Endpoint Factorization for Inclusive Semileptonic Top Quark Decays in Off-Shell Boosted Top Quark Production

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with
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Doktoratskolleg Particles and Interactions

Der Wissenschaftsfonds.

VIENNA - DOCTORAL - SCHOOL - PHYSICS

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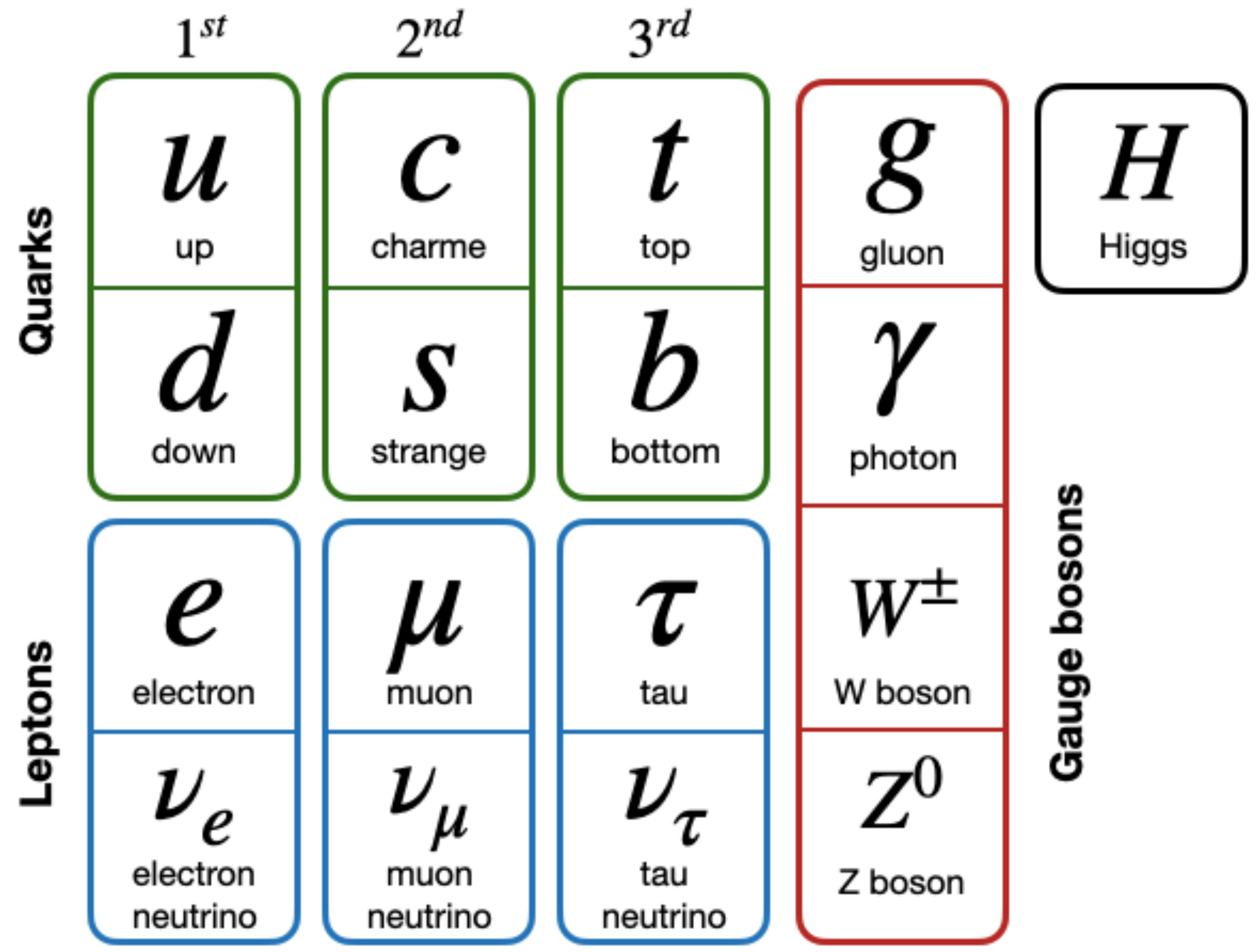
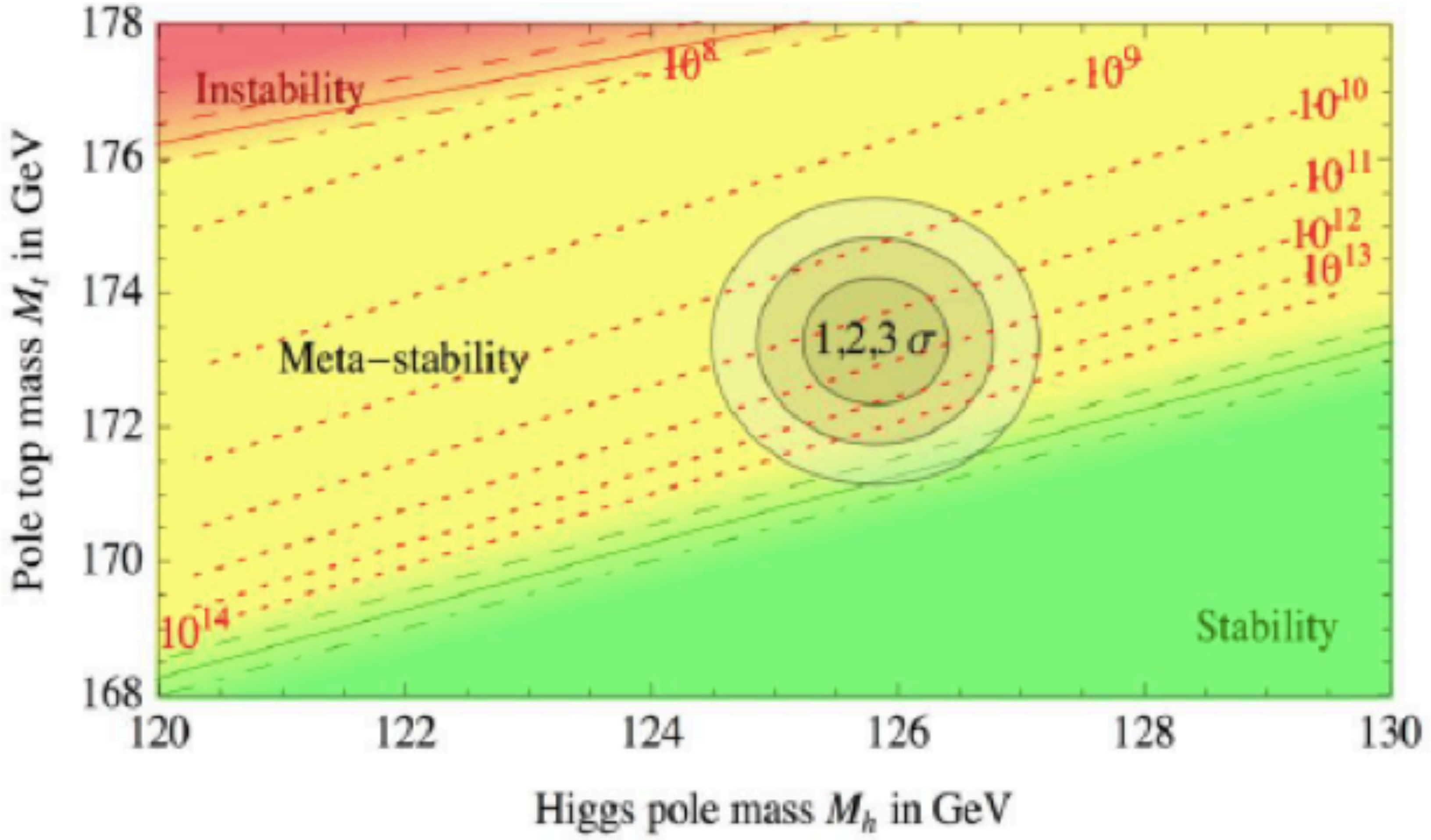
Motivation — Top Quark Physics

- The **top quark** is the heaviest known elementary particle.
- Due to its large mass the top quark plays a key role in consistency checks of the Standard Model and new-physics searches.
- Most precise mass determinations come from **“direct mass measurements”**.
  - Kinematic reconstruction of the top quark decay products.
  - Theoretical computations used for the mass measurements rely on **multipurpose Monte Carlo event generators (MMCs)**.
- Relation between MC top mass parameter m^{MC} and field theoretic mass definitions unresolved $\longrightarrow$ **“top mass interpretation problem”**.

  $\longrightarrow$ First principle study initiated in [Hoang, Plätzer, Samitz JHEP 10 (2018 200)]
- To resolve this issue, significant theoretical progress is required. (improved precision of MC parton showers, **QCD factorization approaches** for boosted top quarks, …)

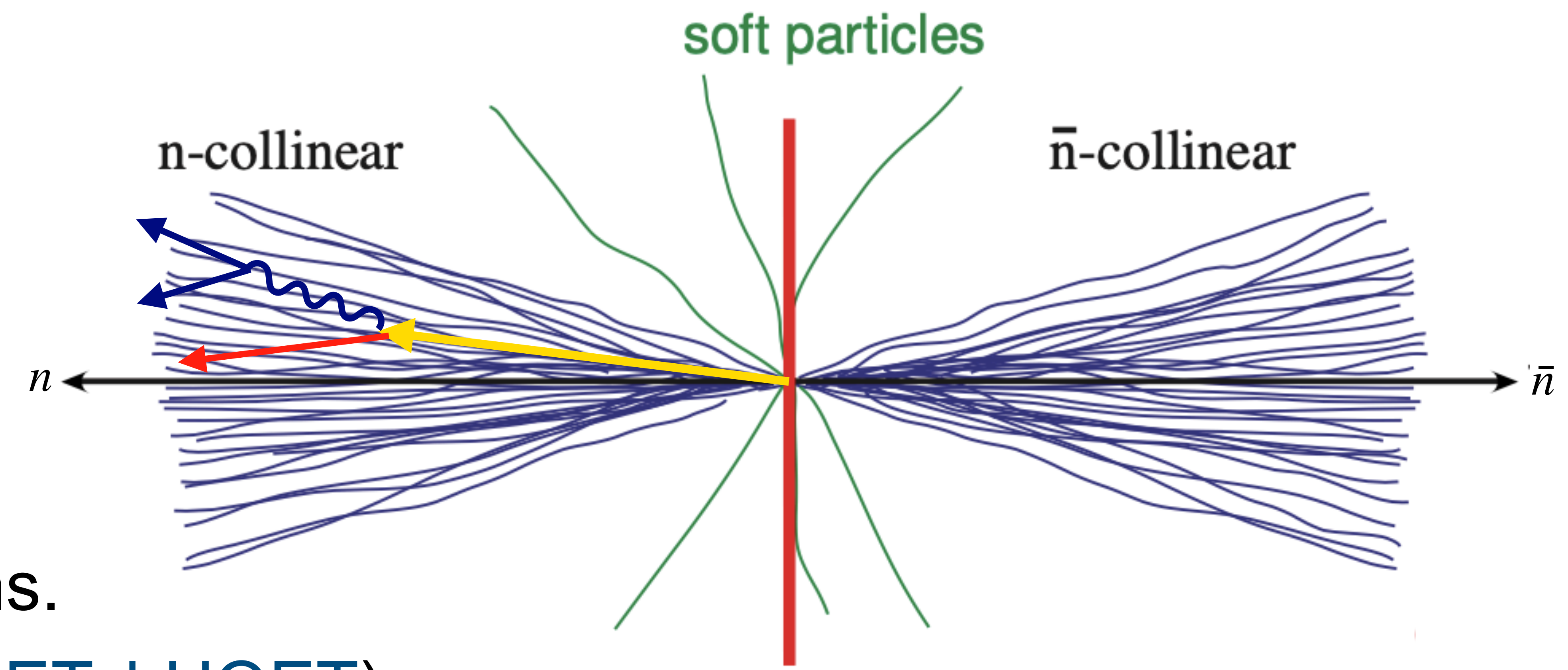




[Degrassi et. al JHEP 1208 (2012) 098]

Motivation — Top Quark Physics

- Studies of top quark production and its decay commonly based on two approaches:
  - Narrow-width (NW) limit:
    - Top quark treated as on-shell particle.
    - Factorization of top production and decay dynamics.
  - Off-shell fixed-order computation:
    - Accounts for non-resonant, non-factorizable and finite lifetime effects.
    - Start-of-the-art: fixed-order NLO QCD.



- New approach: Combine properties of NW limit and off-shell computations.
  - **QCD factorization theorem** for off-shell boosted top quarks ($\longrightarrow$ SCET, bHQET)
    - Merge factorization approaches for boosted top production and semileptonic B decays.
  - Top quark state **based on a measurement** (and not on the concept of a “top particle”).
  - Incorporate resummed QCD corrections for differential top decay observables.

- Aim: Analytic control of top mass dependent decay observables.

Outline

- Effective field theories:
  - Heavy Quark Effective Theory (HQET)
  - Soft-Collinear Effective Theory (SCET)
- Semileptonic $\bar{B}$ meson decays
- Boosted top pair production in e^+e^- collisions
- SCET electroweak + QCD top quark jet function
- Endpoint factorization theorem for inclusive semileptonic top quark decays
- Summary & Outlook

Effective Field Theories

Basic problem: Predictions of QFTs in high-energy physics inevitably involve multiple length scales.

- **EFTs** provide a general theoretical framework to deal with these **multi-scale problems**.
  - Reduction of multi-scale problems to a combination of simpler single-scale problems.
  - Basic principles of EFTs used in a wide range of areas in high-energy physics.

- Crucial ingredients for the construction of EFTs:
  - Identification of the relevant degrees of freedom.
  - Identification of the symmetries that constrain the interactions.
  - Identification of an appropriate expansion parameter. $\longrightarrow$ **Power counting**

- Power counting: Consider e.g. the hierarchy of scales $m_Q \ll \Lambda$

$\longrightarrow$ expansion parameter $\lambda = \dfrac{m_Q}{\Lambda} \ll 1$

$$\Rightarrow \quad \mathcal{L} \to \sum_n \mathcal{L}_{\text{EFT}}^{(n)}$$

Heavy Quark Effective Theory

Basic problem: Predictions of QFTs in high-energy physics inevitably involve multiple length scales.

