QCD Factorization and Resummation from Heavy Jets to Heavy Hadrons

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[2305.15461, 2306.08033, 2404.08622, and work in progress]

Particle Physics Seminar

University of Vienna, June 4, 2024

50 Years, One Lagrangian

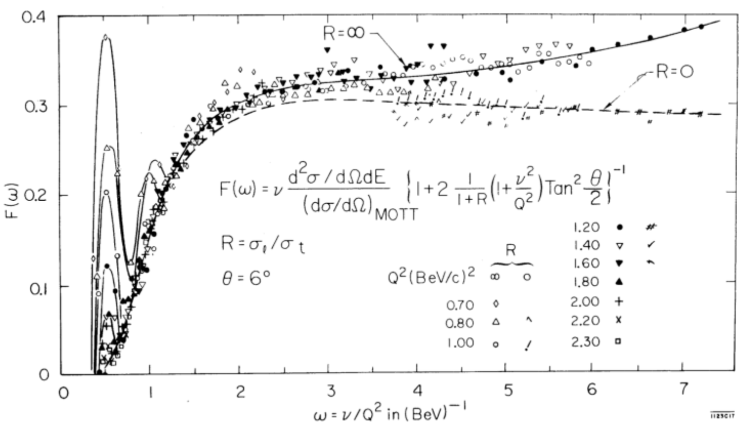
["50 Years of QCD", UCLA Bhaumik Institute for Theoretical Physics, 2023; book: 2212.11107]

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f (\mathrm{i}\not{D} - m_f)\psi_f - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

t

(not to scale)

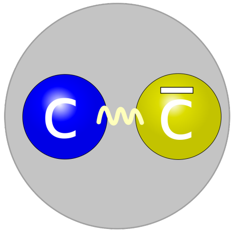
Bjorken Scaling, SLAC 1968



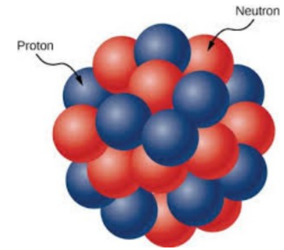
[W. Panofksy, ICHEP 1968, Vienna]

$$\beta_V = -(g^3/16\pi^2)\tfrac{11}{3}C_2(G) + O(g^5)$$

Asymptotic Freedom, 1973

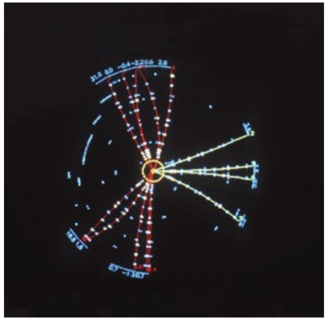


J/ψ, SLAC/BNL 1974

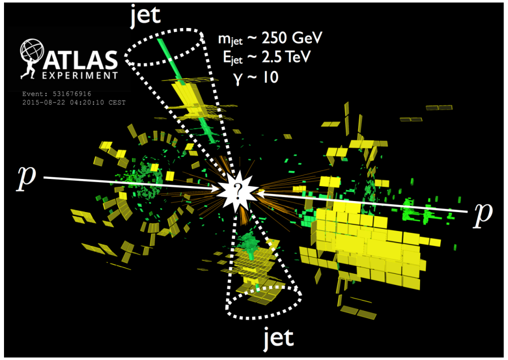


Nuclei

Neutron Stars



Gluon Jets, DESY 1979



10 Open Problems in QCD

[Iain Stewart, Colloquium Talk at REF 2023 Workshop in Madrid]

1. Prove Yang-Mills mass gap
2. First-principles description of hadronization?
3. When does factorization break, exactly?
4. Systematically improvable description of nuclei?
5. Why is the QGP a nearly perfect fluid?

1

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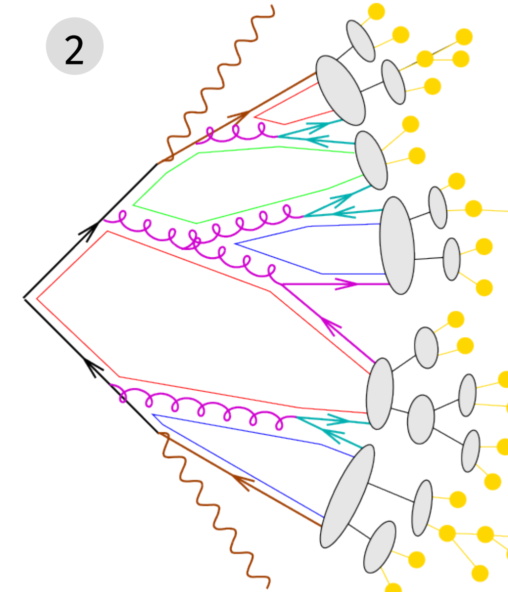
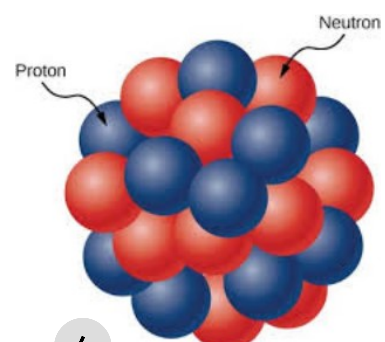


[Figure: S. Hoeche, 50 Years of QCD]



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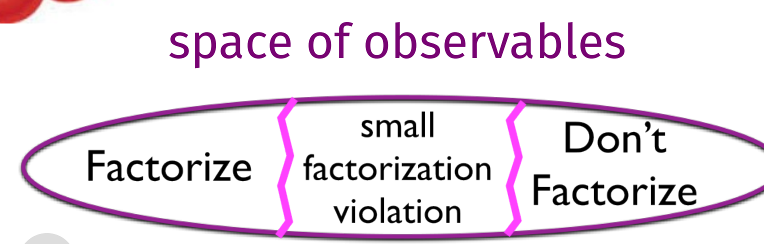

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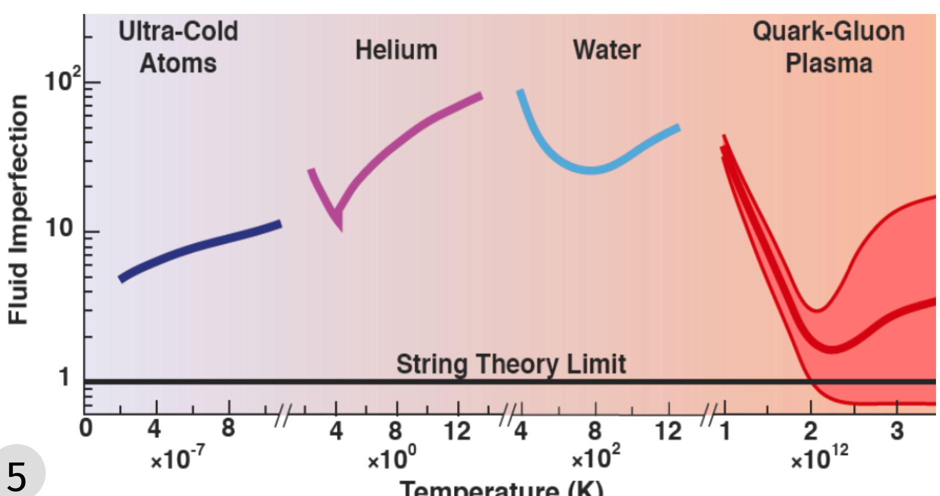

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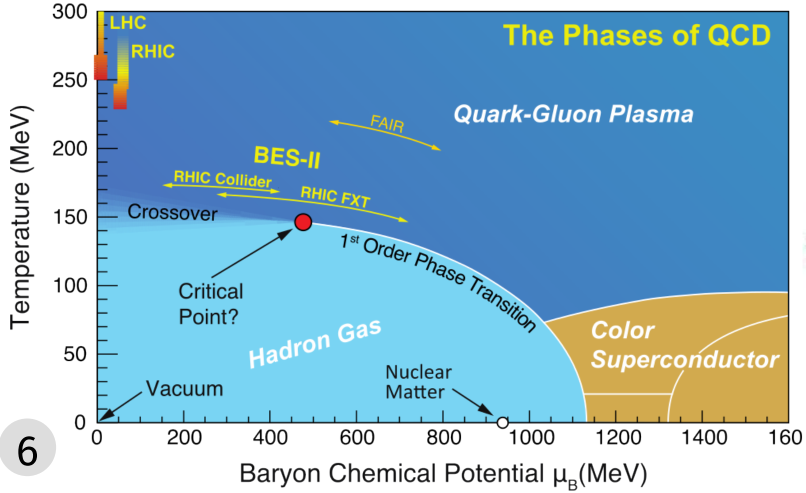

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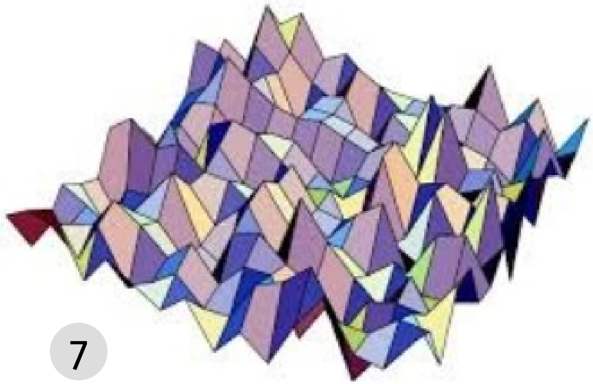

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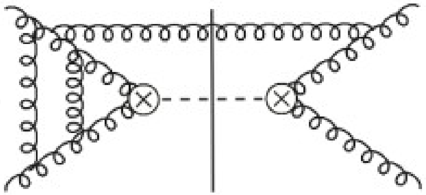

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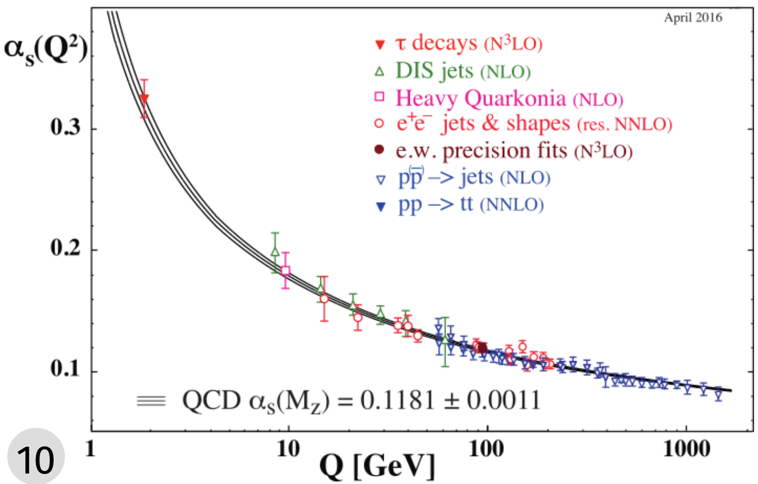
1. Does QCD have a critical point?
2. Lattice QCD sign problem (finite μ_B or t)
3. Dynamics of matter in a neutron star?
4. Ultimate method for perturbative QCD?
5. Best way to precisely measure α_s?



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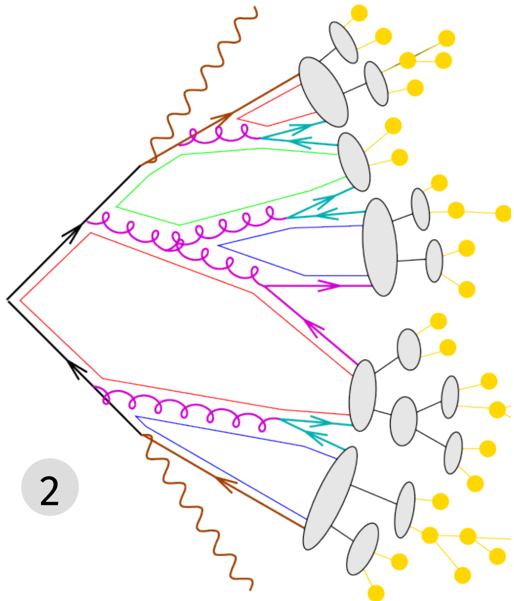

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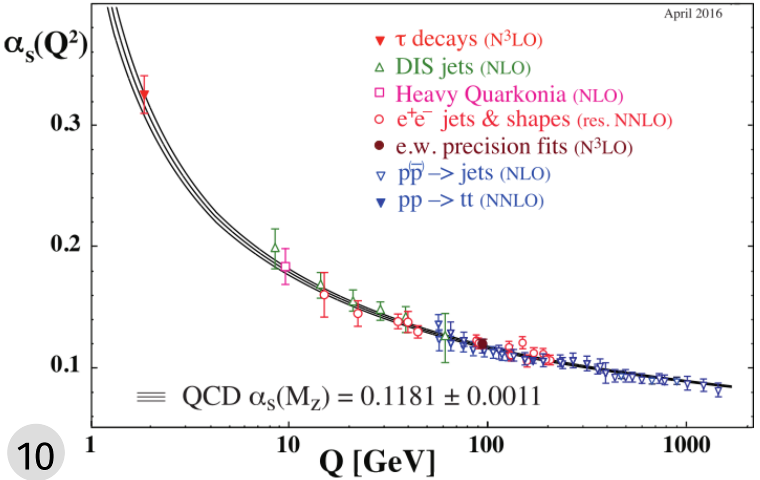

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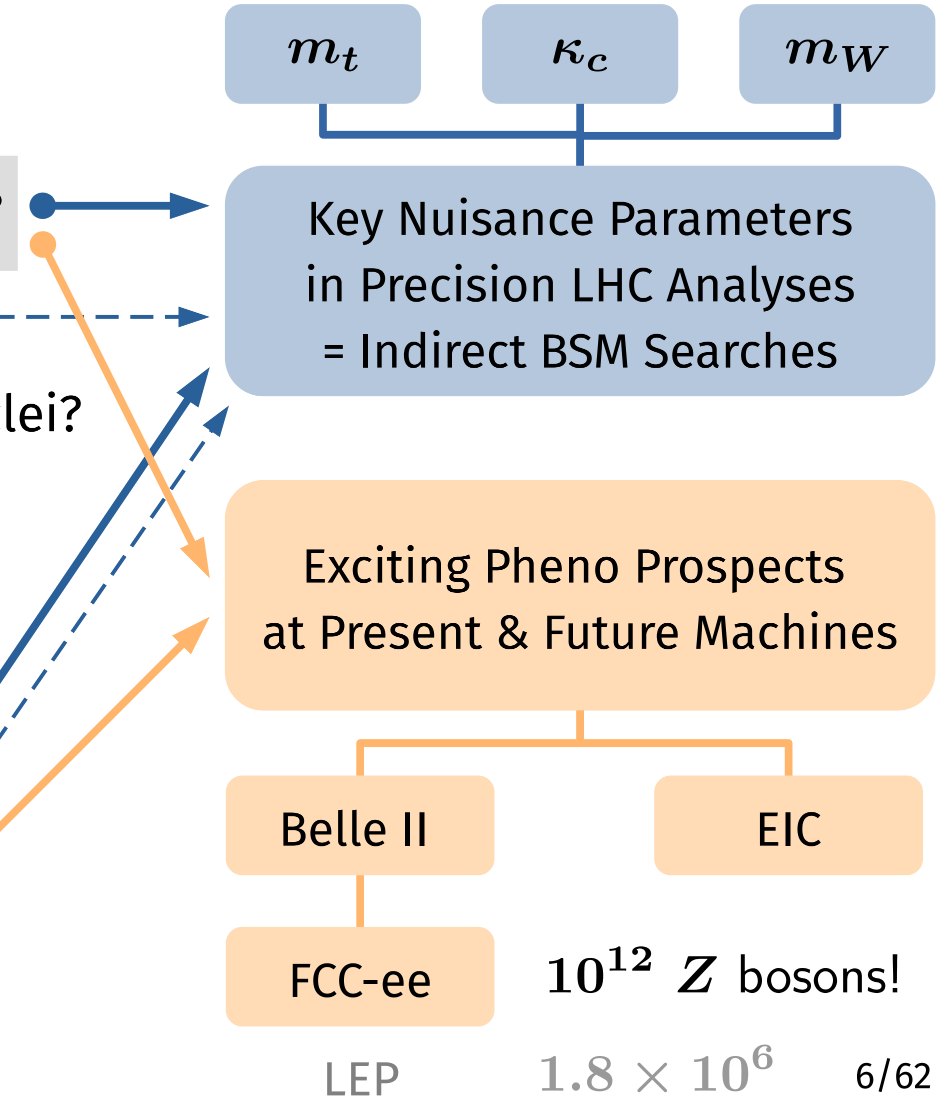
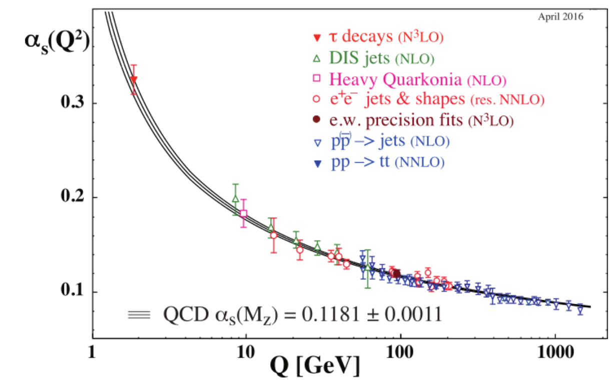


