(High Frequency) Gravitational Waves: How and Where to Find Them

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GOLDBLUM TEST

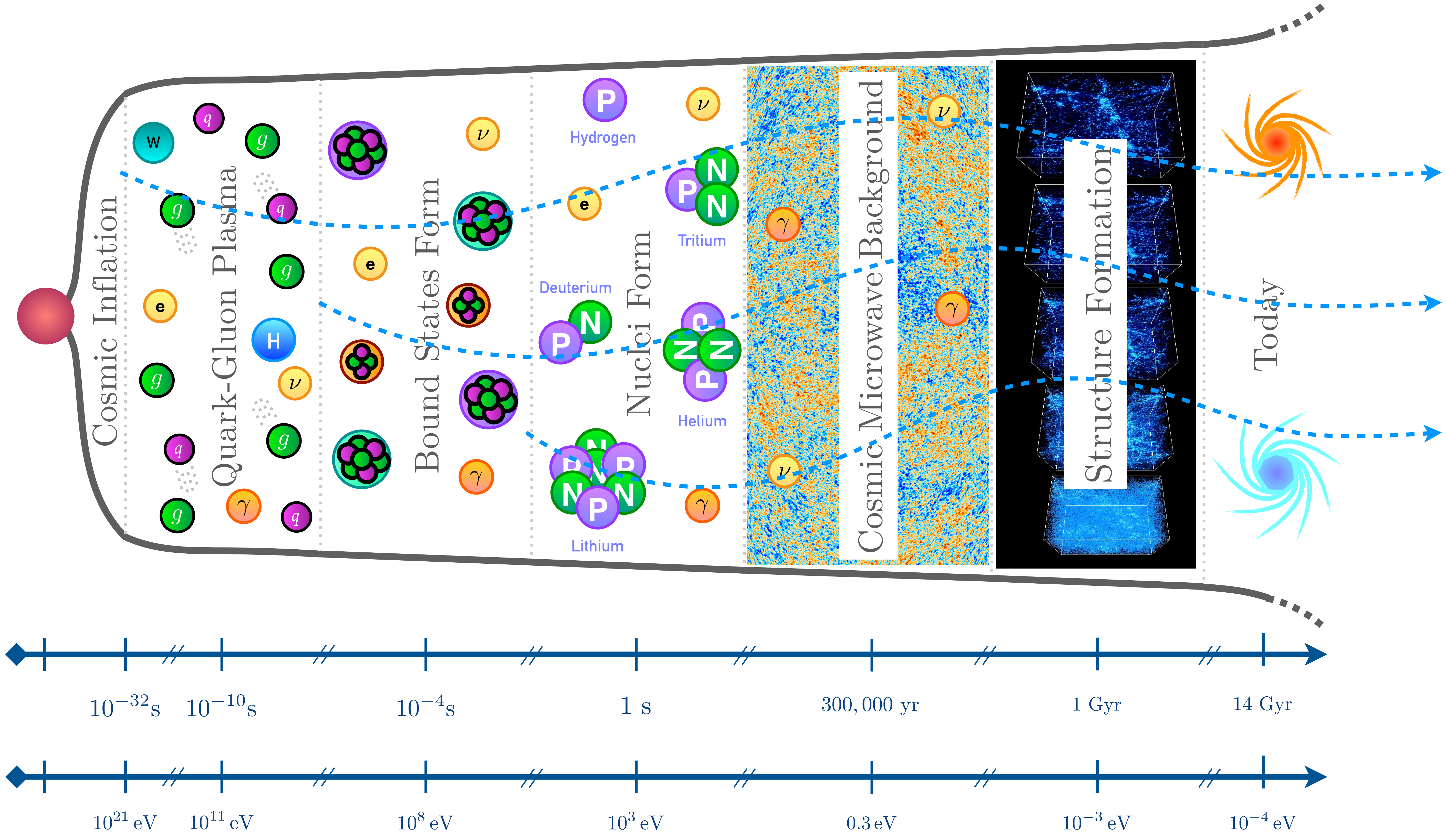
Yeah, yeah, but your scientists were so preoccupied with whether or not they could that they didn't stop to think if they should

Dr. Ian Malcolm (1993)

Why?

Gravitational Waves

So far, GWs telling us about the recent* universe

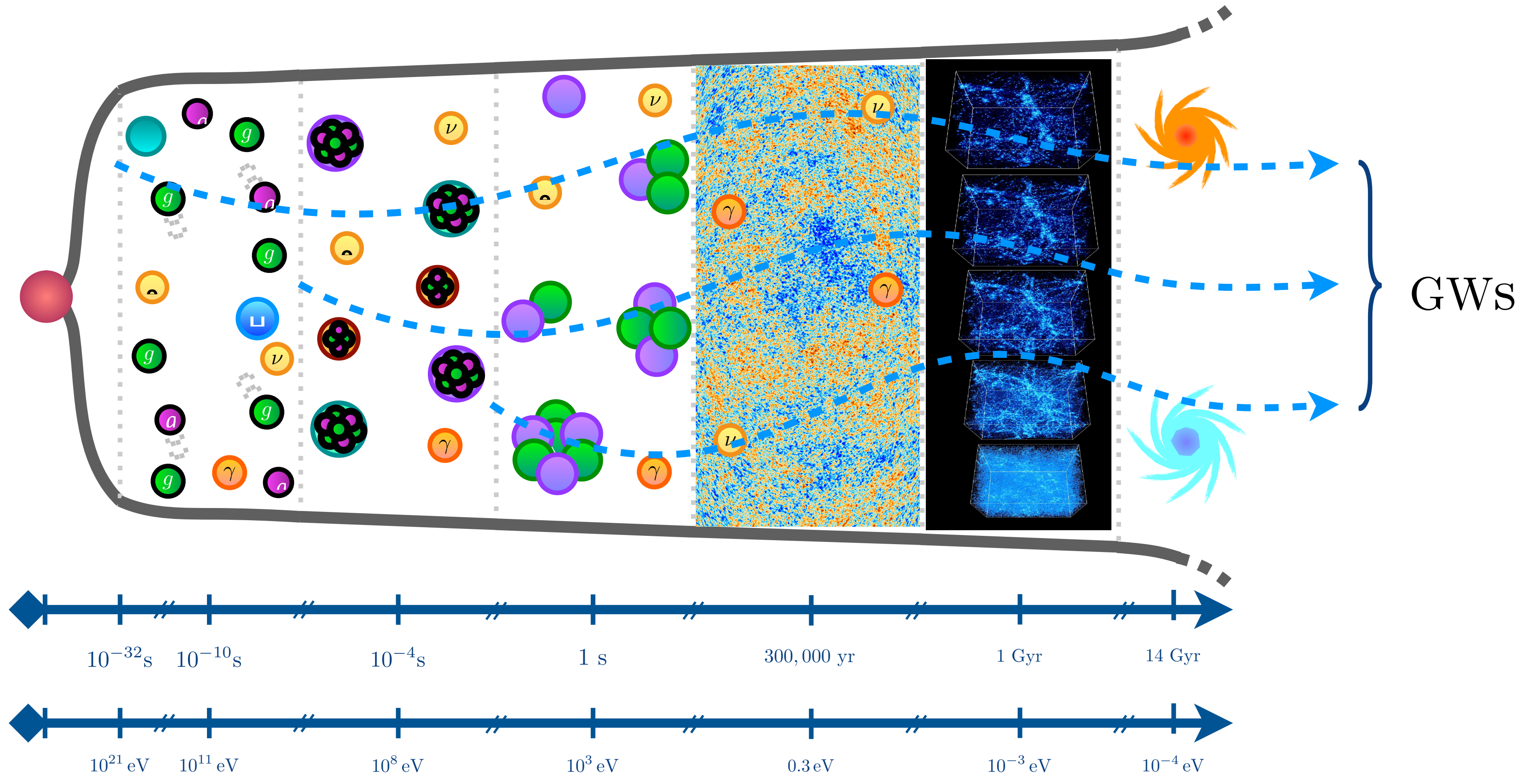


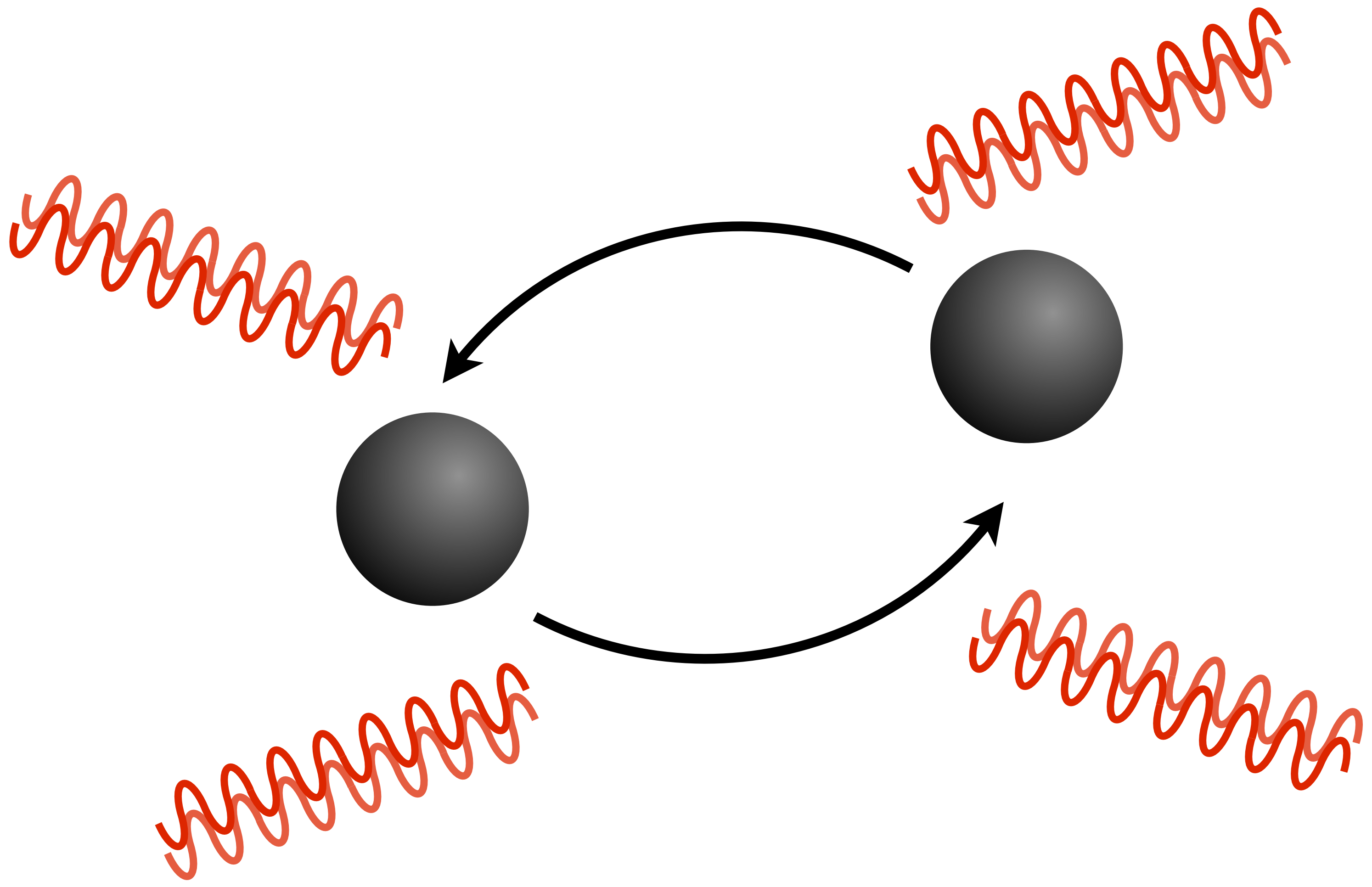
GWs can inform us about earlier epochs

High-Frequency Gravitational Waves

Cosmological GWs

$$\omega_g \Leftrightarrow T_{\text{origin}}$$





Astrophysical GWs

$$\omega_g \Leftrightarrow \Lambda_{\text{origin}}$$

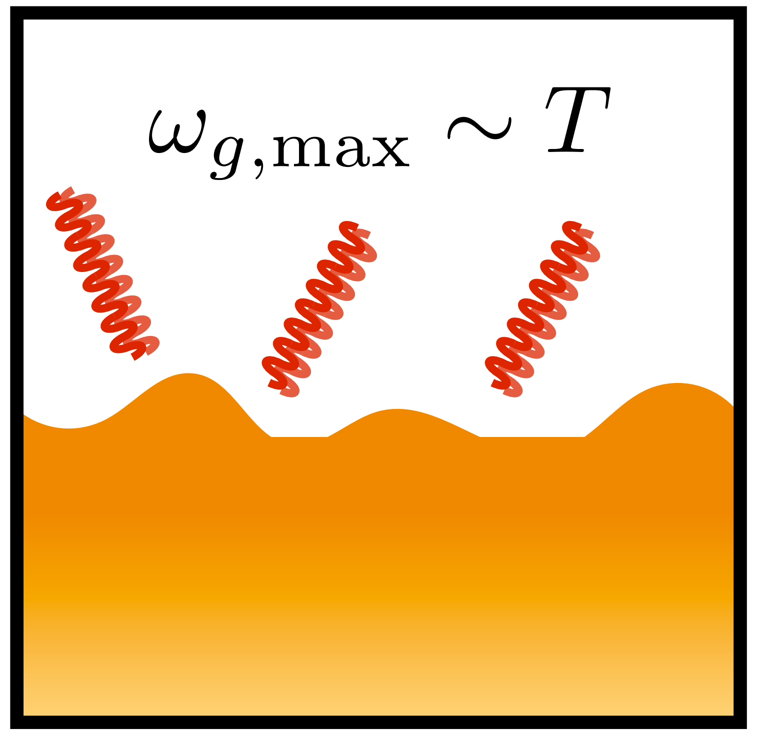
Stochastic Gravitational Waves

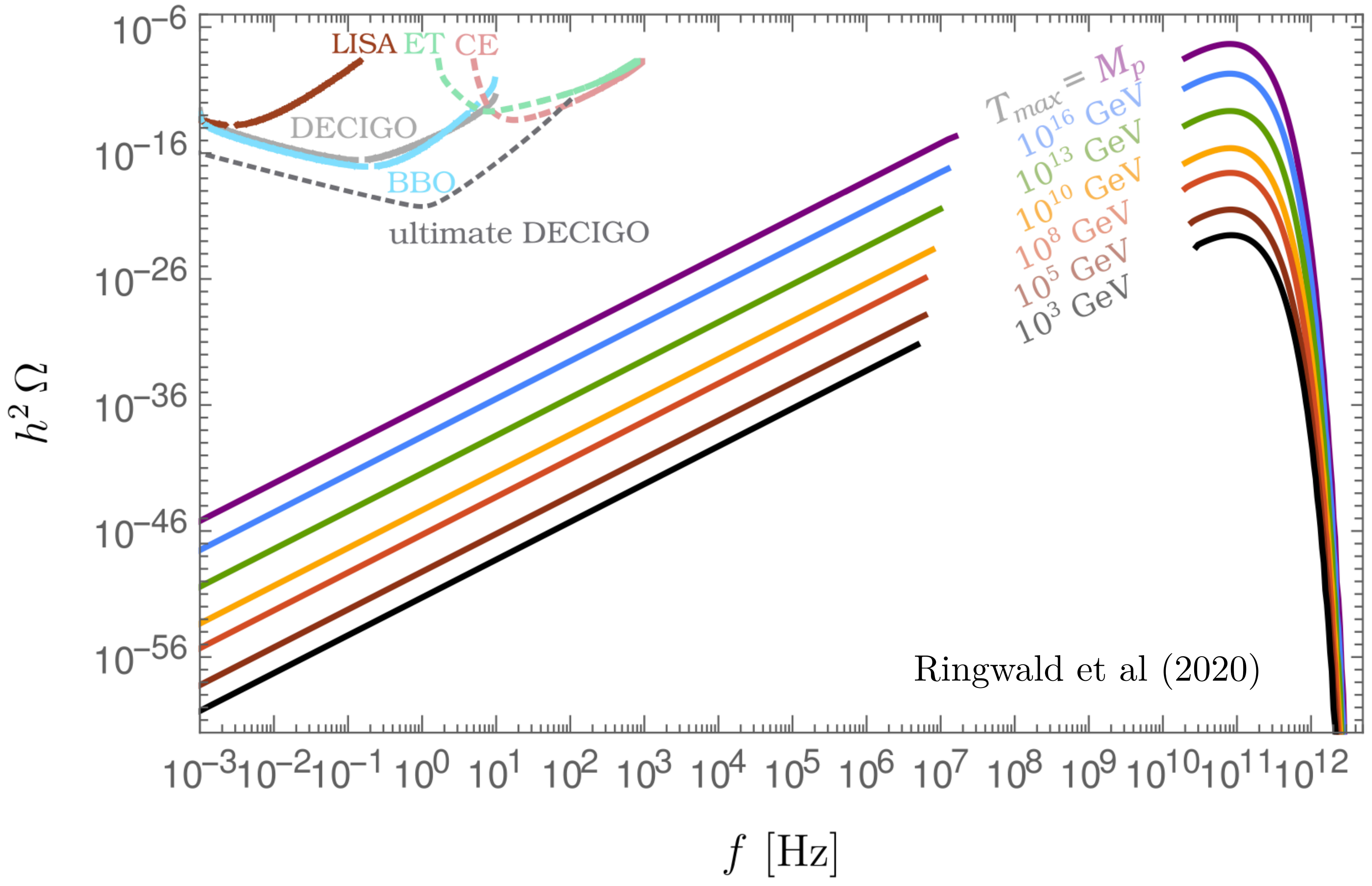
$h\,\hat{h}_{\mu\nu}T^{\mu\nu}$

Standard Model

CGravMB — Ghiglieri & Laine (2015), Ghiglieri et al (2020), Ringwald et al (2020)

Solar fluctuations — Weinberg (1972)

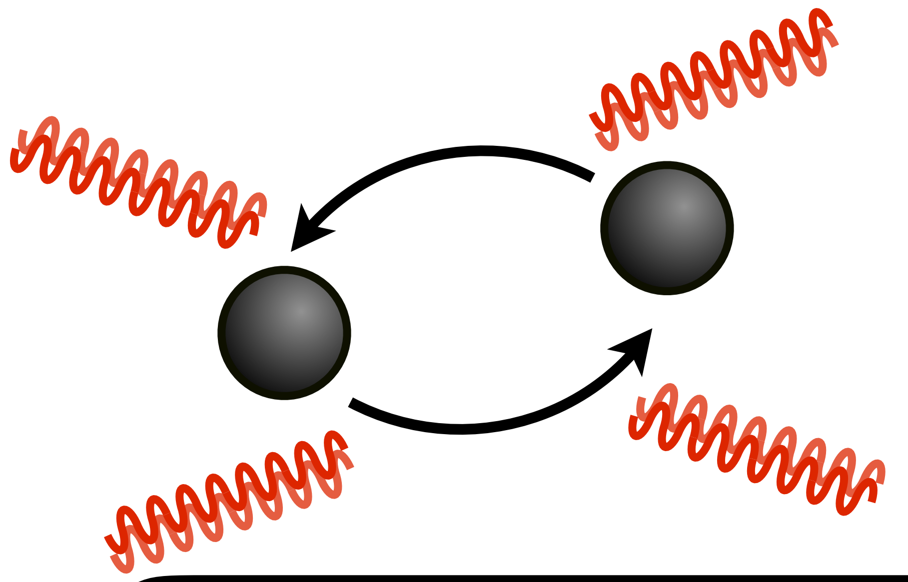




HFGWs probe BSM Cosmology

Astrophysical Gravitational Waves

Binary inspirals:

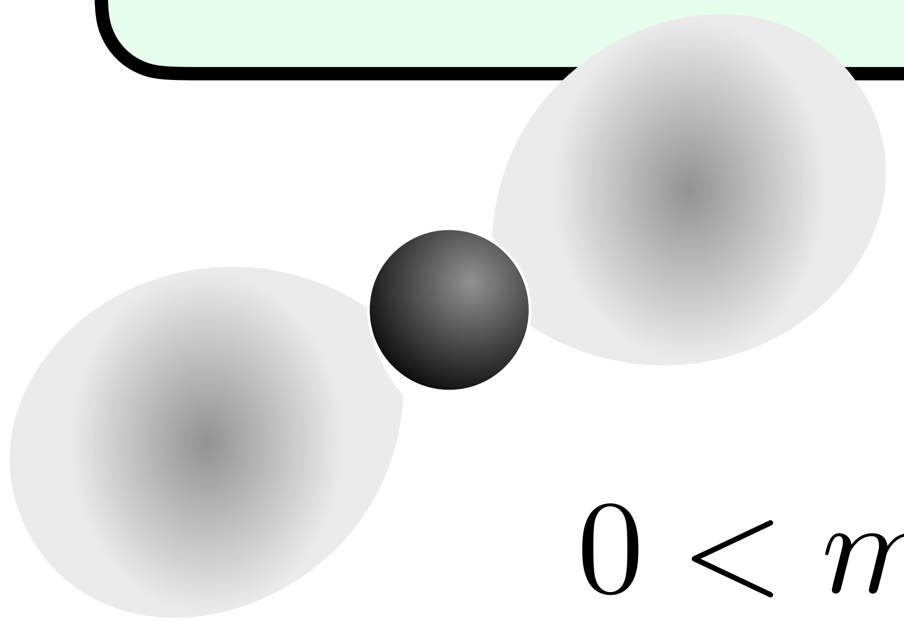


$$\omega_g \lesssim \text{kHz} \Leftrightarrow M_b \gtrsim M_\odot$$

$\omega_g \simeq 14$ GHz

$$\omega_g \gtrsim \text{kHz} \Rightarrow \text{BSM}$$

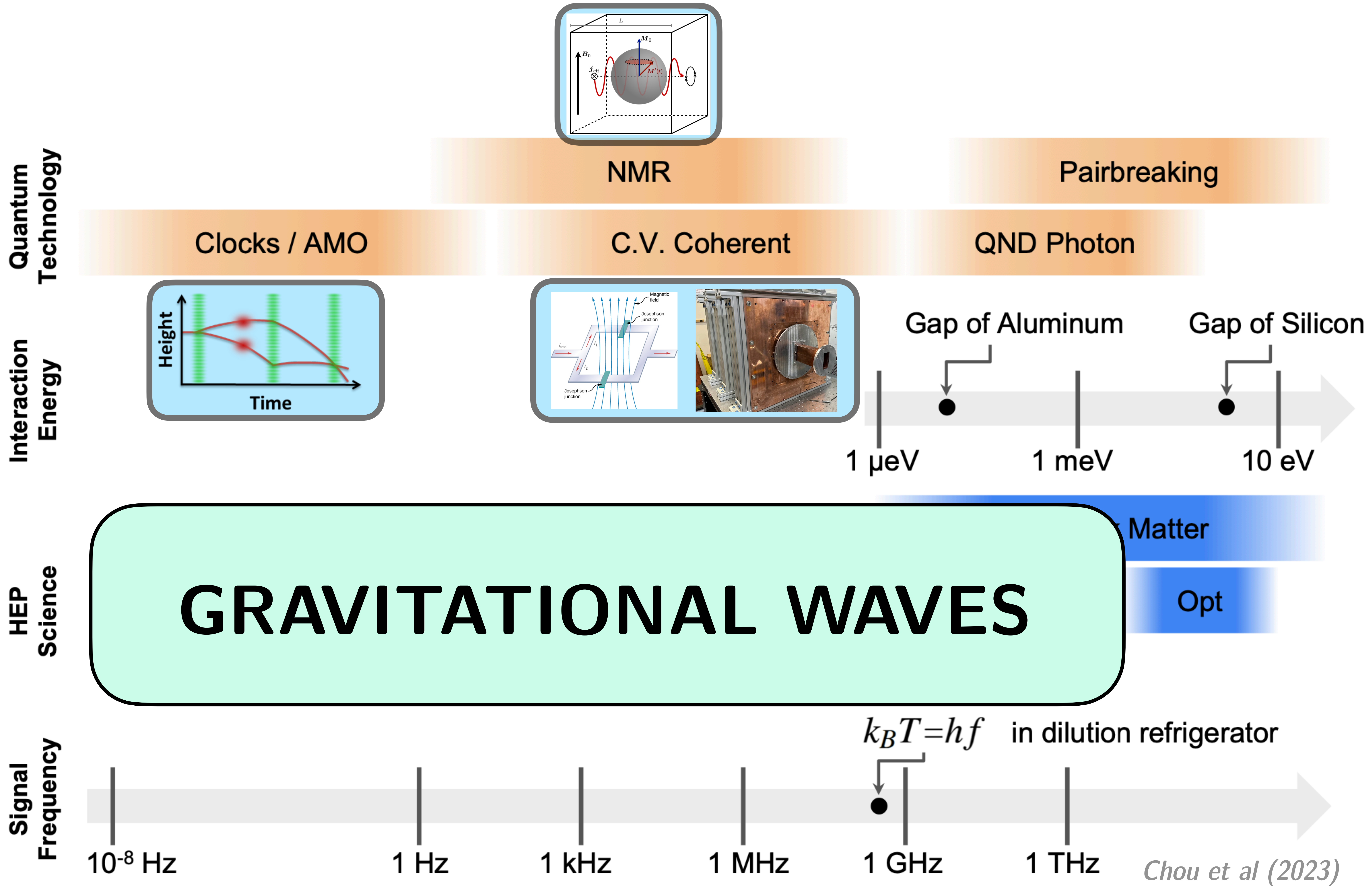
Superradiance:



$$0 < m_a \leq m\omega_+ \quad \omega_+ \sim \frac{1}{4GM_{\rm BH}}$$

$$m_a \sim \mu\text{eV} \times (10^{-4}\ M_\odot / M_{\rm PBH})$$

Maximising Use-case Of Experiments



DETECTION HEURISTICS

How do we measure GWs?