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  - Reduction of multi-scale problems to a combination of simpler single-scale problems.
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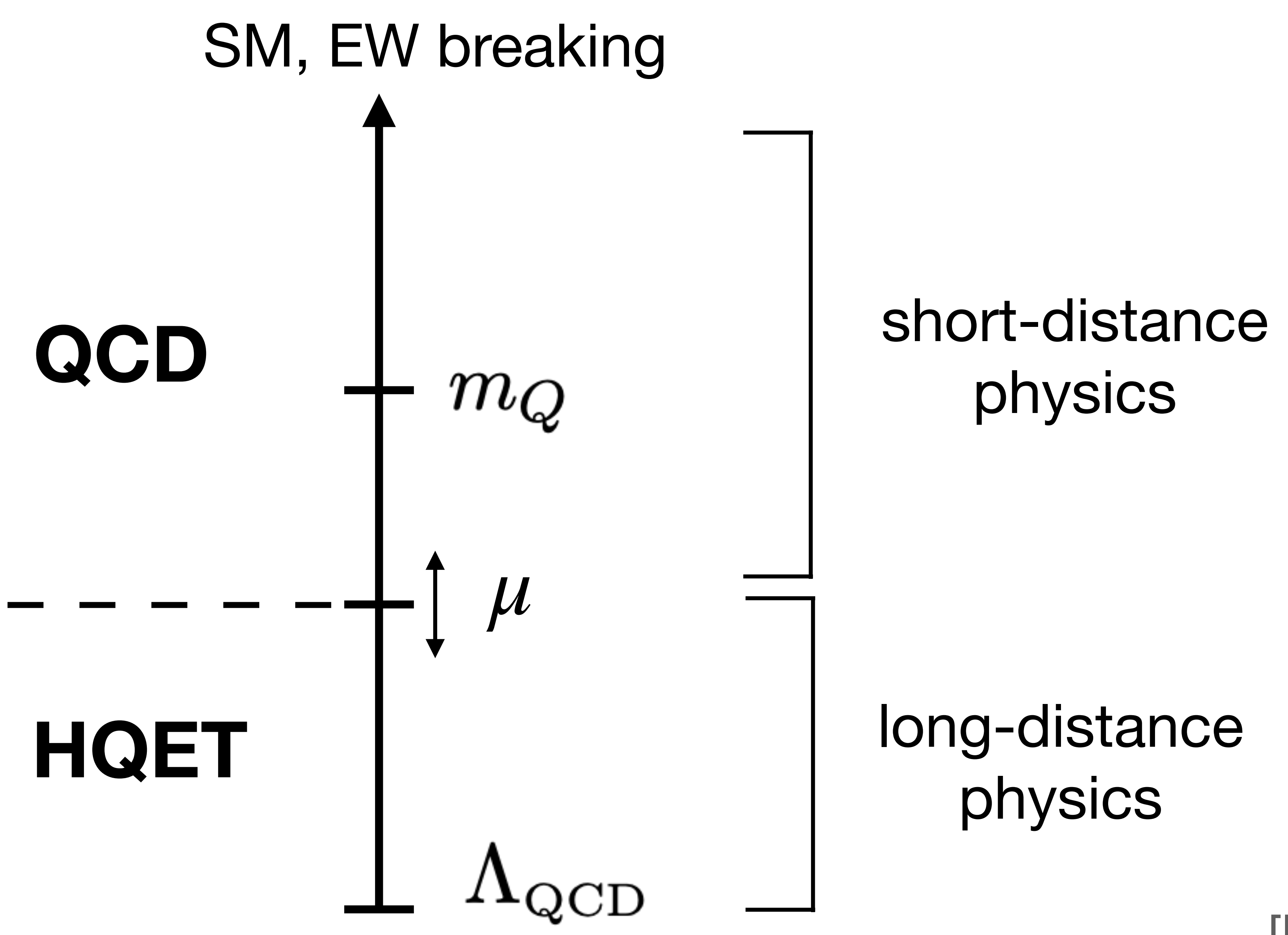
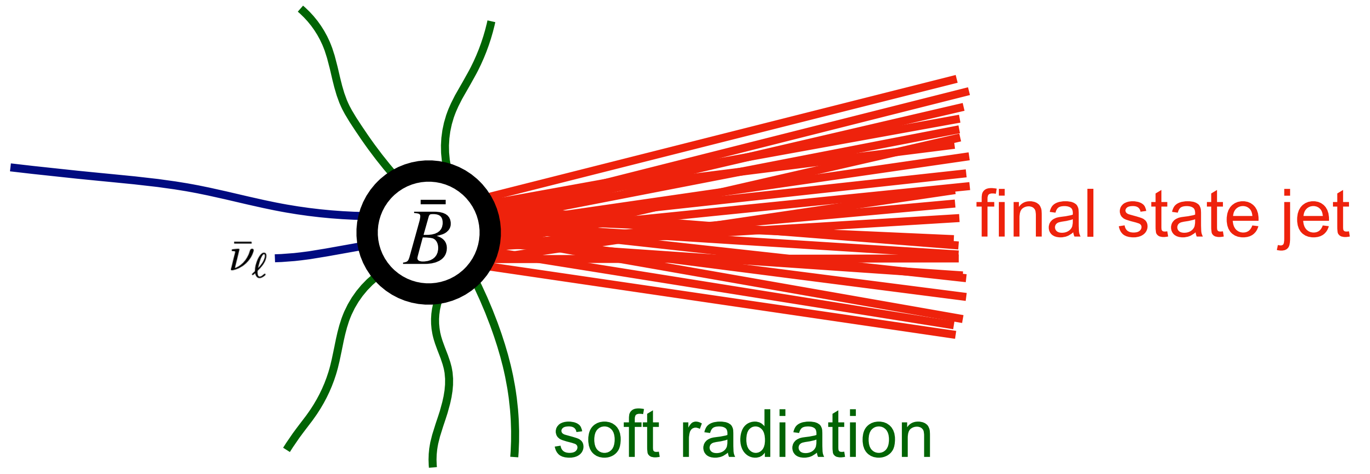
- Heavy Quark Effective Theory: Describes the dynamics of hadrons with one heavy quark with mass m_Q.
  - Separation of perturbative and non-pert. QCD effects due to scale hierarchy $m_Q \gg \Lambda_{\rm QCD}$.
  - Heavy quark symmetry



[Neubert, *Subnucl.Ser.* 34 (1997) 98-165]

Soft-Collinear Effective Theory (SCET)

- SCET: Used to describe energetic QCD processes where the final state particles have large energies compared to their invariant mass.
  - B meson decays to light particles in kinematic endpoint regions, e.g. $\bar{B} \to X_u \, l \, \bar{\nu}$. $(m_X \ll E_X \sim \frac{m_B}{2})$
  - Jet production in pp collisions and e^+e^- collisions. $(m_J \ll E_J)$
  - …

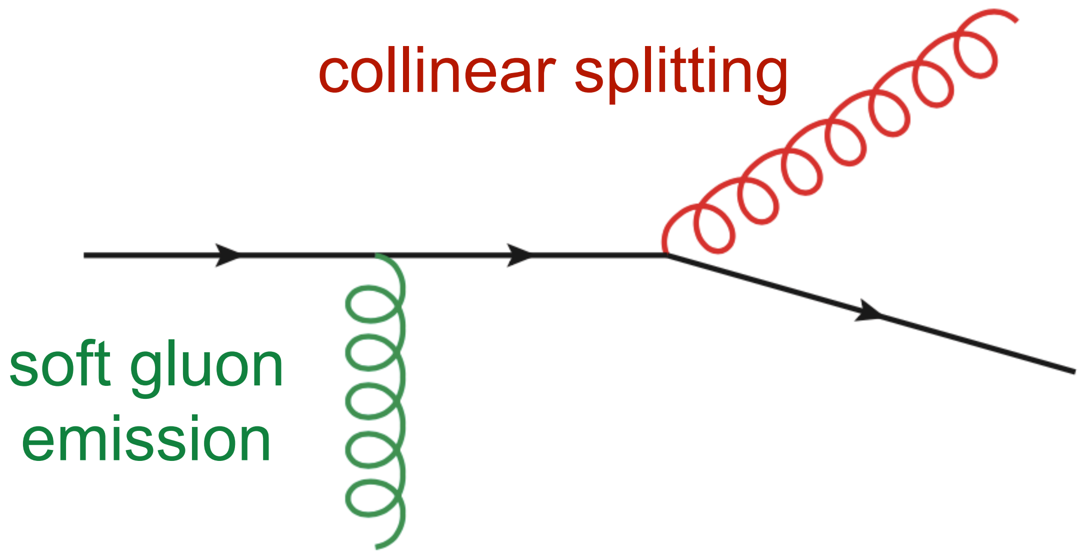


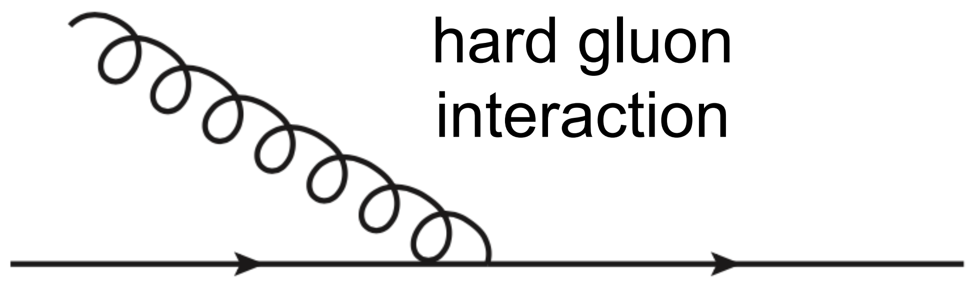
- Momentum modes:

$$\begin{aligned} \text{collinear dofs}: \quad & p_n \sim Q(\lambda^2, 1, \lambda) \\ & p_{\bar{n}} \sim Q(1, \lambda^2, \lambda) \\ \text{soft dofs}: \quad & p_s \sim Q(\lambda^2, \lambda^2, \lambda^2) \\ \text{hard dofs}: \quad & p_h \sim Q(1, 1, 1) \end{aligned}$$

$$p^\mu = \underbrace{n \cdot p}_{p^+} \frac{\bar{n}^\mu}{2} + \underbrace{\bar{n} \cdot p}_{p^-} \frac{n^\mu}{2} + p_\perp^\mu \equiv (p^+, p^-, p_\perp)$$

$$p^2 = p^+ p^- + p_\perp^2$$





- Not described by SCET. $\longrightarrow$ Hard modes integrated out.

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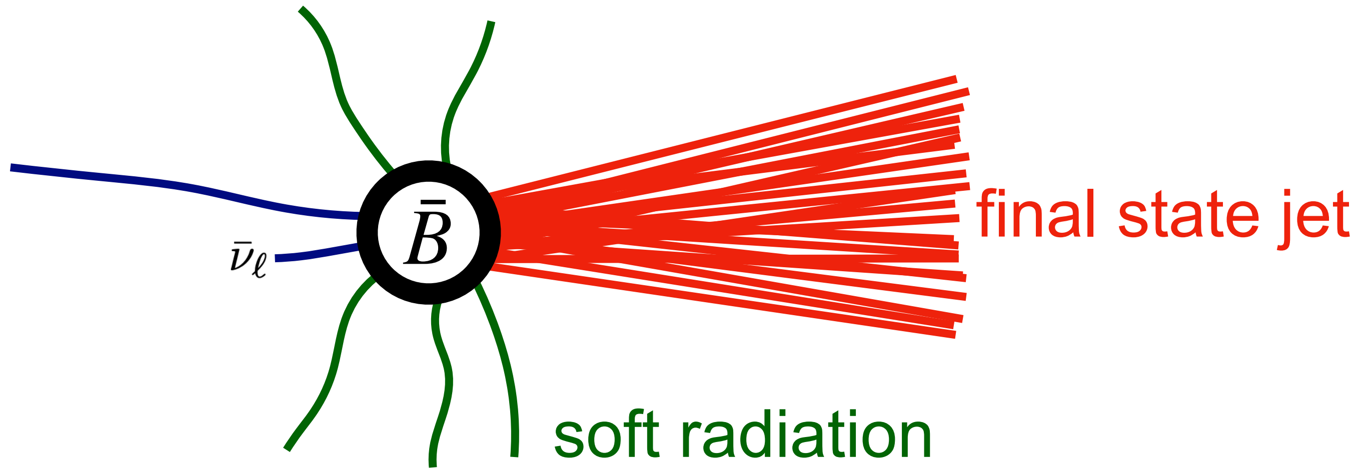
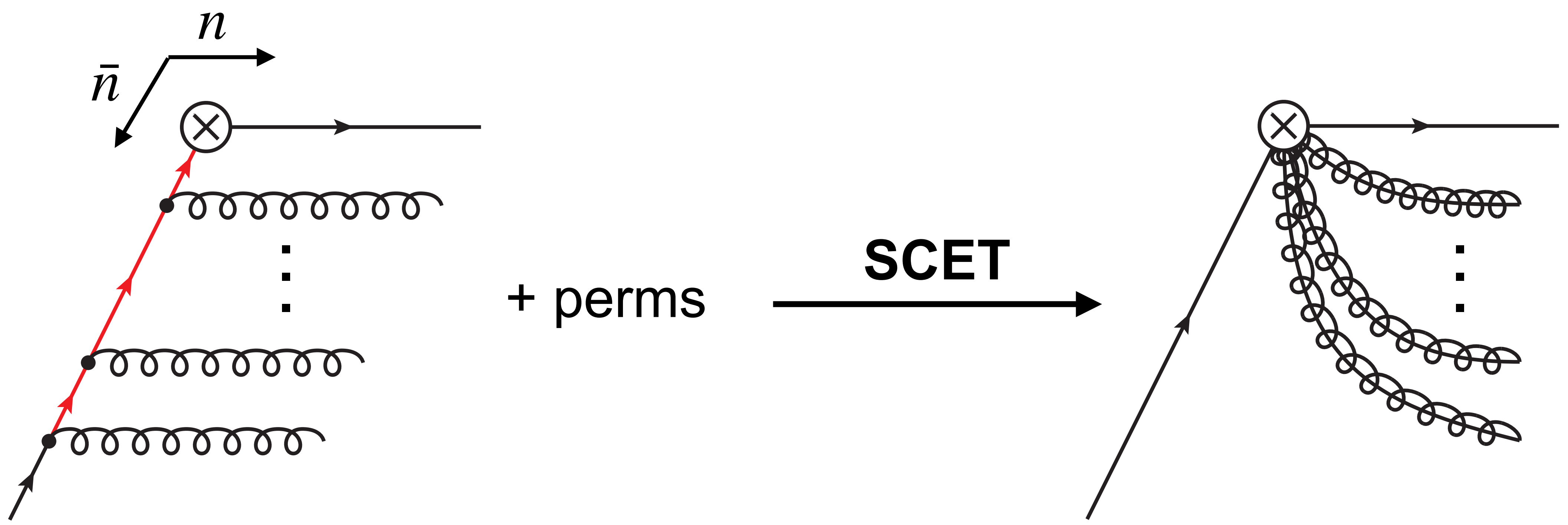


- Leading order collinear quark Lagrangian:

$$\mathcal{L}_n = \bar{\xi}_n \left[in \cdot D_s + gn \cdot A_n + gn \cdot A_s + i\not{D}_n^{\perp} W_n^{\dagger} \frac{1}{\bar{n} \cdot \mathcal{P}} W_n \, i\not{D}_n^{\perp} \right] \frac{\not{\bar{n}}}{2} \xi_n$$

collinear Wilson line



$$W_n(x) = P \exp\left(-ig_s \int_{-\infty}^{0} \mathrm{d}s \; \bar{n} \cdot A_n(s\bar{n} + x) \right)$$

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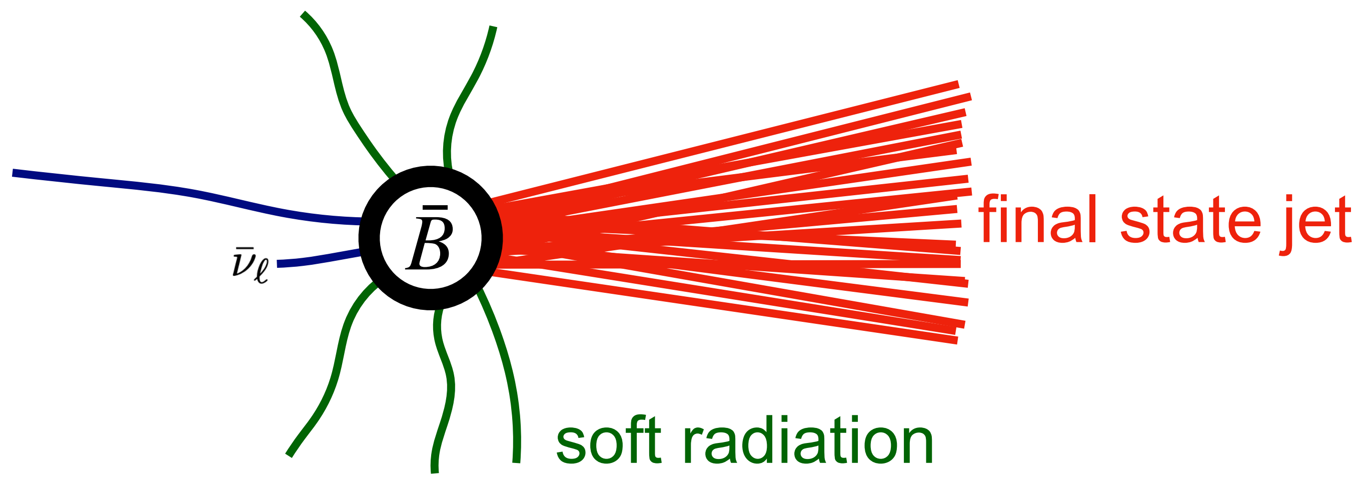