Overview of today's talk

1 NNLL Resummation of Sudakov Shoulder Logarithms in the **Heavy** Jet Mass Distribution

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f(\mathrm{i}\not{D} - m_f)\psi_f - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$$

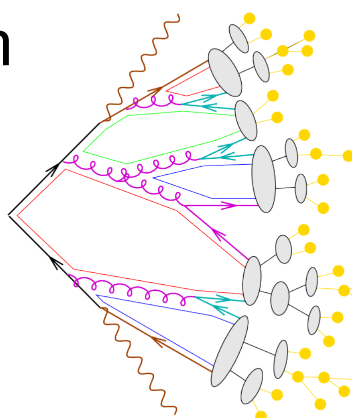
$\alpha_s(m_Z)$



2 Transverse Momentum-Dependent Fragmentation Functions of **Heavy** Quarks & Hadrons

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f(\mathrm{i}\not{D} - m_f)\psi_f - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$$

$m_f = m_{c,b} \gg \Lambda_{\rm QCD}$



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JHEP 09 (2023) 205, 2305.15461, with Arindam Bhattacharya (Harvard), Xiaoyuan Zhang (Harvard), Matt Schwartz (Harvard) and Iain Stewart (MIT)




2 Transverse Momentum-Dependent Fragmentation Functions of **Heavy** Quarks & Hadrons

1. NNLL Resummation of Sudakov Shoulder Logarithms in the **Heavy** Jet Mass Distribution

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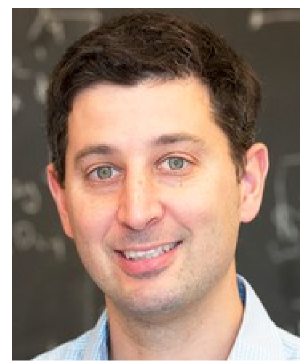
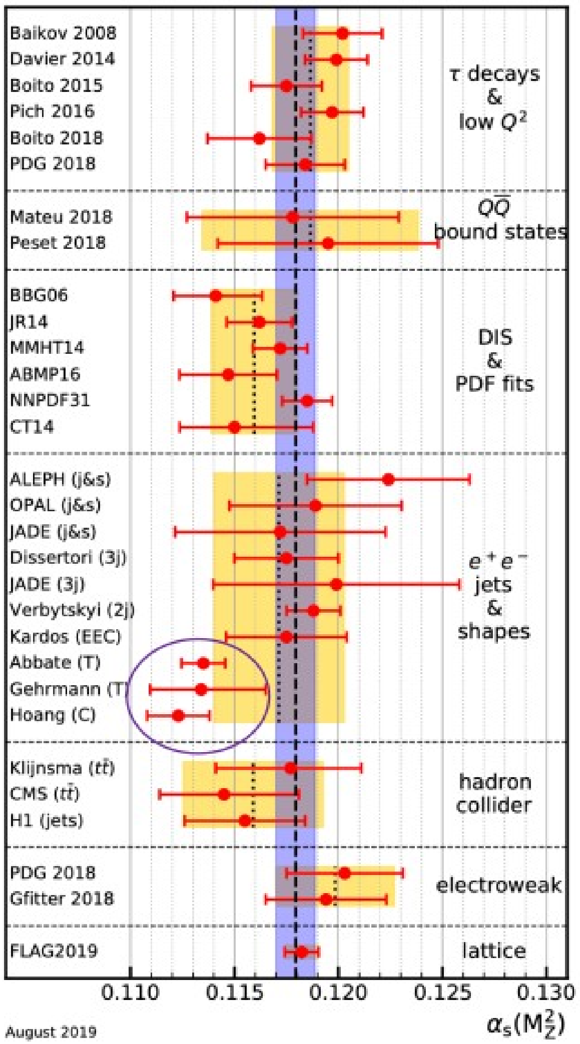



2. Transverse Momentum-Dependent Fragmentation Functions of **Heavy** Quarks & Hadrons

Motivation: α_s from e^+e^- event shapes

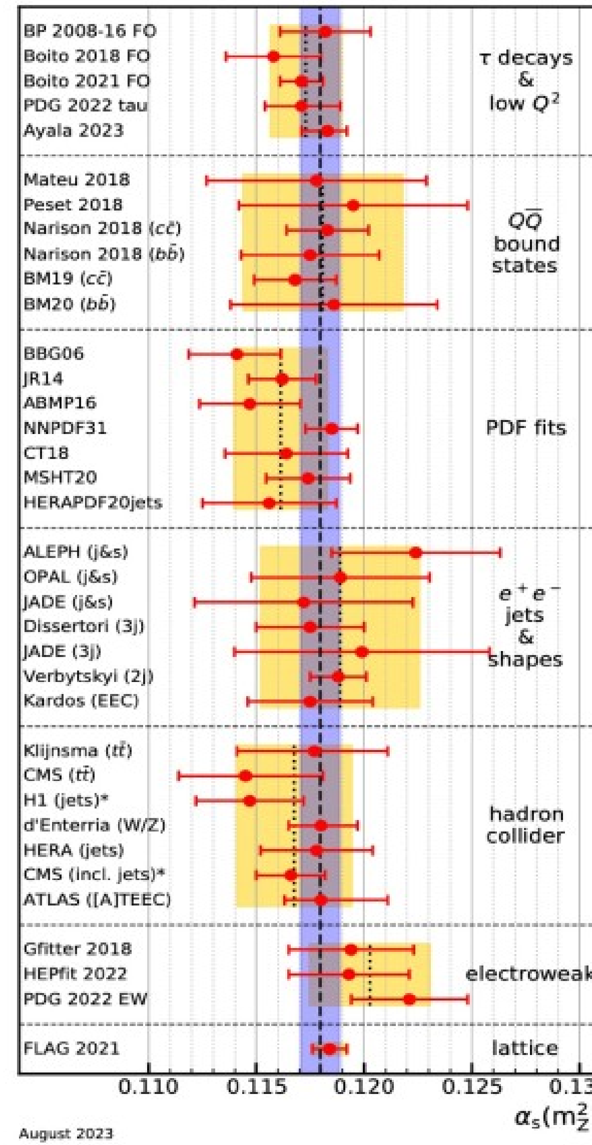
PDG 2019: $\alpha_s(m_Z) = 0.1179 \pm 0.0010$



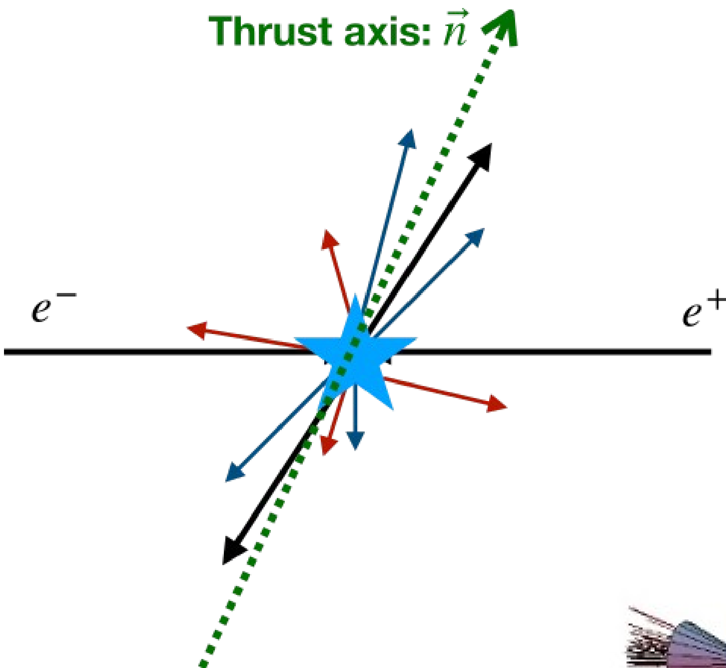
"These findings are inconsistent with the very small experimental, hadronization, and theoretical uncertainties of only 2, 5, and 9 per-mille, respectively, as reported in Refs. [683, 685]. For these reasons, we exclude the results of Refs. [683–685] from the average."

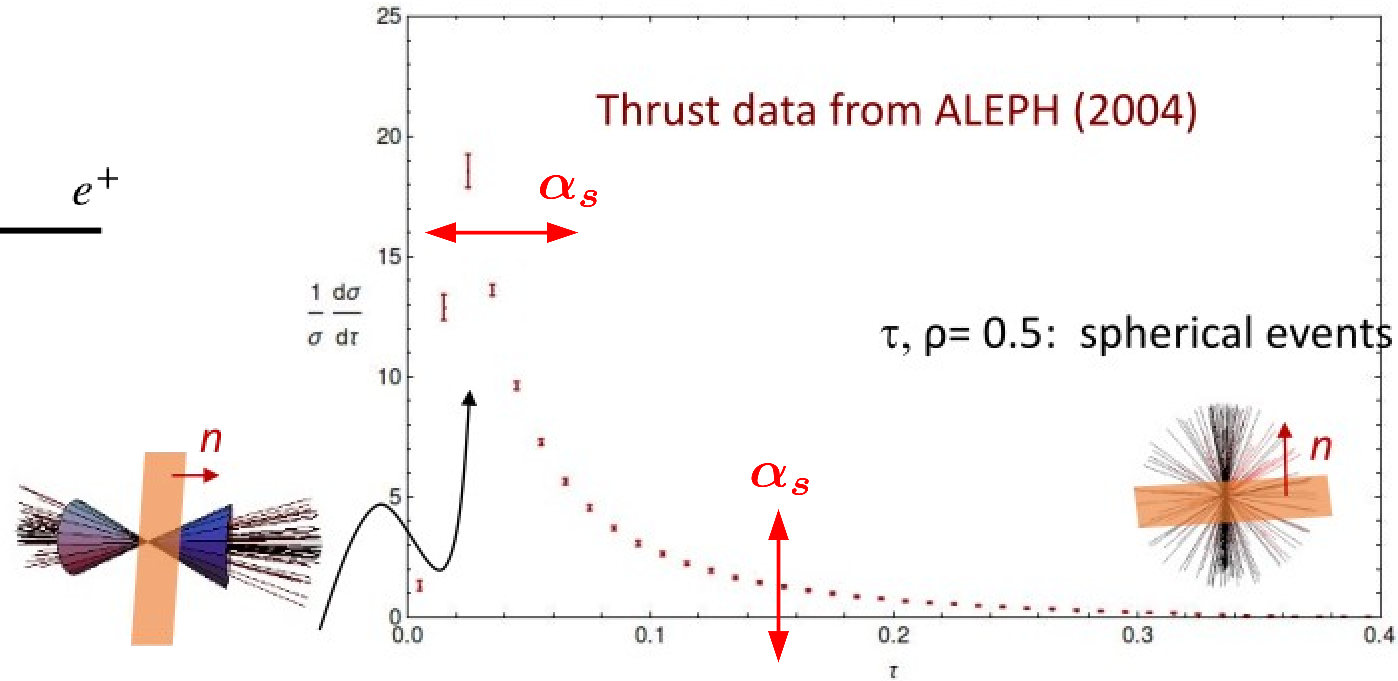
– PDG '23 QCD Review, p. 35

- These are very precise data sets & advanced theory calc's.
- *Goal:* Make sure we leave no stone unturned & use as much e^+e^- data as we have.



PDG 2023: 0.1180 ± 0.0009





τ,ρ << 1: small jets

[Figure: M. Schwartz]

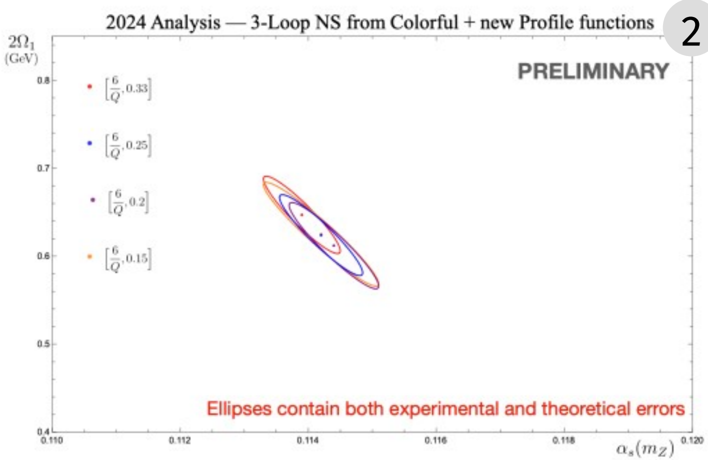
$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

$$\tau \equiv 1 - T$$

A quick word on $\mathcal{O}(\Lambda_{\rm QCD})$ power corrections

the Durham algorithm. The key finding in Ref. [1] is that non-perturbative corrections computed in the three-jet region significantly deviate from those computed in the two-jet limit and hence the aforementioned fits based on power corrections in the two-jet limit result in smaller values of $\alpha_s(m_Z^2)$. Another important observation is that the inclusion of resummation effects introduces a relatively substantial ambiguity outside the two-jet limit. Additionally, other factors such as the choice of mass-scheme used to extend the definition of event shapes to massive hadrons can have significant effects.

[Huston, Rabbertz, Zanderighi, PDG QCD Review, 2312.14015]



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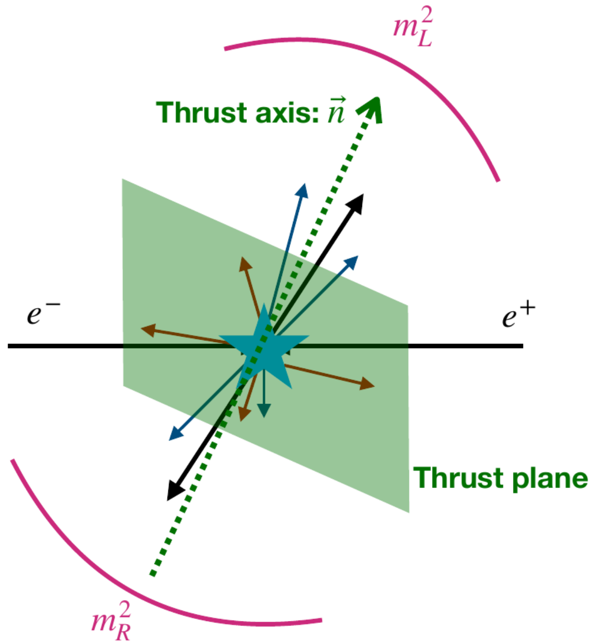
1. Nonperturbative corrections in three-jet region likely different from dijet.
   [Caola et al., 2204.02247; Nason, Zanderighi, 2301.03607]

2. But 2024 thrust update on dijet fit window only still finds low value of αs !
   [Benitez-Rathgeb, Hoang, Mateu, Stewart, Vita, 2024]

$$\alpha_s = 0.1142 \pm 0.0006_{\rm pert} \pm 0.0009_{\rm exp} \pm 0.0004_{\rm had} = 0.1142 \pm 0.0012_{\rm tot}$$

Idea: Use data for Heavy Jet Mass event shape in addition

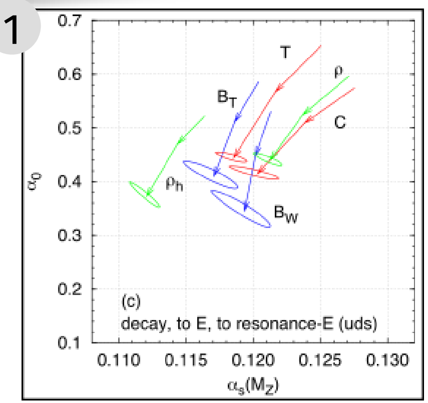


$$\rho = \frac{1}{Q^2}\max\{m_L^2, m_R^2\}$$

20 years ago: [Salam, Wicke, hep-ph/0102343]

Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo simulations that hadronisation corrections for ρ_h have unusual characteristics: in contrast to what is seen in more inclusive variables, the hadronisation depends strongly on the underlying hard configuration. There is therefore a need to develop techniques allowing a more formal approach to the study of such problems.

