FACTORIZATION OF NON-GLOBAL LHC OBSERVABLES

PART 1: RESUMMATION OF SUPER-LEADING LOGARITHMS

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erc European Research Council

AdG EFT4jets

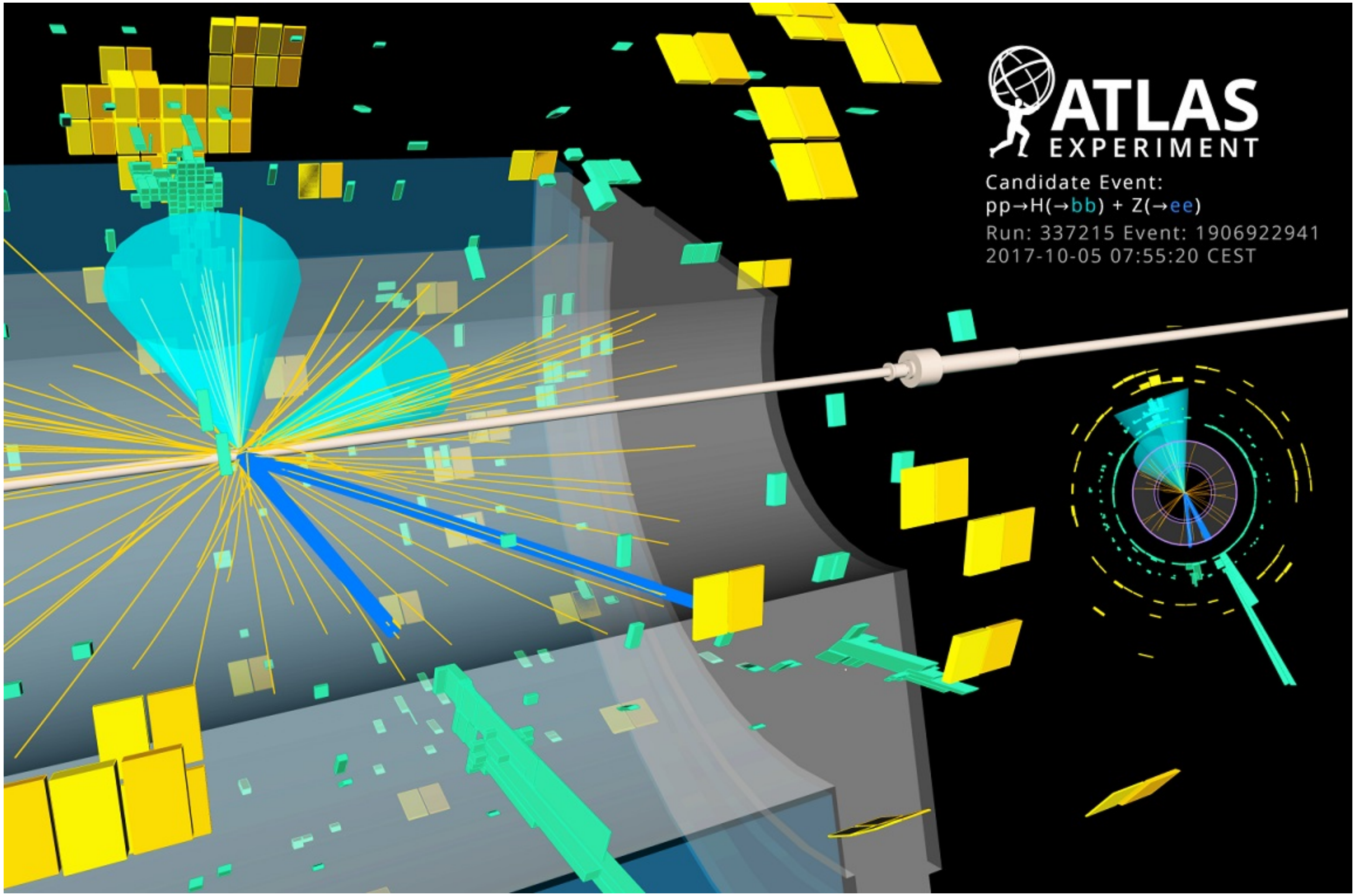
ERWIN SCHRÖDINGER LECTURE I UNIVERSITÄT WIEN — 14 MAY 2024

based on:

Thomas Becher, MN, Dingyu Shao [2107.01212]; Thomas Becher, MN, Dingyu Shao, Michel Stillger [2307.06359]

Philipp Böer, Patrick Hager, MN, Michel Stillger, Xiaofeng Xu [2405.05305]

LARGE LOGARITHMS IN LHC JET PROCESSES



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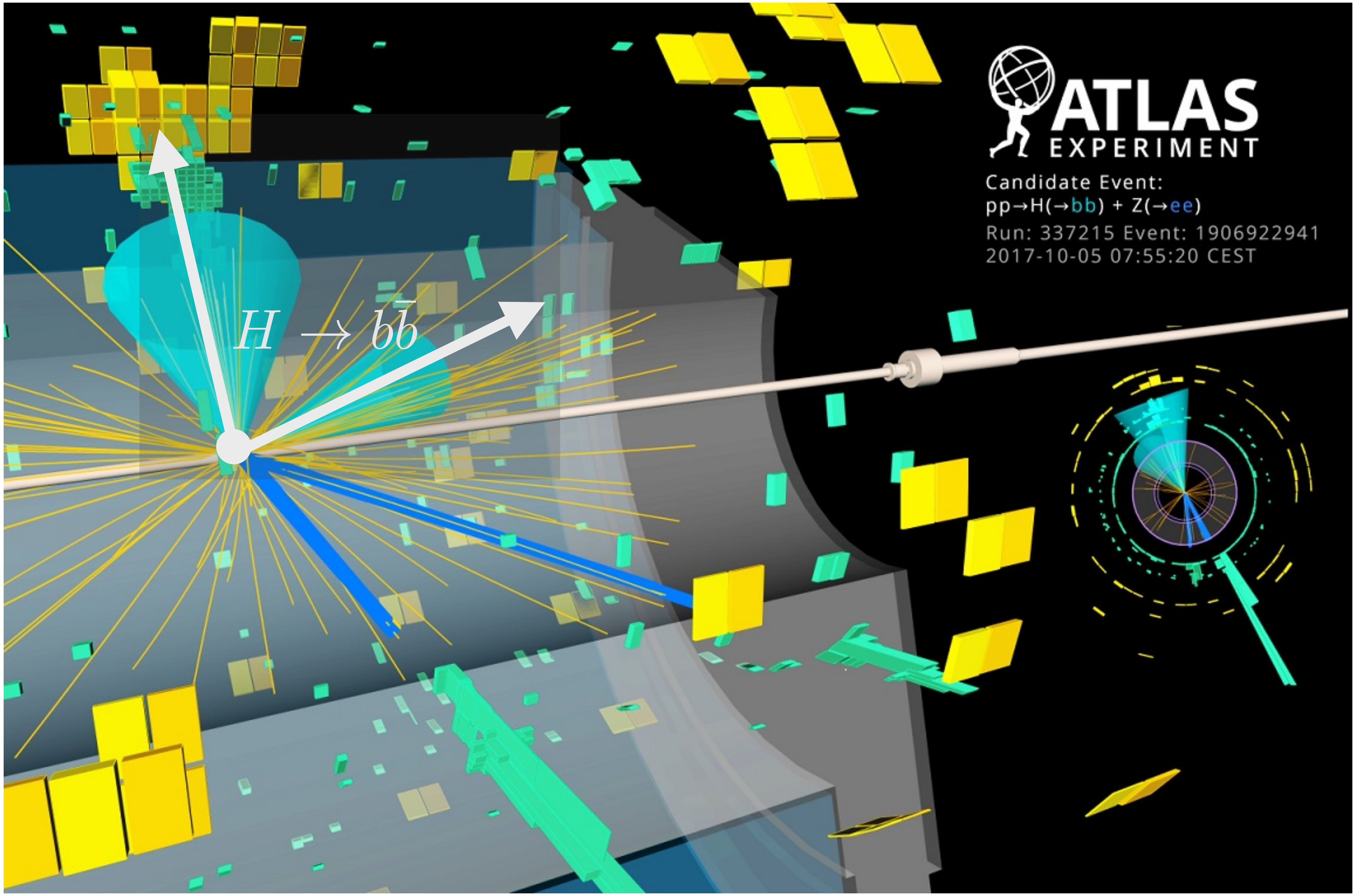
LARGE LOGARITHMS IN LHC JET PROCESSES



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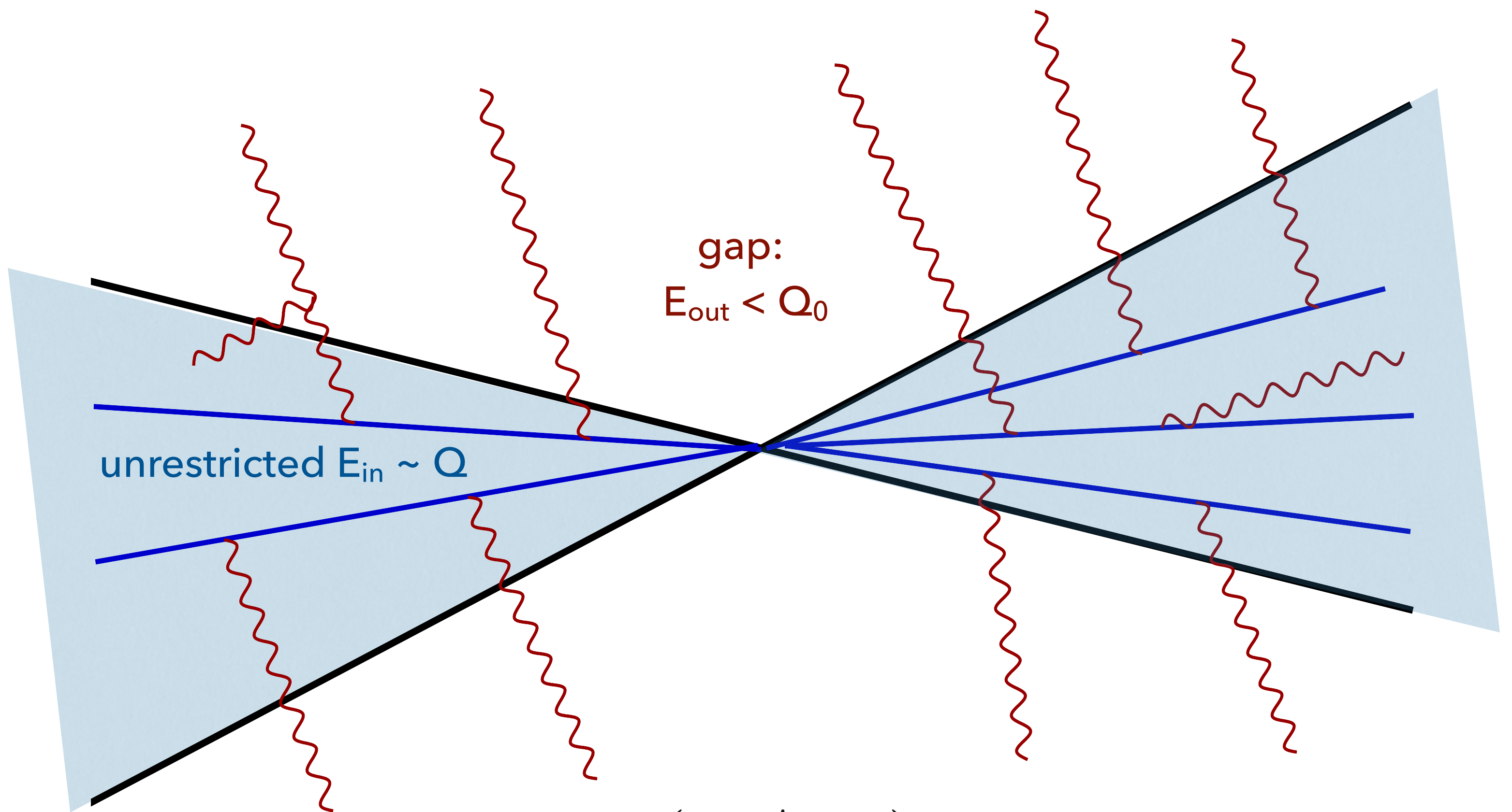
LARGE LOGARITHMS IN LHC JET PROCESSES



