMBER 2015

Strong Loophole-Free Test of Local Realism*

Lynden K. Shalm,[1,†] Evan Meyer-Scott,[2] Bradley G. Christensen,[3] Peter Bierhorst,[1] Michael A. Wayne,[3,4] Martin J. Stevens,[1] Thomas Gerrits,[1] Scott Glancy,[1] Deny R. Hamel,[5] Michael S. Allman,[1] Kevin J. Coakley,[1] Shellee D. Dyer,[1] Carson Hodge,[1] Adriana E. Lita,[1] Varun B. Verma,[1] Camilla Lambrocco,[1] Edward Tortorici,[1] Alan L. Migdall,[4,6] Yanbao Zhang,[2] Daniel R. Kumor,[3] William H. Farr,[7] Francesco Marsili,[7] Matthew D. Shaw,[7] Jeffrey A. Stern,[7] Carlos Abellán,[8] Waldimar Amaya,[8] Valerio Pruneri,[8,9] Thomas Jennewein,[2,10] Morgan W. Mitchell,[8,9] Paul G. Kwiat,[3] Joshua C. Bienfang,[4,6] Richard P. Mirin,[1] Emanuel Knill,[1] and Sae Woo Nam[1,‡]

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[4]*National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA*
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(Received 10 November 2015; published 16 December 2015)

We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute p values as small as 5.9×10^{-9} for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted p value is 2.3×10^{-7}. We therefore reject the hypothesis that local realism governs our experiment.

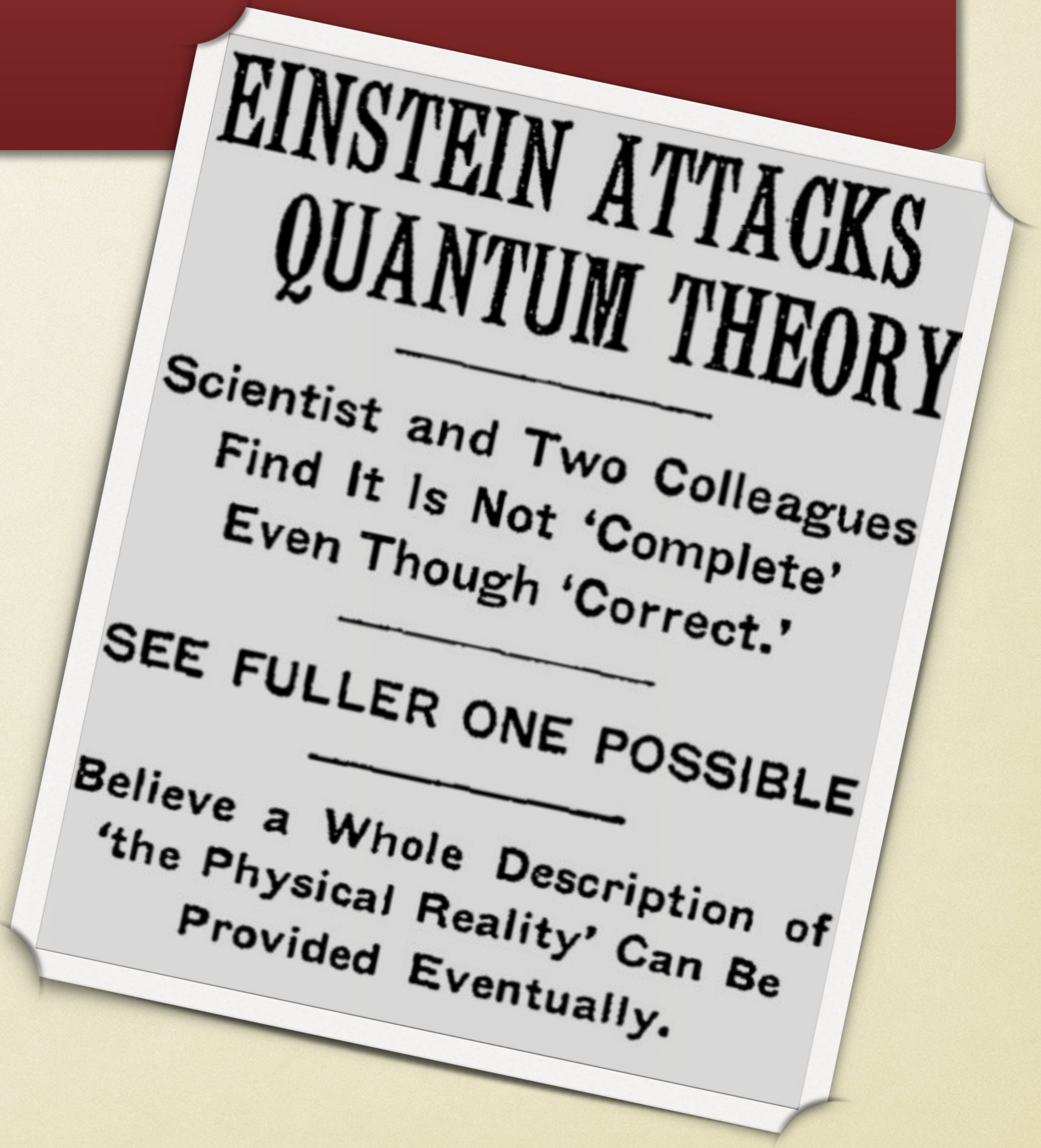
Local, deterministic models satisfy Bell inequality
quantum mechanics does not

Entanglement is just a measurement,
Bell inequality violation is a true discovery

both can be studied at colliders

- high-energy regime
- in the presence of weak and strong interactions
- qubits and qutrits

Where have we already seen entanglement or Bell inequality violation at high energies?


EINSTEIN ATTACKS QUANTUM THEORY

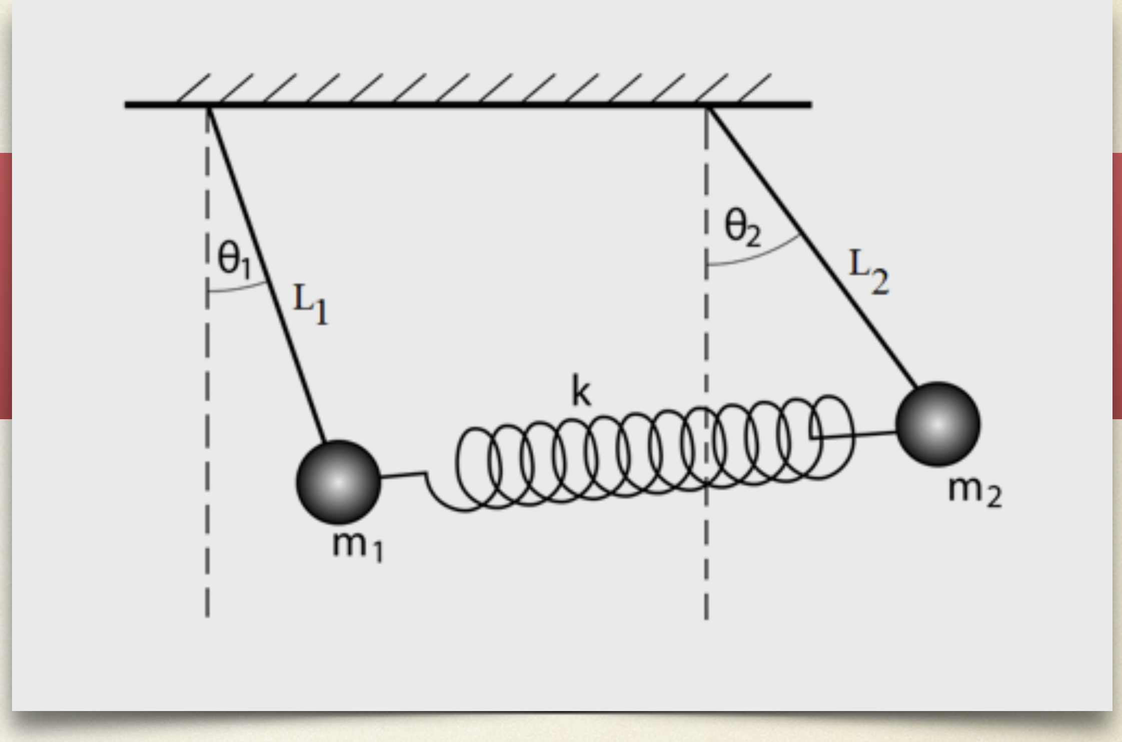
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

New York Times headline
May 4th, 1935

Flavor space

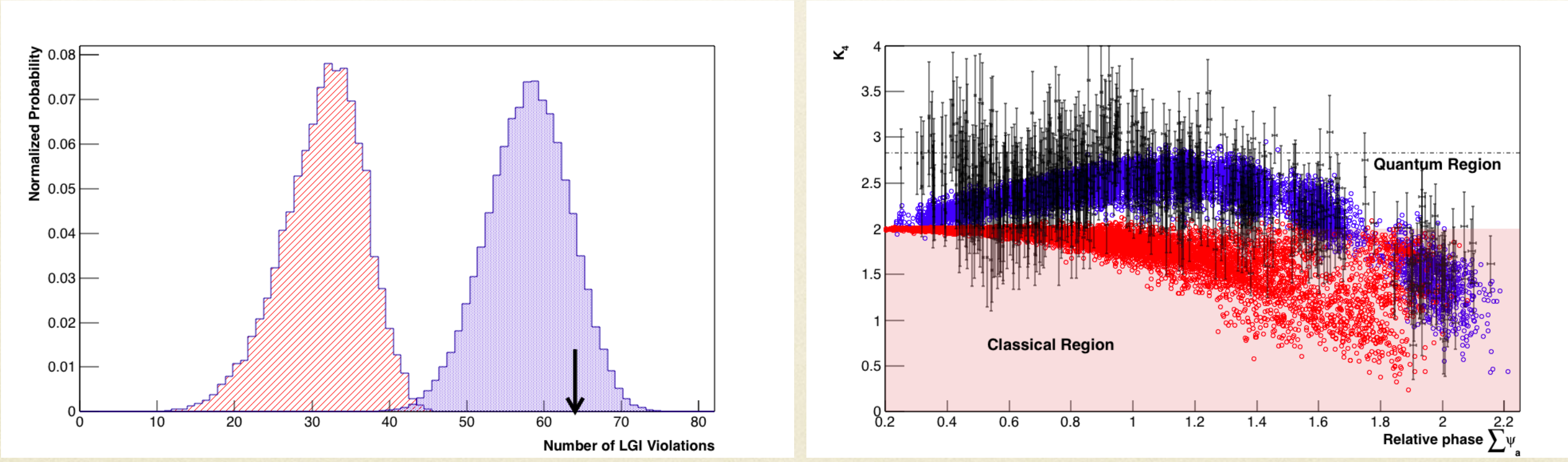


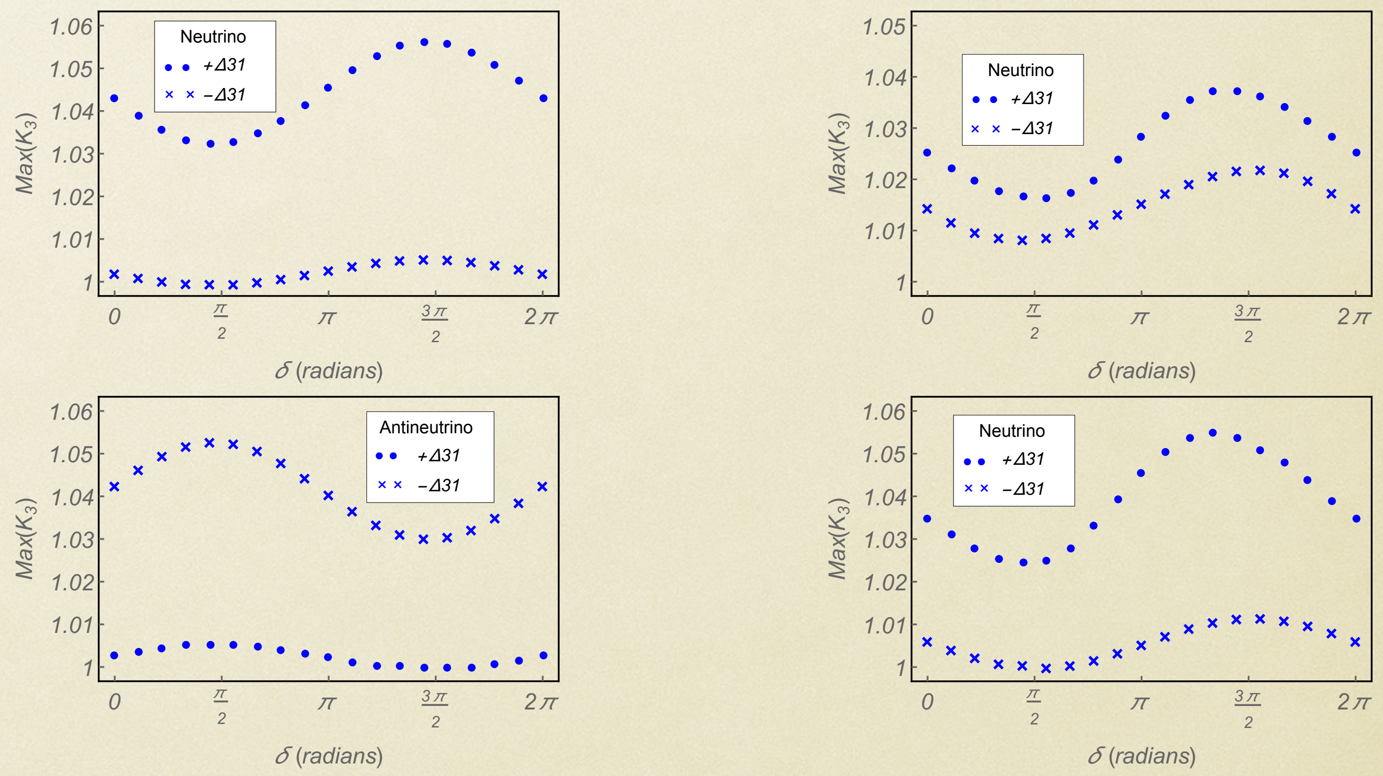
$K^0\bar{K}^0$ and $B^0\bar{B}^0$ oscillations

F Benatti and R Floreanini, Phys. Rev. D57 (1998) R1332, Eur. Phys. J. C13 (2000) 267
A Go, Belle Collaboration, Phys. Phys. Lett. 99 (2007) 131802

neutrino oscillations

Leggett-Garg inequality violation





Minos (6σ)

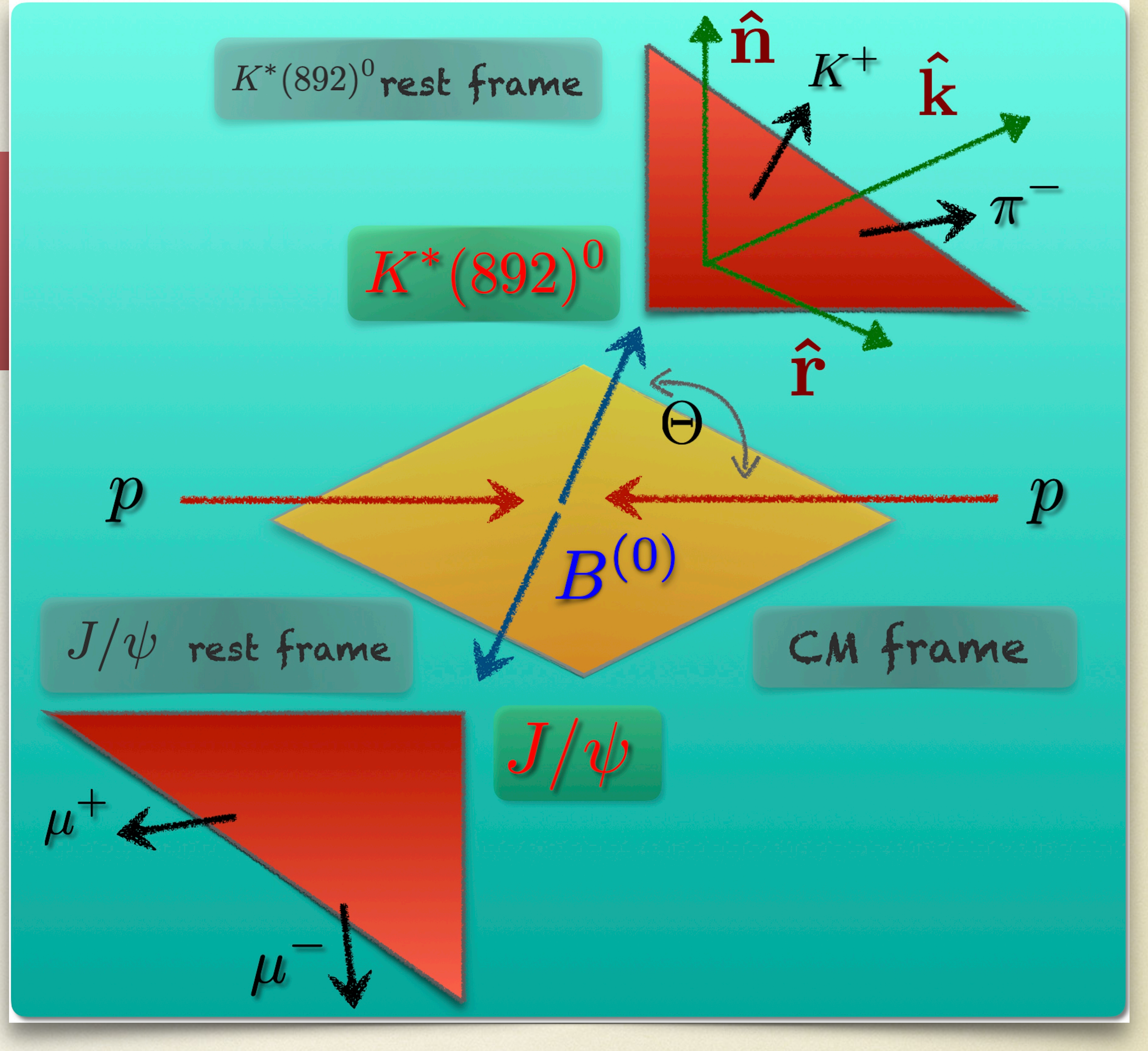
Dune T2K/Noνa

JA Formaggio, DI Kaiser, MM Murskyj and TE Weiss, Phys. Rev. Lett. 117 (2016) 050402

J Naikoo et al, Phys. Rev. D 99 (2019) 095001

2

B-meson decays



	$\mathscr{E}$	$\mathcal{I}_3$	
• $B^0 \to J/\psi\, K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	
• $B^0 \to \phi\, K^*(892)^0$ [22]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	
• $B^0 \to \rho\, K^*(892)^0$ [23]	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$	
• $B_s \to \phi\,\phi$ [24]	0.734 ± 0.037	2.525 ± 0.064	8.2σ
• $B_s \to J/\psi\,\phi$ [25]	0.731 ± 0.032	2.462 ± 0.080	

entanglement ✓ Bell inequality

Parameter	Result
$\lvert A_0\rvert^2$	$0.384 \pm 0.007 \pm 0.003$
$\lvert A_\perp\rvert^2$	$0.310 \pm 0.006 \pm 0.003$
$\delta_\parallel$ [rad]	$2.463 \pm 0.029 \pm 0.009$
$\delta_\perp$ [rad]	$2.769 \pm 0.105 \pm 0.011$

	$\lvert A_0\rvert^2$	$\lvert A_\perp\rvert^2$	$\delta_\parallel$	$\delta_\perp$
$\lvert A_0\rvert^2$	1	−0.342	−0.007	0.064
$\lvert A_\perp\rvert^2$		1	0.140	0.088
$\delta_\parallel$			1	0.179
$\delta_\perp$				1

R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **131**, no.17, 171802 (2023) [arXiv:2304.06198 [hep-ex]].