1



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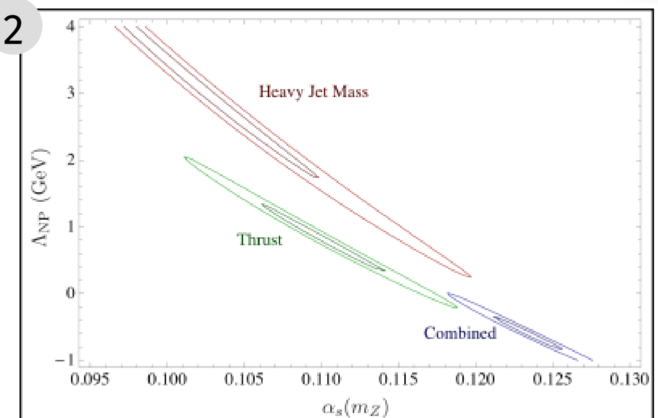


10 years ago: [Becher, Schwartz, 0803.0342] [Chien, Schwartz, 1005.1644]

$\mathrm{N^3LL}$ dijet resummation + NNLO + power correction:

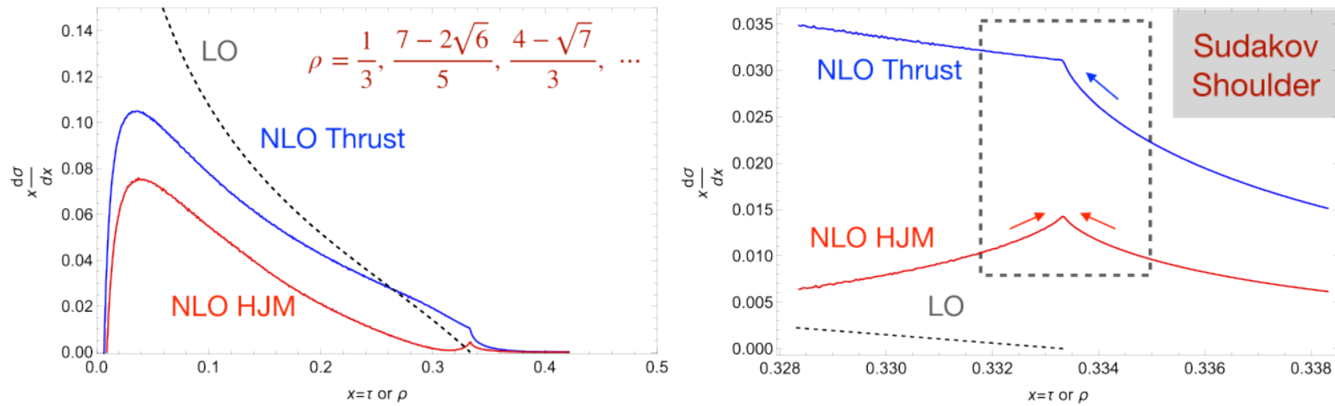
Inconsistence between thrust and heavy jet mass

- Might also shed additional light on 3-jet NP corrections!
  [Caola et al., 2204.02247; Nason, Zanderighi, 2301.03607]

Sudakov Shoulders: What makes HJM special

[Slide credit: Xiaoyuan Zhang, Loopfest '23]

- Thrust and HJM have different kinks order by order in perturbation theory



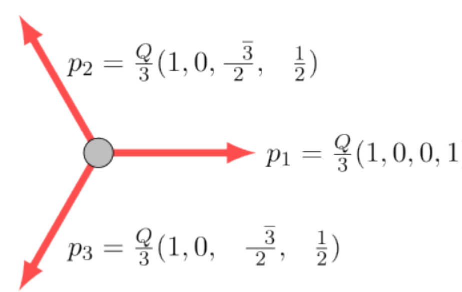
[Catani, Webber, hep-ph/9710333]

- Sudakov shoulders arise from incomplete cancellations between the virtual corrections and real emissions, where the range of event shape grows order-by-order in perturbation theory.
- Start with 3-parton configuration, the event shapes are restricted at each order:

Tree, one-loop virtual: $\tau, \rho \leq \frac{1}{3}$

Real emission: $\tau, \rho \leq \frac{7-2\sqrt{6}}{5} \approx 0.42$

Incomplete cancellation ⇒ divergence, kinks, etc. ⇒ large logarithms



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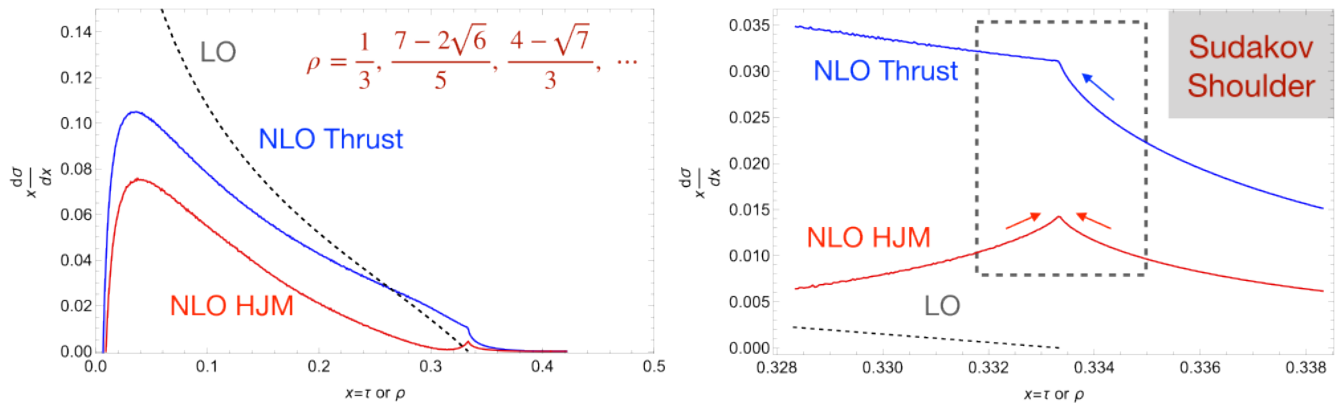

Fixed-order calculations: [Bhattacharya, Schwartz, **XYZ**, 2205.05702]

- Thrust: only right shoulder $\quad t = \tau - \frac{1}{3}$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma}{d\tau} = \frac{\alpha_s}{4\pi}\theta(t)\left\{-6\left(2C_F + C_A\right) t\ln^2 t + \left[6C_F\left(1-4\ln 3\right) + C_A\left(1-12\ln 3\right) + 4n_f T_F\right] t\ln t\right\}$$

- HJM: left shoulder (affects the α_s fit!) and right shoulder $\quad r = \frac{1}{3} - \rho$

$$\frac{1}{\sigma_{LO}}\frac{d\sigma}{d\rho} = \frac{\alpha_s}{4\pi}\theta(r)\left\{-2\left(2C_F + C_A\right) r\ln^2 r + \left[2C_F\left(1+4\ln\frac{4}{3}\right) + C_A\left(\frac{1}{3}+4\ln\frac{4}{3}\right) + \frac{4}{3}n_f T_F\right] r\ln r\right\}$$
$$+\frac{\alpha_s}{4\pi}\theta(-r)\left\{-4\left(2C_F + C_A\right)(-r)\ln^2(-r) + \left[4C_F\left(1-4\ln 6\right) + 2C_A\left(\frac{1}{3}-4\ln 6\right) + \frac{8}{3}n_f T_F\right](-r)\ln(-r)\right\}$$

Sudakov Shoulders: What makes HJM special

[Slide credit: Xiaoyuan Zhang, Loopfest ‘23]

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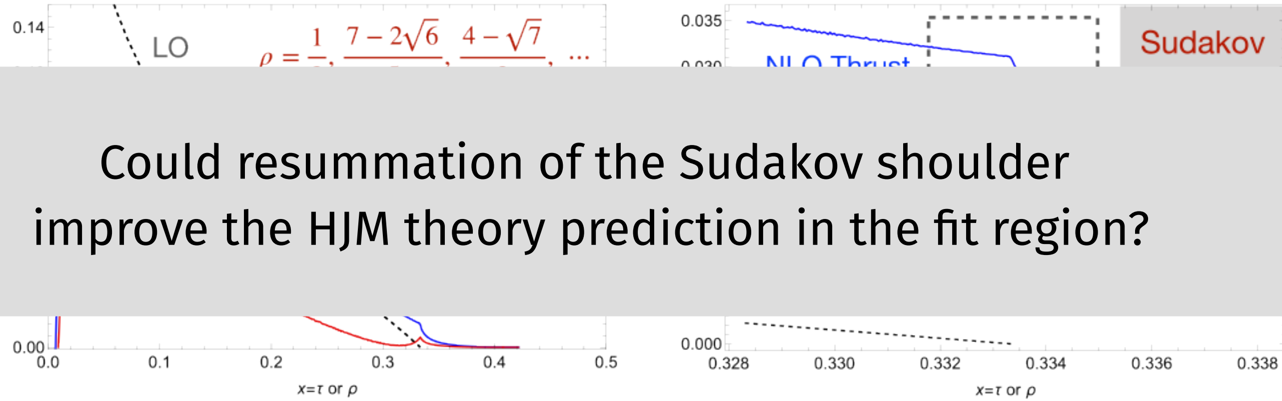

Could resummation of the Sudakov shoulder
improve the HJM theory prediction in the fit region?

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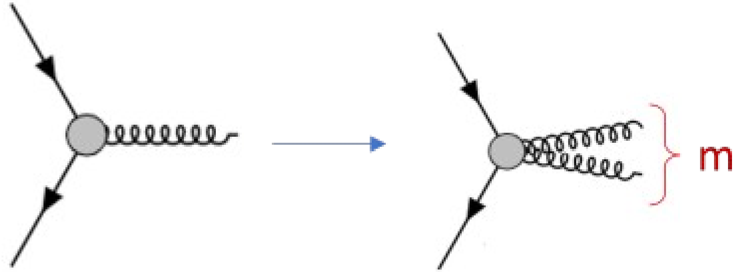
$$+\frac{\alpha_s}{4\pi}\theta(-r)\left\{-4\left(2C_F + C_A\right)(-r)\ln^2(-r) + \left[4C_F\left(1 - 4\ln 6\right) + 2C_A\left(\frac{1}{3} - 4\ln 6\right) + \frac{8}{3}n_f T_F\right](-r)\ln(-r)\right\}$$

$$\frac{d\sigma_{\text{sh}}}{d\rho} = \sum_i \frac{d\sigma_i}{d\rho} = \frac{d\sigma_g}{d\rho} + 2\frac{d\sigma_q}{d\rho}$$



$$\frac{d\sigma}{d\rho} = \int_0^\infty dm_h^2 \int_0^\infty dm_\ell^2 \frac{d^2\sigma}{dm_\ell^2 dm_h^2}(r + m_h^2 - m_\ell^2)\theta(r + m_h^2 - m_\ell^2)$$

$\theta(r)\, r$ → $\delta(r)$

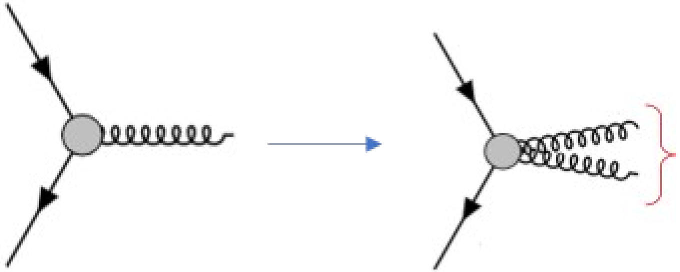
$\downarrow \frac{\mathrm{d}^2}{\mathrm{d}r^2}$

$r = 1/3 - \rho$

$\theta(r)\, r\, \ln r$ → $\left[\frac{\theta(r)}{r}\right]_+$

$$\frac{1}{\sigma_{\text{LO}}}\frac{d^3\sigma_g}{d\rho^3} = H_3(Q^2,\mu)\int \mathrm{d}r_s\, \mathrm{d}m_q^2\, \mathrm{d}m_{\bar{q}}^2\, \mathrm{d}m_g^2\, S_3(Q^2 r_s,\mu)$$
$$\times J_q(m_q^2,\mu)\, J_q(m_{\bar{q}}^2,\mu)\, J_g(m_g^2,\mu)$$
$$\times \delta\left(r - r_s + \frac{m_q^2 + m_{\bar{q}}^2 - m_g^2}{Q^2}\right)$$

$$\frac{d\sigma_{\mathsf{sh}}}{d\rho} = \sum_i \frac{d\sigma_i}{d\rho} = \frac{d\sigma_g}{d\rho} + 2\frac{d\sigma_q}{d\rho}$$



$$\frac{d\sigma}{d\rho} = \int_0^\infty dm_h^2 \int_0^\infty dm_\ell^2 \frac{d^2\sigma}{dm_\ell^2 dm_h^2}(r + m_h^2 - m_\ell^2)\theta(r + m_h^2 - m_\ell^2)$$

$\theta(r)\, r$ → $\delta(r)$

$\frac{\mathrm{d}^2}{\mathrm{d}r^2}$ $\quad r = 1/3 - \rho$

$\theta(r)\, r\, \ln r$ → $\left[\frac{\theta(r)}{r}\right]_+$

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$$\times J_q(m_q^2, \mu)\, J_q(m_{\bar{q}}^2, \mu)\, J_g(m_g^2, \mu)$$
$$\times \delta\left(r - r_s + \frac{m_q^2 + m_{\bar{q}}^2 - m_g^2}{Q^2}\right)$$

FO ✓

Sudakov Landau Poles

[Slide credit: Matt Schwartz]

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) r \left(\frac{rQ}{\mu_s}\right)^{\eta_\ell} \left(\frac{rQ}{\mu_s}\right)^{\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)} \frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

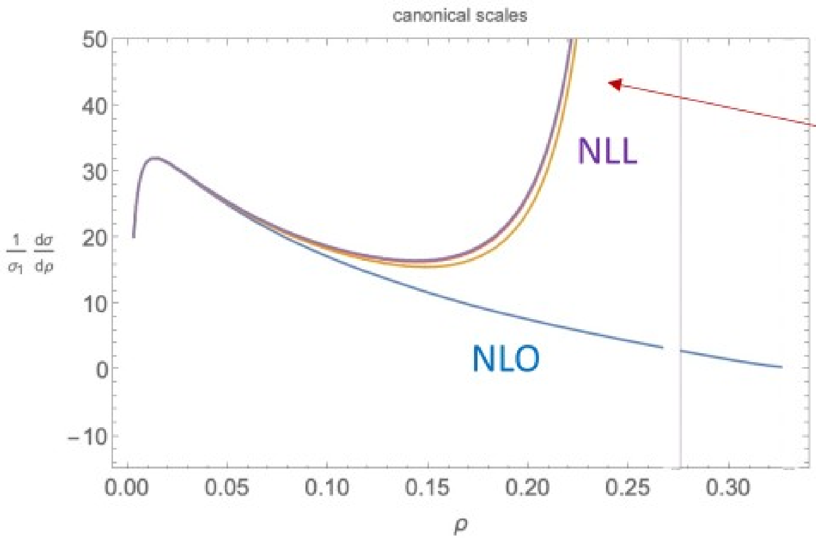
$\eta_\ell = 2C_A A_\Gamma(\mu_j, \mu_s)$ $\quad$ $\eta_h = 4C_F A_\Gamma(\mu_j, \mu_s)$

$\mu_h = Q$
$\mu_j = \sqrt{r}Q$
$\mu_s = rQ$

canonical scales

$$\eta_\ell = -C_A \frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r \qquad \eta_h = -2C_F\frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r$$

Plot it



Blows up at ρ∼0.25

- Arises when

$$\sin(\pi(\eta_\ell+\eta_h)) = 0 \quad \Leftrightarrow \quad \eta_\ell+\eta_h = (C_A+2C_F)\frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r = 1, 2, 3, \cdots$$

- Comes from running associated with cusp anomalous dimension
- We call this a "**Sudakov Landau pole**"
  - Present even if β=0

$$\frac{\mathrm{d}\sigma^{(\mathrm{DDT})}}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}\vec{q}_T} \propto Q^2 \frac{\mathrm{d}}{\mathrm{d}q_T^2}\left[e^{\mathbb{S}}\frac{\Gamma(1+h/2)}{\Gamma(1-h/2)}\right], \qquad h = 4\Gamma_{\mathrm{cusp}}[\alpha_s(q_T)]\ln\frac{q_T}{Q}$$

[Frixione, Nason, Ridolfi, hep-ph/9809367; review: Ebert & Tackmann, 1611.08610]

$$\frac{\mathrm{d}\sigma_{\mathrm{sing}}}{\mathrm{d}Q\mathrm{d}Y\,\mathrm{d}p_T^2} = \sum_{a,b} H_{ab}(Q^2,\mu) \times [B_a B_b S](Q^2, x_a, x_b, \vec{p}_T, \mu)$$

$$[B_a B_b S] \equiv \int \mathrm{d}^2\vec{k}_a\,\mathrm{d}^2\vec{k}_b\,\mathrm{d}^2\vec{k}_s\,\delta^{(2)}(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s) \times B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$

$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)\, \tilde{S}(b_T, \mu, \nu)$$

$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{f}_a^{\mathrm{TMD}}(x_a, b_T, \mu, \zeta_a)\, \tilde{f}_b^{\mathrm{TMD}}(x_b, b_T, \mu, \zeta_b)$$

$\Rightarrow$ Choosing canonical scales as $\mu \simeq \sqrt{\zeta} \simeq 1/b_T$ removes the Sudakov Landau pole.

Position vs. momentum space

[Slide credit: Matt Schwartz]

In momentum space, distribution is complicated and non-analytic

$$f(r) \equiv \frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}\int_0^\infty dx \int_0^\infty dy\; x^{a-1} y^{b-1} \delta(r+y-x)$$
$$= \frac{1}{\Gamma(a+b)}\left[r^{a+b-1}\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + (-r)^{a+b-1}\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

In position space, distribution is remarkably simple

$$\tilde{f}(z) = \int_{-\infty}^{\infty} dr\, f(r) e^{izr} = (-iz)^a (iz)^b$$

- difficult to compute
- must carefully track analytic continuation

No longer has Sudakov landau poles at a+b = 1,2,3...