D'Agnolo & SARE (in prep.)

Displacement Measurements

GW measurement as a displacement measurement

SQL for displacement:

$$S_{xx}^{\mathrm{SQL}} \sim \frac{1}{m\,\Gamma\,\omega}$$

Send mass to infinity — amazing measurement?

$$S_{xx} \sim S_{xx}^{\mathrm{imp}} + |\chi(\omega)|^2 S_{FF}^{\mathrm{ba}} \qquad \chi(\omega) \propto \frac{1}{m}$$

Detector Energy

Detector stores EM energy: $U_{\text{in}} \sim \langle E_0(t) E_0^*(t) \rangle V_{\text{det}}$

In frequency space, effect of GWs on the stored energy more clear

$$U_h \sim \Big(\langle E_0(\omega) E_h^*(\omega) \rangle + \langle E_h(\omega) E_h^*(\omega) \rangle \Big) V_{\text{det}}$$

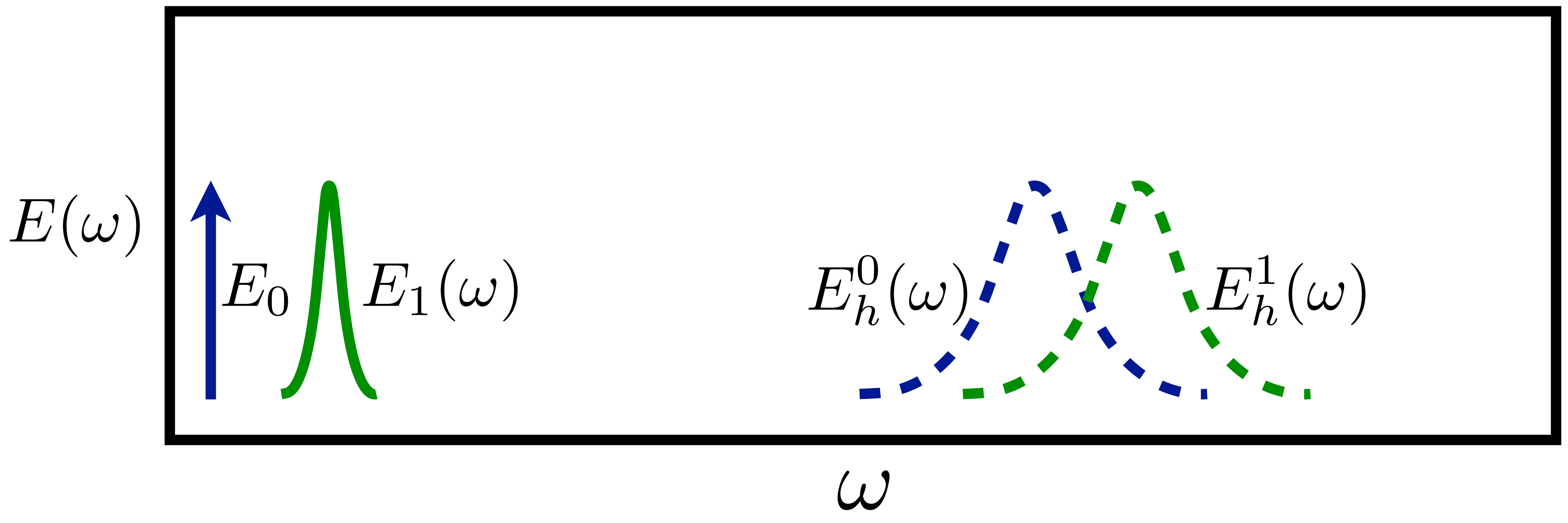
Linear signal: *e.g. interferometers*

Quadratic signal: *e.g. power measurement*

Clearly better, right?

Detector Energy: Quadratic Detector

$$U_h^q \sim \langle E_h(\omega) E_h^*(\omega) \rangle V_{\text{det}}$$



Experiment can be performed such that background energy at detection frequency ∼ zero

$$N_{\text{bg}}^{\gamma} = \left(\frac{1}{2}\right)_{\text{ba}} + \left(\frac{1}{2}\right)_{\text{0pt}}$$

Detector Energy: Quadratic Detector

Minimum detectable power seen by detector

$$P_{\rm min} \sim \frac{(N_{\rm bg}^{\gamma})^{1/2}\,\omega}{t_{\rm int}}$$

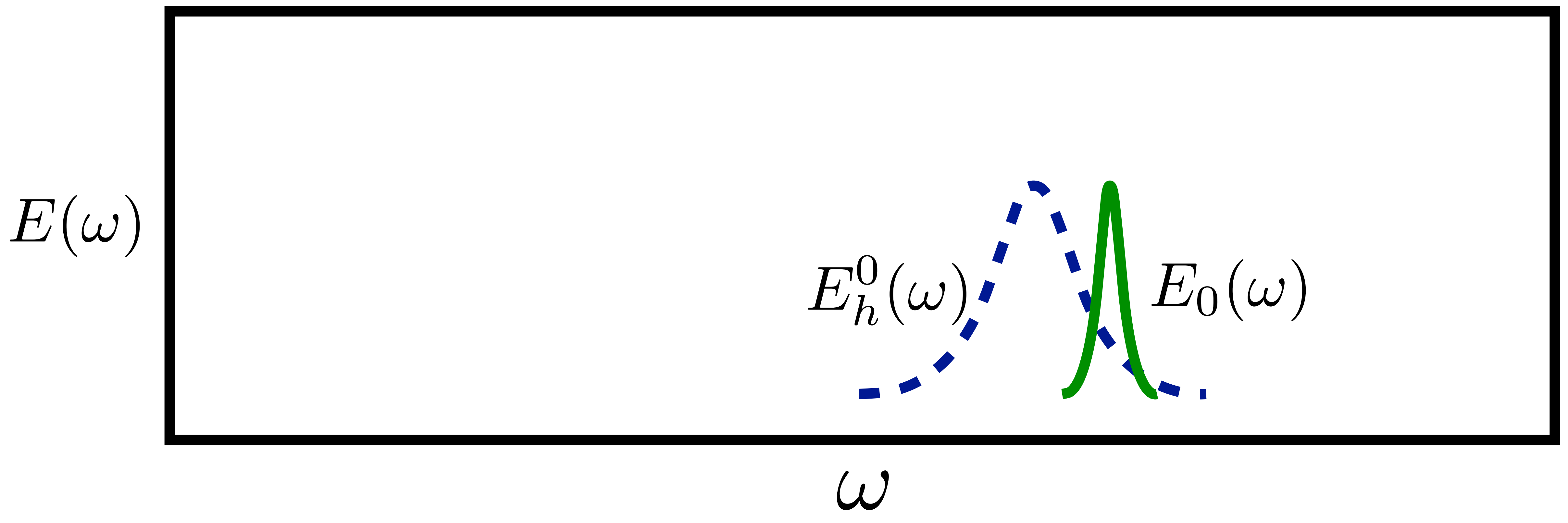
Signal power in quadratic measurement

$$P_{\rm sig} \sim h^2 |\mathcal{T}(\omega)|^2\,\omega\,U_{\rm in}$$

Best possible sensitivity: $h^q_{\rm min} \sim \frac{1}{\sqrt{U_{\rm in} t_{\rm int}}} \frac{1}{\mathcal{T}(\omega)}$

Detector Energy: Linear Detector

$$U_h^l \sim \langle E_0(\omega) E_h^*(\omega) \rangle V_{\rm det}$$



Experiment is performed such that $\langle E_0(\omega) E_0^*(\omega) \rangle \neq 0$

$$N_{\rm bg}^\gamma \sim U_{\rm in}\, t_{\rm int}$$

Detector Energy: Linear Detector

Noise power seen by detector

$$P_{\text{min}} \sim \omega \left(\frac{U_{\text{in}}}{t_{\text{int}}}\right)^{1/2}$$

Signal power in linear measurement

$$P_{\text{sig}} \sim h\,\mathcal{T}(\omega)\,\omega\,U_{\text{in}}$$

Best possible sensitivity: $h^{l}_{\text{min}} \sim \frac{1}{\sqrt{U_{\text{in}} t_{\text{int}}}} \frac{1}{\mathcal{T}(\omega)}$

🤔

Caveats and Takeaway Messages

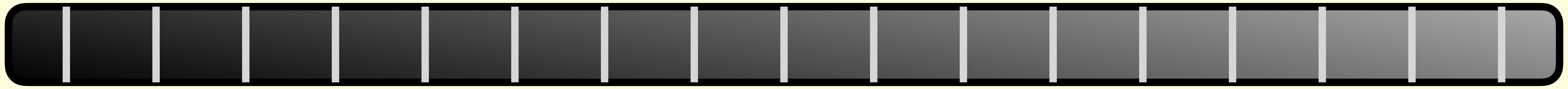
Best possible sensitivity: $h^q_{\rm min} \sim \frac{1}{\sqrt{U_{\rm in} t_{\rm int}}} \frac{1}{\mathcal{T}(\omega)}$

How about that $\mathcal{T}$ransfer function?

Bandwidth and other detector details shoved under $\mathcal{T}$he rug

Stochastic backgrounds (almost) always imply quadratic measurements

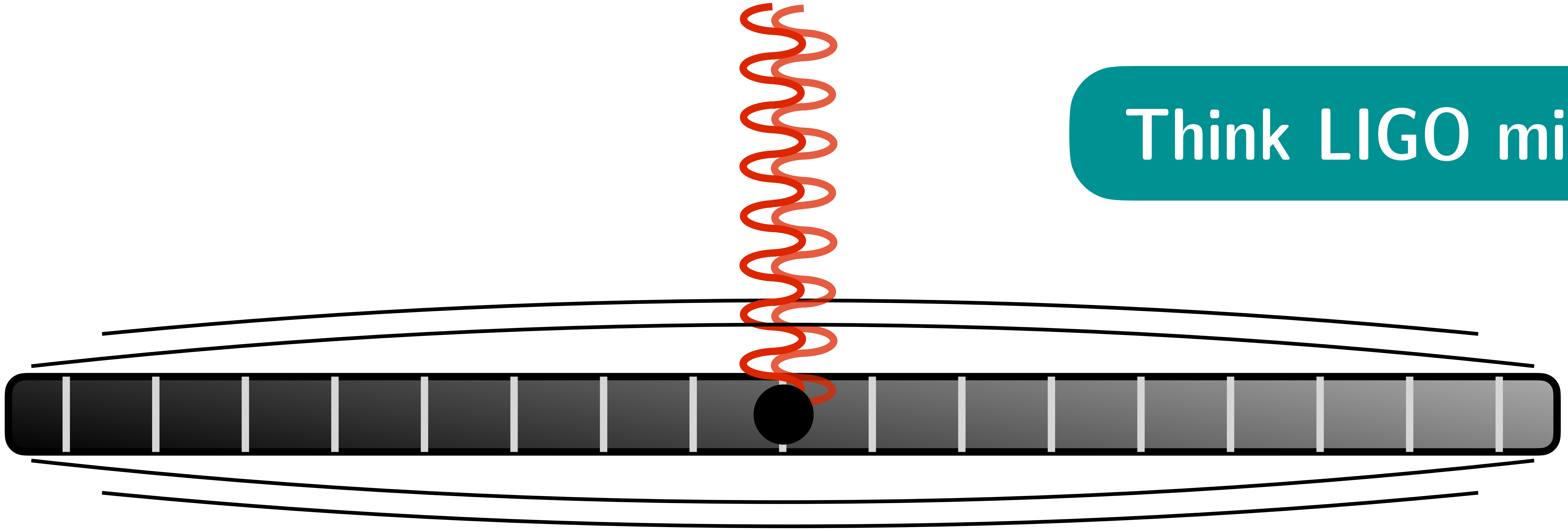
TRANSFER FUNCTIONS



When is the ruler rigid?

When is the ruler rigid?

Incoming GW: long wavelength

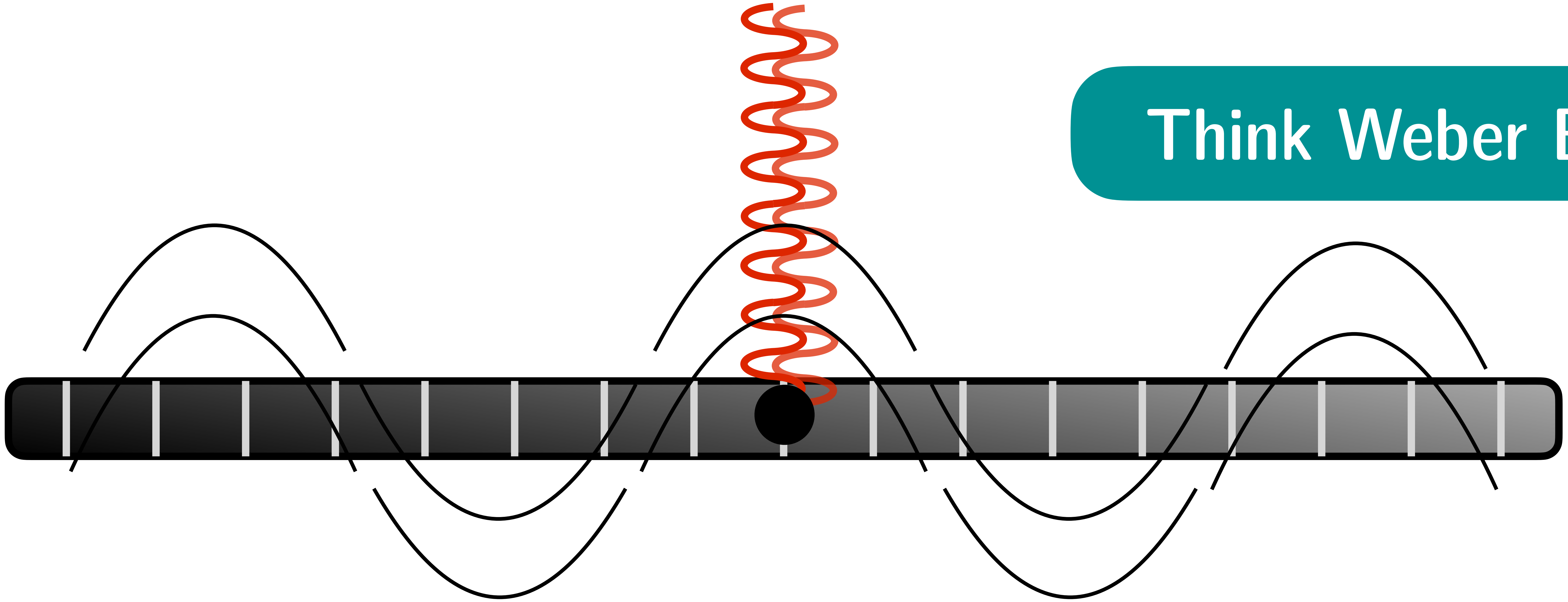


Think LIGO mirror

The whole object moves back and forth

When is the ruler rigid?

Incoming GW: matched wavelength



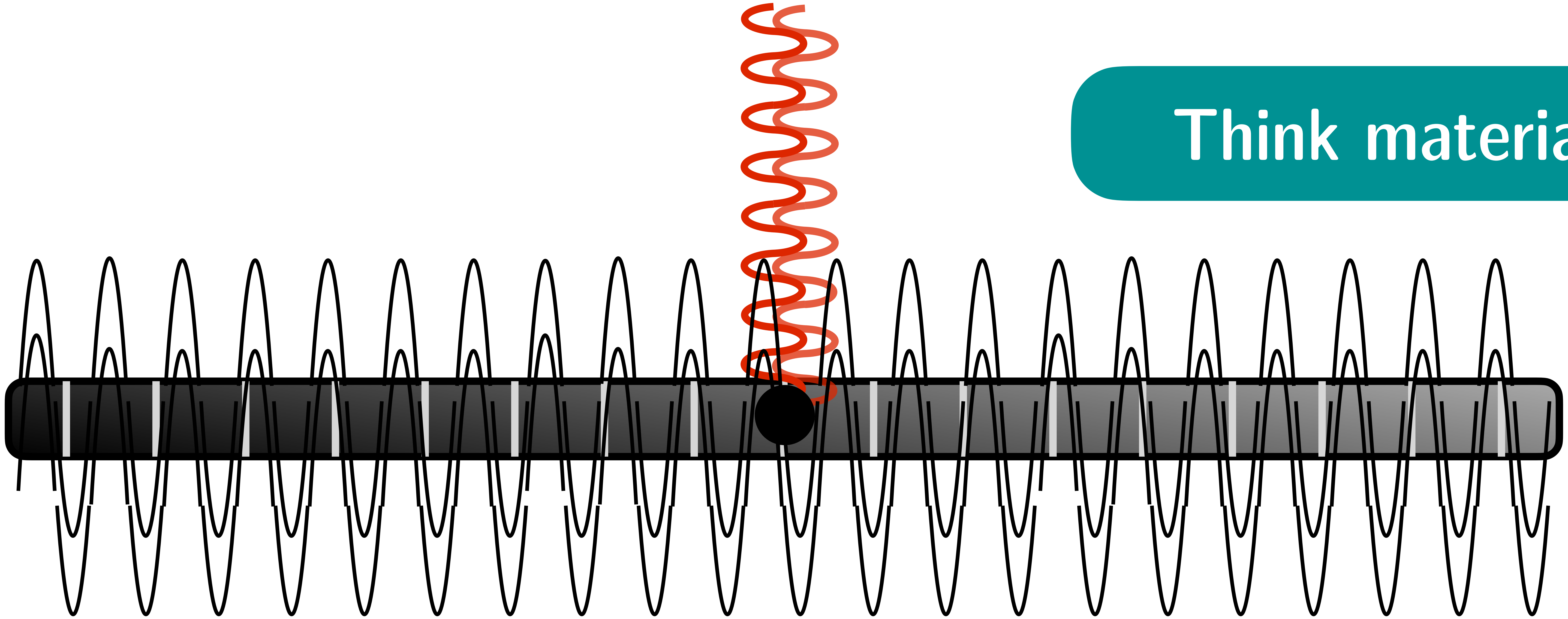
Think Weber Bar

Resolve structure of the ruler, e.g. resonances

When is the ruler rigid?

Incoming GW: short wavelength

Think materials



Ruler fully flexible

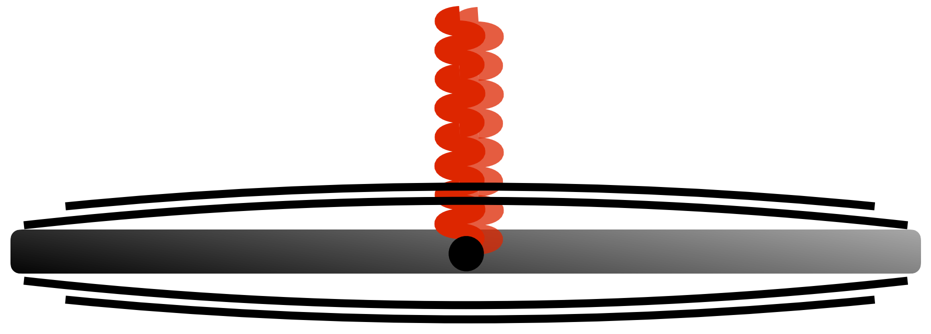
When is the ruler rigid?