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collinear-soft coupling $iD_s^\mu = i\partial^\mu + gA_s^\mu$

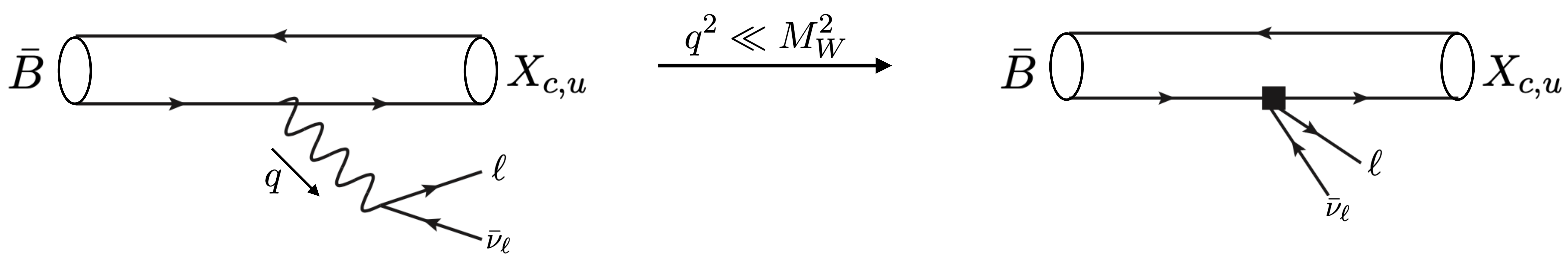
$$\xi_n \to Y_n \, \xi_n, \qquad W_n \to Y_n \, W_n \, Y_n^\dagger$$

$$Y_n(x) = \overline{\mathrm{P}} \exp\left(-ig_s \int_0^\infty \mathrm{d}s \; n \cdot A_s(ns + x) \right)$$ soft Wilson line

$$\mathcal{L}_n = \bar{\xi}_n \, in \cdot \partial_s \frac{\not{\bar{n}}}{2} \xi_n + \ldots$$

Recap: Semileptonic B decays

- Inclusive semileptonic decays $\bar{B} \to X_c\,\ell\,\bar{\nu}_\ell$ and $\bar{B} \to X_u\,\ell\,\bar{\nu}_\ell$ allow extraction of $|V_{cb}|$ and $|V_{ub}|$ from measurements of the decay spectra.



$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}h^2} \sim G_F^2\,L_{\mu\nu}\,W^{\mu\nu}$$

$$W^{\mu\nu} = \frac{1}{2m_B}\frac{1}{\pi}\,\mathrm{Im}\Big[\langle\bar{B}|\;i\!\int \mathrm{d}^4x\,\mathrm{e}^{-iq\cdot x}\,T\{J^{\dagger\mu}(0)J^\nu(x)\}\,|\bar{B}\rangle\Big], \qquad J^\nu = (\bar{c}\,\gamma^\nu P_L\,b)$$

- $\bar{B} \to X_c\,\ell\,\bar{\nu}_\ell$: $W^{\mu\nu}$ can be studied by using a local OPE within HQET.
  - Non-pert. physics encoded in matrix elements of local operators.

- $\bar{B} \to X_u\,\ell\,\bar{\nu}_\ell$: Cuts on E_ℓ or h^2 needed to eliminate $b \to c$ background events.
  - Restriction to phase space region of energetic jets with small invariant mass $\longrightarrow$ **SCET**
  - OPE not applicable in this region $\longrightarrow$ need to rely on factorization tools.

Factorization for Semi-leptonic B decays

- Factorized form of differential decay rate in the endpoint region:

Neubert '94
Korchemsky, Sterman '94
Bauer, Pirjol, Stewart '02

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}h^2} \sim \Gamma_0\, H(E_\ell,\mu) \int \mathrm{d}k^+\, J(k^+,\mu)\, S_{\text{shape}}(m_B - m_b - k^+,\mu)$$

hard function

$$H \sim C^2$$

- perturbative
- $\mu_H \sim m_b$

jet function

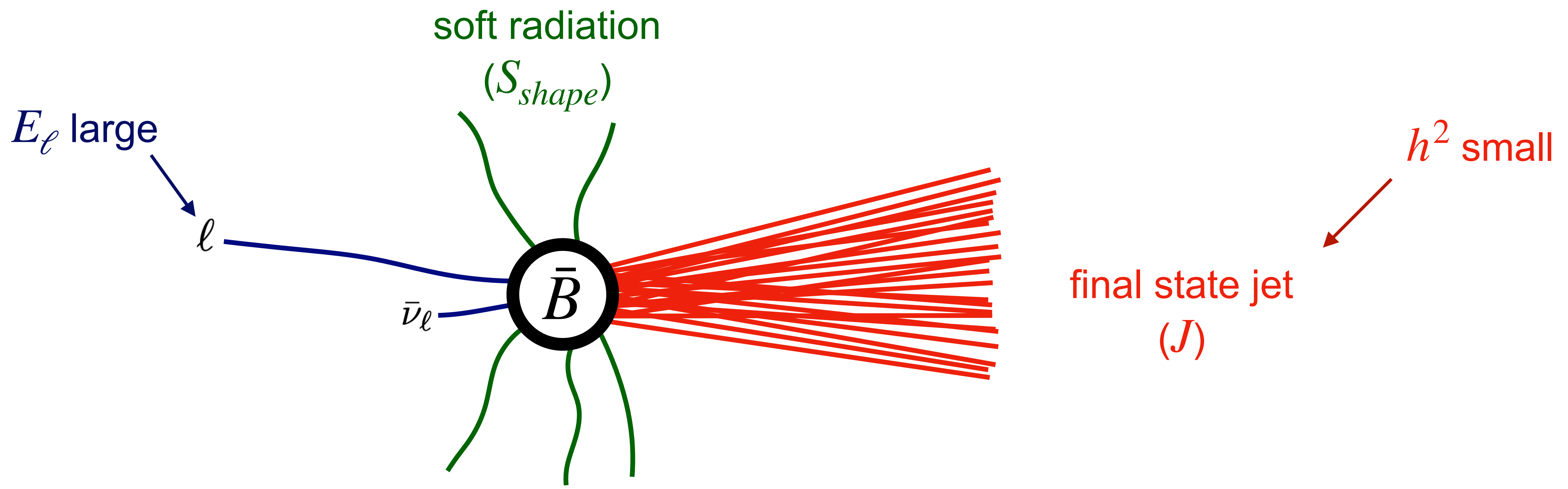
$$J(k) \sim \mathrm{Im}\left[\int \mathrm{d}^4x\, e^{ik\cdot x}\, \langle 0|\, T[\bar{\xi}_n W_n(0)\, \not{\bar{n}}\, W_n^\dagger \xi_n(x)]\, |0\rangle\right]$$

- perturbative
- $\mu_J \sim \sqrt{\Lambda_{\mathrm{QCD}}\, m_b}$

soft function

$$S_{\text{shape}}(k^+) = \langle \bar{B}|\, \bar{h}_v\, Y_n\, \delta(k^+ - in\cdot\partial)\, Y_n^\dagger\, h_v\, |\bar{B}\rangle$$

- non-perturbative
- $\mu_S \gtrsim \Lambda_{\mathrm{QCD}}$



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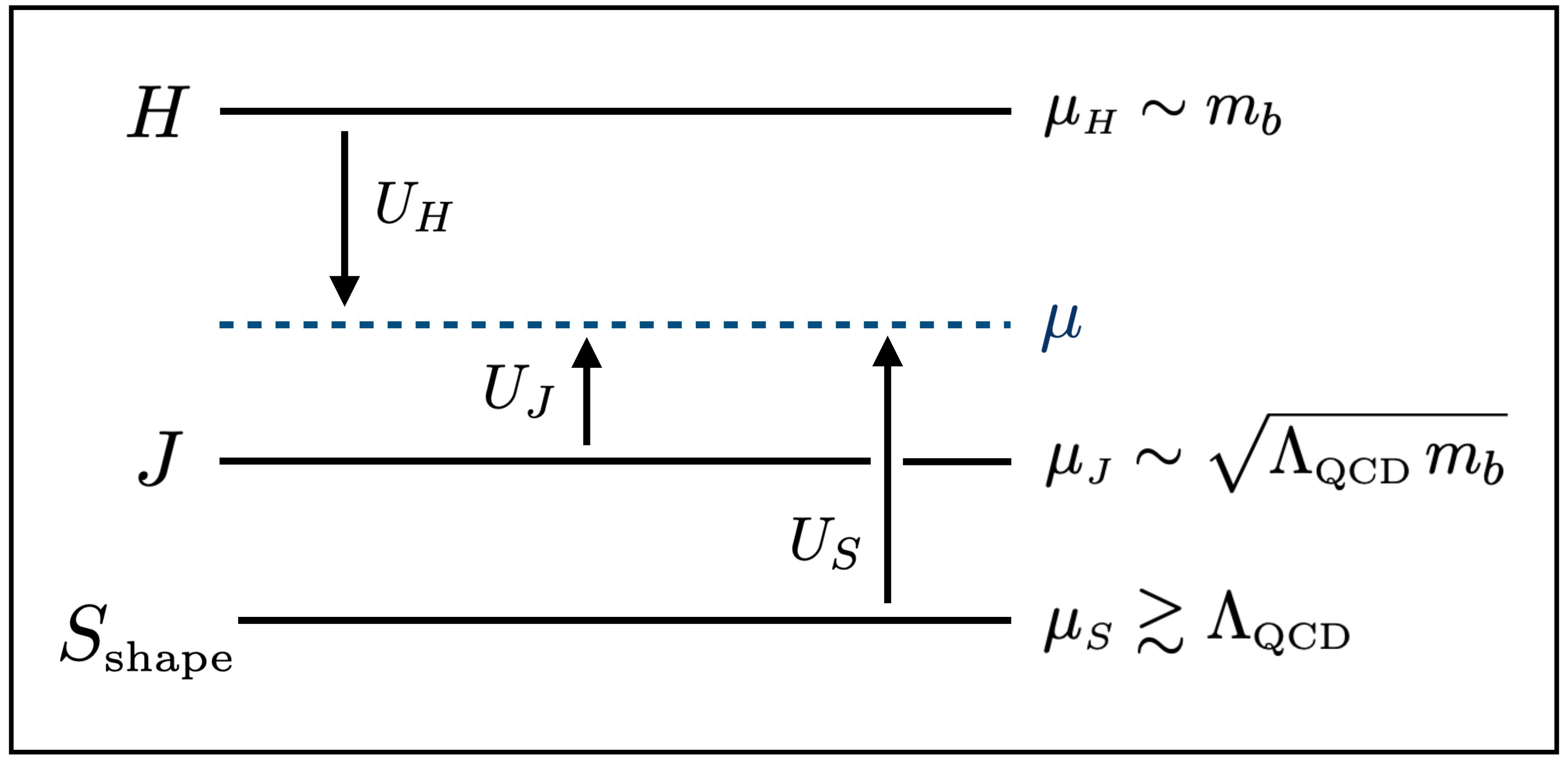
hard function
$H \sim C^2$

soft function
$$S_{\mathrm{shape}}(k^+) = \langle \bar{B} |\, \bar{h}_v\, Y_n\, \delta(k^+ - in\cdot\partial)\, Y_n^\dagger\, h_v\, | \bar{B} \rangle$$

jet function
$$J(k) \sim \mathrm{Im}\left[\int \mathrm{d}^4x\, e^{ik\cdot x}\, \langle 0 |\, T[\bar{\xi}_n W_n(0)\, \not{\bar{n}}\, W_n^\dagger \xi_n(x)]\, | 0 \rangle \right]$$

hard function: $\mu_H \sim m_b$
jet function: $\mu_J \sim \sqrt{\Lambda_{\mathrm{QCD}}\, m_b}$
soft function: $\mu_S \gtrsim \Lambda_{\mathrm{QCD}}$