- Note: must be position space not Laplace space
- In **Laplace space** distribution is **not simple**
  - For 1-sided Laplace transform, need to flip sign in exponent to make integral convergent

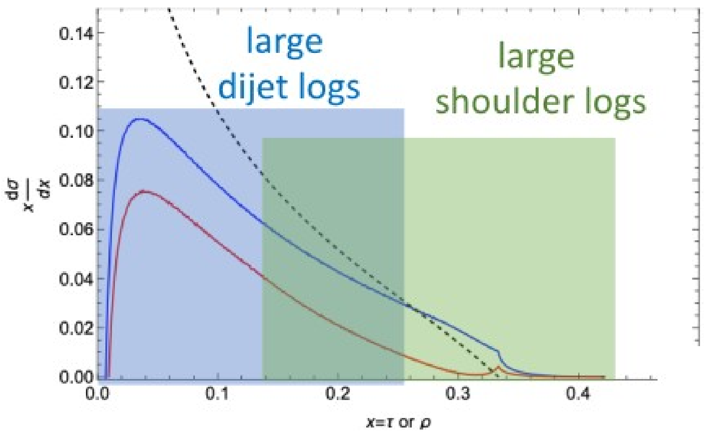
$$\mathcal{L}[f](\nu) = \int_0^\infty dr\; e^{-\nu r} f(r) + \int_{-\infty}^0 dr\; e^{\nu r} f(r) = \nu^{-a-b}\frac{\sin(\pi a)+\sin(\pi b)}{\sin(\pi(a+b))}\,.$$

  - Still has Sudakov Landau pole

Matching to the dijet region

[Slide credit: Matt Schwartz]

We need to match bewteen the shoulder region and the dijet region



Look at fixed order

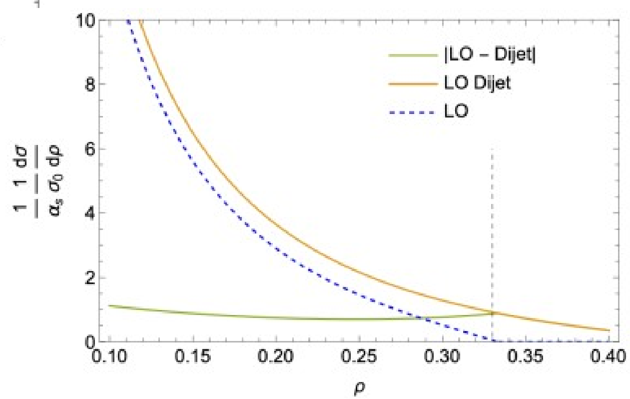
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\rho} = C_F\frac{\alpha_s}{2\pi}\left\{\frac{3(1+\rho)(3\rho-1)}{\rho} + \frac{[4+6\rho(\rho-1)]\ln\frac{1-2\rho}{\rho}}{\rho(1-\rho)}\right]\theta\left(\frac{1}{3}-\rho\right)$$

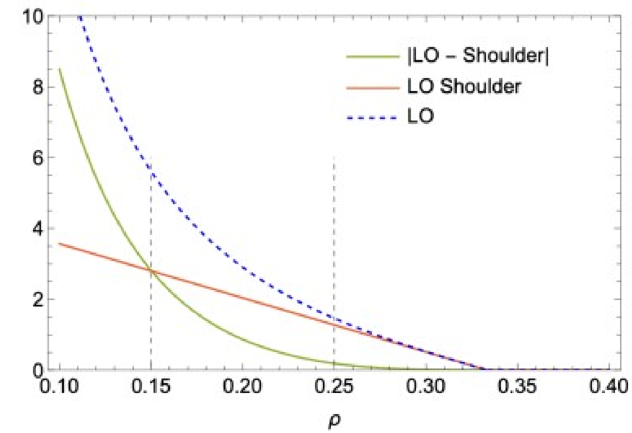
expand near ρ=0

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{dij}}}{d\rho} = C_F\frac{\alpha_s}{2\pi}\left(-\frac{3}{\rho}-\frac{4\ln\rho}{\rho}\right)$$

expand near ρ=⅓

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{shoulder}}}{d\rho} = C_F\frac{\alpha_s}{2\pi}72\left(\frac{1}{3}-\rho\right)\theta\left(\frac{1}{3}-\rho\right)$$





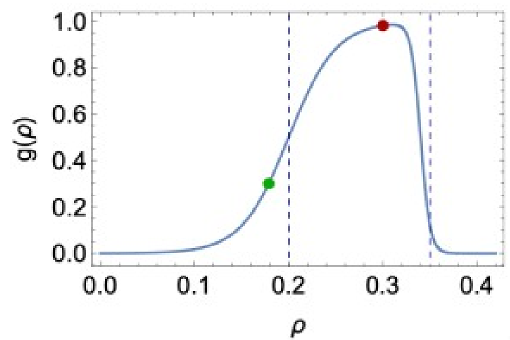
- want pure shoulder for ρ > 0.25
- fade to dijet by ρ < 0.15

[Slide credit: Matt Schwartz]

"Hybrid profiles" [Lustermans, Michel, Tackmann, Waalewijn '19]

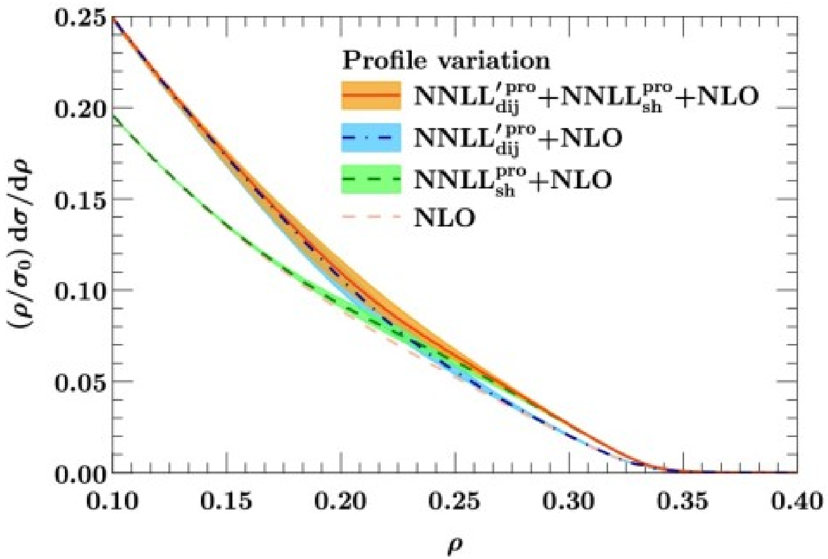
Turn off resummation in shoulder and dijet region by using ρ dependent profile functions for soft and jet scales:

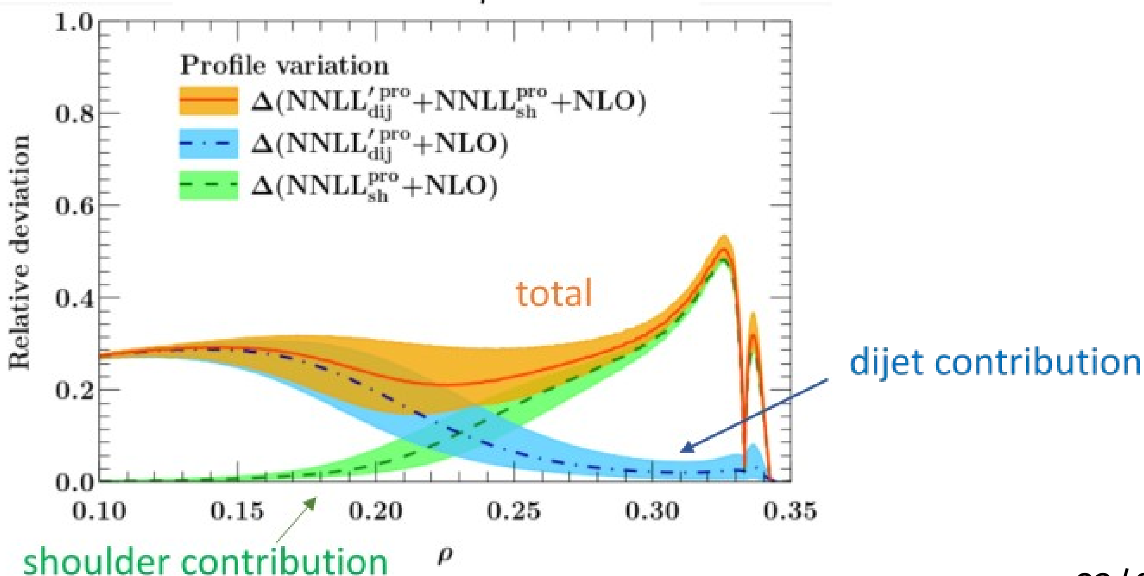
$$\mu_{j,s}^{\rm pro}(z,\rho) = \mu_h^{1-g(\rho)}[\mu_{j,s}^{\rm can}(z)]^{g(\rho)}$$



We use sigmoid functions to

- fade between shoulder and dijet
- fade out resummation in the right shoulder.

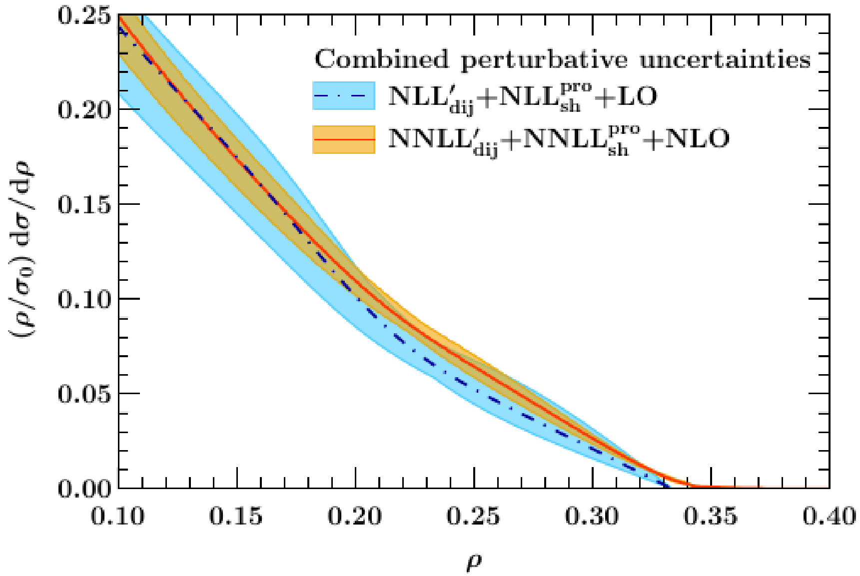


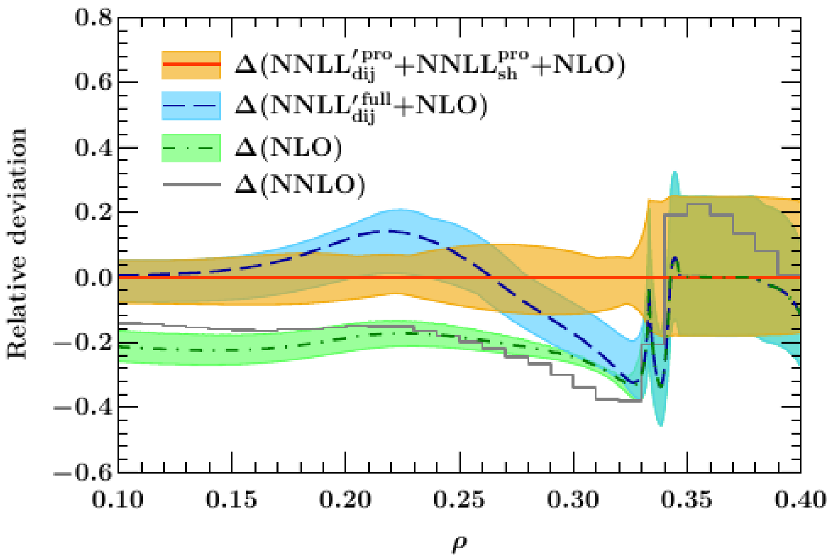


[Slide credit: Xiaoyuan Zhang]

- Combine dijet ($\rho \to 0$) resummation and shoulder ($\rho \to 1/3$) resummation:

$$\frac{d\sigma^{\text{match}}}{d\rho} = \frac{d\sigma^{\text{dij}}(\mu_{\text{dij}}^{\text{pro}})}{d\rho} + \frac{d\sigma^{\text{sh}}(\mu_{\text{sh}}^{\text{pro}})}{d\rho} + \left[\frac{d\sigma^{\text{FO}}(\mu_{\text{FO}})}{d\rho} - \frac{d\sigma^{\text{dij}}(\mu_{\text{FO}})}{d\rho} - \frac{d\sigma^{\text{sh}}(\mu_{\text{FO}})}{d\rho}\right]$$



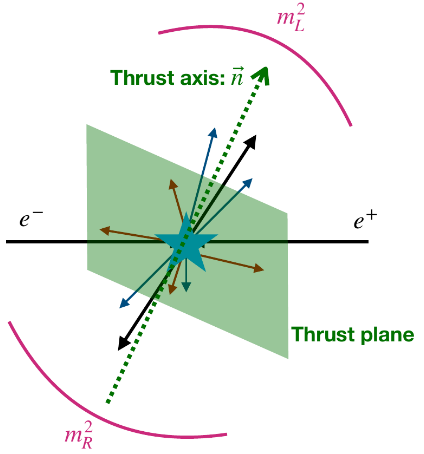


- Conclusion: shoulder resummation provides significant corrections, which could affect the α_s extraction

Summary: Heavy Jet Mass Sudakov Shoulder at NNLL

- LEP event shapes are still a very good place to fit the strong coupling.
- Sudakov shoulders are large logs associated with phase-space boundaries.
- For Heavy Jet Mass, they contribute inside the typical fit region.
- Adapted tools from TMD resummation to solve outstanding issue in momentum-space shoulder resummation and go to NNLL.
  - Effects may be significant within fit region.

Questions so far?



Overview of today's talk

1. NNLL Resummation of Sudakov Shoulder Logarithms in the **Heavy** Jet Mass Distribution

2. Transverse Momentum-Dependent Fragmentation Functions of **Heavy** Quarks & Hadrons

   - JHEP 09 (2023) 205, 2305.15461, with Rebecca von Kuk (DESY) and Zhiquan Sun (MIT)
   - 2404.08622 with R. von Kuk and Z. Sun
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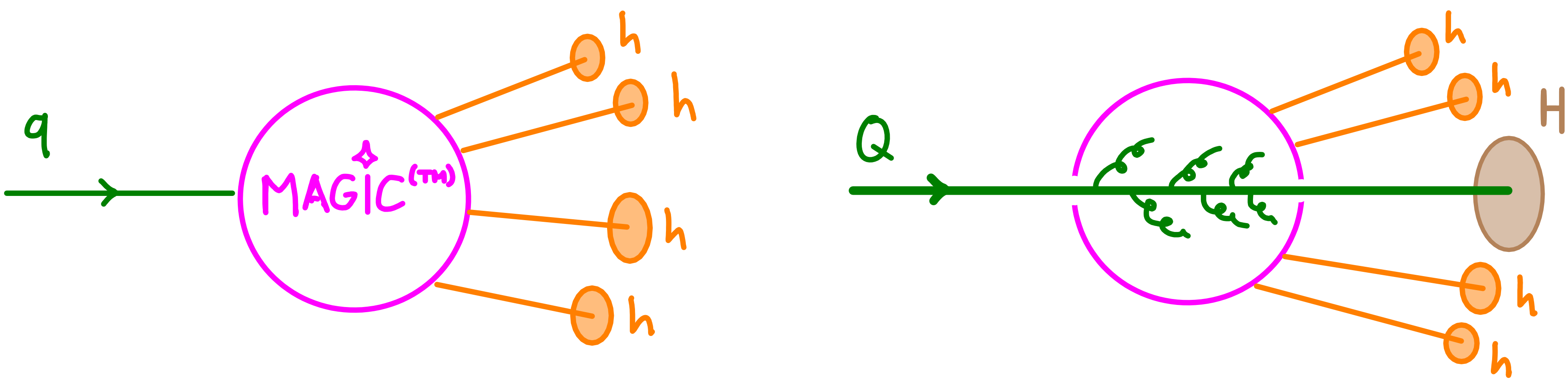
Motivation: Why Heavy Quarks?

- Charm and bottom quarks with $m \equiv m_c, m_b \gg \Lambda_{\text{QCD}}$ are special
- Decay & mixing of heavy hadrons: Most precisely measured [Belle, BaBar, LHCb, ...] and understood strongly coupled system in QCD $\Rightarrow \Lambda^{\cancel{\text{flavor}}}_{\text{new physics}} \gtrsim 100\,\text{TeV}$

This talk: $\Lambda_{\text{QCD}} \sim 100\,\text{MeV}$

Heavy quark TMD fragmentation functions as a powerful probe of hadronization.

- Probe hadronization by "sticking in" a static color source!



Motivation: Why Heavy Quarks?