At the level of equations:

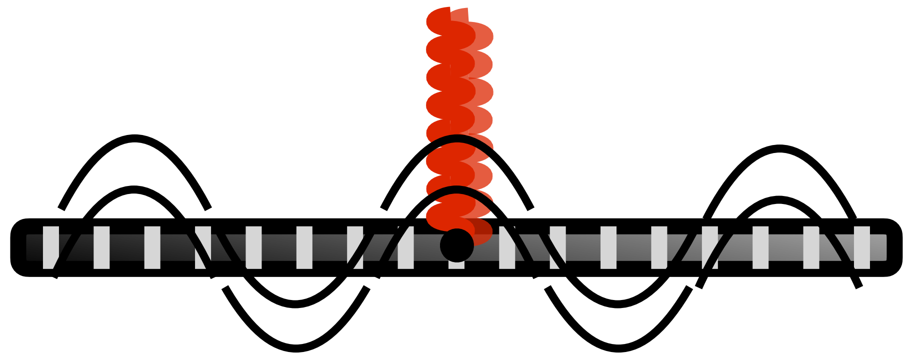
$$\partial_t^2 s_i + k^2 s_i \sim \partial_t^2 h_{ij} s_j$$

NB: don't include for LIGO mirror

$$s_i \sim \frac{\omega_g^2}{k^2} h_{ij} s_j$$

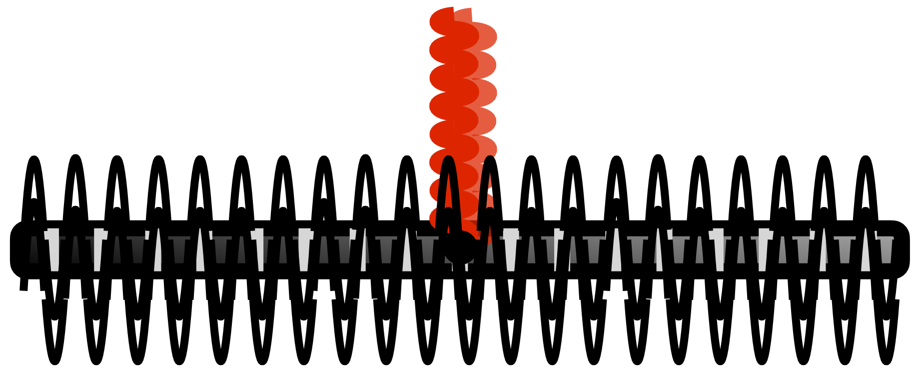


Looks like proper detector frame k∼1/L



Case requires knowledge of detector

$$s_i \sim h_{ij} s_j$$



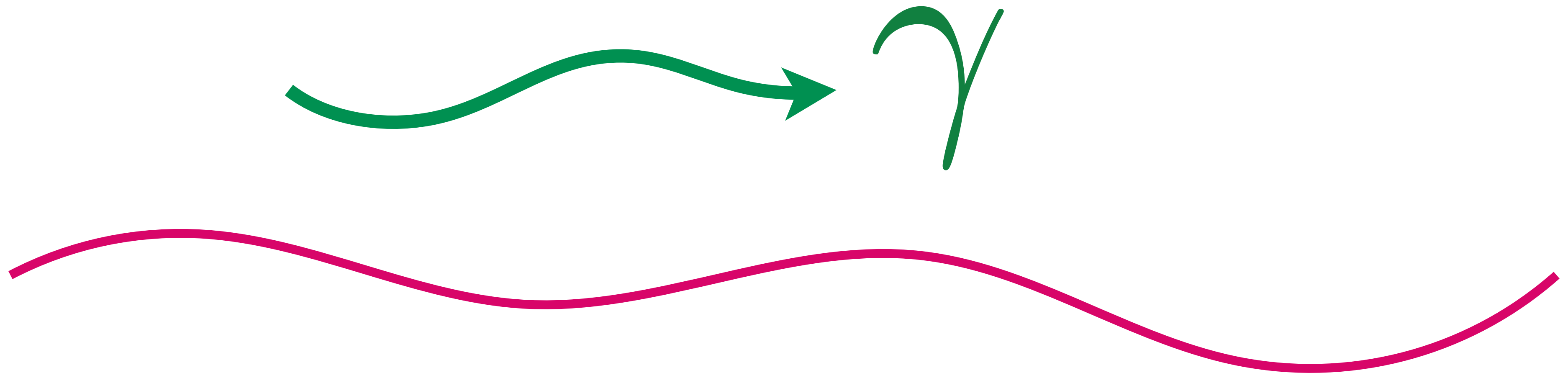
Looks like TT frame

Frame subtleties crucial!

AXION EXPERIMENTS

What are the prospects?

Interactions of Gravitational Waves *with light*



$$S_{\rm EM} = \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + g^{\mu\nu} J_\mu A_\nu \right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \quad \longrightarrow \quad \mathcal{L} \supset \mathcal{O}(hF^2)$$

Equation of motion: $\partial F \sim -\partial \, (h \, F)$

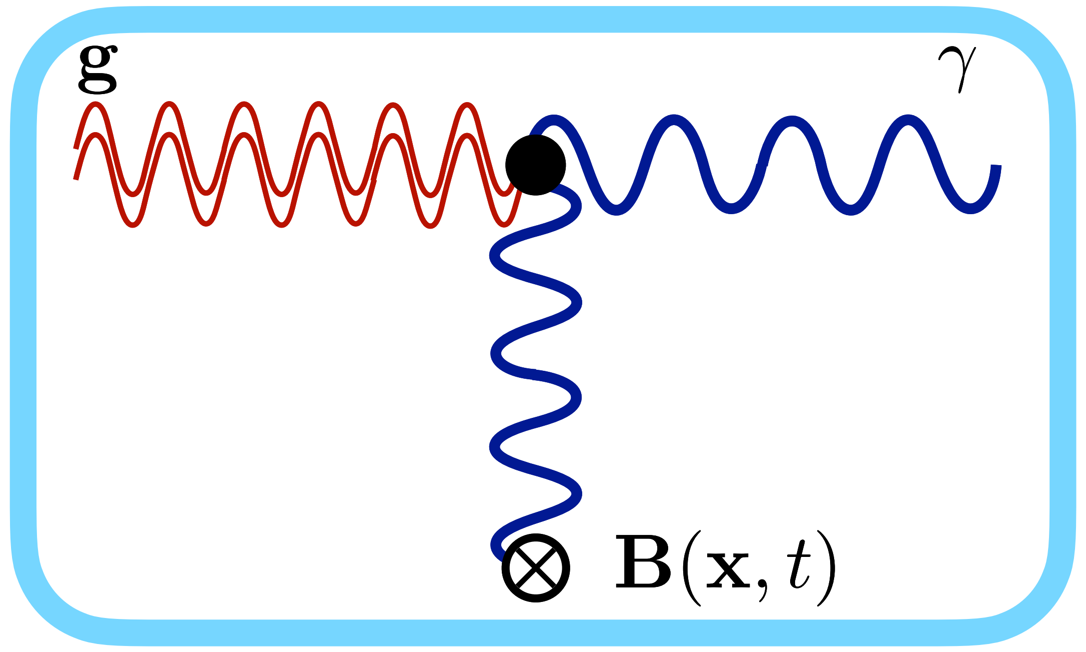
Effective current from spatial or temporal variations of h or F

$$j^\mu_{\rm eff} \equiv \partial_\nu \left(\frac{1}{2} h \, F^{\mu\nu} + h^\nu{}_\alpha \, F^{\alpha\mu} - h^\mu{}_\alpha \, F^{\alpha\nu} \right)$$

Developing Axion $\leftrightarrow$ GW Intuition

Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\rm eff} \equiv \partial_{\nu}\left(\frac{1}{2}\, h\, F^{\mu\nu} + h^{\nu}{}_{\alpha}\, F^{\alpha\mu} - h^{\mu}{}_{\alpha}\, F^{\alpha\nu}\right)$$

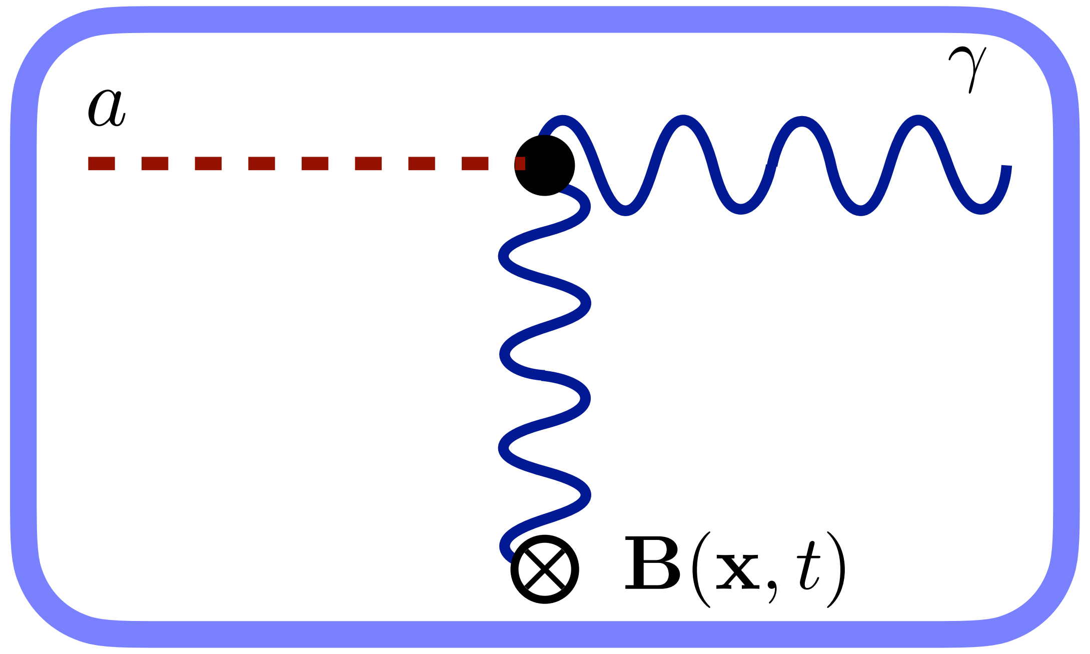


Gertsenshtein effect (1962)

Zeldovich (1973)

$$j_{\rm g} \sim \partial\,(h\,F)$$

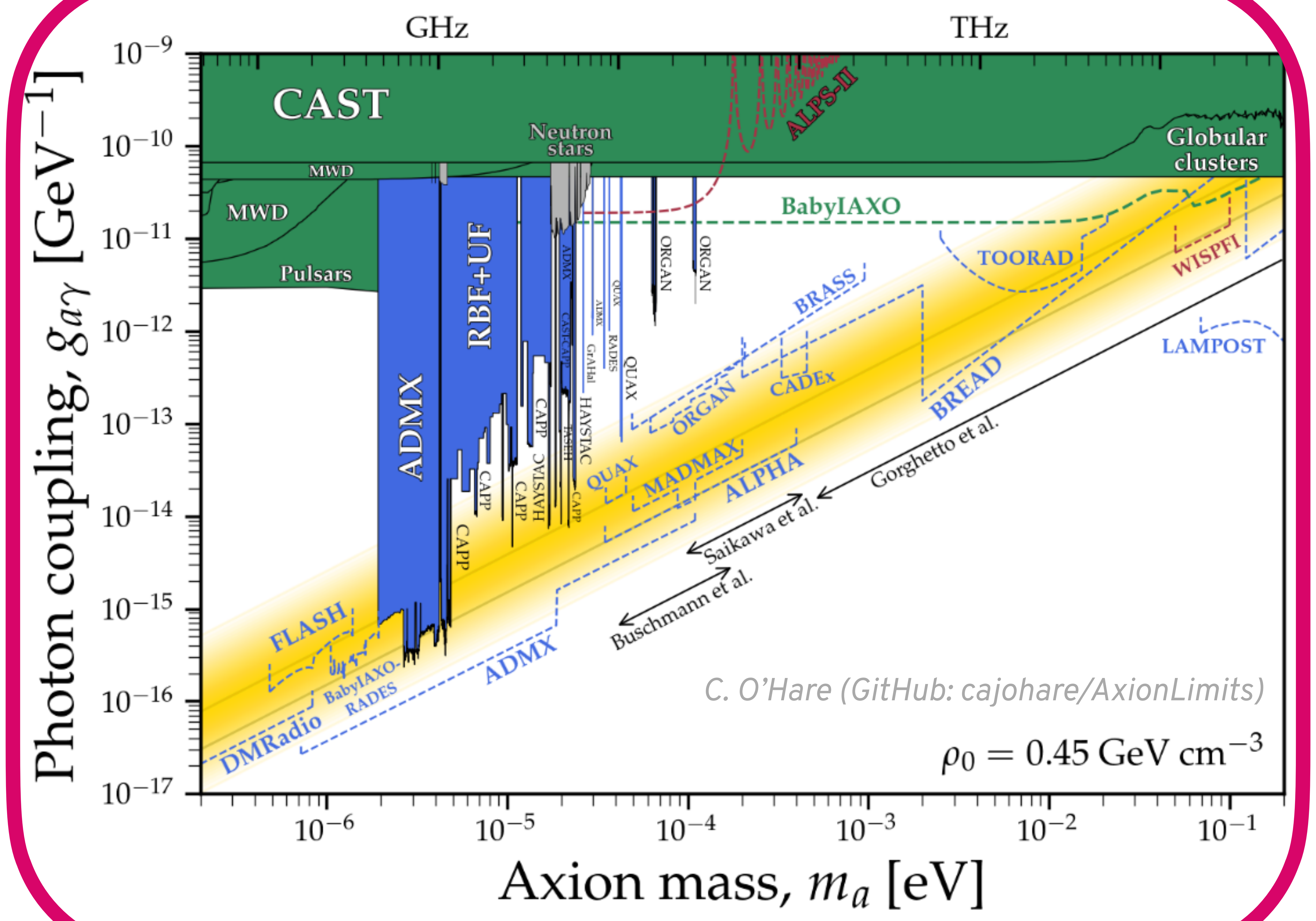
Should be reminiscent of axion physics...



Raffelt & Stodolsky (1988)

$$j_{\rm a} \sim g_{a\gamma\gamma}\partial\,(a\,F)$$

Intuition for EM signal



Estimate sensitivity to GWs by comparing sizes of currents

$$j_{\rm eff}^{\rm axion} \sim g_{a\gamma\gamma}\partial_t(a\mathbf{B}) + \mathcal{O}(v)$$

$$j_{\rm eff}^{\rm axion} \lesssim 10^{-19}\,{\rm T/m}$$

↓

$$j_{\rm eff}^{\rm GW} \sim \partial_t(h\mathbf{B}) + \ldots$$

$$h \lesssim 10^{-21}$$

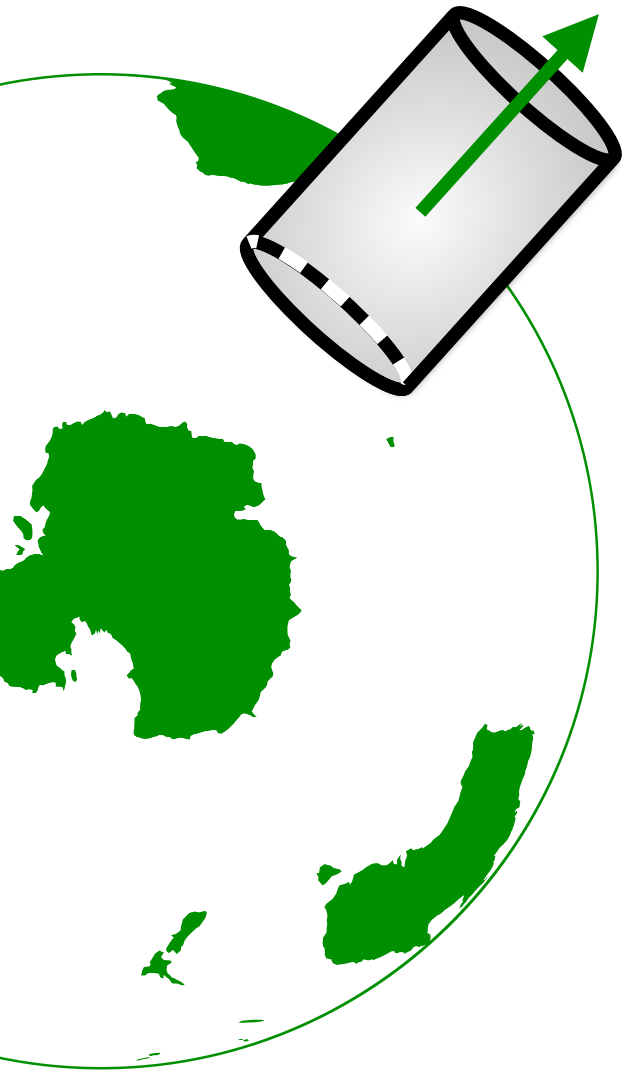
NDA risky...

Framing the Question

GW as sum of plane waves

$$h \propto e^{i\omega_g(t-z)} \longrightarrow \partial_i h_{jk}^{\mathrm{TT}} \sim -\delta_{iz}\partial_t h_{jk}^{\mathrm{TT}}$$

$$x^{k_1}\dots x^{k_r} R_{\mu\nu\rho\sigma,k_1\dots k_r} = (-i\omega_g z)^r R_{\mu\nu\rho\sigma}$$



$$h_{00} = -2R_{0m0n}x^m x^n\left(-\frac{i}{\omega_g z} + \frac{1+e^{-i\omega_g z}}{(\omega_g z)^2}\right)$$

$$h_{0i} = -2R_{0min}x^m x^n\left(-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

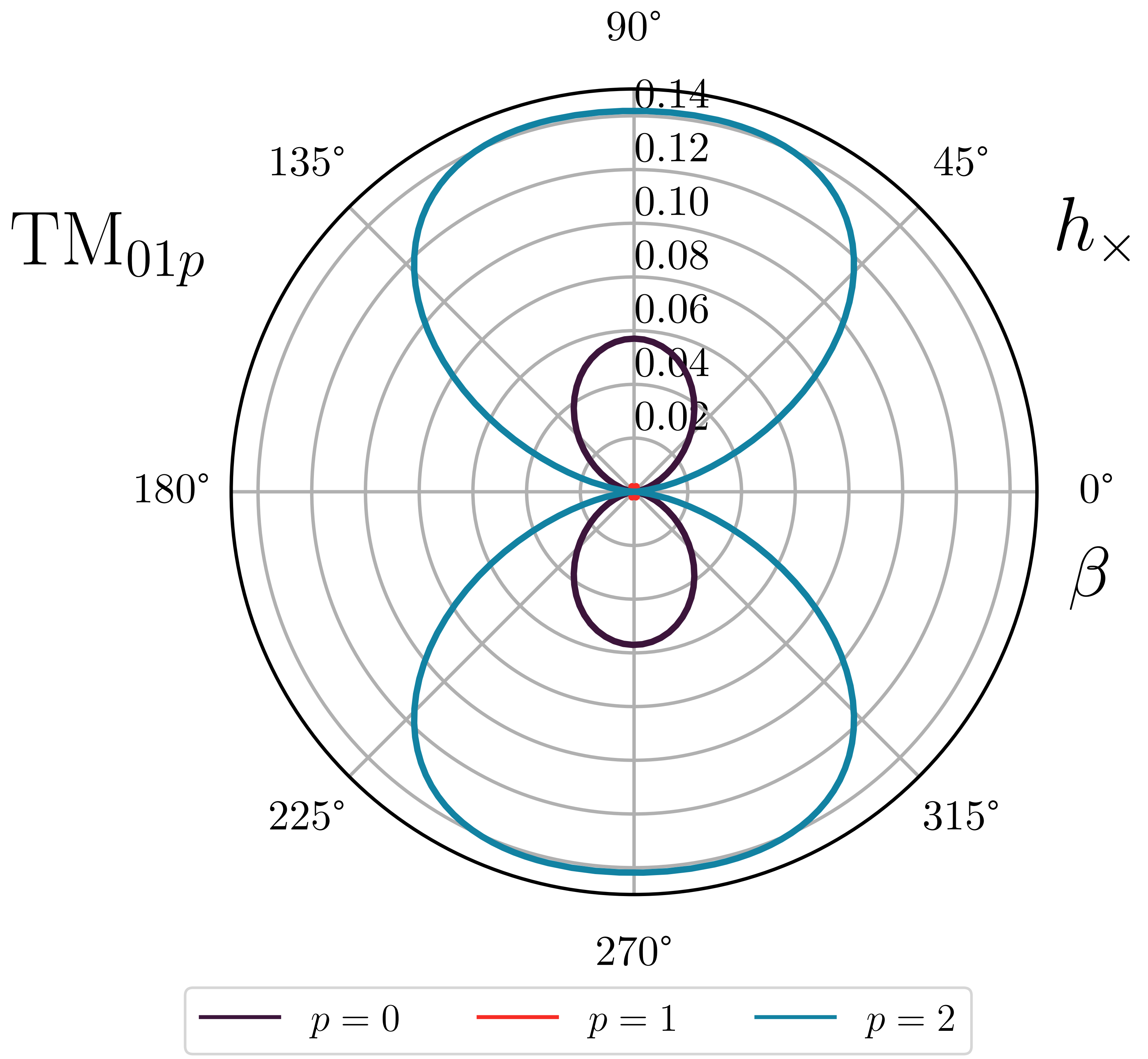
$$h_{ij} = -2R_{imjn}x^m x^n\left(-\frac{1+e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Märzlin (1994)
Rakhmanov (2014)

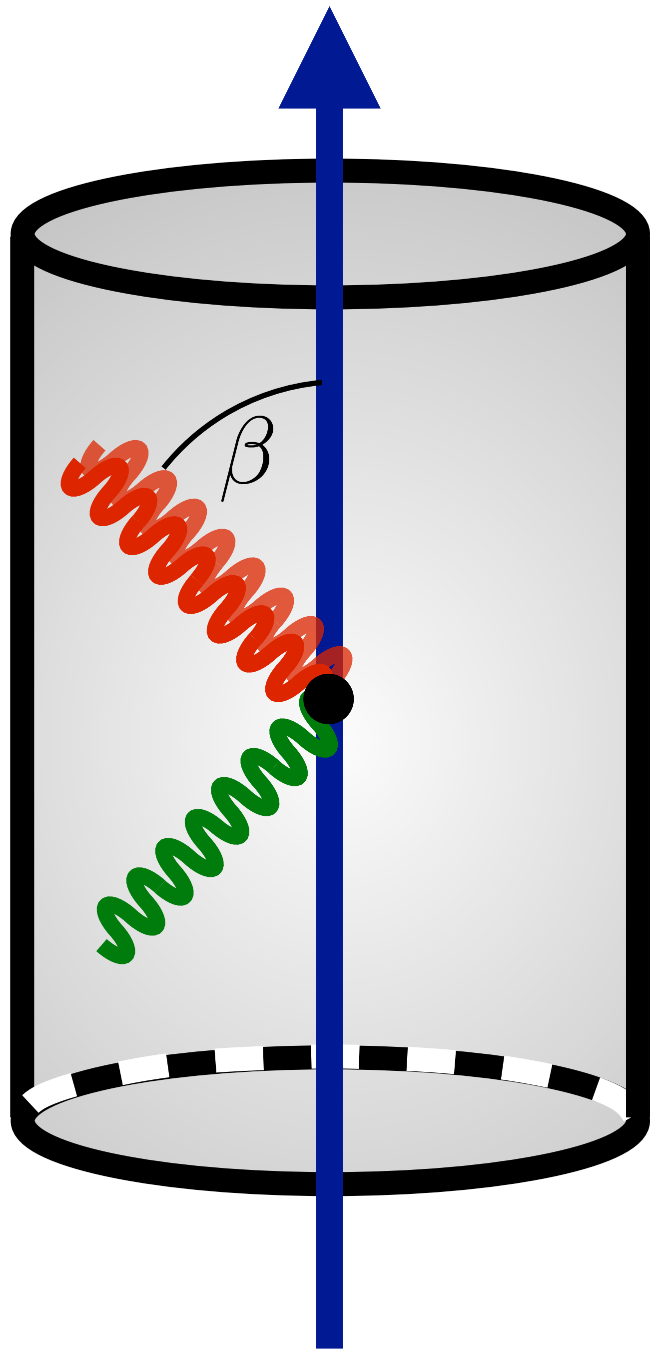
Axion Cavity Modes Couple to GWs

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav} \cdot \mathbf{J}_{\rm eff}$$