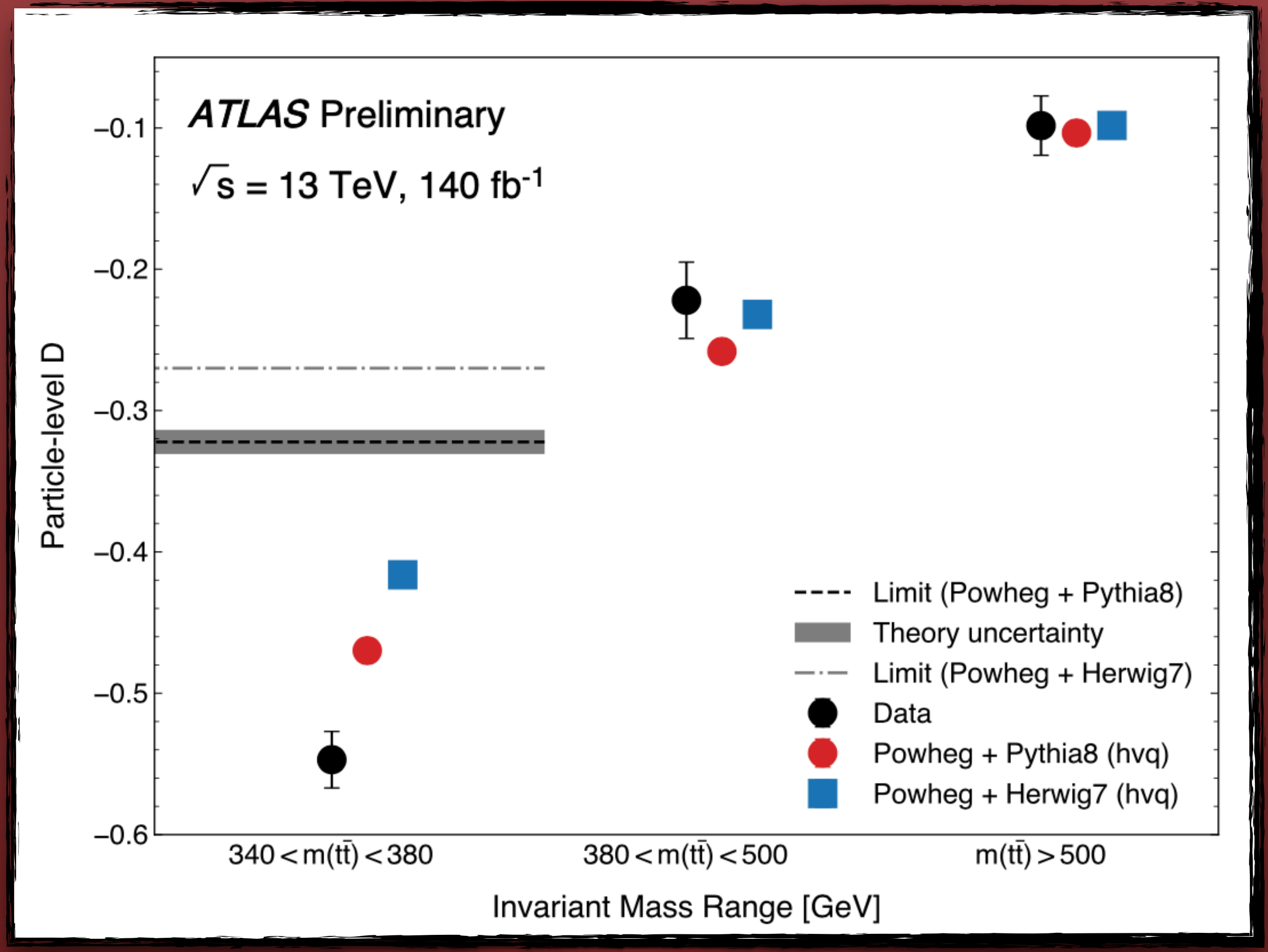
MF, R. Floreanini, E. Gabrielli and L. Marzola, Phys. Rev D 109 (2024) 3, L031104
E. Gabrielli and L. Marzola, arXiv:2406.17772 (2024)

** K Chen et al, Eur. Phys. J. C 84 (2024) 580 $B_c^\pm \to J/\psi\, \rho^\pm$

Pairs of top quarks

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

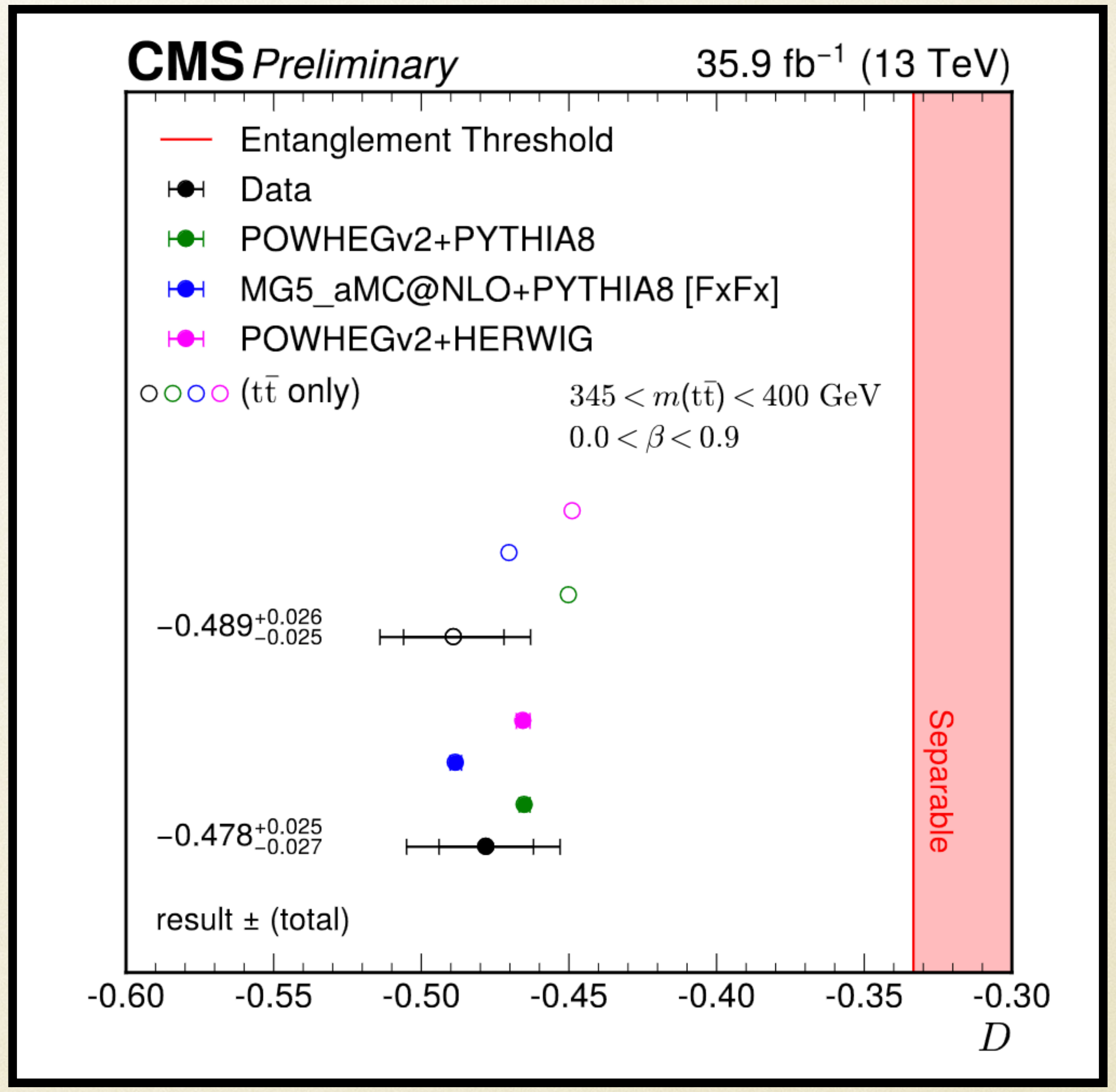
$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right)$$



$$D = -0.547 \pm 0.002\ [\text{stat}] \pm 0.021\ [\text{syst}]$$

ATLAS Collaboration, Nature 633 (2024) 542

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus **136** (2021) 907



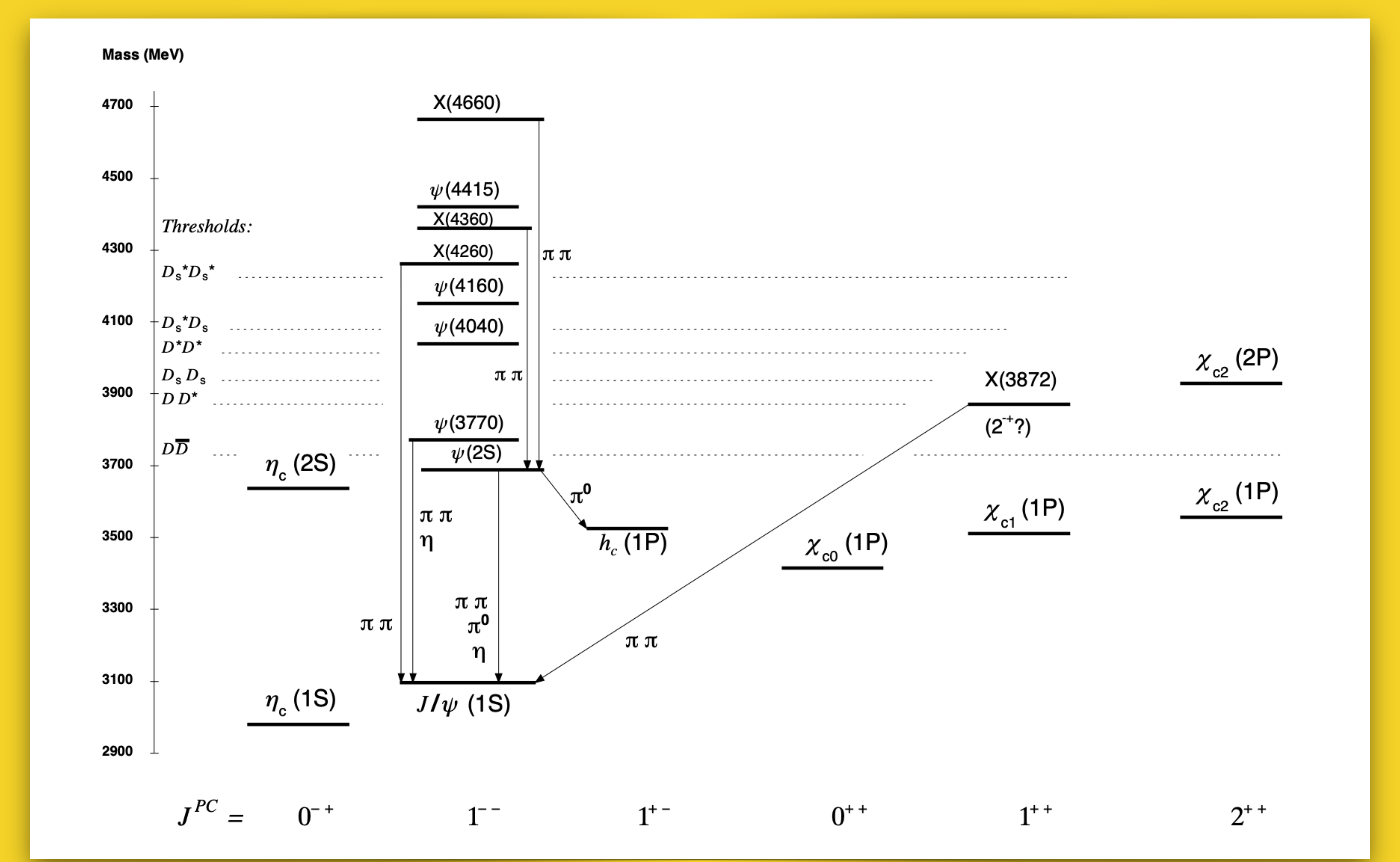
$$D = -0.478^{+0.025}_{-0.027}$$

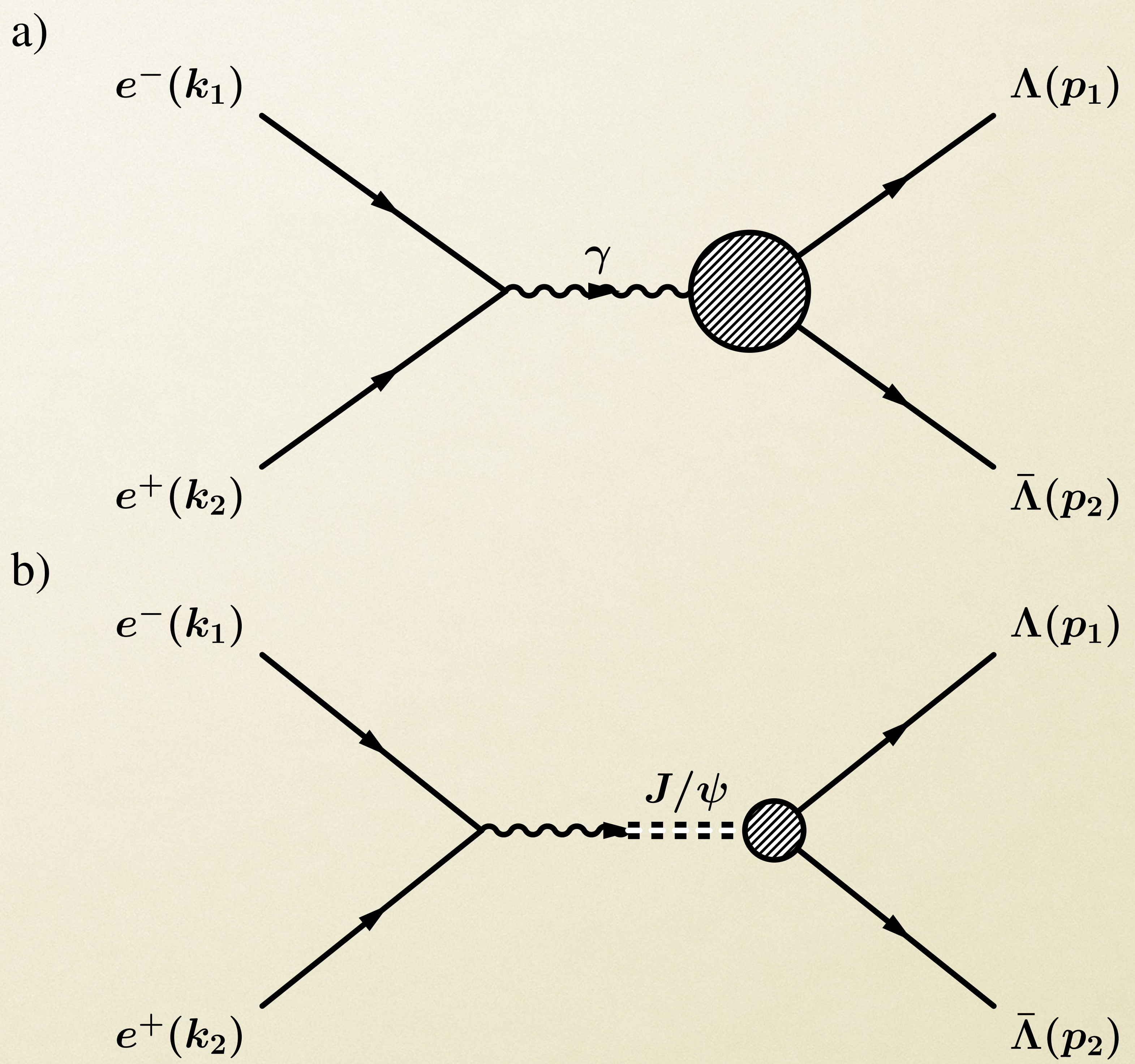
CMS Collaboration, arXiv:2406.03976 (2024)

