Dark matter, bound states, and unitarity

Kallia Petraki

SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257

LPENS
LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE

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Frontiers in particle dark matter searches

(very simplistic summary)

Past decades

Most research focused on

$\mathbf{m_{DM} \sim 100\ GeV \sim m_{W,Z}}$

(e.g. prototypical WIMP scenario)

Current frontiers

Heavy dark matter

$\mathbf{m_{DM} \gtrsim TeV}$

Not constrained by colliders.

→ Experimentally probed by existing / upcoming **telescopes** e.g. HESS, IceCube, CTA, Antares

Light dark matter

$\mathbf{m_{DM} \lesssim few\ GeV}$

Not constrained by older direct detection experiments

→ Development of new generation of **direct detection** experiments

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Heavy ($m_{DM} \gtrsim$ TeV) dark matter

How does the phenomenology of dark matter look like?
(in popular scenarios, e.g. thermal-relic DM)

New type of dynamics emerges:
Long-range interactions

$$\lambda_B \sim \frac{1}{\mu v_{\rm rel}}, \frac{1}{\mu \alpha} \lesssim \frac{1}{m_{\rm mediator}} \sim \text{interaction range}$$

μ: reduced mass ($m_{\rm DM}/2$)

Heavy (m … Te…

Does this occur in models we care about?

- WIMPs with m > few TeV
- WIMPs with m < TeV co-annihilating with coloured/charged particles
- Self-interacting DM

… type … erges:

L … interactions

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What changes when the interactions are long-ranged?

Implications of long-range interactions

Sommerfeld effect

distortion of scattering-state wavefunctions
⇒ affects all cross-sections, incl annihilation

- Freeze-out ⇒ changes correlation of parameters (mass – couplings)
- Indirect detection signals
- Elastic scattering

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Bound states

- **Unstable bound states**
  ⇒ extra annihilation channel
  - Freeze-out
  - Indirect detection
  - Novel low-energy indirect detection signals
  - Colliders

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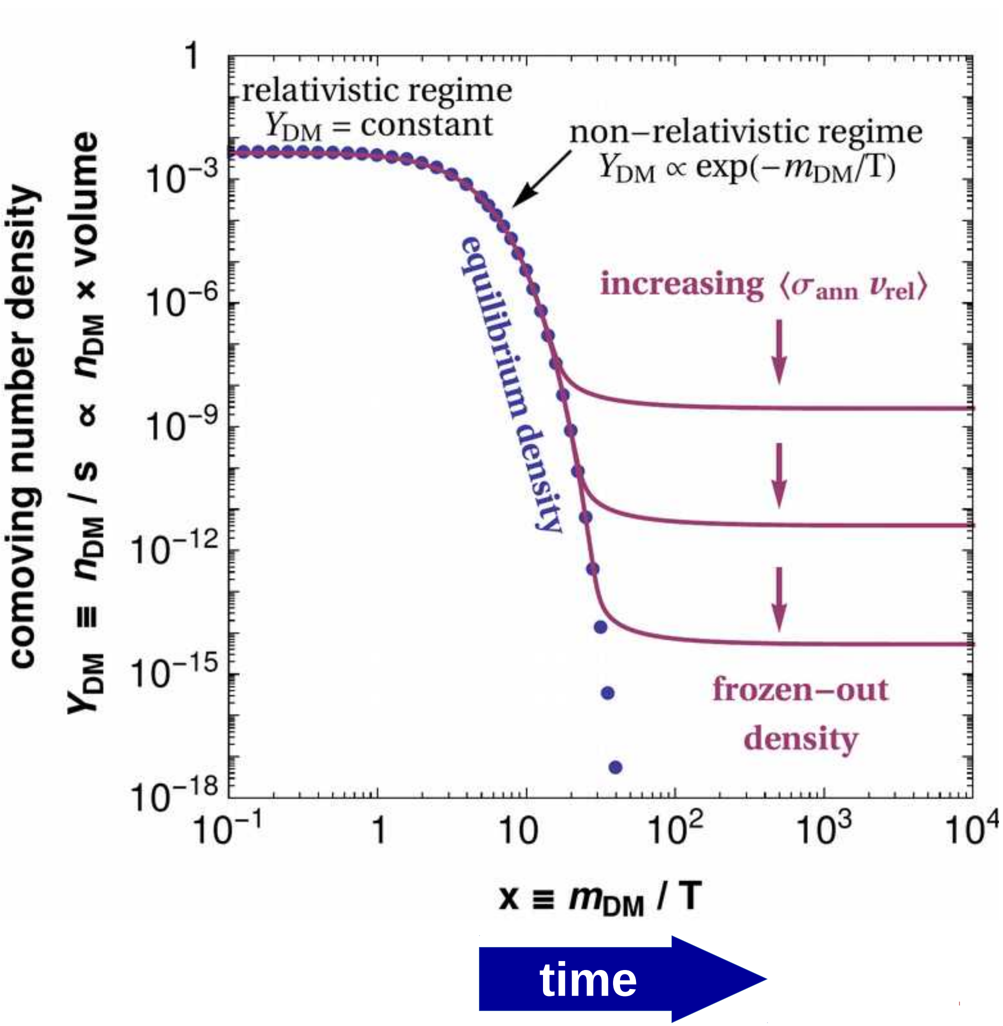
Outline

- Dark matter freeze-out:
  dark U(1) model, Boltzmann eqs
- The origin of non-perturbative effects at perturbative coupling
- Unitarity limit and long-range interactions
- Neutralino-squark co-annihilation scenarios
- Notorious Higgs

Dark matter production via **thermal freeze-out**



$T > m_{DM}$

DM kept in chemical & kinetic equilibrium with the plasma, via

$X + \bar{X} \leftrightarrow f + \bar{f}$

$n_{DM} \sim T^3$ or Y_{DM} = constant

$T < m_{DM}$

$Y_{DM} \propto \exp(-m_{DM}/T)$, while still in equilibrium

$T < m_{DM}/25$

Density too small, annihilations stall

⇒ Freeze-out!

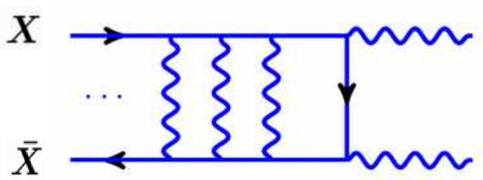
$$\Omega \simeq 0.26 \times \left(\frac{1pb\cdot c}{\sigma_{ann} v_{rel}}\right)$$

1 pb ~ σ_{Weak}
WIMP miracle!

Dark U(1) model: Dirac DM $X,\bar{X}$ coupled to γ_D

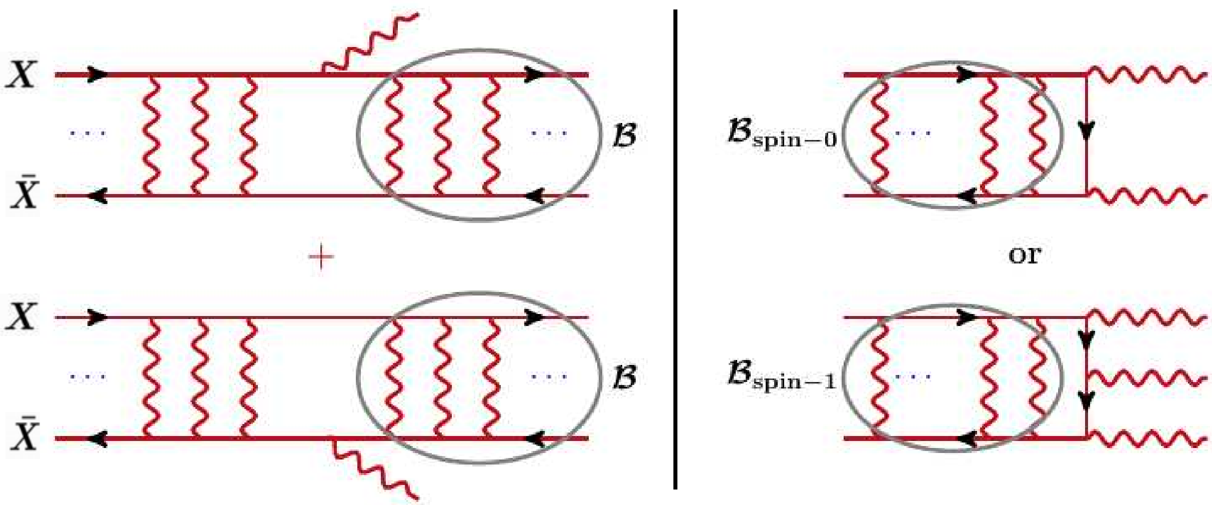
Direct annihilation

$X + \bar{X} \to 2\gamma_D$

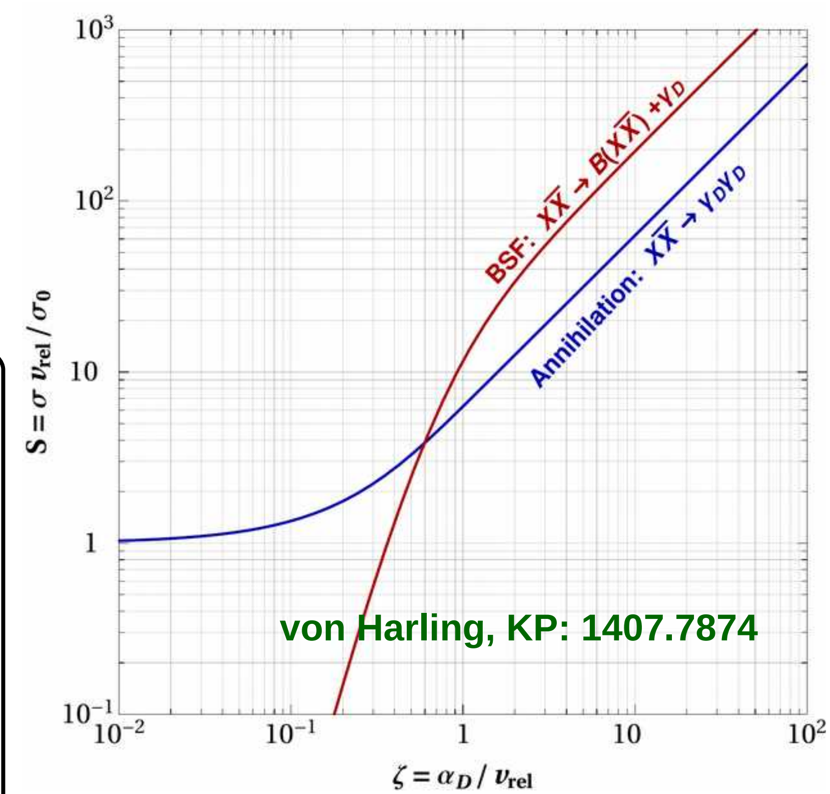


$$\sigma_{\rm ann} v_{\rm rel} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\rm ann}(\alpha_D / v_{\rm rel})$$

Bound-state formation and decay



$$\sigma_{\rm BSF} v_{\rm rel} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\rm BSF}(\alpha_D / v_{\rm rel})$$



$$S_{\rm ann} \simeq \left(\frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}\right) \xrightarrow{\zeta \gtrsim 1} 2\pi\zeta$$

$$S_{\rm BSF} \simeq \left(\frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}\right) \frac{2^9 \zeta^4 e^{-4\zeta \,{\rm arccot} \zeta}}{3(1+\zeta^2)^2} \xrightarrow{\zeta \gtrsim 1} 3.13 \times 2\pi\zeta$$