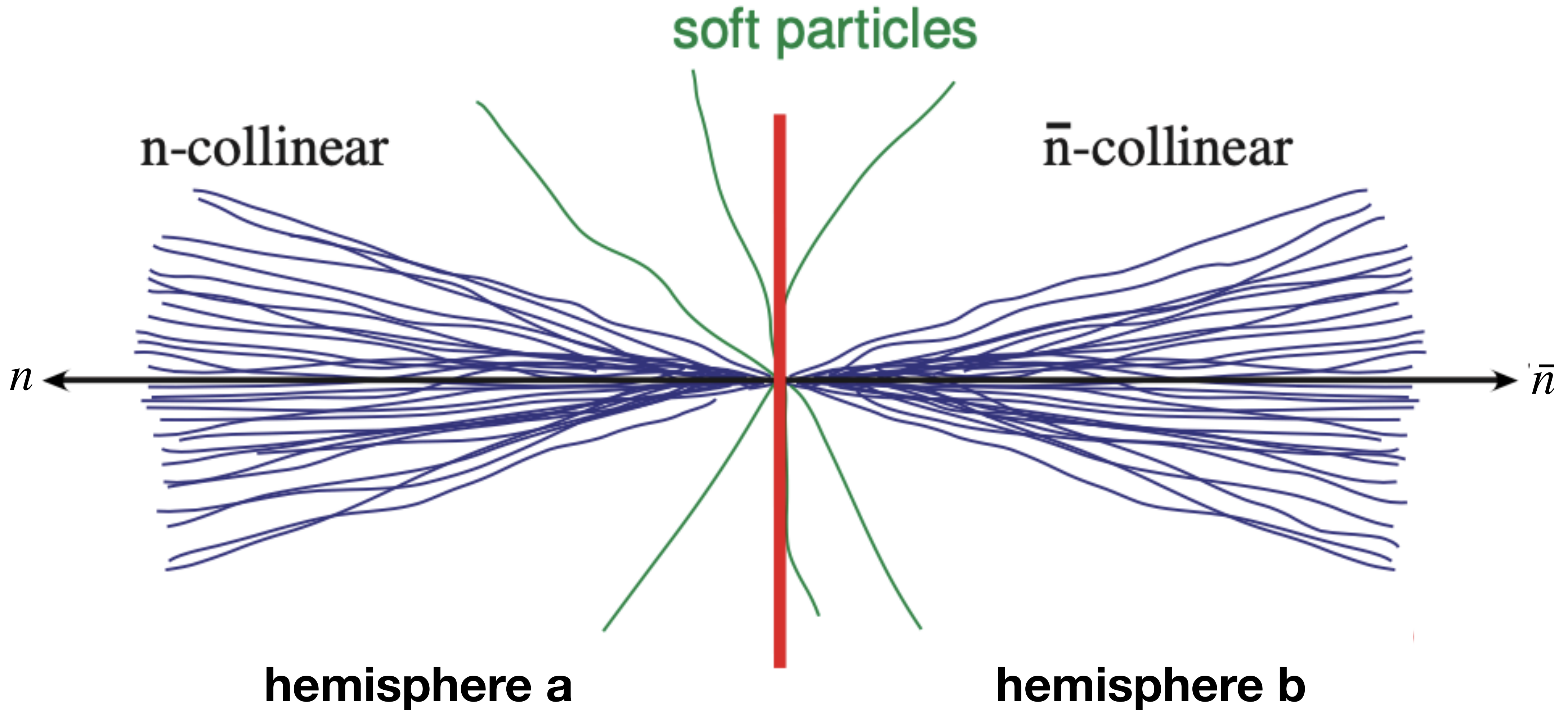


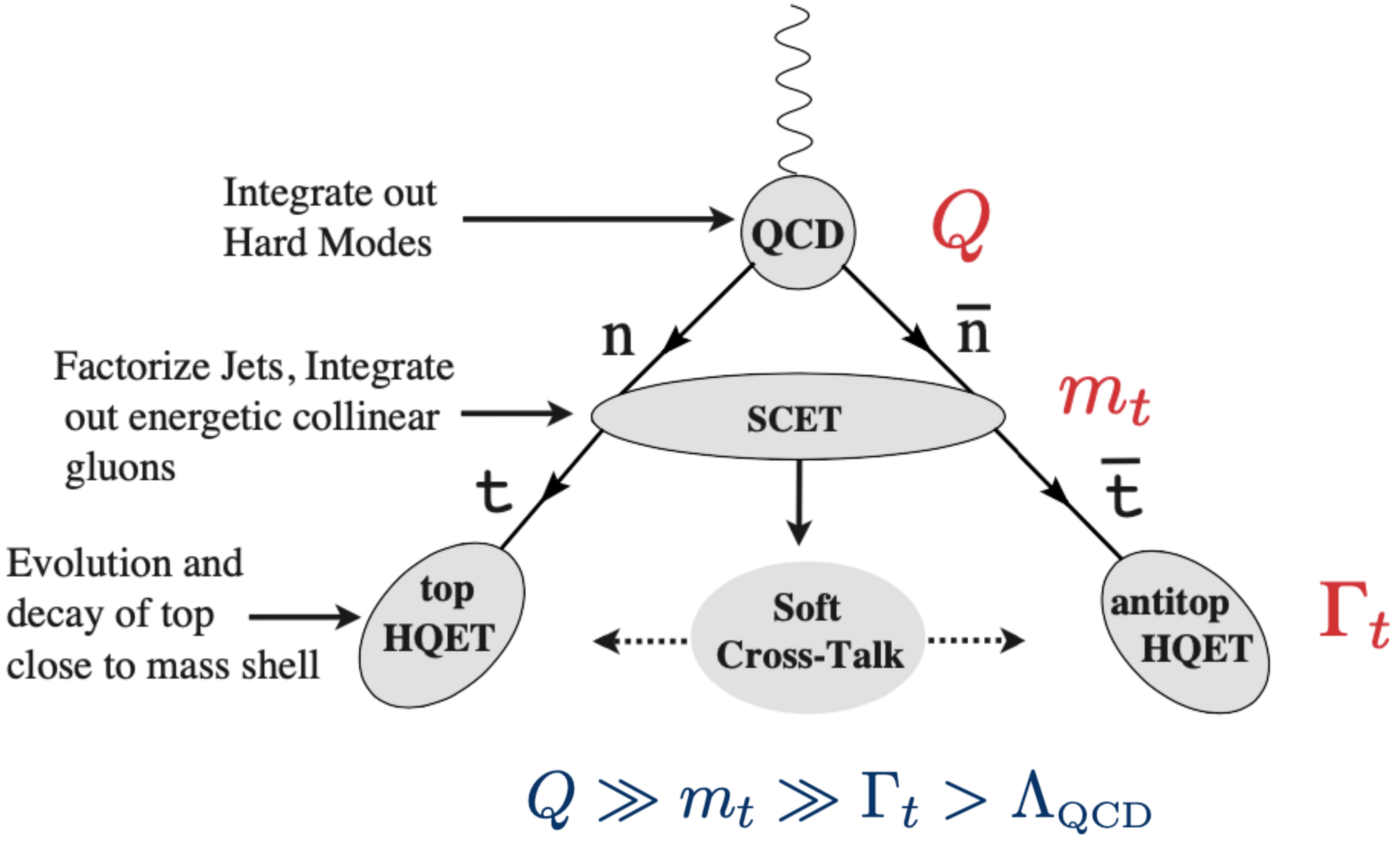
- Each sector depends only on a single physical scale $\longrightarrow$ Large logarithms can be avoided.
- RGEs can be used to evolve the distinct sectors to a common scale.

Recap: Boosted top pair production in e^+e^- collisions

- Factorization approach for inclusive boosted top hemisphere jet production in $e^+e^- \to t\bar{t}$ at c.m. energies $Q \gg m_t$.
  - Dijet region for factorization characterized by $s_{a,b} = M_{a,b}^2 - m_t^2 \ll m_t^2$.
  - Top state defined by measurements of $M_{a,b}$.

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2} \sim \sigma_0\, H_Q(Q,\mu) \int \mathrm{d}l\,\mathrm{d}l'\, J_t(s_a - Ql,\mu)\, J_{\bar{t}}(s_b - Ql',\mu)\, S(l,l',\mu)$$

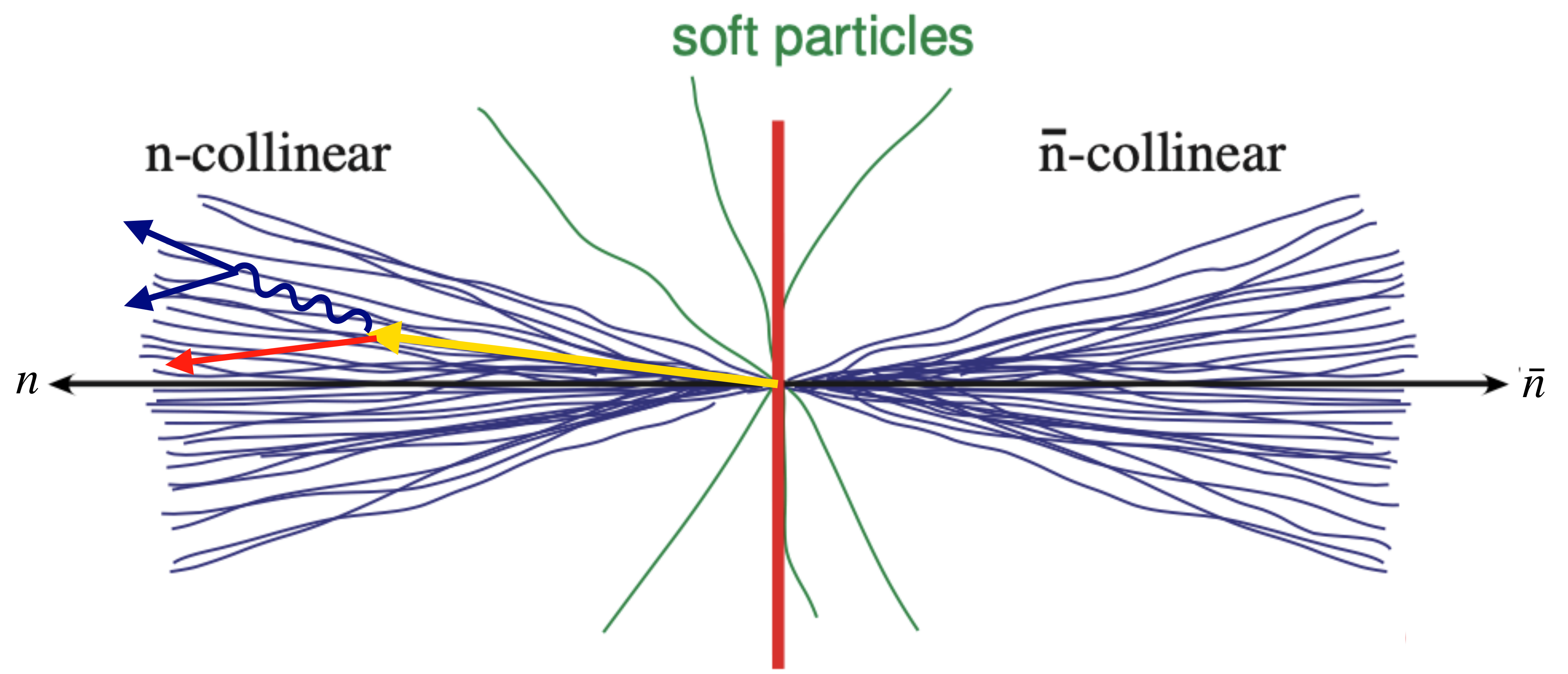




[Fleming, Hoang, Mantry, Stewart: Phys.Rev.D77:074010 (2008)]

Boosted top pair production in e^+e^- collisions

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2} \sim \sigma_0\, H_Q(Q,\mu) \int \mathrm{d}l\,\mathrm{d}l'\, J_t(s_a - Ql,\mu)\, J_{\bar{t}}(s_b - Ql',\mu)\, S(l,l',\mu)$$



- Combination of factorization approaches for top quark production in e^+e^- collision with factorization methods known from heavy quark decays.
  - Combine hemisphere mass measurements with measurements of top decay sensitive observables.
  - Treatment of finite lifetime effects and of the dynamics of the top quark decay products.
  - Study of effects at kinematic endpoint regions.

- Important application: Gauge invariant off-shell top quark decay.

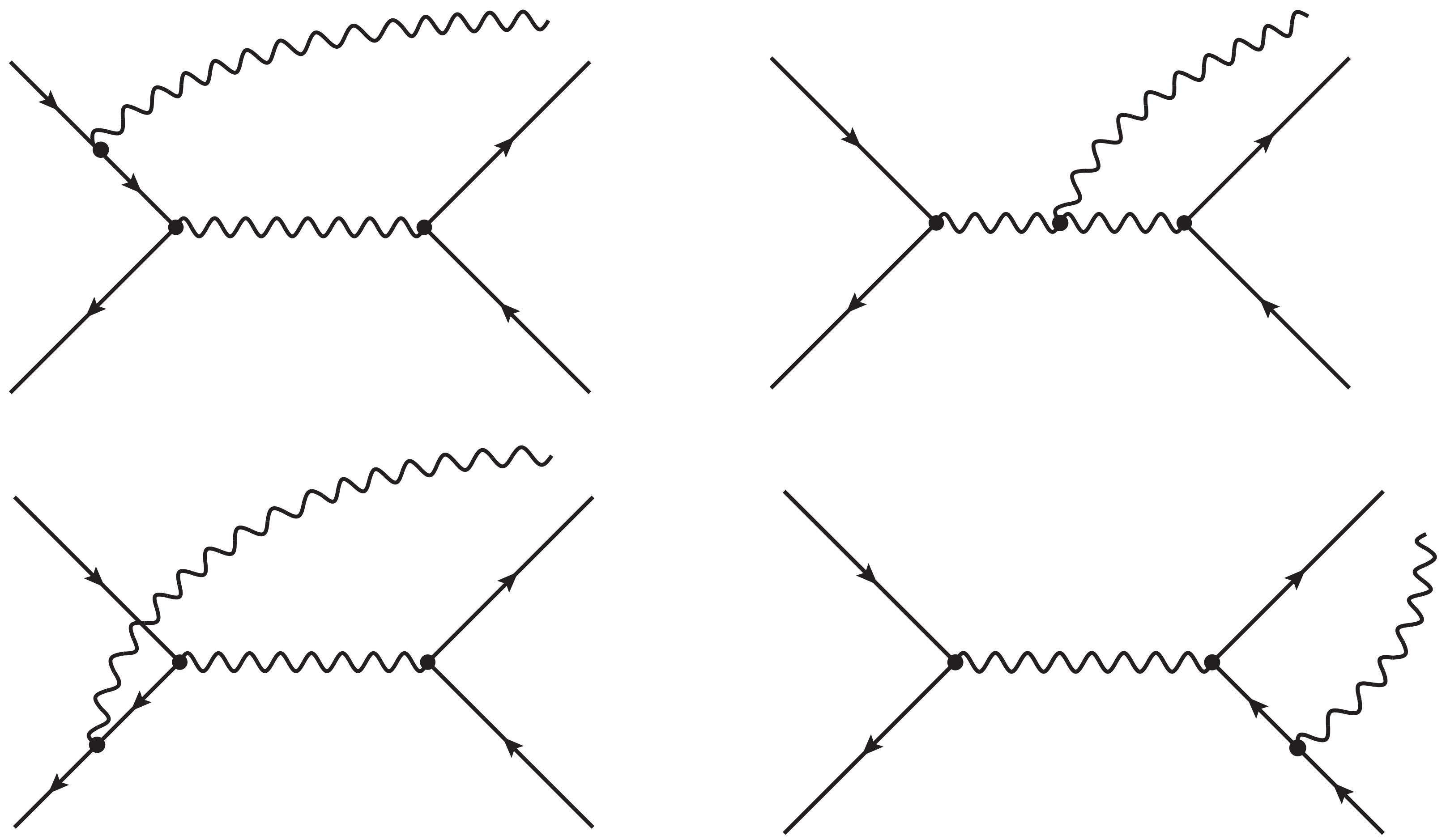
Differential top jet function

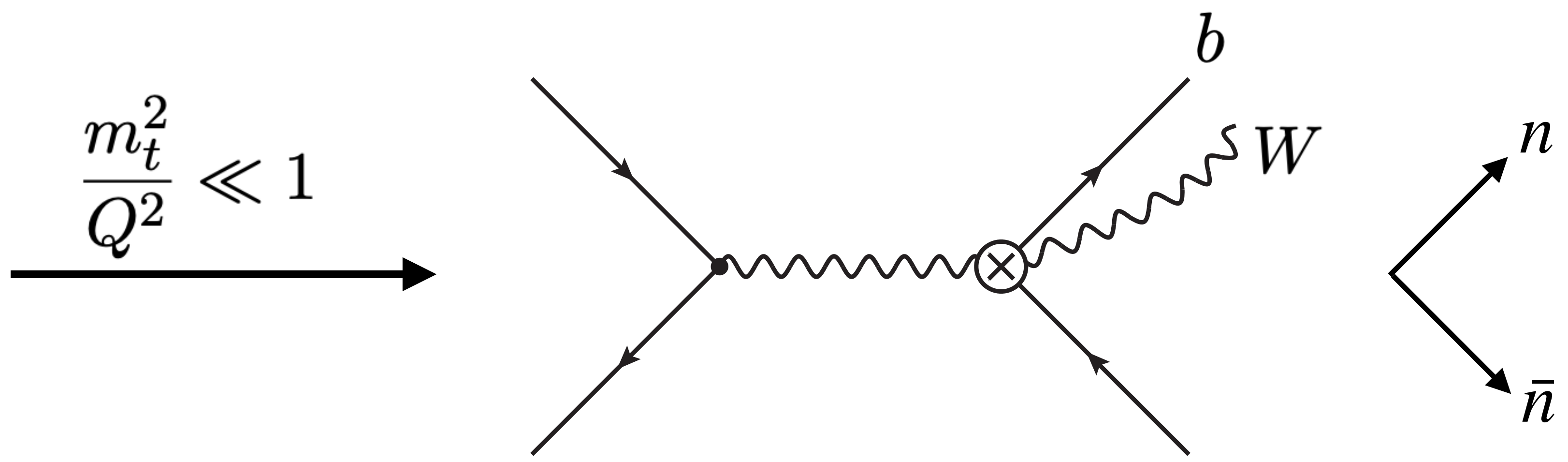
- Scrutiny of the SCET electroweak+QCD jet function describing the top quark decay.

$$J_n^{L/R}(p^2) = \frac{1}{N_c\,(\bar{n}\cdot p)} \underbrace{\sum_X}_{\text{contains phase space integration over final state particles}} (2\pi)^3\,\delta^{(4)}(p - P_X)\,\mathrm{Tr}\Big[\,\langle 0|\,\frac{\not{\bar{n}}}{4}\,\chi_n^{L/R}(0)\,|X\rangle\,\langle X|\,\overline{\chi_n^{L/R}}(0)\,|0\rangle\Big]$$

chiral jet field $\chi_n^{L/R} = W_n^\dagger\,P_{L/R}\,\xi_n$

- Underlying aspect of the collinear sector:





collinear Wilson line

$$\boxed{W_n(x) = P\exp\left(-ig\int_{-\infty}^{0}\mathrm{d}s\;\bar{n}\cdot W^a(s\bar{n}+x)\,\tau^a + \ldots\right)}$$