CMS Collaboration, arXiv:2409.11067 (2024)

4

Charmonium





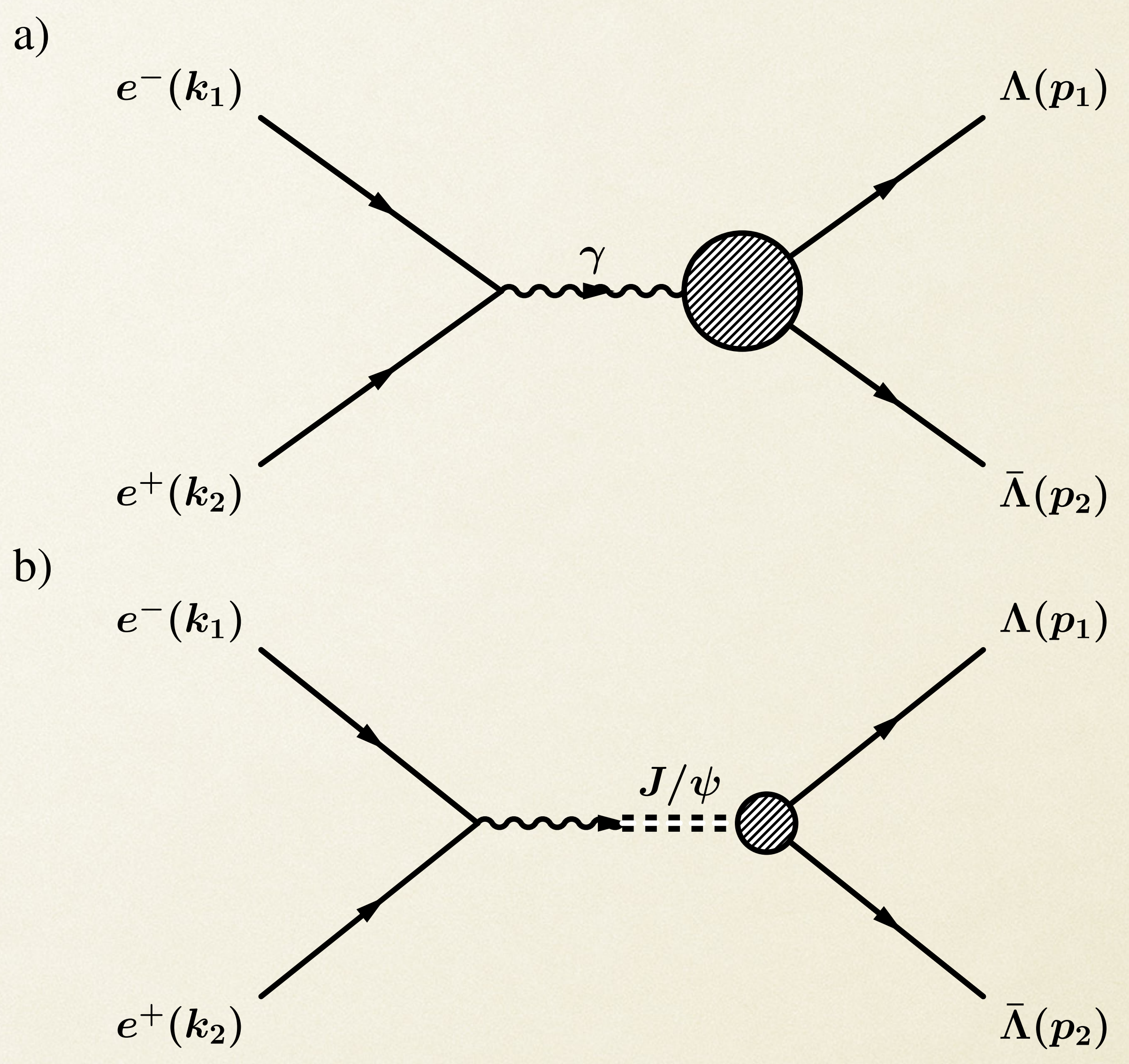
MF, R. Floreanini, E. Gabrielli and L. Marzola, Phys. Rev. D110 (2024) 053008
see, also: S. Wu et al., Phys. Rev. D110 (2024) 054012

$$\xi = (\theta, \Omega_1, \Omega_2),$$

$$\mathcal{W}(\xi) = \mathcal{F}_0(\xi) + \alpha \mathcal{F}_5(\xi) + \alpha_1\alpha_2 \left(\mathcal{F}_1(\xi) + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\mathcal{F}_2(\xi) + \alpha\mathcal{F}_6(\xi)\right) + \sqrt{1-\alpha^2}\sin(\Delta\Phi)\left(\alpha_1\mathcal{F}_3(\xi) + \alpha_2\mathcal{F}_4(\xi)\right), \qquad (6.55)$$

$$\mathcal{F}_0(\xi) = 1$$
$$\mathcal{F}_1(\xi) = \sin^2\theta \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta \cos\theta_1 \cos\theta_2$$
$$\mathcal{F}_2(\xi) = \sin\theta \cos\theta (\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2)$$
$$\mathcal{F}_3(\xi) = \sin\theta \cos\theta \sin\theta_1 \sin\phi_1$$
$$\mathcal{F}_4(\xi) = \sin\theta \cos\theta \sin\theta_2 \sin\phi_2$$
$$\mathcal{F}_5(\xi) = \cos^2\theta$$
$$\mathcal{F}_6(\xi) = \cos\theta_1 \cos\theta_2 - \sin^2\theta \sin\theta_1 \sin\theta_2 \sin\phi_1 \sin\phi_2. \qquad (6.56)$$

G. Faldt and A. Kupsc, Phys. Lett B 772 (2017) 16



maximum likelihood fit

more events
SM needed

$$\rho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} \propto w_{\lambda_1\lambda_2} w^*_{\lambda_1'\lambda_2'} \sum_k D^{(J)*}_{k,\lambda_1-\lambda_2}(0,\Theta,0) D^{(J)}_{k,\lambda_1'-\lambda_2'}(0,\Theta,0)$$

Charmonium spin-0 states

$$\eta_c \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \chi_c^0 \to \Lambda + \bar{\Lambda}$$

$$|\psi_0\rangle \propto w_{\frac{1}{2}\,-\frac{1}{2}}\, |\tfrac{1}{2}, \tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}, -\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

Concurrence	$\mathscr{C} = 1$
Horodecki condition	$\mathfrak{m}_{12} = 2$

N. A. Tornqvist, *Suggestion for Einstein-podolsky-rosen Experiments Using Reactions Like $e^+e^- \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$, Found. Phys.* **11** (1981) 171–177.

N. A. Tornqvist, *The Decay $J/\psi \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$ as an Einstein-Podolsky-Rosen Experiment, Phys. Lett. A* **117** (1986) 1–4.

S. P. Baranov, *Bell's inequali decays $\eta_c \to \Lambda\bar{\Lambda}$, $\chi_c \to \Lambda\bar{\Lambda}$ an Phys. G* **35** (2008) 075002.

$$\chi_c^0 \to \phi + \phi$$

$$|\Psi\rangle = w_{-1\,-1}\, |-1, -1\rangle + w_{0\,0}\, |0\,0\rangle + w_{1\,1}\, |1, 1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\text{stat}} \pm 0.019|_{\text{syst}}\,.$$

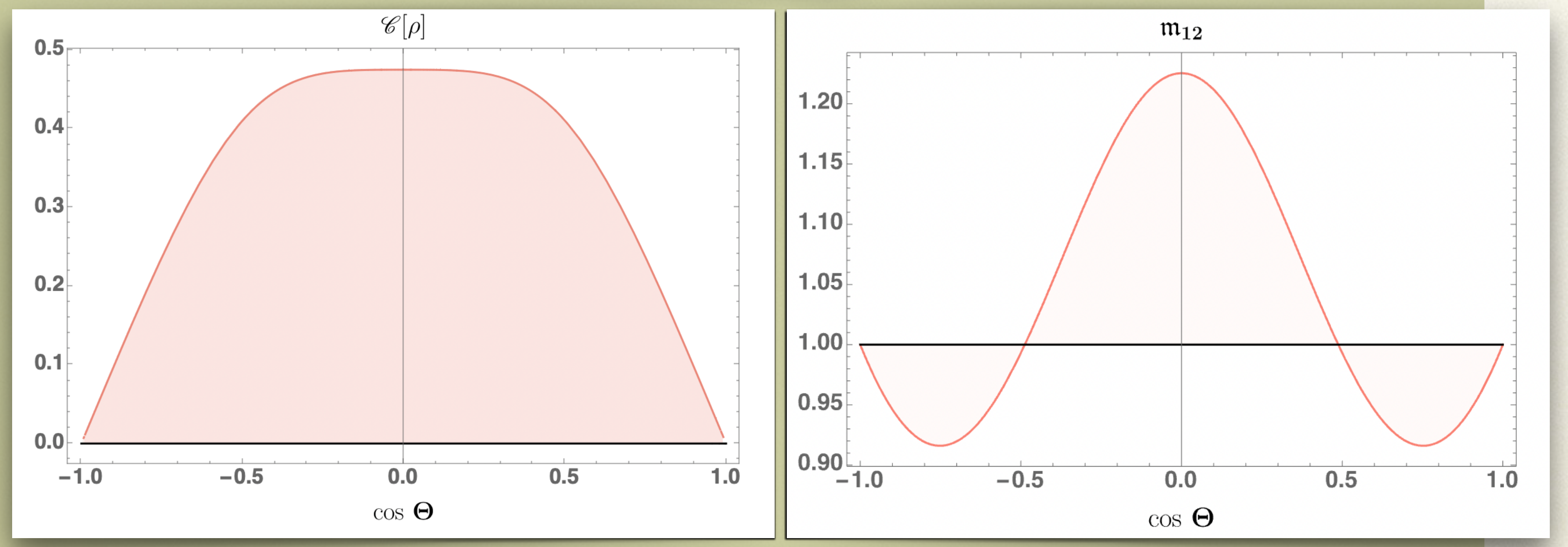
BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$, JHEP* **05** (2023) 069, [arXiv:2301.12922].

Entropy	$\mathscr{E}[\rho] = 0.531 \pm 0.0021$	(255σ)
Bell operator	$\text{Tr}\, \rho_{\phi\phi}\, \mathscr{B} = 2.2961 \pm 0.0165$	(18σ)

Charmonium spin-1 states

$$J/\psi \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \psi(3686) \to \Lambda + \bar{\Lambda}$$

$$\begin{aligned} |\psi_\uparrow\rangle &\propto w_{\frac{1}{2}\frac{1}{2}}\, |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \\ |\psi_\downarrow\rangle &\propto w_{-\frac{1}{2}-\frac{1}{2}}\, |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \\ |\psi_0\rangle &\propto w_{\frac{1}{2}-\frac{1}{2}}\, |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\frac{1}{2}}\, |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle , \end{aligned}$$



$$\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}} \quad \text{and} \quad \Delta\Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}} .$$

BESIII Collaboration, M. Ablikim et al., *Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ-$\bar{\Lambda}$ Pairs, Phys. Rev. Lett.* **129** (2022), no. 13 131801, [arXiv:2204.11058].

Concurrence $\mathscr{C} = 0.475 \pm 0.0039$ (122σ)

Horodecki condition $\mathfrak{m}_{12} = 1.225 \pm 0.004$ (56σ)

Bell inequality violation

decay	$\mathfrak{m}_{12}$	significance
$J/\psi \to \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \to \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \to \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \to \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \to \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \to \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \to \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \to \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2

ongoing work

$$pp \to t\bar{t}$$

A

LHC, data already available

Analysis under way

$$pp \to H \to ZZ^*$$

B

LHC, data already available

Analysis under way

$$e^+e^- \to \tau^+\tau^-$$

C

Belle II, data already available

Analysis under way

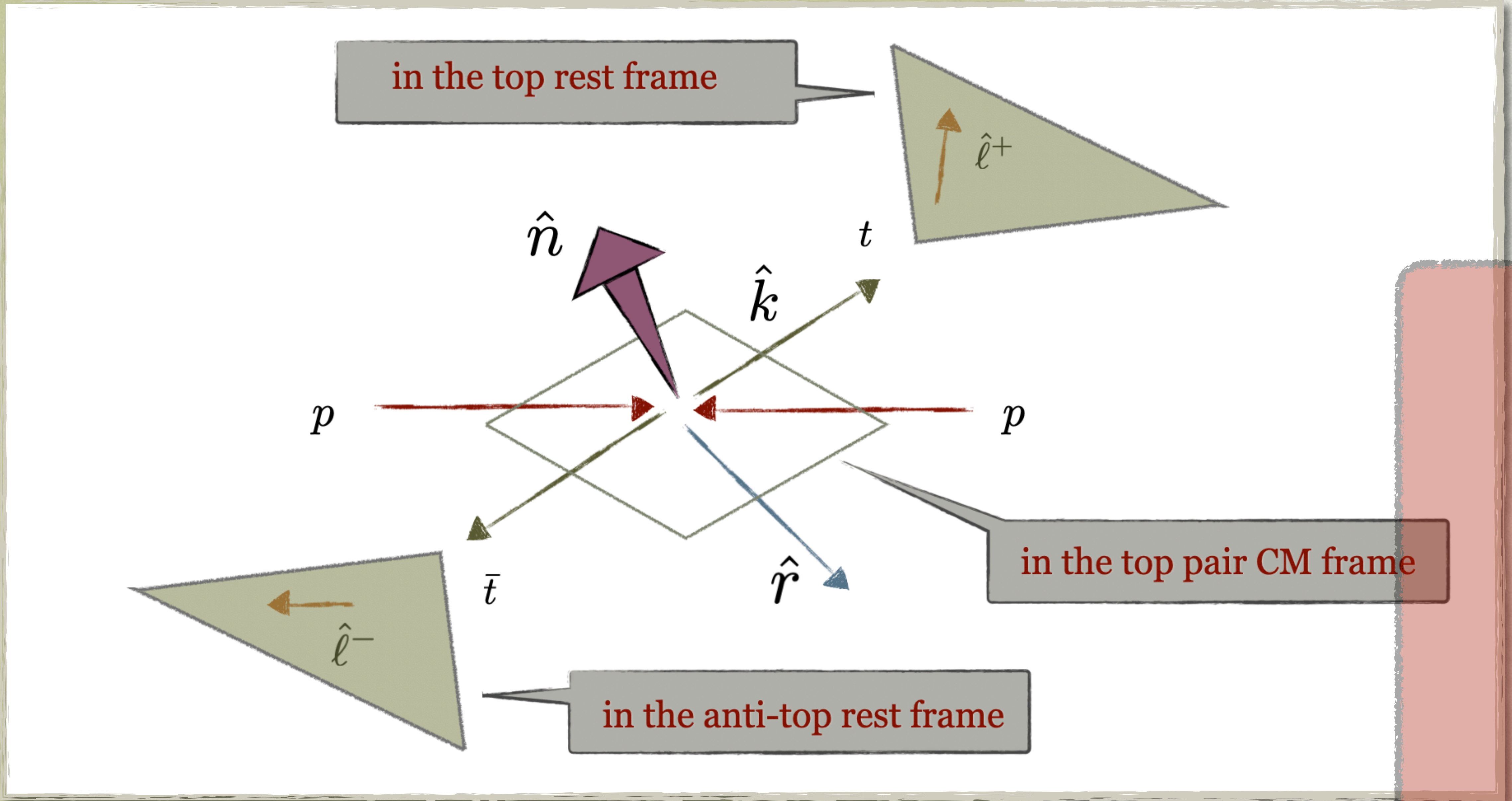
While waiting — let us see some simulations

Top-quark pairs

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

Event generation

MadGraph5 (NNPDF23)
DELPHES (fast simulation ATLAS detector)



exactly two opposite sign lepton of different flavor

at least 2 anti-k_t jets with R=0.4
at least 1 b-tagged jet

$p_T > 25$ GeV $|\eta| < 2.5$ jets

$p_T > 20$ GeV $|\eta| < 2.47$ leptons

neutrino weighting technique

Implementing at the LHC

W. Bernreuther, D. Heisler, and Z. G. Si, J. High Energy Phys. 12 (2015) 026.
Y. Afik and J. R. M. de Nova, Eur. Phys. J. Plus **136**, 907 (2021).

