EW corrections in SMEFT

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Vienna Seminar, 19 March, 2024

Outline for NLO SMEFT

1) The context
2) Renormalisation and interesting features with $h \to b\bar{b}$ an as example
3) NLO corrections and uncertainties in “sane” renormalisation schemes
4) Universal NLO corrections and democratisation of EW input schemes

SMEFT framework

SMEFT: treat SM as a low-energy EFT of a UV complete theory, assuming

- $\Lambda_{\rm NP} \gg \Lambda_{\rm EW}$, i.e. no new light particles
- SU(3)xSU(2)xU(1) broken by vev of SU(2) doublet Higgs field

SMEFT Lagrangian:

$$\mathcal{L}^{\rm SMEFT} = \mathcal{L}^{\rm SM} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu) + (\text{dim-8 and higher})$$

- Q_i are 59 dimension-6 operators (2499 when flavour indices included)
- $C_i \sim 1/\Lambda_{\rm NP}^2$ are Wilson coefficients
- well defined QFT: renormalizable order by order in $1/\Lambda_{\rm NP}^2$

SMEFT in practice

$$\mathcal{L}^{\text{SMEFT}} = \mathcal{L}^{\text{SM}} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu)$$

Two uses:

- bottom up (the present): NP if any $C_i \neq 0 \Rightarrow$ many efforts to see this in data through global fits
- top down (the future): if NP is known, SMEFT is tool for calculating RG-improved cross sections at $E \ll \Lambda_{\text{NP}}$

NLO calculations:

- reduce dependence on renormalisation scheme (for instance μ-dependence in $\overline{\text{MS}}$)
- more robust fits (bottom up), better agreement with data (top-down)

The NLO SMEFT landscape

A rapidly expanding field:

- Current calculations done on case-by-case basis: [Giardino, Dawson, Maltoni, Zhang, Trott, Petriello, Duhr, Schulze, Passarino, Signer, Pruna, Shepherd, Hartmann, Baglio, Lewis, Zhang, Boughezal, Degrande, BP, Vryonidou, Mimasu, Deutschmann, Scott, Dedes, Suxho, Trifyllis, Gomez-Ambrosio, Durieux, Cullen, Gauld, Haisch, Zanderighi, Corbett . . .]
- Future is NLO automation as in SM
  (already available for QCD corrections [Degrande et al. arXiv:2008.11743])

This talk: issues common to all NLO EW calculations in SMEFT

- general procedure with $h \to b\bar{b}$ as an example
  [Cullen, BP, Scott: '19]
- EW input schemes and universal corrections in SMEFT
  [Biekötter, BP, Scott, Smith arXiv:2305.03763, arXiv:2312.08446]

Motivation for $h \to b\bar{b}$ at NLO in SMEFT

1) **Phenomenology**:

- can measure *hbb* coupling at (sub)percent level at Higgs factory
- $\Rightarrow$ NLO SMEFT calculation sets long-term baseline for analysis in EFT

2) **SMEFT development**:

- reveals many non-trivial features of SMEFT at NLO in (relatively) simple setting
- analytic results useful for benchmarking automated codes for NLO SMEFT

$h \to b\bar{b}$ as SMEFT laboratory

Some things we dealt with in full NLO calculation:
[Jonathan Cullen, B.P., Darren Scott: arXiv:1904.06358]

- renormalize e, M_W, M_Z, m_b, C_i, plus external b-quark, h-boson fields (45 C_i appear at NLO, checks 100s of entries in 1-loop anom. dims. [Alonso, Jenkins, Manohar, Trott '13])
- EW gauge invariance: tadpoles and gauge fixing in SMEFT
- find appropriate renormalisation scheme for combining EW and QCD corrections
- understand strong EW input-scheme dependence of finite corrections in SMEFT

Outline for NLO SMEFT

1) The context

2) **Renormalisation and interesting features with $h \to b\bar{b}$ an as example**

3) NLO corrections and uncertainties in "sane" renormalisation schemes

4) Universal NLO corrections and democratisation of EW input schemes

Outline of an NLO calculation

Basic outline:

- specify input parameters and renormalisation scheme
- write down LO and UV counterterm amplitudes
- calculate one-loop matrix elements and UV counterterms (2-point functions apart from operator mixing)
- calculate real emissions of photons, gluons, add together with UV-renormalised virtual corrections to get IR finite answer

In general, every piece of calculation gets dim-4 (SM) and dim-6 (SMEFT) contributions, dim-8 terms are dropped

Will illustrate the procedure with $h \to b\bar{b}$

Input parameters for $h \to b\bar{b}$

In the "α scheme", input parameters are:

$$\{M_W, M_Z, \alpha\}, \quad \{m_f, m_H, V_{ij},\ C_i(\mu), \alpha_s(\mu)\}$$

- C_i and α_s are renormalised in the $\overline{\text{MS}}$ scheme
- M_W, M_Z, m_H, m_t renormalised in on-shell scheme
- renormalisation scheme for $\alpha = e^2/(4\pi)$ and $m_b \neq 0$ kept flexible
- all other $m_f = 0$
- we approximate $V_{ij} = \delta_{ij}$

Will come back to other EW input schemes later:

- $\{M_W, M_Z, G_F\}$ = "α_μ scheme"
- $\{\alpha, M_Z, G_F\}$ = "LEP scheme"
- $\{s_w^{\text{eff}}, M_Z, G_F\}$ = "v_μ^{eff} scheme"
- $\{s_w^{\text{eff}}, M_Z, \alpha\}$ = "v_α^{eff} scheme"

$h \to b\bar{b}$ at LO in SM

$$\mathcal{L}_{\rm yuk} = -y_b^* \bar{q}_L H b_R + \text{h.c.}$$

- SSB: $H = \frac{1}{\sqrt{2}}(0, v_T + h(x))^T$
- mass basis: $y_b = \frac{\sqrt{2} m_b}{v_T}$

$$i\mathcal{M}^{(0)}(h \to b\bar{b}) = -i\bar{u}(p_b)\left(\mathcal{M}_L^{(4,0)} P_L + \mathcal{M}_L^{(4,0)*} P_R\right) v(p_{\bar{b}})\,; \quad \mathcal{M}_L^{(4,0)} = \frac{m_b}{v_T}$$