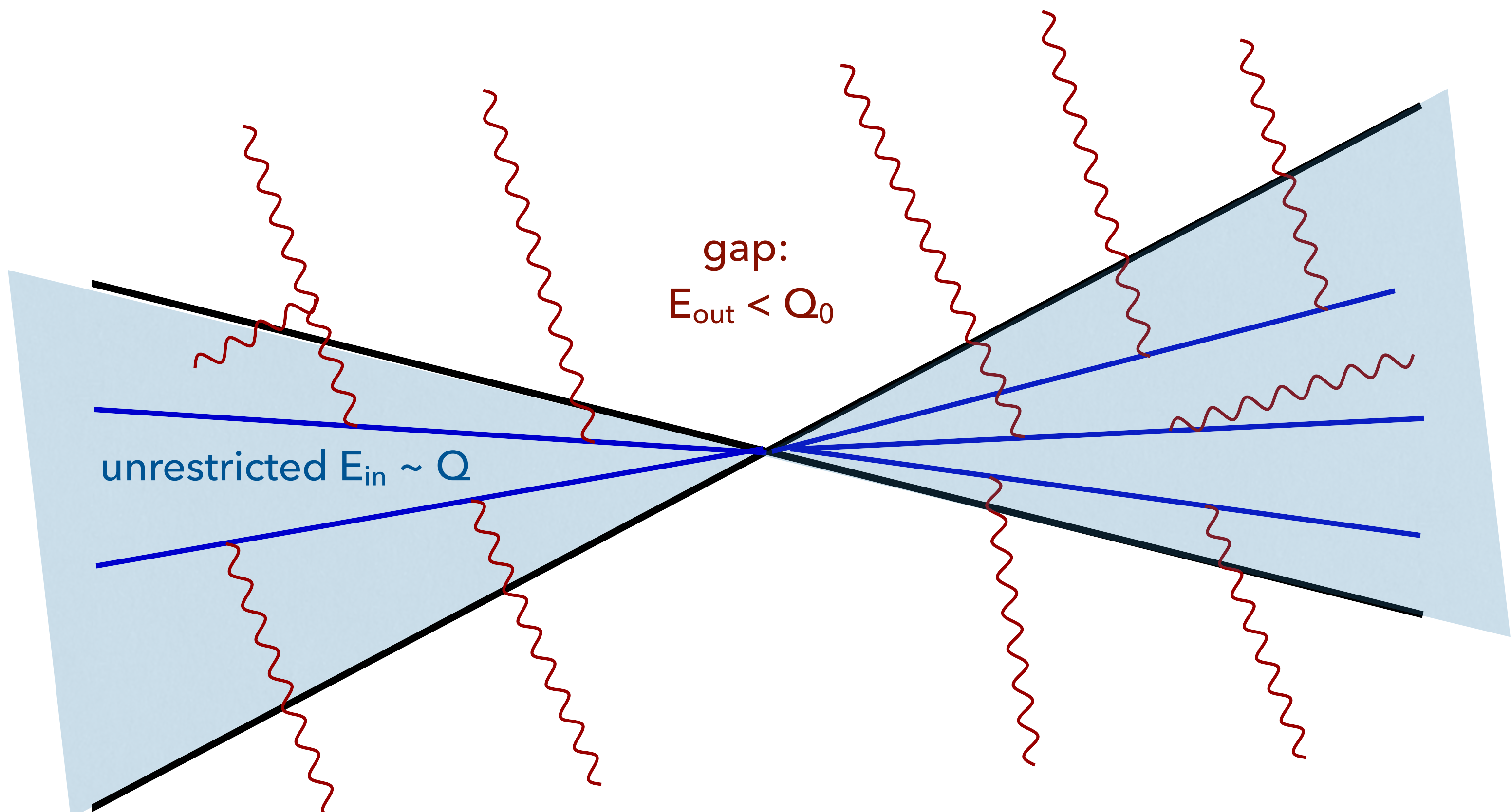
$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion includes "super-leading" logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \dots\right\}$$

state-of-the-art

LARGE LOGARITHMS IN LHC JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

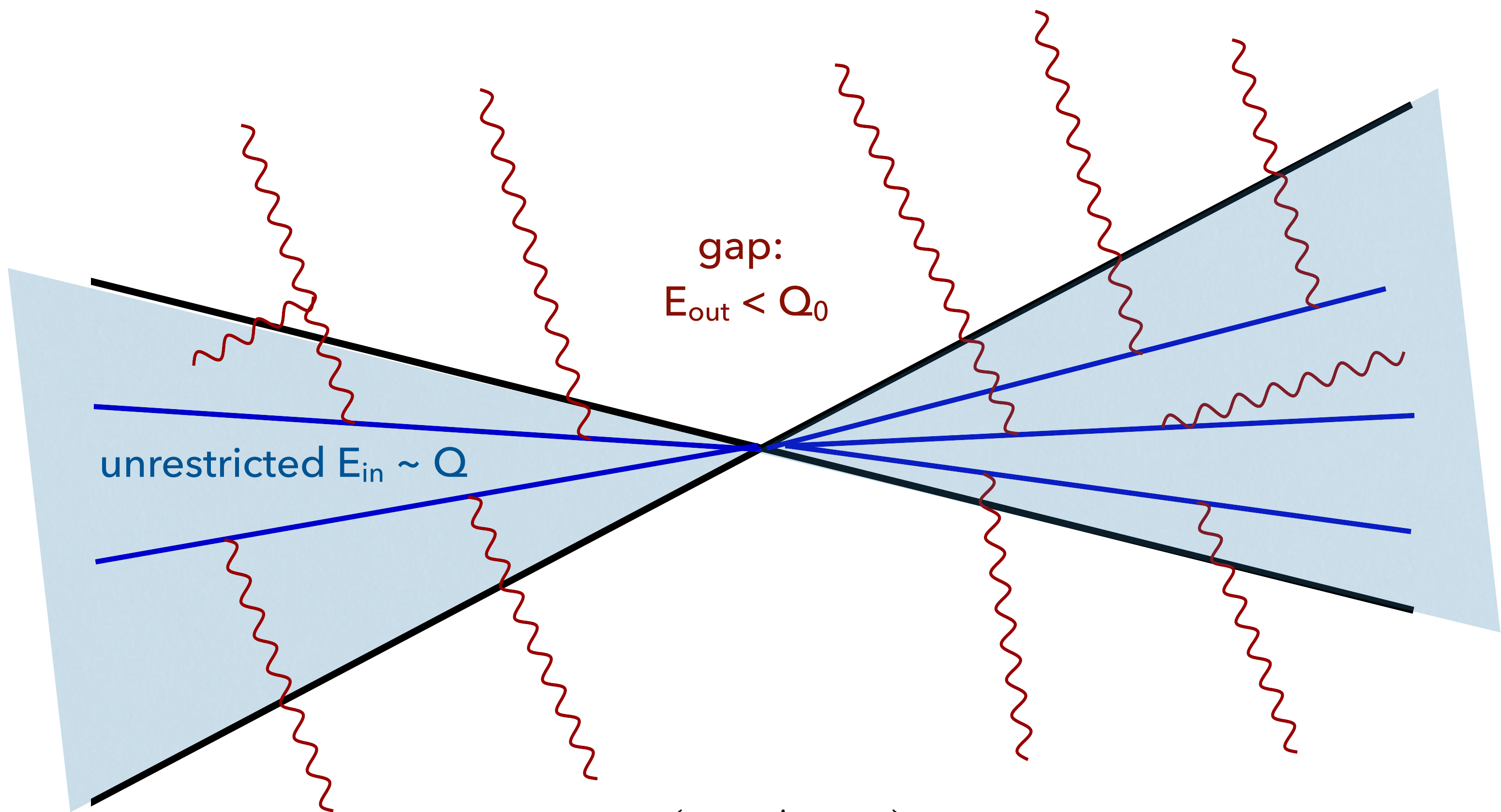
Perturbative expansion includes “super-leading” logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 \, L^5 + \alpha_s^5 \, L^7 + \dots}_{\text{formally larger than } O(1)} \right\}$$

state-of-the-art

J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)

LARGE LOGARITHMS IN LHC JET PROCESSES



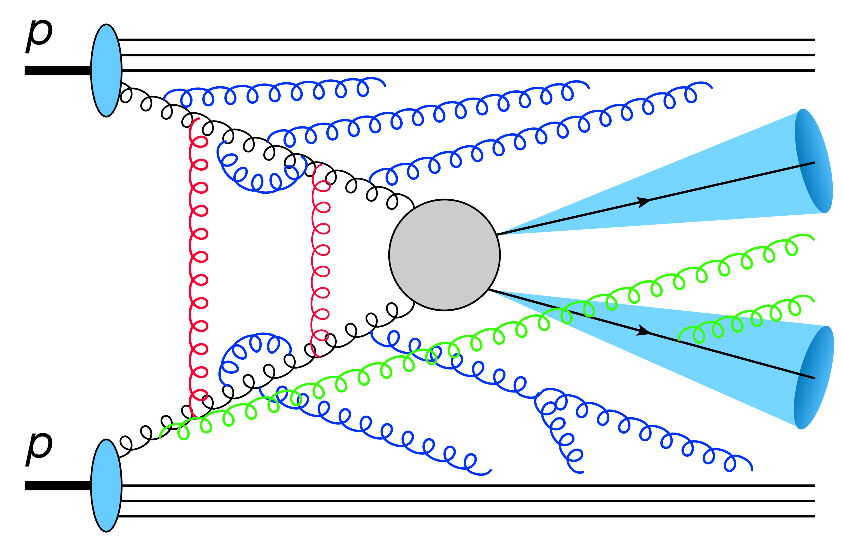
$$L = \ln(Q/Q_0) \gg 1$$

Really, a double logarithmic series starting at 3-loop order:

$$\sigma \sim \sigma_{\rm Born} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + \underbrace{(\alpha_s \pi^2)}_{(\Im\mathrm{m}\, L)^2} \underbrace{\left[\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots\right]}_{\text{formally larger than } O(1)}\right\}$$

COULOMB PHASES BREAK COLOR COHERENCE

Super-leading logarithms



- Breakdown of color coherence due to initial-state soft gluon (Glauber) exchange
  J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)
- Soft anomalous dimension:

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

T. Becher, M. Neubert (2009)

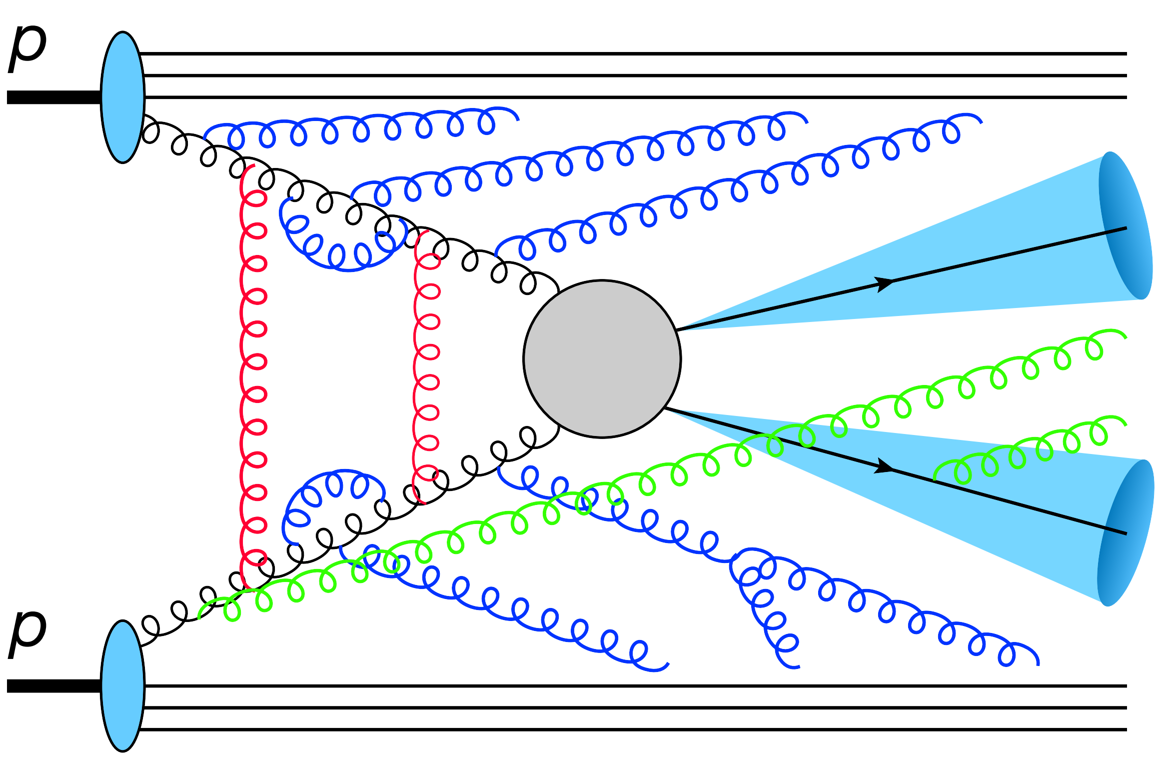
where $s_{ij} > 0$ if particles i and j are both in initial or final state