Thermal freeze-out with bound states

Boltzmann equations

free particles: $$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\rm ann} v_{\rm rel}\rangle \left(n^2 - n^{\rm eq\,2}\right) - \sum_{\mathcal{B}} \left(\langle\sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel}\rangle\, n^2 - \Gamma^{\rm ion}_{\mathcal{B}}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle\sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel}\rangle\, n^2 - \Gamma^{\rm ion}_{\mathcal{B}}\, n_{\mathcal{B}}\right) - \Gamma^{\rm dec}_{\mathcal{B}} \left(n_{\mathcal{B}} - n^{\rm eq}_{\mathcal{B}}\right) - \sum_{\mathcal{B}'\neq\mathcal{B}} \left(\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'} n_{\mathcal{B}} - \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}} n_{\mathcal{B}'}\right)$$

dilution due to expansion of the universe

Processes		Detailed balance
Bound state formation (BSF) Ionisation (ion)	$X + \bar{X} \to \mathcal{B}(X\bar{X}) + \gamma_D$ $\mathcal{B}(X\bar{X}) + \gamma_D \to X + \bar{X}$	$\langle\sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel}\rangle (n^{\rm eq})^2 = \Gamma^{\rm ion}_{\mathcal{B}}\, n^{\rm eq}_{\mathcal{B}}$
Decay (dec)	$\mathcal{B}(X\bar{X}) \to 2\gamma_D$ or $3\gamma_D$	
Transitions (trans)	$\mathcal{B}(X\bar{X}) \to \mathcal{B}'(X\bar{X}) + \gamma_D$ $\mathcal{B}(X\bar{X}) + \gamma_D \to \mathcal{B}'(X\bar{X})$	$\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'}\, n^{\rm eq}_{\mathcal{B}} = \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}}\, n^{\rm eq}_{\mathcal{B}'}$

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dilution due to expansion of the universe

Typically at least one rate is large enough
$$\Gamma_{\mathcal{B}}^{\rm ion} + \Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm trans} \gg H$$
to keep bound states close to equilibrium

$\Rightarrow$ set $dn_{\mathcal{B}}/dt + 3Hn_{\mathcal{B}} \simeq 0$
$\Rightarrow$ get algebraic equations for $n_{\mathcal{B}}$ in terms of n, $n_{\mathcal{B}}^{\rm eq}$
$\Rightarrow$ re-employ it in Boltzmann equation for n

Ellis, Luo, Olive: 1503.07142

Complete treatement: Binder, Filimonova, Petraki, White 2112.00042

Thermal freeze-out with bound states

Boltzmann equations and effective cross-section

free particles: $$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm ann} v_{\rm rel}\rangle \left(n^2 - n^{{\rm eq}\ 2}\right) - \sum_{\mathcal{B}} \left(\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right)$$

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↓

$$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm eff} v_{\rm rel}\rangle \left(n^2 - n^{{\rm eq}\ 2}\right)$$

where, neglecting bound-to-bound transitions,

$$\langle \sigma^{\rm eff} v_{\rm rel}\rangle \equiv \langle \sigma^{\rm ann} v_{\rm rel}\rangle + \sum_{\mathcal{B}} \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \times \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}}$$

Thermal freeze-out with bound states

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efficiency factors

$$r_{\mathcal{B}} = \sum_{\mathcal{B}'} \Gamma^{\rm dec}_{\mathcal{B}'} \left(\mathbb{\Gamma}^{\rm ion} + \mathbb{\Gamma}^{\rm dec} + \mathbb{\Gamma}^{\rm trans} - \mathbb{T} \right)^{-1}_{\mathcal{B}'\mathcal{B}}$$

Complete treatement:
Binder, Filimonova, Petraki, White
2112.00042

Bound-to-bound transitions
only enhance the total effective cross-section!

Thermal freeze-out with bound states

Effective cross-section

$$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm eff} v_{\rm rel}\rangle \left(n^2 - n^{\rm eq\ 2}\right)$$

where, neglecting bound-to-bound transitions,

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At $T \gg$ Binding Energy $\Rightarrow$ $\Gamma_{\mathcal{B}}^{\rm ion} \gg \Gamma_{\mathcal{B}}^{\rm dec}$,

$$\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}} \simeq \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm ion}} = \frac{n_{\mathcal{B}}^{\rm eq}}{(n^{\rm eq})^2}\,\Gamma_{\mathcal{B}}^{\rm dec}$$

$$\simeq \frac{g_{\mathcal{B}}}{g_X^2}\left(\frac{4\pi}{m_X T}\right)^{3/2} \times e^{|E_{\mathcal{B}}|/T}\,\Gamma_{\mathcal{B}}^{\rm dec}$$

$\Downarrow$

Independent of actual BSF cross-section!

$\Gamma_{\mathcal{B}}^{\rm dec} \propto \langle \sigma^{\rm ann} v_{\rm rel}\rangle \rightarrow$ modest increase over the direct annihilation, but increases exponentially as T drops.

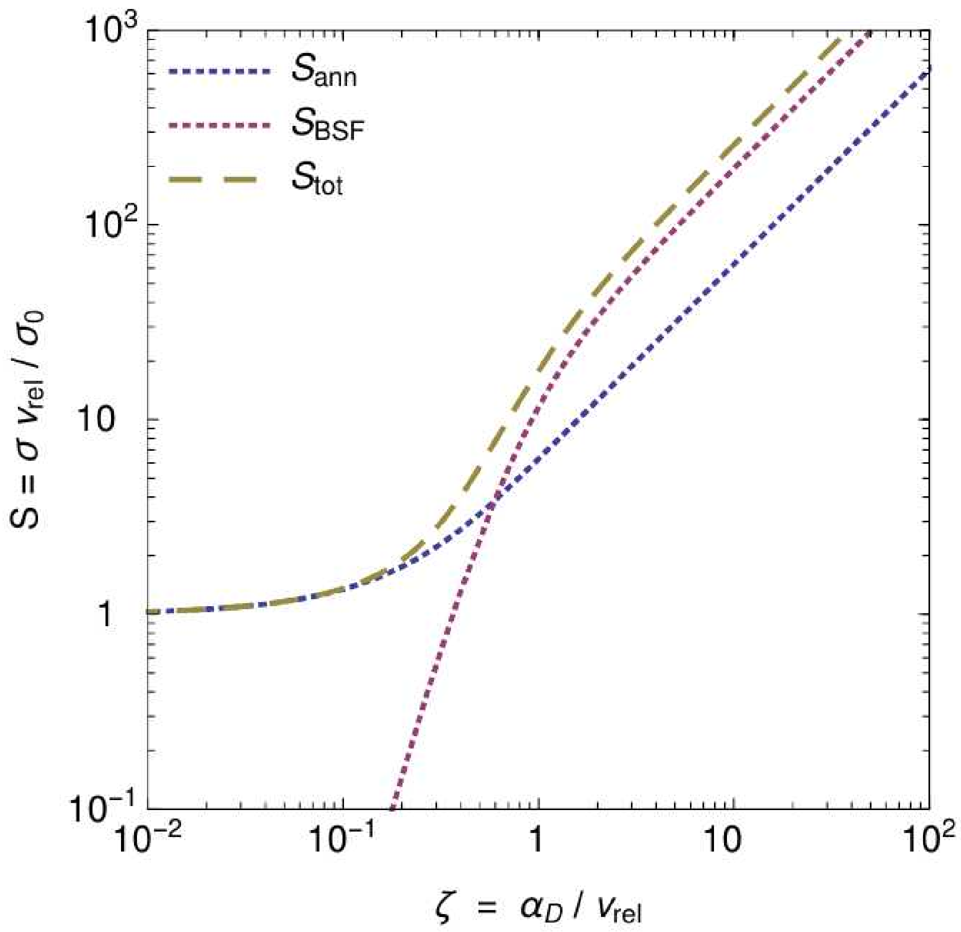
At $T \lesssim$ Binding Energy $\Rightarrow$ $\Gamma_{\mathcal{B}}^{\rm ion} \ll \Gamma_{\mathcal{B}}^{\rm dec}$,

$$\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}} \simeq \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle.$$

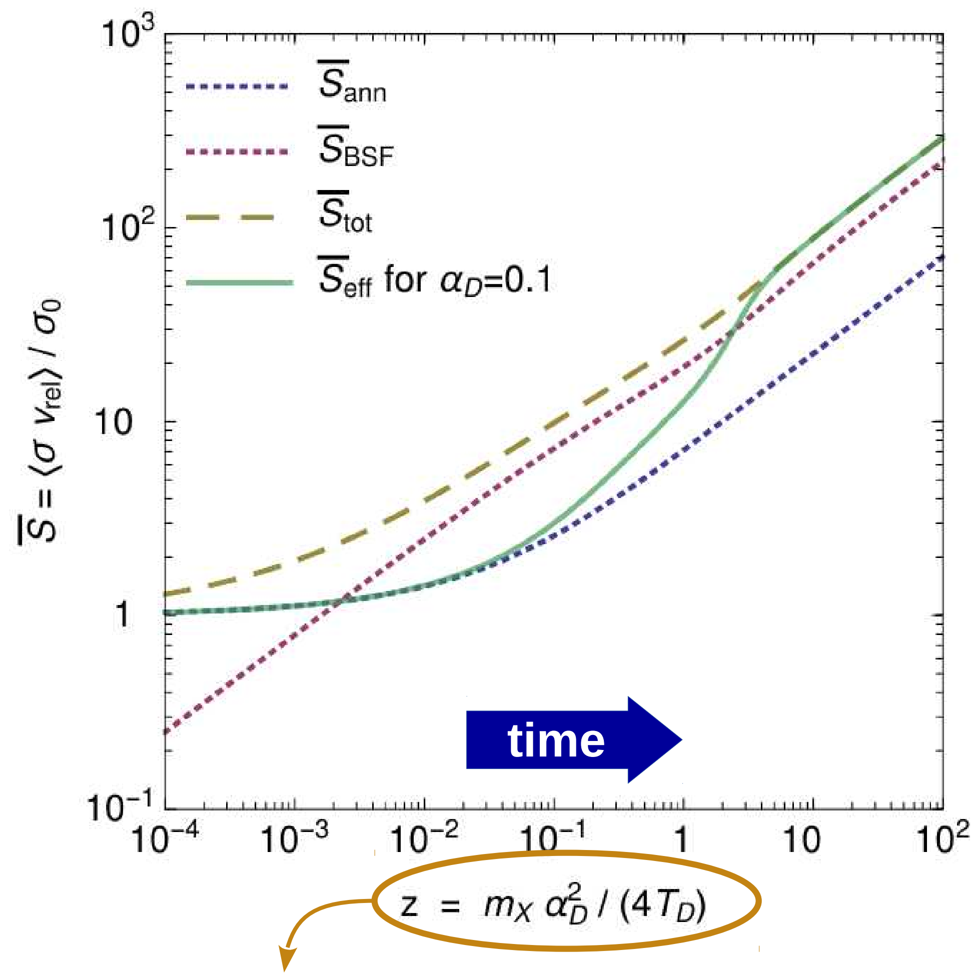
Typically, most of DM destruction due to BSF occurs in this regime.