- eliminate v_T: trade $(g_1, g_2, v_T) \leftrightarrow (M_W, M_Z, e)$ using

$$M_W^2 = \frac{g_2^2 v_T^2}{4}; \quad M_Z^2 = \frac{v_T^2}{4}(g_1^2 + g_2^2); \quad e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$$

$$\Rightarrow v_T = \frac{2 M_W \sin\theta_W}{e}, \quad \cos\theta_W = \frac{M_W}{M_Z}$$

$h \to b\bar{b}$ at LO in SMEFT I

dim-6 Lagrangian:

$$\mathcal{L}^{(6)} = \sum_{i=1}^{59} C_i Q_i$$

- one Q_i relevant for $h \to b\bar{b}$ at LO easy to guess:

$$Q_{bH} = H^\dagger H(\bar{q}_L H b_R + \text{h.c.}) \xrightarrow{\text{SSB}} \frac{1}{2\sqrt{2}} \left(v_T^3 + 3hv_T^2 + 3h^2 v_T + h^3 \right) \left[\bar{b}_L b_R + \text{h.c.} \right]$$

- $y_b = \frac{\sqrt{2}m_b}{v_T} + v_T^2 C_{bH}^*/2$, and hbb coupling modified
- LO amplitude gets dim-6 correction:

$$\mathcal{M}_L^{(6,0)}(h \to b\bar{b}) = \frac{m_b}{v_T} - \frac{v_T^2 C_{bH}^*}{\sqrt{2}}$$

$h \to b\bar{b}$ at LO in SMEFT II

Other Q_i where $Q_i \xrightarrow{\text{SSB}} v_T^2 Q_{i,\text{kin}}^{(4)}$ contribute in less obvious ways

- $Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \xrightarrow{\text{SSB}} -\frac{v_T^2}{2}(\partial_\mu h)^2 + \ldots..$
  $\Rightarrow$ rescales Higgs kinetic term

- $Q_{HD} = \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right) \xrightarrow{\text{SSB}} \frac{v_T^2}{2}\left[(\partial_\mu h)^2 + \frac{v_T^4}{8}(g_2 W_\mu^3 - g_1 B_\mu)^2\right] + \ldots$
  $\Rightarrow$ rescales Higgs kinetic term, modifies gauge-boson mass matrix

- $Q_{HWB} = H^\dagger \sigma^I H W_{\mu\nu}^I B^{\mu\nu} \xrightarrow{\text{SSB}} -\frac{v_T^2}{2} W_{\mu\nu}^3 B^{\mu\nu} + \ldots$
  $\Rightarrow$ introduces new kinetic mixing, modifies rotation to gauge boson mass basis

SMEFT in mass basis

- LO effect 1: Higgs field is rescaled to give canonically normalised kinetic terms:

$$H = \frac{1}{\sqrt{2}}\left(0, v_T + h\left(1 + v_T^2\left[C_{H\Box} - \frac{C_{HD}}{4}\right]\right)\right]^T$$

- LO effect 2: Higgs vev $\langle H^\dagger H\rangle \equiv v_T^2/2$

$$\frac{1}{v_T} = \frac{1}{\hat{v}_T}\left(1 + \hat{v}_T^2 \frac{\hat{c}_w}{\hat{s}_w}\left[C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w} C_{HD}\right]\right); \quad \hat{v}_T = 2M_W \hat{s}_W/e; \quad \hat{c}_w = \frac{M_W}{M_Z}$$

- another example: covariant derivative in mass basis

$$\begin{aligned} D_\mu = \partial_\mu - i\frac{e}{\hat{s}_w}\left[1 + \frac{\hat{c}_w^2 \hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB}\right]\left(\mathcal{W}_\mu^+ \tau^+ + \mathcal{W}_\mu^- \tau^-\right) \\ -i\left[\frac{e}{\hat{c}_w \hat{s}_w}\left(1 + \frac{(2\hat{c}_w^2 - 1)\hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB}\right)\left(\tau^3 - \hat{s}_w^2 Q\right)\right. \\ \left. + e\left(\frac{\hat{c}_w \hat{v}_T^2}{2\hat{s}_w} C_{HD} + \hat{v}_T^2 C_{HWB}\right) Q\right] \mathcal{Z}_\mu - ieQ\mathcal{A}_\mu\,, \end{aligned}$$

- gauge fixing much more involved than SM

The LO $h \to b\bar{b}$ amplitude

- LO decay amplitude

$$i\mathcal{M}^{(0)}(h \to b\bar{b}) = -i\bar{u}(p_b)\left(\mathcal{M}_L^{(0)} P_L + \mathcal{M}_L^{(0)*} P_R\right) v(p_{\bar{b}})$$

- split into dim-4 and dim-6 contributions

$$\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$$

- Explicit results

$$\mathcal{M}_L^{(4,0)} = \frac{m_b}{\hat{v}_T}\,,$$
$$\mathcal{M}_L^{(6,0)} = m_b \hat{v}_T \left[-\frac{\hat{v}_T}{m_b} \frac{C_{bH}^*}{\sqrt{2}} + C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} \right]$$

UV COUNTERTERMS I

- replace fields and parameters in bare LO amplitude by renormalised ones
  - wavefunction renormalisation ($f = h, b_L, b_R$)

$$f^{(0)} = \sqrt{Z_f} f = \left(1 + \frac{1}{2}\delta Z_f\right) f \,,$$

  - masses, electric charge, and Wilson coefficients

$$M^{(0)} = M + \delta M, \qquad e^{(0)} = e + \delta e, \qquad C_i^{(0)} = C_i + \delta C_i$$