- Imaginary part (only at hadron colliders):

$$\boxed{\mathrm{Im}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = +2\pi\,\gamma_{\rm cusp}(\alpha_s)\; \boldsymbol{T}_1\cdot\boldsymbol{T}_2} + (\dots)\,\mathbf{1}$$

irrelevant

THEORY OF JET PROCESSES AT LHC



red: Coulomb gluons

blue: gluons emitted along beams

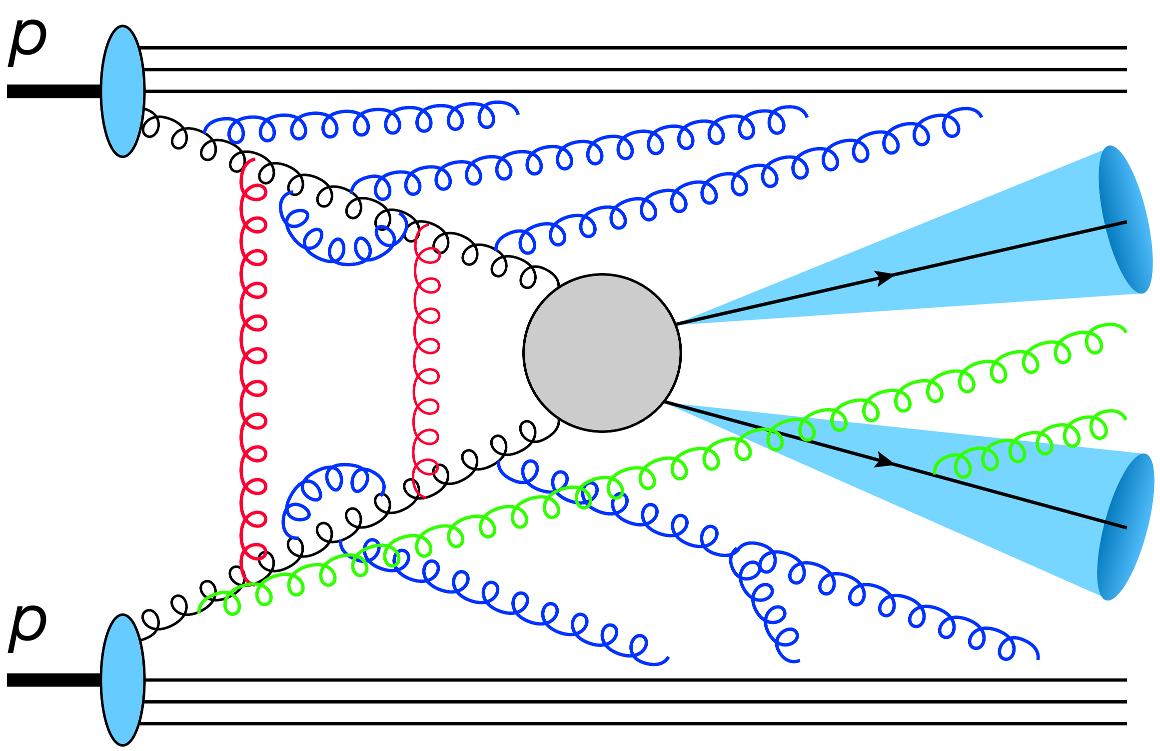
green: soft gluons between jets

Loss of color coherence from initial-state Coulomb interactions

- Weird "super-leading logarithms"

$$d\sigma_{pp\to f}(s) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2\, f_{a/p}(x_1,\mu)\, f_{b/p}(x_2,\mu)\, \underbrace{d\sigma_{ab\to f}(\hat{s}=x_1 x_2 s,\mu)}_{\text{SLLs}}$$

THEORY OF JET PROCESSES AT LHC



red: Coulomb gluons

blue: gluons emitted along beams

green: soft gluons between jets

Loss of color coherence from initial-state Coulomb interactions

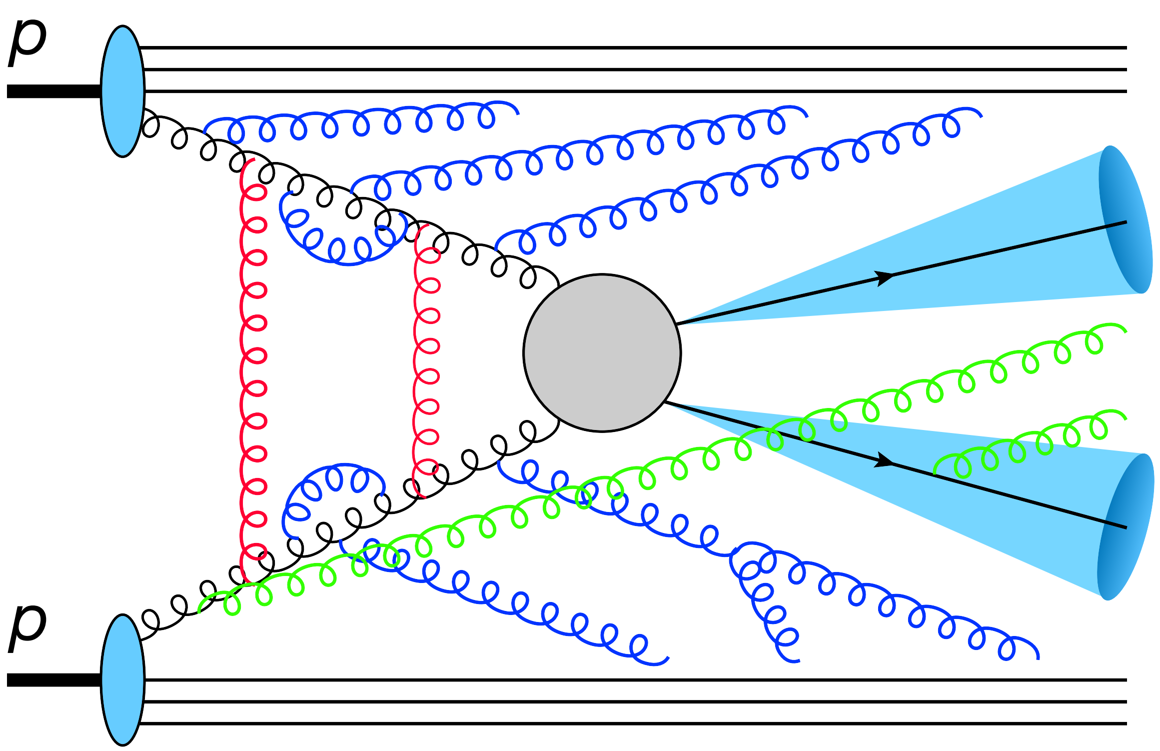
- Weird "super-leading logarithms"
- Breakdown of naive factorization

$$d\sigma_{pp\to f}(s) \neq \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2\, f_{a/p}(x_1,\mu)\, f_{b/p}(x_2,\mu)\, d\sigma_{ab\to f}(\hat{s}=x_1x_2s,\mu)$$

with $\mu \approx \sqrt{\hat{s}} \equiv Q$

SLLs

THEORY OF JET PROCESSES AT LHC



red: Coulomb gluons

blue: gluons emitted along beams

green: soft gluons between jets

Loss of color coherence from initial-state Coulomb interactions

- Weird "super-leading logarithms"
- Breakdown of naive factorization
- Phenomenological consequences?

erc AdG **EFT4jets**

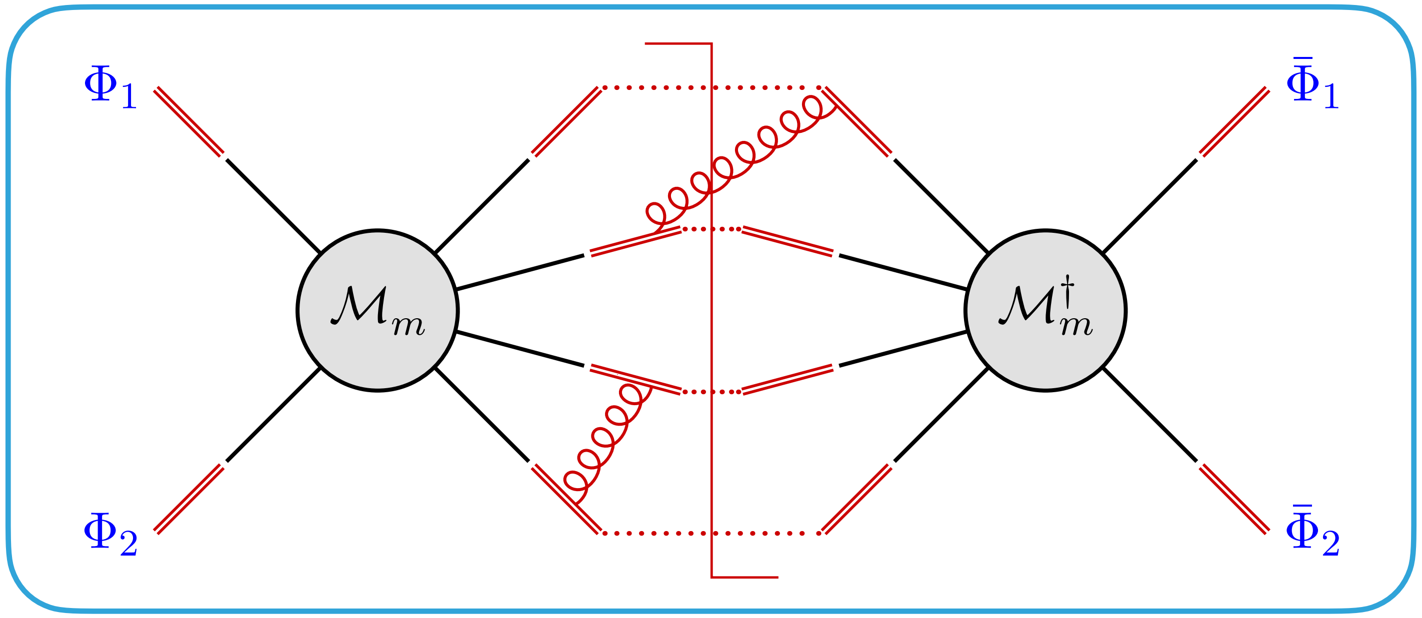
Need for a complete theory of quantum interference effects in jet processes!

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \big\langle \underbrace{\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)}_{\text{high scale}} \otimes \underbrace{\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu)}_{\text{low scale}} \big\rangle$$

T. Becher, M. Neubert, D. Shao (2021)
[see also: T. Becher, M. Neubert, L. Rothen, D. Shao (2015, 2016)]



⇒ new perspective to think about non-global observables!

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \Big\langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \Big\rangle$$

T. Becher, M. Neubert, D. Shao (2021)
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high scale

low scale

Rigorous operator definitions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^{m} \int \frac{dE_i\, E_i^{d-3}}{(2\pi)^{d-2}}\, |\mathcal{M}_m^{ab}(\{\underline{p}\})\rangle\langle\mathcal{M}_m^{ab}(\{\underline{p}\})|\; (2\pi)^d\, \delta\Big(Q - \sum_{i=1}^{m} E_i\Big)\, \delta^{(d-1)}(\vec{p}_{\text{tot}})\, \Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

density matrix involving hard-scattering amplitude in color space

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \rangle$$

T. Becher, M. Neubert, D. Shao (2021)
[see also: T. Becher, M. Neubert, L. Rothen, D. Shao (2015, 2016)]

high scale

low scale

Rigorous operator definitions:

$$\boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,x_1,x_2) = \int_{-\infty}^{\infty} \frac{dt_1}{2\pi}\, e^{-ix_1t_1\bar{n}_1\cdot p_1} \int_{-\infty}^{\infty} \frac{dt_2}{2\pi}\, e^{-ix_2t_2\bar{n}_2\cdot p_2}\, \widetilde{\boldsymbol{\mathcal{W}}}_m(\{\underline{n}\},Q_0,t_1,t_2)$$

with:

$$\begin{aligned}
&\widetilde{\boldsymbol{\mathcal{W}}}_m(\{\underline{n}\},Q_0,t_1,t_2) \\
&= \sum\!\!\!\!\!\!\!\int_{X_s} \mathcal{P}^{(1)}_{\bar{\alpha}\alpha}\, \mathcal{P}^{(2)}_{\bar{\beta}\beta}\, \langle H_1(p_1)H_2(p_2)|\, \bar{\Phi}_1^{\bar{\alpha}}(t_1\bar{n}_1)\, \bar{\Phi}_2^{\bar{\beta}}(t_2\bar{n}_2)\, \boldsymbol{S}_1^\dagger(n_1)\,\dots\,\boldsymbol{S}_m^\dagger(n_m)\,|X_s\rangle \\
&\quad\times \langle X_s|\, \boldsymbol{S}_1(n_1)\,\dots\,\boldsymbol{S}_m(n_m)\, \Phi_1^{\alpha}(0)\, \Phi_2^{\beta}(0)\, |H_1(p_1)H_2(p_2)\rangle\, \theta(Q_0 - E_{\text{out}}^{\perp})
\end{aligned}$$

soft Wilson lines

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \big\rangle$$

T. Becher, M. Neubert, D. Shao (2021)
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high scale

low scale

Renormalization-group equation:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\},Q,\mu) = -\sum_{m\le l} \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)\, \boldsymbol{\Gamma}_{ml}^H(\{\underline{n}\},Q,\mu)$$

operator in color space and in the
infinite space of parton multiplicities

All-order summation of large logarithmic corrections, including the super-leading logarithms!