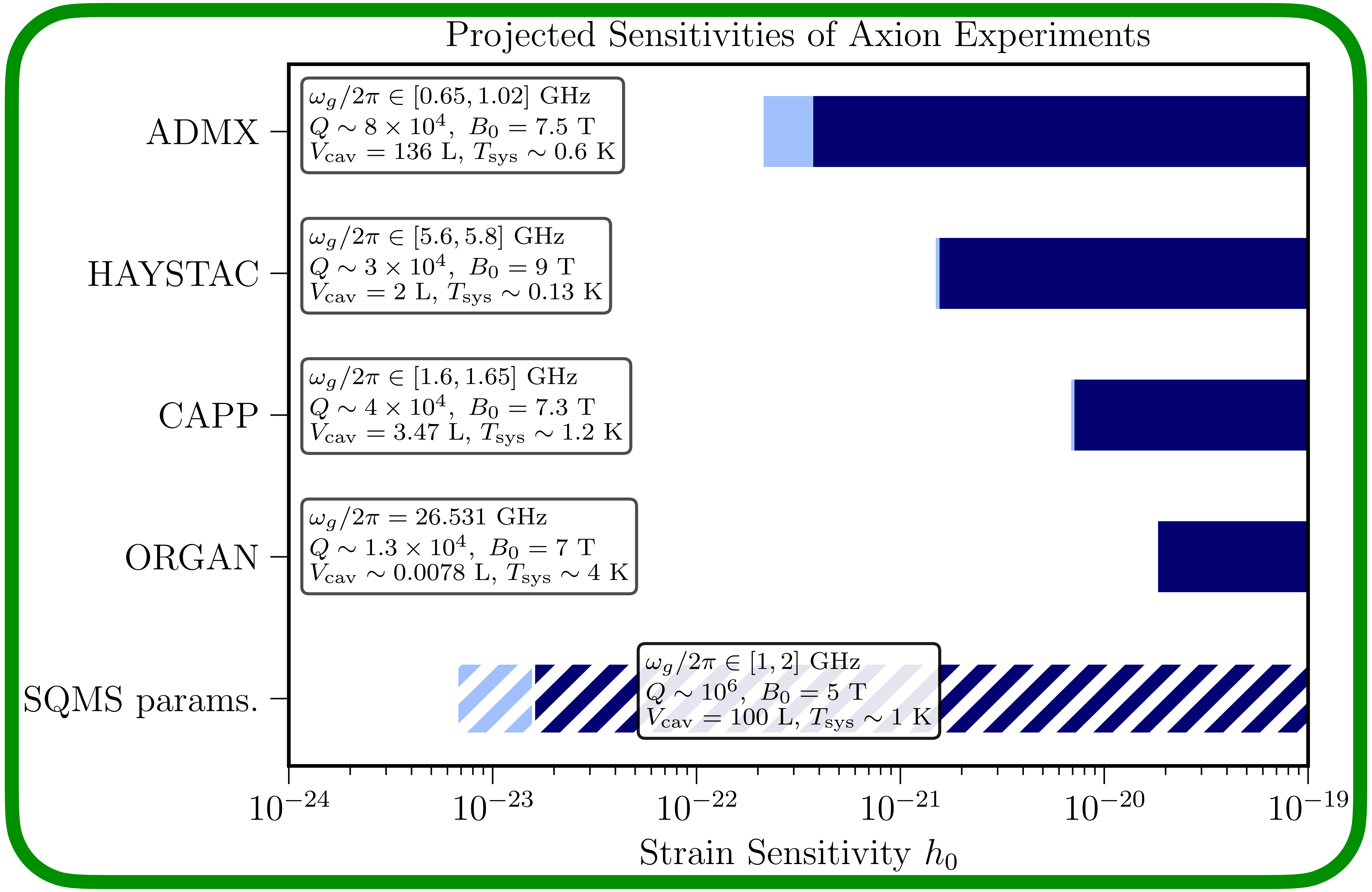


Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

But TM modes not optimal...



Axion Cavity Sensitivity



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

$$P_{\rm sig} = \frac{1}{2} Q\,\omega_g^3\,V_{\rm cav}^{5/3}\,(\eta_n\,h_0\,B_0)^2$$

c.f. axion power:

$$P_{\rm sig}^a \sim Q\,\omega\,V\,(\eta_a g_{a\gamma\gamma} a B_0)^2$$

$$\mathcal{T} \sim Q\eta_0(\omega_g V_{\rm cav}^{1/3}) \sim 10^5$$

Why did our NDA fail?

Axion conversion in a b.g. magnetic field:

$$\boldsymbol{E}_a \sim -g_{a\gamma\gamma} a\, \boldsymbol{B_0} e^{-i\omega t}$$

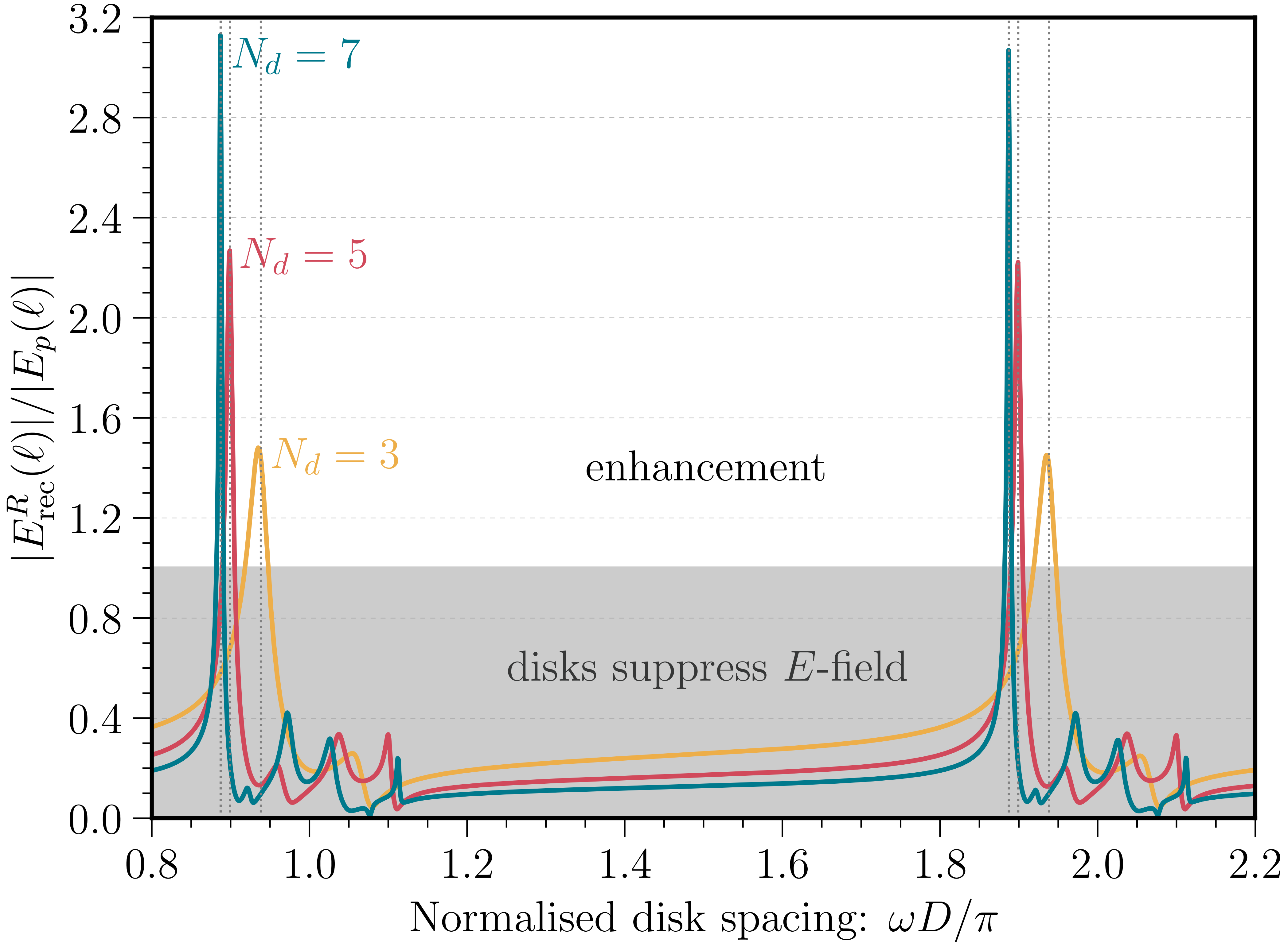
GW conversion in a b.g. magnetic field (in TT gauge):

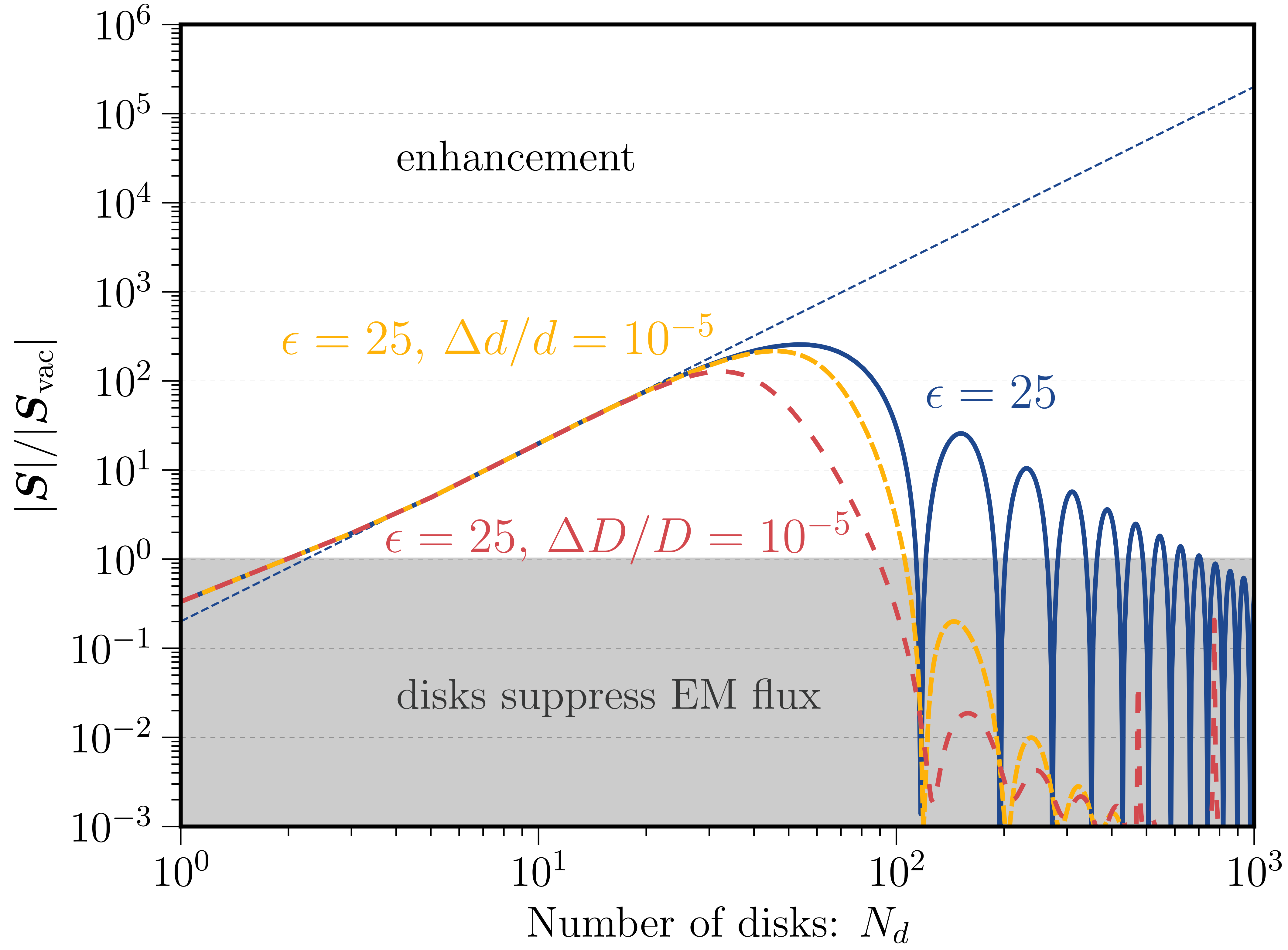
$$\boldsymbol{E}_v^p = -\frac{B_0}{2}\left[i\omega x(h_\times \hat{\boldsymbol{p}} + h_+ \hat{\boldsymbol{s}}) + h_\times s_\theta \hat{\boldsymbol{k}}\right] e^{-i\omega(t-\hat{\boldsymbol{k}}\cdot\boldsymbol{x})}$$

Domcke, SARE, Kopp (2024)

Consequence of mass degeneracy of photon and GW in vacuum

Dielectric Haloscopes





Disks giveth, but disks also taketh away

On resonance, flux density enhanced by ∼200

Domcke, SARE, Kopp (2024)

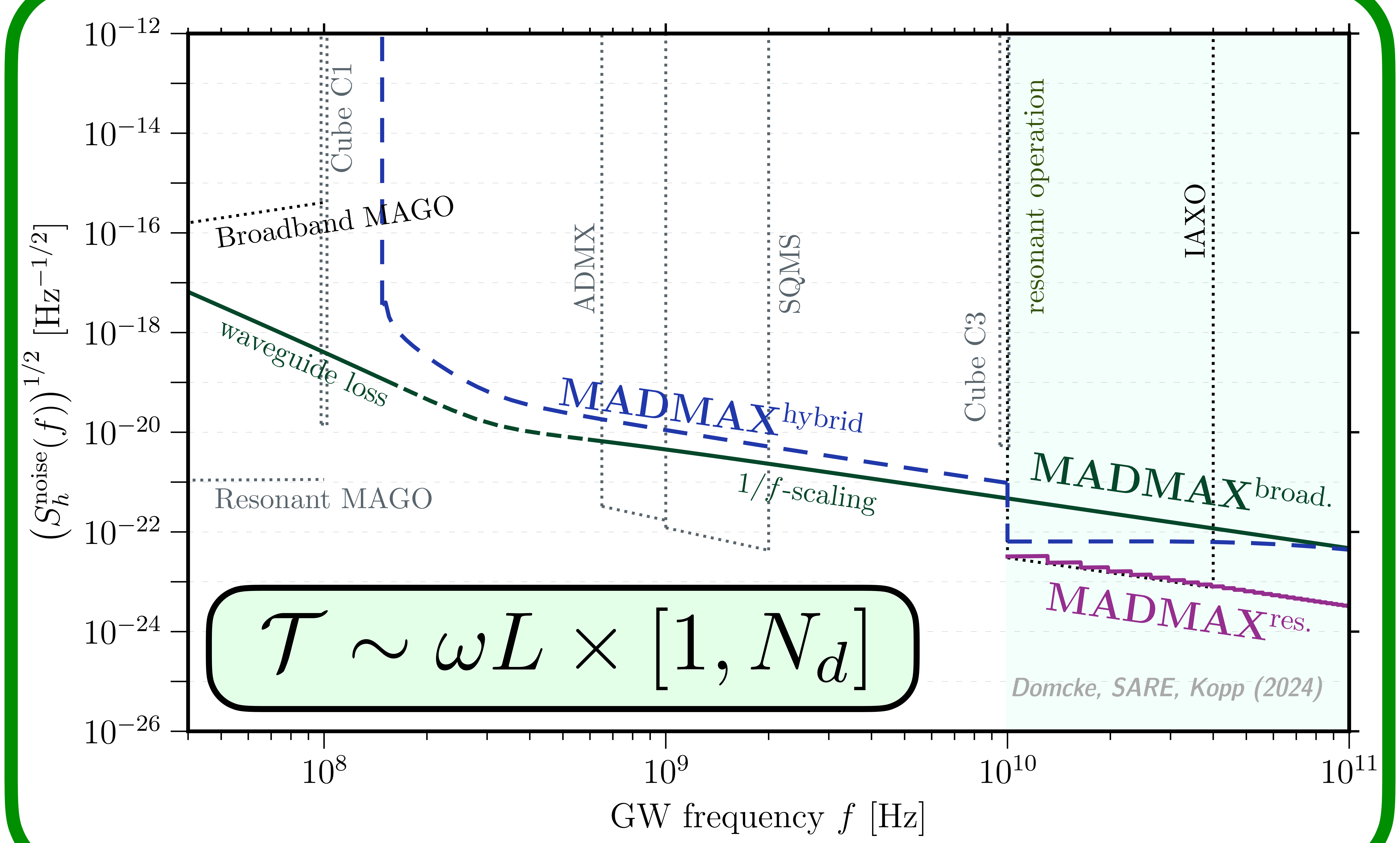
Dielectric* Haloscopes

Fully resonant approach requires scan, but improves sensitivity by ~ 10

Hybrid w/ half disks, half vacuum

Take out disks, fully broadband

Note ωL enhancement from vacuum conversion



No-Disc MADMAX for PBHs

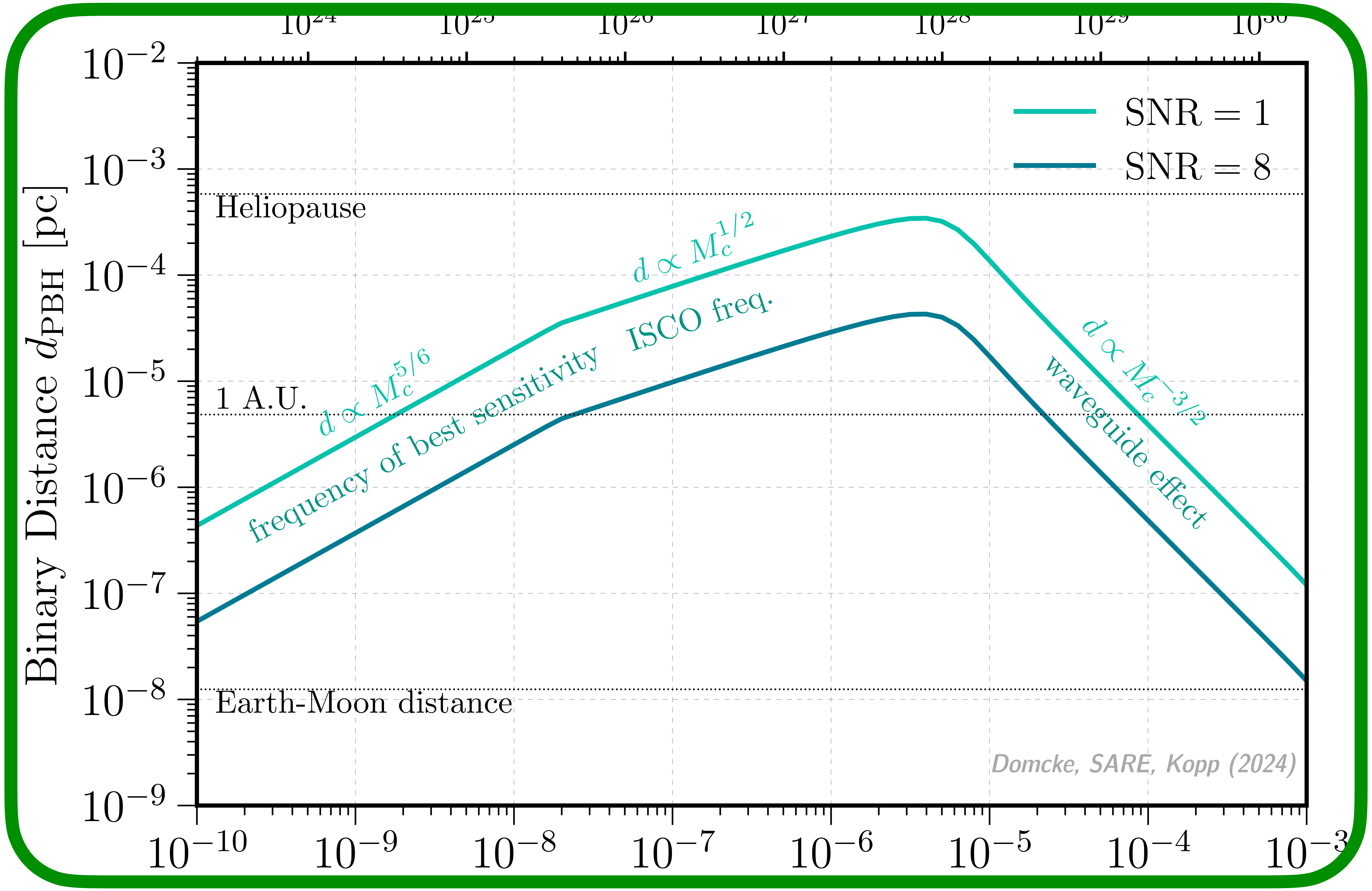
Take out disks, fully broadband

Typical distance to binary

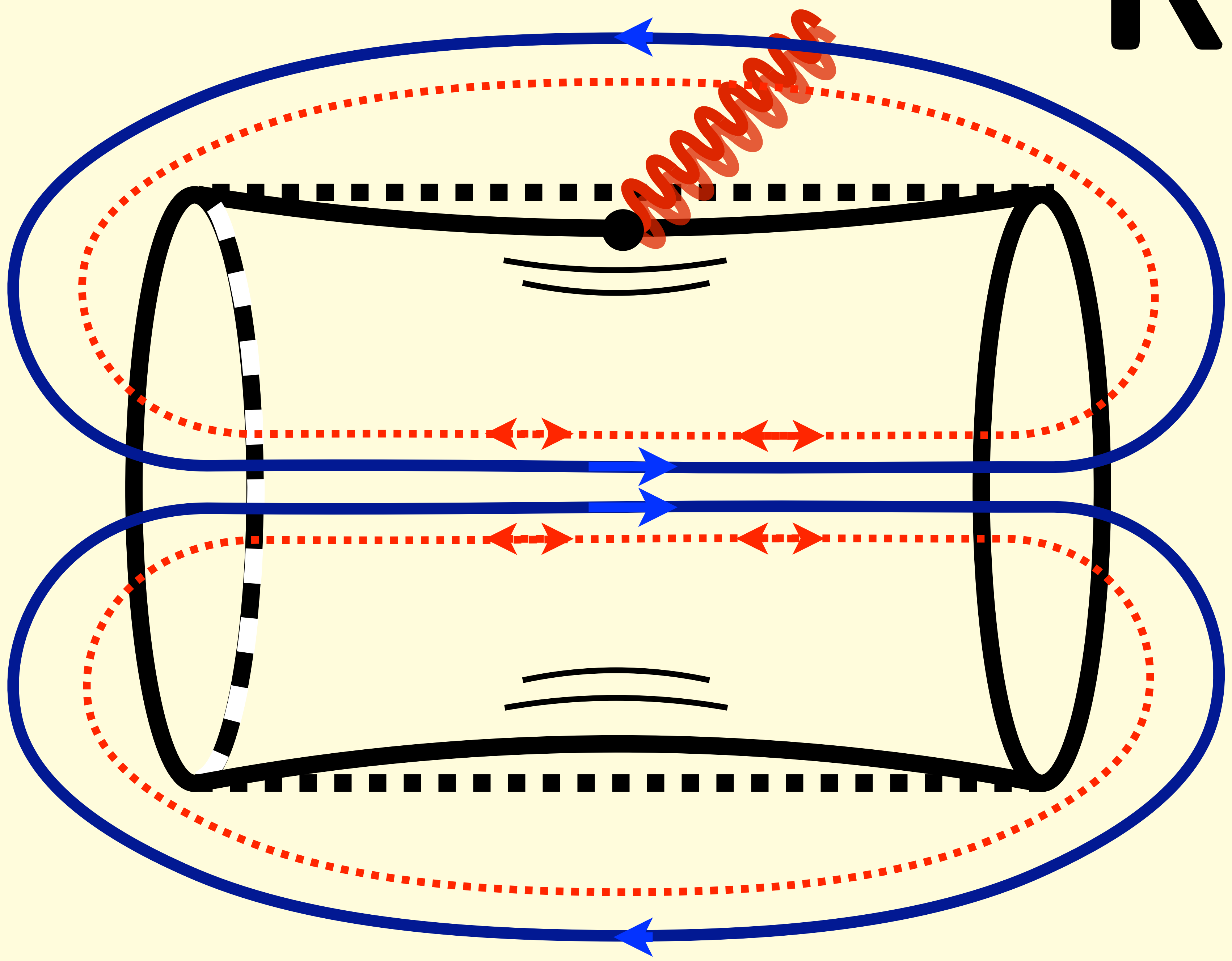
$\sim$ 10 kpc

Franciolini, Maharana, Muia (2022)