- expand to linear order in counterterms, separating dim-4 and dim-6 contributions
- mass, charge, field counterterms obtained from two-point functions in mass basis
- δC_i related to operator renormalisation, obtained in symmetric phase in [Jenkins, Manohar, Trott, Alonso].

$$\delta C_i = \frac{1}{2\epsilon} \sum_{i=1}^{59} \gamma_{ij} C_j$$

  - $\gamma_{ij} \equiv \gamma_{ij}(g_1, g_2, \lambda, Y_f)$ adapted to broken phase by re-expressing in terms of the input parameters $M_W, M_Z, v_T, e, m_H, m_b ...$

UV counterterms II: counterterm amplitude

dimension-4 counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(4)}}{m_b} - \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)$$

dimension-6 counterterm is

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} =& \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right) \\ &+ \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right) \\ &- \frac{\hat{v}_T^2}{\sqrt{2}} C_{bH}^*\left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w}C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} \\ &+ m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)\end{aligned}$$

where

$$\frac{\delta\hat{v}_T}{\hat{v}_T} \equiv \frac{\delta M_W}{M_W} + \frac{\delta\hat{s}_w}{\hat{s}_w} - \frac{\delta e}{e}$$

and

$$\frac{\delta\hat{s}_w}{\hat{s}_w} = -\frac{\hat{c}_w^2}{\hat{s}_w^2}\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right), \quad \delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} = -\frac{1}{\hat{c}_w\hat{s}_w}\left(\frac{\delta\hat{s}_w^{(4)}}{\hat{s}_w}\right)$$

Calculational procedure

One-loop $h \to bb$ matrix elements and two-point functions for counterterms involve many Feynman diagrams and dim-6 operators

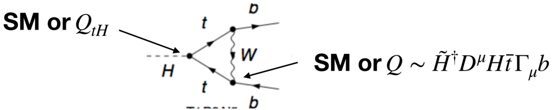
- we used normal chain of **automation**: Feynrules (in-house model file, including gauge fixing and ghosts), Feynarts, FormCalc, Package X
- all loop integrals obtained analytically in terms of Passarino-Veltmann integrals and also given in terms of standard functions in electronic files with paper

Decay rate also requires real emission corrections $h \to bb(g, \gamma)$

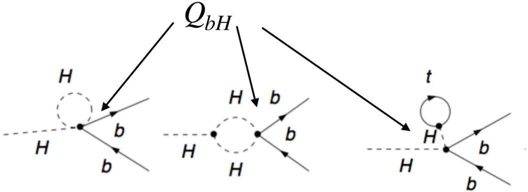
- squared matrix elements generated with automated tools
- 3-body phase space integrals done by hand

Examples of NLO corrections

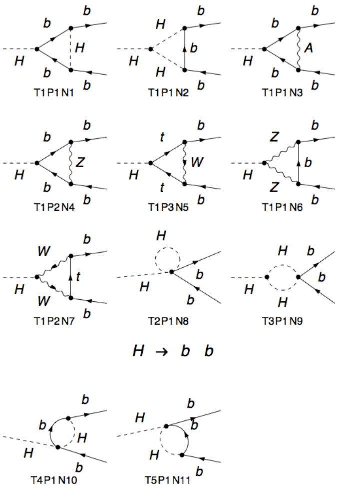
- Type 1: SM-like diagrams

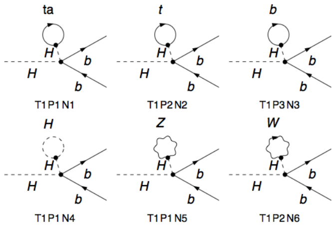


- Type 2: non-SM-like diagrams



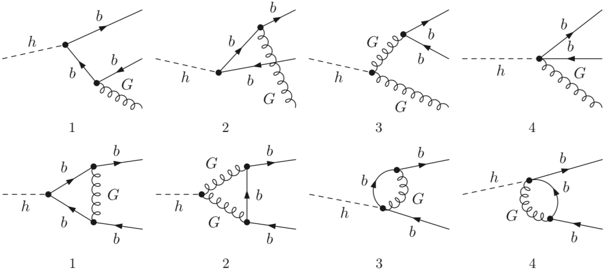
Example in unitary gauge: SM and $Q_{bH} \sim H^\dagger H \bar{b}_L b_R H + \mathrm{h.c.}$



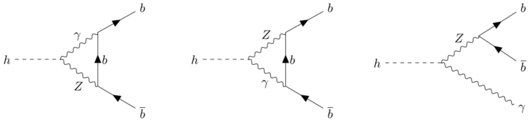


QCD-QED CORRECTIONS

- QCD corrections by far simplest to calculate [Gauld, B.P., Scott '16]
- UV-renormalised one-loop amplitudes have IR divergences canceled by real emissions



- most corrections involving photons can obtained analogously, exception is graphs involving $h\gamma Z$ vertex



(a) (b) (c)

Analytic structure of $h\gamma Z$ corrections

Performing loop and phase-space integrals:

$$\Gamma_{h\gamma Z} \propto v_b \left[2(C_{HB}-C_{HW})\hat{c}_w\hat{s}_w + C_{HWB}(\hat{c}_w^2-\hat{s}_w^2)\right] F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2}\right)$$