Effective cross-section in dark U(1) model

Cross-sections



Thermally averaged cross-sections

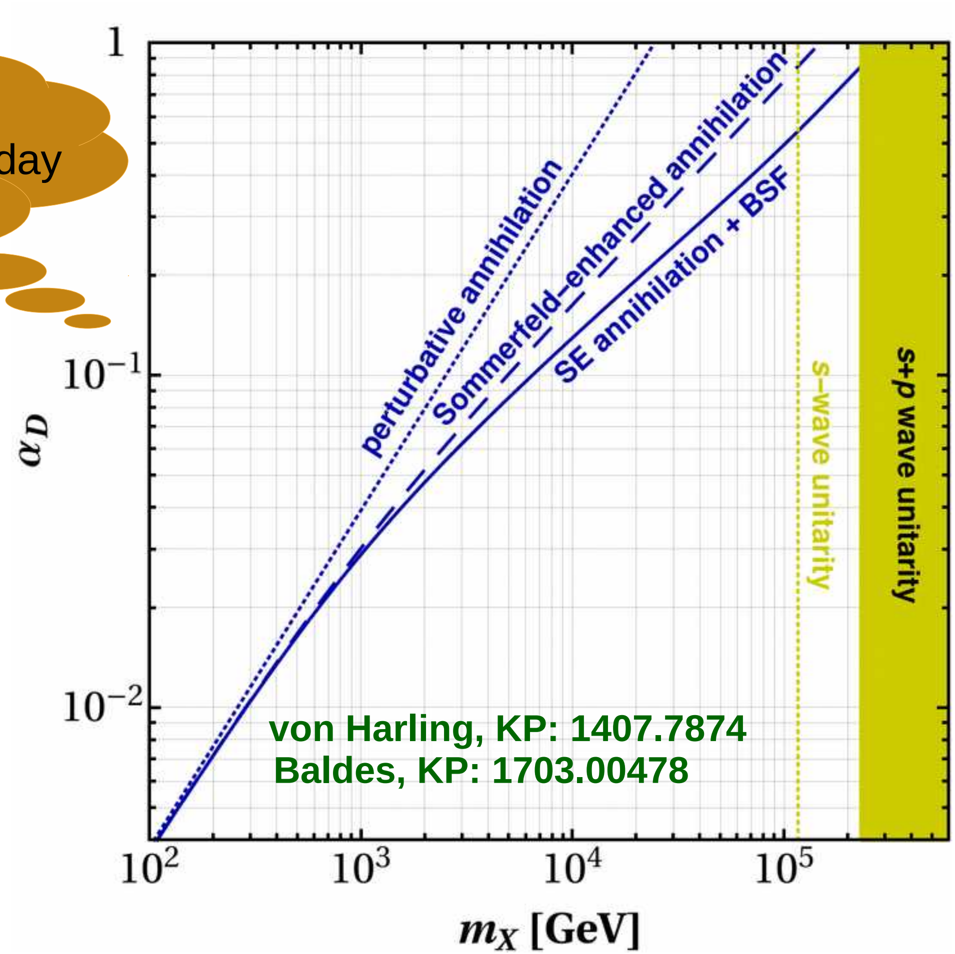


binding energy / temperature

Thermal freeze-out with long-range interactions

Dark U(1) model: Dirac DM $X,\bar{X}$ coupled to γ_D

Important because it determines DM interactions today (direct, indirect detection)



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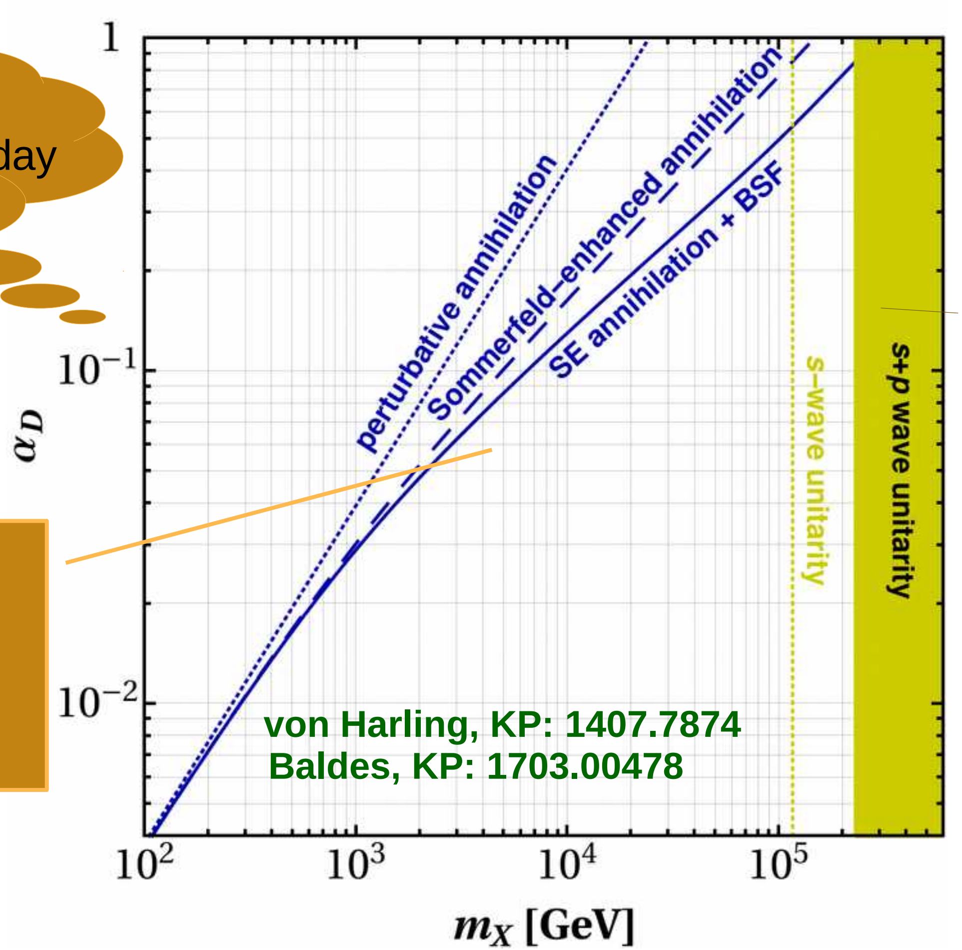


Long-range effects indeed become at $m_{DM} \gtrsim$ few TeV.

Verifies expectation from unitarity arguments!

Dominant annihilation mode: ***s*-wave**.

Dominant BSF mode: ***p*-wave**

Same order!

Higher partial waves Important / dominant in multi-TeV regime.

DM may be even heavier!

A corollary

Saha equilibrium for metastable bound states

$$\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\text{eq}}} = \left(\frac{n_{\text{free}}}{n_{\text{free}}^{\text{eq}}}\right)^2 - \left[\left(\frac{n_{\text{free}}}{n_{\text{free}}^{\text{eq}}}\right)^2 - 1\right] r_{\mathcal{B}}$$

Binder, Filimonova, Petraki, White 2112.00042

$$r_{\mathcal{B}} = \sum_{\mathcal{B}'} \Gamma^{\text{dec}}_{\mathcal{B}'} \left(\mathbb{\Gamma}^{\text{ion}} + \mathbb{\Gamma}^{\text{dec}} + \mathbb{\Gamma}^{\text{trans}} - \mathbb{T}\right)^{-1}_{\mathcal{B}'\mathcal{B}}$$

$r_{\mathcal{B}} = 0$

$r_{\mathcal{B}} = 1$

$$\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\text{eq}}} = \left(\frac{n_{\text{free}}}{n_{\text{free}}^{\text{eq}}}\right)^2$$

Standard Saha equilibrium

$$\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\text{eq}}} = 1$$

Particles with decay rate > Hubble

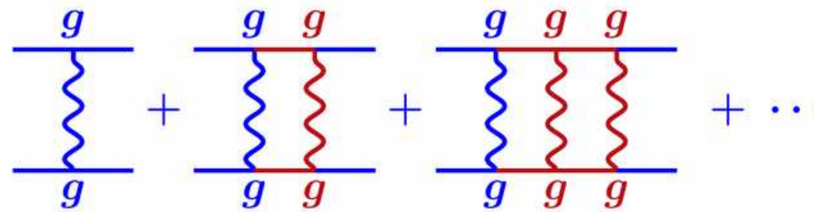
The origin of non-perturbative effects at perturbative coupling

What just happened?

Making sense of the ladder diagrams

Every mediator exchange introduces an
$\alpha = g^2/(4\pi)$ suppression in the amplitude.
How did we get an enhancement and bound states?

Bound-state ladder



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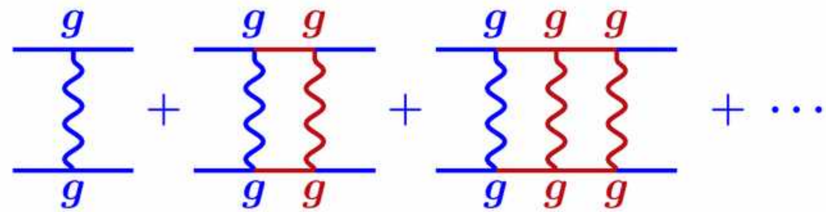

Energy and momentum exchange scale with α!

- Momentum transfer: $|\vec{q}| \sim \mu\alpha$.
- Energy transfer: $q^0 \sim |\vec{q}|^2/\mu \sim \mu\alpha^2$.
- Off-shellness of interacting particles: $q^0 \sim |\vec{q}|^2/\mu \sim \mu\alpha^2$.

one boson exchange $\sim \alpha \times \dfrac{1}{(\mu\alpha)^2} \propto \dfrac{1}{\alpha}$

each added loop $\sim \alpha \times \displaystyle\int dq^0 d^3q \, \frac{1}{q_1 - m_1} \frac{1}{q_2 - m_2} \frac{1}{q_\gamma^2}$

$$\sim \alpha \times (\mu\alpha^2)(\mu\alpha)^3 \frac{1}{\mu\alpha^2} \frac{1}{\mu\alpha^2} \frac{1}{(\mu\alpha)^2}$$

$$\sim 1$$

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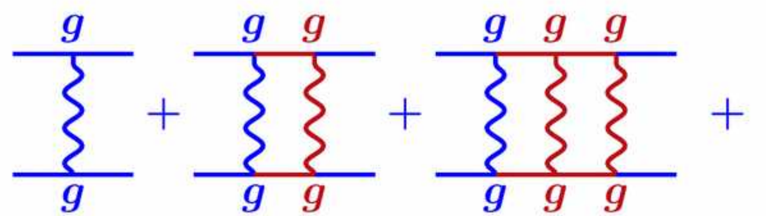

1/α scaling responsible for non-perturbative effects

(not largeness of coupling)

Energy and momentum exchange scale with α!

- Momentum transfer: $|\vec{q}| \sim \mu\alpha$.
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What just happened?
Making sense of the ladder diagrams

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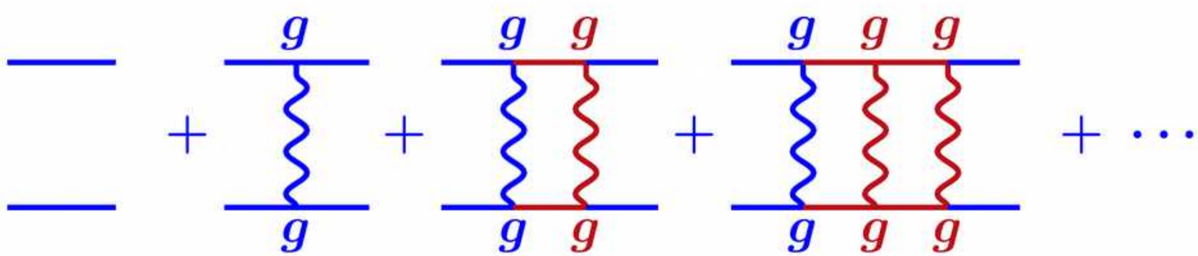
Scattering state ladder



Energy and momentum exchange scale with both α and $v_{\rm rel}$!

$\mu v_{\rm rel}$ is the *expectation value* of the momentum in CM frame,
the quantum uncertainty scales with α.

The Sommerfeld effect appears when
quantum uncertainty $\sim$ *expectation value.*

Unitarity limit and long-range interactions

Partial-wave unitarity limit

$$S^\dagger S = 1 \quad \xrightarrow{S=1+iT} \quad -i(T - T^\dagger) = T^\dagger T$$

Project on a partial wave and
insert complete set of states on RHS

$$\Downarrow$$

$$\sigma_{\text{inel}}^{(\ell)} \leqslant \frac{\pi(2\ell+1)}{k_{\text{cm}}^2} \quad \xrightarrow{\text{non-rel}} \quad \frac{\pi(2\ell+1)}{\mu^2 v_{\text{rel}}^2} \quad \xrightarrow{\mu=M_{\text{DM}}/2} \quad \frac{4\pi(2\ell+1)}{M_{\text{DM}}^2 v_{\text{rel}}^2}$$

[Griest, Kamionkowski (1990); Hui (2001)]

Physical meaning:
saturation of probability for inelastic scattering

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

Implies upper bound on the mass of thermal-relic DM

Griest, Kamionkowski (1990)

$$\sigma_{\rm ann} v_{\rm rel} \simeq 2.2 \times 10^{-26}\ {\rm cm^3/s} \leqslant \frac{4\pi}{M^2_{\rm DM} v_{\rm rel}}$$

$$\langle v^2_{\rm rel}\rangle^{1/2} = (6T/M_{\rm DM})^{1/2} \xrightarrow[M_{\rm DM}/T \approx 25]{\text{freeze-out}} 0.49$$

$$\Rightarrow M_{\rm uni} \simeq \begin{cases} 117\ \text{TeV}, & \text{self-conjugate DM} \\ 83\ \text{TeV}, & \text{non-self-conjugate DM} \end{cases}$$

Two assumptions to be questioned

1. "one does not expect $\sigma v_{\rm rel} \propto 1/v_{\rm rel}$ for annihilation channels in a non-relativistic expansion."
2. The s-wave yields the dominant contribution to the annihilation cross-section.