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\xi_{ab} = \cos\theta_+^a \cos\theta_-^b$$

	label	$\hat{\mathbf{a}}$	$\hat{\mathbf{b}}$
transverse	n	$\text{sign}(y_p)\ \hat{\mathbf{n}}_p$	$-\text{sign}(y_p)\ \hat{\mathbf{n}}_p$
r axis	r	$\text{sign}(y_p)\ \hat{\mathbf{r}}_p$	$-\text{sign}(y_p)\ \hat{\mathbf{r}}_p$
helicity	k	$\hat{\mathbf{k}}$	$-\hat{\mathbf{k}}$

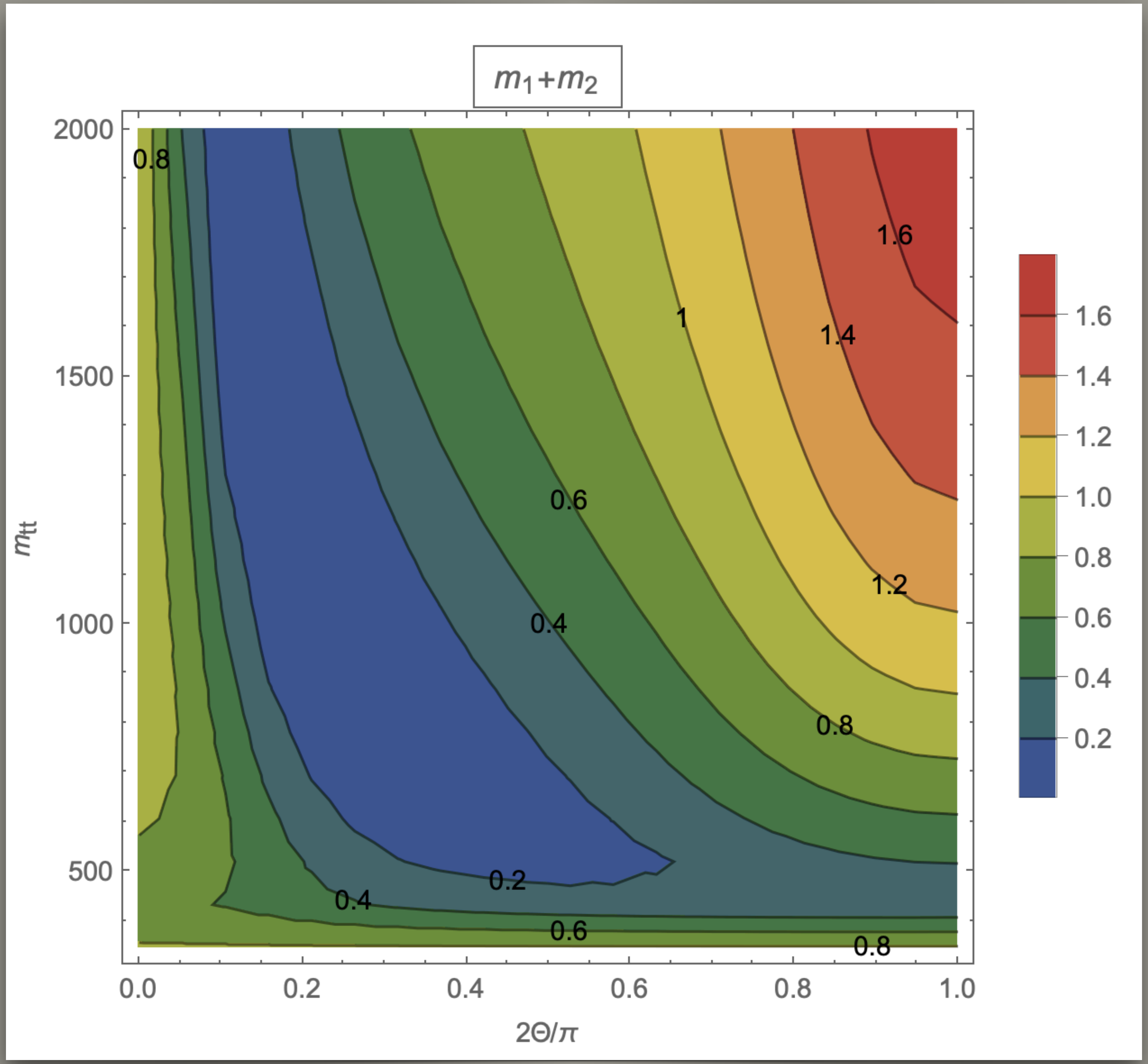
3 x 3 matrix

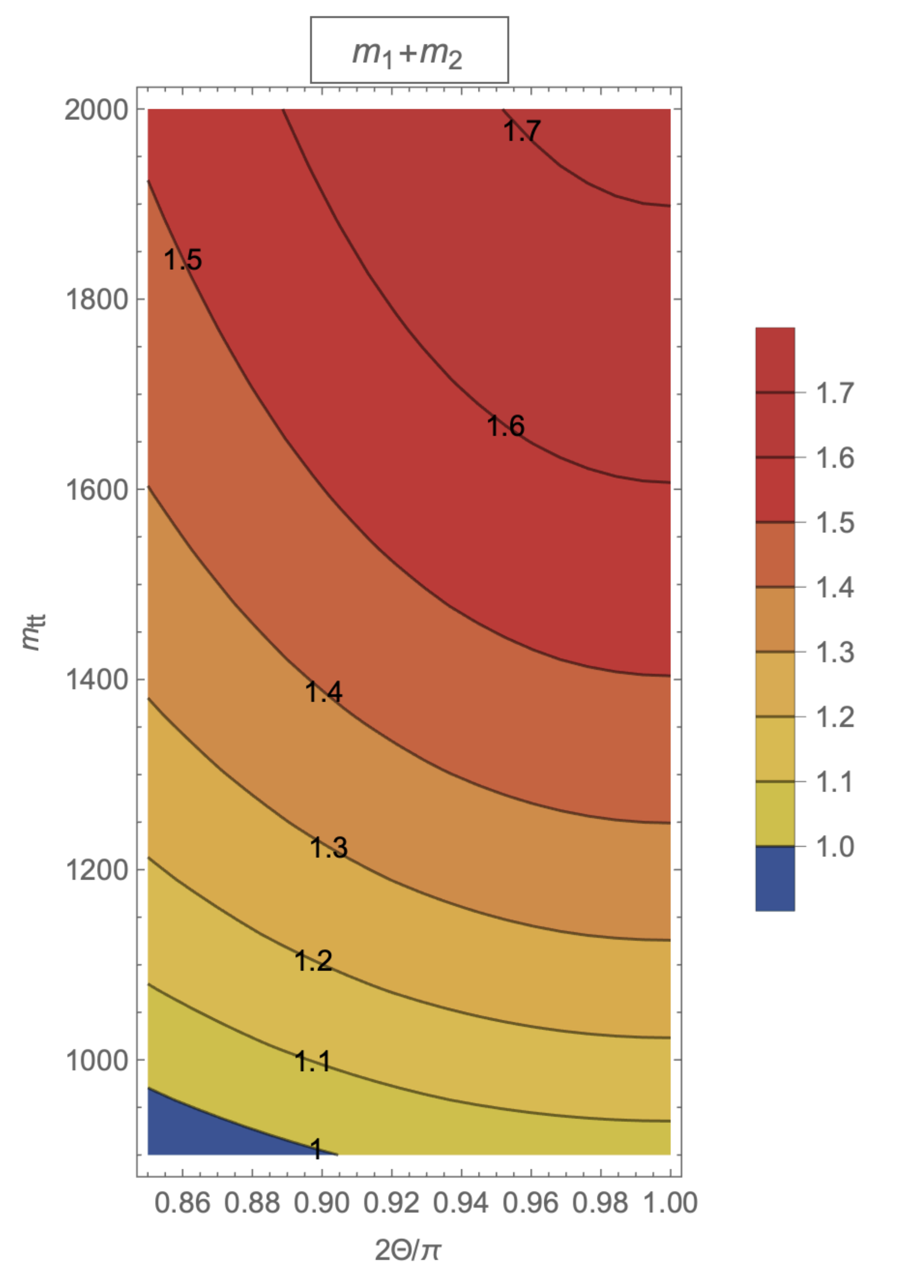
$$C_{ab}\left[\sigma(m_{t\bar{t}}, \cos\Theta)\right] = -9\ \frac{1}{\sigma}\int d\xi_{ab}\frac{d\sigma}{d\xi_{ab}}\xi_{ab}$$

diagonalization for each value
of invariant mass and scattering angle

$$m_1 + m_2 > 1$$

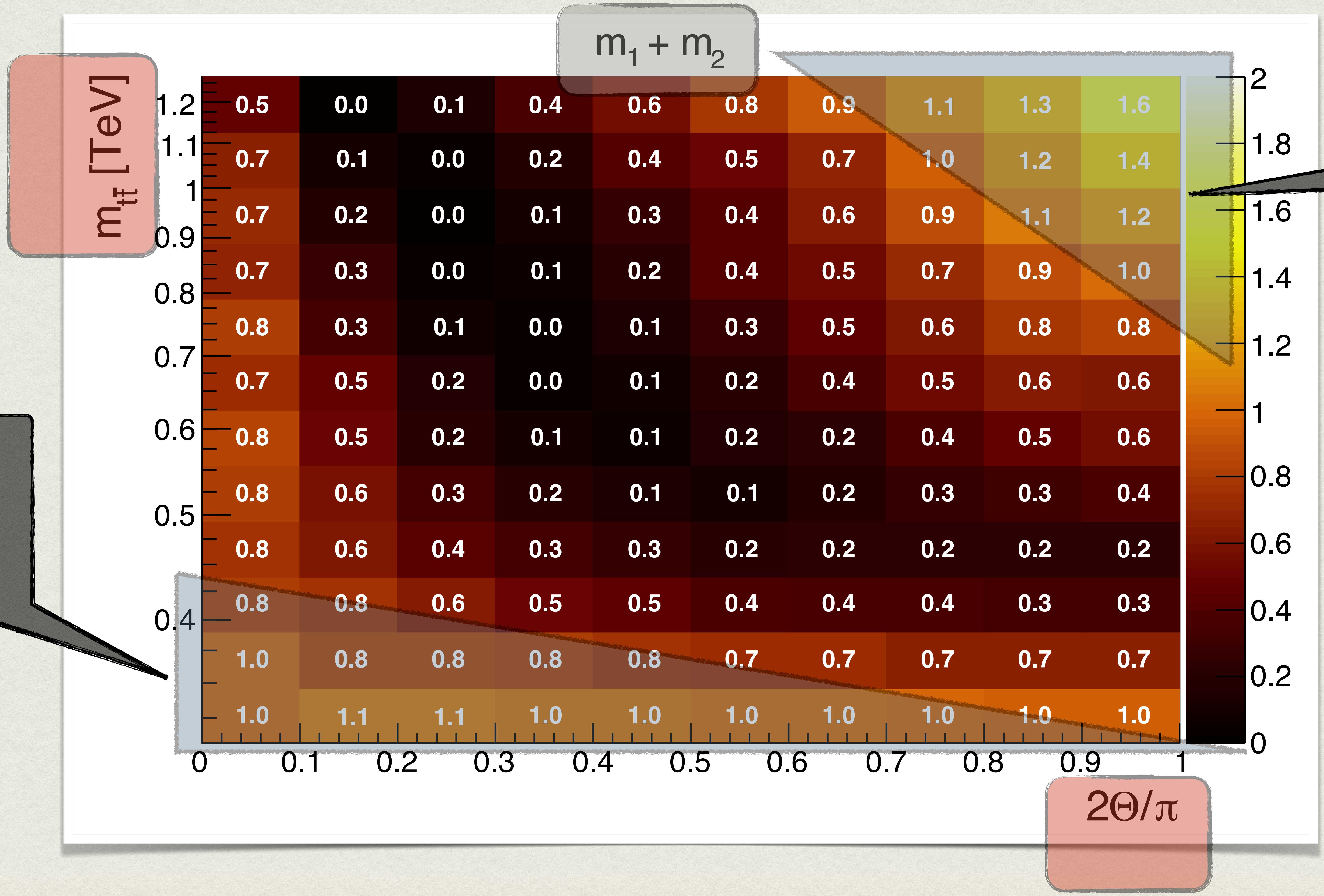
the study of entanglement leads to that of Bell inequalities violation





MC analysis

M. Fabbrichesi, R. Floreanini, and G. Panizzo, *Testing Bell Inequalities at the LHC with Top-Quark Pairs*, *Phys. Rev. Lett.* **127** (2021), no. 16 161801, [arXiv:2102.11883].



both qq and gg give top pair max. entangled

only gg gives top pair max. entangled

Results

bins

$$\frac{2\,\Theta}{\pi} \gtrsim 0.7 \qquad m_{t\bar{t}} \gtrsim 0.9\ \text{TeV}$$