Youtube: @electricandmagneticfields2314

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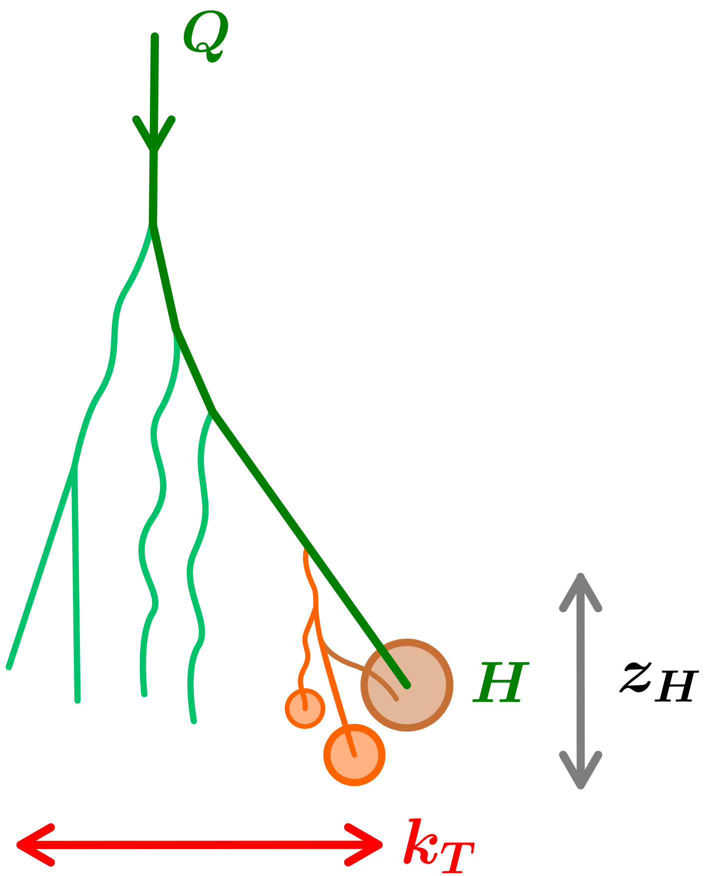
TMD vs. collinear heavy-quark FFs



- Longitudinal z_H distribution (collinear heavy-quark FF) is well understood

  [B. Mele, P. Nason, Nucl. Phys. B 361 (1991) 626]

  [R. L. Jaffe, L. Randall, Nucl. Phys. B 412 (1994) 79]

  [A. F. Falk, M. E. Peskin, Phys. Rev. D 49 (1994) 3320]

  [M. Neubert, 0706.2136]

  [M. Fickinger, S. Fleming, C. Kim and E. Mereghetti, JHEP 11 (2016) 095]

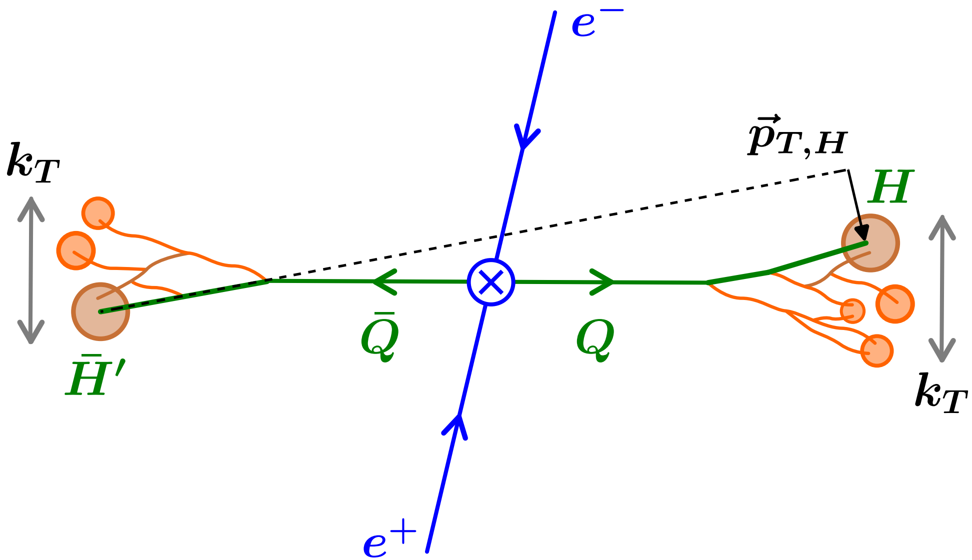
- Recently implemented in `Herwig 7`!

  [M. Masouminia, P. Richardson, 2312.02757]

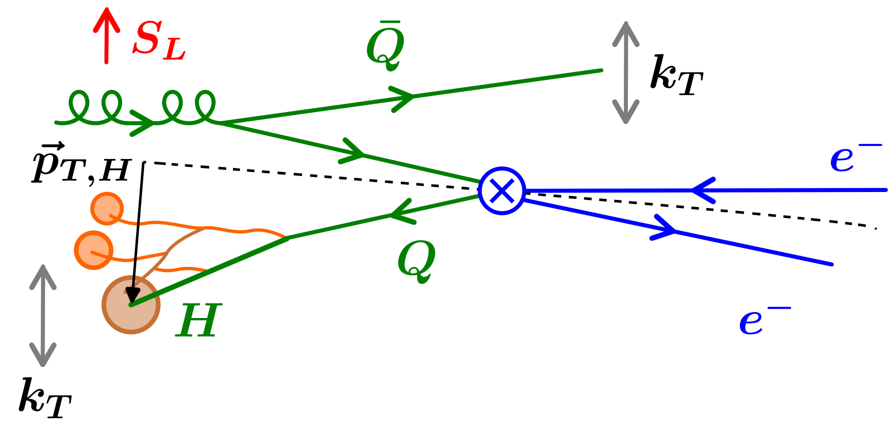
- Heavy-quark TMD FFs $\Rightarrow$ new!

Typical Processes and TMD Observables

$e^+e^- \to H\bar{H}'X$



$eN \to HX$



"unpolarized TMD FF"

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\vec{p}_{H,T}} \propto H\Big[(1+\cos^2\theta)\, D_{1\,H/Q} \otimes D_{1\,\bar{H}/\bar{Q}} + \tfrac{1}{2}\sin^2\theta\cos(2\phi_H)\, H^{\perp}_{1\,H/Q} \otimes H^{\perp}_{1\,H/Q}\Big]$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\vec{p}_{H,T}} \propto H\Big[f_{1\,Q/N} \otimes D_{1\,H/Q} + f(y)\sin(2\phi_H) S_L\, h^{\perp}_{1L\,Q/N} \otimes H^{\perp}_{1\,H/Q}\Big]$$

"Collins FF"

- Form of TMD factorization is *unchanged* as long as $m, k_T \ll Q$
  - Work out how mass modifies TMD FFs/PDFs depending on hierarchy of m and k_T

… great playground for effective field theory!

Lightning review: HQET, spectroscopy & collinear FFs

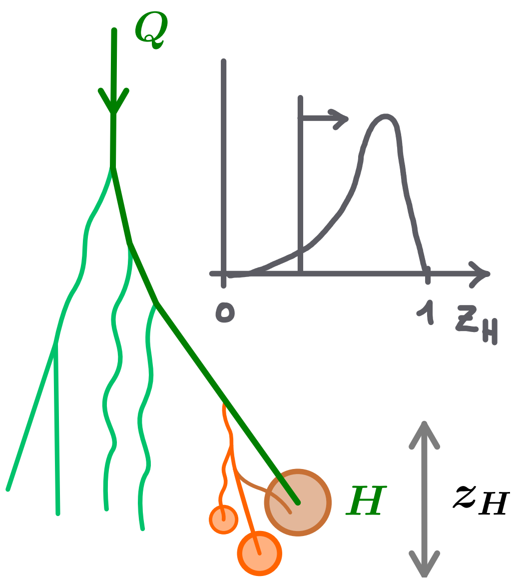
Integrate out QCD modes far off quark mass shell ⇒ heavy-quark effective theory:

$$\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v \, v \cdot D \, h_v + \mathcal{L}_{\mathrm{light}} + \mathcal{O}\Big(\frac{1}{m}\Big) \,, \quad v^\mu = P_H^\mu / M_H \,, \quad v^2 = 1$$

Spectroscopy:

- Flavor symmetry, $\mathcal{L}_{\mathrm{HQET}}(\not{m})$ ⇒ $m_D - m_c = m_B - m_b + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/m)$
- Spin symmetry, $\big[\mathcal{L}_{\mathrm{HQET}}, \vec{S}_Q\big] = 0$ ⇒ $m_{D^*} = m_D + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/m)$

$s_\ell = \frac{1}{2} \; : \quad D = \frac{1}{\sqrt{2}}(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle) \qquad D^* = |{\uparrow\uparrow}\rangle, \frac{1}{\sqrt{2}}(|{\uparrow\downarrow}\rangle + |{\downarrow\uparrow}\rangle), |{\downarrow\downarrow}\rangle$



Collinear FFs:

- $1 - z_H \gg \Lambda_{\mathrm{QCD}}/m \; : \; D_{H/Q}(z_H) = d_{Q/Q}(z_H) \, \chi_H$

  [Mele, Nason '91; general shape function case: Fickinger et al. '16]
- Flavor symmetry: $\chi_D = \chi_B$
- Spin symmetry + parity: $P(h_\ell = +\frac{1}{2}) = P(h_\ell = -\frac{1}{2})$

  ⇒ $\chi_{D^*} = 3\chi_D$, and similarly $\chi_{\Sigma_c^*} = 2\chi_{\Sigma_c}, \ldots$
- No interference between light helicities ⇒ need TMDs!

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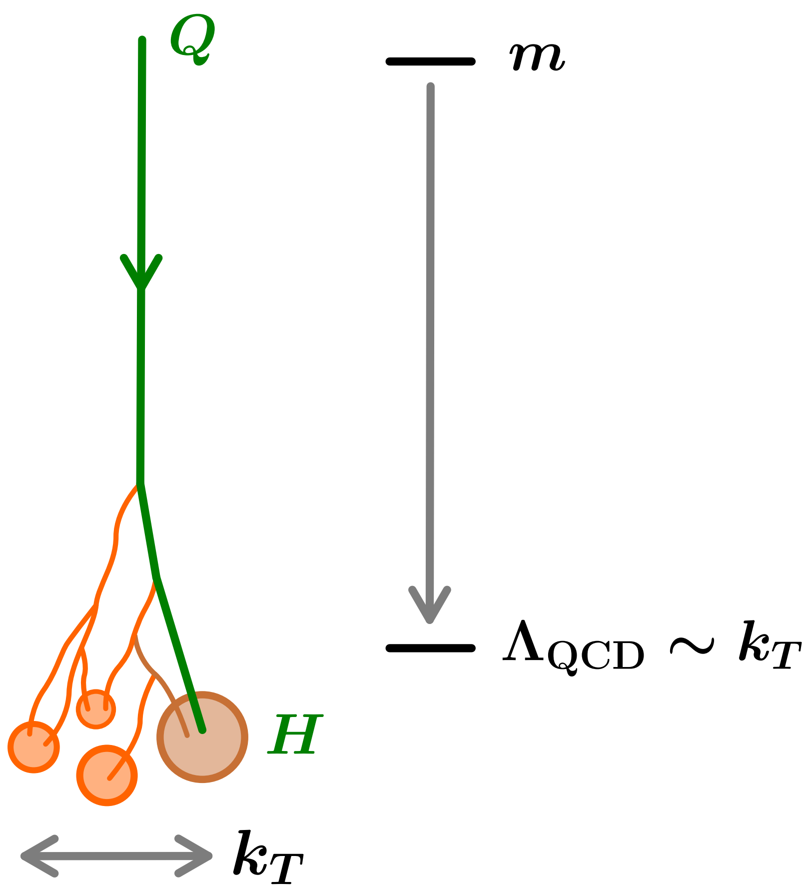
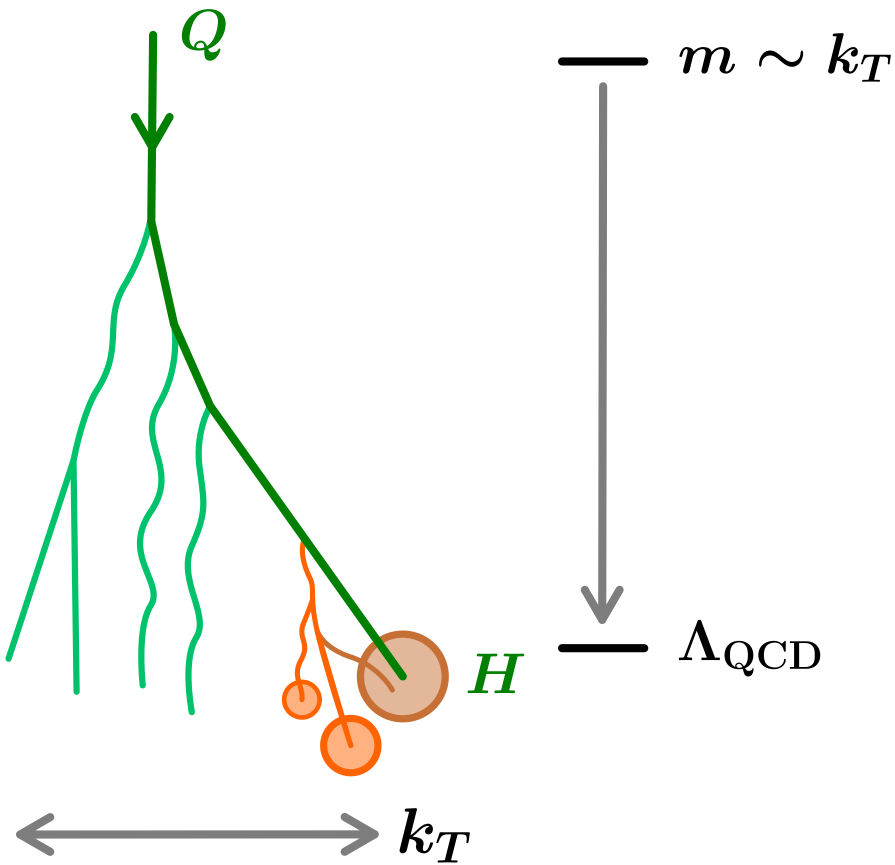
Heavy-quark TMD FFs: Two parametric regimes

Regime 1



$\Lambda_{\text{QCD}} \sim k_T \ll m$

Regime 2

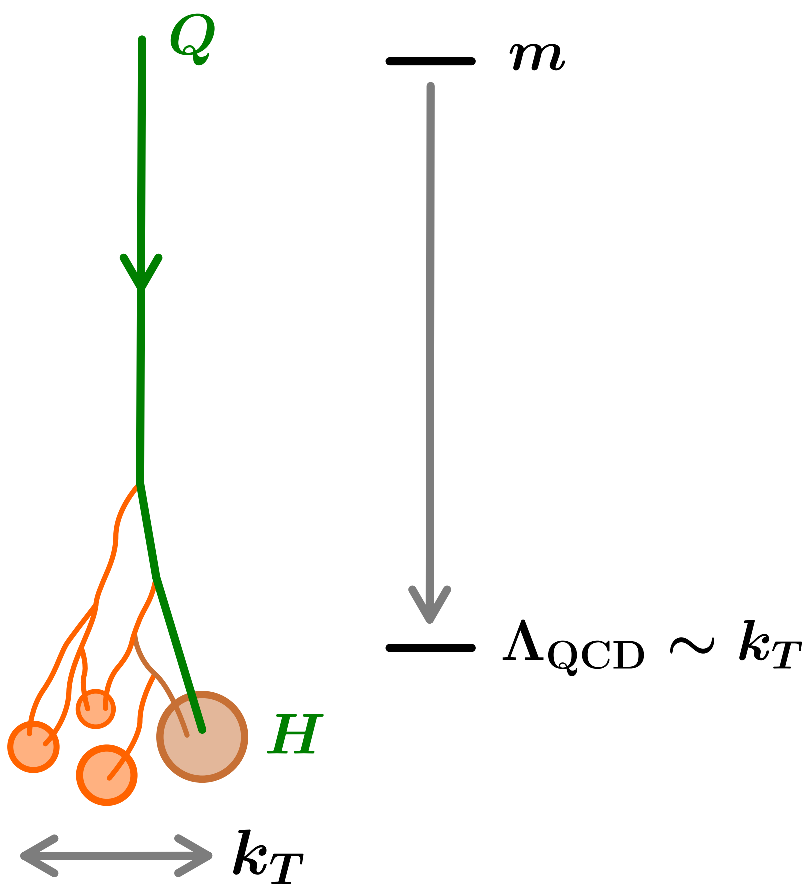


$\Lambda_{\text{QCD}} \ll k_T \sim m$

Heavy-quark TMD FFs *not* suppressed, unlike PDFs.