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

Implies upper bound on t... of thermal-relic DM

What are the underlying dynamics
of heavy thermal-relic DM?

What interactions can approach / attain the unitarity limit?

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...ions
...questioned

1. "one does not expect $\sigma v_{\rm rel} \propto 1/v_{\rm rel}$ for annihilation channels in a ... yields the dominant contribution to the annihilation cross-section.

What are the implications for experiments?

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

1) Velocity dependence of $\sigma_{\rm uni}$

Assuming $\sigma v_{\rm rel}$ = constant, setting it to maximal (inevitably for a fixed $v_{\rm rel}$) and thermal averaging is formally incorrect!

⇒ Unitarity violation at larger $v_{\rm rel}$, non-maximal cross-section at smaller $v_{\rm rel}$.

Sommerfeld-enhanced inelastic processes exhibit exactly this velocity dependence at large couplings / small velocities, e.g. in QED

$$\sigma^{\ell=0}_{\rm ann} v_{\rm rel} \simeq \frac{\pi\alpha_D^2}{M_{\rm DM}^2} \times \frac{2\pi\alpha_D/v_{\rm rel}}{1-\exp(-2\pi\alpha_D/v_{\rm rel})} \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}}$$

⇒ Velocity dependence of $\sigma_{\rm uni}$ definitely *not* unphysical!

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

Parametric

1) ~~Velocity~~ dependence of σ_{uni}

What can we learn?

For a contact-type interaction, mediated by heavy particle with $m_{\rm med} \gtrsim M_{\rm DM}$,

$$\sigma_{\rm ann} v_{\rm rel} \sim \frac{\alpha_D^2 M_{\rm DM}^2}{m_{\rm med}^4} \lesssim \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}}.$$

Approaching unitarity limit requires large coupling (no surprise)

$$\alpha_D \sim m_{\rm med}^4 / M_{\rm DM}^4 \gtrsim 1.$$

Calculation violates unitarity if

$$m_{\rm med} < \alpha_D^{1/2} M_{\rm DM} \lesssim \alpha_D M_{\rm DM}.$$

Comparison between physical scales
⇒ violation signals new effect at play!

Hisano, Matsumoto, Nojiri, hep-ph/0212022
Baldes, KP: 1703.00478

Partial-wave unitarity limit

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Comparison between physical scales
⇒ violation signals new effect at play!

Including the Sommerfeld enhancement, for a light mediator, e.g. dark QED

$$\sigma_{\rm ann} v_{\rm rel} \simeq \frac{2\pi^2 \alpha_D^3}{M^2_{\rm DM} v_{\rm rel}} \lesssim \frac{4\pi}{M^2_{\rm DM} v_{\rm rel}}.$$

Unitarity indicates range of validity

$$\alpha_D \lesssim 0.86$$

Only numerical bound on a dimensionless coupling
⇒ include (resummed) higher order corrections

Hisano, Matsumoto, Nojiri, hep-ph/0212022
Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

1) Velocity dependence of σ_{uni}

Proper thermal average and taking into account delayed chemical decoupling

$$M_{\rm uni} \simeq \begin{cases} 117\ \text{TeV}, & \text{self-conjugate DM} \\ 83\ \text{TeV}, & \text{non-self-conjugate DM} \end{cases}$$

$$M_{\rm uni} \simeq \begin{cases} 198\ \text{TeV}, & \text{self-conjugate DM} \\ 138\ \text{TeV}, & \text{non-self-conjugate DM} \end{cases}$$

s-wave annihilation

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}$$

2) Higher partial waves

In direct annihilation processes, *s*-wave dominates.

- For contact-type interactions, higher ℓ are $v_{\rm rel}^{2\ell}$ suppressed:

$$\sigma_{\rm ann} v_{\rm rel} = \sum_{\ell} \sum_{r=0}^{\infty} c_{\ell r}\, v_{\rm rel}{}^{2\ell+2r}$$

- For long-range interactions:

$$\sigma^{(\ell=0)} v_{\rm rel} \sim \frac{\pi\alpha_D^2}{M_{\rm DM}^2} \times \left(\frac{2\pi\alpha_D/v_{\rm rel}}{1-e^{-2\pi\alpha_D/v_{\rm rel}}}\right) \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}}$$

$$\sigma^{(\ell=1)} v_{\rm rel} \sim \frac{\pi\alpha_D^2}{M_{\rm DM}^2} v_{\rm rel}^2 \times \left(\frac{2\pi\alpha_D/v_{\rm rel}}{1-e^{-2\pi\alpha_D/v_{\rm rel}}}\right)\left(1+\frac{\alpha_D^2}{v_{\rm rel}^2}\right) \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^5}{M_{\rm DM}^2 v_{\rm rel}}$$

Same $v_{\rm rel}$ scaling (as expected from unitarity!), albeit $v_{\rm rel}^2 \to \alpha_D^2$ suppression.

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma_{\text{inel}}^{(\ell)} v_{\text{rel}} \leqslant \sigma_{\text{uni}}^{(\ell)} v_{\text{rel}} = \frac{4\pi(2\ell+1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

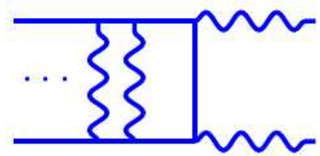
2) Higher partial waves

In direct annihilation processes, *s*-wave dominates.

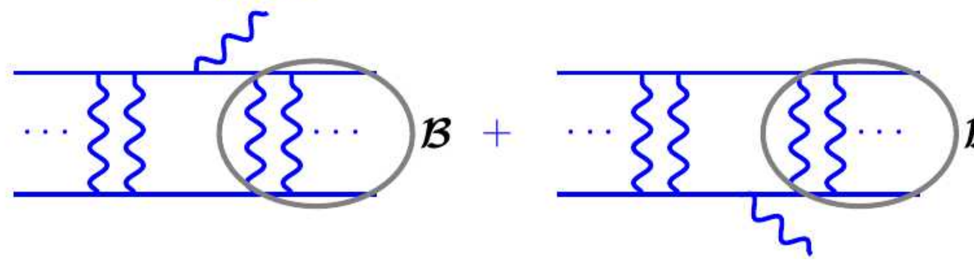
However, DM may annihilate via formation and decay of bound states.

dark QED

$$\sigma_{\text{ann}}^{(\ell=0)} v_{\text{rel}} \xrightarrow{\alpha_D \gg v_{\text{rel}}} \frac{2\pi^2 \alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}}$$



$$\sigma_{\text{BSF}}^{(\ell=1)} v_{\text{rel}} \xrightarrow{\alpha_D \gg v_{\text{rel}}} 3.13 \times \frac{2\pi^2 \alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}}$$



Baldes, KP: 1703.00478

Partial-wave unitarity limit

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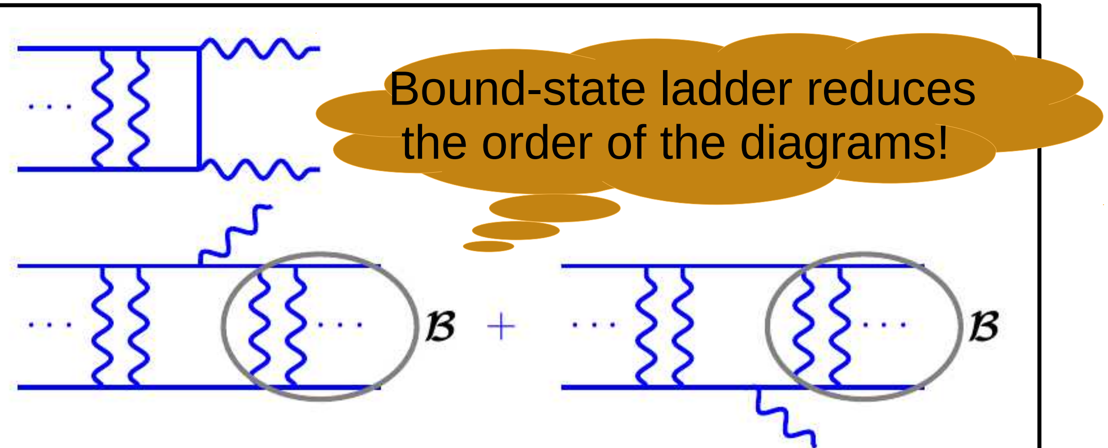
$$\sigma_{\rm BSF}^{(\ell=1)} v_{\rm rel} \xrightarrow{\alpha_D \gg v_{\rm rel}} 3.13 \times \frac{2\pi^2 \alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}}$$



Both s-wave and p-wave saturate their unitarity limit at $\alpha_D \simeq 0.86$.

$\Rightarrow$ Consider combined bound on the DM mass, $M_{\rm uni}^{s+p} \simeq 276$ TeV.

Higher partial waves important for DM destruction in early universe
$\Rightarrow$ higher $M_{\rm uni}$

Baldes, KP: 1703.00478

Partial-wave unitarity limit

$$\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}$$

Can be approached or attained only by long-range interactions

Baldes, KP: 1703.00478

Generic conclusion:

In viable thermal-relic DM scenarios, expect long-range behaviour at $m_{DM} \gtrsim$ few TeV!

- **Freeze-out**
  Sommerfeld & BSF alter predicted mass – coupling relation. Important for all experimental probes.
- **Indirect detection**
  Sommerfeld & BSF must be considered in computing signals. Novel lower energy signals produced in BSF.

Neutralino-squark co-annihilation scenarios

Squark-neutralino co-annihilation scenarios

- Degenerate spectrum → soft jets → evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by "effective" Boltzmann equation

$$n_{\rm tot} = n_{\rm LSP} + n_{\rm NLSP}$$

$$\sigma^{\rm eff}_{\rm ann} = [\, n^2_{\rm LSP}\, \sigma^{\rm LSP}_{\rm ann} + n^2_{\rm NLSP}\, \sigma^{\rm NLSP}_{\rm ann} + n_{\rm LSP}\, n_{\rm NLSP}\, \sigma^{\rm LSP-NLSP}_{\rm ann}\,]/n^2_{\rm tot}$$

Scenario probed in colliders.
Important to compute DM density accurately!
→ **QCD corrections**

DM coannihilation with scalar colour triplet

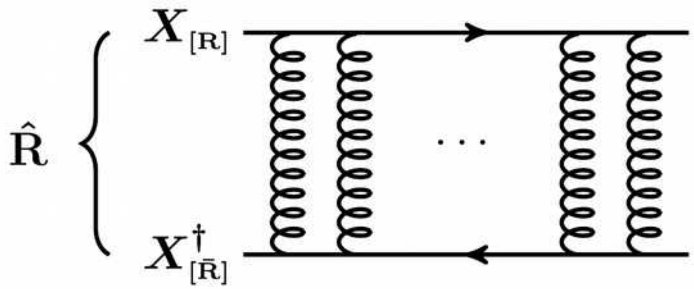
MSSM-inspired toy model

$$\begin{aligned}\mathcal{L} \;\supset\;& \frac{1}{2}\overline{\chi^c}\, i\partial\!\!\!/ \chi - \frac{1}{2} m_\chi\, \overline{\chi^c}\chi \\ &+ \left[(\partial_\mu + i g_s G^a_\mu T^a) X\right]^\dagger \left[(\partial^\mu + i g_s G^{a,\mu} T^a) X\right] - m_X^2 |X|^2 \\ &+ (\chi \leftrightarrow X, X^\dagger) \text{ interactions in chemical equilibrium during freeze-out}\end{aligned}$$