1.1	1.3	1.6
1.0	1.2	1.4
0.9	1.1	1.2

null hypothesis: $m_1 + m_2 \leq 1$

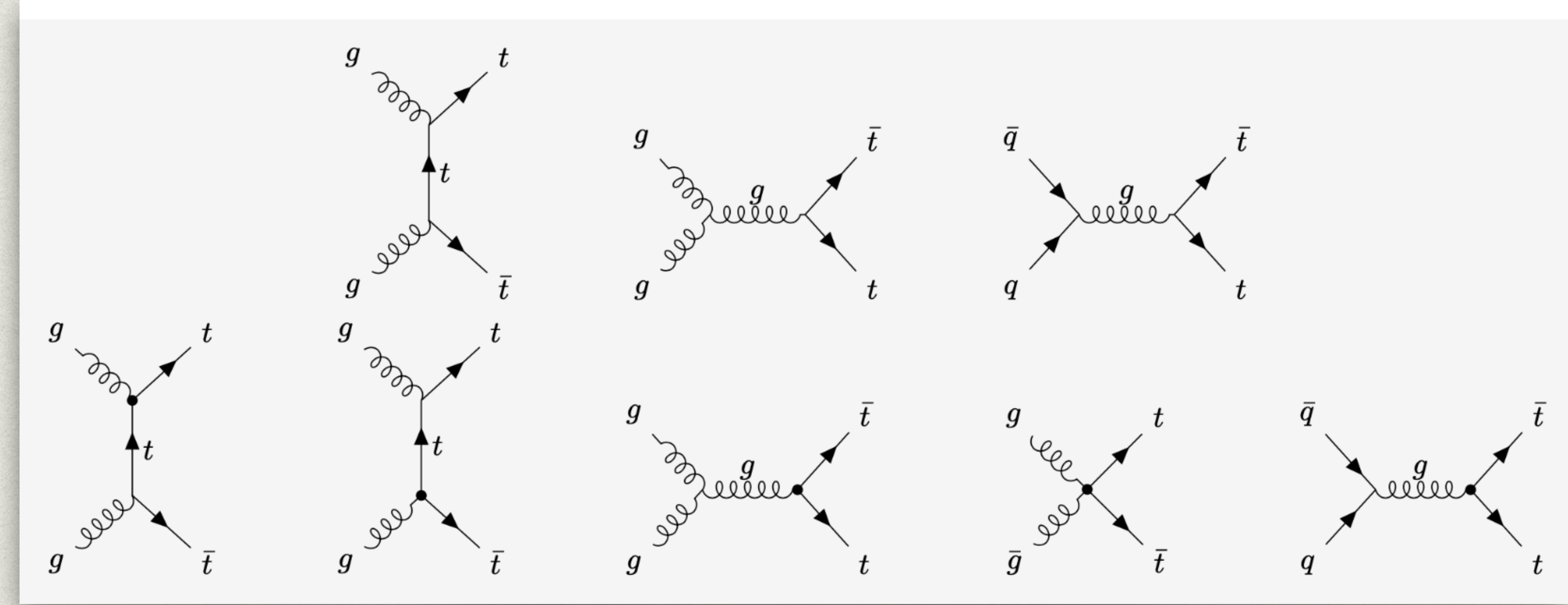
Hypothesis test

$$\chi^2 = \sum_i \frac{(1 - m_1^i - m_2^i)^2}{s_i^2}$$

violation: 98% CL w/ Run II data (139 fb-1)
99.99% CL with Run III

systematic uncertainties (e.g. from unfolding) not included

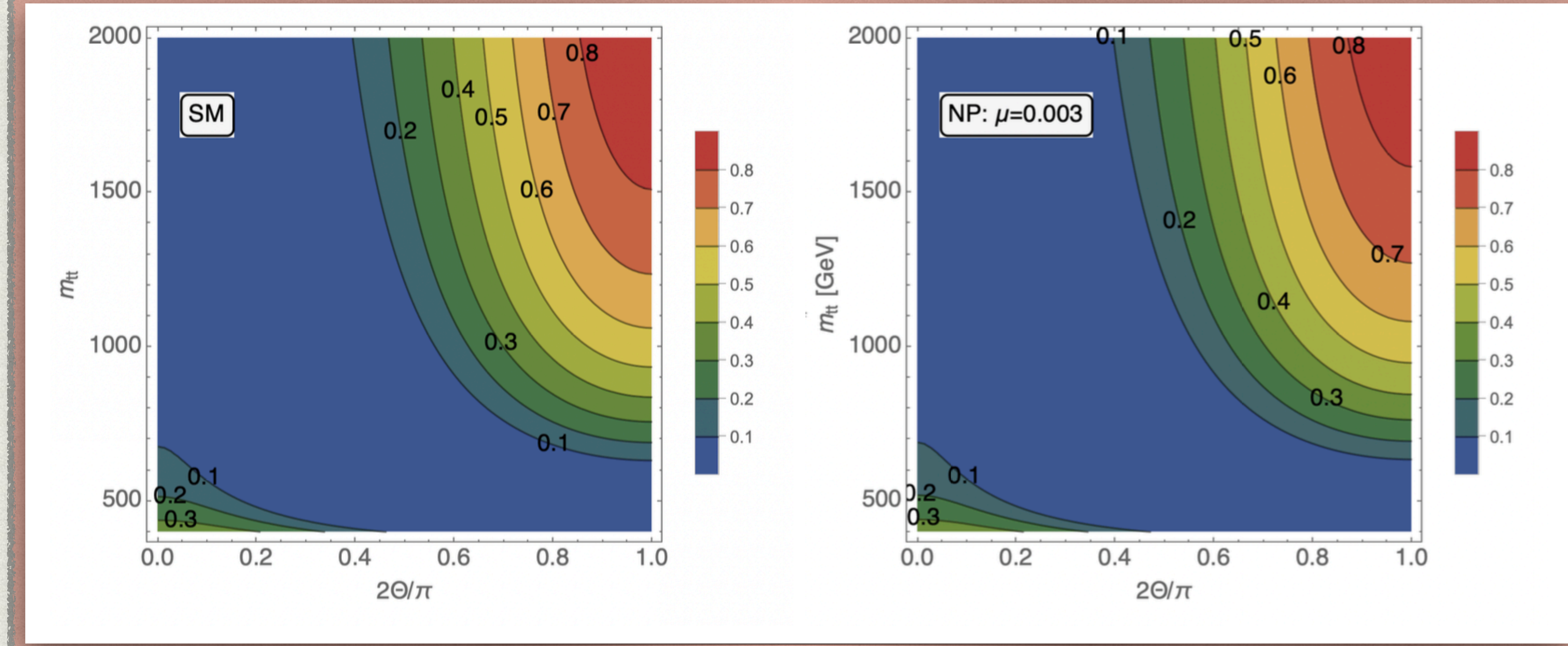
putting quantum observables to work

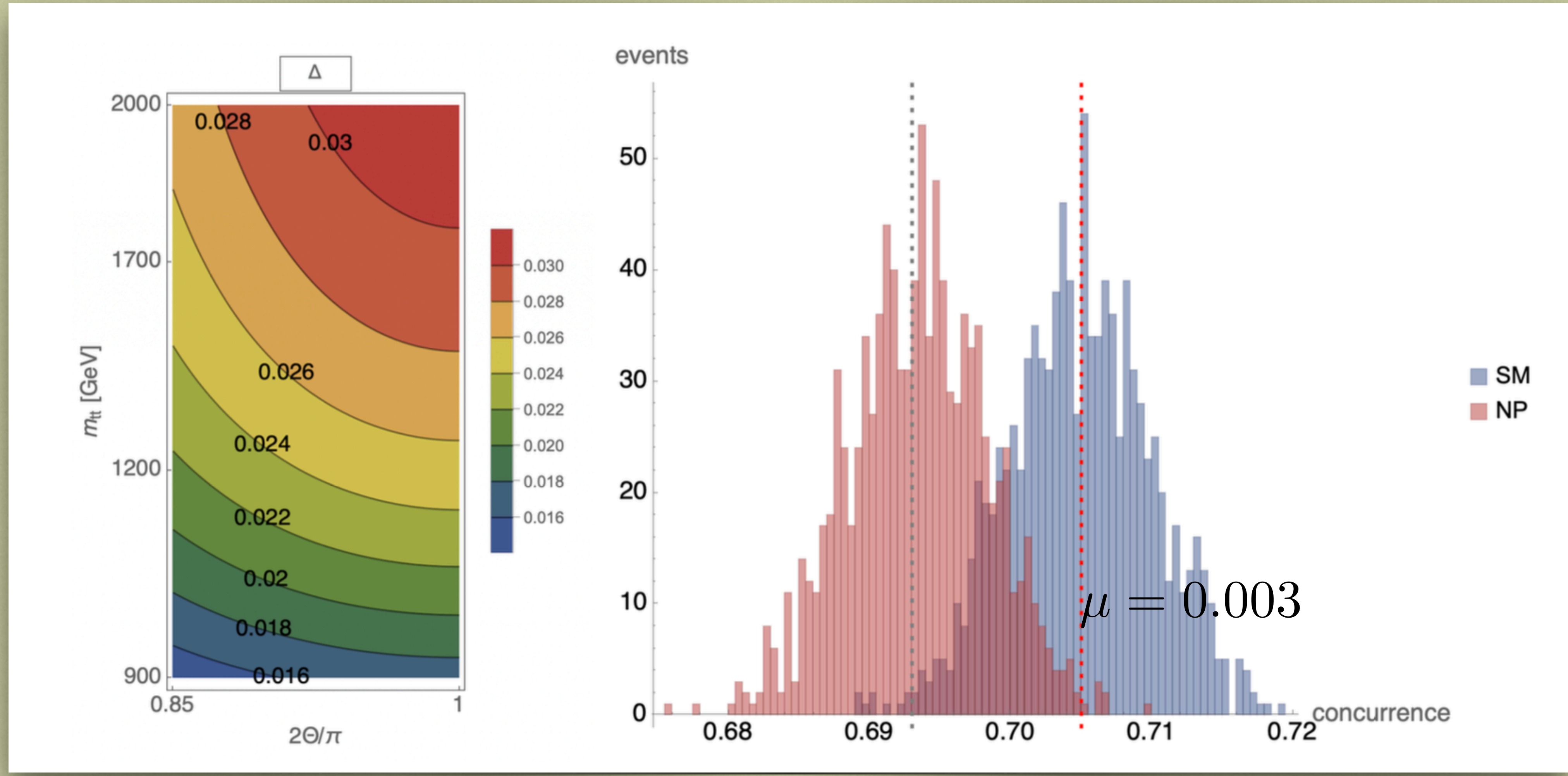


R. Aoude, E. Madge, F. Maltoni, and L. Mantani, *Quantum SMEFT tomography: Top quark pair production at the LHC*, *Phys. Rev. D* **106** (2022), no. 5 055007, [arXiv:2203.05619].

M. Fabbrichesi, R. Floreanini, and E. Gabrielli, *Constraining new physics in entangled two-qubit systems: top-quark, tau-lepton and photon pairs*, *Eur. Phys. J. C* **83** (2023), no. 2 162, [arXiv:2208.11723].

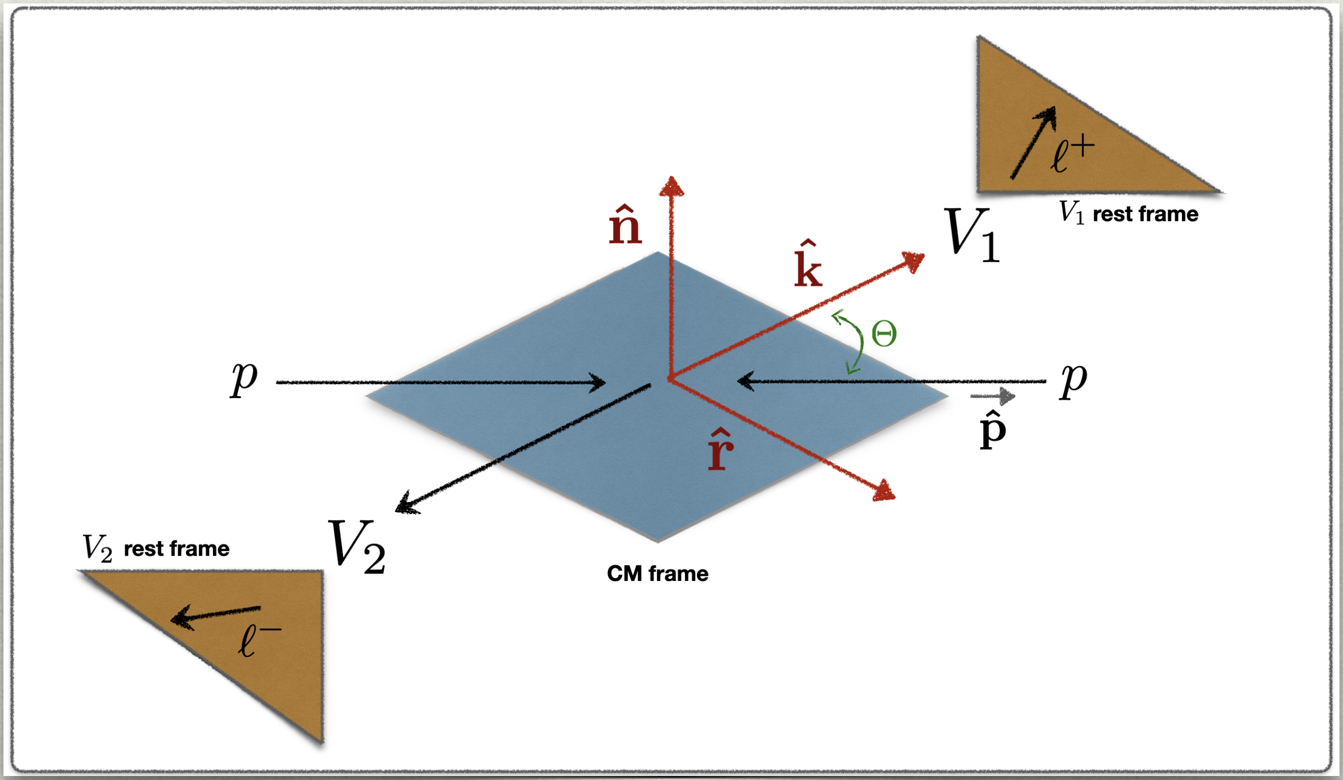
$$\mathcal{L}_{\text{dipole}} = -\mu \, \frac{g_s}{2m_t} \, \bar{t} \, \sigma^{\mu\nu} \, T^a \, t \, G^a_{\mu\nu} \, .$$





$$N = 463 \; (2.4\sigma)$$

Higgs to gauge bosons



A. J. Barr, *Testing Bell inequalities in Higgs boson decays*, *Phys. Lett. B* **825** (2022) 136866, [arXiv:2106.01377].

J. A. Aguilar-Saavedra, A. Bernal, J. A. Casas, and J. M. Moreno, *Testing entanglement and Bell inequalities in H→ZZ*, *Phys. Rev. D* **107** (2023), no. 1 016012, [arXiv:2209.13441].

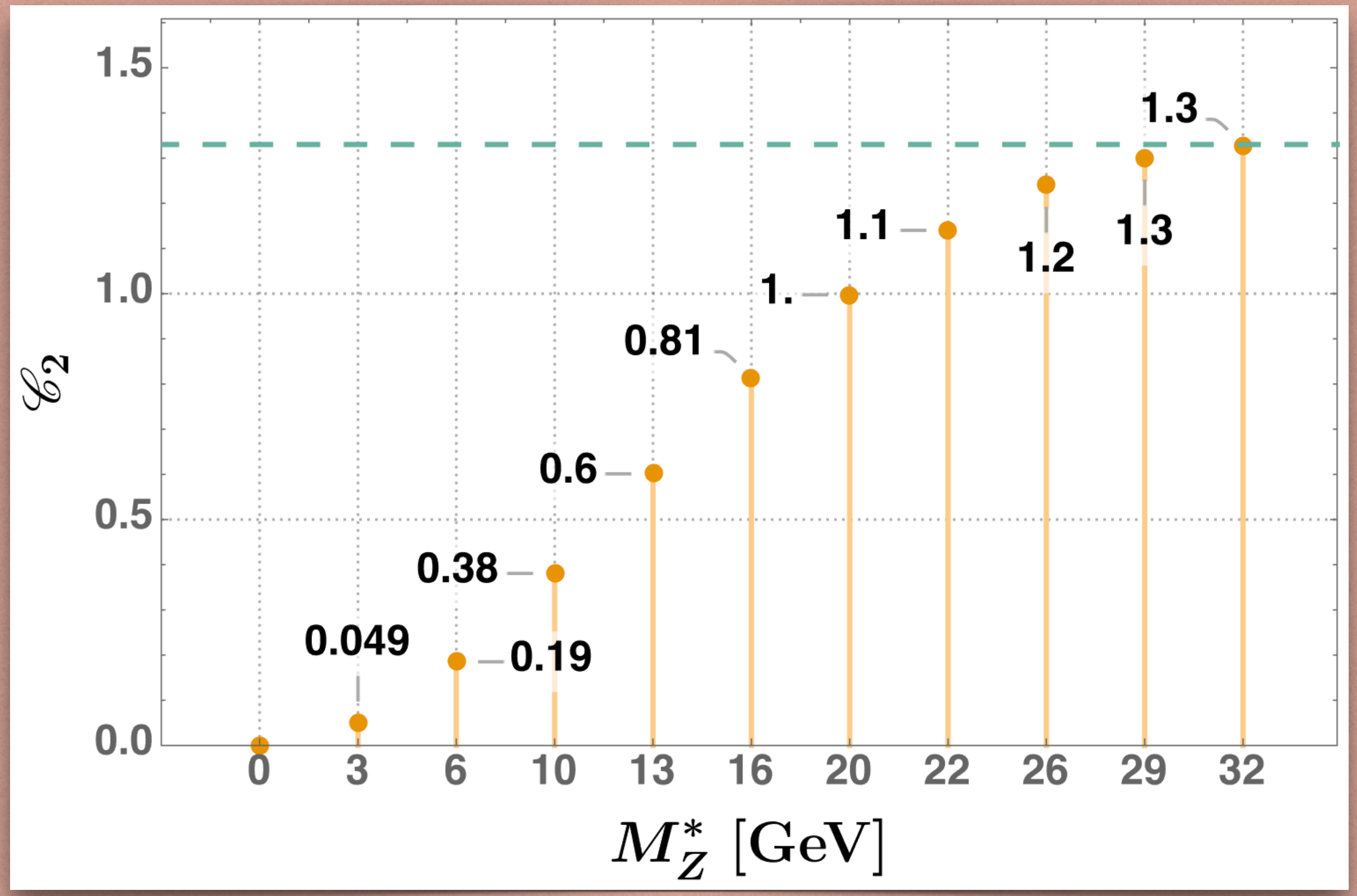
M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Bell inequalities and quantum entanglement in weak gauge boson production at the LHC and future colliders*, *Eur. Phys. J. C* **83** (2023), no. 9 823, [arXiv:2302.00683].

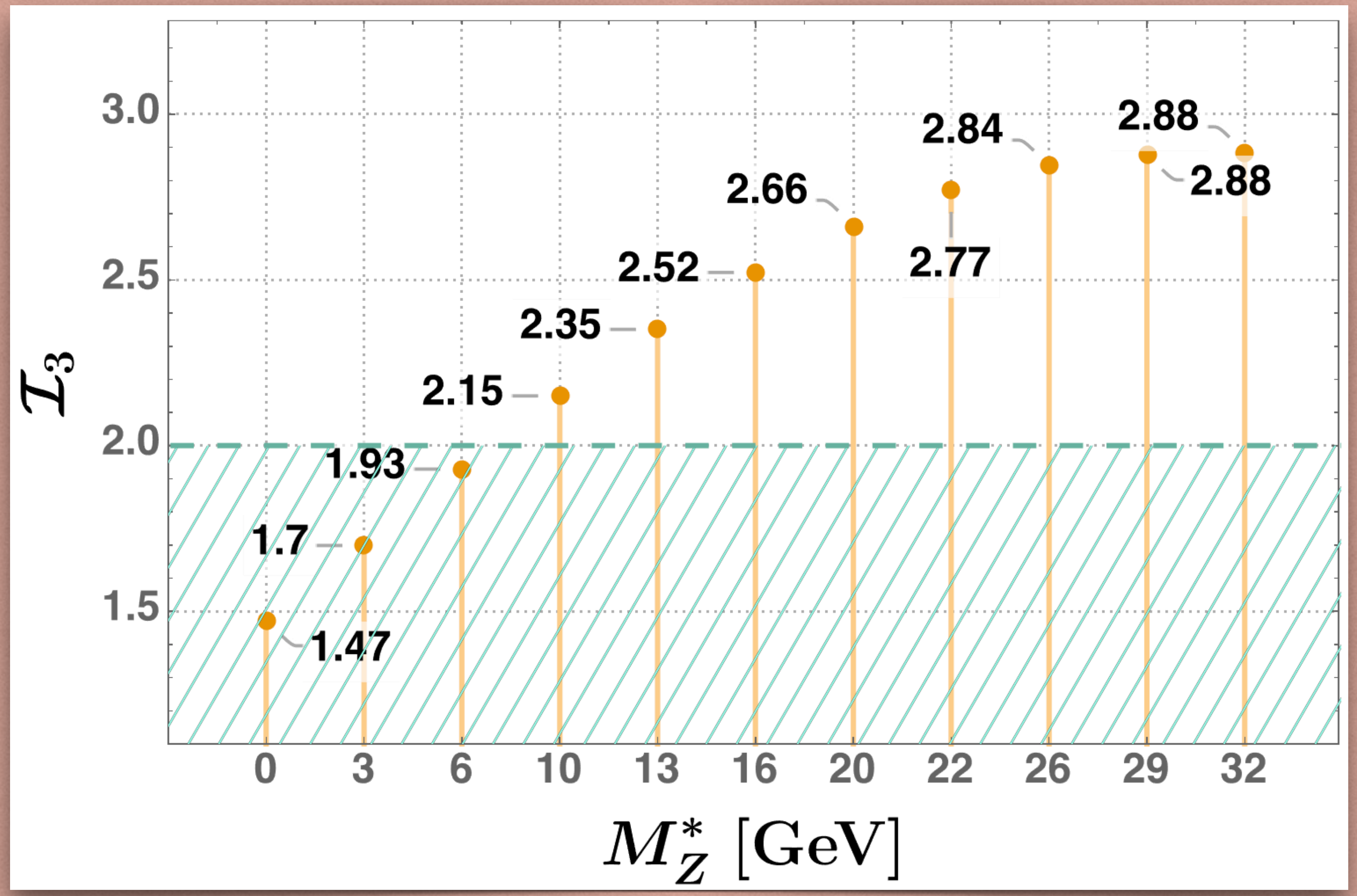
$$H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)\,,$$

$$\rho_H = 2\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$|\Psi_H\rangle = \frac{1}{\sqrt{2+\gamma^2}}\left[|{+-}\rangle - \gamma\,|0\,0\rangle + |{-+}\rangle\right]$$

$$\gamma = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$





putting quantum observables to work

$$\mathcal{L}_{HVV} = g\, M_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} M_Z Z_\mu Z^\mu H$$
$$- \frac{g}{M_W}\left[\frac{a_W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \frac{\widetilde{a}_W}{2} W^+_{\mu\nu} \widetilde{W}^{-\mu\nu} + \frac{a_Z}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{\widetilde{a}_Z}{4} Z_{\mu\nu} \widetilde{Z}^{\mu\nu}\right] H\,,$$

$$\mathscr{E}_{ent} = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right],$$

$$\mathscr{C}_{odd} = \frac{1}{2} \sum_{\substack{a,b \\ a<b}} \left| h_{ab} - h_{ba} \right|,$$