RESUMMATION OF SUPER-LEADING LOGARITHMS

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Low-energy matrix element:

$$\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_s)$$

- Hard-scattering functions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \boldsymbol{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- Expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

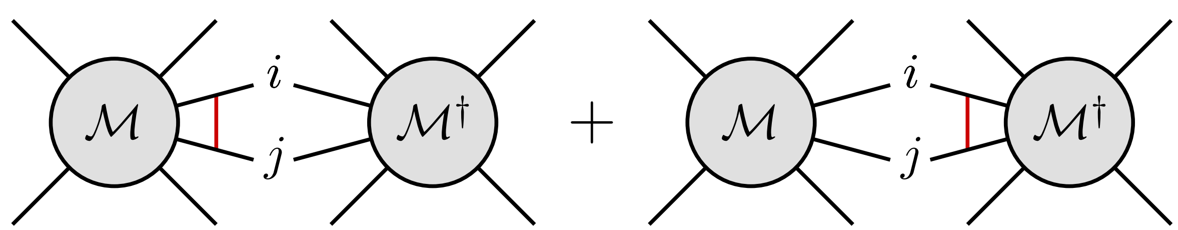
RESUMMATION OF SUPER-LEADING LOGARITHMS

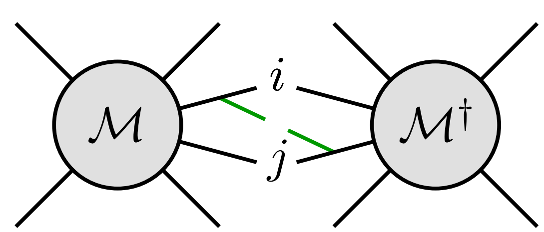
Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Anomalous-dimension matrix:

$$\mathbf{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{2+M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- Action on hard functions:

$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}_m = \sum_{(ij)}$ 

$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_m = \sum_{(ij)}$ 

RESUMMATION OF SUPER-LEADING LOGARITHMS

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

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- Virtual and real contributions contain collinear singularities, which must be regularized and subtracted

$$\mathbf{\Gamma}^H(\xi_1,\xi_2) = \delta(1-\xi_1)\,\delta(1-\xi_2)\,\underbrace{\mathbf{\Gamma}^S}_{\text{soft / soft-collinear part}} + \mathbf{\Gamma}_1^C(\xi_1)\,\delta(1-\xi_2) + \delta(1-\xi_1)\,\mathbf{\Gamma}_2^C(\xi_2)$$

soft / soft-collinear part

collinear parts

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

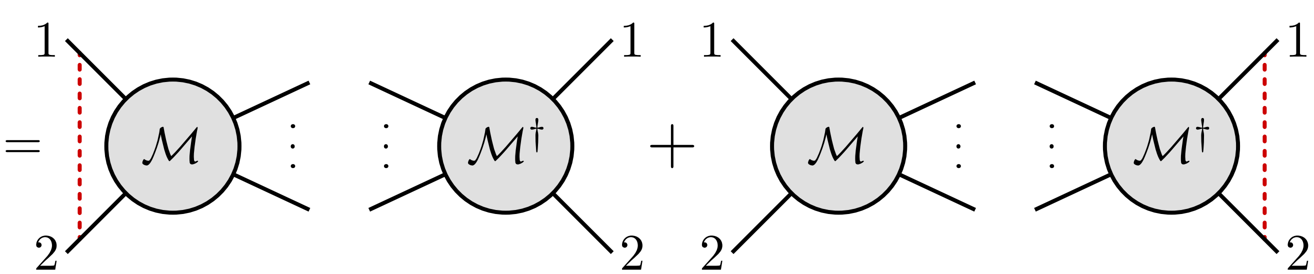
$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boxed{\boldsymbol{V}^G + \boldsymbol{\Gamma}^c} \ln\frac{\mu^2}{\hat{s}}$$

Glauber phase

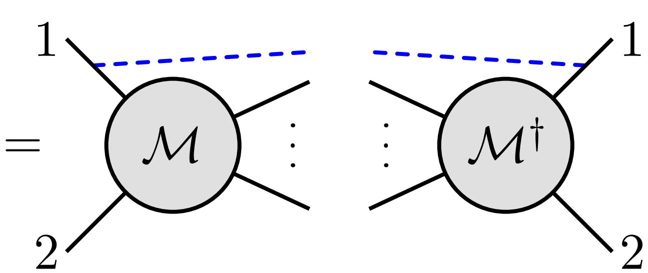
soft emission (collinear div. subtracted)

collinear emission

where:

$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G =$ 

$$\boldsymbol{V}^G = -2i\pi \left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_1^c =$ 

new color space of emitted gluon

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2} \left[C_i \, \boldsymbol{1} - \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \, \delta(n_k - n_i) \right]$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

Glauber phase

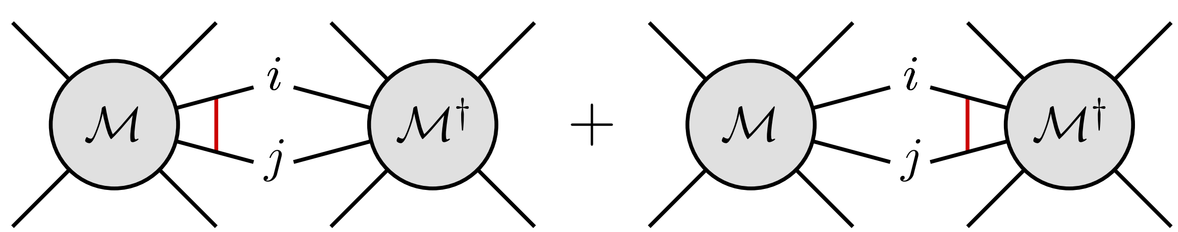
$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \boxed{\overline{\boldsymbol{\Gamma}}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{Q^2}$$

soft emission
(collinear div. subtracted)

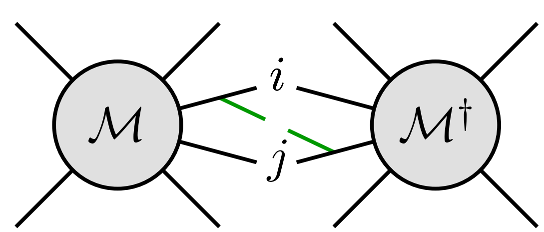
collinear emission

where:

$$\boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

Glauber phase

soft emission (collinear div. subtracted)

collinear emission

where:

$$\overline{\boldsymbol{\Gamma}} = 2\sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi}\,\overline{W}_{ij}^k - 4\sum_{(ij)} \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}\,\overline{W}_{ij}^k\,\Theta_{\text{hard}}(n_k)$$

$$\overline{W}_{ij}^k = W_{ij}^k - \frac{1}{n_i\cdot n_k}\,\delta(n_i - n_k) - \frac{1}{n_j\cdot n_k}\,\delta(n_j - n_k)\,; \qquad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

subtracted dipole emitter

dipole emitter

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of** $\mathbf{\Gamma}^c$

- Three properties simplify the calculation:

  - color coherence in absence of Glauber phases:

$$\boldsymbol{\mathcal{H}}_m\,\mathbf{\Gamma}^c\,\overline{\mathbf{\Gamma}} = \boldsymbol{\mathcal{H}}_m\,\overline{\mathbf{\Gamma}}\,\mathbf{\Gamma}^c$$

  - collinear safety:

$$\langle\boldsymbol{\mathcal{H}}_m\,\mathbf{\Gamma}^c\otimes\mathbf{1}\rangle = 0$$

  - cyclicity of the trace:

$$\big\langle\boldsymbol{\mathcal{H}}_m\,\boldsymbol{V}^G\otimes\mathbf{1}\big\rangle = 0$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of** Γ^c

- Under the color trace, insertions of Γ_c are non-zero only if they come in conjunction with (at least) two Glauber phases and one $\overline{\Gamma}$
- Relevant color traces at $\mathcal{O}(\alpha_s^{n+3}L^{2n+3})$:

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_{2\,\to\,M}\,(\mathbf{\Gamma}^c)^r\,\boldsymbol{V}^G\,(\mathbf{\Gamma}^c)^{n-r}\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}\otimes\mathbf{1}\right\rangle$$

- Kinematic information contained in $(M+1)$ angular integrals from $\overline{\Gamma}$:

$$J_j = \int\frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\text{veto}}(n_k)\,; \quad \text{with} \quad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\,n_j\cdot n_k}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

$$C_{rn} = -256\pi^2 \, (4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{S}_i \right\rangle \right]$$

T. Becher, M. Neubert, D. Shao, M. Stillger (2023)

Basis of color structures:

$$\boldsymbol{O}_1^{(j)} = f_{abe} f_{cde} \, \boldsymbol{T}_2^a \left\{ \boldsymbol{T}_1^b, \boldsymbol{T}_1^c \right\} \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_2^{(j)} = d_{ade} d_{bce} \, \boldsymbol{T}_2^a \left\{ \boldsymbol{T}_1^b, \boldsymbol{T}_1^c \right\} \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_3^{(j)} = \boldsymbol{T}_2^a \left\{ \boldsymbol{T}_1^a, \boldsymbol{T}_1^b \right\} \boldsymbol{T}_j^b - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_4^{(j)} = 2C_1 \, \boldsymbol{T}_2 \cdot \boldsymbol{T}_j - 2C_2 \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_j$$

$$\boldsymbol{S}_1 = f_{abe} f_{cde} \left\{ \boldsymbol{T}_1^b, \boldsymbol{T}_1^c \right\} \left\{ \boldsymbol{T}_2^a, \boldsymbol{T}_2^d \right\}$$

$$\boldsymbol{S}_2 = d_{ade} d_{bce} \left\{ \boldsymbol{T}_1^b, \boldsymbol{T}_1^c \right\} \left\{ \boldsymbol{T}_2^a, \boldsymbol{T}_2^d \right\}$$

$$\boldsymbol{S}_3 = d_{ade} d_{bce} \left[\boldsymbol{T}_2^a \left(\boldsymbol{T}_1^b \boldsymbol{T}_1^c \boldsymbol{T}_1^d \right)_+ + (1 \leftrightarrow 2) \right]$$

$$\boldsymbol{S}_4 = \left\{ \boldsymbol{T}_1^a, \boldsymbol{T}_1^b \right\} \left\{ \boldsymbol{T}_2^a, \boldsymbol{T}_2^b \right\}$$

$$\boldsymbol{S}_5 = \boldsymbol{T}_1 \cdot \boldsymbol{T}_2$$

$$\boldsymbol{S}_6 = \boldsymbol{1}$$

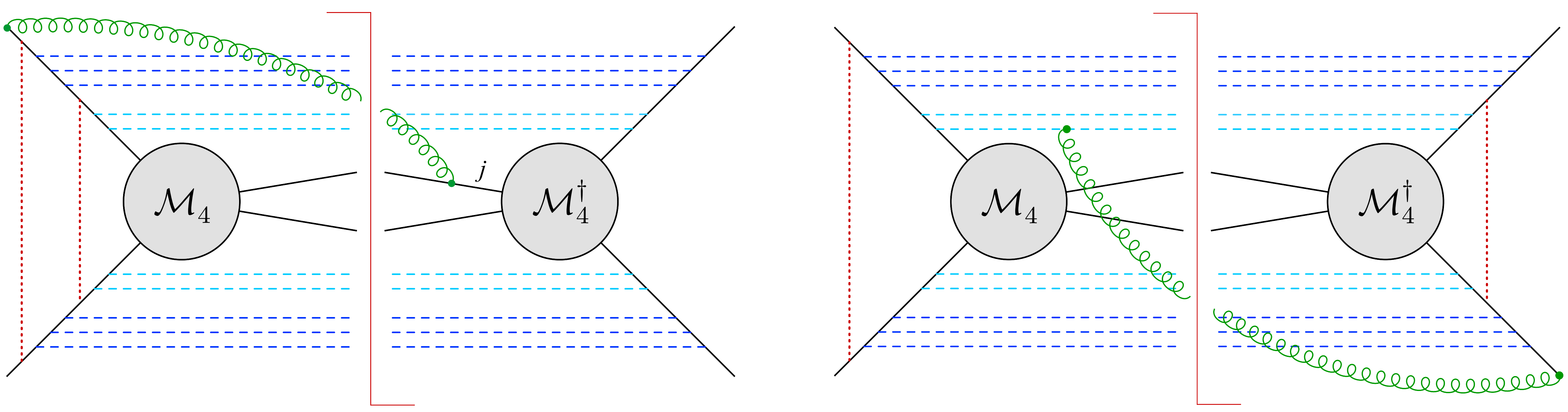
RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

$$C_{rn} = -256\pi^2\,(4N_c)^{n-r}\left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)}\,\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\boldsymbol{O}_i^{(j)}\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)}\,\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\boldsymbol{S}_i\rangle\right]$$

T. Becher, M. Neubert, D. Shao, M. Stillger (2023)

Basis of color structures:



RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

$$C_{rn} = -256\pi^2 \,(4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{S}_i \right\rangle \right]$$

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Coefficient functions:

$$c_1^{(r)} = 2^{r-1}\left[\,(3N_c+2)^r + (3N_c-2)^r\,\right]$$

$$c_2^{(r)} = 2^{r-2}N_c \left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} - \frac{(2N_c)^{r+1}}{N_c^2-4} \right]$$

$$c_3^{(r)} = 2^{r-1}\left[\,(3N_c+2)^r - (3N_c-2)^r\,\right]$$

$$c_4^{(r)} = 2^{r-1} \left[\frac{(3N_c+2)^r}{N_c+1} + \frac{(3N_c-2)^r}{N_c-1} - \frac{2N_c^{r+1}}{N_c^2-1} \right]$$

$$d_1^{(r)} = 2^{3r-1}\left[\,(N_c+1)^r + (N_c-1)^r\,\right] - 2^{r-1}\left[\,(3N_c+2)^r + (3N_c-2)^r\,\right]$$

$$d_2^{(r)} = 2^{3r-2}N_c \left[\frac{(N_c+1)^r}{N_c+2} + \frac{(N_c-1)^r}{N_c-2} \right] - 2^{r-2}N_c \left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} \right]$$

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$$d_4^{(r)} = 2^{3r-1}\left[\,(N_c+1)^r - (N_c-1)^r\,\right] - 2^{r-1}\left[\,(3N_c+2)^r - (3N_c-2)^r\,\right]$$

$$\begin{aligned} d_5^{(r)} = 2^r\,(C_1+C_2) &\left[\frac{N_c+2}{N_c+1}\,(3N_c+2)^r - \frac{N_c-2}{N_c-1}\,(3N_c-2)^r - \frac{2N_c^{r+1}}{N_c^2-1} \right] \\ &- \frac{2^{r-1}N_c}{3}\left[(N_c+4)\,(3N_c+2)^r + (N_c-4)\,(3N_c-2)^r - (2N_c)^{r+1} \right] \end{aligned}$$

$$\begin{aligned} d_6^{(r)} = 2^{3r+1}C_1C_2 &\left[\,(N_c+1)^{r-1} + (N_c-1)^{r-1}\,\right](1-\delta_{r0}) \\ &- 2^{r+1}C_1C_2 \left[\frac{(3N_c+2)^r}{N_c+1} + \frac{(3N_c-2)^r}{N_c-1} - \frac{2N_c^{r+1}}{N_c^2-1} \right] \end{aligned}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