(isospin $+1/2$ for top quark)

isospin generators

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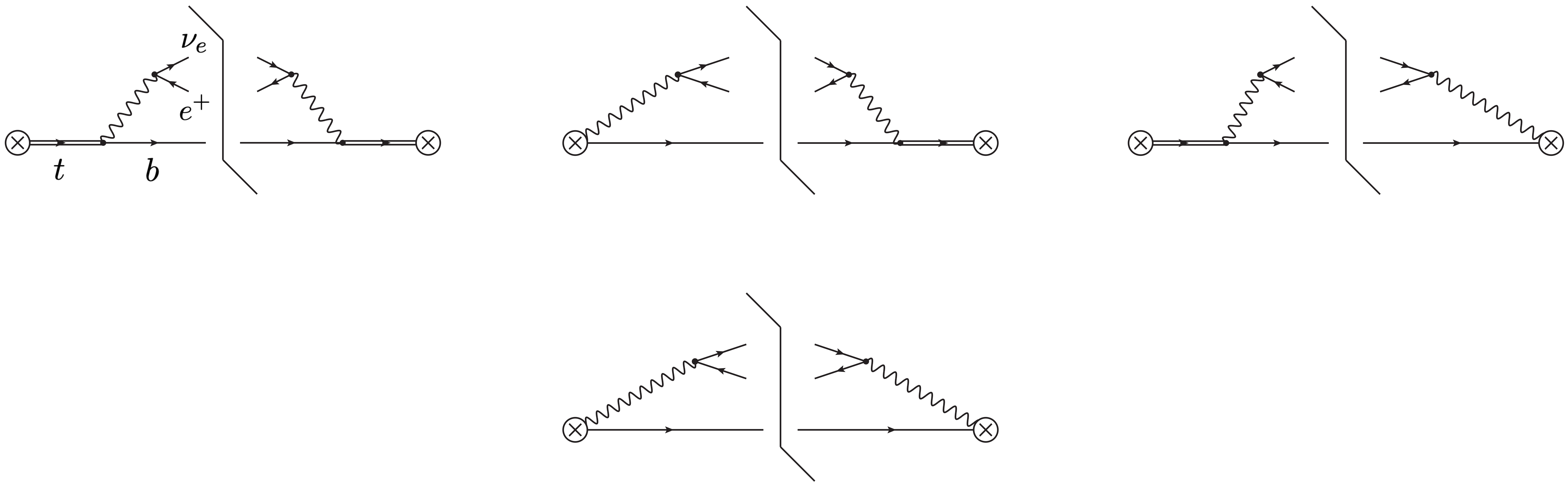
- Scrutiny of the SCET electroweak+QCD jet function describing the top quark decay.

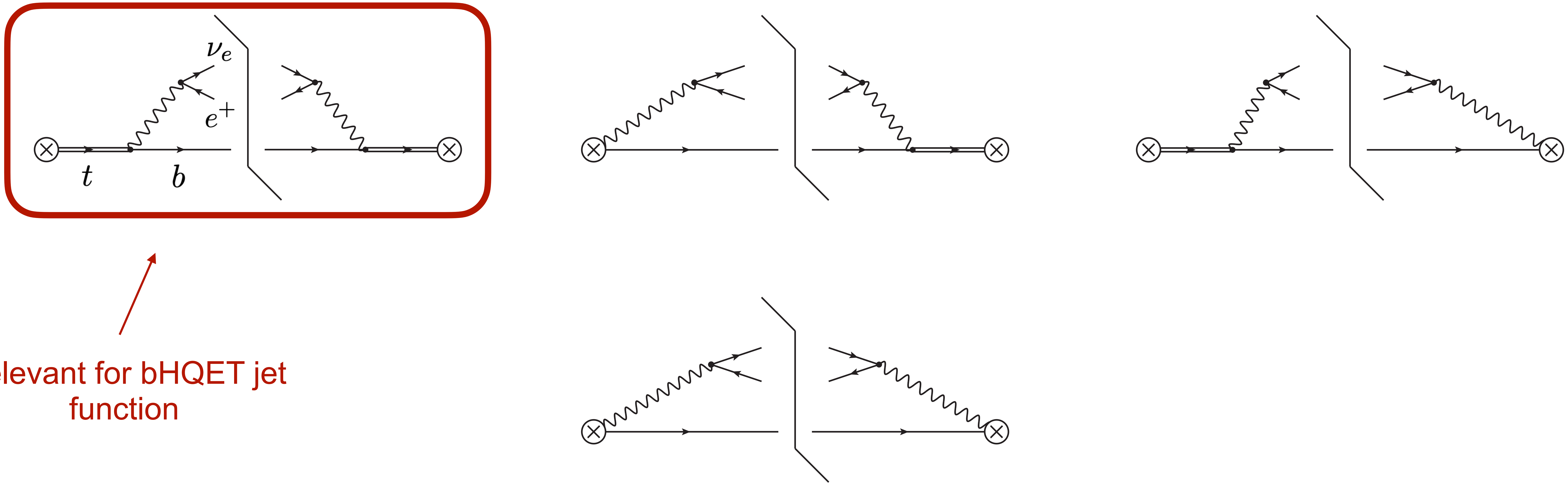
$$J_n^{L/R}(p^2) = \frac{1}{N_c\,(\bar{n}\cdot p)} \sum_X (2\pi)^3\, \delta^{(4)}(p - P_X)\, \mathrm{Tr}\Big[\langle 0|\, \frac{\bar{\not{n}}}{4}\, \chi_n^{L/R}(0)\, |X\rangle\, \langle X|\, \overline{\chi_n^{L/R}}(0)\, |0\rangle \Big]$$



- Scrutiny of the SCET electroweak+QCD jet function describing the top quark decay.

$$J_n^{L/R}(p^2) = \frac{1}{N_c\,(\bar{n}\cdot p)} \sum_X (2\pi)^3\, \delta^{(4)}(p - P_X)\, \mathrm{Tr}\left[\, \langle 0|\, \frac{\bar{\not{n}}}{4}\, \chi_n^{L/R}(0)\, |X\rangle \, \langle X|\, \overline{\chi_n^{L/R}}(0)\, |0\rangle \right]$$

DR



Relevant for bHQET jet function

Differential top jet function

- Scrutiny of the SCET electroweak+QCD jet function describing the top quark decay.

$$J_n^{L/R}(p^2) = \frac{1}{N_c\,(\bar{n}\cdot p)} \sum_X (2\pi)^3\,\delta^{(4)}(p - P_X)\,\mathrm{Tr}\Big[\,\langle 0|\,\frac{\bar{\not{n}}}{4}\,\chi_n^{L/R}(0)\,|X\rangle\,\langle X|\,\overline{\chi_n^{L/R}}(0)\,|0\rangle\Big]$$

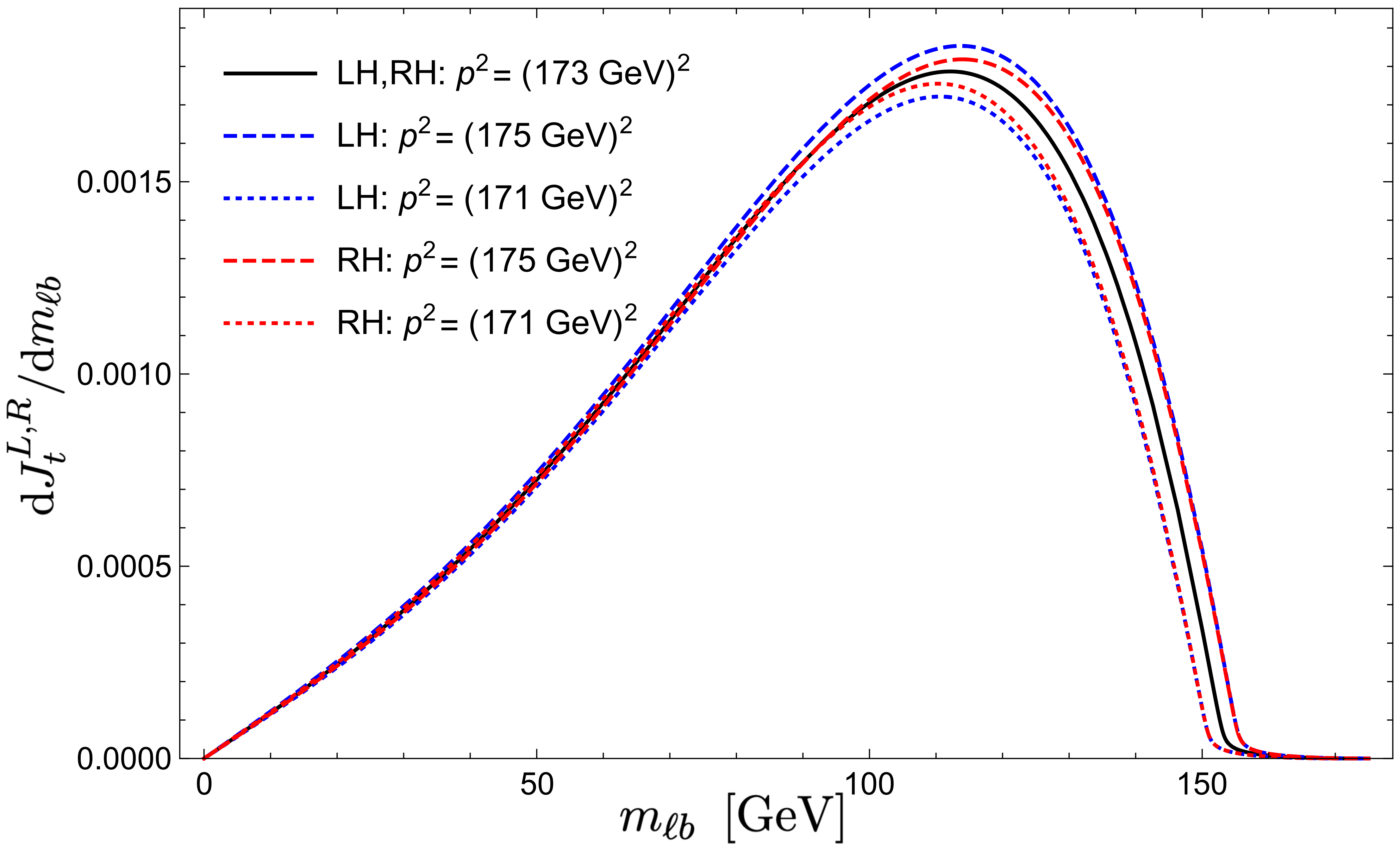
- **Result**: (valid for **boosted** top quarks)

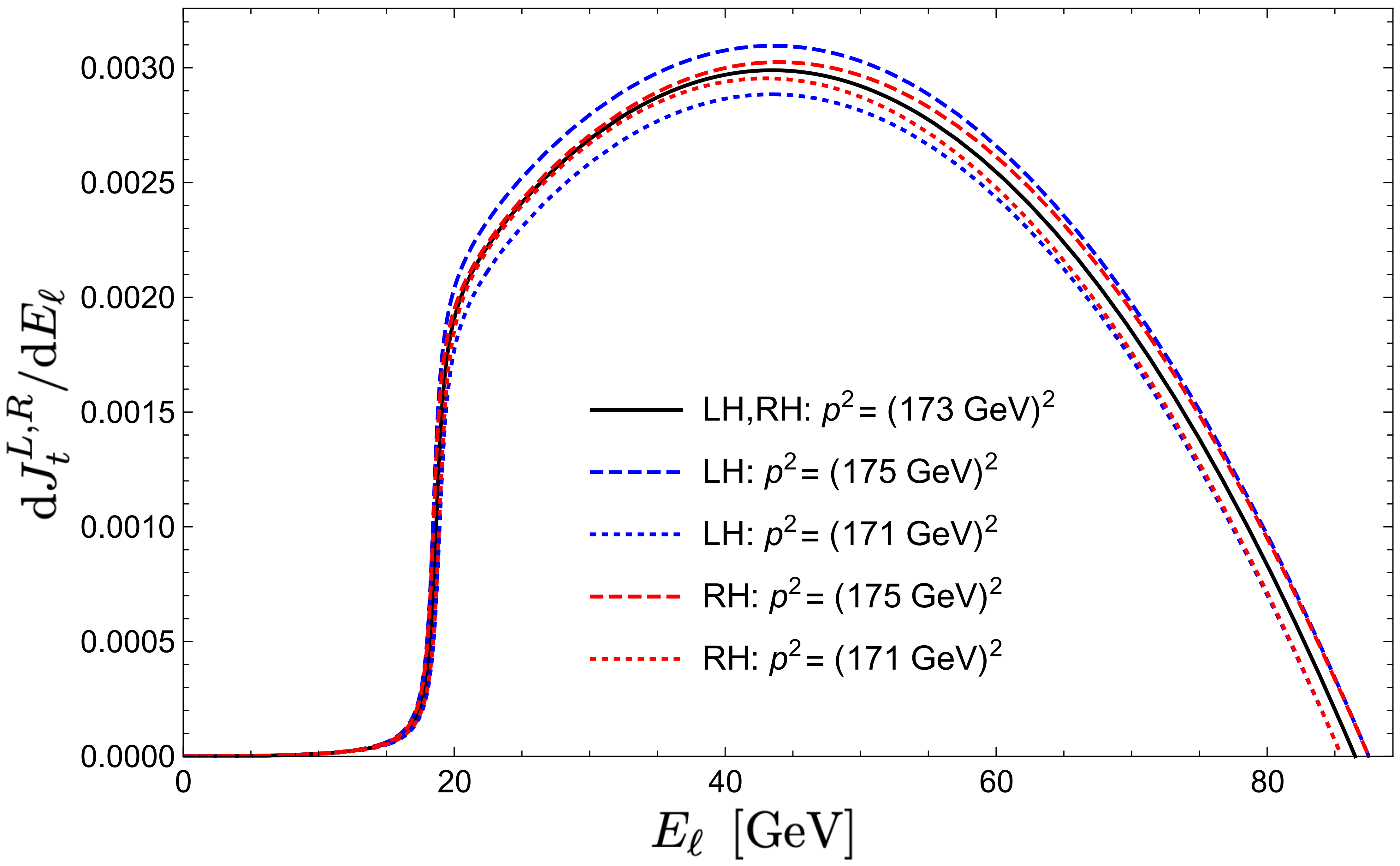
- Universal, process independent and gauge invariant jet function.
- Generalization of the concept of an on-shell top including off-shell effects.
- Accounts for spin correlations.
- Possible application: Top spin measurements for off-shell top decays.
- Excellent approximation for off-shell top production.