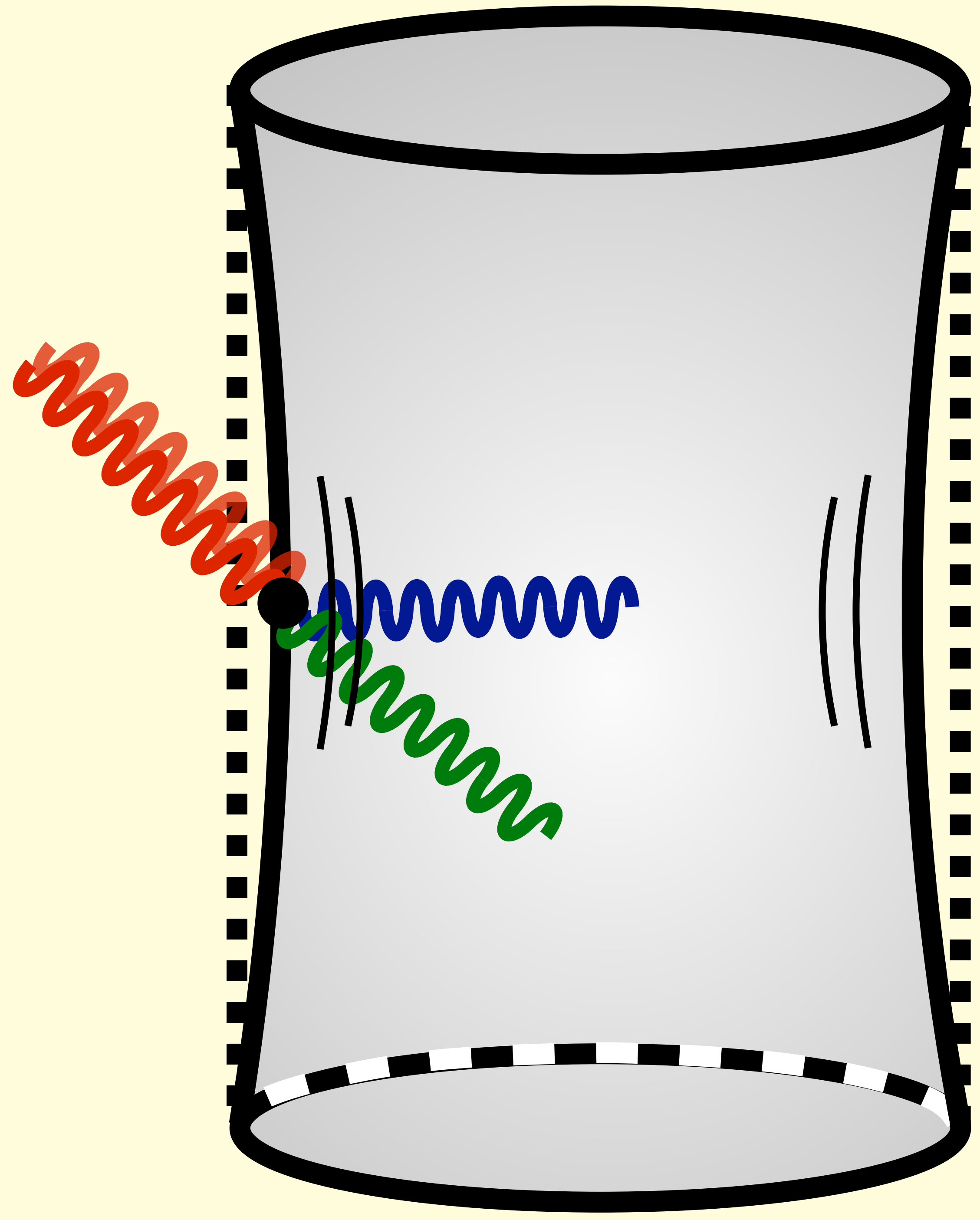
Improves on resonant cavity



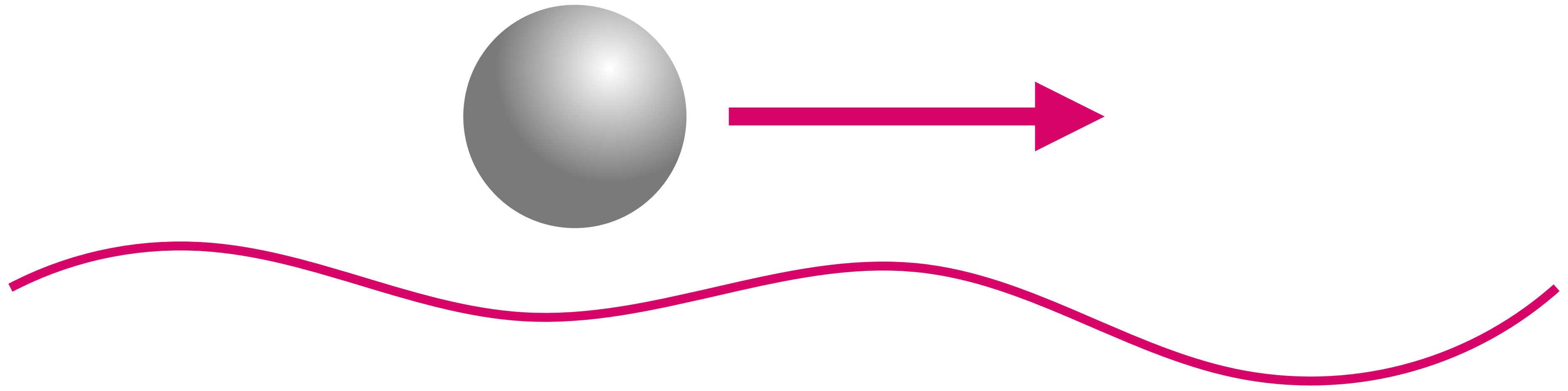
WHAT ABOUT MATTER RULERS?



Unusual Weber Bars



Interactions with Matter

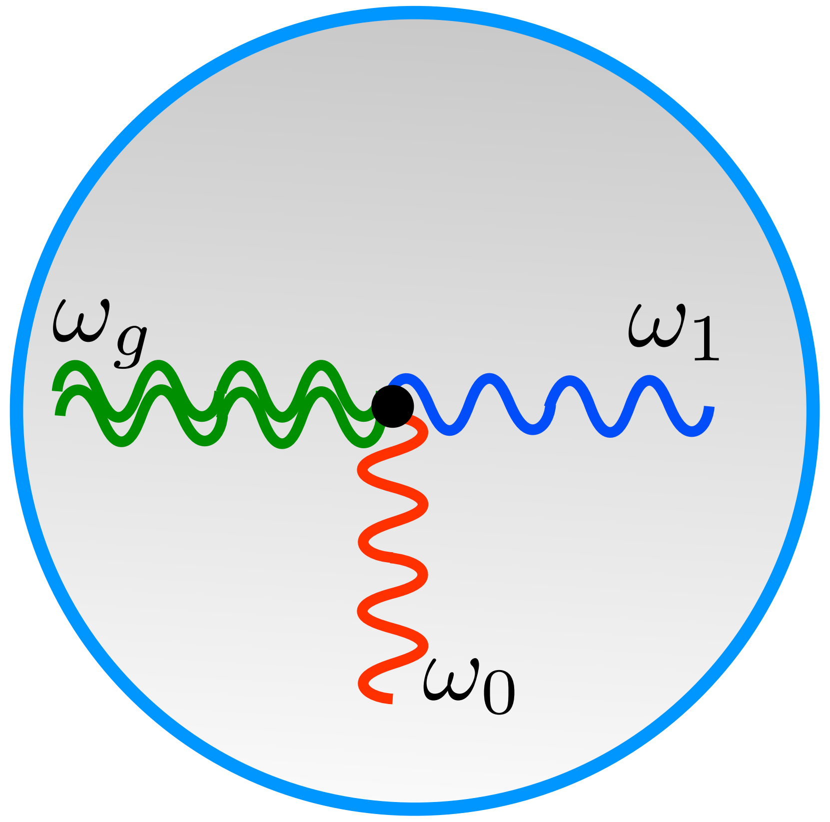


$$S = -\int dt\; m\sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2x_i}{d\tau^2} \simeq -\partial_i \Gamma^j_{00} x^i \longrightarrow \frac{d^2x_i}{d\tau^2} \simeq -\frac{F_i}{m} \longrightarrow F_i \simeq \frac{m}{2}\ddot{h}^{\mathrm{TT}}_{ij} x^i$$

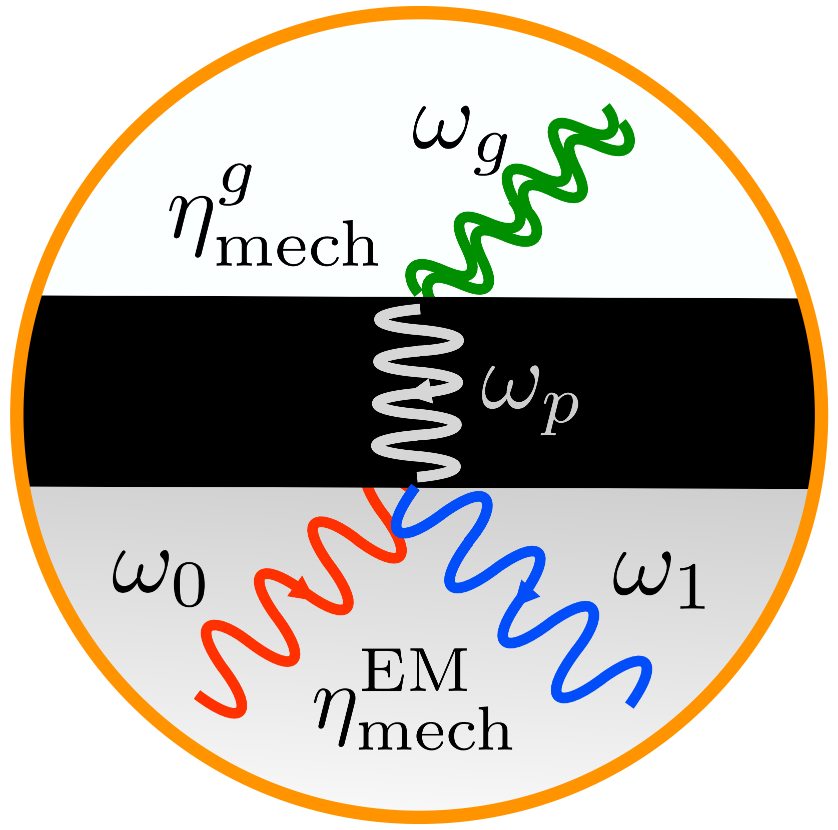
EM and Mechanical signals



Parametrics of the EM signal: $E_{\rm sig}^{\rm (EM)} \sim Q_{\rm em}\,(\omega_g\,L_{\rm cav})^2\,h^{\rm TT}\,E_0$

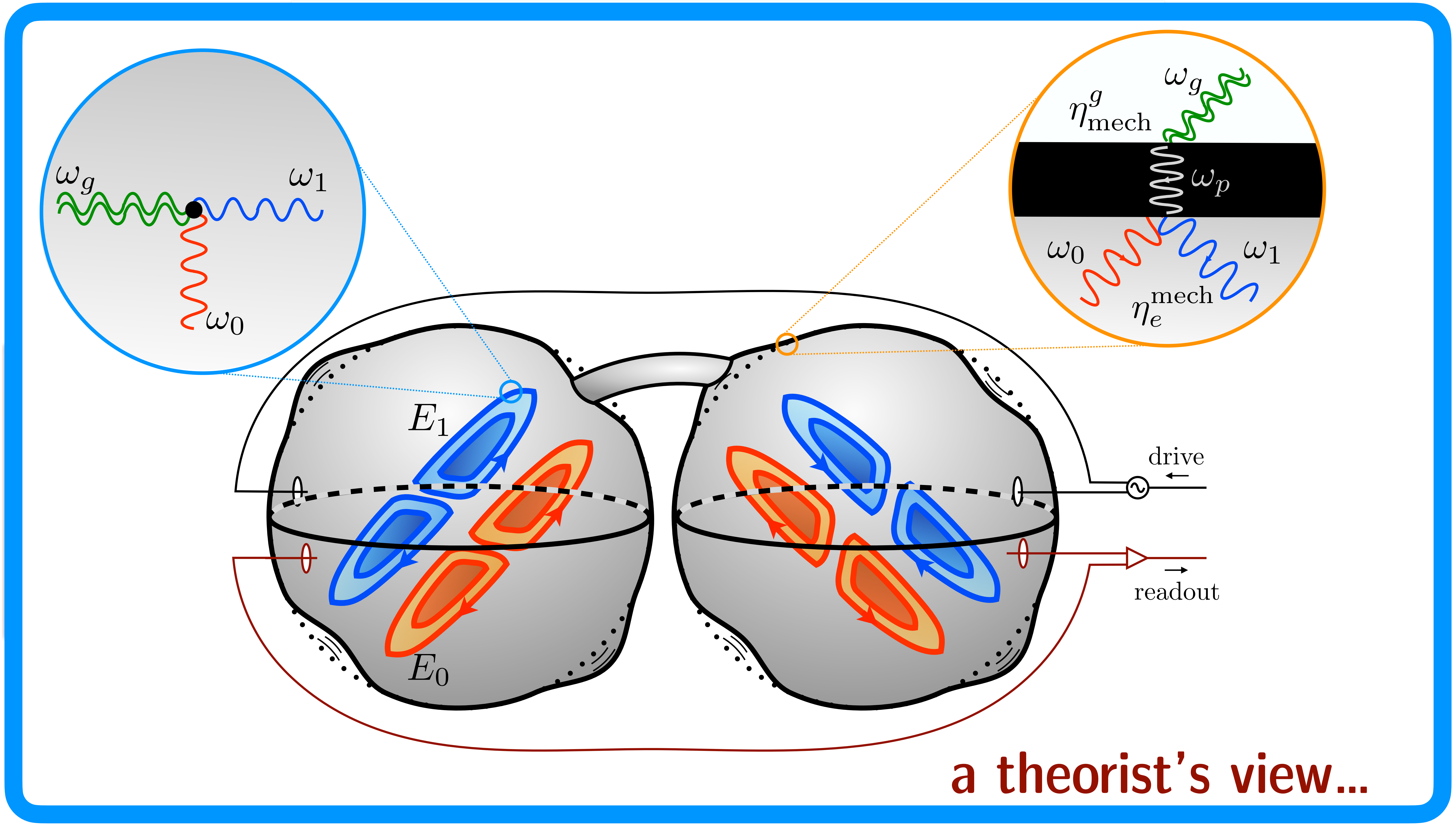
Enhanced by $1/c_s^2 \gg 1$ *(!)*

Mechanical signal: $E_{\rm sig}^{\rm (mech)} \sim Q_{\rm em}\,h^{\rm TT}\,E_0\,\min\left(1\,,\,\frac{\omega_g\,L_{\rm cav}}{c_s}\right)^2$

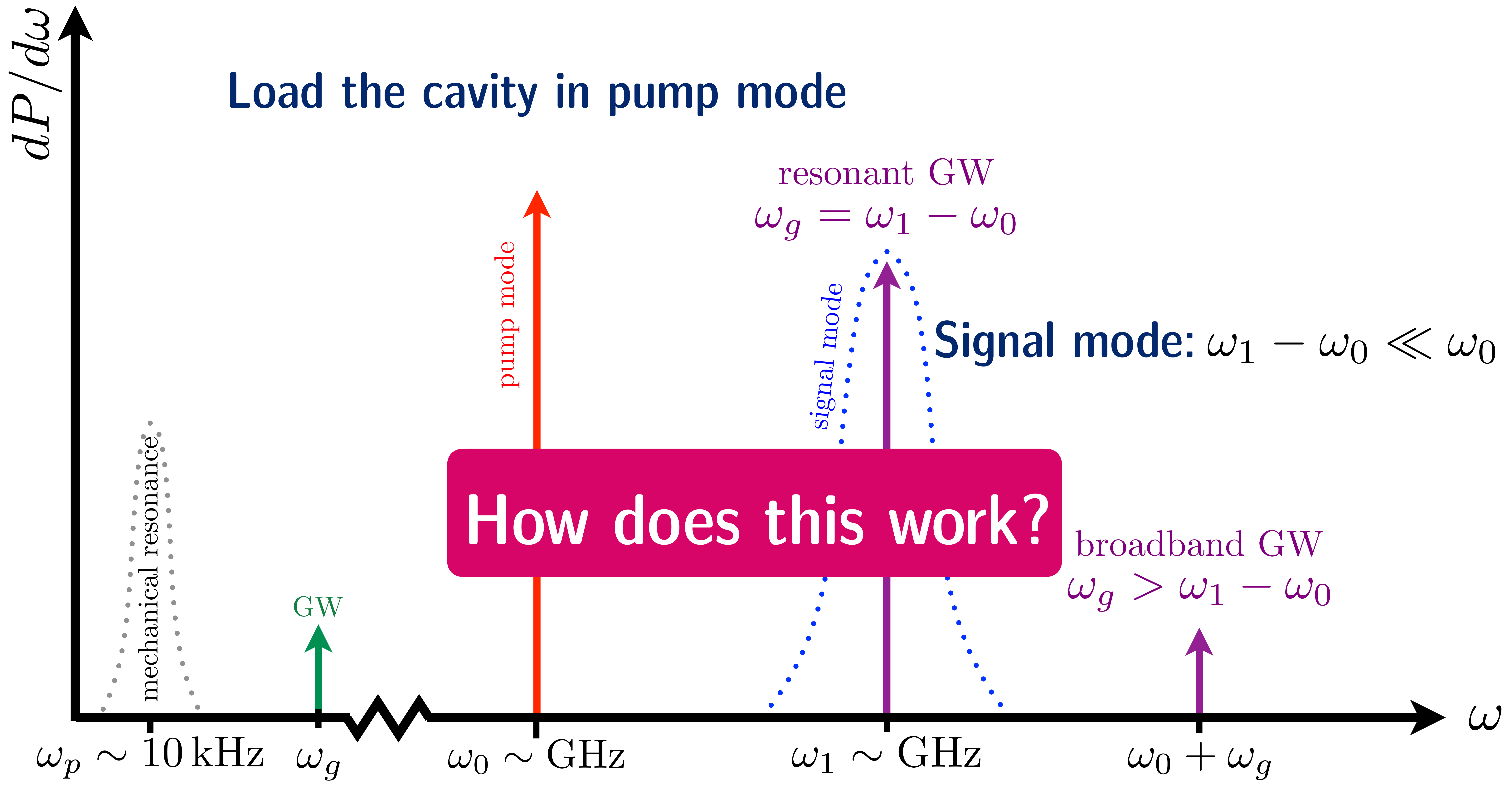


Mechanical modes less "rigid" than EM modes

MAGO 2.0: Mechanical and EM Signals



MAGO 2.0


$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
How does this work?
broadband GW
$\omega_g > \omega_1 - \omega_0$
ω
$\omega_p \sim 10\,\mathrm{kHz}$
ω_g
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$
$\omega_0 + \omega_g$

MAGO 2.0

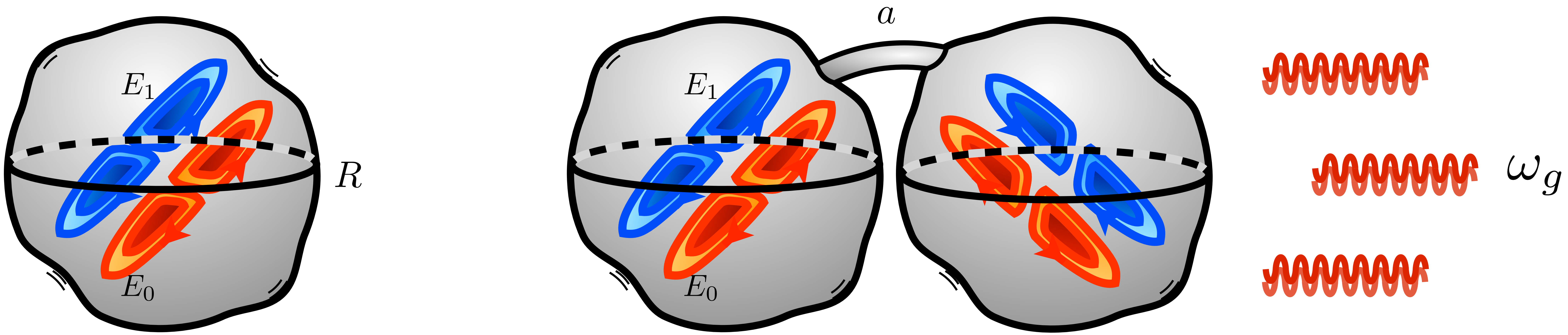
$\omega_a \sim \mathrm{GHz} + \Delta\omega$