$$\begin{aligned}
F_{h\gamma Z}\left(z,\hat{\mu}^2,b\right) = {} & \frac{3}{4}\beta(8z-5) - \beta^3\left(\frac{39}{4}+\frac{z}{b}\right) - \frac{4}{3}\beta^2\pi^2\bar{z} + \frac{4}{3}\pi^2 z\bar{z} + 6\beta\Bigg(\beta^2-\frac{2}{3}z \\
& + \frac{(2b-\beta^2)z^2}{12b^2}\Bigg)\ln(b) + 2(\beta^2-z)\bar{z}\ln(x_z)^2 - 4\beta_z z\bar{z}\ln(x_{\beta z}) \\
& + \ln(x)\Bigg(-\frac{1}{8}\left(15+7\beta^4+8z(4z-7)+\beta^2(2+8z)\right) + 2(z-\beta^2)\bar{z}\ln(x_z) \\
& + 4(\beta^2-z)\bar{z}\ln(1-xx_z) + 2(\beta^2-z)\bar{z}\ln(x_{\beta z})\Bigg) \\
& + \ln(x_z)\left(\frac{\beta\beta_z z\left(\beta^2(2b+z)-2bz\right)}{2b^2} + 2(z-\beta^2)\bar{z}\ln(x_{\beta z})\right) \\
& + 4\beta z\bar{z}\ln(\bar{z}) + \frac{\beta^3(\beta^2+2b)z^2\ln(z)}{2b^2} - 6\beta^3\ln\left(\hat{\mu}^2\right) \\
& + 4(\beta^2-z)\bar{z}\left(\mathrm{Li}_2\left(\frac{x}{x_z}\right) + \mathrm{Li}_2\left(xx_z\right)\right)
\end{aligned}$$

where

$$\beta=\sqrt{1-4b}\,,\quad \beta_z=\sqrt{1-\frac{4b}{z}}\,,\quad x=\frac{1-\beta}{1+\beta},\quad x_z=\frac{1-\beta_z}{1+\beta_z}\,,\quad x_{\beta z}=\frac{\beta-\beta_z}{\beta+\beta_z}\,,\quad \bar{z}=1-z$$

Cross-checks and features

Results involve 45 different Wilson coefficients (generally complex). Cross-checks:

- all UV and IR poles cancel (and μ-dependence consistent with RG eqns)
- SM results reproduced from dim. 4 terms
- all results calculated in unitary and Feynman gauge with full agreement

Interesting features:

- structure of wave-function renormalisation of b-quark field
- Higgs-Z mixing
- Ward identities and electric charge renormalisation
- structure of tadpole contributions

Fermion w.f. renormalisation in SMEFT

- Decompose two-point function of fermion f as

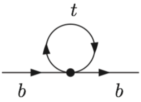
$$\Gamma^f(p) = i(\not{p} - m_f) + i\left[\not{p}\left(P_L\Sigma_f^L(p^2) + P_R\Sigma_f^R(p^2)\right) + m_f\left(\Sigma_f^S(p^2)P_L + \Sigma_f^{S*}(p^2)P_R\right)\right]$$

- In on-shell renormalisation scheme

$$\delta Z_f^L = -\widetilde{\mathrm{Re}}\,\Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2) - m_f^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Big|_{p^2=m_f^2},$$

$$\delta Z_f^R = -\widetilde{\mathrm{Re}}\,\Sigma^{f,R}(m_f^2) - m_f^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Big|_{p^2=m_f^2}$$

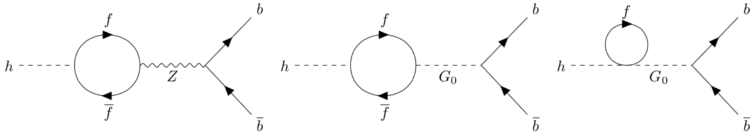
- $\Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2)$ vanishes in SM, but is proportional $\mathrm{Im}(C_i)$ in SMEFT.
- appears in many places in renormalisation of amplitude – example:



$$Z_b^L = \frac{1}{\epsilon}\left[-\frac{m_t^3}{m_b}\left((2N_c+1)\left(C_{qtqb}^{(1)} - C_{qtqb}^{(1)*}\right) + c_{F,3}\left(C_{qtqb}^{(8)} - C_{qtqb}^{(8)*}\right)\right)\right] + \text{finite}$$

Higgs-Z (Goldstone) mixing

- unlike SM, in SMEFT Higgs can mix into Z and neutral Golstone boson G_0



- h-G_0 mixing is that between real and imaginary parts of doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} i \phi^+(x) \\ \left[1 + C_{H,\text{kin.}}\right] h(x) + i \left[1 - \frac{\hat{v}_T^2}{4} C_{HD}\right] \phi^0(x) + v_T \end{pmatrix}$$

- mixing is therefore proportional to imaginary parts of Wilson coefficients and reads

$$\eta_5 = \frac{\sqrt{2}}{\hat{v}_T} \mathrm{Im}\left[N_c m_b C_{bH} - N_c m_t C_{tH} + m_\tau C_{\tau H} + \dots\right]$$

- this term exactly cancels one appearing in renormalisation of Q_{fH} (i.e. that in $\dot{C}_{bH}$ calculated in[Jenkins, Manohar, Trott '13])

Electric charge renormalisation

SM: $ff\gamma$ vertex related to two-point fcns through Ward identities:

- result

$$\frac{\delta e^{(4)}}{e} = \frac{1}{2}\frac{\partial \Sigma_T^{AA(4)}(k^2)}{\partial k^2}\bigg|_{k^2=0} - \frac{(v_f^{(4)} - a_f^{(4)})}{Q_f}\frac{\Sigma_T^{AZ(4)}(0)}{M_Z^2}$$

- $v_f^{(4)} - a_f^{(4)} = -Q_f \hat{s}_w/\hat{c}_w \Rightarrow \delta e^{(4)}$ independent of fermion charge

SMEFT: determine counterterm directly from $ff\gamma$ vertex (not using Ward identities)

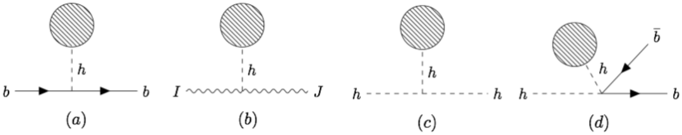
- result

$$\frac{\delta e^{(6)}}{e} = \frac{1}{2}\frac{\partial \Sigma_T^{AA(6)}(k^2)}{\partial k^2}\bigg|_{k^2=0} + \frac{1}{M_Z^2}\left(\frac{\hat{s}_w}{\hat{c}_w}\Sigma_T^{AZ(6)}(0) - \frac{\hat{v}_T^2}{4\hat{c}_w\hat{s}_w}C_{HD}\Sigma_T^{AZ(4)}(0)\right)$$