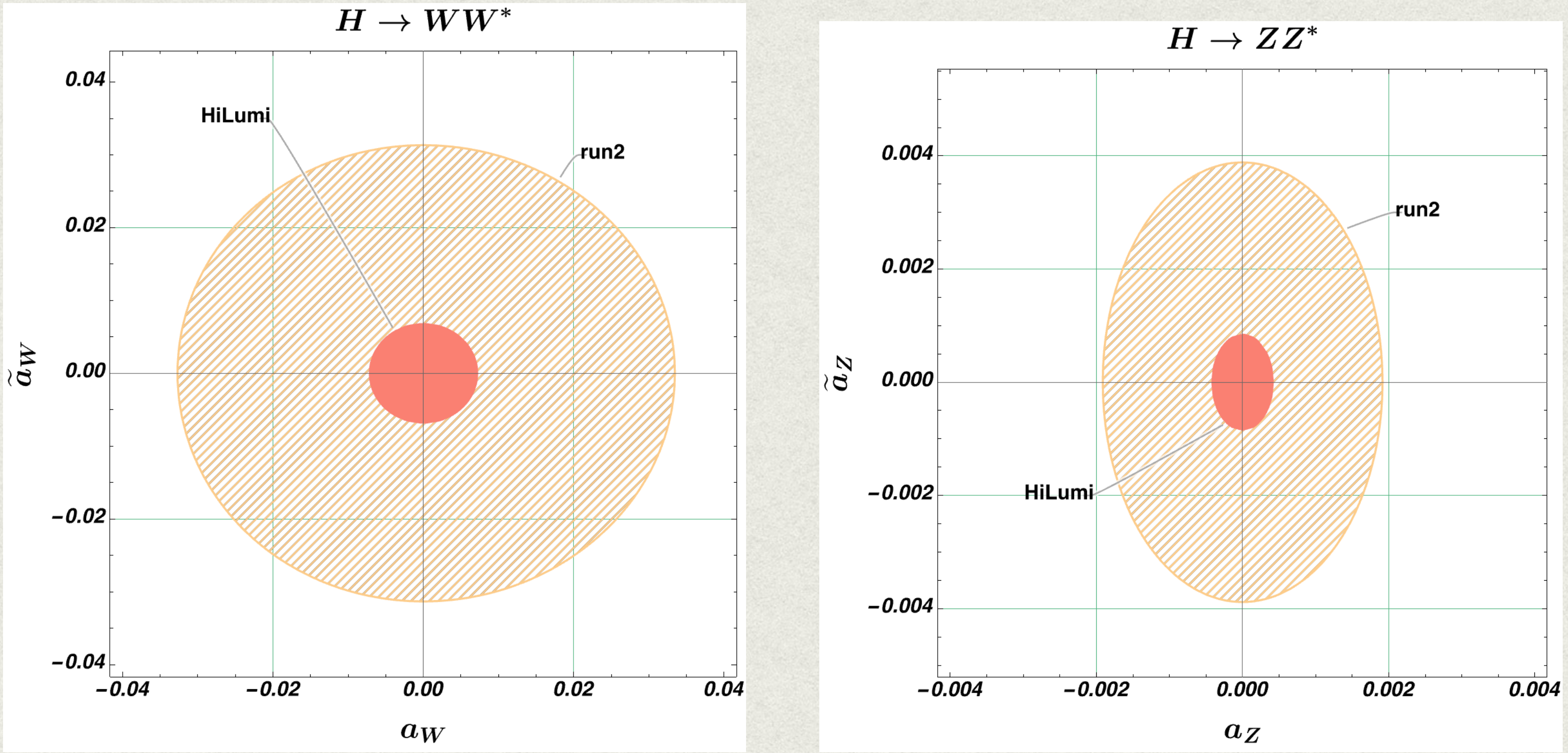


Figure 2: Allowed values for the anomalous couplings a_V and $\widetilde{a}_V$ obtained by using the observables $\mathscr{C}_{odd}$ and $\mathscr{E}_{ent}$. The hatched area use the LHC run2 data ($\mathcal{L} = 140$ fb^{-1}), the purple ones show the HiLumi projection ($\mathcal{L} = 3$ ab^{-1}). The limits, all given at a 95% confidence level, only hold prior to the inclusion of backgrounds.

LHC	*run2*	*HiLumi*
	$\lvert a_W\rvert \leq 0.033$	$\lvert a_W\rvert \leq 0.0070$
	$\lvert \widetilde{a}_W\rvert \leq 0.031$	$\lvert \widetilde{a}_W\rvert \leq 0.0068$
	$\lvert a_Z\rvert \leq 0.0019$	$\lvert a_Z\rvert \leq 0.00040$
	$\lvert \widetilde{a}_Z\rvert \leq 0.0039$	$\lvert \widetilde{a}_Z\rvert \leq 0.00086$

Table 1: 95% confidence intervals for the anomalous couplings obtained by marginalization of the two-parameter plots in Fig. 2. when taken to be independent.

LHC	*run2*	*HiLumi*
	$\lvert a_Z\rvert \leq 0.0028$	$\lvert a_Z\rvert \leq 0.00062$
	$\lvert \widetilde{a}_Z\rvert \leq 0.0039$	$\lvert \widetilde{a}_Z\rvert \leq 0.00086$

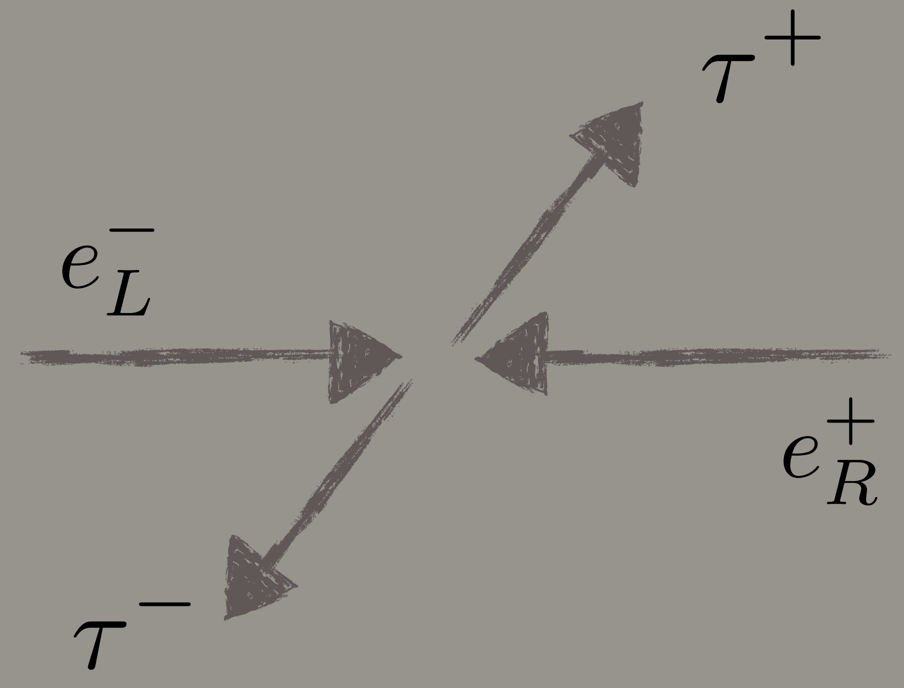
Table 2: 95% confidence intervals for the anomalous couplings obtained by marginalization of the two-parameter plots in Fig. 3. when taken to be independent.

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Stringent bounds on HWW and HZZ anomalous couplings with quantum tomography at the LHC*, *JHEP* **09** (2023) 195, [arXiv:2304.02403].

C

Tau leptons

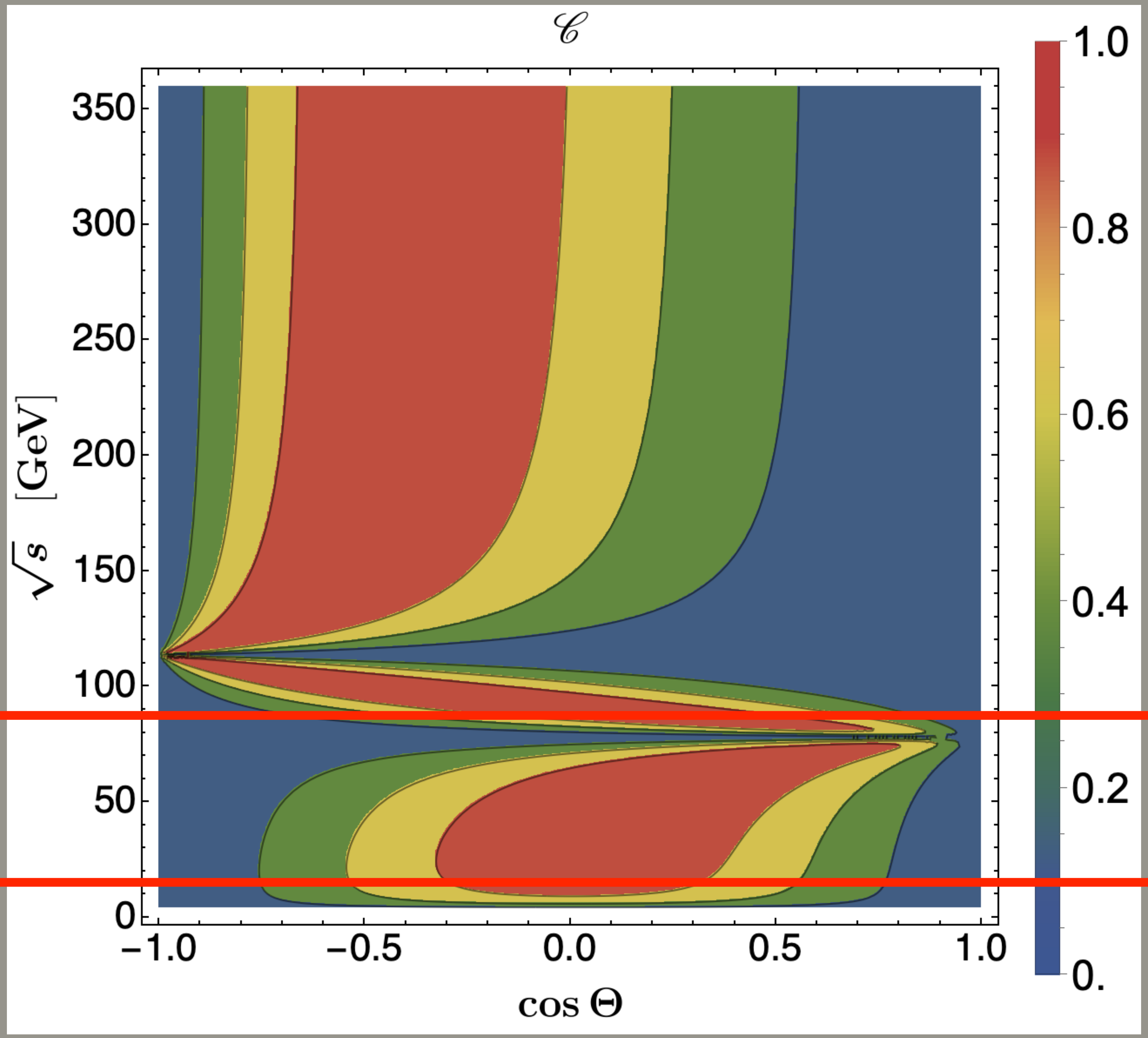
- heaviest lepton
- non-vanishing impact parameter
- simple hadronic decays



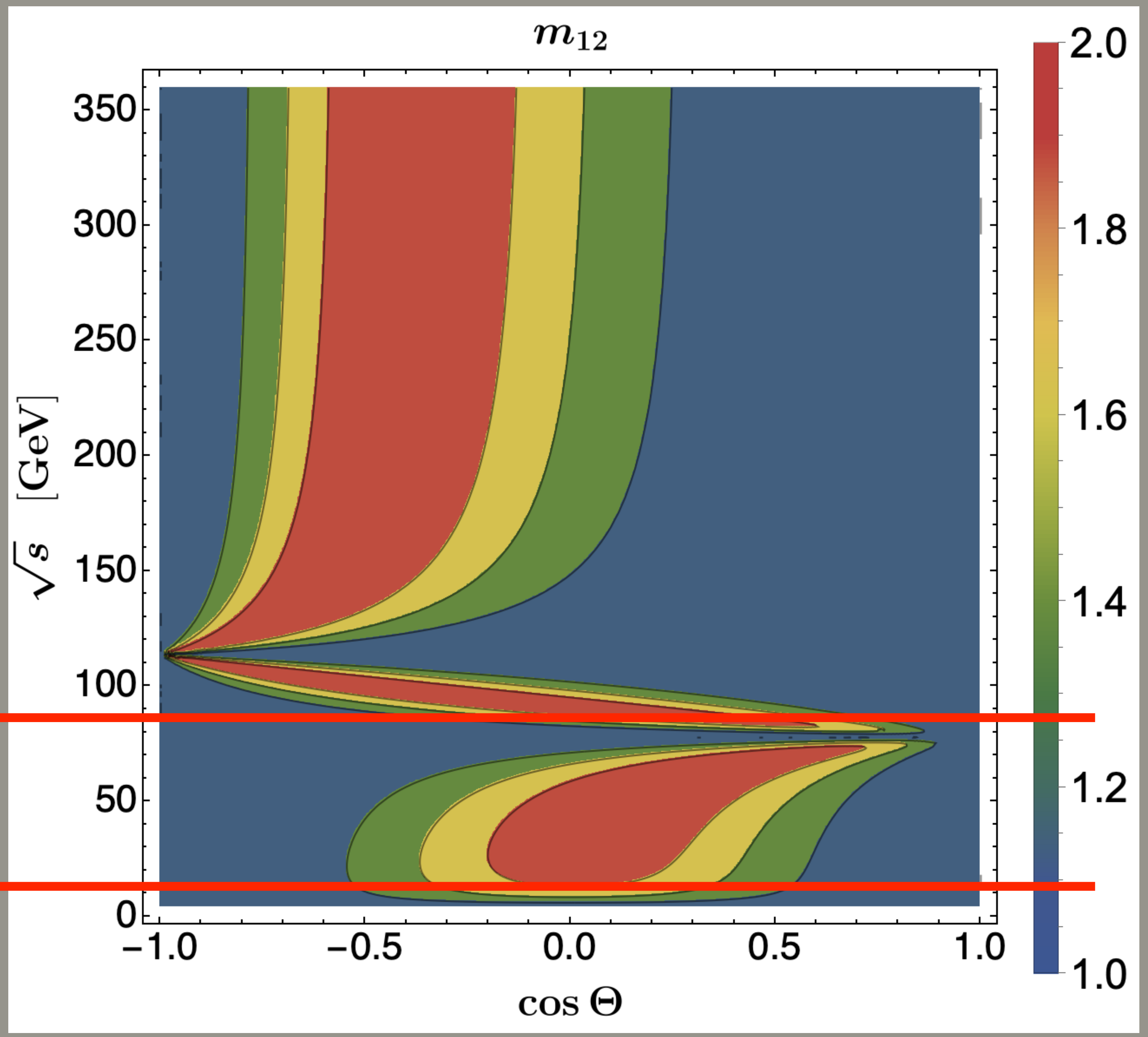
FCC-ee

Belle II

Concurrence



Horodecki condition



$$e^+e^- \to Z,\gamma \to \tau^+\tau^- \to \pi^+\pi^-\nu_\tau\,\bar{\nu}_\tau$$

$$p^\mu_{\tau^+} + p^\mu_{\tau^-} = p^\mu_{e^+e^-}$$

$$(p_{\tau^+} - p_{\pi^+})^2 = m_\nu^2 = 0 \quad \text{and} \quad (p_{\tau^-} - p_{\pi^-})^2 = m_\nu^2 = 0$$
$$p^2_{\tau^+} = m_\tau^2 \quad \text{and} \quad p^2_{\tau^-} = m_\tau^2\,.$$

$$\mathbf{d}_{min} = \mathbf{d} + \frac{[(\mathbf{d}\cdot\mathbf{n}_+)(\mathbf{n}_-\cdot\mathbf{n}_+) - \mathbf{d}\cdot\mathbf{n}_-]\,\mathbf{n}_- + [(\mathbf{d}\cdot\mathbf{n}_-)(\mathbf{n}_-\cdot\mathbf{n}_+) - \mathbf{d}\cdot\mathbf{n}_+]\,\mathbf{n}_+}{1-(\mathbf{n}_-\cdot\mathbf{n}_+)^2}\,.$$

Event generation

MadGraph5
fast simulation of detector smearing

Belle II: K. Ehatäht, M. Fabbrichesi, L. Marzola, and C. Veelken, *Probing entanglement and testing Bell inequality violation with* $e^+e^- \to \tau^+\tau^-$ *at Belle II*, *Phys. Rev. D* **109** (2024), no. 3 03200 [arXiv:2311.17555].

FCC: MF and L. Marzola, Phys. Rev. D 110 (2024) 076004

statistical errors

FCC Collaboration, A. Abada et al., *FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2*, *Eur. Phys. J. ST* **228** (2019), no. 2 261–623.