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- Series of SLLs, starting at 3-loop order:

$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \, \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

from scale integrals (at fixed coupling)

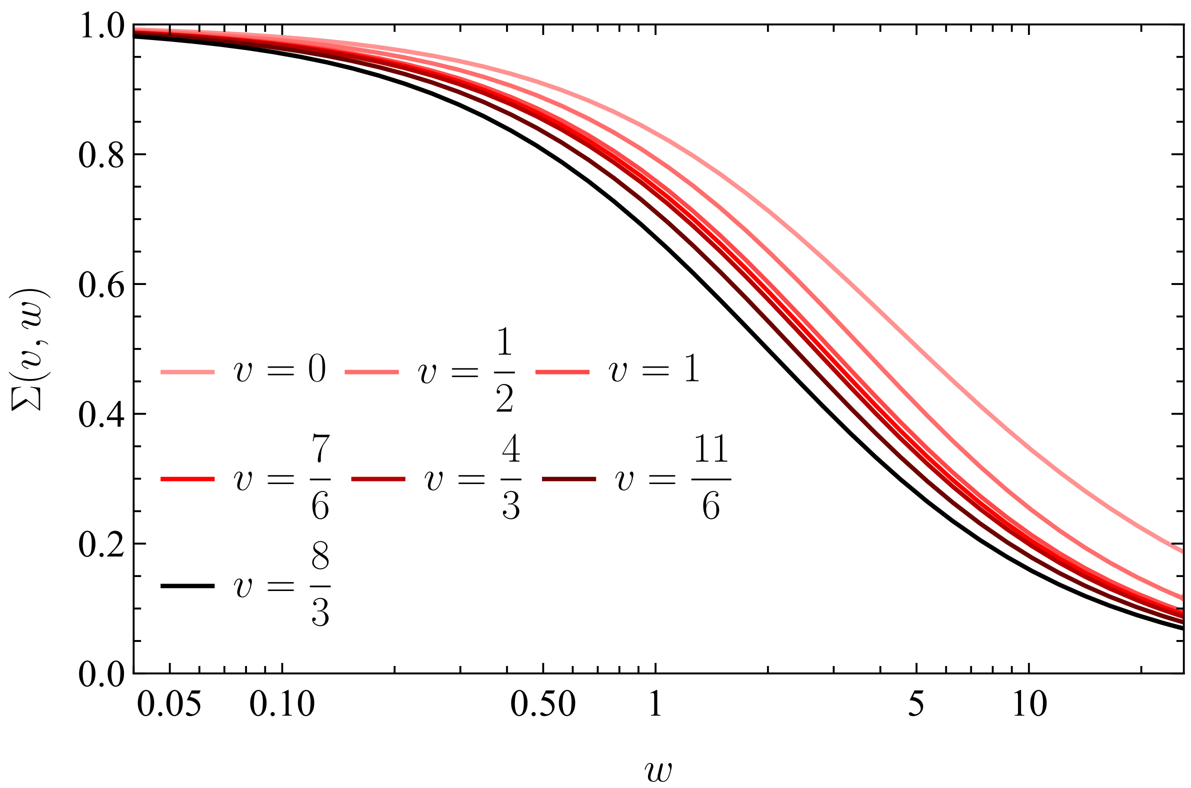
- Reproduces all that is known about SLLs (and much more…)

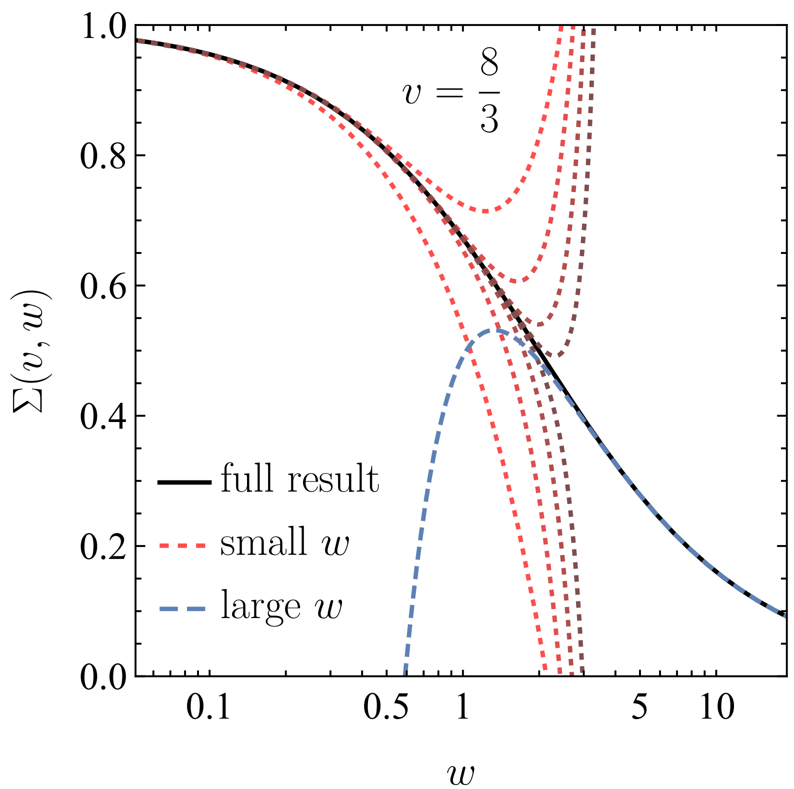
RESUMMATION OF SUPER-LEADING LOGARITHMS

Contribution to partonic cross sections

- Infinite series can be expressed in closed form in terms of a prefactor times Kampé de Fériet functions $\Sigma(v_i, w)$ with $w = \frac{N_c \alpha_s}{\pi} L^2$ and

$$v_0 = 0\,, \qquad v_1 = \frac{1}{2}\,, \qquad v_2 = 1\,, \qquad v_{3,4} = \frac{3N_c \pm 2}{2N_c}\,, \qquad v_{5,6} = \frac{2\,(N_c \pm 1)}{N_c}$$





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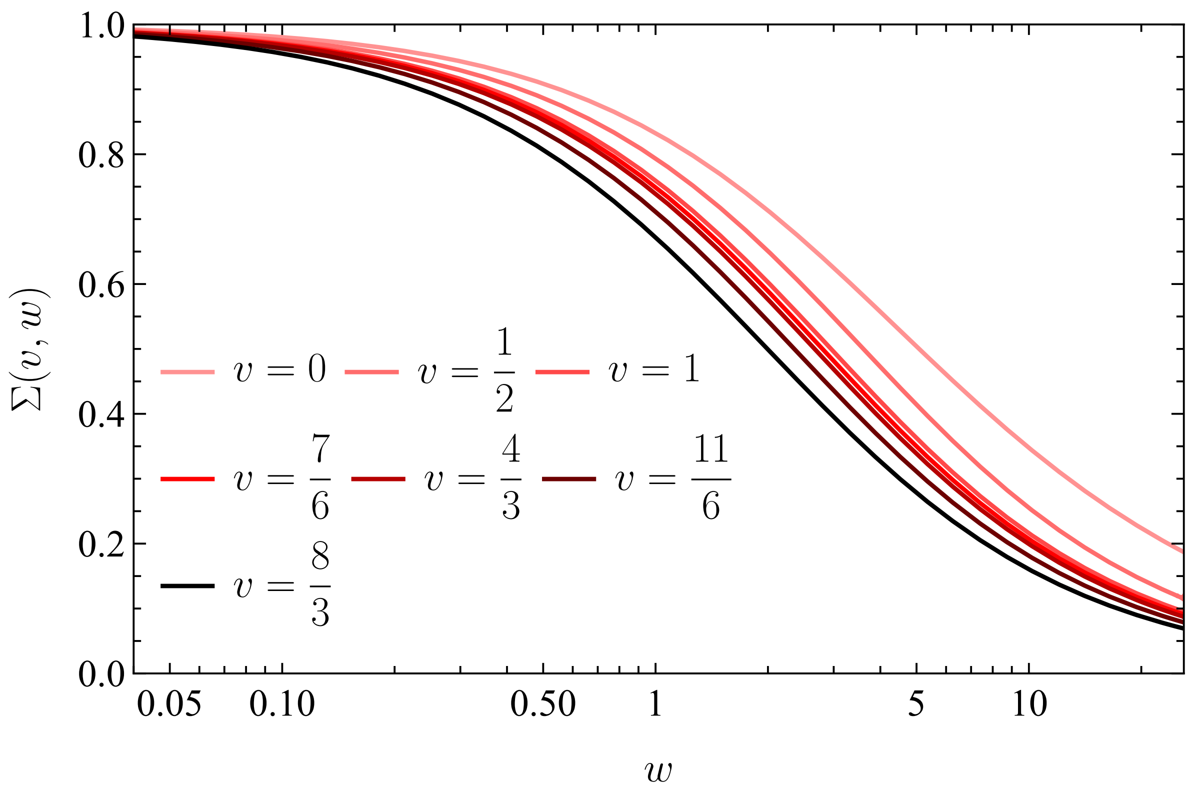

Asymptotic behavior for $w \gg 1$:

$$\Sigma_0(w) = \frac{3}{2w}\Big(\ln(4w) + \gamma_E - 2\Big) + \frac{3}{4w^2} + \mathcal{O}(w^{-3})$$

$$\Sigma(v,w) = \frac{3\arctan\left(\sqrt{v-1}\right)}{\sqrt{v-1}\,w} - \frac{3\sqrt{\pi}}{2\sqrt{v}\,w^{3/2}} + \mathcal{O}(w^{-2})$$

$\Rightarrow$ much slower fall-off than Sudakov form factors ~ e^{-cw}

PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Partonic channels contributing to $pp \to$ 2 jets (gap between jets)

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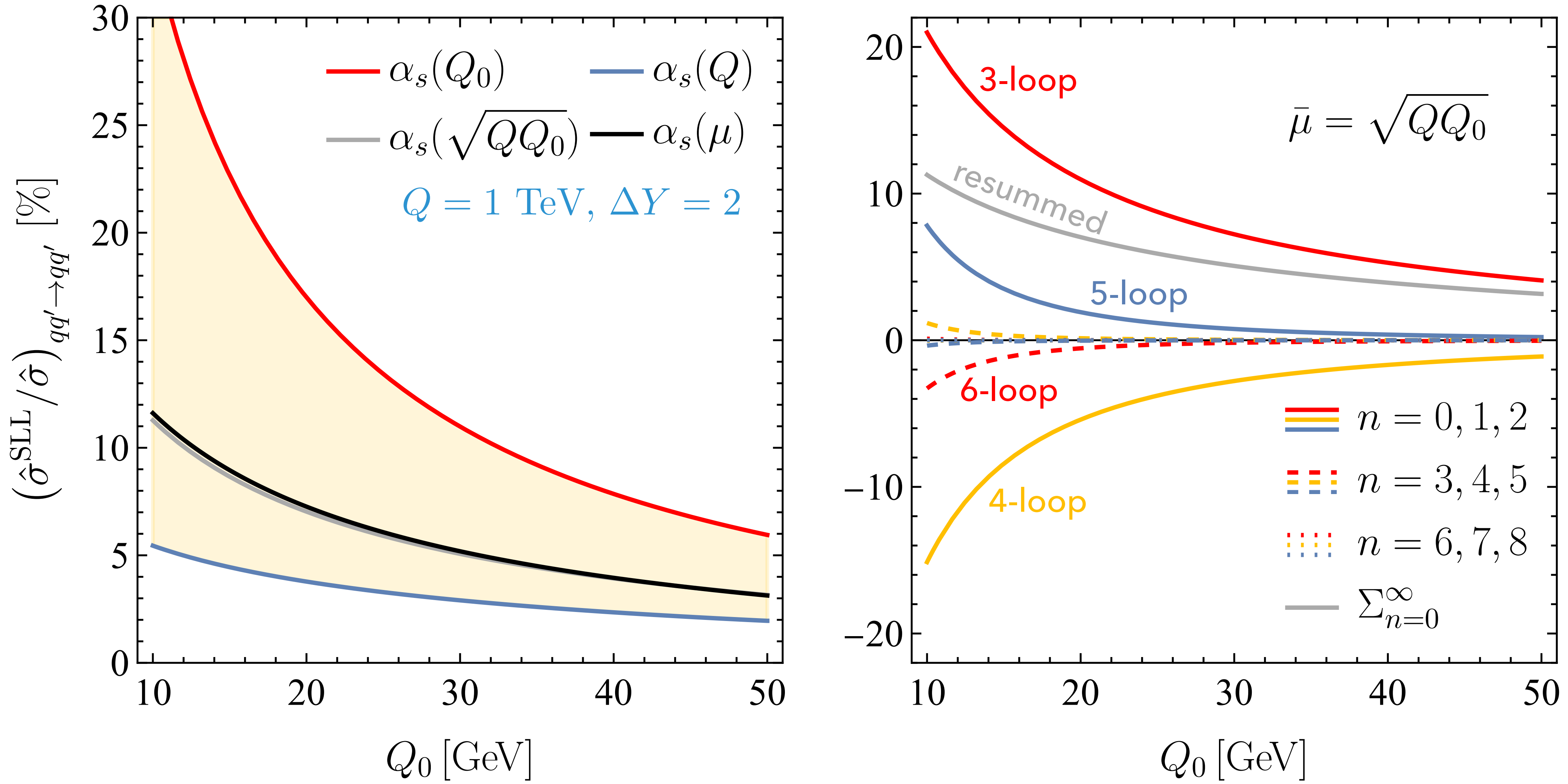