Differential top jet function

- $m_{\ell b}$ and E_ℓ distribution for differential top jet function including all tree level contributions:

$$m_t = 173\,\mathrm{GeV}, \qquad \Gamma_t = 1.43\,\mathrm{GeV}$$

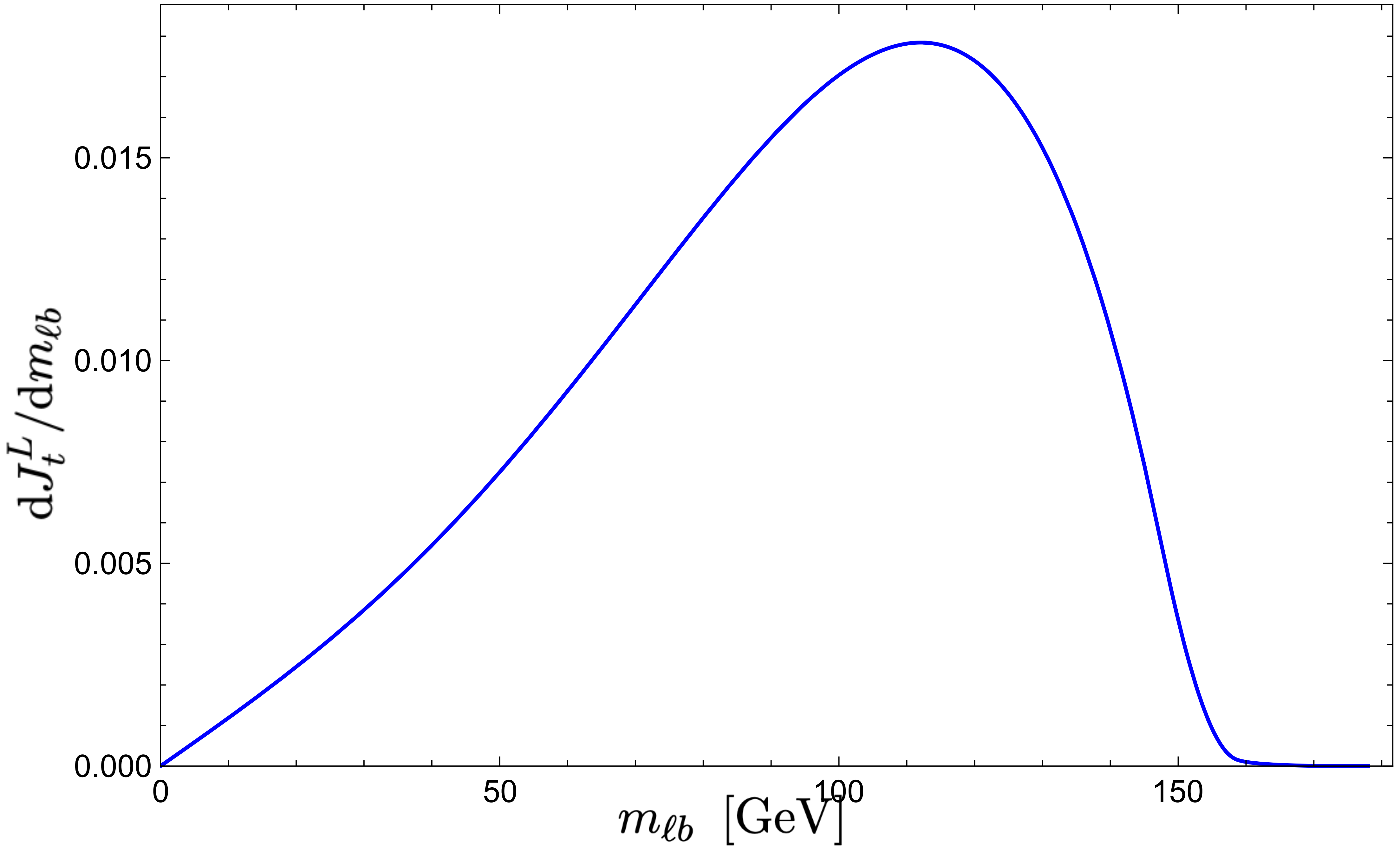


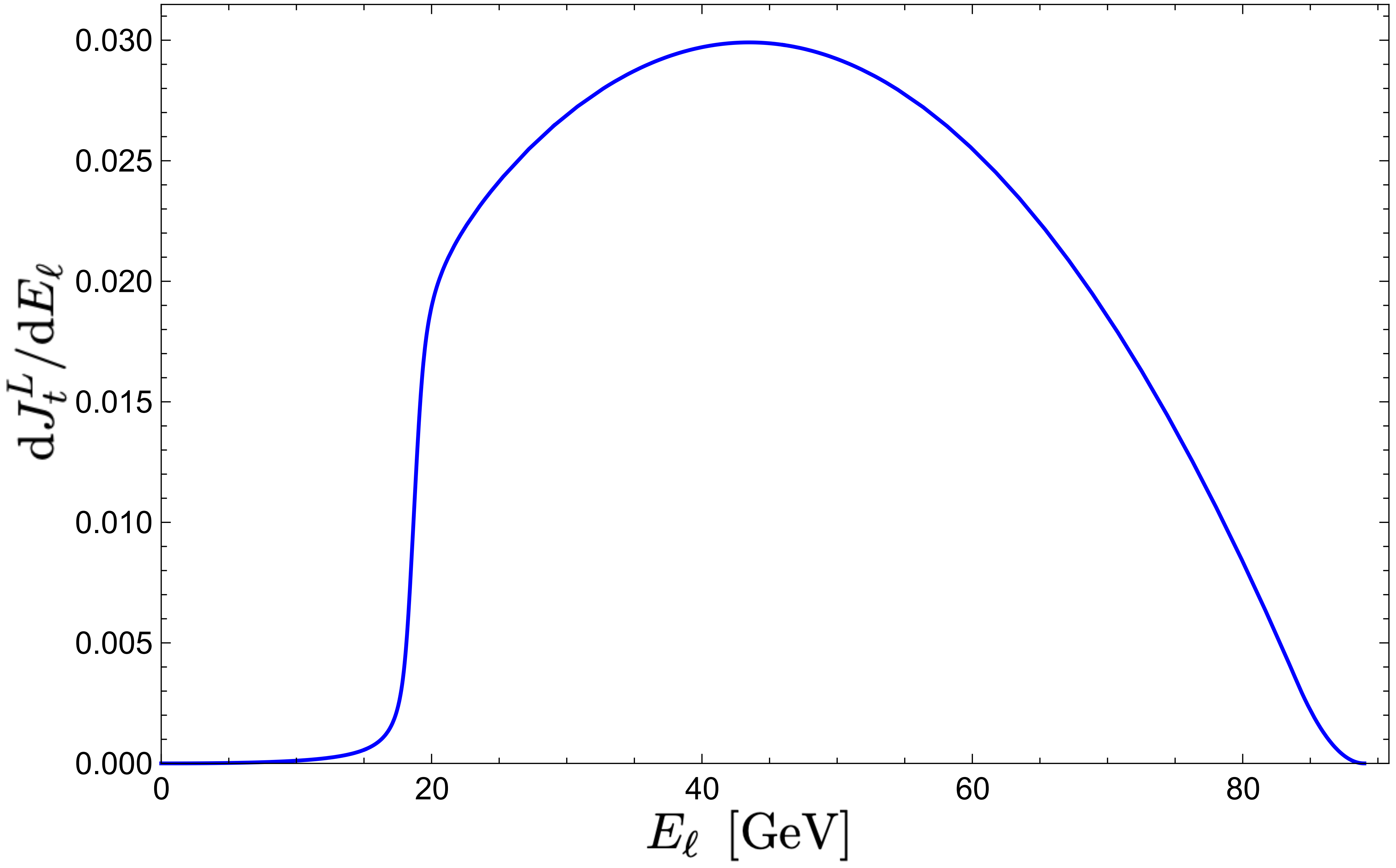


Differential top jet function

- $m_{\ell b}$ and E_ℓ distribution for differential top jet function including all tree level contributions:

$$m_t = 173\,\mathrm{GeV}, \qquad \Gamma_t = 1.43\,\mathrm{GeV}, \qquad \sqrt{p^2} \in [168, 178]$$

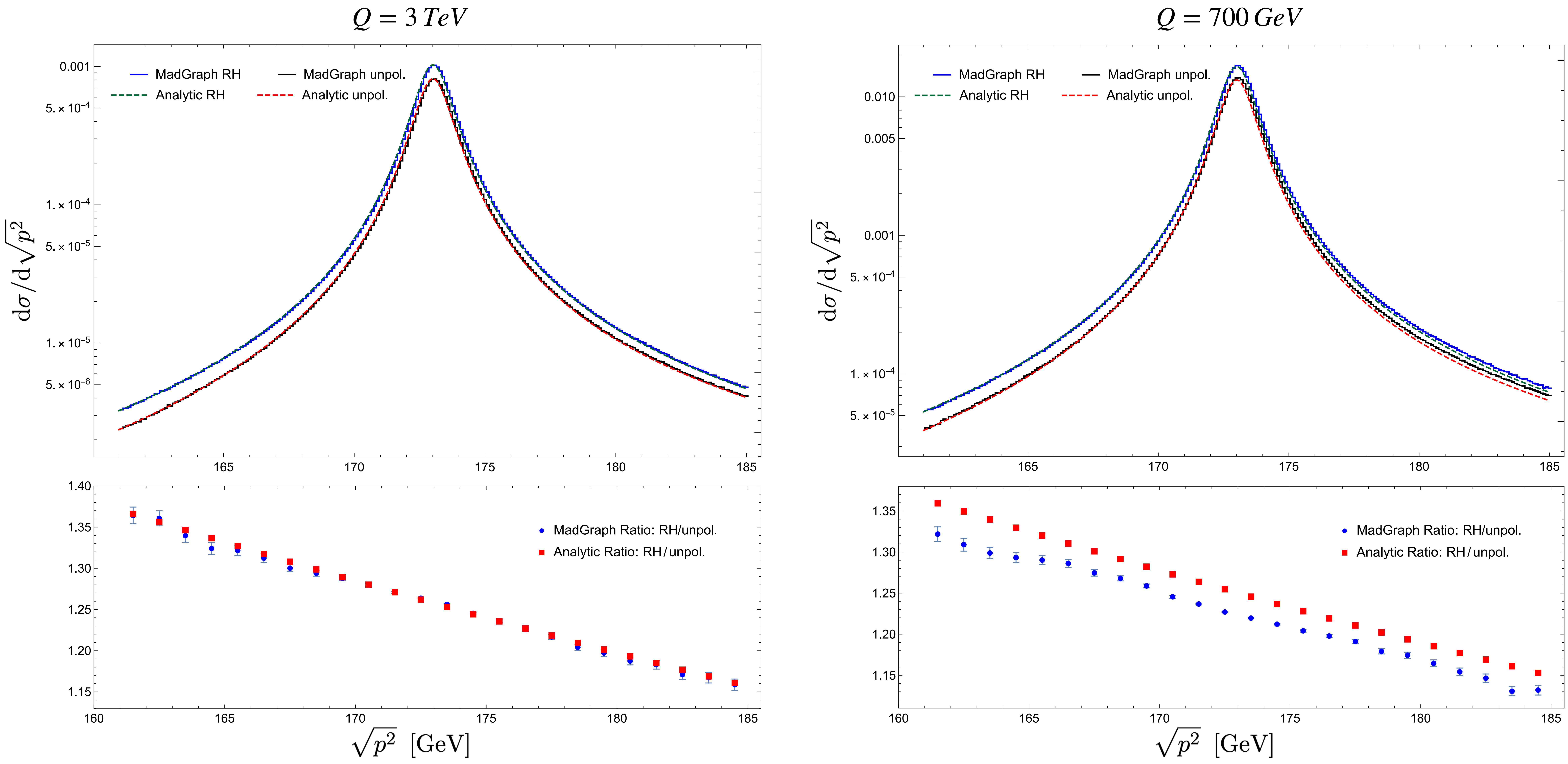




Differential top jet function

- Comparison with MadGraph prediction for $e^+e^- \to \bar{t}\, b\, W^+$ at tree level:

[A. H. Hoang, S. Plätzer, CR, I. Ruffa, work in progress]



Factorization of top quark jet function

- We want to study QCD corrections in the endpoint region.

$$J_t^{\rm SCET} = H_m\, J_t^{\rm bHQET}$$

(SCET ⟶ bHQET)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2\,\mathrm{d}E_\ell} \sim \sigma_0\, H_Q\, H_m \left[J_t^{\rm bHQET}(E_\ell) \otimes J_{\bar t}^{\rm bHQET} \otimes S\right]$$

$$J_t^{\rm bHQET} \sim H\left[J_b \otimes S_{ucs}\right]$$

- Starting point: *bHQET top jet function* $(\hat{s} \equiv (p^2 - m^2)/m = 2v \cdot k)$

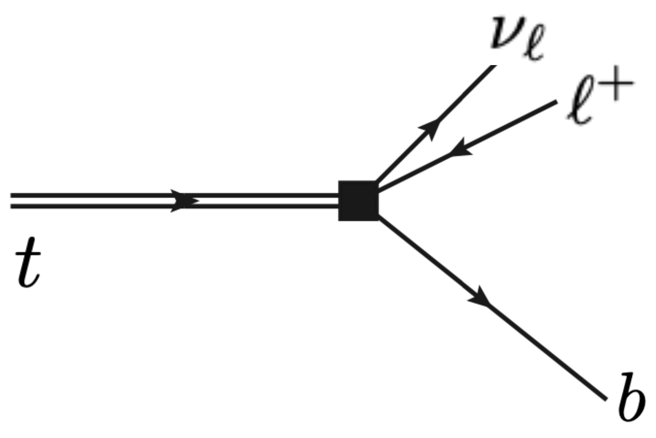
$$J_t^{\rm bHQET}(\hat{s}) \propto \sum_X (2\pi)^3\, \delta^{(4)}(mv + k - P_X)\, \mathrm{Tr}\Big[\langle 0|\, \overline{T}\, \frac{\bar{n}\!\!\!/}{4}\, [W_n^\dagger h_v](0)\, |X\rangle\, \langle X|\, T\, [\bar{h}_v W_n](0)\, |0\rangle \Big]$$

contains phase space integration over final state particles

heavy top quark field

1. Insert decay operator describing the top quark decay:

$$C_\Gamma\, \mathcal{O}_\Gamma = \left(\frac{4G_F}{\sqrt{2}}\right)^2 (\bar{b}\gamma^\mu P_L t)\,(\bar{\nu}_\ell \gamma_\mu P_L \ell)$$



2. Match QCD current onto heavy-to-light EFT currents:

$$\mathcal{J}^\mu_{\rm QCD} = \bar{b}\gamma^\mu P_L t \longrightarrow \mathcal{J}^\mu \sim \sum_i \int \mathrm{d}\omega\, C_i(\omega)\, \left(\bar{\chi}_{n',\omega} \Gamma_i^\mu h_v\right), \qquad \Gamma_i^\mu = \left\{ \gamma^\mu P_L,\, v^\mu P_R,\, \frac{n'^\mu}{n'\cdot v} P_R \right\}$$

light b quark jet field

b jet direction [new aspect!]