$\omega \sim \mathrm{GHz}$

GW

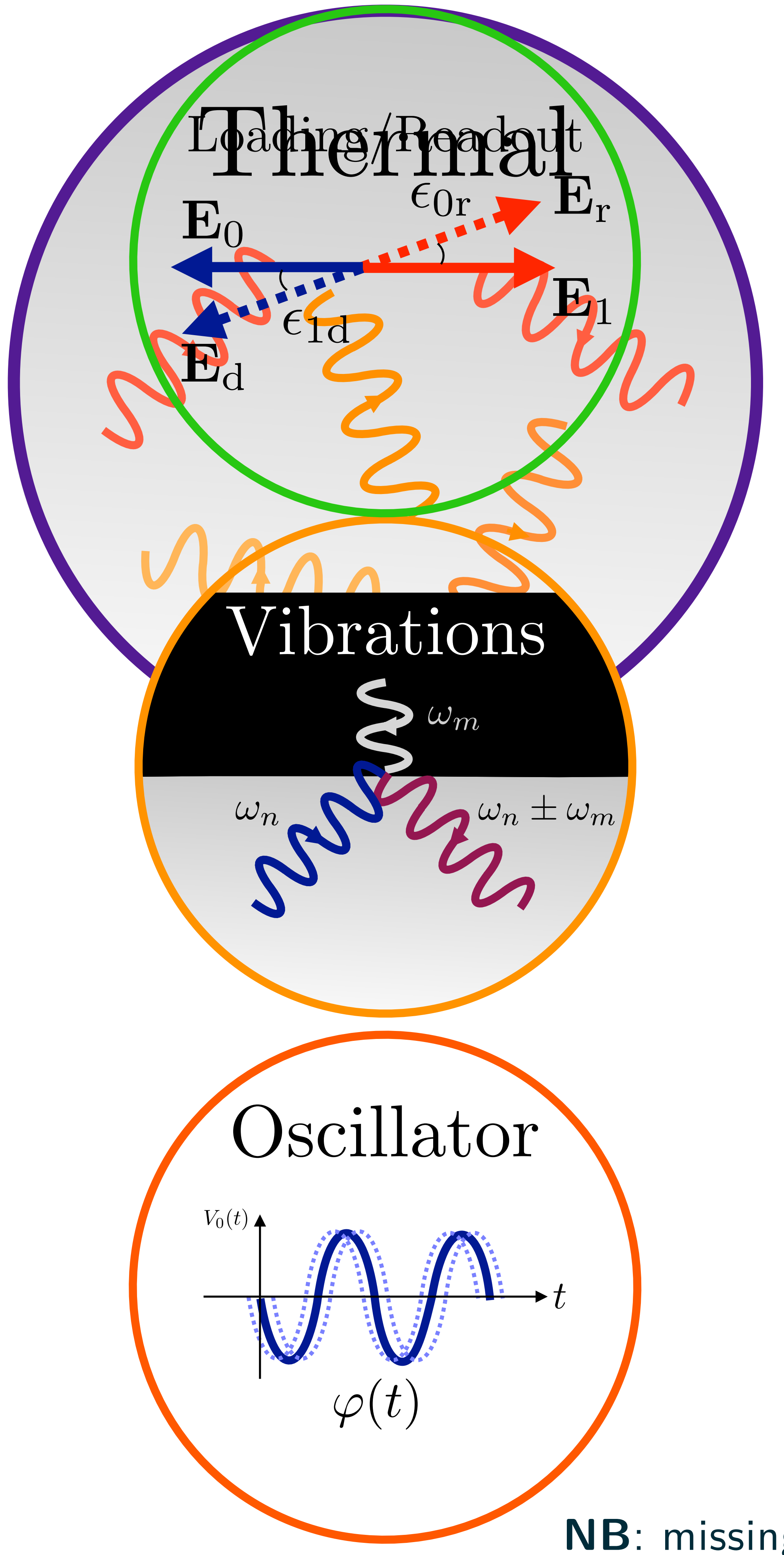
$\omega_s \sim \mathrm{GHz} - \Delta\omega$

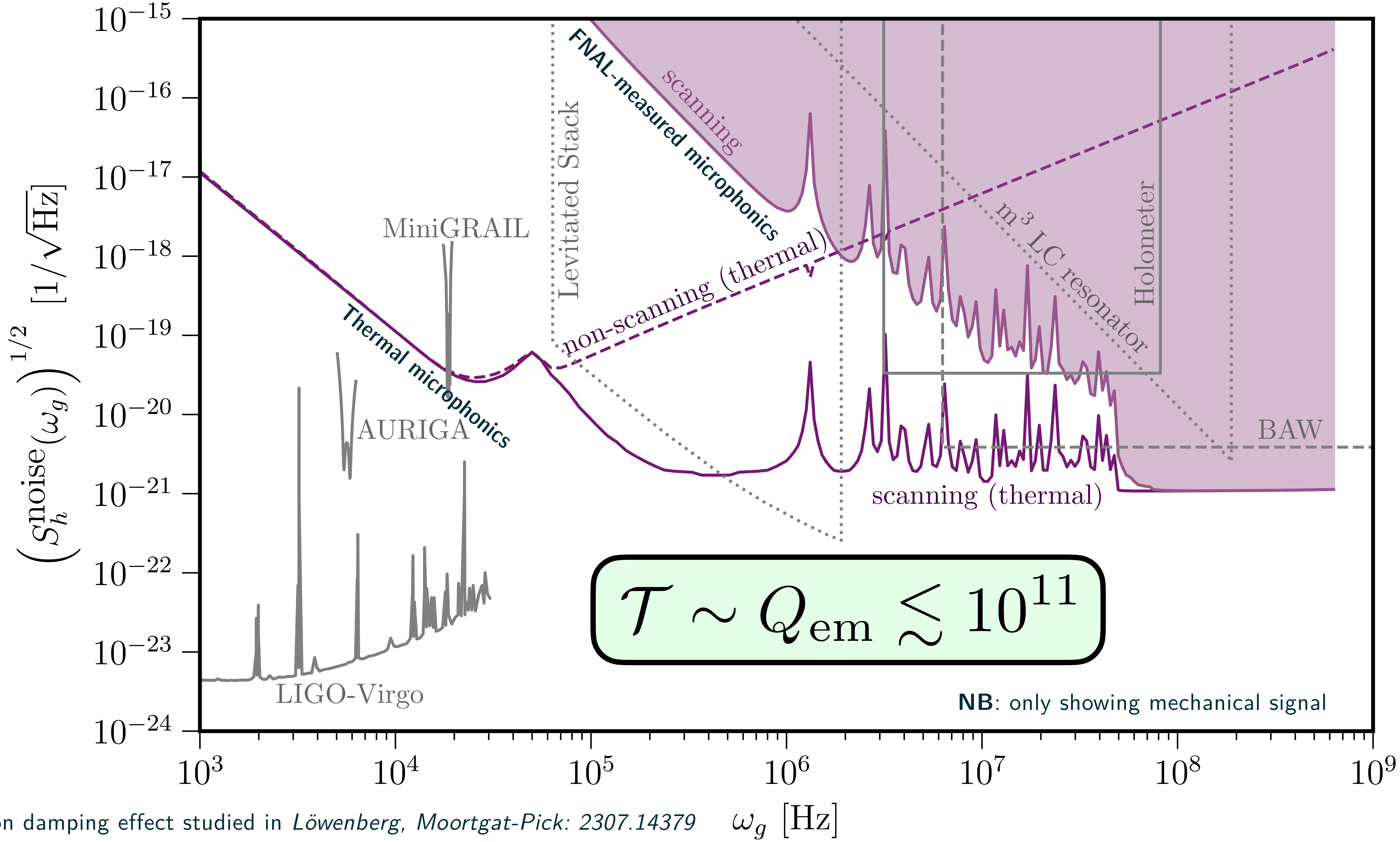
$\Delta\omega \sim (a/R)^3$



Noise in MAGO 2.0

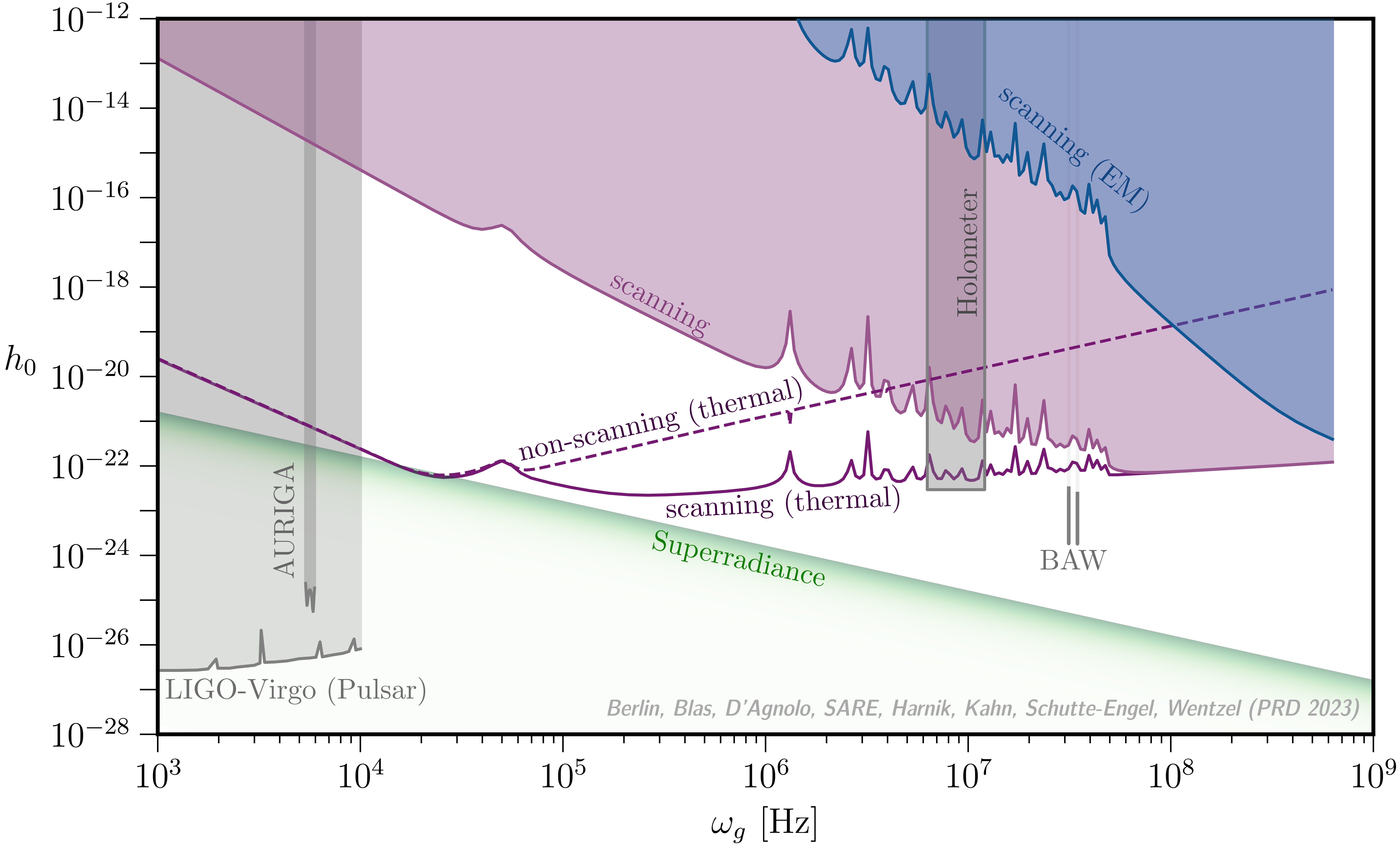
Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel, Wentzel (PRD 2023)





NB: missing radiation damping effect studied in *Löwenberg, Moortgat-Pick: 2307.14379*

MAGO 2.0 sensitivity to coherent GWs



Magnetic Weber Bar

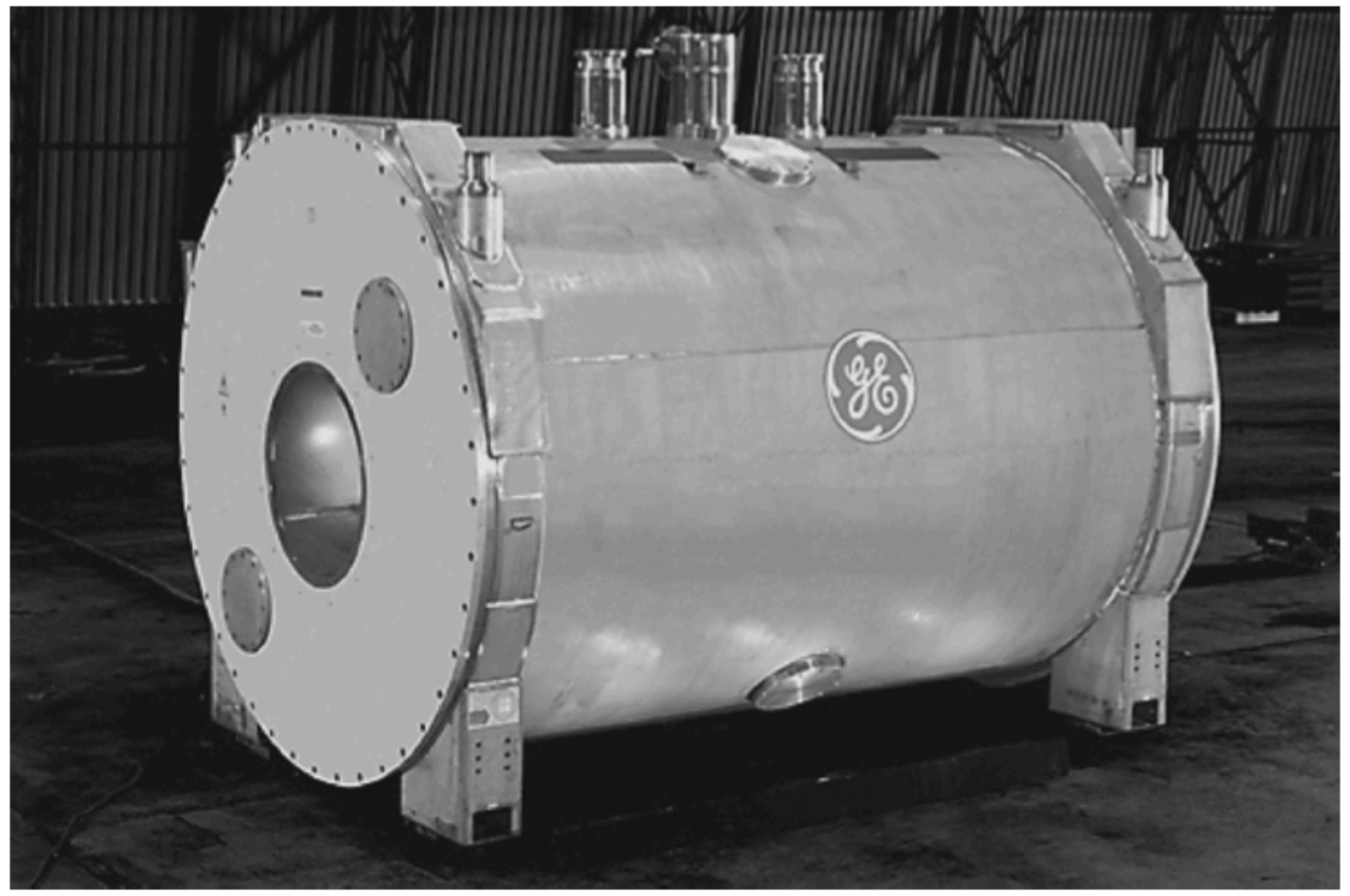


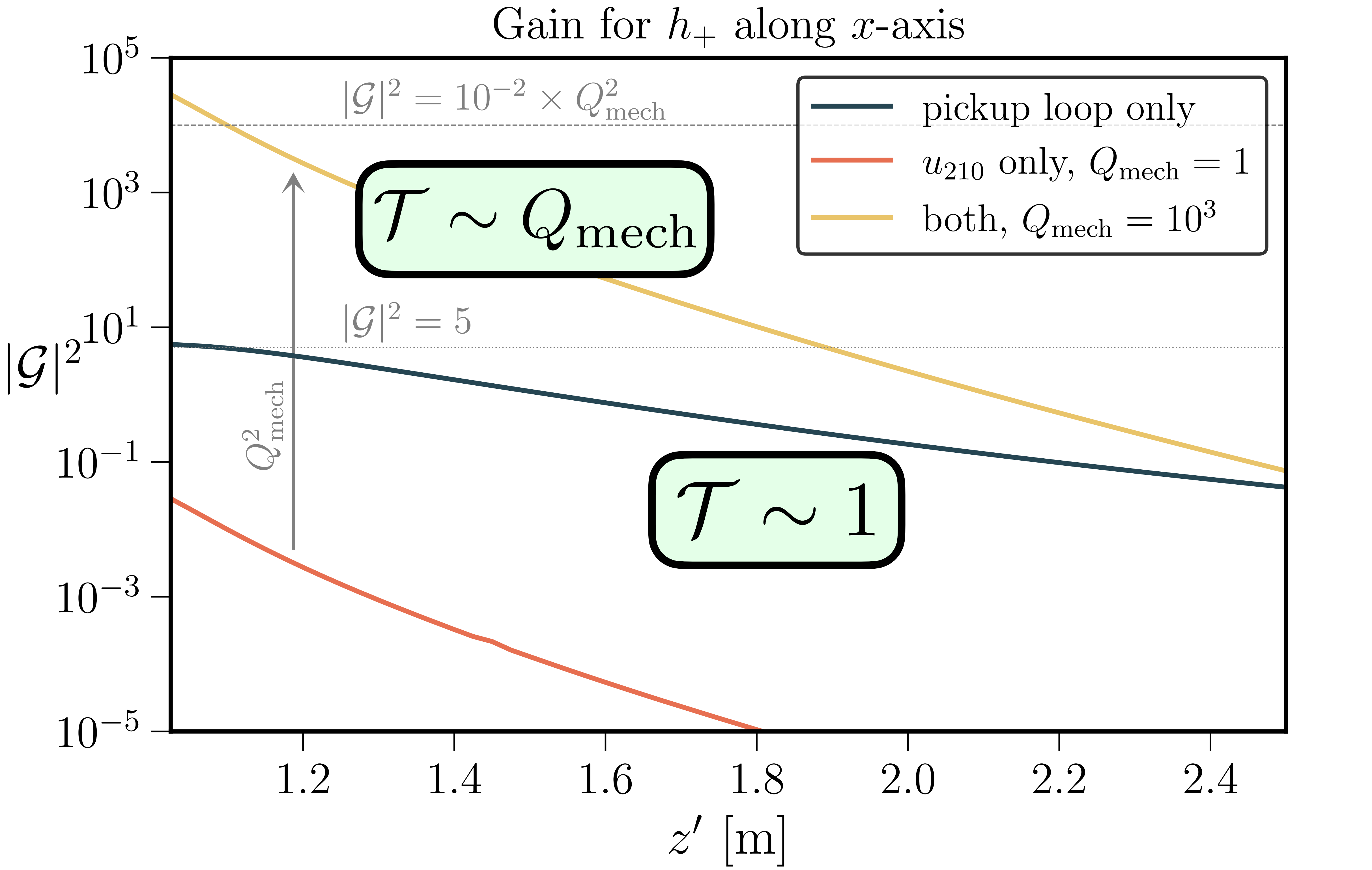
Fig. 10. GE 9.4 T MRI magnet before shipment.

TABLE II
PARAMETERS OF GE 9.4 T MRI MAGNET

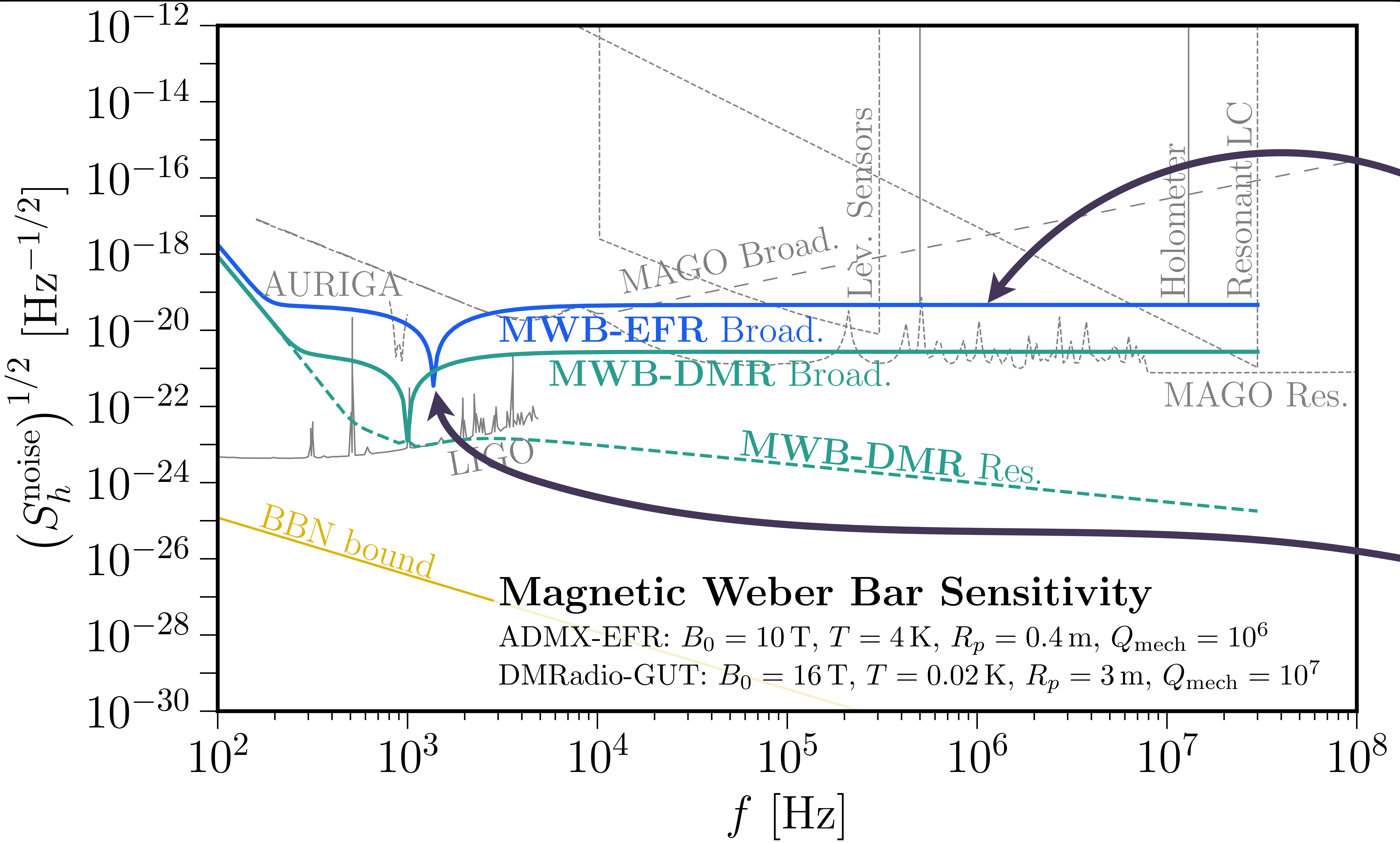
Central Field B_0 (T)	9.4
B_{peak}/B_0	1.024
Uniformity at 40cm DSV, peak-to-peak	5 ppm
Stored energy (MJ)	140
Conductor length (km)	540
Conductor weight (ton)	30
Magnet weight (ton)	45
Magnet length (m)	3.1
Room shielding weight (ton)	520

140 MJ stored energy $\leftrightarrow S_h^{1/2} \sim 10^{-21}\,\text{Hz}^{-1/2}$

(up to transfer function)



Magnetic Weber Bar



Intuition confirmed, with small penalty from transfer function

Enhancement from mechanical resonance transfer function

Domcke, SARE, Rodd (2024)

See also*: Carney, Higgins (IQOQI), Marocco, Wentzel (2024)*

STOCHASTIC SOURCES

What are the prospects?