DM coannihilation with scalar colour triplet

MSSM-inspired toy model

Long-range interaction



$$\mathrm{R} \otimes \bar{\mathrm{R}} = \sum_{\hat{\mathrm{R}}} \hat{\mathrm{R}} = 1 \oplus \mathrm{adj} + \cdots$$

$$V(r) = -\,\alpha_{g,[\hat{\mathrm{R}}]} \,/\, r$$

$$\alpha_{g,[\hat{\mathrm{R}}]} = \alpha_s \times \left[C_2(\mathrm{R}) - C_2(\hat{\mathrm{R}})/2 \right]$$

where $\alpha_s = g_s^2/(4\pi)$

for SU(3)

$$3 \otimes \bar{3} = 1 \oplus 8$$

$\alpha_{g,[1]} = +\,(4/3)\alpha_s$ attractive

$\alpha_{g,[8]} = -\,(1/6)\alpha_s$ repulsive

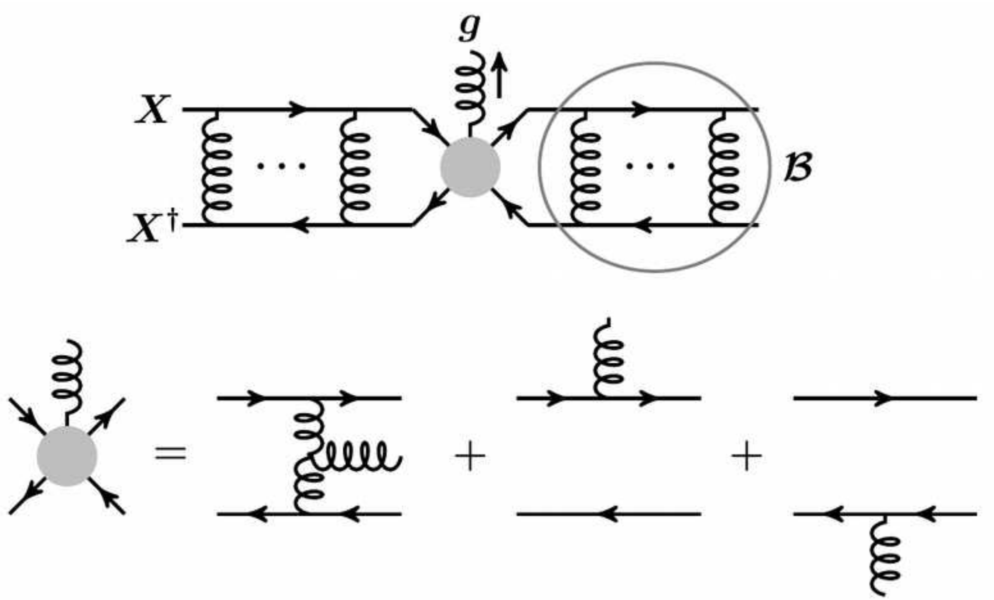
with $\alpha_s \sim 0.1$ at $m_X \sim$ TeV

Kats, Schwartz 0912.0526

DM coannihilation with scalar colour triplet

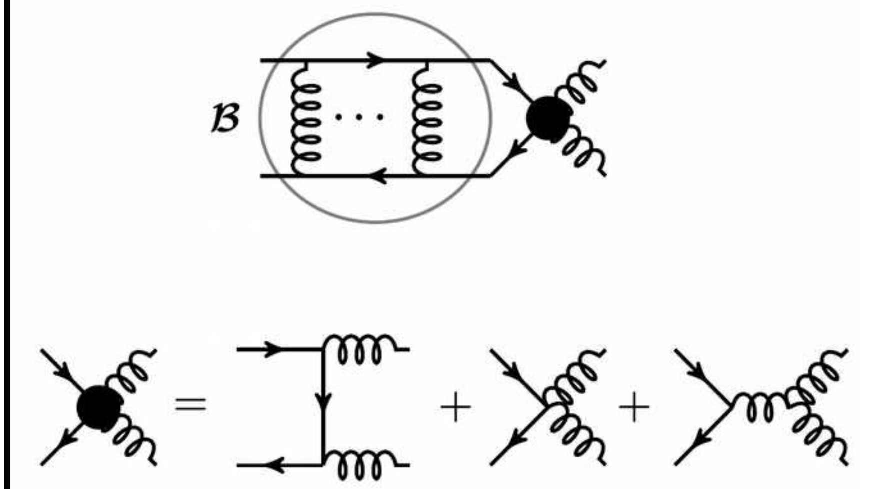
MSSM-inspired toy model

Bound-state formation and decay



Allowed transitions

$$(X + X^\dagger)_{[8]} \rightarrow (X + X^\dagger)_{[1]} + g_{[8]}$$
$$(X + X^\dagger)_{[1]} \rightarrow (X + X^\dagger)_{[8]} + g_{[8]}$$
$$(X + X^\dagger)_{[8]} \rightarrow (X + X^\dagger)_{[8]} + g_{[8]}$$



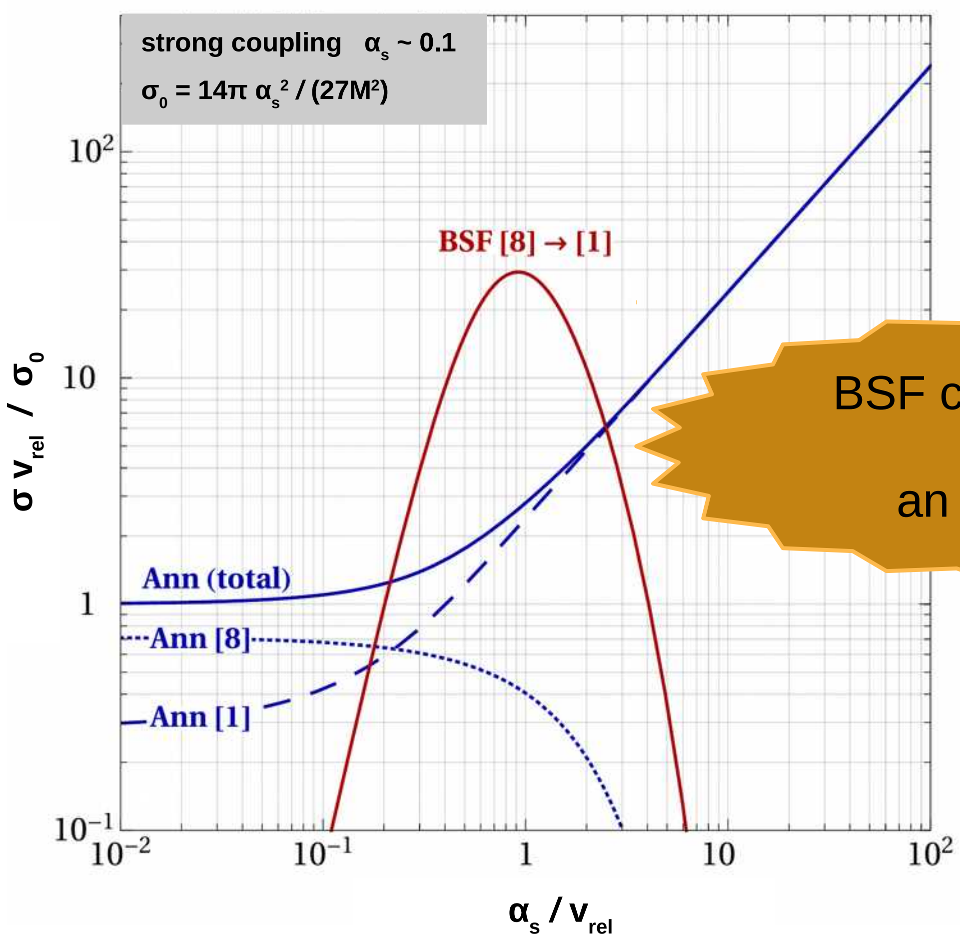
Harz, KP 1805.01200: Cross-sections for radiative BSF in non-Abelian theories

In agreement with Brambilla, Escobedo, Ghiglieri, Vairo 1109.5826:
Gluo-dissociation of quarkonium in pNRQCD

DM coannihilation with scalar colour triplet

MSSM-inspired toy model

Bound-state formation *vs* Annihilation

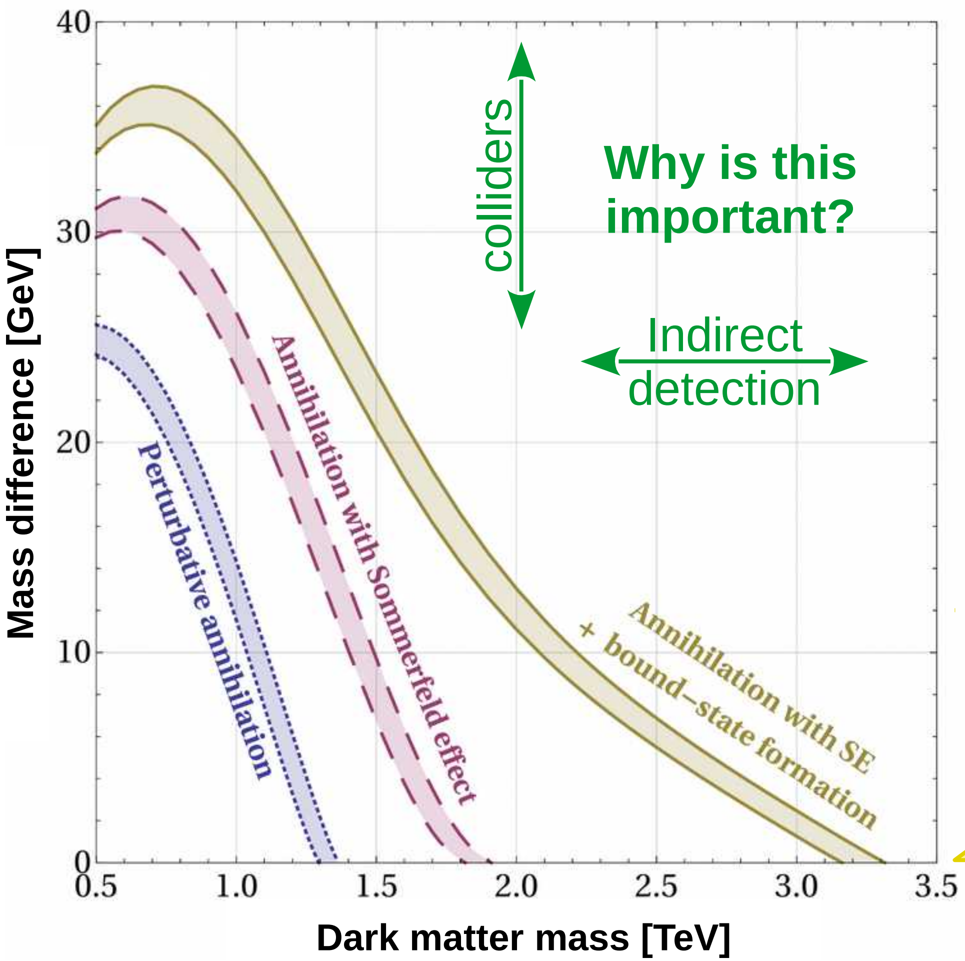


BSF can exceed Annihilation
by more than
an order of magnitude!

Harz, KP: 1805.01200

DM coannihilation with scalar colour triplet

MSSM-inspired toy model



Effect on relic density: much much larger than obs uncertainty in Ω_{DM}

Not the final picture!