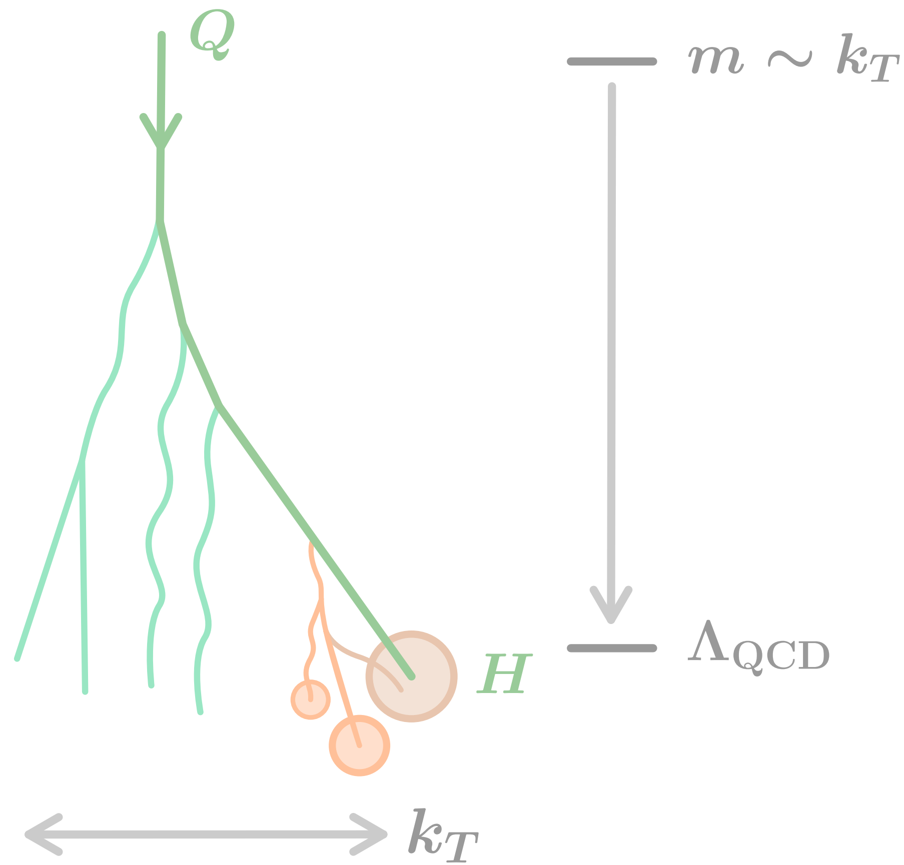
Heavy-quark TMD FFs: Two parametric regimes

Regime 1



$\Lambda_{\rm QCD} \sim k_T \ll m$

Regime 2



$\Lambda_{\rm QCD} \ll k_T \sim m$

Heavy-quark TMD FFs *not* suppressed, unlike PDFs.

Regime 1, $\Lambda_{\rm QCD} \sim k_T \ll m$: Setup

Strategy: Match TMD FF correlator onto (boosted) HQET to integrate out $\mu \sim m$.

$$
\begin{aligned}
&\Delta^{\beta\beta'}_{H/Q}(z_H, b_\perp) \\
&= \frac{1}{2 z_H N_c} \int \frac{\mathrm{d}b^+}{4\pi}\, e^{\mathrm{i} b^+ (P_H^-/z_H)/2}\, \mathrm{Tr} \sum_X \langle 0 | [W^\dagger \psi_Q^\beta](b) | H X \rangle \langle H X | [\bar\psi_Q^{\beta'} W](0) | 0 \rangle \\
&\xrightarrow{\text{HQET}} \frac{\delta(1-z_H)}{\bar n \cdot v}\, C_m(m)\, \boxed{\frac{1}{2N_c}\, \mathrm{Tr} \sum_X \langle 0 | [W^\dagger h_v^\beta](b_\perp) | H_v X \rangle \langle H_v X | [\bar h_v^{\beta'} W](0) | 0 \rangle} \\
&\qquad\qquad\qquad\qquad\qquad \equiv F_H^{\beta\beta'}(b_\perp)
\end{aligned}
$$

Project out unpolarized and Collins TMD FF:

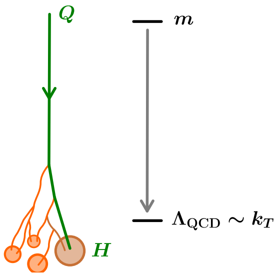
$$
D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1-z_H)\, C_m\Big(m, \mu, \frac{\zeta}{m^2}\Big)\, \chi_{1,H}(b_T, \mu, \sqrt{\zeta}/m) + \mathcal{O}\Big(\frac{1}{m}\Big)
$$

$$
b_T M_H\, H^{\perp(1)}_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1-z_H)\, C_m\Big(m, \mu, \frac{\zeta}{m^2}\Big)\, \chi^\perp_{1,H}(b_T, \mu, \sqrt{\zeta}/m) + \mathcal{O}\Big(\frac{1}{m}\Big)
$$

[Matching coefficient C_m at NNLO: Hoang, Pathak, Pietrulewicz, Stewart ’15]

New scalar bHQET TMD “fragmentation factors”:

$$
\chi_{1,H}(b_T) = \frac{1}{2}\, \mathrm{tr}\, F_H(b_\perp)\,, \qquad \chi^\perp_{1,H}(b_T) = \frac{1}{2}\, \mathrm{tr}\Big[\frac{\not{b}_\perp}{b_T} \not{z}\, F_H(b_\perp)\Big]
$$



Regime 1, $\Lambda_{\rm QCD} \sim k_T \ll m$: Decoupling the heavy quarks

Field redefinition to decouple spin & color degrees of freedom of heavy quark:

$$h_v(x) = Y_v(x)\, h_v^{(0)}(x) \qquad Y_v(x) = P\Big[\exp\Big(ig\int_0^\infty \mathrm{d}s\, v\cdot A(x+vs)\Big)\Big]$$

$$\mathcal{L}_{\rm HQET} = \bar{h}_v^{(0)}(iv\cdot\partial)h_v^{(0)} + \mathcal{L}_{\rm light}$$

[Korchemsky, Radyushkin '92]

$$h_v(x)\,|s_Q, h_Q; s_\ell, h_\ell, f_\ell; X\rangle = u(v, h_Q)\, Y_v(x)\, |s_\ell, h_\ell, f_\ell; X\rangle$$

$$\sum_{h_H} |H_v, h_H; X\rangle\langle H_v, h_H; X| = \sum_{h_H}\Big(\sum_{h_Q}\sum_{h_\ell} |h_Q; s_\ell, h_\ell, f_\ell; X\rangle\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H\rangle\Big) \times \Big(\sum_{h'_Q}\sum_{h'_\ell}\langle s_H, h_H | s_Q, h'_Q; s_\ell, h'_\ell\rangle\langle h'_Q; s_\ell, h'_\ell, f_\ell; X|\Big)$$

$$\Rightarrow F_H(b_\perp) = \frac{1}{2}\sum_{h_H}\sum_{h_Q, h'_Q}\sum_{h_\ell, h'_\ell} u(v, h_Q)\,\bar{u}(v, h'_Q)\,\langle\ldots|\ldots\rangle\langle\ldots|\ldots\rangle\,\rho_{\ell, h_\ell h'_\ell}(b_\perp)$$

$$\rho_{\ell, h_\ell h'_\ell}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\int_X \langle 0|[W^\dagger Y_v](b_\perp)|s_\ell, h_\ell, f_\ell; X\rangle\langle s_\ell, h'_\ell, f_\ell; X|[Y_v^\dagger W](0)|0\rangle$$

$\Rightarrow$ Light spin density matrix encodes *all* nonperturbative physics within hadron multiplet.

Regime 1, $\Lambda_{QCD} \sim k_T \ll m$: Results for unpolarized TMD FF

Taking the trace $\mathrm{tr}\big[F_H(b_\perp)\big] \propto \mathrm{tr}\big[u(v,h_Q)\bar{u}(v,h'_Q)\big]$ sets $h_Q = h'_Q$:

$$D_{1\,H/Q}(z_H, b_T) \propto \chi_{1,H}(b_T) = \frac{1}{2}\sum_{h_H}\sum_{h_Q}\sum_{h_\ell} \big|\langle s_Q, h_Q; s_\ell, h_\ell \big| s_H, h_H\rangle\big|^2 \rho_{\ell, h_\ell h_\ell}(b_\perp)$$

$$\Rightarrow \boxed{\frac{1}{N_{H/\ell}}\chi_{1,H}(b_T) = \chi_{1,\ell}(b_T) \equiv \sum_{H\in M_\ell}\chi_{1,H}(b_T) = \sum_{h_\ell}\rho_{\ell,h_\ell h_\ell}(b_\perp)} \qquad (1)$$

M_ℓ : spin symmetry multiplet (same light spin & flavor)

e.g.: $s_\ell = \frac{1}{2}$: $\quad \chi_{1,D}(b_T,\mu,\zeta) = \frac{1}{4}\chi_{1,\ell}(b_T,\mu,\zeta)\,, \qquad \chi_{1,D^*}(b_T,\mu,\zeta) = \frac{3}{4}\chi_{1,\ell}(b_T,\mu,\zeta)$

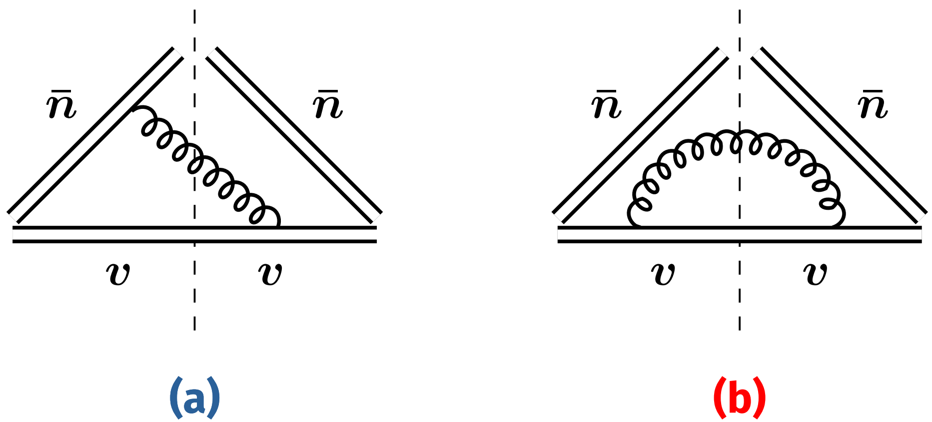
$s_\ell = 1$: $\quad \chi_{1,\Sigma_c}(b_T,\mu,\zeta) = \frac{1}{3}\chi_{1,\ell}(b_T,\mu,\zeta)\,, \qquad \chi_{1,\Sigma_c^*}(b_T,\mu,\zeta) = \frac{2}{3}\chi_{1,\ell}(b_T,\mu,\zeta)$

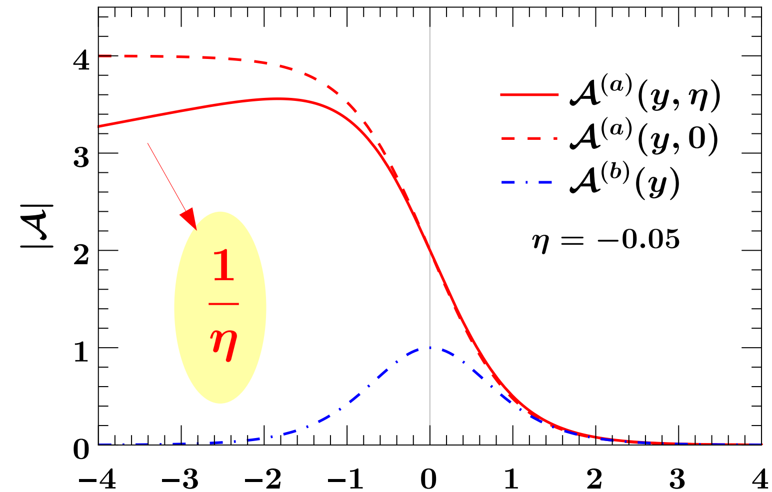
$$\Rightarrow \boxed{\chi_1(b_T) \equiv \sum_H \chi_{1,H}(b_T) = \frac{1}{N_c}\mathrm{Tr}\,\langle 0\big|\big[W^\dagger Y_v\big](b_\perp)\;\big[Y_v^\dagger\, W\big](0)\big|0\rangle} \qquad (2)$$

- Square & combine with soft factor $\Rightarrow$ total $e^+e^- \to H\bar{H}X$ TMD cross section
  - Theoretically simplest real-life fragmentation observable – 100% Wilson loops!

    Semi-inclusive states are gone! How hard can this be on the lattice? 😉

$$\chi_1^{\rm bare}(b_T, \epsilon, \eta, \bar{n}\cdot v) = \frac{1}{N_c}\,{\rm Tr}\langle 0 \big| W_\eta^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W_\eta(0) \big| 0 \rangle$$





gluon rapidity y in heavy-quark rest frame

$$\chi_1(b_T, \mu, \rho) = \lim_{\epsilon\to 0} Z_{\chi_1}^{-1}(\mu, \rho, \epsilon) \lim_{\eta \to 0}\Big[\chi_1^{\rm bare}(b_T, \epsilon, \eta, \rho)\, \sqrt{S^{(n_\ell)}}(b_T, \epsilon, \eta, \nu)\Big]$$

$\rho \equiv \bar{n}\cdot v$

$$= 1 + \frac{\alpha_s C_F}{4\pi}(-L_b)\big(4\ln\rho - 2\big) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\Lambda_{\rm QCD}^2 b_T^2)$$

Regime 1, $\Lambda_{\rm QCD} \sim k_T \ll m$: NLO results and renormalization

$$\chi_1^{\rm bare}(b_T, \epsilon, \eta, \bar{n}\cdot v) = \frac{1}{N_c} \mathrm{Tr}\langle 0 | W_\eta^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W_\eta(0) | 0\rangle$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \chi_1(b_T, \mu, \rho) = \gamma_{\chi_1}[\alpha_s(\mu), \rho] = \gamma_\mu^q(\mu, \zeta) - \gamma_{C_m}(\mu, m, \zeta)$$

TMD μ anom. dim.

$$\rho \frac{\mathrm{d}}{\mathrm{d}\rho} \ln \chi_1(b_T, \mu, \rho) = \gamma_\zeta^{(n_\ell)}(b_T, \mu)$$

CS kernel (n_ℓ light quarks)

$$\rho \frac{\mathrm{d}}{\rho} \gamma_{\chi_1}[\alpha_s(\mu), \rho] = \mu \frac{\mathrm{d}}{\mu} \gamma_\zeta^{(n_\ell)}(\mu, b_T) = -2\Gamma_{\rm cusp}[\alpha_s(\mu)]$$

$$\sqrt{\zeta} = \bar{n}\cdot P_H / z \stackrel{!}{=} m\, \bar{n}\cdot v = m\,\rho$$

$$\chi_1(b_T, \mu, \rho) = \lim_{\epsilon\to 0} Z_{\chi_1}^{-1}(\mu, \rho, \epsilon) \lim_{\eta \to 0}\Big[\chi_1^{\rm bare}(b_T, \epsilon, \eta, \rho) \sqrt{S^{(n_\ell)}}(b_T, \epsilon, \eta, \nu)\Big]$$

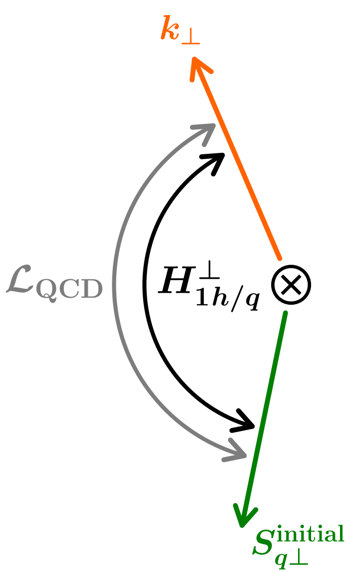
$$= 1 + \frac{\alpha_s C_F}{4\pi}(-L_b)\big(4 \ln\rho - 2\big) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\Lambda_{\rm QCD}^2 b_T^2)$$

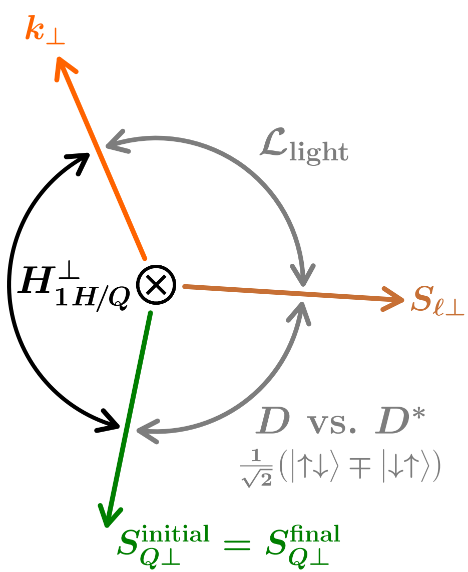
$\rho \equiv \bar{n}\cdot v$

$$H_{1\,H/Q}^{\perp(1)}(z_H, b_T) \propto \chi_{1,H}^{\perp}(b_T) = \frac{1}{2}\,\mathrm{tr}\Big[\frac{\not{b}_\perp}{b_T}\not{z}\,F_H(b_\perp)\Big]$$

$\Rightarrow$ Collins function given by off-diagonal entries (transverse polarization) of $\rho_{\ell,h_\ell h'_\ell}$.

e.g.: $s_\ell = 1/2\,,\ s_H = 0\ :\quad \chi_{1,D}^{\perp}(b_T) = \frac{1}{4}\big[\rho_{\ell,-+}(b_\perp) - \rho_{\ell,+-}(b_\perp)\big]$