Factorization of top quark jet function

3. Phase space factorization: (Define total hadronic momentum $h = \sum_n p_n$ and leptonic momentum $q = p_\ell + p_{\nu_\ell}$.)

$$\mathrm{d}\Pi_{n+2}(p) = \frac{\mathrm{d}q^2}{2\pi}\frac{\mathrm{d}h^2}{2\pi}\,\mathrm{d}\Pi_2(p;q,h)\,\mathrm{d}\Pi_2(q;p_\ell,p_{\nu_\ell})\,\mathrm{d}\Pi_n(h)\,, \qquad \mathrm{d}\Pi_n(h) = \prod_n \frac{\mathrm{d}p_n}{(2\pi)^3}\frac{1}{2E_n}\,(2\pi)^4\,\delta^{(4)}\Big(h - \sum_n p_n\Big)$$

4. Decoupling of n'-collinear and ultracollinear-soft degrees of freedom:

$$\mathcal{L}_{n'} = \bar{\xi}_{n'}\, in'\cdot D_s\,\frac{\bar{n}'}{2}\,\xi_{n'} + \ldots$$

$$\xi_{n'} \to Y_{n'}\,\xi_{n'}\,, \qquad W_{n'} \to Y_{n'}\,W_{n'}\,Y^\dagger_{n'}\,,$$

$$Y_{n'}(x) = \overline{\mathrm{P}}\exp\Big(-ig_s\int_0^\infty \mathrm{d}s\; n'\cdot A_s(n's+x)\Big)$$

soft Wilson line

$$\mathcal{L}_{n'} = \bar{\xi}_{n'}\, in'\cdot \partial_s\,\frac{\bar{n}'}{2}\,\xi_{n'} + \ldots$$

$$\Rightarrow \quad J_t(\hat{s}) \sim \int \frac{\mathrm{d}q^2}{2\pi}\frac{\mathrm{d}h^2}{2\pi}\,\mathrm{d}\Pi_2(p;q,h)\,\mathrm{d}\Pi_2(q;p_\ell,p_{\nu_\ell})\,L_{\mu\nu}(p_\ell,p_{\nu_\ell})\,K^{\mu\nu}(h,\hat{s},\bar{n}\cdot n')$$

Factorization of top quark jet function

$$K^{\mu\nu} \sim \int \mathrm{d}^4 z_1\, \mathrm{d}^4 z_2 \int \mathrm{d}\Pi_n(h) \sum_{j,j'} \int \mathrm{d}\omega\, \mathrm{d}\omega'\, C_j(\omega) C_{j'}(\omega')$$
$$\times \mathrm{Tr}\Big[\langle 0|\, \overline{T}[\bar{h}_v Y_{n'} \bar{\Gamma}^{\mu}_{j'} \chi_{n',\omega'}](z_2)\, [W_n^\dagger \bar{n\!\!\!/}\, h_v](0)\, |X_{n'}, X_{uc}\rangle\langle X_{n'}, X_{uc}|\, T[\bar{h}_v W_n](0)\, [\bar{\chi}_{n',\omega} \Gamma^{\nu}_j Y^\dagger_{n'} h_v](z_1)\, |0\rangle \Big]$$

5. Rearranging the momentum modes in $K^{\mu\nu}$ into distinct sectors

- SCET Fierz for light b quark jet fields $\chi_{n'}$
- HQET Fierz for heavy top quark jet fields h_v
- Color Fierz
- …

⟶ Factorized form of hadronic tensor:

$\hat{h}^+ = n' \cdot h$

$$K^{\mu\nu} \sim \sum_{j,j'} C_j(\hat{h}^-)\, C_{j'}(\hat{h}^-)\, \mathrm{Tr}\left[\frac{P_v}{2} \bar{\Gamma}^{\mu}_{j'} \frac{n\!\!\!/'}{2} \Gamma^{\nu}_j\right] \int \mathrm{d}\hat{r}^+\, \hat{h}^-\, J_b(\hat{h}^- \hat{r}^+)\, S_{ucs}(\hat{h}^+ - \hat{r}^+, \hat{s}, \gamma^2)$$

Factorization of top quark jet function

$$K^{\mu\nu} \sim \int \mathrm{d}^4 z_1\, \mathrm{d}^4 z_2 \int \mathrm{d}\Pi_n(h) \sum_{j,j'} \int \mathrm{d}\omega\, \mathrm{d}\omega'\, C_j(\omega) C_{j'}(\omega')$$

$$\times \mathrm{Tr}\big[\, \langle 0|\, \overline{T}[\bar{h}_v Y_{n'} \bar{\Gamma}^{\mu}_{j'} \chi_{n',\omega'}](z_2)\, [W_n^\dagger\, \bar{n}\!\!\!/\, h_v](0)\, |X_{n'}, X_{uc}\rangle\langle X_{n'}, X_{uc}|\, T[\bar{h}_v W_n](0)\, [\bar{\chi}_{n',\omega} \Gamma^{\nu}_{j} Y^{\dagger}_{n'} h_v](z_1)\, |0\rangle\, \big]$$

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light b quark jet function

$$J_b(k) \sim \mathrm{Im}\left[\int \mathrm{d}^4 x\, \mathrm{e}^{ik\cdot x}\, \langle 0|\, T[\bar{\chi}_{n'}(0)\, \bar{n}\!\!\!/'\, \chi_{n'}(x)]\, |0\rangle \right]$$

Factorization of top quark jet function

$$K^{\mu\nu} \sim \int \mathrm{d}^4 z_1\, \mathrm{d}^4 z_2 \int \mathrm{d}\Pi_n(h) \sum_{j,j'} \int \mathrm{d}\omega\, \mathrm{d}\omega'\, C_j(\omega) C_{j'}(\omega')$$
$$\times \mathrm{Tr}\Big[\langle 0 | \overline{T}[\bar{h}_v Y_{n'} \bar{\Gamma}^{\mu}_{j'} \chi_{n',\omega'}](z_2)\, [W_n^\dagger \not{\bar{n}}\, h_v](0)\, | X_{n'}, X_{uc}\rangle\langle X_{n'}, X_{uc} |\, T[\bar{h}_v W_n](0)\, [\bar{\chi}_{n',\omega} \Gamma^{\nu}_{j} Y^\dagger_{n'} h_v](z_1)\, | 0 \rangle \Big]$$

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“bHQET ultracollinear-soft function”

⟶ New ingredient

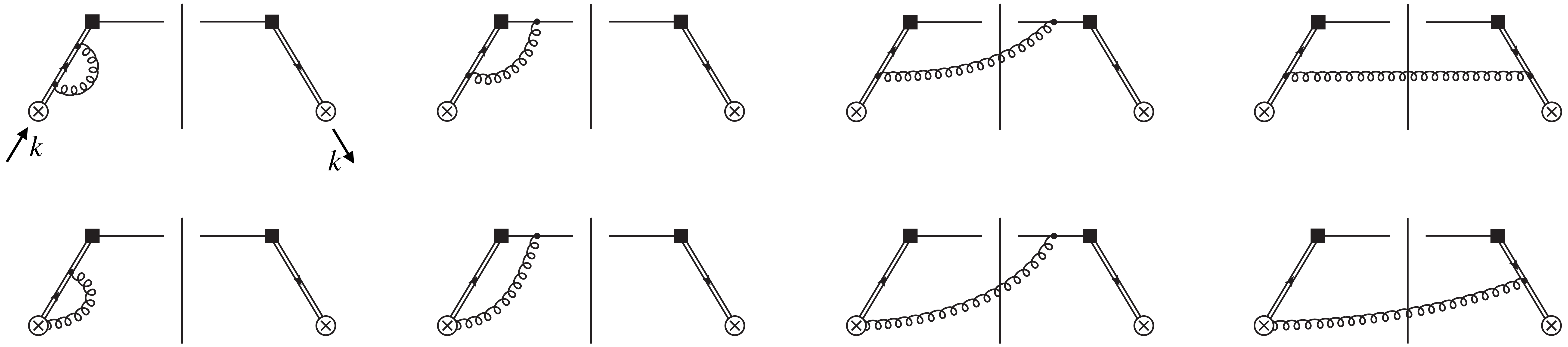
Ultracollinear-Soft Function

[A. H. Hoang, CR work in progress]

$\hat{a}^+ \equiv \hat{a}^+(\hat{s}, h^2, q^2)$ $\qquad \gamma^2 = 2/(\bar{n}\cdot n')$

$$\boxed{\begin{aligned} S_{ucs}(\hat{a}^+, \hat{s}, \gamma^2) &= \frac{1}{N_c}\int d^4y_1 \int d^4y_2 \sum_{X_{uc}} \delta(\hat{k}^+_{uc} - \hat{a}^+)\, e^{-ik\cdot(y_1-y_2)} \\ &\times \langle 0|\,\overline{T}\,[\bar{h}_v\, Y_{n'}]^b_\beta(0)\,[W_n^\dagger\, h_v]^a_\alpha(y_2)\,|X_{uc}\rangle\,\langle X_{uc}|\,T\,[\bar{h}_v\, W_n]^a_\alpha(y_1)\,[Y^\dagger_{n'}\, h_v]^b_\beta(0)\,|0\rangle \end{aligned}}$$

- New ingredient
- Can be computed perturbatively.
- Top width acts as infrared cutoff.
- Describes Fermi motion of the decaying top in the measured state with mass M_a.
- Generalizes the shape function for heavy meson decays



Ultracollinear-Soft Function

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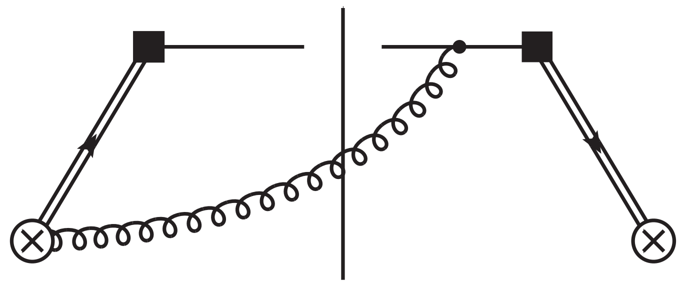

$\hat{p}_g^+ = n'\cdot p_g$

$$I \sim \alpha_s \frac{2\,(\bar{n}\cdot n')}{\hat{s} - i\Gamma}\,\mu^{2\epsilon}\int \frac{\mathrm{d}^{d-1}p_g}{(2\pi)^{d-1}}\,\frac{1}{2E_{p_g}}\,\delta(\hat{p}_g^+ - \hat{a}^+)\,\frac{1}{\bar{n}\cdot p_g + i0}\,\frac{1}{\hat{p}_g^+ - i0}\,\frac{1}{\frac{1}{2}(\hat{s}+i\Gamma) - v\cdot p_g}$$

$\hat{s} = 2v\cdot k$

Ultracollinear-Soft Function

[A. H. Hoang, CR work in progress]

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- $S_{ucs}@\,\mathcal{O}(\alpha_s):$

$$S_{ucs} = \frac{\alpha_s C_F}{4\pi}\, f(\hat{a}^+, \hat{s}, \gamma^2)$$

$f(\hat{a}^+, \hat{s}, \gamma^2) \in$

$\frac{\delta(\hat{a}^+)}{\lvert\hat{s}+i\Gamma\rvert^2}$	$\left[\frac{1}{\hat{a}^+}\right]_+ \frac{1}{\lvert\hat{s}+i\Gamma\rvert^2}\left[\log^n\!\left(\frac{\hat{a}^+-\hat{s}-i\Gamma}{\mu}\right)+c.c.\right]$	$\frac{i}{\Gamma}\,\frac{1}{\lvert\hat{s}+i\Gamma\rvert^2}\left[\log^n\!\left(\frac{\hat{a}^+-\hat{s}-i\Gamma}{\mu}\right)-c.c.\right]$
$\frac{\delta(\hat{a}^+)}{\lvert\hat{s}+i\Gamma\rvert^2}\,\log^2(\gamma^2-1)$	$\left[\frac{1}{\hat{a}^+}\right]_+ \frac{1}{\lvert\hat{a}^+\gamma^2-\hat{s}-i\Gamma\rvert^2}\left[\log^n\!\left(\frac{\hat{a}^+(\gamma^2)-\hat{s}-i\Gamma}{\mu}\right)+c.c.\right]$	$\frac{i}{\Gamma}\,\frac{1}{\lvert\hat{s}+i\Gamma\rvert^2}\,\frac{\hat{s}\,\gamma^2}{\lvert\hat{a}^+\gamma^2-\hat{s}-i\Gamma\rvert^2}\left[(\hat{a}^+\gamma^2-\hat{s}+i\Gamma)\log^n\!\left(\frac{\hat{a}^+(\gamma^2)-\hat{s}-i\Gamma}{\mu}\right)-c.c.\right]$
$\frac{\delta(\hat{a}^+)}{\lvert\hat{s}+i\Gamma\rvert^2}\left[\log^n\!\left(\frac{-\hat{s}-i\Gamma}{\mu}\right)+c.c.\right]$	$\left[\frac{\log(\hat{a}^+)}{\hat{a}^+}\right]_+ \frac{1}{\lvert\hat{a}^+\gamma^2-\hat{s}-i\Gamma\rvert^2}$	$\frac{1}{\lvert\hat{s}+i\Gamma\rvert^2}\,\frac{\gamma^2}{\lvert\hat{a}^+\gamma^2-\hat{s}-i\Gamma\rvert^2}\left[(\hat{s}+i\Gamma)\log^n\!\left(\frac{\hat{a}^+(\gamma^2)-\hat{s}-i\Gamma}{\mu}\right)+c.c.\right]$

Ultracollinear-Soft Function

[A. H. Hoang, CR work in progress]

$\hat{a}^+ \equiv \hat{a}^+(\hat{s}, h^2, q^2)$ $\qquad \gamma^2 = 2/(\bar{n}\cdot n')$

$$\boxed{\begin{aligned} S_{ucs}(\hat{a}^+, \hat{s}, \gamma^2) = \frac{1}{N_c} \int \mathrm{d}^4 y_1 \int \mathrm{d}^4 y_2 \sum_{X_{uc}} \delta(\hat{k}^+_{uc} - \hat{a}^+)\, \mathrm{e}^{-ik\cdot(y_1 - y_2)} \\ \times \langle 0 | \overline{T}\, [\bar{h}_v\, Y_{n'}]^b_\beta(0)\, [\, W_n^\dagger\, h_v]^a_\alpha(y_2)\, | X_{uc}\rangle \, \langle X_{uc} | \, T\, [\bar{h}_v\, W_n]^a_\alpha(y_1)\, [\, Y^\dagger_{n'}\, h_v]^b_\beta(0)\, | 0 \rangle \end{aligned}}$$

- Renormalization of S_{ucs} :

$$S^{\mathrm{bare}}_{ucs}(\hat{a}^+, \hat{s}, \gamma^2) = \int \mathrm{d}\hat{s}' \; \underbrace{Z(\hat{s} - \hat{s}')}_{Z_{J_t} + Z_{\mathrm{shape}}} \; S_{ucs}(\hat{a}^+, \hat{s}', \gamma^2)$$

- S_{ucs} needs to reproduce all UV divergences of the inclusive jet function and the shape function

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- S_{ucs} needs to reproduce all UV divergences of the inclusive jet function and the shape function

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}h^2} \sim H(E_\ell, \mu) \int \mathrm{d}k^+\, J(k^+, \mu)\, S_{\mathrm{shape}}(m_B - m_b - k^+, \mu)$$

$$\frac{\mathrm{d}^3 J_t}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}h^2} \sim \overbrace{L_{\mu\nu} \sum_{j,j'} C_j\, C_{j'}\, \mathrm{Tr}\left[\frac{P_v}{2}\bar{\Gamma}^\mu_{j'}\frac{\not{n}'}{2}\Gamma^\nu_j\right]}^{=H} \int \mathrm{d}\hat{r}^+\, J_b(\hat{h}^- \hat{r}^+)\, S_{ucs}(\hat{h}^+ - \hat{r}^+, \hat{s}, \gamma^2)$$

Factorization for boosted top pair production

- New factorization theorem for boosted top quark pair production in e^+e^- collisions including the effects of the top quark decay:

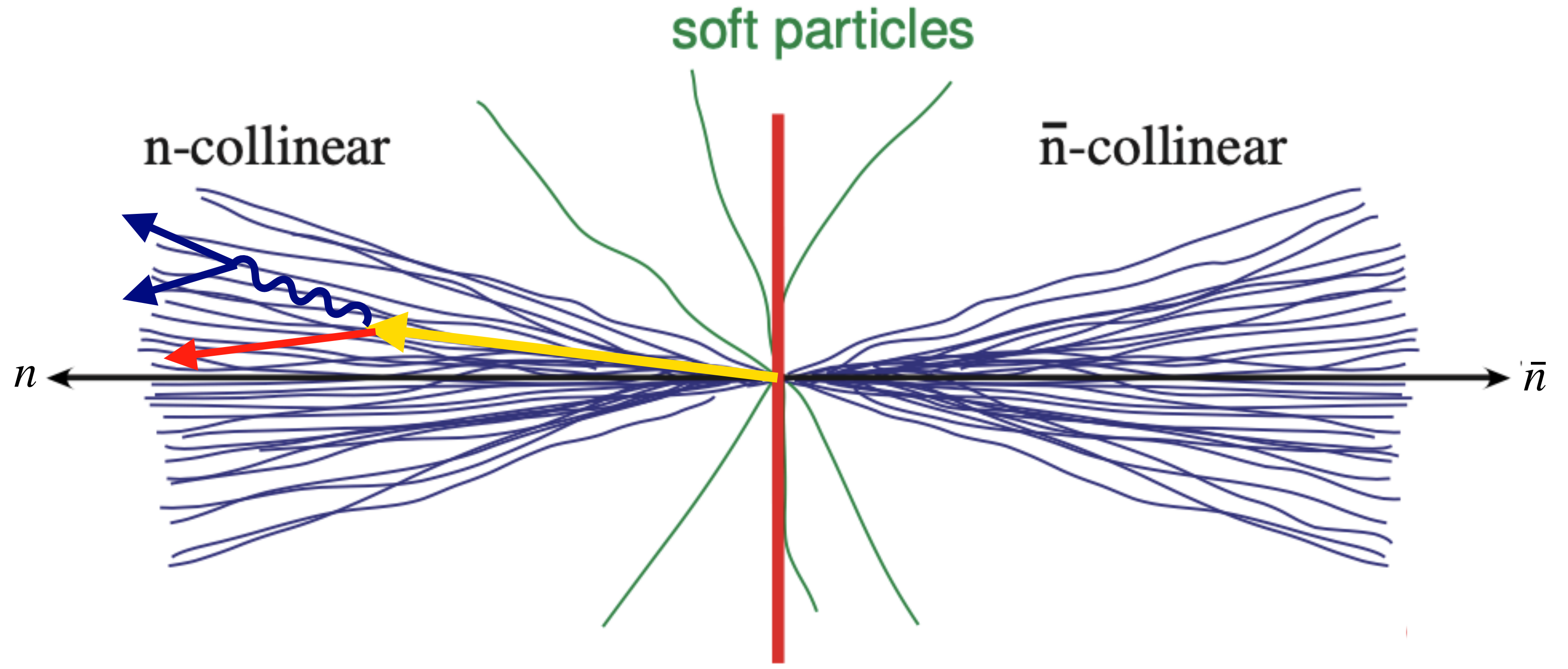
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2} \sim \sigma_0\, H_Q(Q,\mu)\, H_m(m_t,\mu) \int \mathrm{d}\ell^+\,\mathrm{d}\ell^-\, J_t\!\left(\hat{s}_a - \frac{Q\ell^+}{m_t},\mu\right) J_{\bar{t}}\!\left(\hat{s}_b - \frac{Q\ell^-}{m_t},\mu\right) S(\ell^+,\ell^-,\mu)$$

$$\boxed{\begin{aligned}\frac{\mathrm{d}^3 J_t}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}h^2} \sim & \int \mathrm{d}\Pi_2(p;h,q)\,\mathrm{d}\Pi_2(q;p_\ell,p_{\nu_\ell})\,\delta\!\left(E_\ell - \frac{p\cdot p_\ell}{\sqrt{p^2}}\right)\\ & \times L_{\mu\nu}\sum_{j,j'} C_j\, C_{j'}\, \mathrm{Tr}\left[\frac{P_v}{2}\bar{\Gamma}^{\mu}_{j'}\frac{\not{n}'}{2}\Gamma^{\nu}_{j}\right]\int \mathrm{d}\hat{r}^+\,\hat{h}^-\,J_b(\hat{h}^-\hat{r}^+)\,S_{ucs}(\hat{h}^+-\hat{r}^+,\hat{s},\gamma^2)\end{aligned}}$$

inclusive

differential

- Describes semileptonically decaying top quarks within the top jet function.
- Allows to study top decay sensitive observables in the endpoint region.
- Generalizes the results of known shape functions for heavy meson decays.



Summary & Outlook

Summary:

- We aim to combine properties of the NW limit and off-shell computations for decaying top quark studies.
  - Merge existing factorization theorems for boosted off-shell top production in e^+e^- collisions and for heavy quark decays in the endpoint region.
  - Top state **defined by measurements** (and not by NW limit).
  - Leads to a gauge-invariant jet function for boosted top quarks including off-shell effects (up to leading order in m_t/Q).
  - (b)HQET expansion allows to study NLO QCD corrections to decay sensitive observables beyond the commonly employed NW limit in the endpoint region.
  - Generalizes known results of shape functions for heavy meson decays.

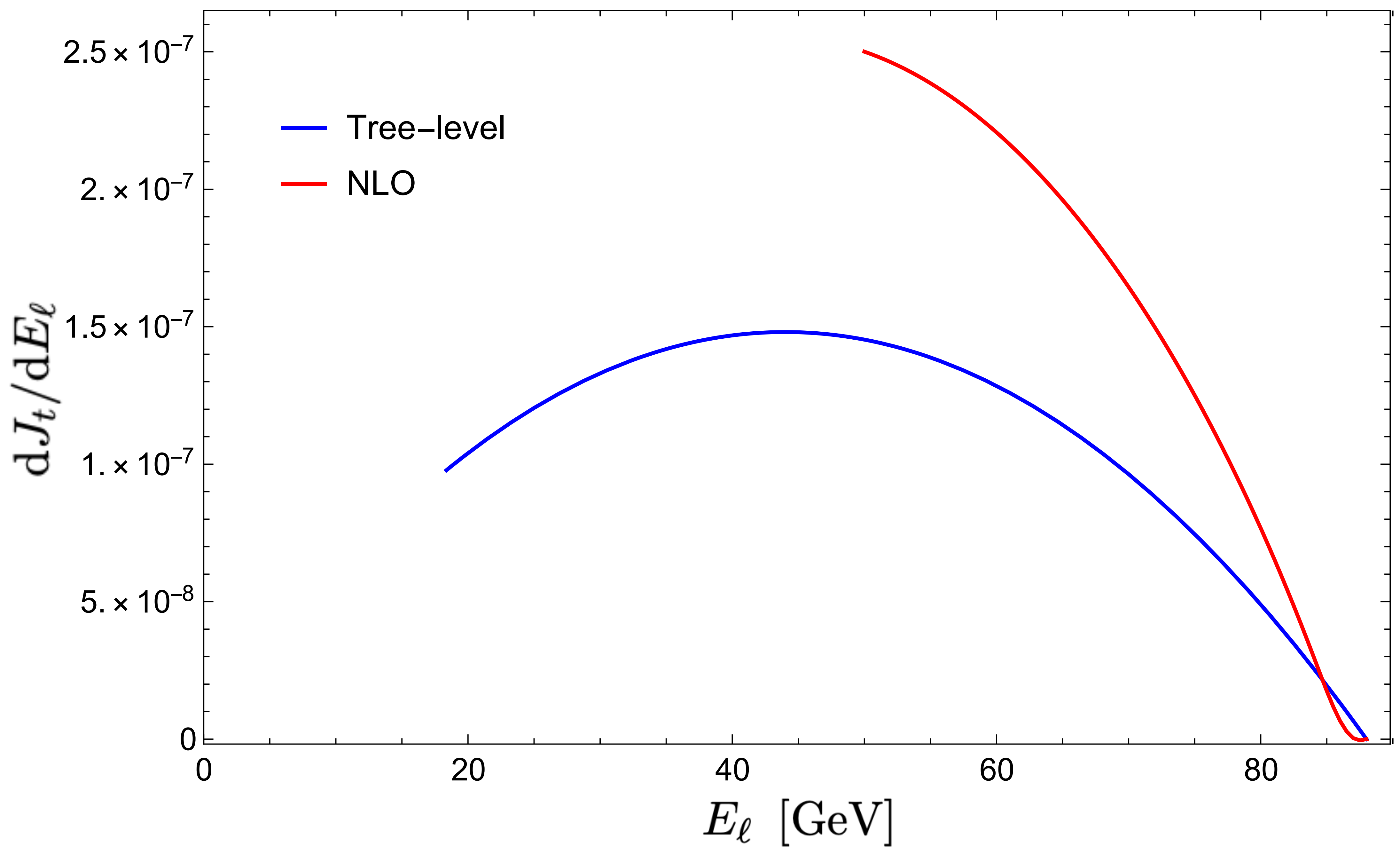
Outlook:

- We want to study the NLO QCD corrections for off-shell and boosted top decays in the endpoint region.
  $\longrightarrow$ Analysis of our factorized approach with NLL resummation.
- Comparison with predictions from Monte-Carlo event generators.

Back-up Slides

NLO QCD Corrections (Preliminary)

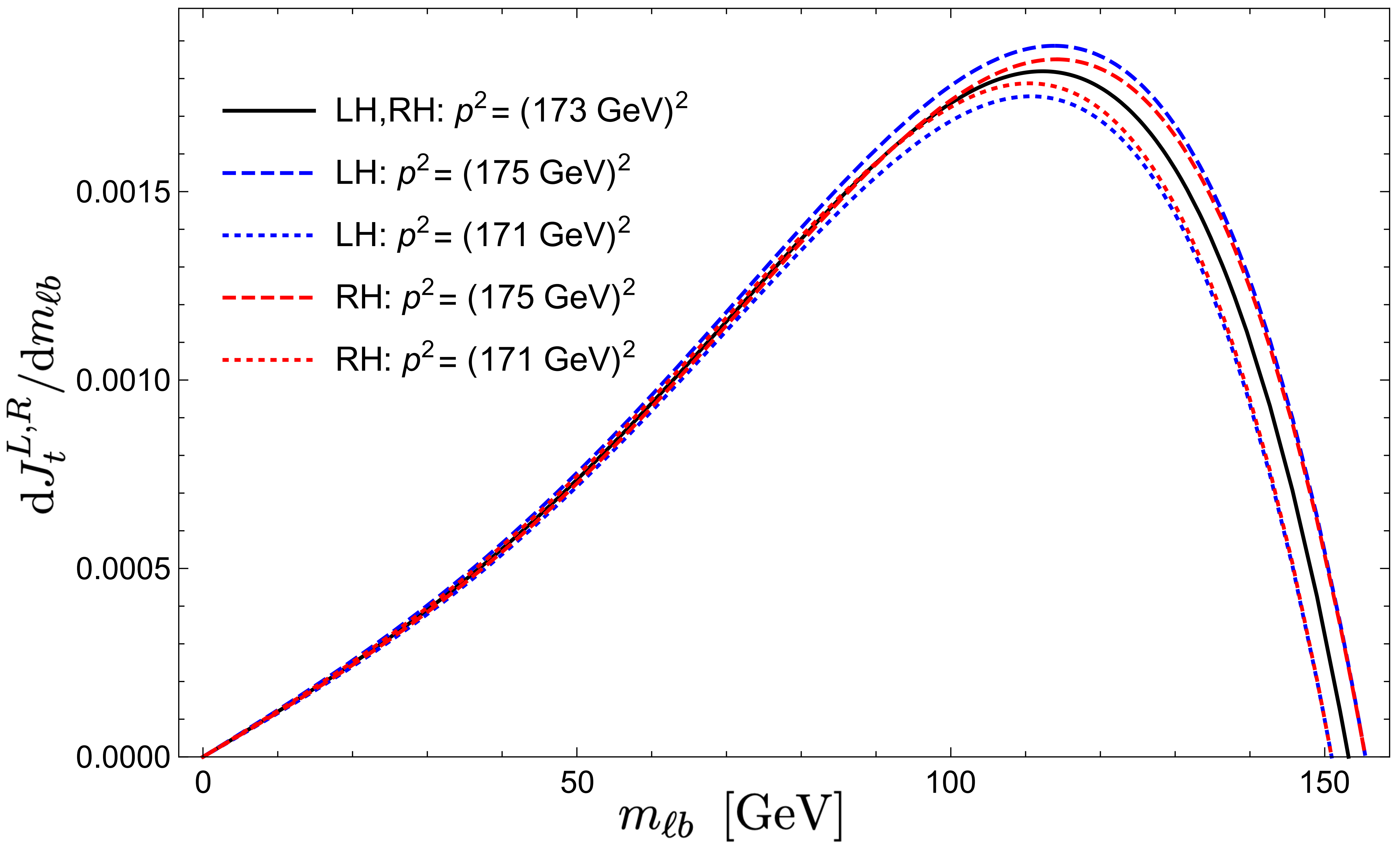
- E_ℓ distribution for differential top jet function including NLO QCD corrections to
  - b quark jet function
  - ultra collinear-soft function
  - convolution with large angle soft contributions

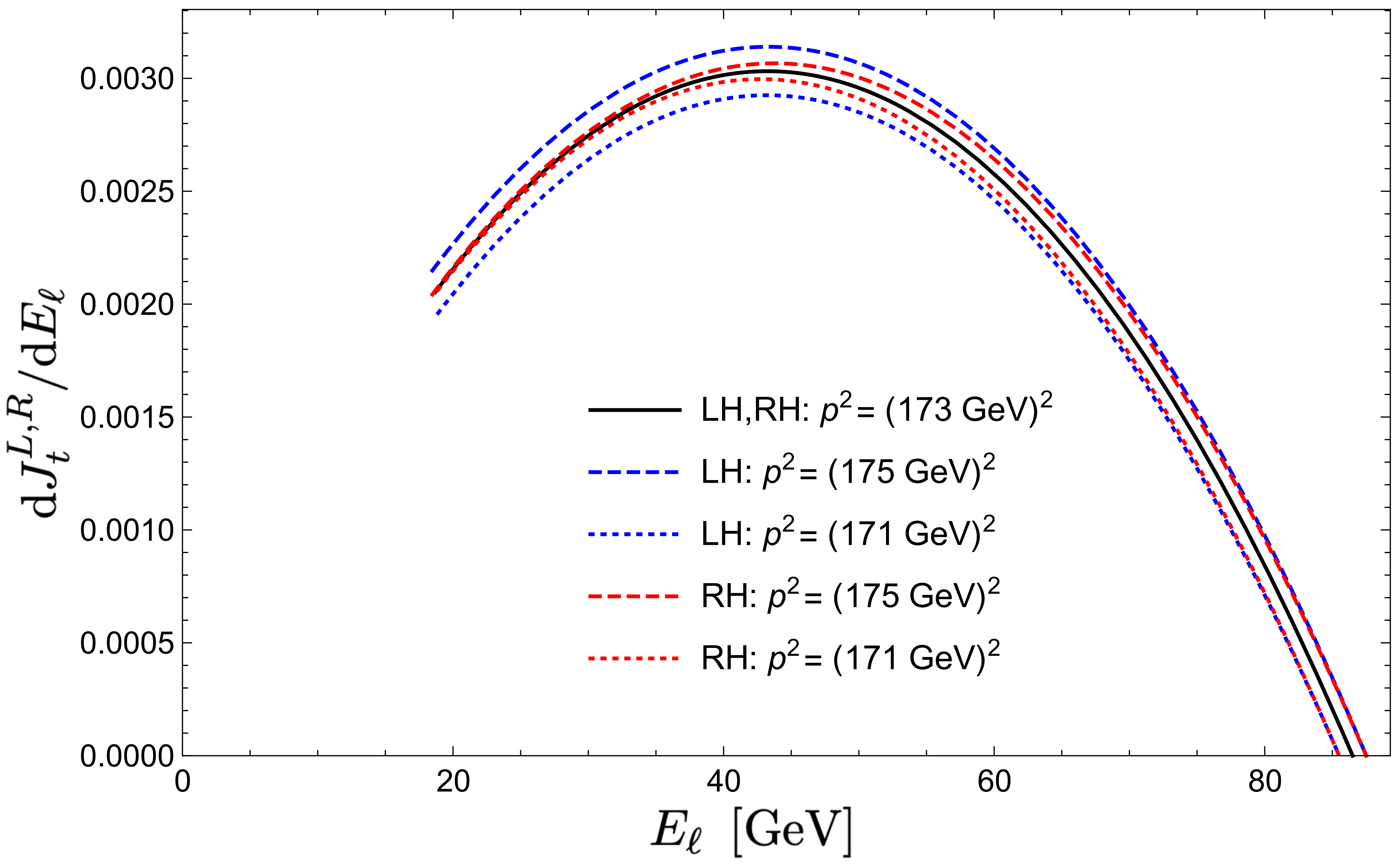


Differential top jet function

- $m_{\ell b}$ and E_ℓ distribution for differential top jet function including all tree level contributions:

$$m_t = 173\,\mathrm{GeV}, \qquad \Gamma_t = 1.43\,\mathrm{GeV},$$
$$\Gamma_W \to 0$$





Differential top jet function

- $m_{\ell b}$ and E_ℓ distribution for differential top jet function including all tree level contributions:

$$m_t = 173\,\mathrm{GeV}, \qquad \sqrt{p^2} \in [168, 178]\,\mathrm{GeV}$$
$$\Gamma_t = 1.43\,\mathrm{GeV}, \qquad \Gamma_W \to 0$$