We model the detector resolution with the following uncertainties:

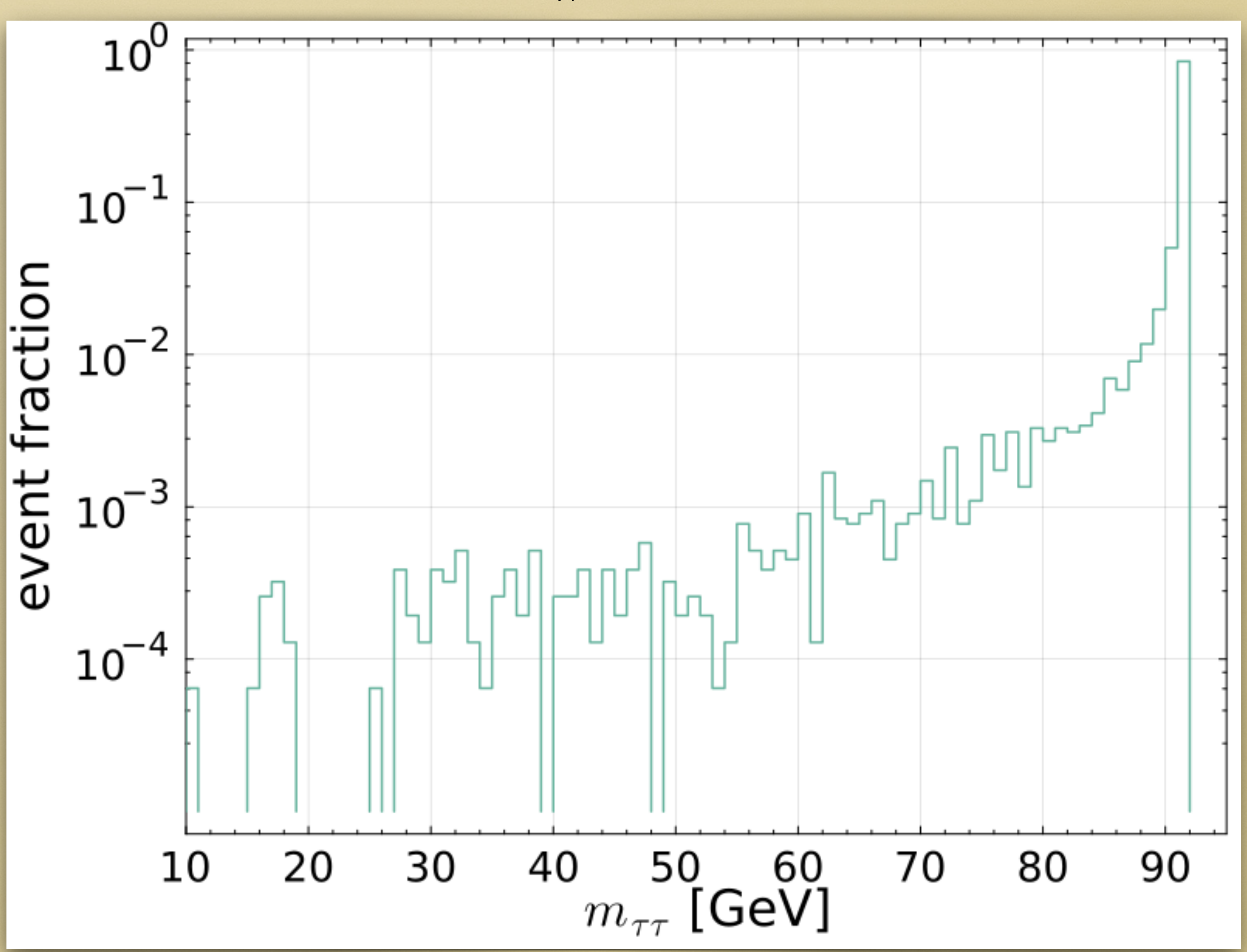
$$\frac{\sigma_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{p_T}{\text{GeV}} \quad \text{and} \quad \sigma_{\theta,\phi} = 0.1 \times 10^{-3}\,\text{rad}$$

for the tracks proper and

$$\sigma_b = 3\,\mu\text{m} \oplus \frac{15\,\mu\text{m}}{\sin^{2/3}\Theta}\,\frac{\text{GeV}}{p_T}$$

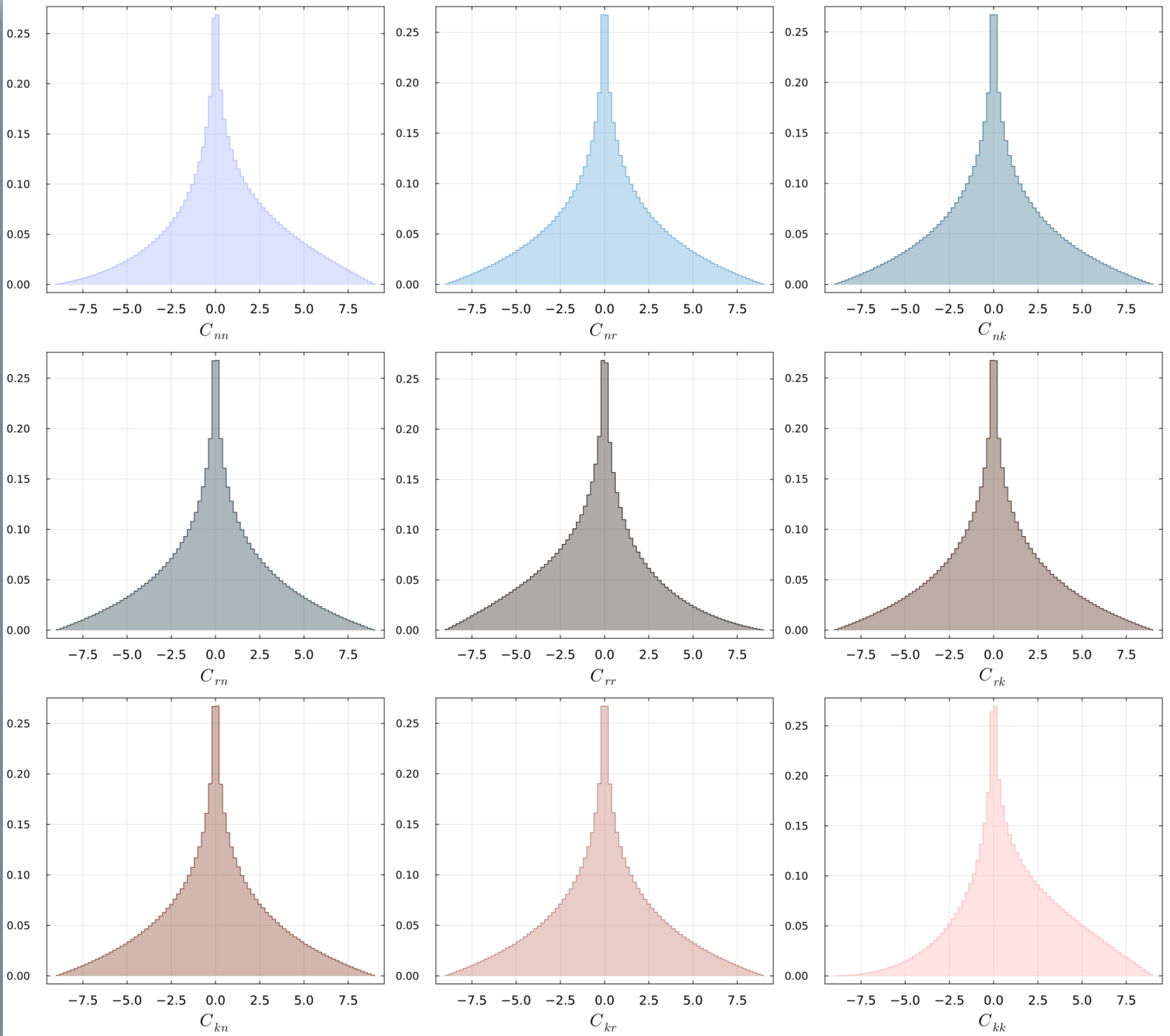
more statistical errors

C. Bierlich et al., *A comprehensive guide to the physics and usage of PYTHIA 8.3*, *SciPost Phys. Codeb.* **2022** (2022) 8, [arXiv:2203.11601].

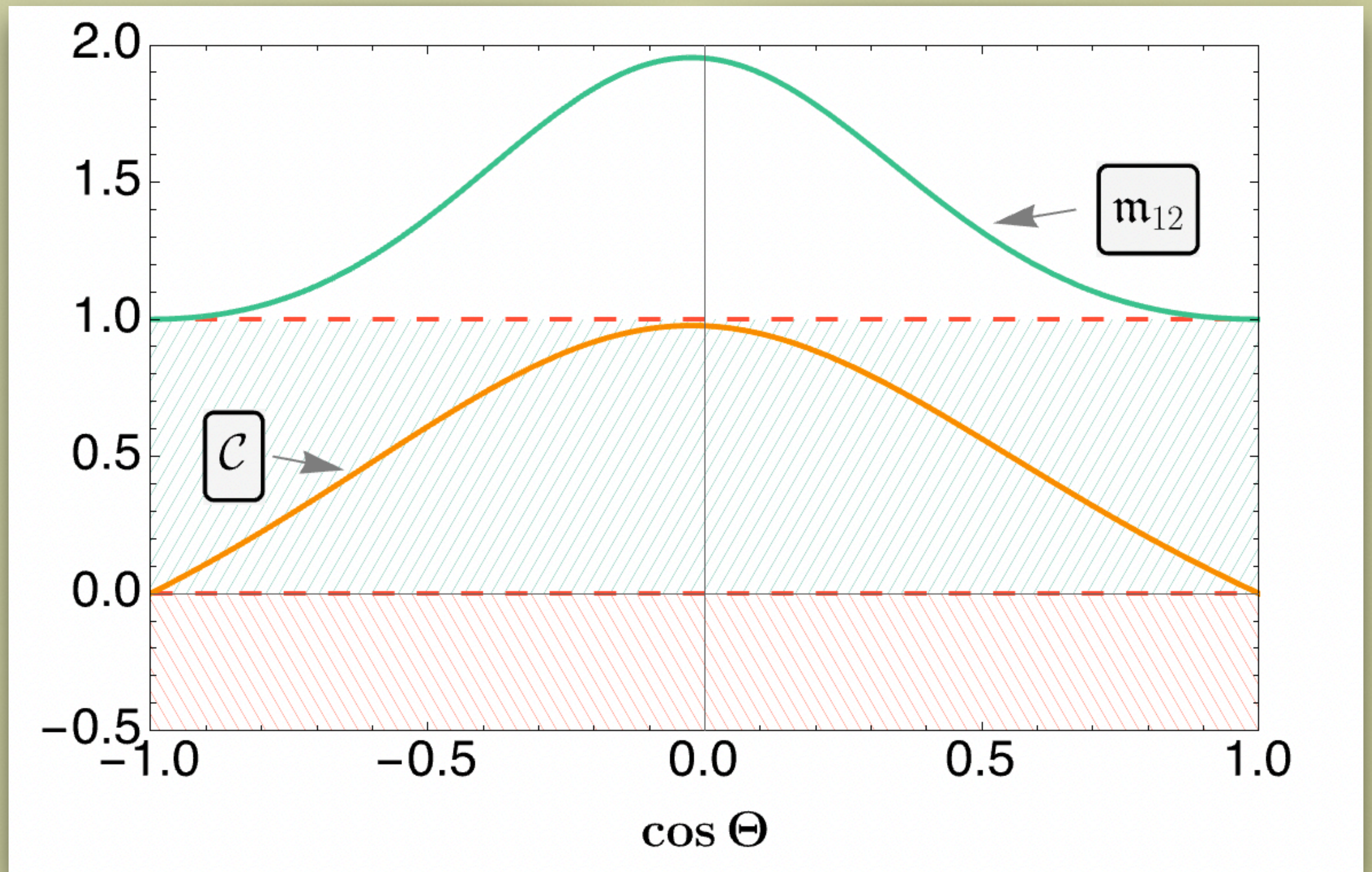


plus systematic errors

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_i^+\,\mathrm{d}\cos\theta_j^-} = \frac{1}{4}\left(1 + \mathrm{C}_{ij}\,\cos\theta_i^+\,\cos\theta_j^-\right)$$



$$\rho = \frac{1}{4}\left[\mathbb{1}\otimes\mathbb{1} + \sum_i \mathrm{B}_i^+\,(\sigma_i\otimes\mathbb{1}) + \sum_j \mathrm{B}_j^-\,(\mathbb{1}\otimes\sigma_j) + \sum_{i,j}\mathrm{C}_{ij}\,(\sigma_i\otimes\sigma_j)\right]$$



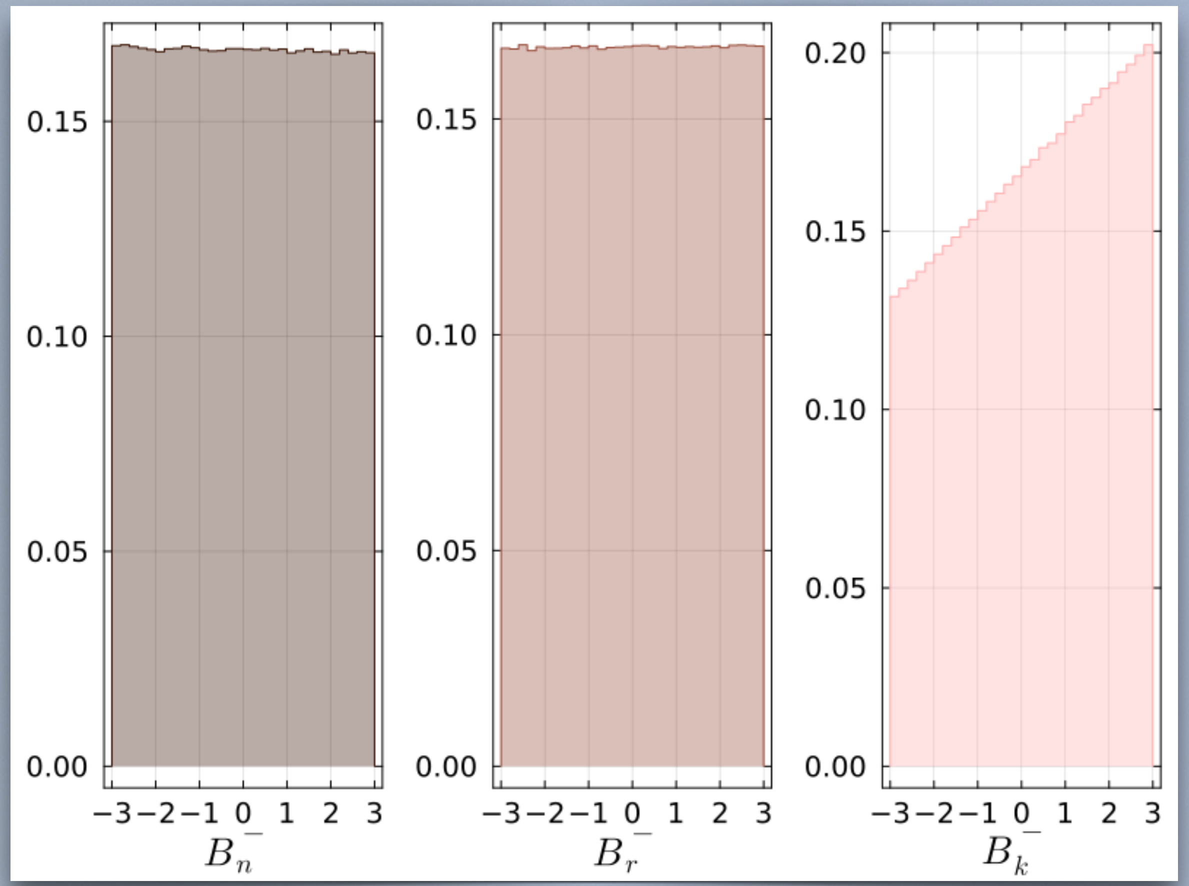
$$\mathscr{C} = 0.4805 \pm 0.0063|_{\mathrm{stat}} \pm 0.0012|_{\mathrm{syst}}\,,$$