- For operators $Q_{Hf} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{f}\gamma^\mu f)$, one has $v_f^{(6)} - a_f^{(6)} = 2C_{Hf}\hat{v}_T^2/4\hat{c}_w\hat{s}_w$
  $\Rightarrow$ Naive generalization of SM result fails

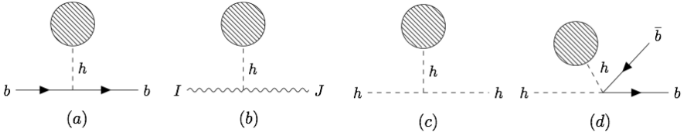
Tadpoles I

- we used FJ tadpole scheme [Fleischer, Jegerlehner '80]
- discussion in [Denner, Jenniches, Lang, Sturm '16] shows FJ scheme implemented by simply calculating all tadpole contributions to n-point functions
- tadpoles needed for $h \to bb$ in SMEFT



- we calculated tadpoles in Feynman and unitary gauge and found expected results:
  (1) tadpoles cancel in on-shell scheme
  (2) mass and electric charge counterterms, and matrix elements +wavefunction renormalisation separately gauge invariant after adding tadpoles
- structure of tadpoles contributions in SMEFT richer than in SM

Tadpoles II



$IJ \in \{\gamma\gamma, \gamma Z, WW, ZZ\}$

- (a) contributes to δm_b in SM and SMEFT, but also to δZ_b^L in SMEFT

$$\delta Z^L_{b,\mathrm{tad.}} = -\frac{i\sqrt{2}\hat{v}_T^2}{m_H^2 m_b} \mathrm{Im}(C_{bH}) T^{(4)}$$

- (b) contributes to $\delta M_W, \delta M_Z$ in SM and SMEFT, also to δe in SMEFT ($IJ = \gamma\gamma$)

$$\frac{\delta e^{\mathrm{cl.4},(6)}}{e} = \frac{1}{16\pi^2}\left[c_{h\gamma\gamma} A_0(m_H^2) + 4\hat{c}_w \hat{s}_w C_{HWB}\left(4M_W^2 - 3A_0(M_W^2)\right)\right] - 2c_{h\gamma\gamma}\frac{\hat{v}_T}{m_H^2} T^{(4)}_{\mathrm{un.}}$$

- (c) contributes to δZ_h in SMEFT (through C_{HD} and $C_{H\Box}$), but not in SM
- (d) contributes to $h \to bb$ matrix element in SMEFT, but not in SM

Outline for NLO SMEFT

1) The context

2) Renormalisation and interesting features with $h \to b\bar{b}$ an as example

3) **NLO corrections and uncertainties in “sane” renormalisation schemes**

4) Universal NLO corrections and democratisation of EW input schemes

Meaningful results

To quote meaningful results, need to

- fix a renormalisation scheme (preferably one where radiative corrections minimal)
- assign an uncertainty to uncalculated higher orders (usually through scale variations)

Enhanced NLO corrections I: QCD corrections

- QCD/QED corrections generate $\ln m_b/m_H$ terms when $\mu = m_H$:

$$\frac{\Gamma^{(1)}_{g,\gamma}}{\Gamma^{(4,0)}} \approx \ln^2\left(\frac{m_b^2}{m_H^2}\right)\frac{\hat{v}_T^2}{\pi}\left(C_F\alpha_s C_{HG} + Q_b^2\alpha c_{h\gamma\gamma}\right)$$
$$+\; c_{m_b}\ln\left(\frac{m_b^2}{m_H^2}\right)\frac{3}{2}\left(\frac{C_F\alpha_s + Q_b^2\alpha}{\pi}\right)\left[1 + 2\hat{v}_T^2\left(C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right)\right.\right.$$
$$\left.\left. + \frac{\hat{c}_w}{\hat{s}_w}C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}}{2\sqrt{2}}\right)\right]$$

- double logs of IR origin remain and are in fact largest NLO correction
- $c_{m_b} = 1$ in on-shell scheme, $c_{m_b} = 0$ ($\overline{\text{MS}}$ scheme) for m_b.
- $\Rightarrow$ **QCD/QED prefers $\overline{\textbf{MS}}$ scheme for** m_b (running mass resums single UV logs)

Enhanced NLO corrections II: tadpoles

- in $\overline{\text{MS}}$ scheme tadpoles don't cancel in decay rate and contribute m_t^4 enhanced corrections. Example, in SM in $m_t \to \infty$ limit

$$\overline{\text{MS}}\text{ scheme:} \qquad \frac{\overline{\Gamma}_t^{(4,1)}}{\Gamma^{(4,0)}} \approx -\frac{N_c}{2\pi^2}\frac{m_t^4}{\hat{v}_T^2 m_H^2} \approx -15\%$$

$$\text{on-shell scheme:} \qquad \frac{[\Gamma_t]^{\text{O.S.}(4,1)}}{\Gamma^{(4,0)}} = \frac{m_t^2}{16\pi^2\hat{v}_T^2}\left(-6 + N_c\frac{7-10\hat{c}_w^2}{3\hat{s}_w^2}\right) \approx -3\%$$

- similar behaviour in SMEFT contributions to decay rate
- **EW corrections prefer on-shell scheme to avoid large tadpole corrections**

Combining EW and QCD corrections is a non-trivial problem. Would like to calculate QCD in $\overline{\text{MS}}$ scheme, but EW in on-shell scheme...