Harz, KP: 1805.01200

Squark-neutralino co-annihilation scenarios

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- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by "effective" Boltzmann equation

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Scenario probed in colliders.
Important to compute DM density accurately!
→ **QCD corrections**

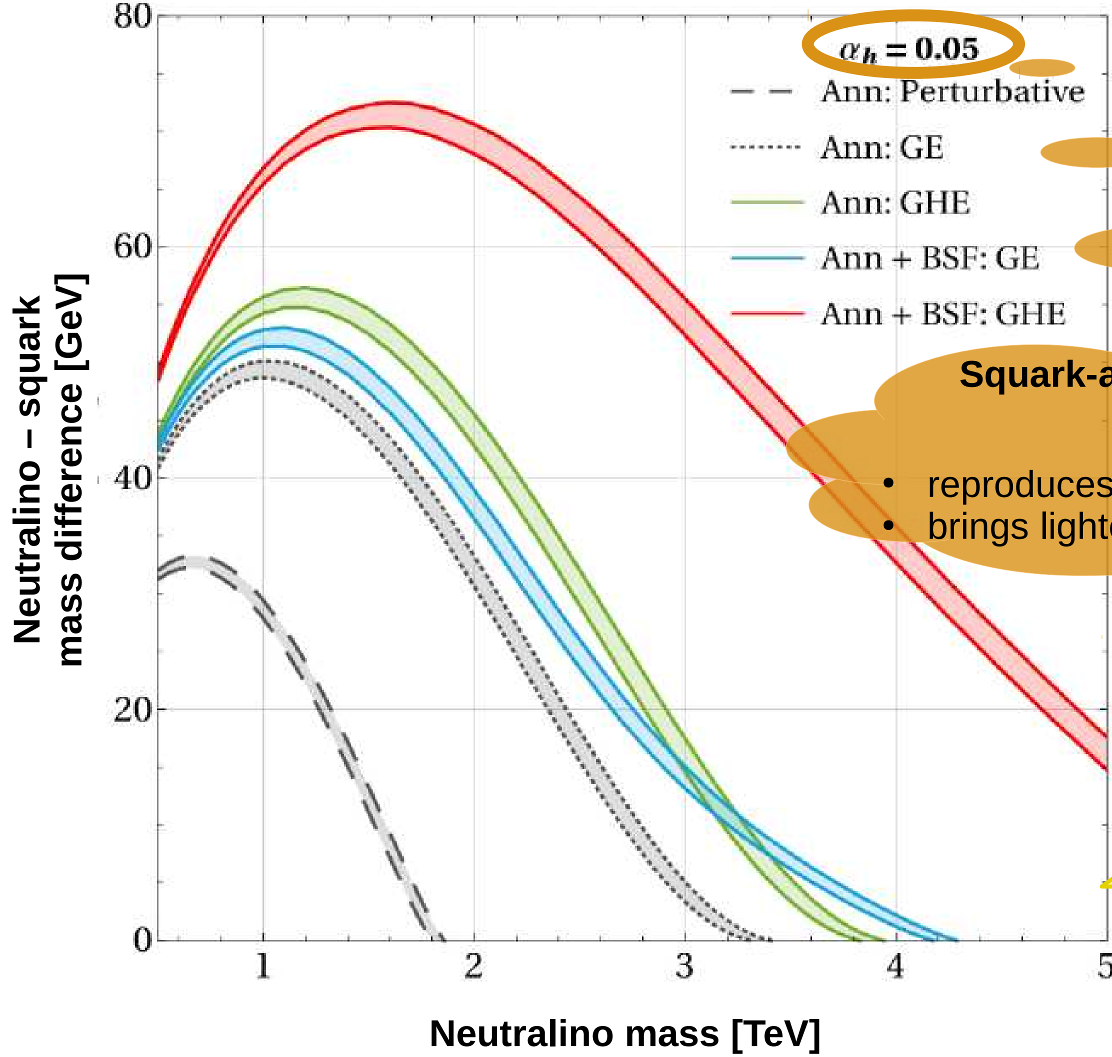
The Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation **Harz, KP: 1711.03552**
- Binding of bound states **Harz, KP: 1901.10030**

DM coannihilation with scalar colour triplet

MSSM-inspired toy model

The effect of the Higgs-mediated potential



Squark-antisquark-Higgs coupling

Large α_h

- reproduces measured Higgs mass
- brings lightest stop close in mass with LSP

Not the final picture!

Harz and KP: 1711.03552, 1901.10030

The Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation — Harz, KP: 1711.03552
- Binding of bound states — Harz, KP: 1901.10030

- **Formation of bound states via Higgs (*doublet*) emission ?**

Capture via emission of neutral scalar suppressed,
due to selection rules: quadruple transitions

March-Russel, West 0812.0559
KP, Postma, Wiechers: 1505.00109
An, Wise, Zhang: 1606.02305
KP, Postma, de Vries: 1611.01394

Capture via emission of charged scalar [or its Goldstone mode]
very very rapid: monopole transitions !

Ko,Matsui,Tang: 1910:04311
Oncala, KP: 1911.02605
Oncala, KP: 2101.08666
Oncala, KP: 2101.08667

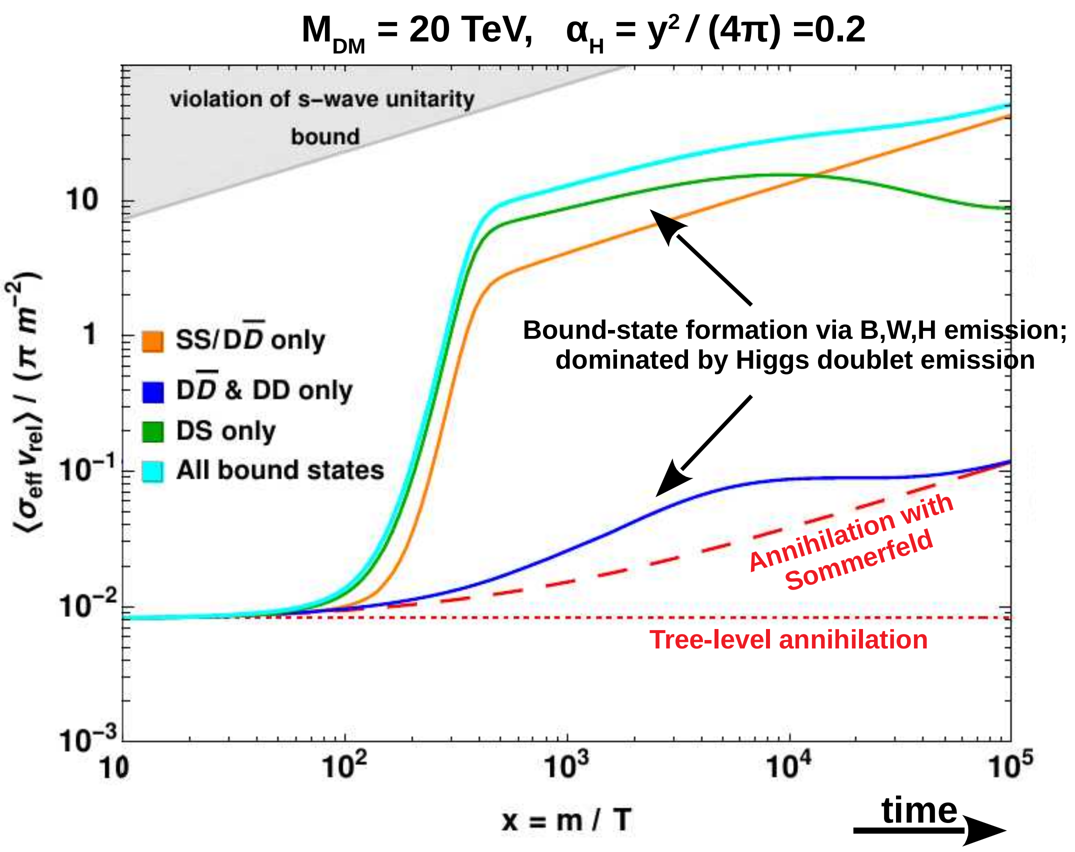
Sudden change in effective Hamiltonian precipitates transitions.
Akin to atomic transitions precipitated by β decay of nucleus.

Renormalisable Higgs-portal WIMP models

Singlet-***D***oublet coupled to the Higgs: $L \supset -y\,\bar{D}\,H\,S$

$m_D \simeq m_S \rightarrow D$ and S co-annihilate.

Freeze-out begins before the EWPT if m_{DM} > 5TeV



Oncala, KP: 2101.08666/7

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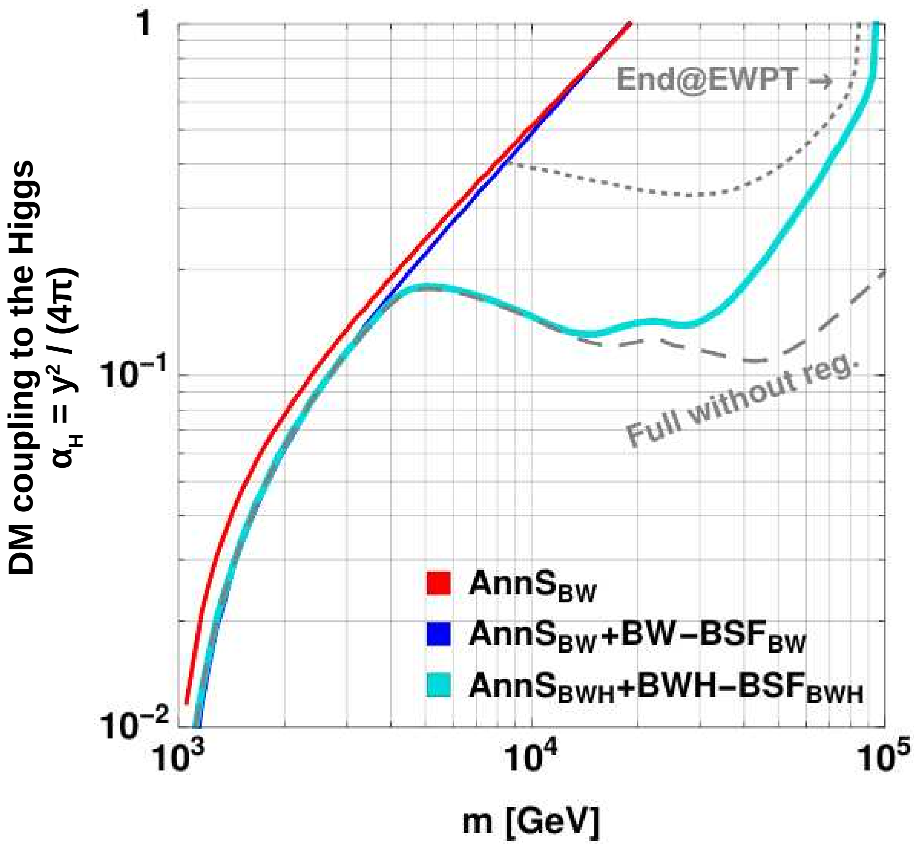


Huge effect!

~ 10^2 in relic density!

Impels reconsideration of Higgs-portal models (incl. neutralino-squark coann scenarios)

Oncala, KP: 2101.08666/7

Conclusions

- **Bound states impel complete reconsideration of thermal decoupling at / above the TeV scale: *emergence of a new type of inelasticity***

  Unitarity limit can be approached / realised only by long-range interactions
  ⇒ bound states play very important role! Baldes, KP: 1703.00478

  There is no unitarity limit on the mass of thermal relic DM!

- **Experimental implications:**
  - DM heavier than anticipated: multi-TeV probes very important.
  - Indirect detection:

    Enhanced rates due to BSF
    Novel signals: low-energy radiation emitted in BSF
    Indirect detection of asymmetric DM
  - Colliders: improved detection prospects due increased mass gap in coannihilation scenarios

- **Further existing/upcoming work:** excited bound states, restoring unitarity, Higgs extra slides

Radiative capture into bound states

- Emission of **force mediator** (boson that generates long-range potential)

  attn: there may be multiple force mediators.

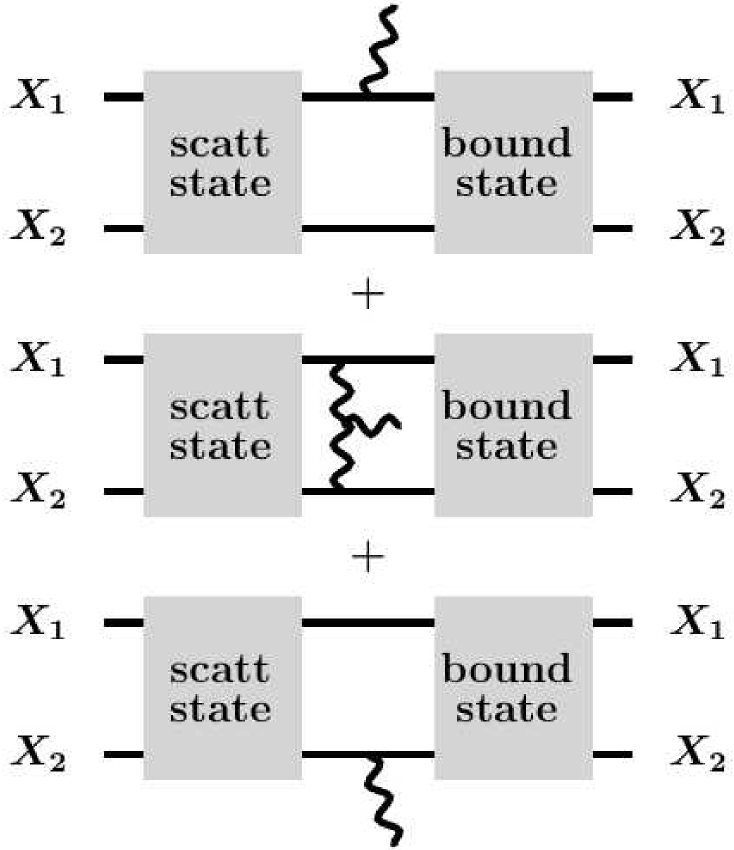
- Emission of another (light enough) **particle that does not contribute to the long-range potential**.