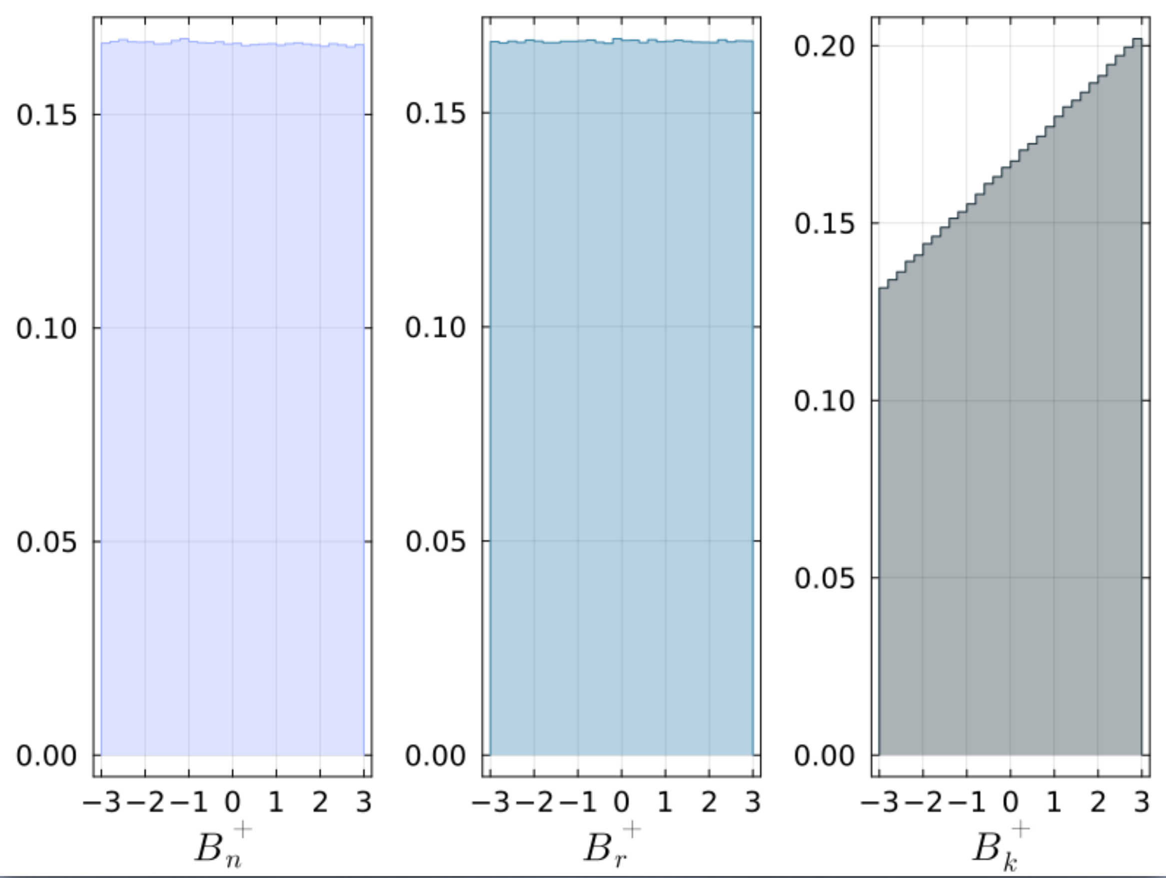
$$\mathfrak{m}_{12} = 1.239 \pm 0.017|_{\mathrm{stat}} \pm 0.008|_{\mathrm{syst}}\,,$$

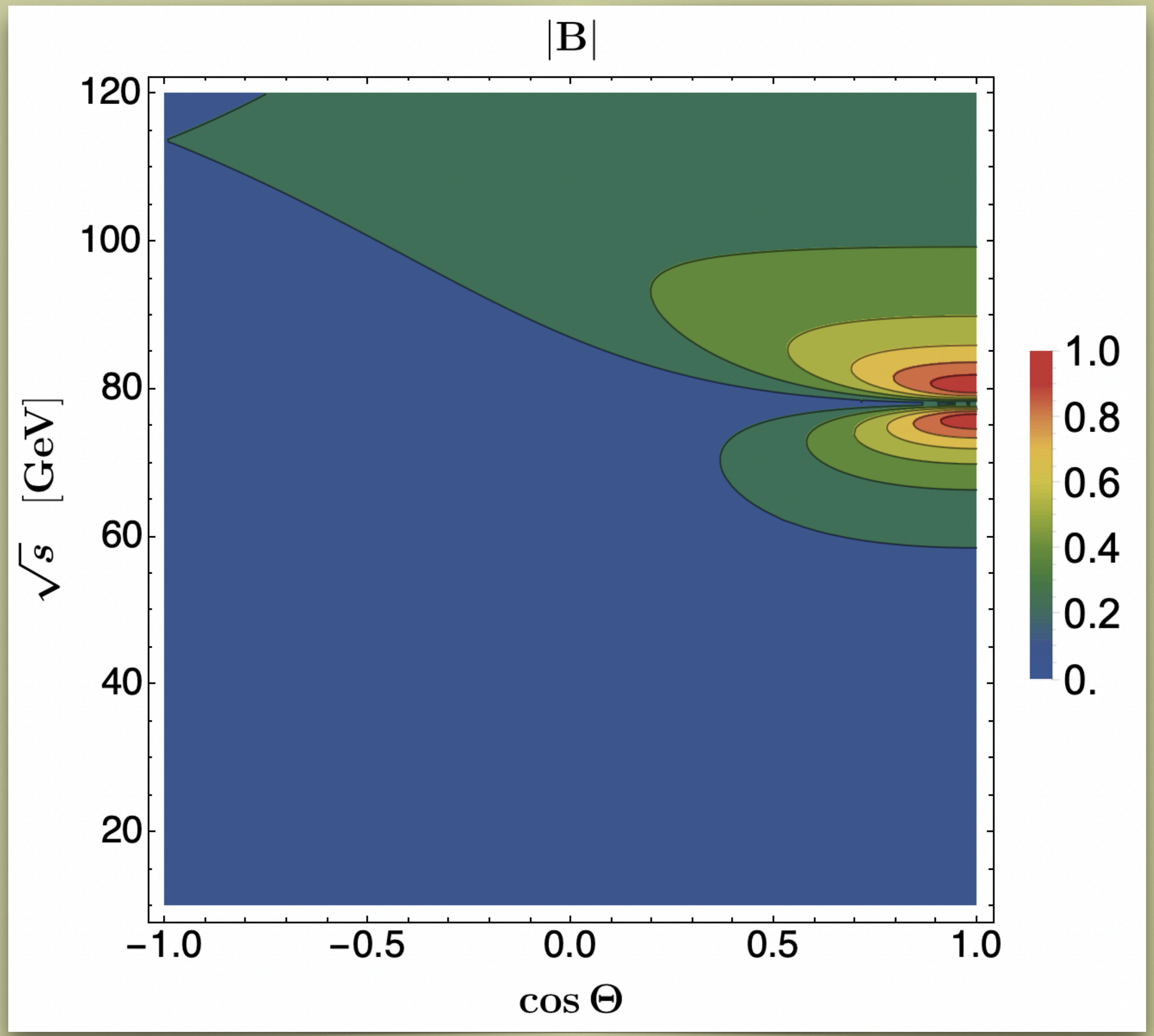
J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079, [arXiv:1405.0301].

K. Hagiwara, T. Li, K. Mawatari, and J. Nakamura, *TauDecay: a library to simulate polarized tau decays via FeynRules and MadGraph5*, *Eur. Phys. J. C* **73** (2013) 2489, [arXiv:1212.6247].

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_i^\pm} = \frac{1}{2}\left(1 \mp \mathrm{B}_i^\pm \cos\theta_i^\pm\right)$$







$$\langle P\rangle_\tau = \frac{1}{2}(\mathrm{B}_k^+ + \mathrm{B}_k^-) = 0.2203 \pm 0.0044|_{\mathrm{stat}} \pm 0.0008|_{\mathrm{syst}}\,,$$

$$P_\tau(\cos\Theta) = \frac{\mathcal{A}_\tau\,(1+\cos^2\Theta) + 2\cos\Theta\,\mathcal{A}_e}{1+\cos^2\Theta + 2\cos\Theta\,\mathcal{A}_\tau\mathcal{A}_e}\,,$$

$$\mathcal{A} = \mathcal{A}_e = \mathcal{A}_\tau = \frac{2\,(1-4\sin^2\theta_W)}{1+(1-4\sin^2\theta_W)^2}\,,$$

$$\sin^2\theta_W = 0.2223 \pm 0.0006|_{\mathrm{stat}} \pm 0.0001|_{\mathrm{syst}}\,,$$

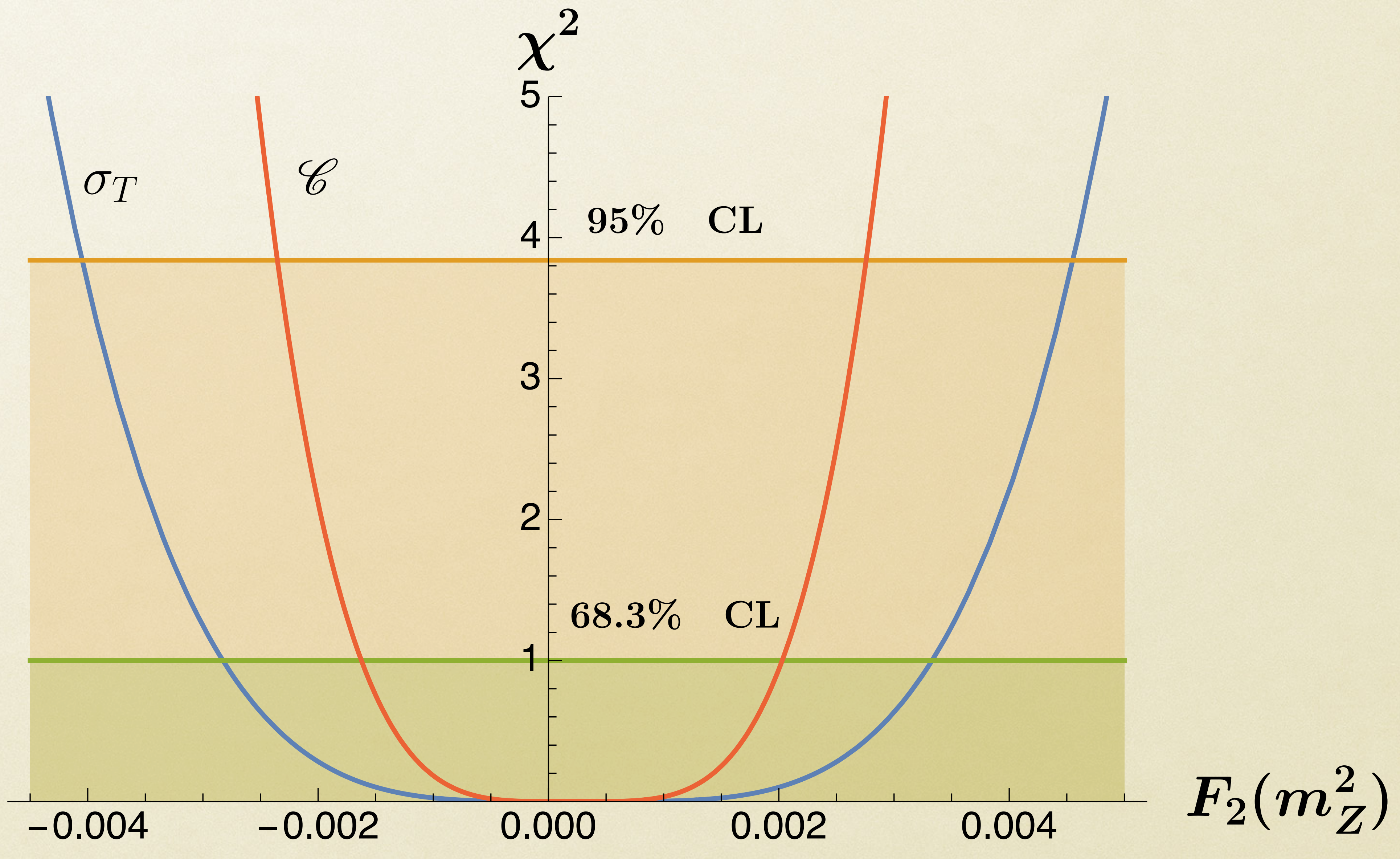
putting quantum observables to work

$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\,\Gamma^\mu(q^2)\,\tau\,Z_\mu(q) = i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\left[\gamma^\mu F_1^V(q^2) + \gamma^\mu\gamma_5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m_\tau}F_3(q^2)\right]\tau\,Z_\mu(q)$$

$$\mathscr{C}_{odd} = \sum_{i<j}\left|\,\mathrm{C}_{ij} - \mathrm{C}_{ji}\,\right|,$$

$$\sigma_T = \frac{1}{64\pi^2\,s}\int d\Omega\frac{|\mathcal{M}|^2}{4}\sqrt{1-\frac{4m_\tau^2}{s}},$$

$$\mathscr{C} = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$$



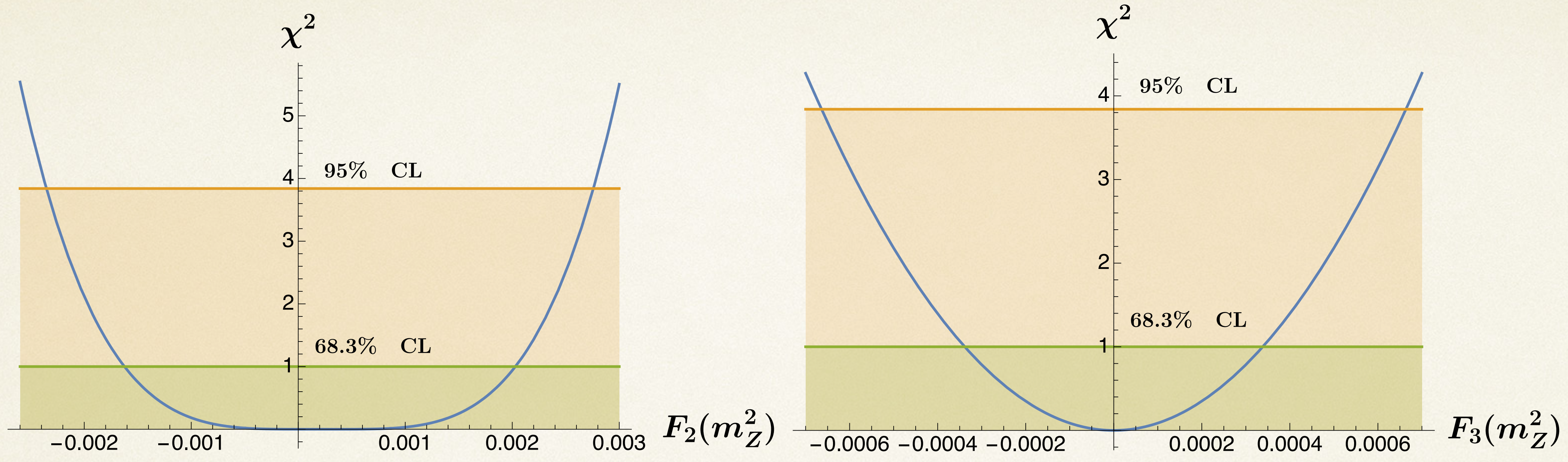


Figure 3.4: χ^2 test for the form factor $F_2(m_Z^2)$ and $F_3^V(m_Z^2)$. The limits for $F_2(m_Z^2)$ are obtained by means of the concurrence, those for the form factor $F_3^V(m_Z^2)$ by means of the operator $\mathscr{C}_{odd}$.

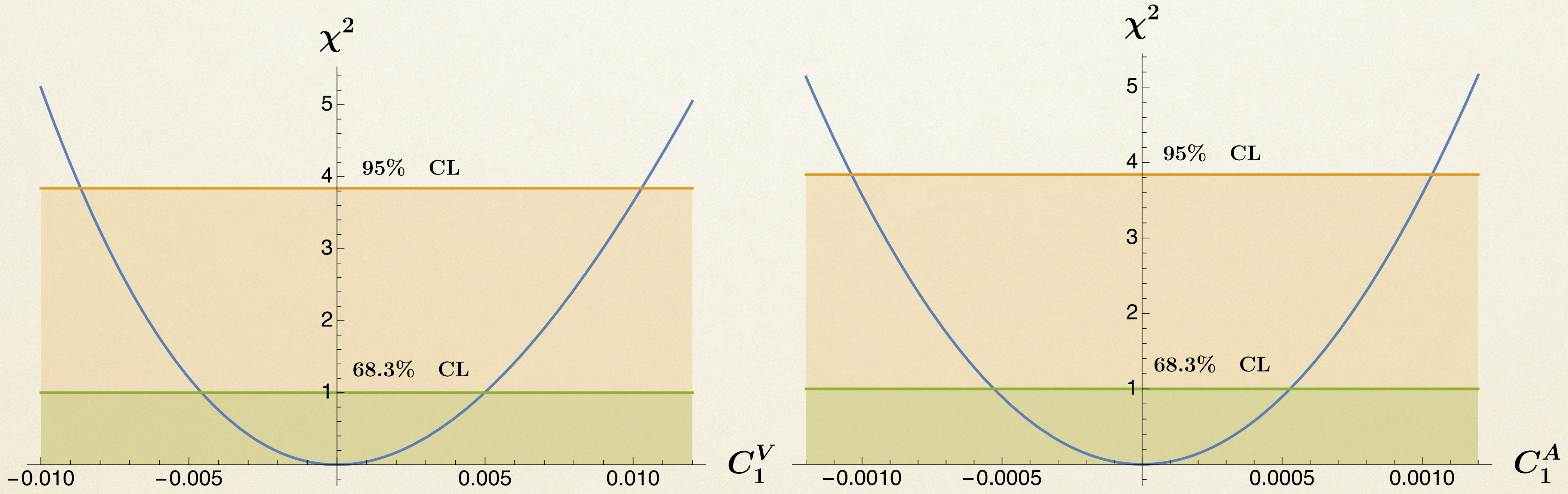


Figure 3.5: χ^2 test for the form factor $F_1^V(m_Z^2)$ (left) and $F_1^A(m_Z^2)$ (right). Both limits are obtained by means of the cross section.

$\mathscr{O}_a$	σ_a^I	limits I ($L = 17.6$ fb^{-1})	σ_a^{II}	limits II ($L = 150$ ab^{-1})
$\mathscr{C}$	0.006	$-0.002 \leq F_2(m_Z^2) \leq 0.003$	0.001	$-0.001 \leq F_2(m_Z^2) \leq 0.001$
$\mathscr{C}_{odd}$	0.009	$-0.001 \leq F_3(m_Z^2) \leq 0.001$	0.006	$-0.0004 \leq F_3(m_Z^2) \leq 0.0005$
σ_T	0.05 pb	$-0.009 \leq C_1^V \leq 0.010$	0.02 pb	$-0.004 \leq C_1^V \leq 0.004$
σ_T	0.05 pb	$-0.001 \leq C_1^A \leq 0.001$	0.02 pb	$-0.0004 \leq C_1^A \leq 0.0004$