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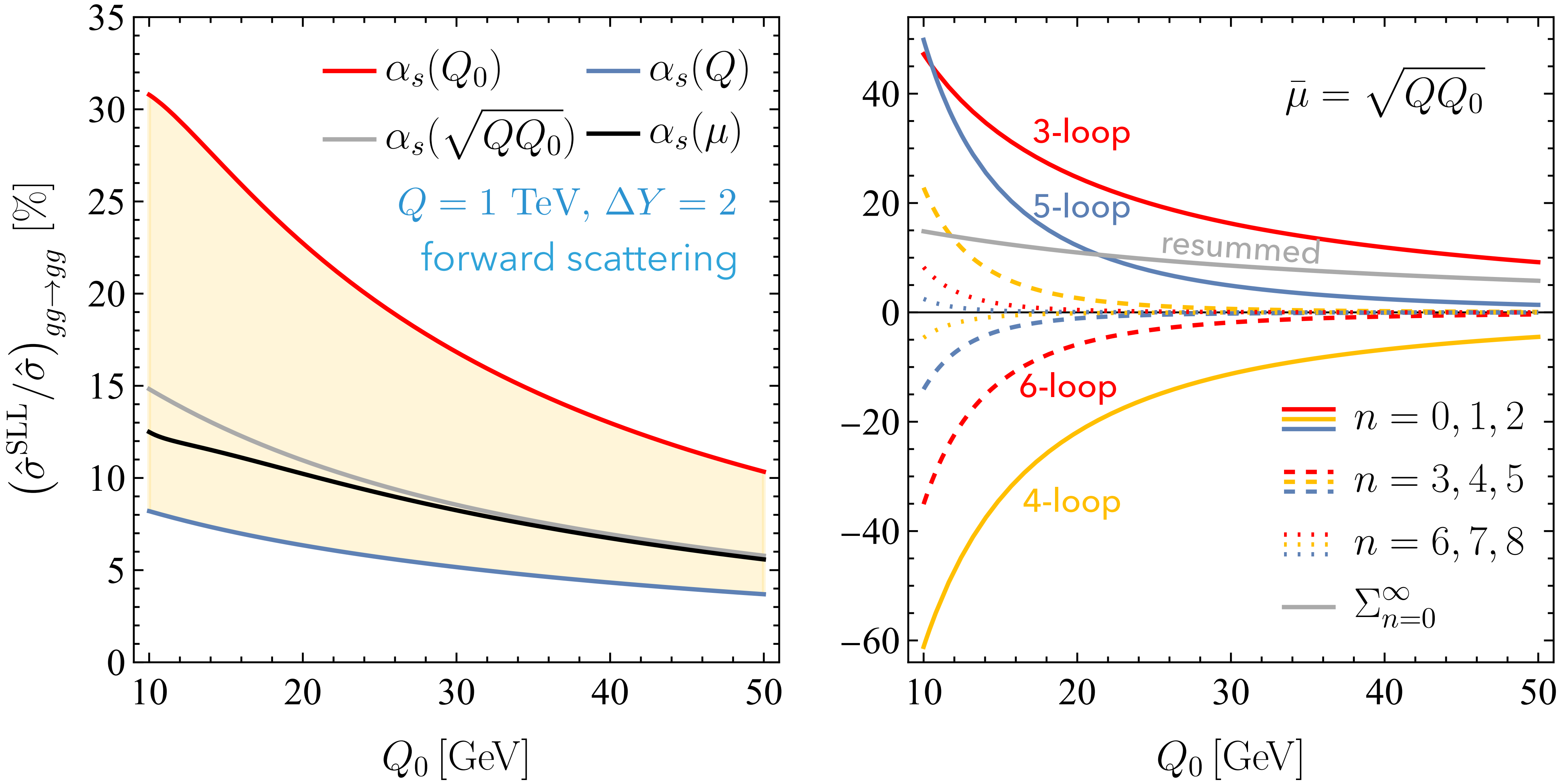

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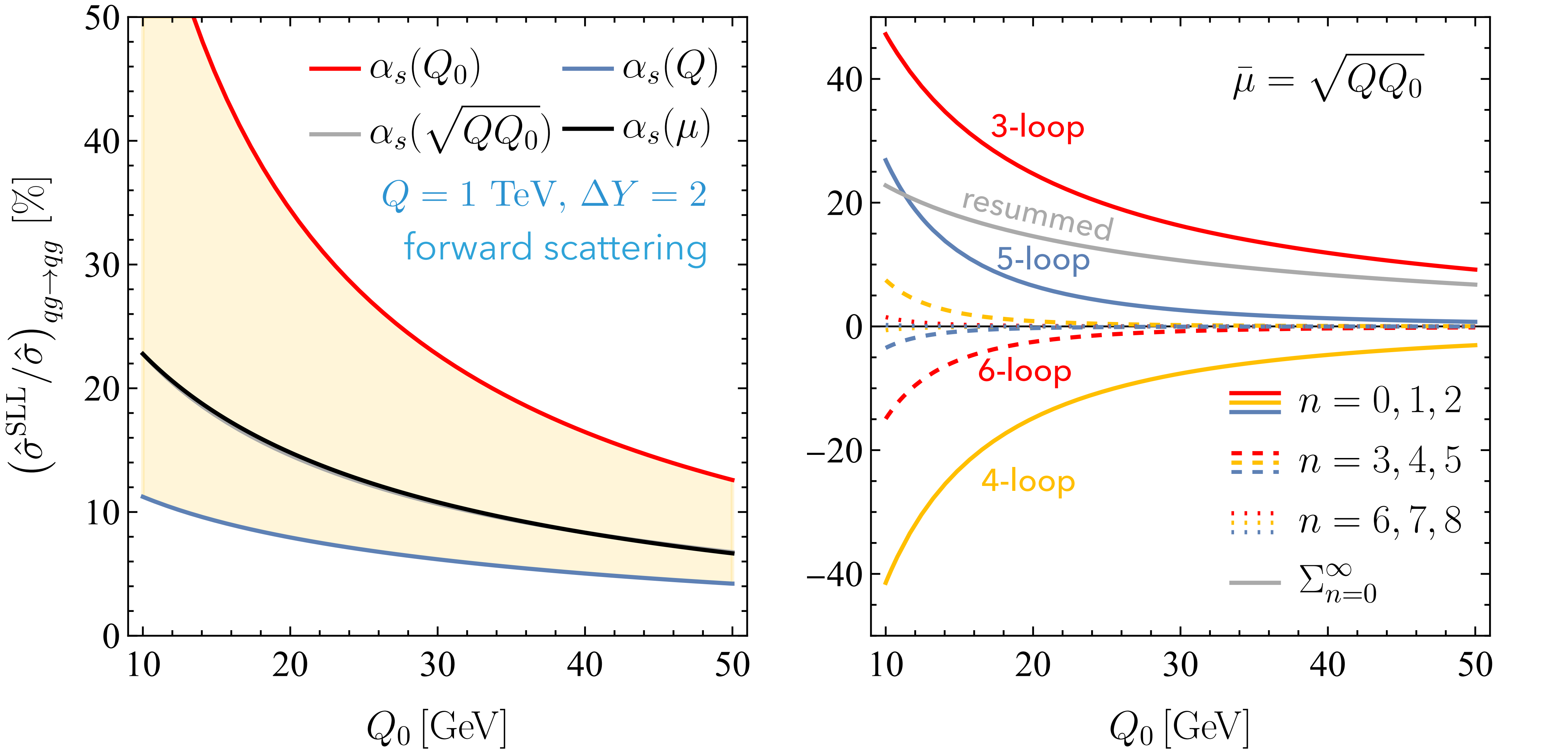

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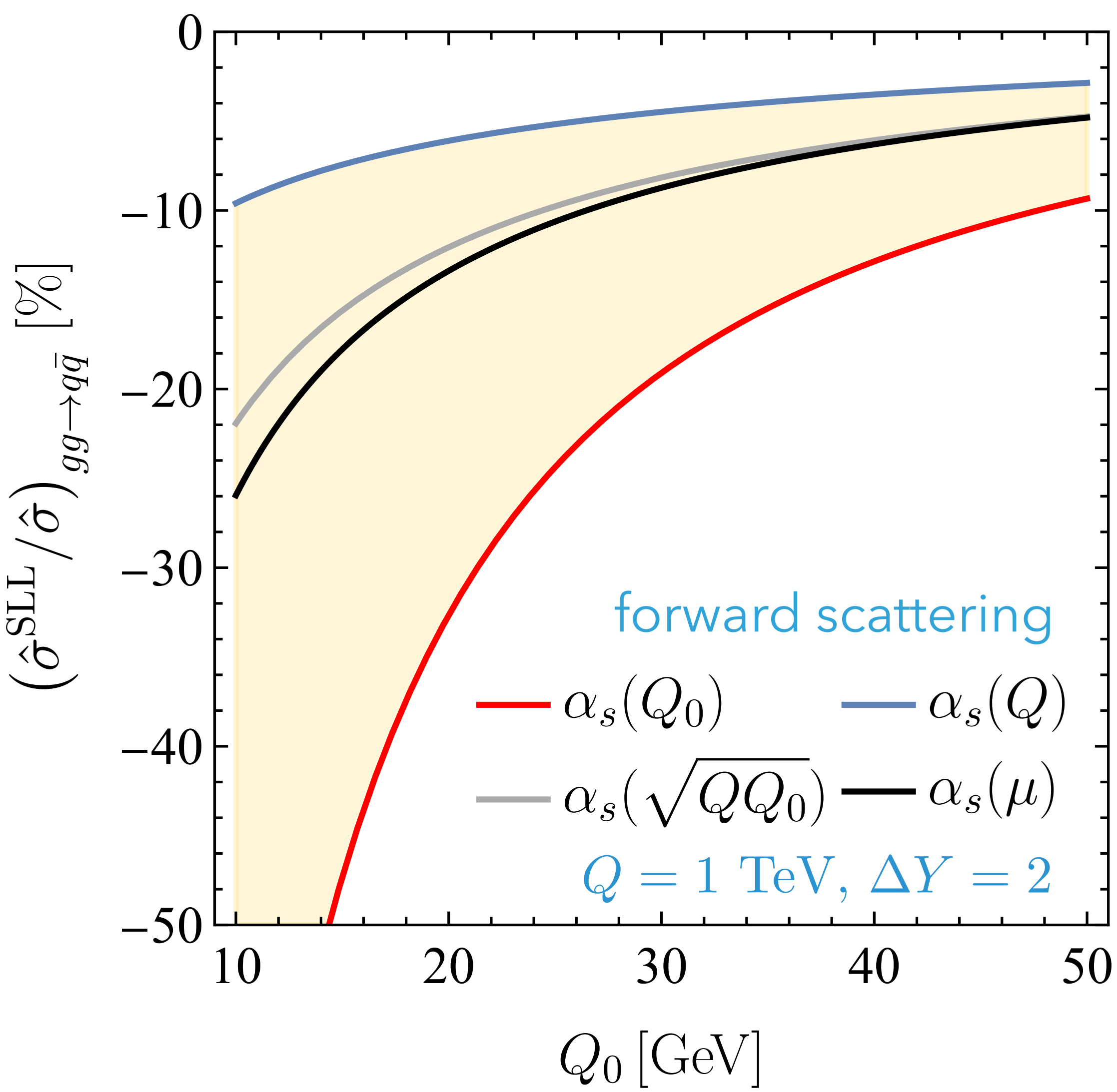