Decoupling relations I

- decoupling relations connect $\overline{\text{MS}}$ parameters in SM, with those in low-energy theory where top and heavy bosons integrated out:

$$\overline{m}_b(\mu) = \zeta_b(\mu, m_t, m_H, M_W, M_Z)\overline{m}_b^{(\ell)}(\mu)$$

- decoupling constant ζ_b contain contributions from heavy particles.
- ζ_b calculated by relating on-shell mass with $\overline{\text{MS}}$ masses in SM and low-energy theories:

$$m_b = z_b^{-1}(\mu, m_b, m_t, m_H, M_W, M_Z)\overline{m}_b(\mu) = \left[z_b^{(\ell)}(\mu, m_b)\right]^{-1} \overline{m}_b^{(\ell)}(\mu)$$

$$\Rightarrow \zeta_b(\mu, m_t, m_H, M_W, M_Z) = \left.\frac{z_b(\mu, m_b, m_t, m_H, M_W, M_Z)}{z_b^{(\ell)}(\mu, m_b)}\right|_{m_b \to 0}$$

- works analogously for electric charge. The connection between low energy parameters and experiment are:

from B-physics: $\overline{m}_b^{(\ell)}(\overline{m}_b^{(\ell)}) \approx 4.2\ \mathrm{GeV}$

from LEP: $\overline{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z)\left(1 + \frac{100\alpha}{27\pi}\right), \quad \alpha(M_Z) \approx 1/129,\ \alpha \approx 1/137$

Decoupling relations II

- dim.4 contributions to ζ_i well known, we calculated dim.6 corrs. Example:

$$\zeta_e^{(4,1)} = \frac{\alpha}{\pi}\left[-\frac{1}{12} - \frac{7}{8}\ln\left(\frac{\mu^2}{M_W^2}\right) + \frac{N_c}{6}Q_t^2\ln\left(\frac{\mu^2}{m_t^2}\right)\right]$$

$$\zeta_e^{(6,1)} = \frac{\alpha}{\pi}\left[\sqrt{2}\hat{v}_T m_t N_c Q_t\left(\hat{c}_w\frac{\mathrm{Re}(C_{tB})}{e} + \hat{s}_w\frac{\mathrm{Re}(C_{tW})}{e}\right)\ln\left(\frac{\mu^2}{m_t^2}\right) + 9\frac{C_W}{e}\hat{s}_w M_W^2\ln\left(\frac{\mu^2}{M_W^2}\right)\right] + \frac{\delta e^{\mathrm{cl.4(6)}}}{e}\Bigg|_{\mathrm{fin.},\, m_b\to 0}$$

- relation between NLO decay rate using low-energy parameters vs. SM params:

$$\overline{\Gamma}_\ell^{(4,1)} = \overline{\Gamma}^{(4,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right),$$

$$\overline{\Gamma}_\ell^{(6,1)} = \overline{\Gamma}^{(6,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(6,1)} + \zeta_e^{(6,1)}\right) + 2\overline{\Gamma}^{(6,0)}\zeta_b^{(4,1)} + \sqrt{2}C_{bH}\frac{(\overline{v}^{(\ell)})^3}{\overline{m}_b^{(\ell)}}\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right)$$

- illustrative results: QCD-QED corrections and EW corrections in $m_t \to \infty$ limit:

$$\overline{\Gamma}_{\ell,g,\gamma} = \overline{\Gamma}_{g,\gamma}\,, \qquad \overline{\Gamma}_{\ell,t} = [\Gamma_t]^{\mathrm{O.S.}}$$

  - interpretation: QCD-QED corrections in $\overline{\mathrm{MS}}$ scheme (UV logs resummed), heavy-particle EW corrections in on-shell (no large tadpoles)

Numerical results I: inputs and uncertainty estimate

- numerical values of parameters are $(\overline{v}^{(\ell)}(m_H) \equiv 2M_W \hat{s}_w / \overline{e}^{(\ell)}(m_H))$

m_H	125 GeV	$\overline{m}_b^{(\ell)}(m_H)$	3.0 GeV
m_t	173 GeV	$\overline{e}^{(\ell)}(m_H)$	$\sqrt{4\pi/128}$
M_W	80.4 GeV	$\overline{v}^{(\ell)}(m_H)$	240 GeV
M_Z	91.2 GeV	$\alpha_s(m_H)$	0.1

- use dimensionless coefficients

$$\tilde{C}_i(m_H) \equiv \Lambda_{\rm NP}^2 C_i(m_H),$$

  then dim.6 contributions suppressed by v^2/Λ^2 (not necessary to specify Λ)
- use $\mu = m_H$ in Wilson coefficients and $\overline{\rm MS}$ parameters by default
- estimate higher-order corrections from scale variations or using different renormalisation schemes

Estimating scale uncertainties

- the C_i are unknown. Therefore, use RG eqns to express $C_i(\mu_C)$ in terms of $C_i(m_H)$
- in practice will use $\mu = m_H/2, 2m_H$, so need only fixed-order solutions: ($\dot{C} = \frac{dC}{d\ln\mu}$)

$$
\begin{aligned}
C_i(\mu_C) &= C_i(m_H) + \ln\left(\frac{\mu_C}{m_H}\right)\dot{C}_i(m_H)\,, \\
\overline{m}_b^{(\ell)}(\mu_R) &= \overline{m}_b^{(\ell)}(m_H)\left[1 + \gamma_b(m_H)\ln\left(\frac{\mu_R}{m_H}\right)\right], \\
\overline{\alpha}^{(\ell)}(\mu_R) &= \overline{\alpha}^{(\ell)}(m_H)\left[1 + 2\gamma_e(m_H)\ln\left(\frac{\mu_R}{m_H}\right)\right],
\end{aligned}
$$

- **note**: it is possible (and preferable) to vary μ_C and μ_R independently in order to get a conservative uncertainty estimate, by evaluating

$$
\begin{aligned}
\overline{\Gamma}_\ell^{(6,0)}(\mu_R,\mu_C) &= \overline{\Gamma}_\ell^{(6,0)}(\mu_C)\Big|_{\overline{p}(\mu_C)\to\overline{p}(\mu_R)}\,, \\
\overline{\Gamma}_\ell^{(6,1)}(\mu_R,\mu_C) &= \Bigg\{\overline{\Gamma}_\ell^{(6,1)}(\mu_C) + 2\left[\ln\left(\frac{\mu_C}{m_H}\right) - \ln\left(\frac{\mu_R}{m_H}\right)\right]\Bigg(\gamma_b(\mu_C)\overline{\Gamma}_\ell^{(6,0)}(\mu_C) \\
&\quad + \frac{C_{bH}(\mu_C)}{\sqrt{2}}\frac{(\overline{v}^{(\ell)})^3(\mu_C)}{\overline{m}_b^{(\ell)}(\mu_C)}\overline{\Gamma}_\ell^{(4,0)}(\mu_C)\left[\gamma_b(\mu_C)+\gamma_e(\mu_C)\right]\Bigg)\Bigg\}\Bigg|_{\overline{p}(\mu_C)\to\overline{p}(\mu_R)}
\end{aligned}
$$