$\Rightarrow$ $$\sum_{H\in M_\ell} \chi_{1,H}^{\perp}(b_T, \mu, \zeta) = 0$$

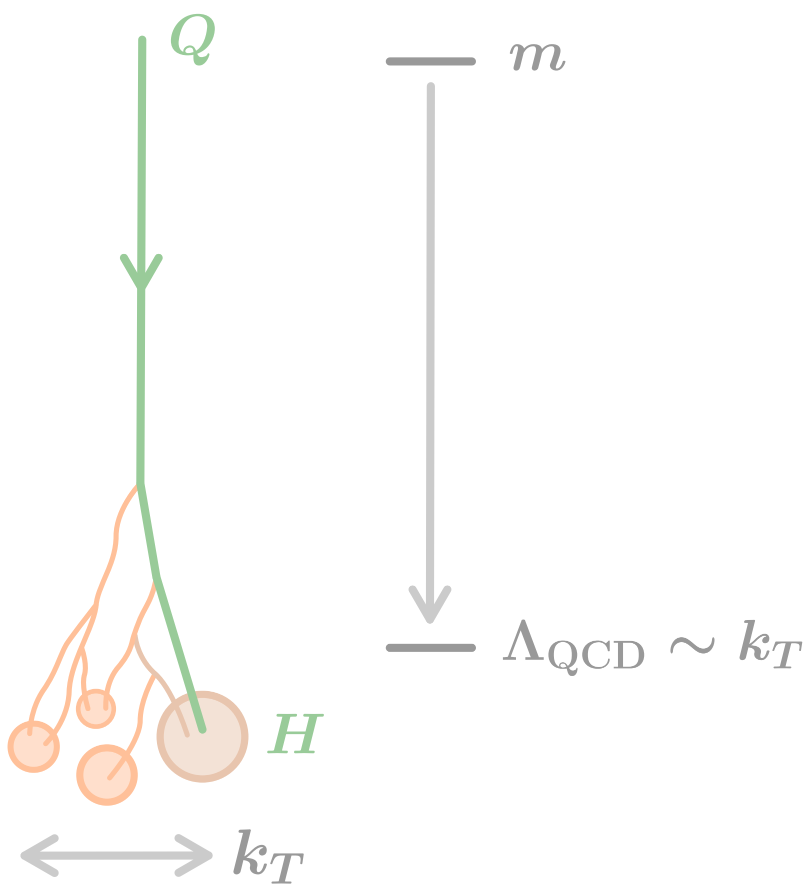
M_ℓ : spin symmetry multiplet (same light spin & flavor)

$$H_{1\,\Lambda_c/c}^{\perp} = 0$$
$$H_{1\,D/c}^{\perp} = -H_{1\,D^*/c}^{\perp}$$
$$H_{1\,\Sigma_c/c}^{\perp} = -H_{1\,\Sigma_c^*/c}^{\perp}$$
$$\vdots$$

- Heavy-quark limit lets us prove a much stronger result than the so-called Schäfer-Teryaev sum rule.
  (only subset of hadrons in sum, pointwise in k_T, holds at renormalized level)

Heavy-quark TMD FFs: Two parametric regimes

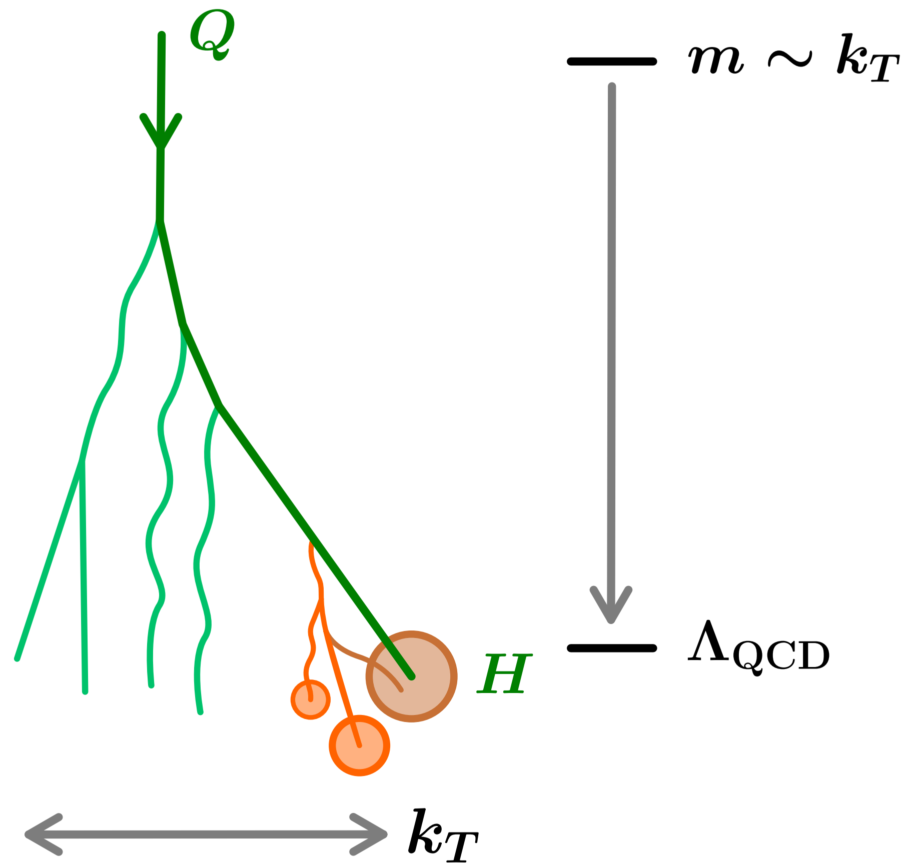
Regime 1



$\Lambda_{\mathrm{QCD}} \sim k_T \ll m$

Heavy-quark TMD FFs *not* suppressed, unlike PDFs.

Regime 2



$\Lambda_{\mathrm{QCD}} \ll k_T \sim m$

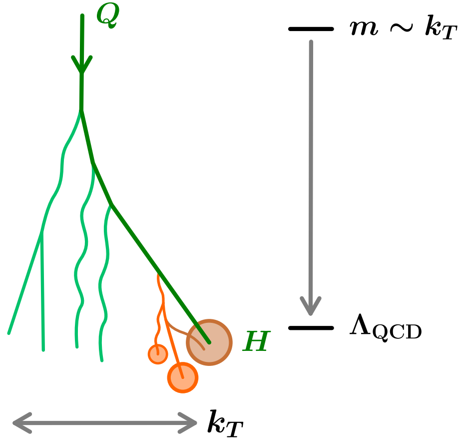
Regime 2, $\Lambda_{\rm QCD} \ll k_T \sim m$: Unpolarized TMD FF

Strategy: Match onto bHQET to integrate out $\mu \sim k_T \sim m$.

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)\, \chi_H + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{m}\Big) + \mathcal{O}(\Lambda_{\rm QCD} b_T)$$

- New perturbative matching coefficient: partonic heavy-quark TMD FF

$$d_{1\,Q/Q}(z_H, b_T) = \mathrm{tr}\Big[\frac{\not{n}}{2}\,\Delta_{Q/Q}(z_H, b_\perp)\Big] = \delta(1 - z_H) + \mathcal{O}(\alpha_s)$$



Recall: χ_H is known to satisfy spin-symmetry relations.

$\Rightarrow$ E.g. $D_{1\,\Sigma_c^*/c} = 2 D_{1\,\Sigma_c/c} + \mathcal{O}(\Lambda_{\rm QCD}/m_c)$
for all k_T & to all orders in α_s.

Consistency relation for $\Lambda_{\rm QCD} \ll k_T \ll m$:

$$d_{1\,Q/Q}(z_H, b_T, \mu, \zeta) = \delta(1 - z_H)\, C_m(m, \mu, \zeta)\, \chi_1^{\rm pert}\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big) + \mathcal{O}\Big(\frac{1}{b_T m}\Big)$$

Regime 2, $\Lambda_{\mathrm{QCD}} \ll k_T \sim m$: Collins FF

Strategy: Two-step matching onto bHQET to integrate out $\mu \sim k_T, m$.

1. Use known twist-3 matching for light quarks to integrate out k_T:

$$b_T M_H\, H_{1\,H/Q}^{\perp(1)}(z_H, b_T) = b_T \hat{H}_{H/Q}(z_h) + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 b_T^2)$$

$$\hat{H}_{H/Q}(z_h) \equiv \frac{z_H^2}{2N_c} \int \frac{\mathrm{d}x^+}{4\pi}\, e^{\mathrm{i}x^+(P_H^-/z_H)/2}\, \mathrm{Tr}\,\mathrm{tr} \sum_X \Big\{ \langle 0 | W^\dagger(x)$$
$$\times\, \sigma_{\alpha -}\big[\mathrm{i}D_\perp^\alpha(x) + g\mathcal{G}_\perp^\alpha(x)\big]\psi_Q(x)\big|HX\rangle\langle HX\big|\bar{\psi}_Q(0)W(0)|0\rangle + \mathrm{h.c.}\Big\}$$

[Mulders, Tangerman ’95; Boer, Mulders ’97; for NLO at $k_T > 0$, see Yuan, Zhou ’09]

2. Match twist-3 matrix element onto bHQET to integrate out m:

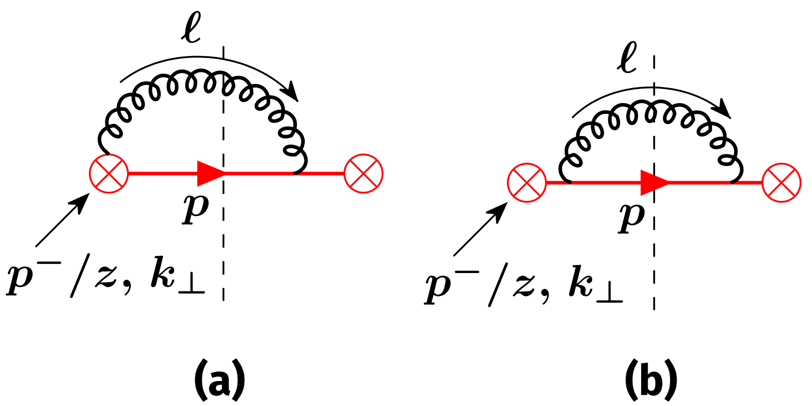
$$\hat{H}_{H/Q}(z_h) \to \delta(1 - z_H)\, \chi_{H,G}$$
$$\chi_{H,G} \equiv \frac{1}{2N_c}\, \mathrm{Tr}\,\mathrm{tr} \sum_X \Big\{ \langle 0 | W^\dagger \sigma_{\beta\alpha} z^\beta \big[\mathrm{i}D_\perp^\alpha + g\mathcal{G}_\perp^\alpha\big] h_v \big|H_v X\rangle\langle H_v X\big| \bar{h}_v W |0\rangle + \mathrm{h.c.}\Big\}$$

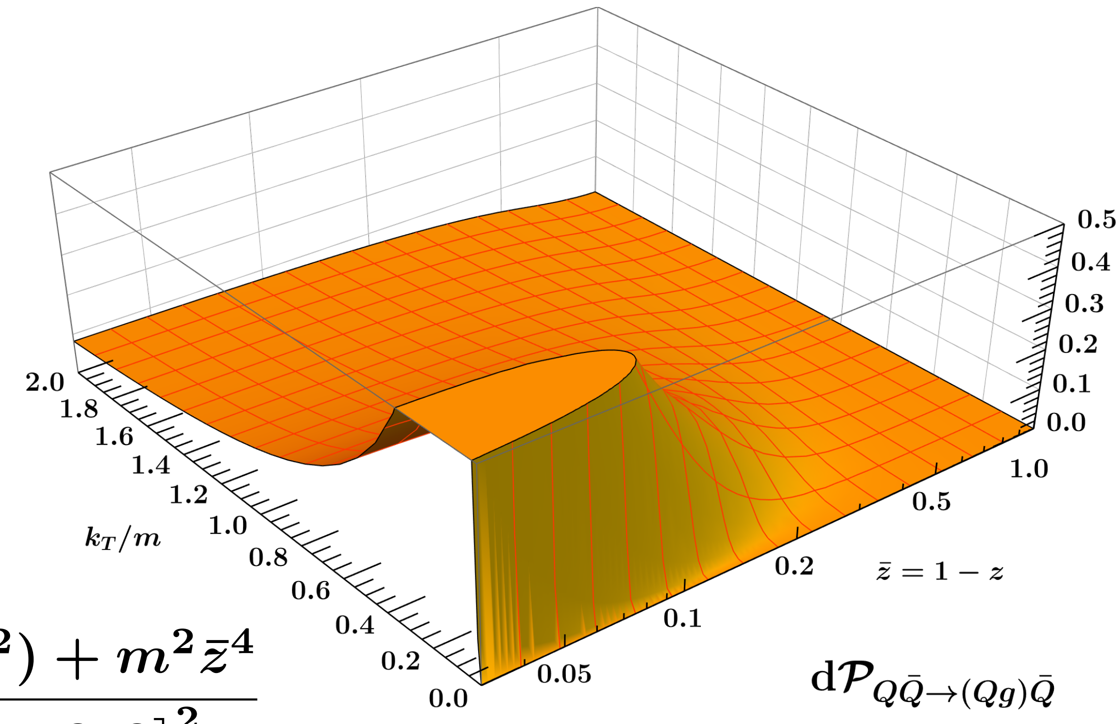
$\Rightarrow$

$$b_T M_H\, H_{1\,H/Q}^{\perp(1)}(z_H, b_T) = \delta(1 - z_H)\, b_T\, \chi_{H,G} + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 b_T^2)$$

- Easy to show: $\sum_{H \in M_\ell} \chi_{H,G} = 0$
  - New sum rule continues to hold!

Regime 2, $\Lambda_{\rm QCD} \ll k_T \sim m$: Setup at NLO



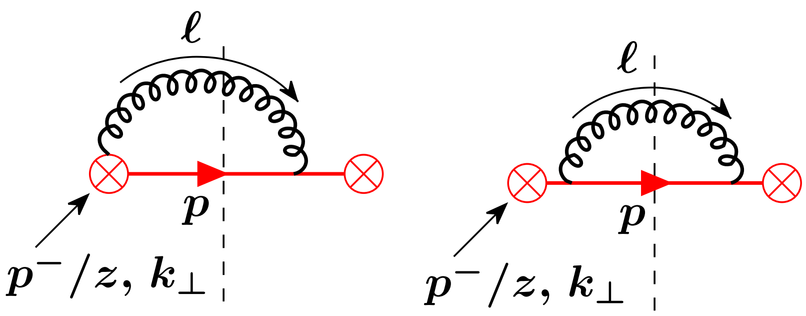


$\bar{z} \equiv z_g = 1 - z > 0$ and $k_T > 0$:

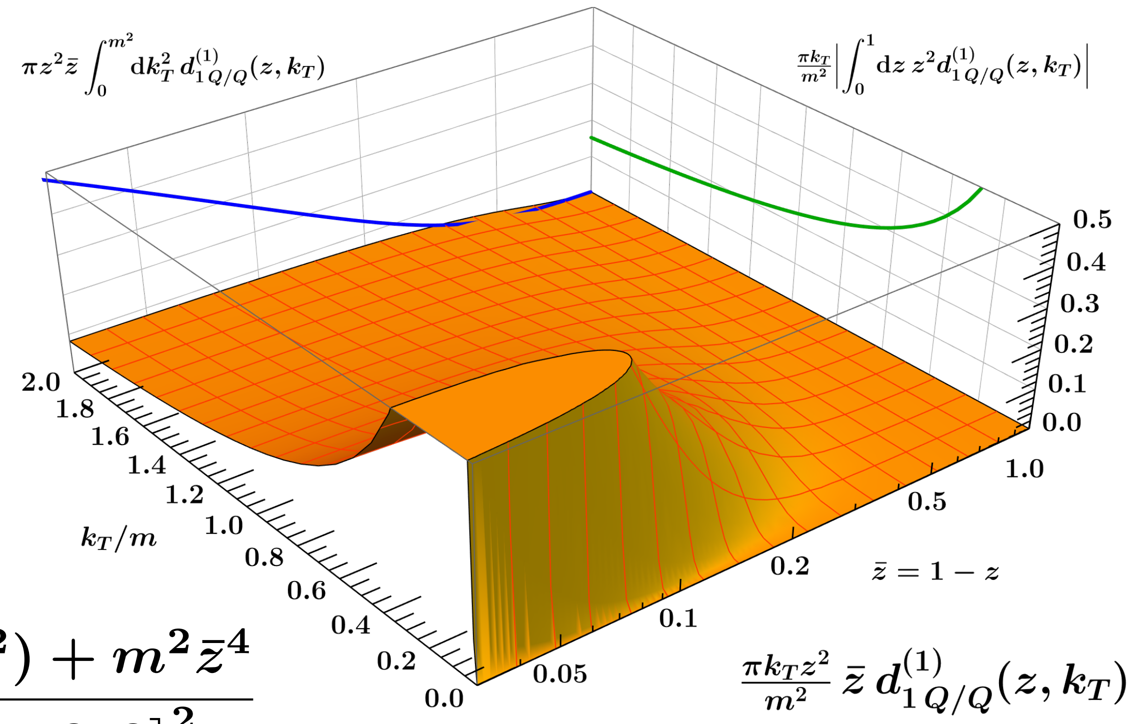
$$\frac{\mathrm{d}\mathcal{P}_{Q\bar{Q}\to(Qg)\bar{Q}}}{\mathrm{d}z\,\mathrm{d}(k_T^2)} = \frac{\alpha_s C_F}{2\pi}\,\frac{k_T^2 z^2(1+z^2) + m^2\bar{z}^4}{\bar{z}\big[k_T^2 z^2 + m^2\bar{z}^2\big]^2}$$

⇒ Task: Regulate the quasi-collinear splitting probability.