Properties of radiated particle determine:

- **(angular momentum) selection rules**
- **strength and energy dependence of cross-sections**

Radiative capture into bound states

I. vector emission



p ~ μ α

$$\mathcal{M}_{\rm BSF} \sim \int d^3p\, \tilde{\psi}^*_{\rm Bound}(p)\, p\, \tilde{\psi}_{\rm Scatt}(p)$$

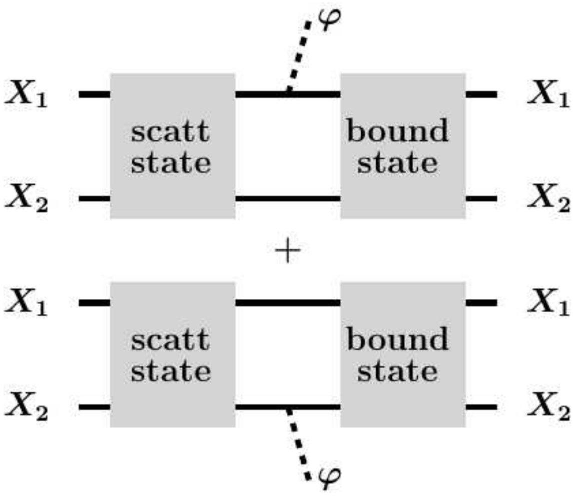
$$\sim \int d^3r\, [\nabla \psi^*_{\rm Bound}(r)]\, \psi_{\rm Scatt}(r)$$

Dipole transition: $|\Delta \ell| = 1$

Many works, from Quarkonia and Dark Matter sides

Radiative capture into bound states

II. (neutral) scalar emission



$p \sim \mu \alpha \quad >> \quad P_\varphi \sim \mu \alpha^2 / 2$

$$\mathcal{M}_{\rm BSF} \sim \int d^3p \; \tilde{\psi}^*_{\rm Bound}(p) \left[y_1 \, \tilde{\psi}_{\rm Scatt}\left(p + \frac{m_2}{m_1+m_2} P_\varphi\right) + y_2 \, \tilde{\psi}_{\rm Scatt}\left(p - \frac{m_1}{m_1+m_2} P_\varphi\right)\right]$$

$$\sim \int d^3r \; \psi^*_{n\ell m}(r) \left[y_1 \, \psi_{\rm Scatt}(r) \; e^{-\frac{m_2}{m_1+m_2} P_\varphi \cdot r} + y_2 \, \psi_{\rm Scatt}(r) \; e^{+\frac{m_1}{m_1+m_2} P_\varphi \cdot r}\right]$$

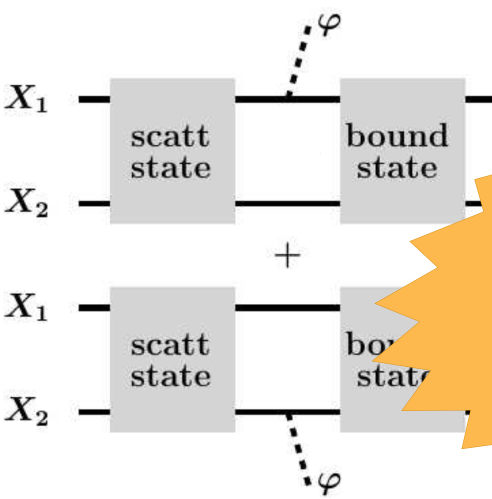
$P_\varphi \, r \sim \alpha$

monopole $\Delta\ell = 0:$	$\int d^3p \; \psi^*_{\rm Bound}(r) \; \psi_{\rm Scatt}(r) \; (y_1 + y_2)$	cancels due to orthogonality of wavefunctions
dipole $\Delta\ell = 1:$	$\int d^3p \; \psi^*_{\rm Bound}(r) \; \psi_{\rm Scatt}(r) (P_\varphi \cdot r) \left(-\frac{y_1 m_2}{m_1+m_2} + \frac{y_2 m_1}{m_1+m_2}\right)$	cancels for $\frac{y_1}{m_1} = \frac{y_2}{m_2}$, suppressed by α
quadrapole $\Delta\ell = 2:$	$\int d^3p \; \psi^*_{\rm Bound}(r) \; \psi_{\rm Scatt}(r) (P_\varphi \cdot r)^2 \; \frac{1}{2} \left[\left(\frac{y_1 m_2}{m_1+m_2}\right)^2 + \left(\frac{y_2 m_1}{m_1+m_2}\right)^2 \right]$	suppressed by α^2

Petraki, Postma, Wiechers: 1505.00109
Petraki, Postma, de Vries: 1611.01394

Radiative capture into bound states

II. (neutral) scalar emission



Quite suppressed,
… except if …
we break the rules

…μ α >> $P_\varphi \sim \mu\,\alpha^2/2$

$$\ldots + \frac{m_2}{m_1+m_2}P_\varphi\Big) + y_2\,\tilde{\psi}_{\rm Scatt}\Big(p - \frac{m_1}{m_1+m_2}P_\varphi\Big)\Big]$$

$$\ldots_{\rm catt}(r)\; e^{-\frac{m_2}{m_1+m_2}P_\varphi\cdot r} + y_2\,\psi_{\rm Scatt}(r)\; e^{+\frac{m_1}{m_1+m_2}P_\varphi\cdot r}\Big]$$

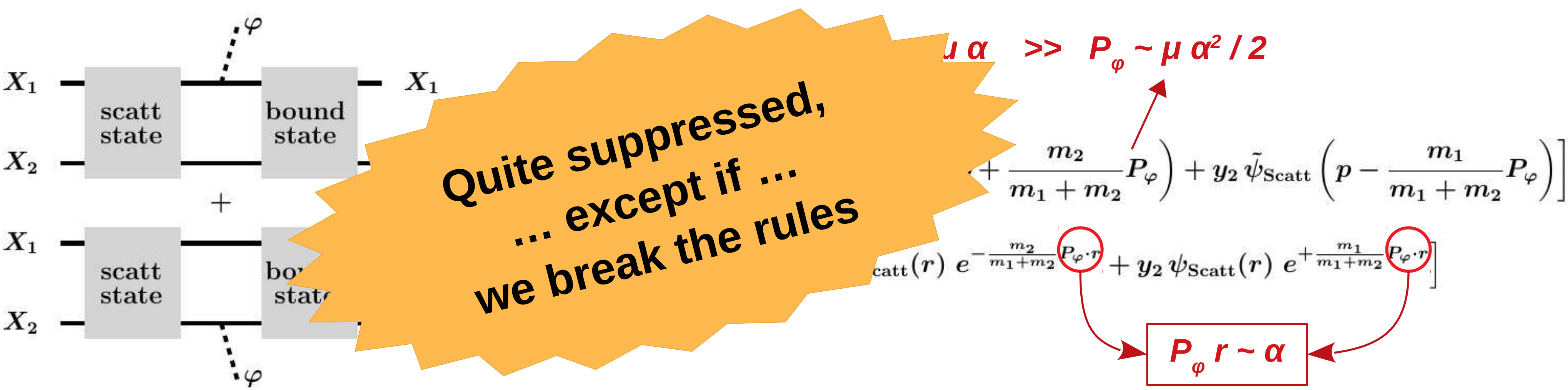
$P_\varphi\, r \sim \alpha$

monopole $\Delta\ell = 0:$	$\int d^3p\;\psi^*_{\rm Bound}(r)\;\psi_{\rm Scatt}(r)\,(y_1+y_2)$	cancels due to orthogonality of wavefunctions
dipole $\Delta\ell = 1:$	$\int d^3p\;\psi^*_{\rm Bound}(r)\;\psi_{\rm Scatt}(r)(P_\varphi\cdot r)\left(-\frac{y_1 m_2}{m_1+m_2}+\frac{y_2 m_1}{m_1+m_2}\right)$	cancels for $\frac{y_1}{m_1}=\frac{y_2}{m_2}$, suppressed by α
quadrapole $\Delta\ell = 2:$	$\int d^3p\;\psi^*_{\rm Bound}(r)\;\psi_{\rm Scatt}(r)(P_\varphi\cdot r)^2\,\frac{1}{2}\left[\left(\frac{y_1 m_2}{m_1+m_2}\right)^2+\left(\frac{y_2 m_1}{m_1+m_2}\right)^2\right]$	suppressed by α^2

Petraki, Postma, Wiechers: 1505.00109
Petraki, Postma, de Vries: 1611.01394

Radiative capture into bound states

II. (neutral) scalar emission



$$\text{monopole}\quad \Delta\ell = 0: \quad \int d^3p\ \psi^*_{\text{Bound}}(r)\ \psi_{\text{Scatt}}(r)\,(y_1 + y_2) \qquad \text{cancels due to orthogonality of wavefunctions}$$

$$\text{dipole}\quad \Delta\ell = 1: \quad \int d^3p\ \psi^*_{\text{Bound}}(r)\ \psi_{\text{Scatt}}(r)(P_\varphi \cdot r)\left(-\frac{y_1 m_2}{m_1+m_2} + \frac{y_2 m_1}{m_1+m_2}\right) \qquad \text{cancels for } \frac{y_1}{m_1} = \frac{y_2}{m_2}\text{, suppressed by } \alpha$$

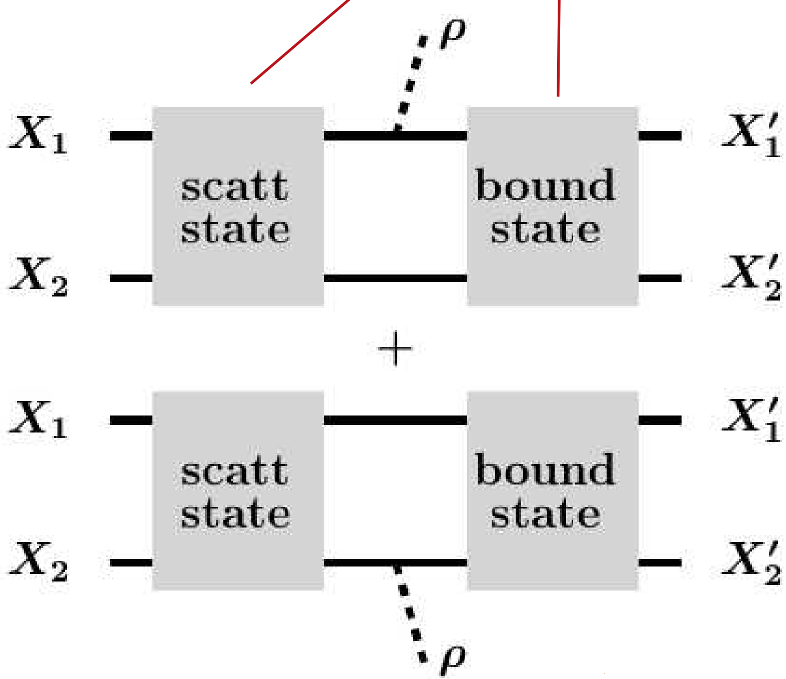
$$\text{quadrapole}\quad \Delta\ell = 2: \quad \int d^3p\ \psi^*_{\text{Bound}}(r)\ \psi_{\text{Scatt}}(r)(P_\varphi \cdot r)^2\ \frac{1}{2}\left[\left(\frac{y_1 m_2}{m_1+m_2}\right)^2 + \left(\frac{y_2 m_1}{m_1+m_2}\right)^2\right] \qquad \text{suppressed by } \alpha^2$$