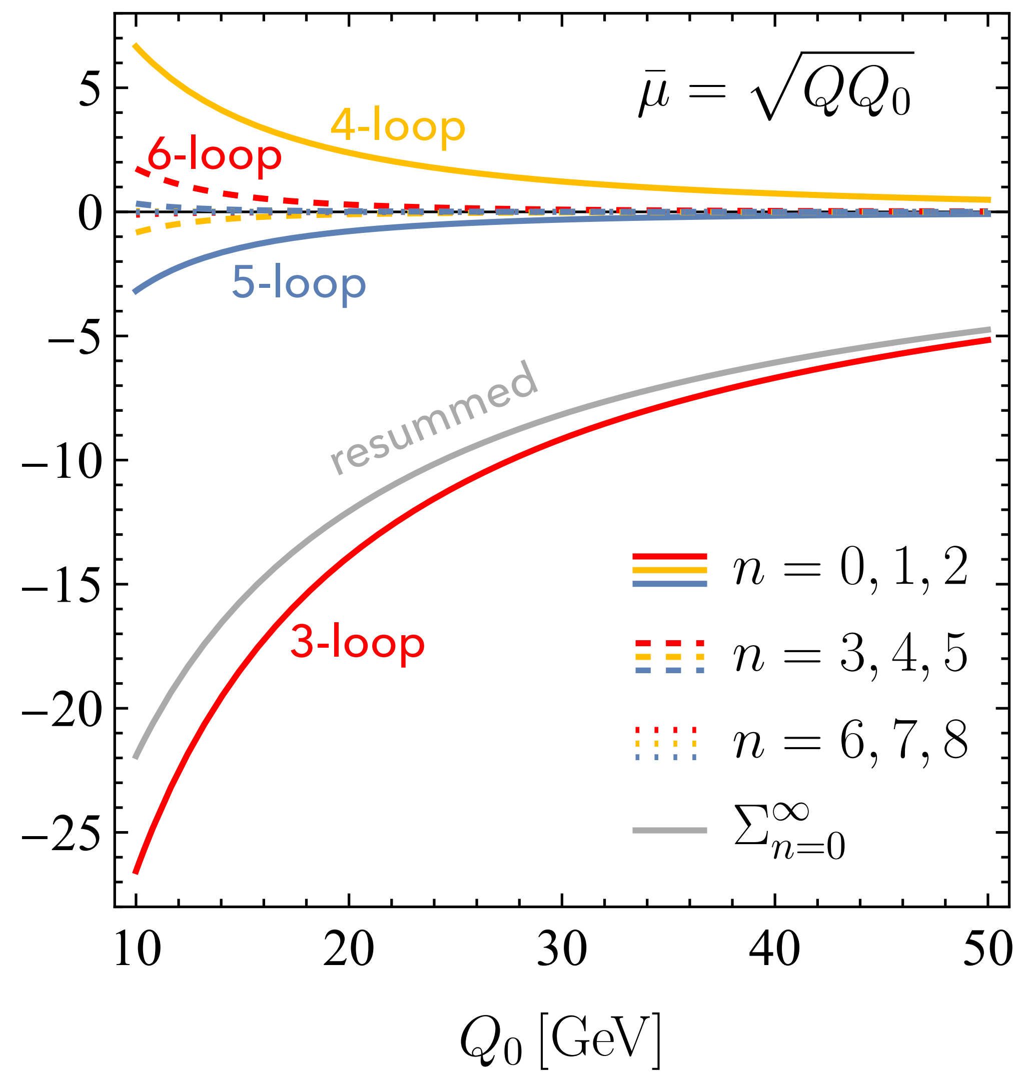

A MORE POWERFUL FORMALISM

Rewrite the evolution kernel (ordered exponential) for the SLLs

- Expand out all terms except the log-enhanced soft-collinear piece:

P. Böer, P. Hager, M. Neubert, M. Stillger, X. Xu (2024)

$$\boldsymbol{U}_{\rm SLL}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

cusp anomalous dimension

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

where:

$$\boldsymbol{U}_c(\mu_i,\mu_j) = \exp\left[\boldsymbol{\Gamma}^c\int_{\mu_j}^{\mu_i}\frac{d\mu}{\mu}\,\gamma_{\rm cusp}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right]$$

$$\mu_h = Q$$

matrix on the space of basis operators

resums all double-logarithmic terms

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$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

- All double-logarithmic terms are exponentiated!
- One scale integral for each insertion of $\boldsymbol{V}^G$ and $\overline{\boldsymbol{\Gamma}}$
- Easy to include running-coupling effects
- Asymptotic behavior of $\boldsymbol{U}_c(\mu_i,\mu_j)$ determines the asymptotic behavior of the resummed series

A MORE POWERFUL FORMALISM

Introduce a color basis

- Simplest case of (anti-)quark-initiated scattering processes:

$$\boldsymbol{X}_1 = \sum_{j>2} J_j \, i f^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_4 = \frac{1}{N_c} \, J_{12} \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \,,$$

$$\boldsymbol{X}_2 = \sum_{j>2} J_j \, (\sigma_1 - \sigma_2) \, d^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_5 = J_{12} \, \boldsymbol{1} \,,$$

$$\boldsymbol{X}_3 = \frac{1}{N_c} \sum_{j>2} J_j \, (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \,,$$

where $\sigma_i = -1 \; (+1)$ for an initial-state quark (anti-quark), and all structures are normalized such that their trace with a hard function is at most of $O(N_c^0)$ in the large-N_c limit

A MORE POWERFUL FORMALISM

Introduce a color basis

- Represent $\boldsymbol{\Gamma}^c, \boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{\Gamma}^c \to N_c\,\mathbb{\Gamma}^c \qquad \text{with} \qquad \mathbb{\Gamma}^c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -\frac{C_F}{N_c} & 0 & 0 \end{pmatrix}$$

Positive eigenvalues: {0, 1/2, 1}
(additional ones for initial-state gluons)

Recall:

$$\boldsymbol{U}_{\mathrm{SLL}}(\{\underline{n}\}, \mu_h, \mu_s) = \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \int_{\mu_s}^{\mu_2} \frac{d\mu_3}{\mu_3}$$
$$\times\, \boldsymbol{U}_c(\mu_h, \mu_1)\, \gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_1)\big)\, \boldsymbol{V}^G\, \boldsymbol{U}_c(\mu_1, \mu_2)\, \gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_2)\big)\, \boldsymbol{V}^G\, \frac{\alpha_s(\mu_3)}{4\pi}\, \overline{\boldsymbol{\Gamma}}$$

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- Represent $\boldsymbol{\Gamma}^c, \boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{U}_c(\mu_i,\mu_j) \;\rightarrow\; \mathbb{U}_c(\mu_i,\mu_j) = \begin{pmatrix} U_c(1;\mu_i,\mu_j) & 0 & 0 & 0 & 0 \\ 0 & U_c(1;\mu_i,\mu_j) & 0 & 0 & 0 \\ 0 & 0 & U_c(\frac{1}{2};\mu_i,\mu_j) & 0 & 0 \\ 0 & 0 & 2\left[U_c(\frac{1}{2};\mu_i,\mu_j) - U_c(1;\mu_i,\mu_j)\right] & U_c(1;\mu_i,\mu_j) & 0 \\ 0 & 0 & \frac{2C_F}{N_c}\left[1 - U_c(\frac{1}{2};\mu_i,\mu_j)\right] & 0 & 1 \end{pmatrix}$$

Generalized Sudakov factors: $U_c(v;\mu_i,\mu_j) = \exp\left[v\,N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\,\gamma_{\text{cusp}}\big(\alpha_s(\mu)\big) \ln\frac{\mu^2}{\mu_h^2}\right] \le 1$

Recall:

$$\boldsymbol{U}_{\text{SLL}}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$
$$\times\, \boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

double-log terms (SLLs) always lead to suppression!

A MORE POWERFUL FORMALISM

Introduce a color basis

- Represent $\boldsymbol{\Gamma}^c, \boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{V}^G \to i\pi N_c\,\mathbb{V}^G \quad \text{with} \quad \mathbb{V}^G = \begin{pmatrix} 0 & -2\,\delta_{q\bar{q}}\,\frac{N_c^2-4}{N_c^2} & \frac{4}{N_c^2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Recall:

$$\boldsymbol{U}_{\rm SLL}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$
$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

A MORE POWERFUL FORMALISM

Introduce a color basis

- Represent $\boldsymbol{\Gamma}^c, \boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}} \to 16i\pi X_1 \equiv 16i\pi X^T \varsigma \qquad\qquad \varsigma = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Recall:

$$\boldsymbol{U}_{\rm SLL}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \int_{\mu_s}^{\mu_2} \frac{d\mu_3}{\mu_3}$$

$$\times\, \boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

A MORE POWERFUL FORMALISM

Introduce a color basis

- This yields:

$$\sigma_{2\to M}^{\mathrm{SLL}}(Q_0) = \sum_{\text{partonic channels}} \int d\xi_1 \int d\xi_2\, f_1(\xi_1,\mu_s)\, f_2(\xi_2,\mu_s)\, \big\langle \boldsymbol{\mathcal{H}}_{2\to M}(\mu_h)\, \boldsymbol{X}^T \big\rangle\, \mathbb{U}_{\mathrm{SLL}}(\mu_h,\mu_s)\, \varsigma$$

5 process-dependent color traces

with:

$$\mathbb{U}_{\mathrm{SLL}}(\mu_h,\mu_s) = 16i\pi\, N_c \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \int_{\mu_s}^{\mu_2} \frac{d\mu_3}{\mu_3}\, \frac{\alpha_s(\mu_3)}{4\pi} \times \mathbb{U}_c(\mu_h,\mu_1)\, \gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_1)\big)\, \mathbb{V}^G\, \mathbb{U}_c(\mu_1,\mu_2)\, \gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_2)\big)$$

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RG-IMPROVED SLL RESUMMATION

Perform the scale integrals in terms of the running coupling

- Generalized Sudakov factors in RG-improved perturbation theory:

2-loop cusp anomalous dimension and β-function

$$U_c(v;\mu_i,\mu_j) = \exp\left[v N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\, \gamma_{\text{cusp}}(\alpha_s(\mu)) \ln\frac{\mu^2}{\mu_h^2} \right]$$

$$= \exp\left\{ \frac{\gamma_0 v N_c}{2\beta_0^2} \left[\frac{4\pi}{\alpha_s(\mu_h)} \left(\frac{1}{x_i} - \frac{1}{x_j} - \ln\frac{x_j}{x_i} \right) + \left(\frac{\gamma_1}{\gamma_0} - \frac{\beta_1}{\beta_0} \right) \left(x_i - x_j + \ln\frac{x_j}{x_i} \right) + \frac{\beta_1}{2\beta_0} \left(\ln^2 x_j - \ln^2 x_i \right) \right] \right\}$$

with $x_i \equiv \alpha_s(\mu_i)/\alpha_s(\mu_h)$ and:

$$U_c(v;\mu_i,\mu_j)\, U_c(v;\mu_j,\mu_k) = U_c(v;\mu_i,\mu_k)\,, \qquad U_c(0;\mu_i,\mu_j) = 1$$

- Encounter products of two Sudakov factors:

$$U_c(v^{(1)},v^{(2)};\mu_h,\mu_1,\mu_2) \equiv U_c(v^{(1)};\mu_h,\mu_1)\, U_c(v^{(2)};\mu_1,\mu_2)$$

RG-IMPROVED SLL RESUMMATION

Explicit form of the evolution function for SLLs

- RG-improved perturbation theory:

$$\mathbb{U}_{\mathrm{SLL}}(\mu_h,\mu_s)\,\varsigma = -\frac{32\pi^2}{\beta_0^3}\,N_c\int_1^{x_s}\frac{dx_2}{x_2}\,\ln\frac{x_s}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}\begin{pmatrix} 0 \\ -\frac{1}{2}\,U_c(1;\mu_h,\mu_2) \\ U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2) \\ 2\left[U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)-U_c(1;\mu_h,\mu_2)\right] \\ \frac{2C_F}{N_c}\left[U_c(1;\mu_1,\mu_2)-U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\right] \end{pmatrix}$$

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- Fixed-coupling approximation:

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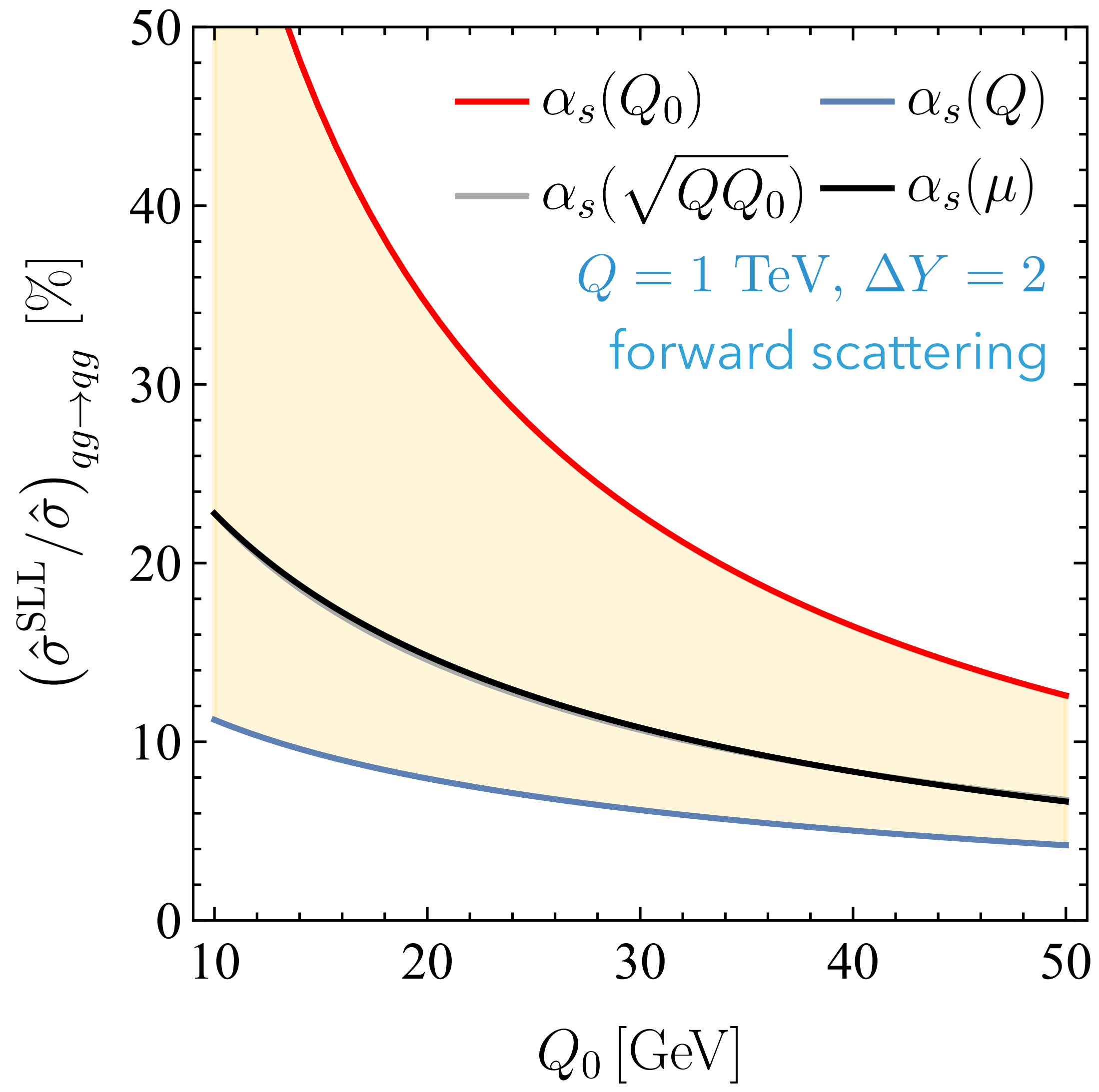
$$\mathbb{U}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = -\frac{2\pi^2}{3}\,N_c\left(\frac{\alpha_s}{\pi}\,L\right)^3\begin{pmatrix} 0 \\ -\frac12\,\Sigma(1,1;w) \\ \Sigma(\frac12,1;w) \\ 2\left[\Sigma(\frac12,1;w)-\Sigma(1,1;w)\right] \\ \frac{2C_F}{N_c}\left[\Sigma(0,1;w)-\Sigma(\frac12,1;w)\right]\end{pmatrix}$$