Stochastic GW SNR

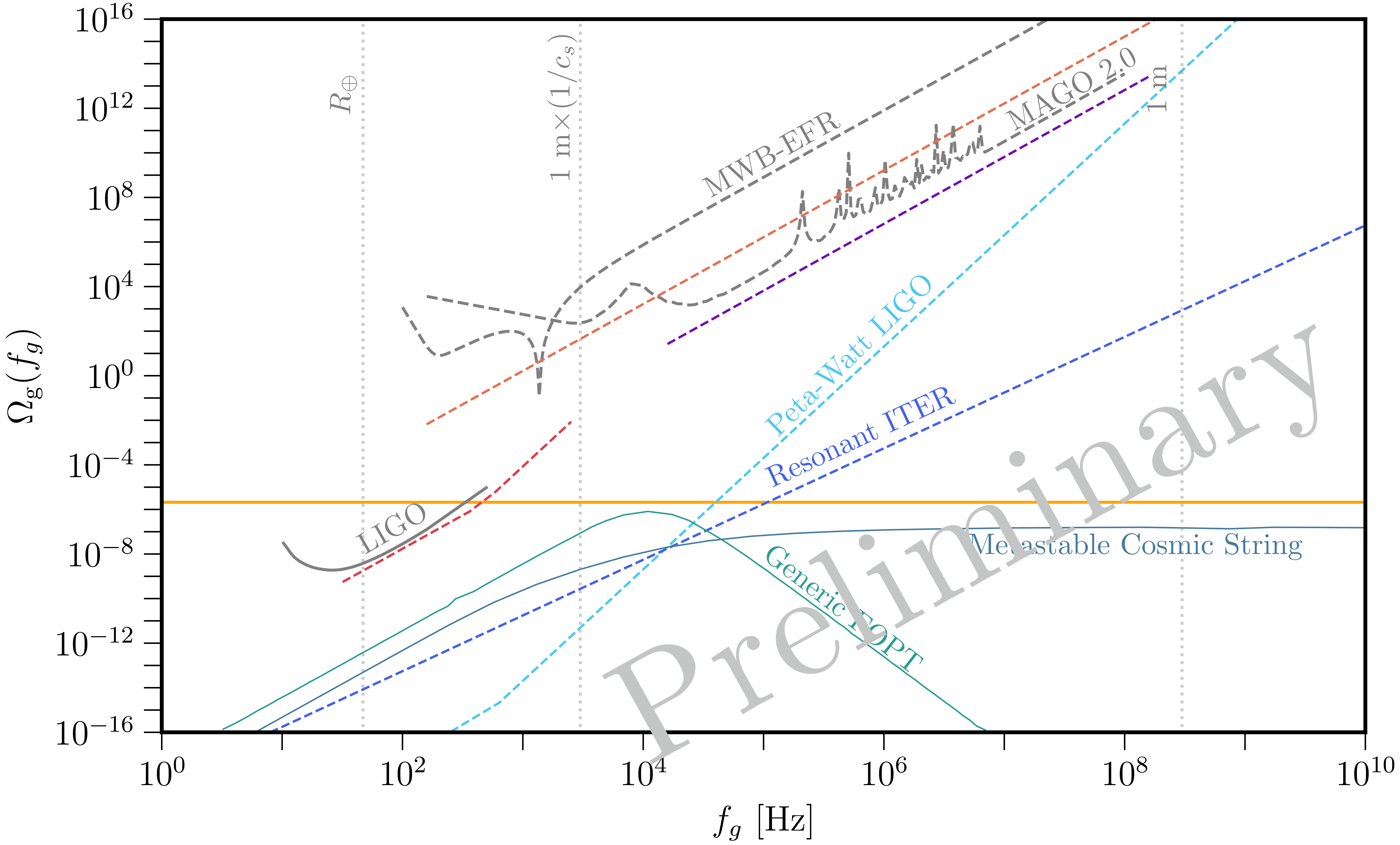
Where is the problem?

Detectors measure strain — all exquisite w/ $h \lesssim 10^{-20}$!

Stochastic GWs depend on *energy density*

$$\rho_g \sim M_{\rm pl}^2\, f^2\, h^2 \quad \Rightarrow \quad \Omega_g \propto \frac{f^2 h^2}{H_0^2}$$

Stochastic GWs



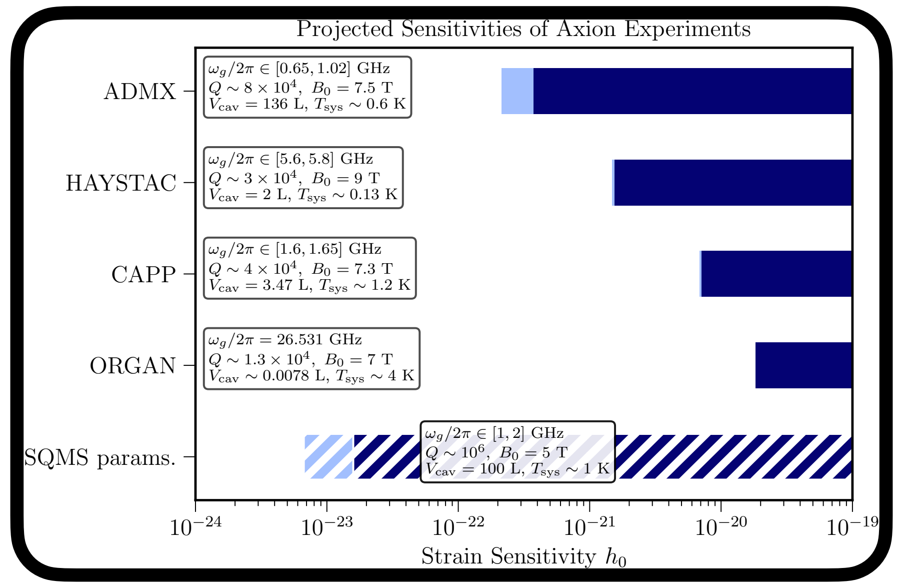
D'Agnolo, SARE (in prep)

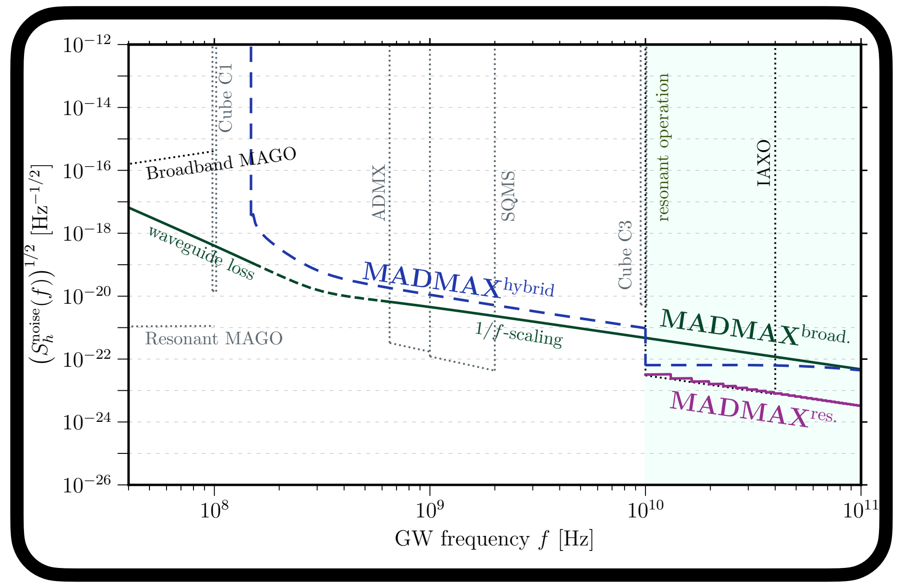
Frequency growth of energy density hurts...

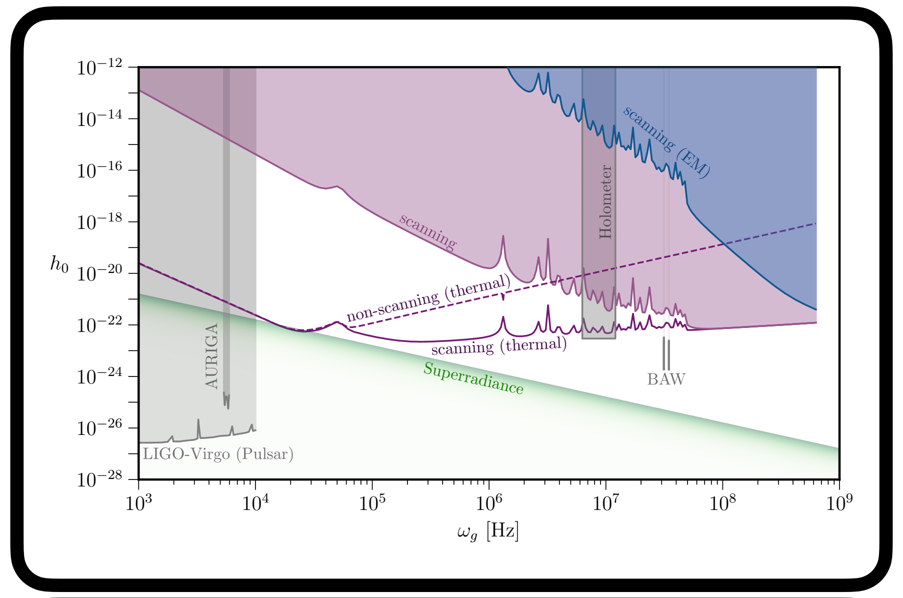
Cross-correlation of detectors

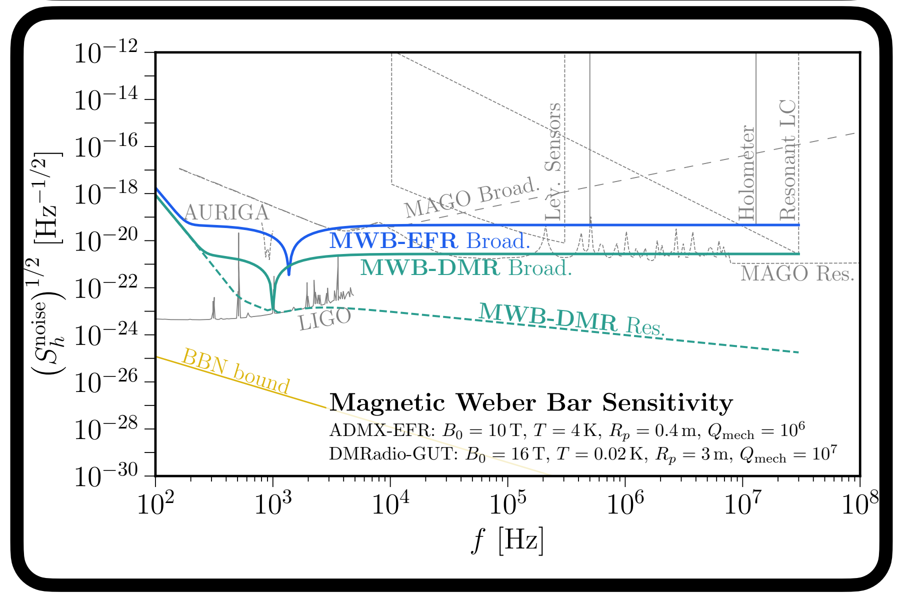
$$\Omega_g \propto N_{\rm det}^{-1}$$

Summary









Have we found the optimal detection approach?

— maximise stored energy and/or transfer function

Advances in readout

— networks, quantum techniques?

synergies w/ Axion searches, QC(?)

see e.g. GravNET (https://www.pi.uni-bonn.de/gravnet/en)

PI: Matthias Schott

Further signals above kHz?

Stochastic GWs require dramatic technology

BACKUP

Framing the Question

A more detailed estimate requires some GR

GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

Riemann tensor invariant at O(h):

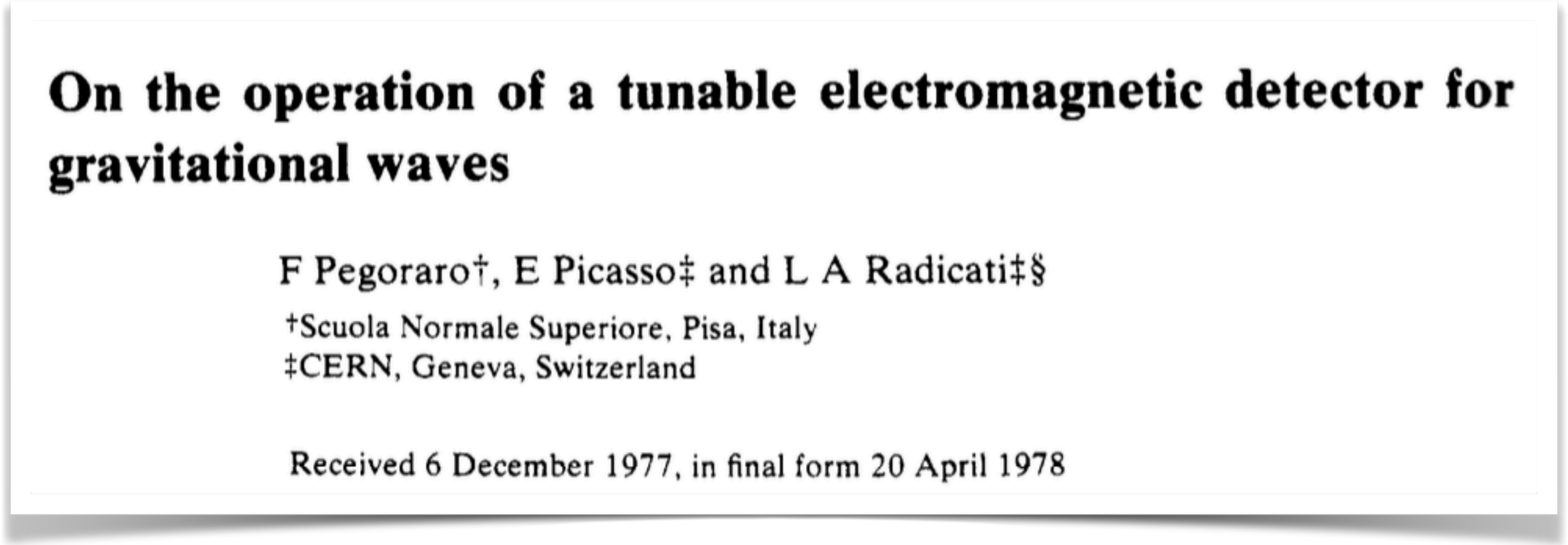
$$R_{0i0j} = -\frac{1}{2}\partial_t^2 h_{ij}^{\mathrm{TT}},$$

$$R_{0ijk} = \frac{1}{2}\partial_t\left(\partial_k h_{ij}^{\mathrm{TT}} - \partial_j h_{ik}^{\mathrm{TT}}\right),$$

$$R_{ikjl} = \frac{1}{2}\left(\partial_k\partial_j h_{il}^{\mathrm{TT}} + \partial_i\partial_l h_{jk}^{\mathrm{TT}} - \partial_i\partial_j h_{kl}^{\mathrm{TT}} - \partial_k\partial_l h_{ij}^{\mathrm{TT}}\right)$$

MAGO 2.0

Revive an old idea from the 1970s, and a prototype from the 2000s

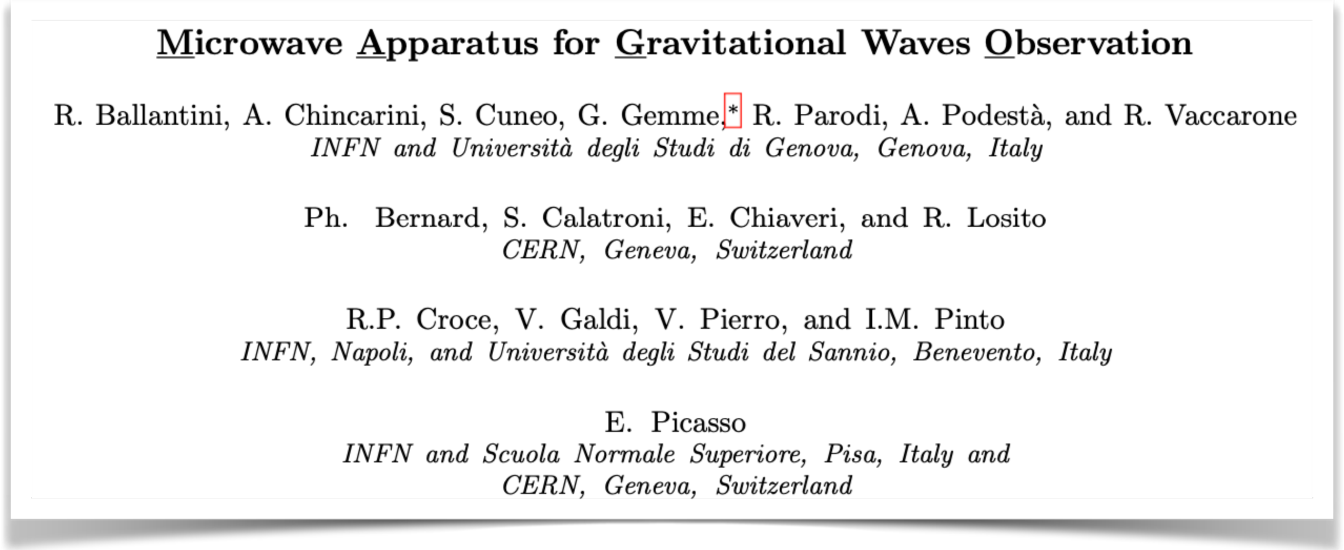


On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro†, E Picasso‡ and L A Radicati‡§

†Scuola Normale Superiore, Pisa, Italy
‡CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978



Microwave Apparatus for Gravitational Waves Observation

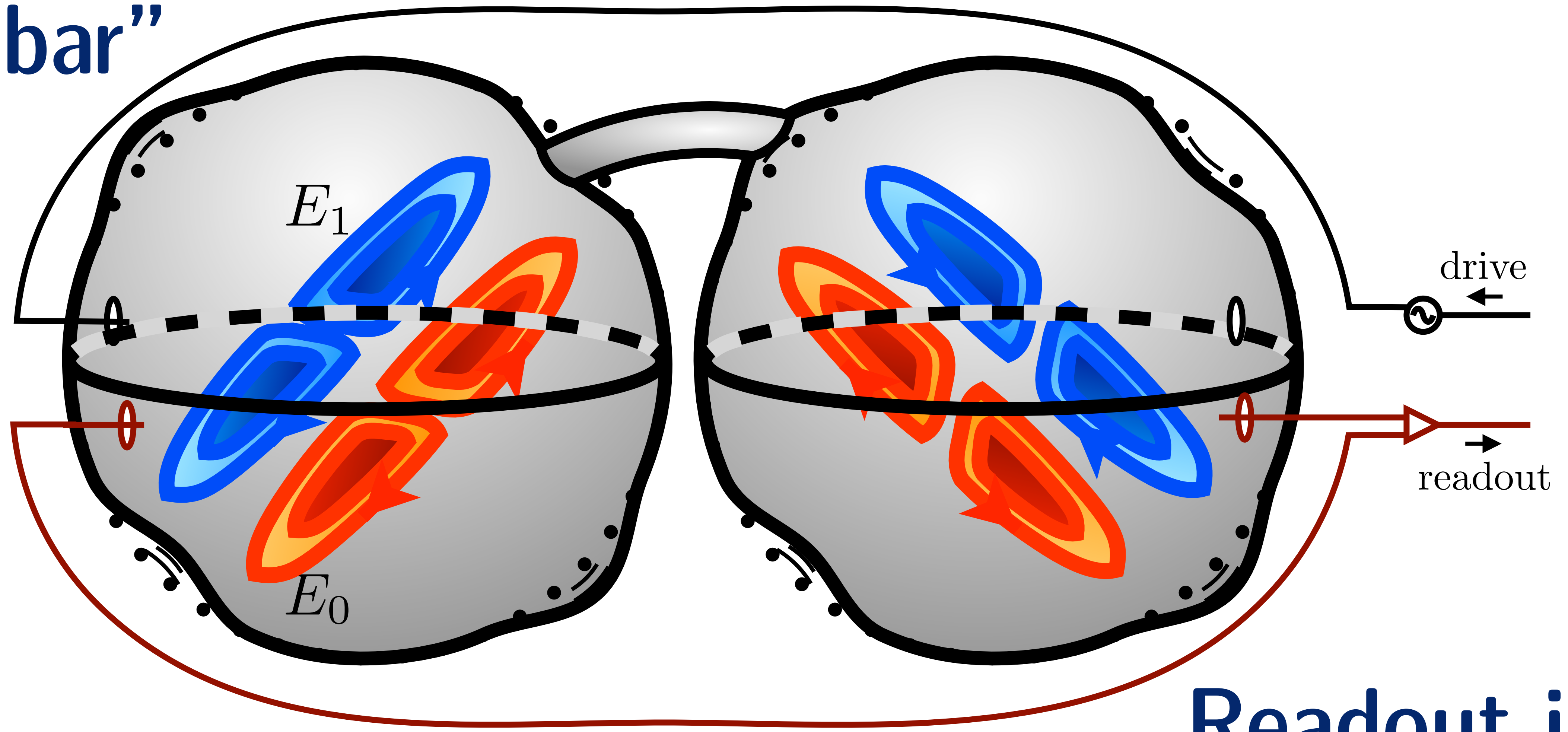
R. Ballantini, A. Chincarini, S. Cuneo, G. Gemme,* R. Parodi, A. Podestà, and R. Vaccarone
INFN and Università degli Studi di Genova, Genova, Italy

Ph. Bernard, S. Calatroni, E. Chiaveri, and R. Losito
CERN, Geneva, Switzerland

R.P. Croce, V. Galdi, V. Pierro, and I.M. Pinto
INFN, Napoli, and Università degli Studi del Sannio, Benevento, Italy

E. Picasso
INFN and Scuola Normale Superiore, Pisa, Italy and CERN, Geneva, Switzerland

Cavity walls are a “Weber bar”