Petraki, Postma, Wiechers: 1505.00109
Petraki, Postma, de Vries: 1611.01394

Radiative capture into bound states

III. <u>charged</u> scalar emission

If ρ charged under some symmetry,
in / out potentials different



not orthogonal

$$\mathcal{M}_{\rm BSF} \sim (y_1 + y_2) \int d^3p \; \tilde{\psi}^*_{\rm Bound}(p) \; \tilde{\psi}_{\rm Scatt}(p)$$

$$\sim (y_1 + y_2) \int d^3r \; \psi^*_{\rm Bound}(r) \; \psi_{\rm Scatt}(r)$$

Monopole transitions $\Delta\ell = 0$

no momentum
suppression
b/c of scalar vertex

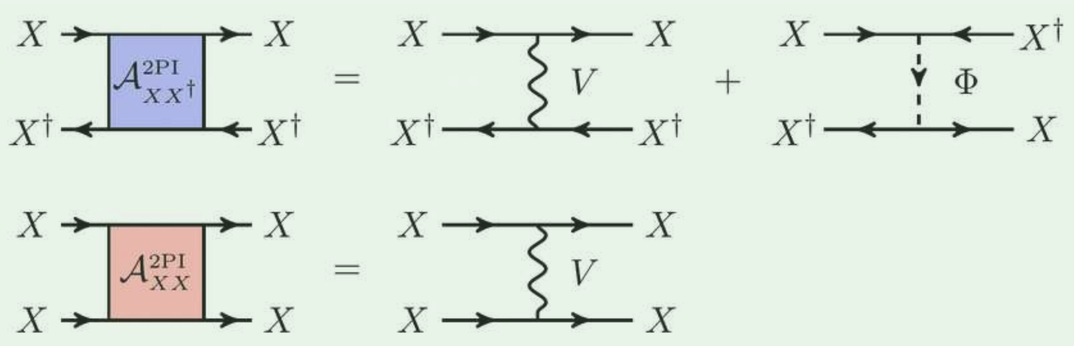
Extremely fast transitions!

Oncala, Petraki: 1911.02605

Scalar DM *X,X†* coupled to doubly charged light scalar mediator *Φ*

$$\mathcal{L} \supset \; -igX^\dagger V^\mu(\partial_\mu X) \; - i2g\Phi^\dagger V^\mu(\partial_\mu \Phi) \; - \frac{y m_X}{2} XX\Phi^\dagger + h.c.$$

$$m_X \gg m_\Phi$$



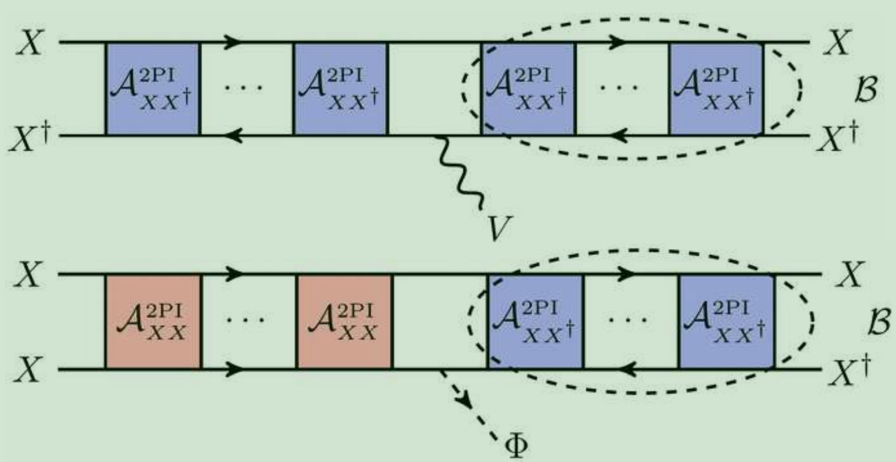
$$U_{XX^\dagger}(r) \; = \; -\frac{\alpha_V}{r} \; - (-1)^\ell \frac{\alpha_\Phi}{r} e^{-m_\Phi r}$$

$$U_{XX}(r) \; = \; +\frac{\alpha_V}{r}$$

Potential differs even in the global symmetry limit $\alpha_V \to 0$

BSF_V

BSF_Φ



dipole transition

$$\mathcal{M} \sim 2g \int d^3p \; \psi^*_{n\ell m}(r) \nabla \phi_k(r) \sim 2g \int d^3p \; \psi^*_{n\ell m}(r) r \phi_k(r)$$

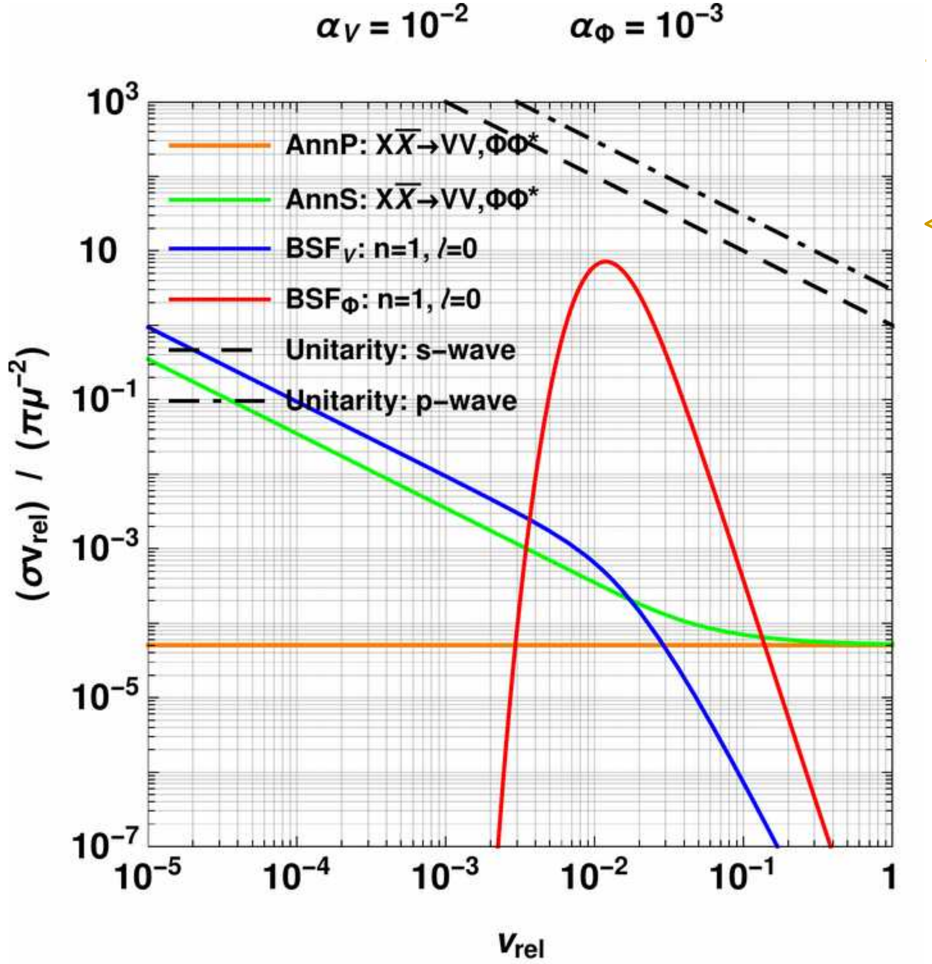
Change in effective Hamiltonian
⇒ monopole transition

$$\mathcal{M} \sim 2y \int d^3p \; \psi^*_{n\ell m}(r) \; \phi_k(r)$$

extremely fast!

Oncala, KP: 1911.02605

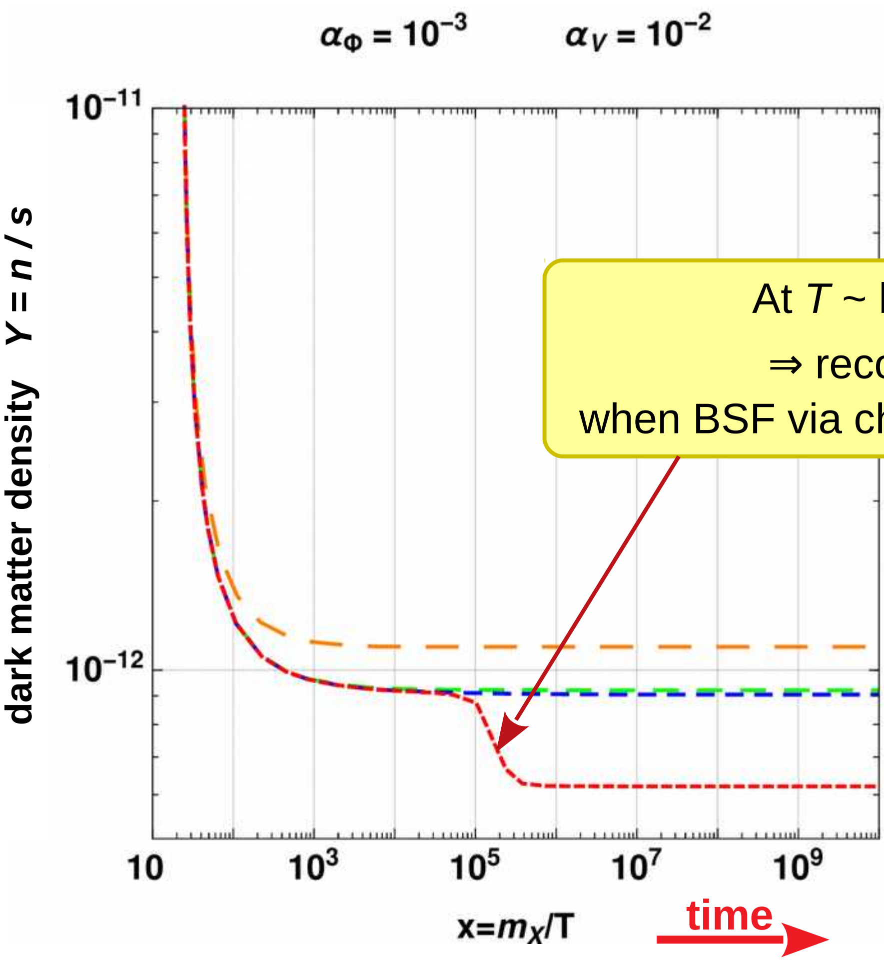
Scalar DM $X, X^\dagger$ coupled to doubly charged light scalar mediator Φ



BSF$_\Phi$ very large, even for small values of α_Φ, α_V !

Oncala, KP: 1911.02605

Scalar DM $X,X^{\dagger}$ coupled to doubly charged light scalar mediator Φ

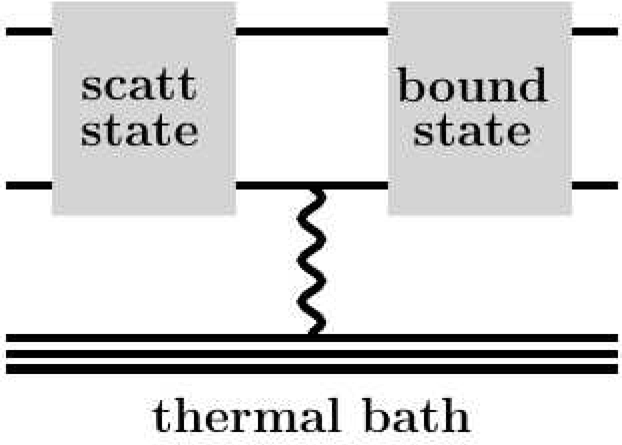


At T ~ binding energy << m_X/30
⇒ recoupling of DM destruction
when BSF via charged scalar emission considered

Up to 2 orders of magnitude reduction in the relic density !

Oncala, KP: 1911.02605

Capture into bound states via
scattering on relativistic thermal bath



$$\sigma_{\rm BSF}^{\rm scatt} \sim \sigma_{\rm BSF}^{\rm rad} \times R$$

$$R \sim (T/\omega_{\rm rad})^3 \sim (T/|\mathcal{E}_{\mathcal{B}}|)^3$$

typically does not affect
DM density significantly

Abelian gauge theories: Binder, Mukaida, Petraki 1910.11288
Non-Abelian gauge theories: Binder, Blobel, Harz, Mukaida 2002.07145
Scalar mediators: Oncala, Petraki 2101.08666