Kampé de Fériet functions

$$w=\frac{N_c\,\alpha_s(\bar\mu)}{\pi}\,L^2$$

PHENOMENOLOGICAL IMPACT OF RG IMPROVEMENT

SLL resummation with controlled scale uncertainties



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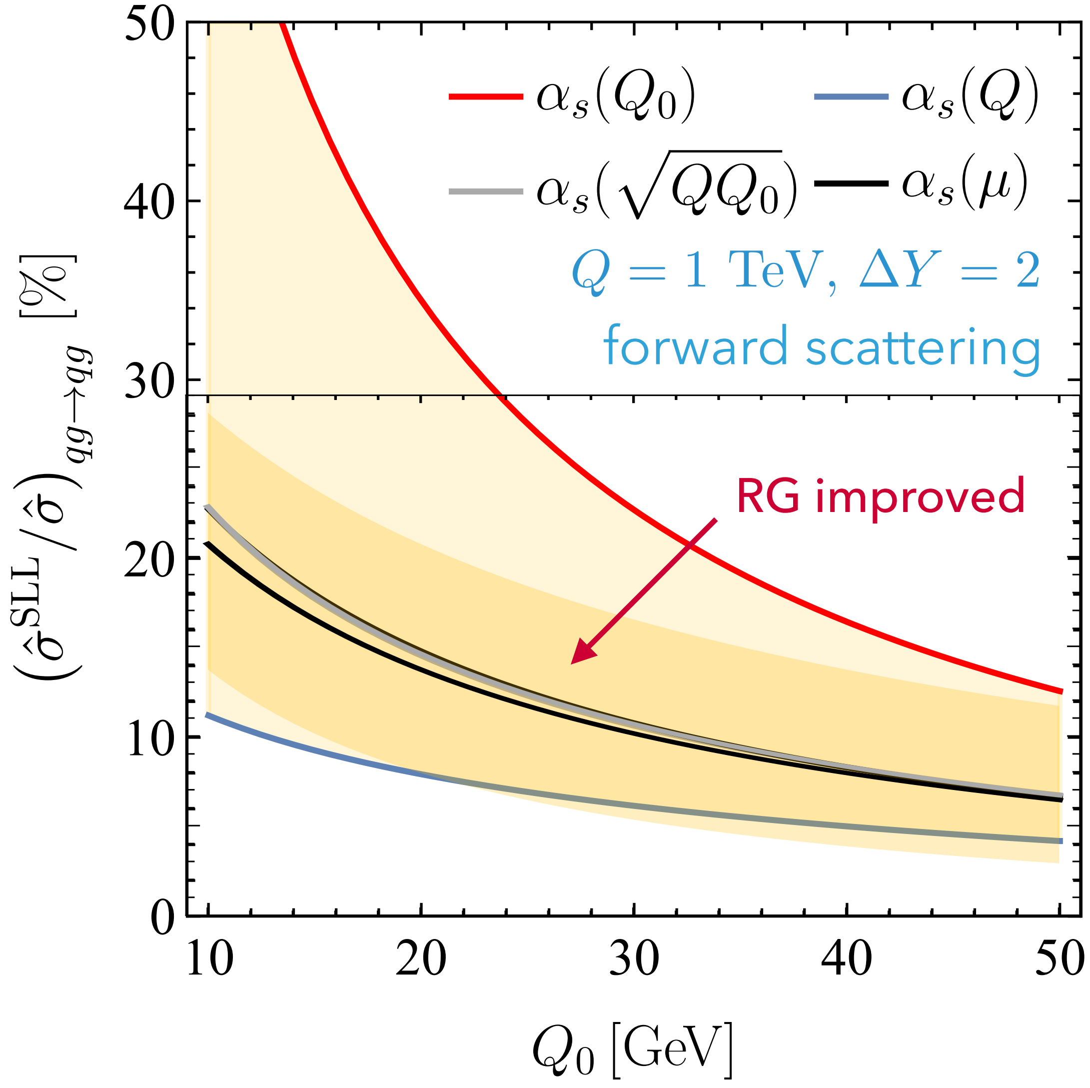
PHENOMENOLOGICAL IMPACT OF RG IMPROVEMENT

SLL resummation with controlled scale uncertainties



P. Böer, P. Hager, M. Neubert, M. Stillger, X. Xu (2024)

EXPLORING UNCHARTERED TERRITORY

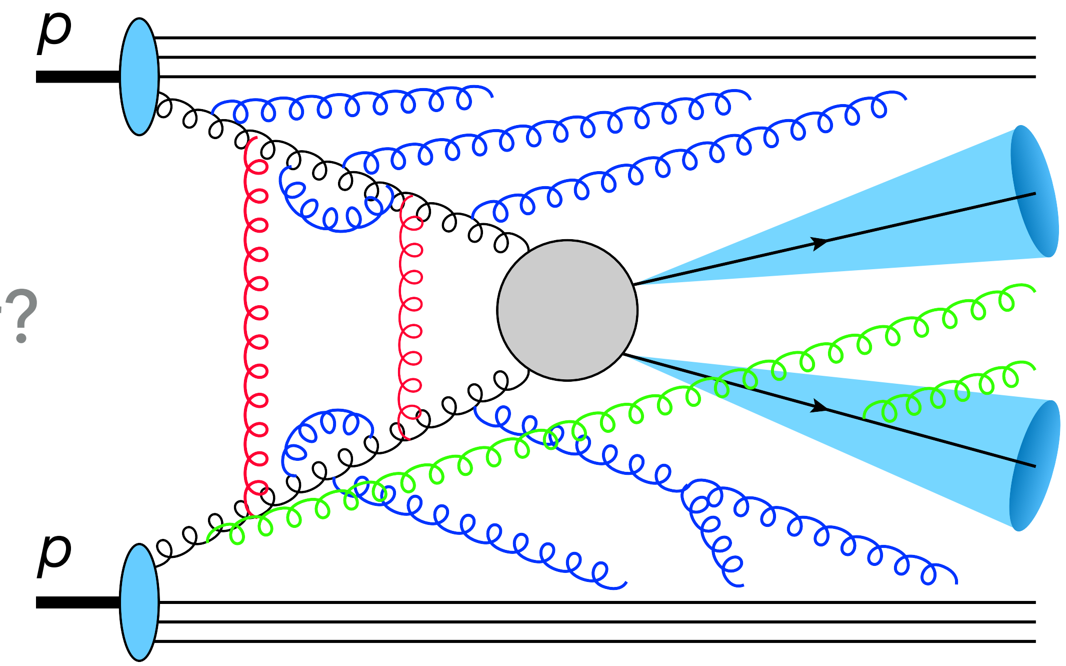
Important open questions

- How to include multiple Glauber phases and multiple soft emissions (single-log effects), and how large is their effect?

EXPLORING UNCHARTERED TERRITORY

Important open questions

- How to include multiple Glauber phases and multiple soft emissions (single-log effects), and how large is their effect?
- Can collinear factorization violations be understood in a quantitative way?
  At what scale (Q_0 or $\Lambda_{\rm QCD}$) do they occur?



EXPLORING UNCHARTERED TERRITORY

Important open questions

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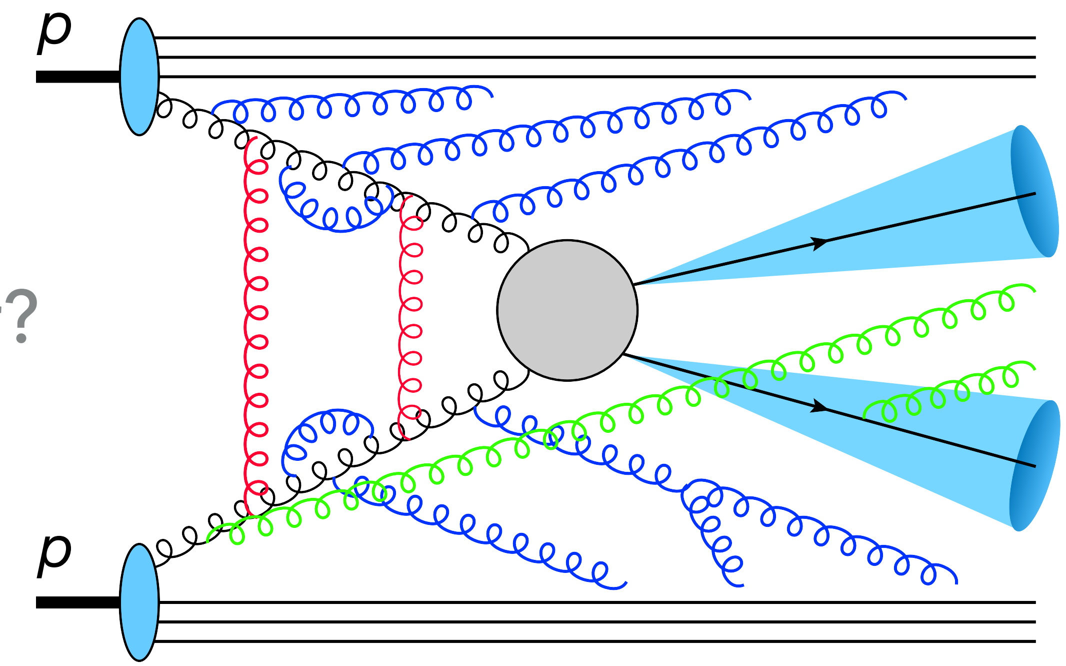
- Can collinear factorization violations be understood in a quantitative way? At what scale (Q_0 or Λ_{QCD}) do they occur?
- What are the implications for LHC phenomenology?



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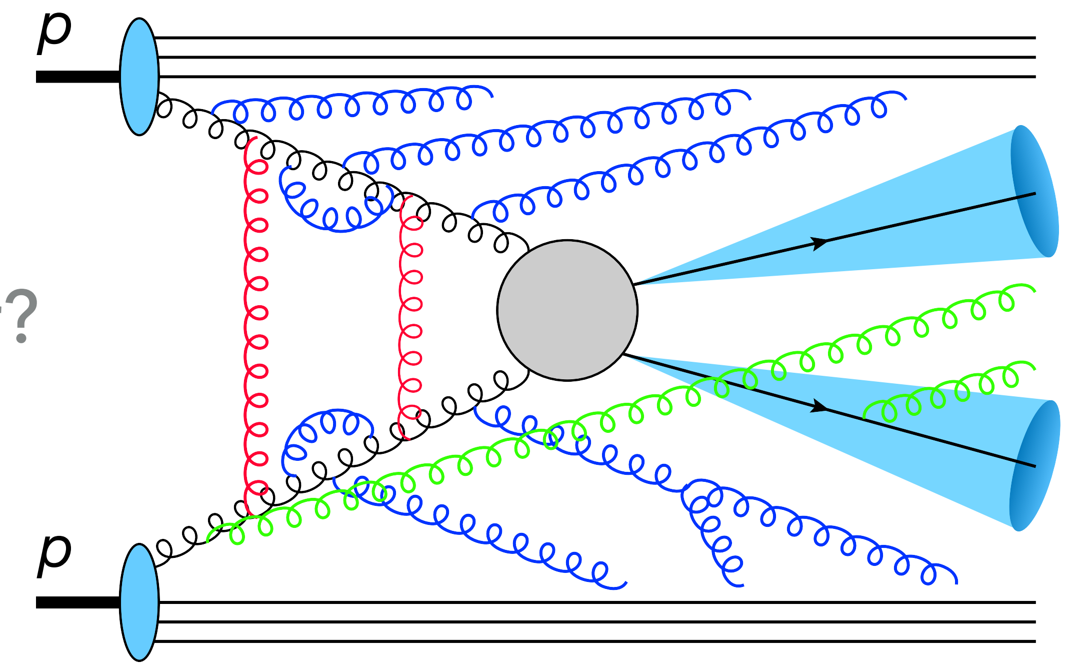

- Results are relevant for future improvements of parton showers with quantum interference