Readout in a quiet mode of cavity

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

$$\mathbf{U}(\mathbf{x},t) = u_p(t)\,\mathbf{U}_p(\mathbf{x})$$

Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p}\,\dot{u}_p + \omega_p^2\,u_p \simeq -\frac{1}{2}\,\omega_g^2\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\,h_0\,e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{TT}_{ij}}{V_{\rm cav}^{1/3}\,V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x}\,U_p^{*i}\,x^j$$

$$\langle\mathbf{U}_p\rangle \sim h_0\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\times\begin{cases}\frac{\omega_g^2}{\omega_g^2-\omega_p^2}\ , & |\omega_g-\omega_p|\gg\omega_p/Q_p\\ Q_p\ , & |\omega_g-\omega_p|\ll\omega_p/Q_p\end{cases}$$

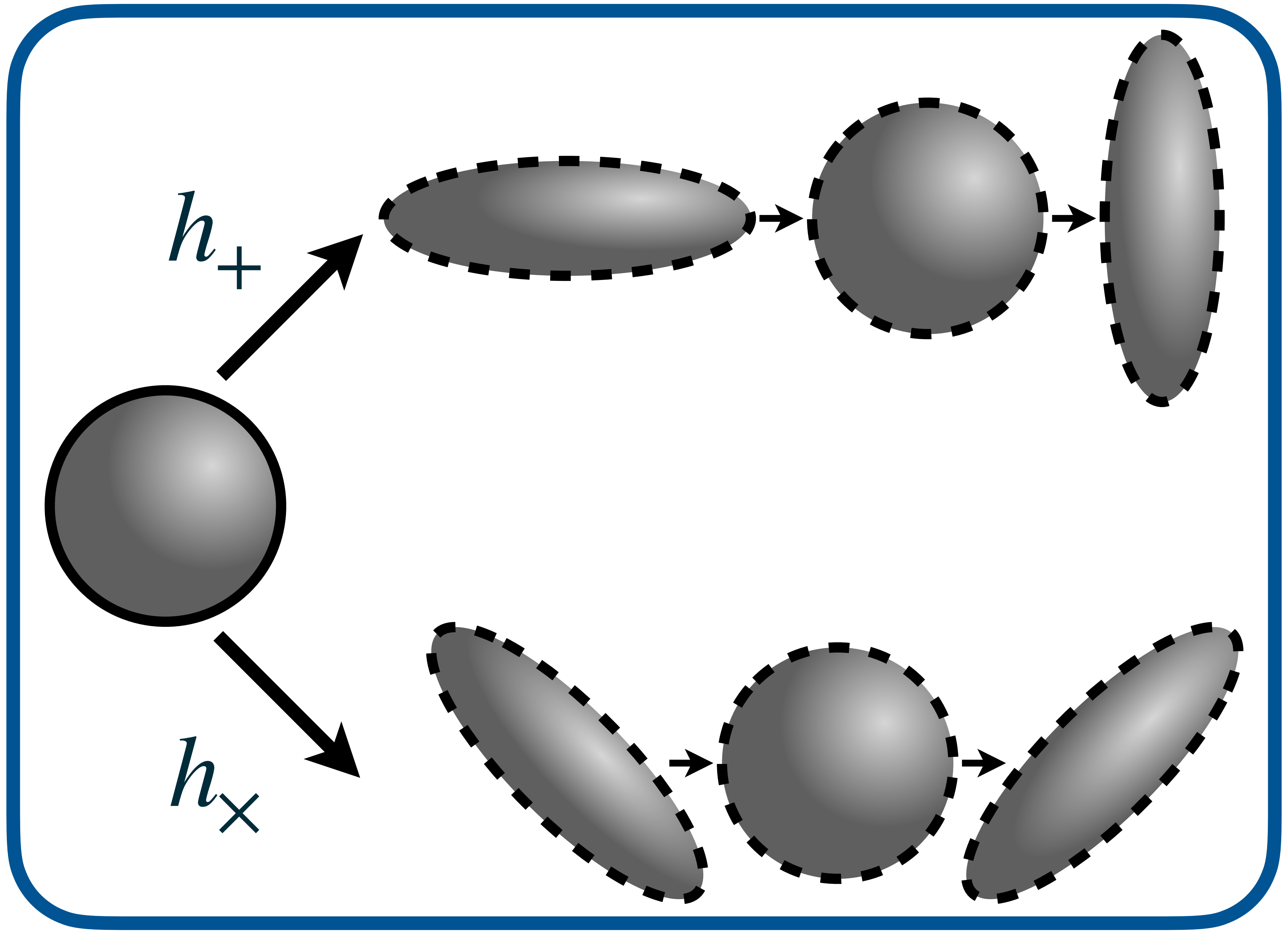
Tiny displacement $\ll$ nm

Cur Cavis?* pt. 2

MAGO 2.0

* "Why Cavities?" in Latin

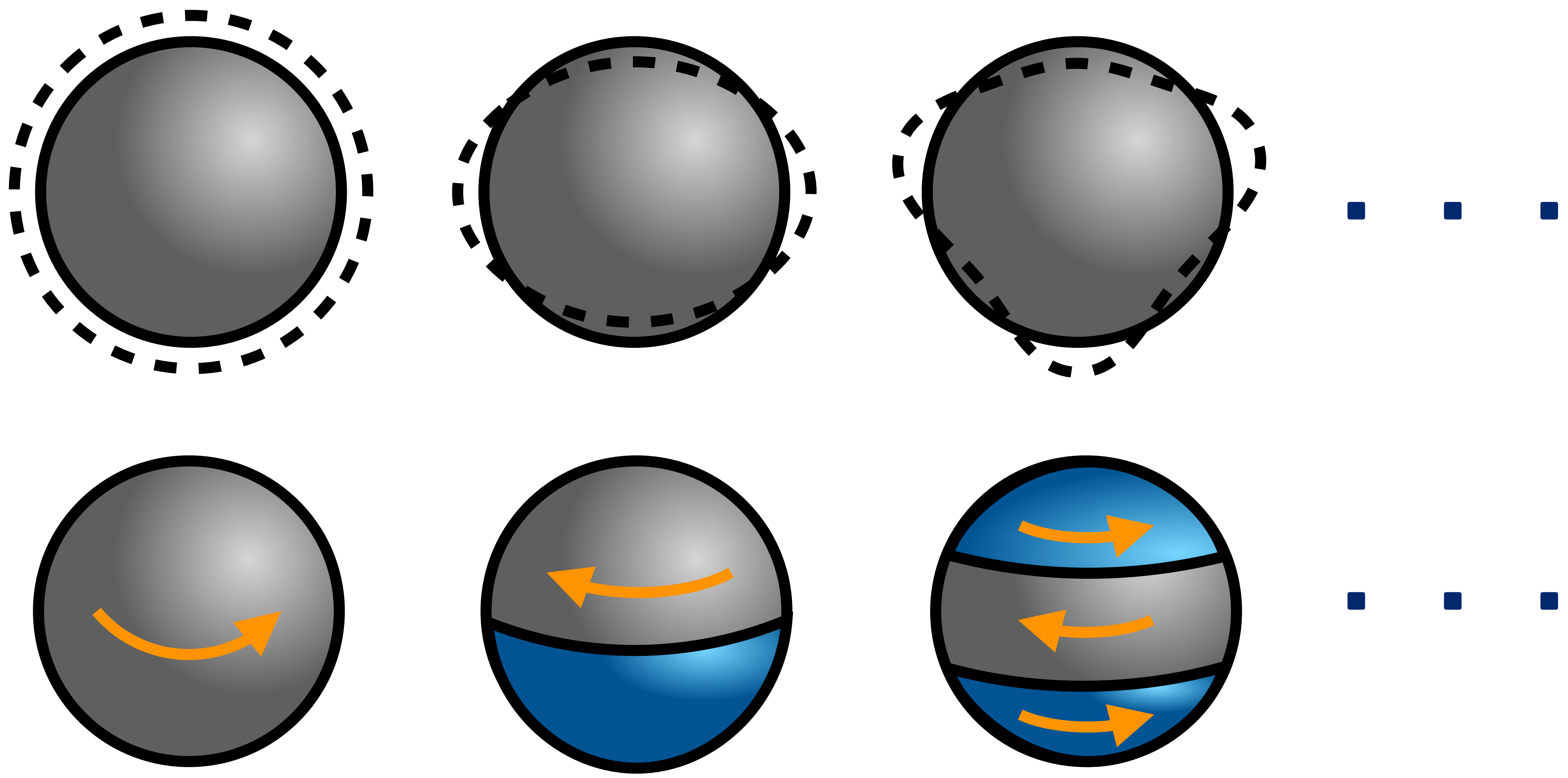
Gravitational Wave and a Hollow Sphere



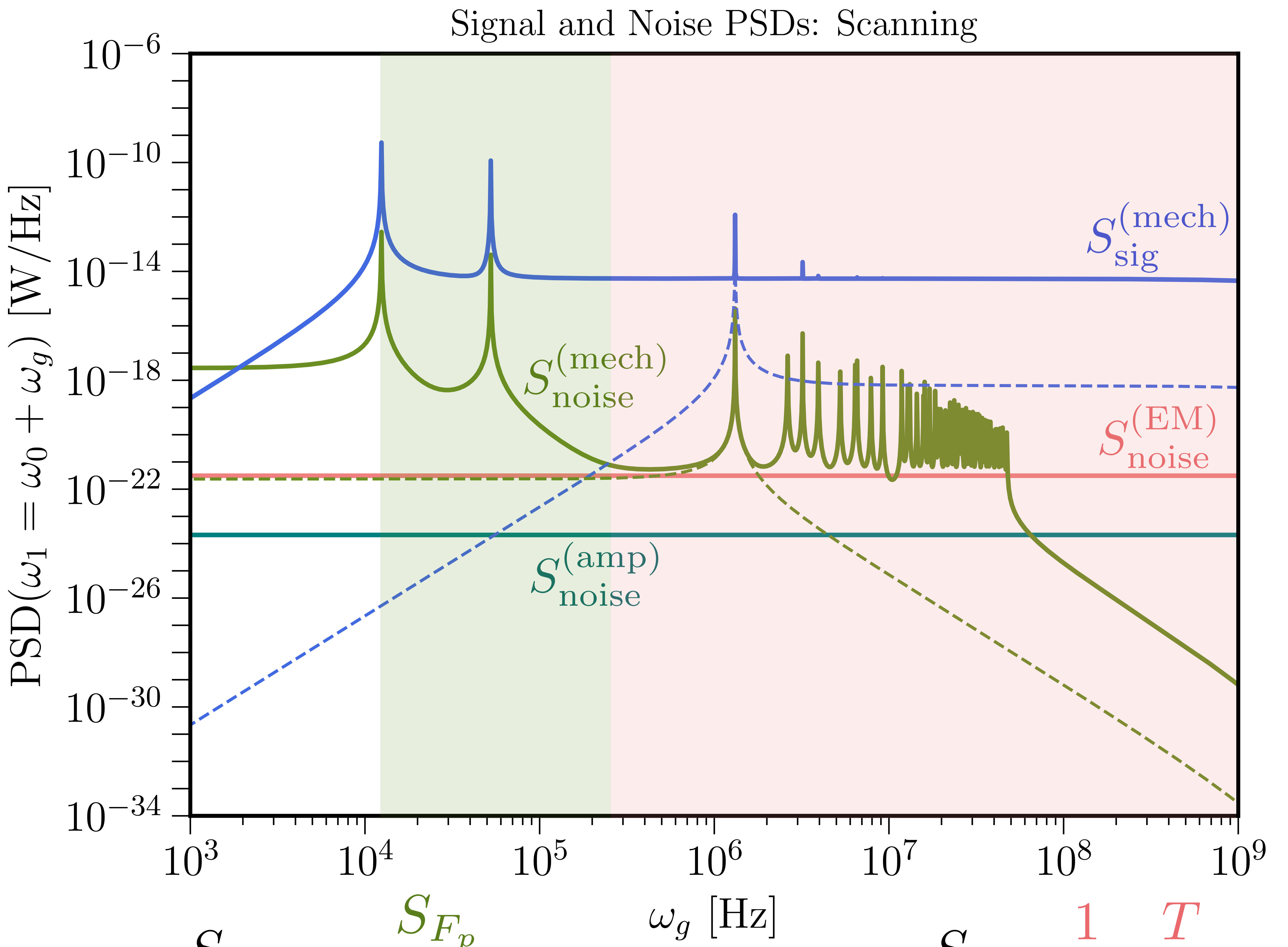
TT frame intuition

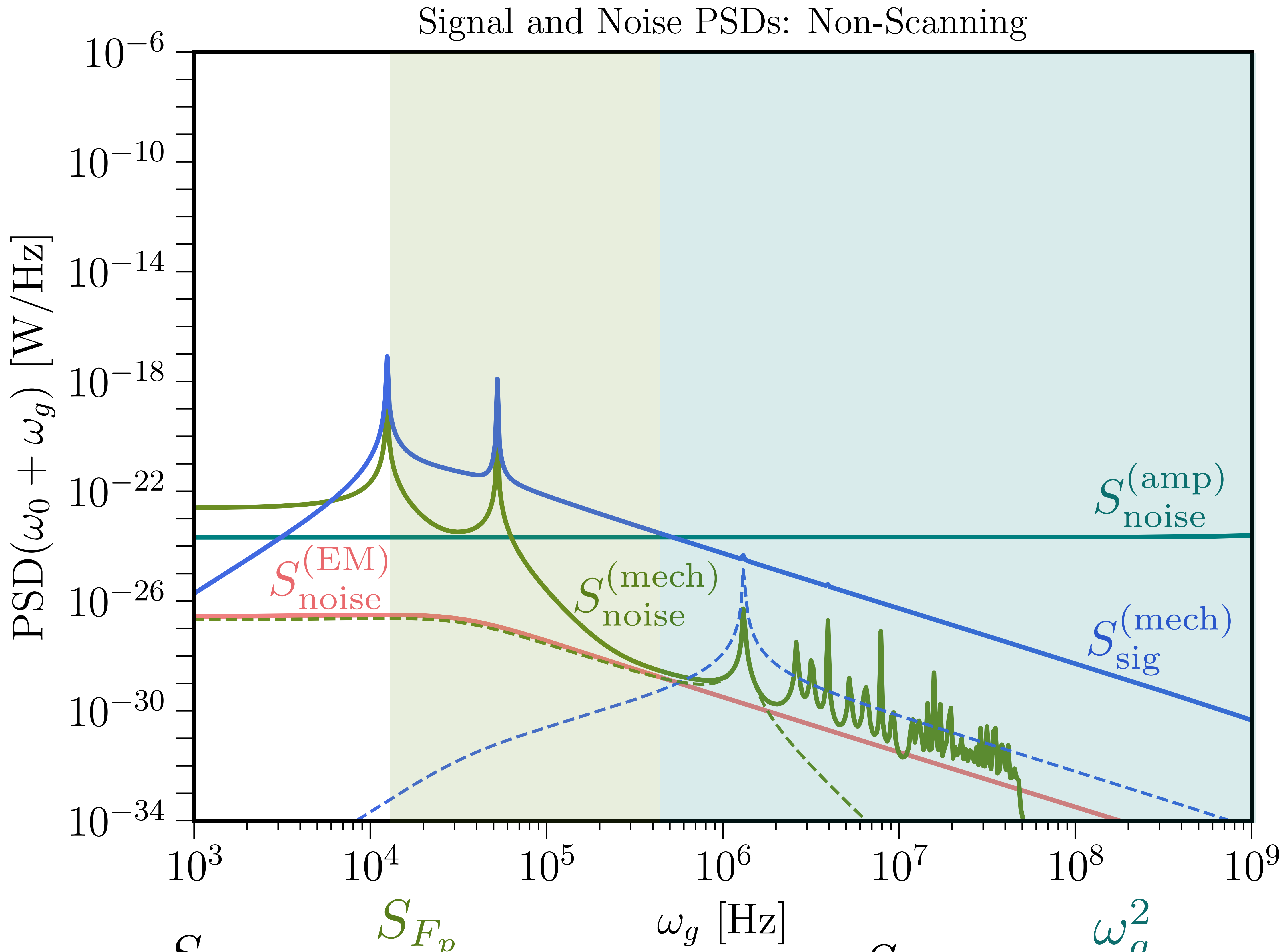
Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \underbrace{\nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1}}_{\text{Spheroidal}} + \underbrace{i\,\mathbf{L}\phi_{T_2}}_{\text{Toroidal}}\,.$$



Noise in MAGO 2.0





$$S_h \sim \frac{S_{F_p}}{\omega_g^4 \, (ML)^2}$$

Mech < MHz

$$S_h \sim \frac{1}{Q}\frac{T}{\omega_0 U_0}$$

EM Th. > MHz

$$S_h \sim \frac{S_{F_p}}{\omega_g^4 \, (ML)^2}$$

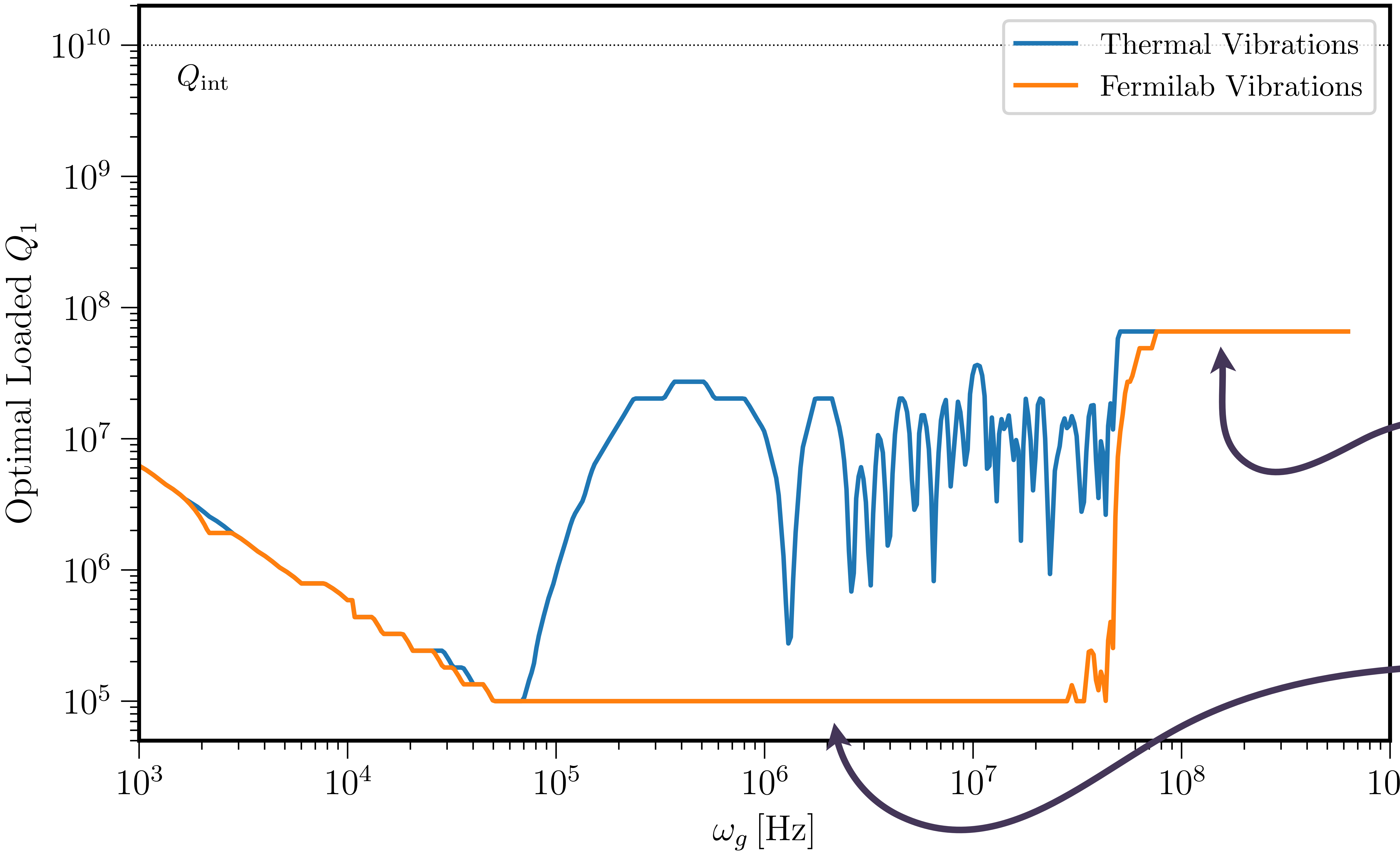
Mech < MHz

$$S_h \sim \frac{\omega_g^2}{\hbar\omega_0(\omega_0 U_0)}$$

Amp > MHz

NB: missing radiation damping effect studied in *Löwenberg, Moortgat-Pick: 2307.14379*

Optimal Scanning



Integration time:

$$t_{\rm int} \sim t_e \min\left(\frac{\omega_1}{Q_1\,\omega_g}\,,\,1\right)$$

Thermal:

$$Q_1 \sim Q_{\rm int}(\omega_1/T)$$

Vibrations:

$$Q_1 \sim Q_1^{\rm min}$$

HEURISTICS

Prospects for stochastic GWs?

Stochastic GW SNR

Stochastic backgrounds have zero mean — necessarily quadratic measurement

Single detector: $\mathrm{SNR} \sim \dfrac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)}$

Two detectors: $\mathrm{SNR} \sim \left(t_{\mathrm{int}} \int df \, \left(\dfrac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)} \right)^2 \right)^{1/2}$

$$S_{\mathrm{sig}}(\omega) \sim |\mathcal{T}(\omega)|^2 S_h(\omega)$$

Comparing LIGO to MAGO

LIGO in this language:

$$\mathcal{T}_{\rm LIGO}(\omega) \sim \frac{k_L}{\omega_p}\left(1+\left(\frac{\omega}{\omega_p}\right)^2\right)^{-1/2} \qquad \mathcal{T}_{\rm LIGO}(\omega) \sim 10^{11}$$

Thermal-noise limited MAGO (on EM resonance):

$$\mathcal{T}_{\rm MAGO}(\omega) \sim |\eta_{\rm mech}||\eta_{\rm mech}^{\rm EM}|\sqrt{Q_{\rm int}\frac{\omega_{\rm sig}}{T}}\left(\frac{1}{\Delta\omega_{\rm osc}\, t_{\rm int}}\right)^{1/4} \qquad \mathcal{T}_{\rm MAGO}(\omega) \sim 10^3$$

Comparing LIGO to MAGO

Best possible sensitivity: $$h^q_{\rm min} \sim \frac{1}{\sqrt{U_{\rm in} t_{\rm int}}} \frac{1}{\mathcal{T}(\omega)}$$

LIGO stored energy:

$$U_{\rm in}^{\rm LIGO} \sim 10^{-10}\ {\rm J}$$

$$h_{\rm min}^{\rm LIGO} \sim 5 \times 10^{-23} \left(\frac{\omega}{2\pi \times 10^6\ {\rm Hz}} \right)$$

MAGO 2.0 energy:

$$U_{\rm in}^{\rm MAGO} \sim 10^{4}\ {\rm J}$$

$$h_{\rm min}^{\rm MAGO} \sim 10^{-23}$$