$\overline{p}(\mu) \in \{\overline{\alpha}^{(\ell)}(\mu),\ \overline{m}_b^{(\ell)}(\mu),\ \alpha_s(\mu)\}$

Numerical results

- results in units of LO SM: $\Delta(\mu_C, \mu_R) \equiv \Gamma(\mu_C\,, \mu_R)/\Gamma_{\rm SM}^{\rm LO}(m_H, m_H)$
- varying $\mu_C, \mu_R = m_H$ by factors of 2 and adding in quadrature:

$$\Delta^{\rm LO}(m_H, m_H) = (1 \pm 0.08) + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\rm NP}^2}\Bigg\{ (3.74 \pm 0.36)\tilde{C}_{HWB} + (2.00 \pm 0.21)\tilde{C}_{H\Box} - (1.41 \pm 0.07)\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + (1.24 \pm 0.14)\tilde{C}_{HD} \pm 0.35\tilde{C}_{HG} \pm 0.19\tilde{C}_{Hq}^{(1)} \pm 0.18\tilde{C}_{Ht} \pm 0.11\tilde{C}_{Hq}^{(3)} + \dots \Bigg\}$$

$$\Delta^{\rm NLO}(m_H, m_H) = 1.13^{+0.01}_{-0.04} + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\rm NP}^2}\Bigg\{ \left(4.16^{+0.05}_{-0.14}\right)\tilde{C}_{HWB} + \left(2.40^{+0.04}_{-0.09}\right)\tilde{C}_{H\Box} + \left(-1.73^{+0.04}_{-0.03}\right)\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + \left(1.33^{+0.01}_{-0.04}\right)\tilde{C}_{HD} + \left(2.75^{+0.49}_{-0.48}\right)\tilde{C}_{HG} + \left(-0.12^{+0.04}_{-0.01}\right)\tilde{C}_{Hq}^{(3)} + \left(-0.08^{+0.05}_{-0.01}\right)\tilde{C}_{Ht} + \left(0.06^{+0.00}_{-0.05}\right)\tilde{C}_{Hq}^{(1)} + \left(0.00^{+0.07}_{-0.04}\right)\frac{\tilde{C}_{tG}}{g_s} + \dots \Bigg\}$$

- in general, scale uncertainties in LO result overlap with NLO one, and scale uncertainties decrease between LO and NLO
- exception is C_{HG}, which gives large corrections unrelated to RG eqns.
- scale variation of C_{HG} gives rise to C_{tG} with size indicative of 2-loop QCD

Corrections to LO results in $h \to b\bar{b}$

	SM	C_{HWB}	$C_{H\Box}$	C_{bH}	C_{HD}
NLO QCD-QED	18.2%	17.9%	18.2%	18.2%	18.2%
NLO large-m_t	-3.1%	-4.6%	3.2%	3.5%	-9.0%
NLO remainder	-2.2%	-1.9%	-1.2 %	0.6%	-2.0%
NLO correction	12.9%	11.3%	20.2%	22.3%	7.1%

Table: Size of NLO corrections to different terms in LO decay rate, split into QCD-QED, large m_t, and remaining components.

- applying SM K-factor to dim.6 coefficients bad approximation for EW corrections
- this is generally the case, also for other decays such as $W \to \ell\nu$ and $Z \to \ell^+\ell^-$
- nonetheless, possible to decipher patterns across the C_i, input schemes, and decays [Biekötter, BP, Scott, Smith arXiv:2305.03763]

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3 DECAYS IN 3 SCHEMES

EW corrections ($\alpha_s = 0$ in $h \to bb$)

$h \to b\bar{b}$	SM	$C_{H\Box}$	C_{HD}	$C_{\underset{33}{dH}}$	C_{HWB}	$C^{(3)}_{\underset{jj}{Hl}}$	$C_{\underset{1221}{ll}}$
α-scheme: $\{M_W, M_Z, \alpha\}$	-5.2 %	2.1%	-11.0%	4.2%	-6.7%	-	-
α_μ-scheme: $\{M_W, M_Z, G_F\}$	-0.8 %	2.1%	2.0%	1.9%	-	0.9%	-0.8%
LEP scheme: $\{\alpha, M_Z, G_F\}$	-0.7 %	2.1%	1.6%	1.9%	-	0.7%	-0.9%

$W \to \tau\nu$	SM	C_{HD}	C_{HWB}	$C^{(3)}_{\underset{jj}{Hl}}$	$C_{\underset{1221}{ll}}$	$C^{(3)}_{\underset{33}{Hl}}$
α	-4.2%	-1.7%	-3.0%	—	—	2.2%
α_μ	-0.3%	—	—	2.5%	-0.2%	2.2%
LEP	2.0%	8.1%	3.2%	5.1%	2.5%	4.6%

$Z \to \tau\tau$	SM	C_{HD}	C_{HWB}	$C_{\underset{33}{He}}$	$C^{(1)}_{\underset{33}{Hl}}$	$C^{(3)}_{\underset{33}{Hl}}$	$C^{(3)}_{\underset{jj}{Hl}}$	$C_{\underset{1221}{ll}}$
α	-4.0%	-10.6%	-5.4%	7.7%	0.3%	-0.5%	—	—
α_μ	$< 0.1\%$	71.1%	-27.2%	7.6%	0.1%	-0.4%	2.9%	0.6%
LEP	1.0%	7.8%	17.4%	2.0%	4.7%	4.2%	6.9%	4.5%

Is there any rhyme or reason to the pattern across C_i?

Connecting schemes

Start with $\mathcal{L}_{\rm bare}(M_W, M_Z, v_T, \dots)$, and renormalise v_T as

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\sigma^2}\left[1 - v_\sigma^2 \Delta v_\sigma^{(6,0,\sigma)} - \frac{1}{v_\sigma^2}\Delta v_\sigma^{(4,1,\sigma)} - \Delta v_\sigma^{(6,1,\sigma)}\right]; \quad \sigma \in \{\alpha, \mu\}$$

$$v_\alpha \equiv \frac{2M_W s_w}{\sqrt{4\pi\alpha}}, \qquad v_\mu \equiv \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} \equiv \frac{2M_W s_w}{\sqrt{4\pi\alpha_\mu}}$$

- for α scheme $\{M_W, M_Z, \alpha\}$: use $\sigma = \alpha$ and determine Δv_α from charge ren.
- for α_μ scheme $\{M_W, M_Z, G_F\}$: use $\sigma = \mu$ and determine Δv_μ from muon decay
- for LEP scheme $\{\alpha, M_Z, G_F\}$: start with α_μ scheme, and then eliminate M_W using

$$\frac{v_\alpha^2}{v_\mu^2} - 1 \equiv \Delta r = v_\mu^2 \Delta r^{(6,0)} + \frac{1}{v_\mu^2}\Delta r^{(4,1)} + \Delta r^{(6,1)}$$

where $\boldsymbol{\Delta r^{(i,j)}}$ **are finite** and related to $\Delta v_{\mu\alpha} = \Delta v_\mu - \Delta v_\alpha$

$$\Delta r^{(6,0)} = \Delta v_{\mu\alpha}^{(6,0)}, \quad \Delta r^{(4,1)} = \Delta v_{\mu\alpha}^{(4,1)}, \quad \Delta r^{(6,1)} = \Delta v_{\mu\alpha}^{(6,1)} + 2\Delta v_\mu^{(4,1,\mu)}\Delta v_{\mu\alpha}^{(6,0)}$$

TOP LOOPS AND UNIVERSAL CORRECTIONS

- Δr is physical, Δv_σ is not. However, in large-m_t limit in SM:

$$\frac{1}{v_{T,0}^2}\bigg|_{m_t\to\infty} = \frac{1}{v_\sigma^2}\left[1+\frac{1}{v_\sigma^2}\left(\Delta r_t^{(4,1)}\delta_{\alpha\sigma} - 2\Delta M_{W,t}^{(4,1)}\right)\right]\;;\quad \sigma\in\{\alpha,\mu\}$$

$$\frac{\Delta r_t^{(4,1)}}{v_\alpha^2} = -\frac{c_w^2}{s_w^2}\frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \approx -3.5\%\,, \qquad \frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \equiv \frac{3}{16\pi^2}\frac{m_t^2}{v_\alpha^2} \approx 1\%$$

- universal correction $\Delta r_t^{(4,1)}$ in α-scheme comes along with LO (can resum!)
- we generalised this to include universal scheme-dependent corrections in SMEFT through a substitution procedure on LO [arXiv:2305.03763], for example

$$\frac{1}{v_T^2} \to \frac{1}{v_\sigma^2}\left[\underbrace{1+v_\sigma^2 K_t^{(6,0,\sigma)} + \frac{K_t^{(4,1,\sigma)}}{v_\sigma^2} + K_t^{(6,1,\sigma)}}_{\mathrm{LO}_K} + (\text{divergent and unphysical stuff})\right]$$

- the K_t are physical top-loop corrections that always come along with LO
- $\Rightarrow$ re-organise pert. theory. to include them already in "LO_K" approximation

NLO corrections to LO_K results

$W \to \tau\nu$	SM	C_{HD}	C_{HWB}	$C^{(3)}_{\substack{Hl \\ jj}}$	$C_{\substack{ll \\ 1221}}$	$C^{(3)}_{\substack{Hl \\ 33}}$
α	−0.9%	1.1%	0.6%	—	—	2.2%
α_μ	−0.3%	—	—	0.6%	−0.2%	2.2%
LEP	0.0 %	1.9%	0.9 %	0.1%	0.2%	2.5%

$Z \to \tau\tau$	SM	C_{HD}	C_{HWB}	$C_{\substack{He \\ 33}}$	$C^{(1)}_{\substack{Hl \\ 33}}$	$C^{(3)}_{\substack{Hl \\ 33}}$	$C^{(3)}_{\substack{Hl \\ jj}}$	$C_{\substack{ll \\ 1221}}$
α	−0.9%	−1.4%	−0.1%	3.3%	2.0%	1.3%	—	—
α_μ	0.0%	11.2%	−3.4%	3.2%	1.8%	1.3%	0.8%	0.0%
LEP	0.0%	2.3%	−3.0%	2.5%	2.5%	2.0%	0.8%	0.0%

$h \to b\bar{b}$	SM	$C_{H\Box}$	C_{HD}	$C_{\substack{dH \\ 33}}$	C_{HWB}	$C^{(3)}_{\substack{Hl \\ jj}}$	$C_{\substack{ll \\ 1221}}$
α	-1.9 %	2.1%	2.5%	2.5%	-1.5%	-	-
α_μ	-0.8 %	2.1%	2.0%	1.9%	-	0.9%	-0.8%
LEP	-0.8 %	2.1%	1.6%	1.9%	-	0.7%	-0.9%

Corrections smaller and less scheme dependent compared to pure fixed order

Summary

- EW corrections in SMEFT involve many interesting features compared to SM
- With universal corrections understood, global fits in different EW input schemes provide important consistency checks [Biekötter, BP, Smith, in progress]
- Next major step will be implementation into event generators [Maskos, BP, Rahaman, Schönherr, in progress] and automated EW