Regime 2, $\Lambda_{\rm QCD} \ll k_T \sim m$: Setup at NLO



(a) (b)



$\bar z \equiv z_g = 1 - z > 0$ and $k_T > 0$:

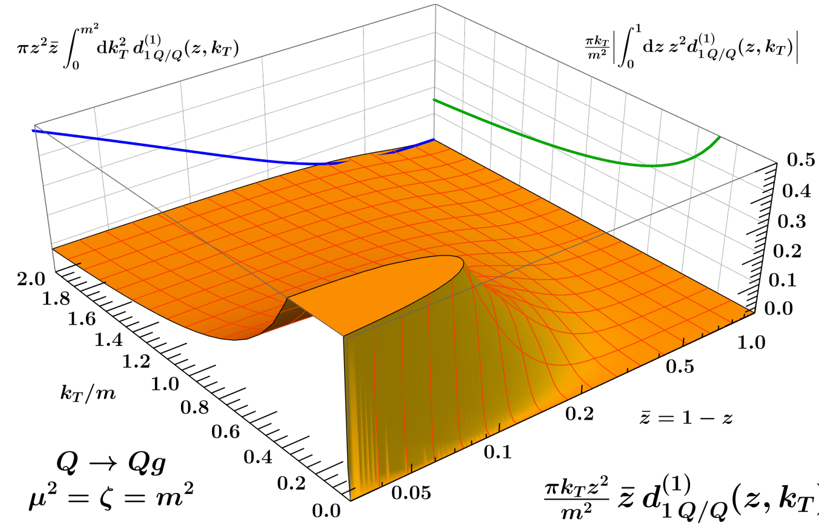
$$\frac{\mathrm{d}\mathcal{P}_{Q\bar Q\to(Qg)\bar Q}}{\mathrm{d}z\,\mathrm{d}(k_T^2)} = \frac{\alpha_s C_F}{2\pi}\,\frac{k_T^2 z^2(1+z^2)+m^2\bar z^4}{\bar z\left[k_T^2 z^2 + m^2 \bar z^2\right]^2}$$

$$\iint = \frac{\alpha_s C_F}{4\pi}\left(5 - \frac{\pi}{2} - \frac{2\pi^2}{3}\right)$$

$$d^{(a)}_{1\,Q/Q} = \frac{\alpha_s C_F}{4\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{\mu}{k_T}\right)^{2\epsilon}\left(\frac{\sqrt{\zeta}}{\nu}\right)^{-\eta}\frac{1}{\pi z^{2-2\epsilon}}\,\frac{z^\eta}{\bar z^{1+\eta}}\,\frac{4z^3}{k_T^2 z^2 + m^2\bar z^2}$$

$$d^{(b)}_{1\,Q/Q} = \frac{\alpha_s C_F}{4\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{\mu}{k_T}\right)^{2\epsilon}\frac{1}{\pi z^{2-2\epsilon}}\,2z^2\bar z\left[\frac{k_T^2 z^2 + m^2(1-4z+z^2) - \epsilon(k_T^2 z^2 + m^2\bar z^2)}{(k_T^2 z^2 + m^2\bar z^2)^2}\right]$$

Regime 2, $\Lambda_{\mathrm{QCD}} \ll k_T \sim m$: Expand in 2D plus distributions



- $\dfrac{z^\eta}{\bar{z}^{1+\eta}} = -\dfrac{\delta(\bar{z})}{\eta} + \mathcal{O}(\eta^0)$ cancels as for massless

$\Rightarrow$ Left to expand: $f(x, z, \epsilon)$ $\qquad x \equiv k_T^2/m^2$

Recall: $f(x, \epsilon) = \left[f(x, 0) + \mathcal{O}(\epsilon)\right]_+ + \delta(x)\, F(\epsilon)$

$$F(\epsilon) = \int_0^1 \mathrm{d}x' \, f(x', \epsilon) = \frac{1}{\epsilon} + \ldots$$

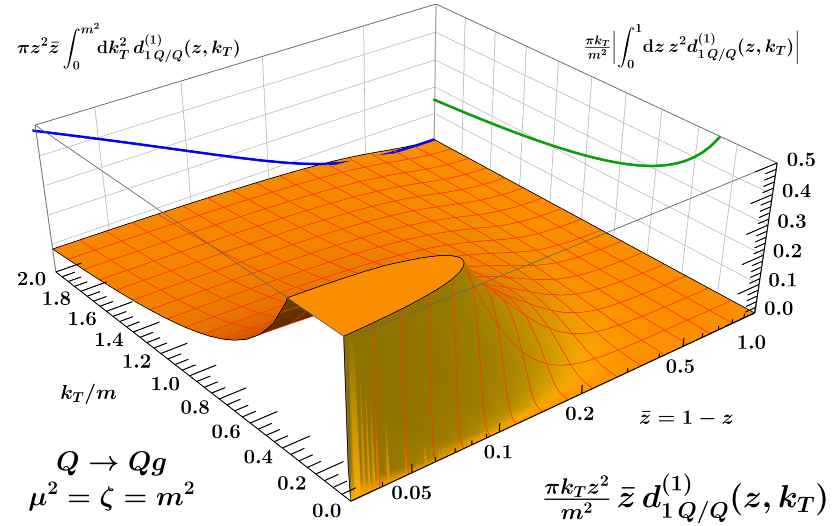
Generalize to 2D

$$f(x, z, \epsilon) = \left[f(x, z, 0) + \mathcal{O}(\epsilon)\right]_{+,+} + \delta(x)\left[F_x(z, \epsilon)\right]_+ + \delta(\bar{z})\left[F_z(x, \epsilon)\right]_+ + \delta(x)\,\delta(\bar{z})\, F_{xz}(\epsilon)$$

$$F_x(z, \epsilon) \equiv \int_0^1 \mathrm{d}z' \, f(x, z', \epsilon) \qquad F_z(x, \epsilon) \equiv \int_0^1 \mathrm{d}x' \, f(x', z, \epsilon) \qquad F_{xz}(\epsilon) \equiv \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z' \, f(x, z', \epsilon)$$

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z \, \left[f(x, z)\right]_{+,+} g(x, z) \equiv \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z \, f(x, z) \left[g(x, z) - g(0, z) - g(x, 0) + g(0, 0)\right]$$

Regime 2, $\Lambda_{\rm QCD} \ll k_T \sim m$: Results in k_T space



$$
\begin{aligned}
& d^{(1)}_{1\,Q/Q}(z, k_T, \mu, \zeta) \\
&= \frac{\alpha_s C_F}{4\pi} \frac{1}{\pi z^2} \Bigg\{ \delta(\bar{z}) \Bigg[2 \ln \frac{\zeta}{\mu^2} \mathcal{L}_0(k_T^2, \mu^2) \\
&\quad + 2 \ln \frac{\mu^2}{m^2} \mathcal{L}_0(k_T^2, m^2) \\
&\quad - \ln^2 \frac{\mu^2}{m^2} \delta(k_T^2) + 3 \ln \frac{\mu^2}{m^2} \delta(k_T^2) \Bigg] \\
&\quad + \frac{1}{m^2} \left[\frac{2 x z^4 (1 + z^2) + 2 z^2 \bar{z}^4}{\bar{z} (x z^2 + \bar{z}^2)^2} \right]_{+,+} + \delta(k_T^2) \left[\frac{2(1+z^2)}{\bar{z}} \ln\!\left(1 + \frac{z^2}{\bar{z}^2}\right) - \frac{4 z^3}{\bar{z}(1 - 2\bar{z}z)} \right]_+ \\
&\quad + \delta(\bar{z}) \frac{1}{m^2} \left[- \frac{2 + 3x + 4x^2 + 3x^3 + \pi\sqrt{x}(2 + 7x + x^2) + x(1 + 7x + 2x^2) \ln x}{x(1+x)^3} \right]_+ \\
&\quad + \delta(\bar{z})\, \delta(k_T^2) \left(5 - \frac{\pi}{2} - \frac{2\pi^2}{3}\right) \Bigg\}
\end{aligned}
$$

$x \equiv k_T^2/m^2$

→ Confirms expected renormalization by $\gamma_\mu(\zeta, \not{m}, \mu)$ and $\gamma_\zeta(k_T, m, \mu)$

[Secondary quark mass effects in the CS kernel show up at NNLO.]

Regime 2, $\Lambda_{\rm QCD} \ll k_T \sim m$: Results in b_T space

Plus distributions domains align with primary variables $\Rightarrow$ easy to take integral transforms!

$$\int_0^\infty \mathrm{d}(k_T^2)\, J_0(b_T k_T)\, \big[f(k_T^2, z)\big]_{+,+} = \Big[\int_0^\infty \mathrm{d}(k_T^2)\big[J_0(b_T k_T) - 1\big] f(k_T^2, z)\Big]_+$$

only z

$$\Downarrow$$

$$d_{1\,Q/Q}^{(1)}(z, b_T, \mu, \zeta) = \frac{\alpha_s C_F}{4\pi}\frac{1}{z^2}\Big\{\delta(1-z)\Big[-2L_b \ln\frac{\zeta}{m^2} + 4\ln^2\frac{\mu}{m} + 6\ln\frac{\mu}{m} - L_y^2 + 4 - \frac{\pi^2}{6}\Big]$$
$$- 4(1+L_y)\,\mathcal{L}_0(1-z) - 8\mathcal{L}_1(1-z) + \tilde{\mathcal{R}}(z, b_T m)\Big\}$$

$$\tilde{\mathcal{R}}(z, y) \equiv y\int_0^\infty \mathrm{d}t\, J_1(ty)\, \mathcal{R}(z, t^2)$$
$$= \frac{4}{\bar z}\Big[1 + L_y + (1+z^2)K_0\Big(\frac{y\bar z}{z}\Big) - y\bar z K_1\Big(\frac{y\bar z}{z}\Big) + 2\ln\bar z\Big] = \mathcal{O}(\bar z^0)$$

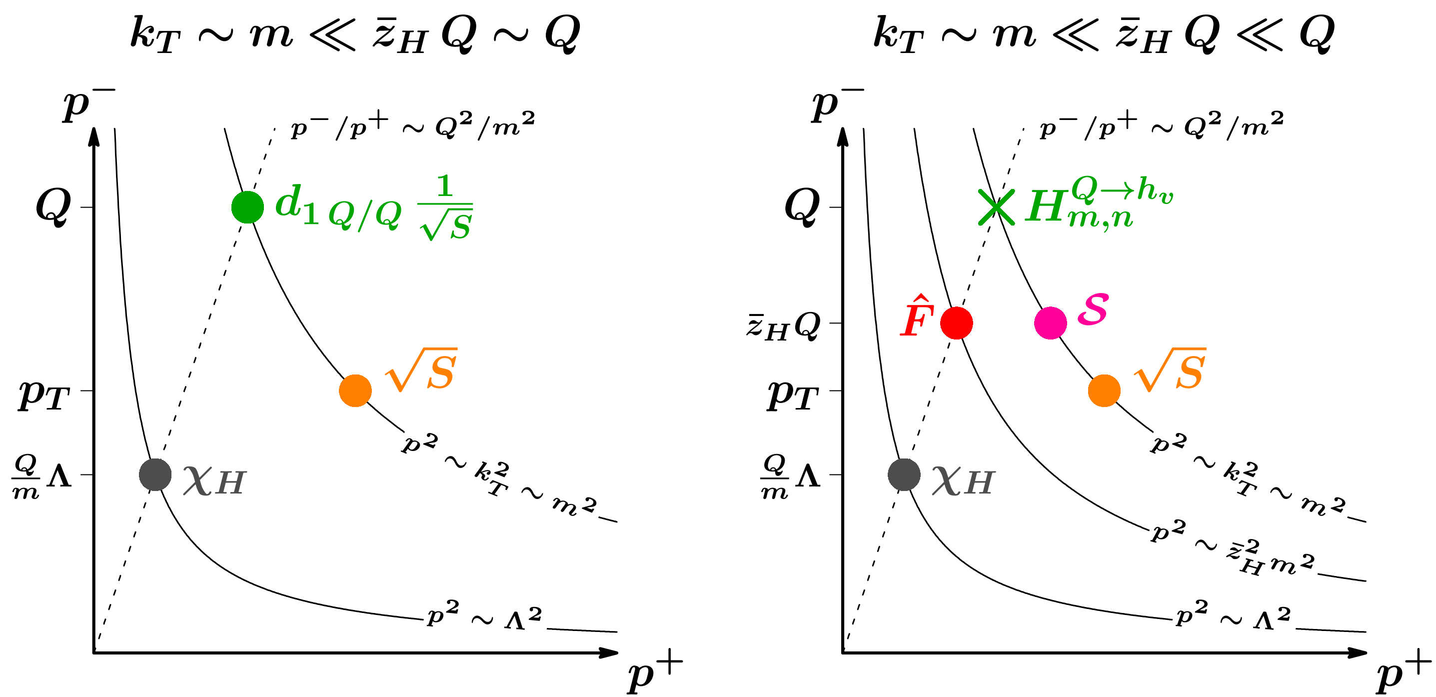
$$L_y \equiv \frac{m^2 b_T^2}{(2e^{-\gamma_E})^2} \qquad L_b \equiv \frac{\mu^2 b_T^2}{(2e^{-\gamma_E})^2}$$

Full agreement with Dai, Kim Leibovich, 2310.19207, who used a completely orthogonal organization of k_T singularities. ✓

Why so simple as $z \to 1$? Why only logarithms of $m\, b_T$?

Joint resummation for massive quark fragmentation

[Joint resummation: Laenen, Sterman, Vogelsang '01; Lustermans, Waalewijn, Zeune '16; Kang, Samanta, Shao, Zeng '22]



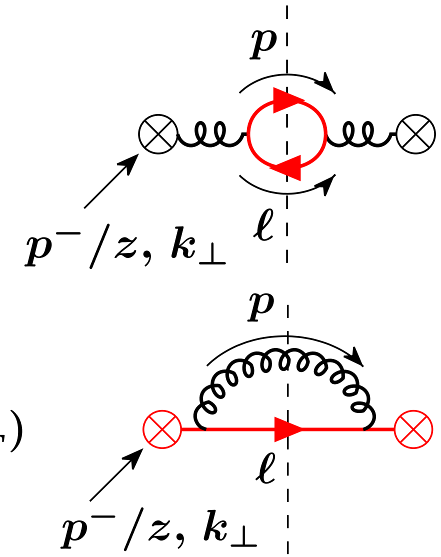
$$d_{1\,Q/Q}\Big(1-\frac{k^-}{\omega}, b_T, \mu, \omega^2\Big) = H_{m,n}^{Q\to h_v}\Big(m, \mu, \frac{\omega}{\nu}\Big)\, \mathcal{S}(k^-, b_T, m, \mu, \nu) \otimes_{k^-} \hat{F}\big(k^-, \bar{n}\cdot v\,\mu\big)$$
$$\times \sqrt{S}(b_T, m, \mu, \nu) + \mathcal{O}\big[(1-z_H)^0\big]$$

✓

[Full two-loop RHS known: Becher, Neubert '06; Pietrulewicz, Samitz, Spiering, Tackmann '17]

Not discussed today

- Explicit check of large-mass limit vs. bHQET at NLO ✓
- Nonvalence channels & small-mass limit ✓



$$d_{1\,Q/i}(z, b_T, \mu, \zeta) = \frac{1}{z^2} \sum_j \mathcal{J}_{j/i}(z, b_T, \mu, \zeta) \otimes_z d_{Q/j}(z, \mu) + \mathcal{O}(m^2 b_T^2)$$

$$\mathcal{J}_{k/i}(z, b_T, m, \mu, \zeta) = \frac{1}{z^2} \sum_j \mathcal{J}_{j/i}(z, b_T, \mu, \zeta) \otimes_z \mathcal{M}_{k/j,T}(z, m, \mu) + \mathcal{O}(m^2 b_T^2)$$

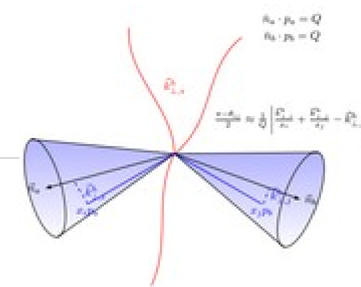
[NLO massless matching coefficients: Echevarria, Idilbi, Scimemi, 1402.0869]

- Full mass dependence in NNLL Energy-Energy-Correlator in the back-to-back limit:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}z_\chi} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(1)}_{ee\to q\bar q}}{\mathrm{d}z_\chi} + 2\mathcal{H}^{(0)}_{ee\to Q\bar Q}\,\frac{\alpha_s C_F}{4\pi}\Big\{8(2\ln\rho - 1)\,\mathcal{L}_0(\bar z_\chi) + 8\Big(2\ln^2\rho + \ln\rho + \frac{2\pi^2}{3} - 2\Big)\,\delta(\bar z_\chi)$$
$$+ 4\rho^2\,\frac{\pi(-3 - 18x + x^2) + \sqrt{x}(9 + x - 9x^2 - x^3) - \sqrt{x}(1 + 24x + 9x^2 + 2x^3)\ln x}{\sqrt{x}(1+x)^4}\Big\}$$
$$+ \mathcal{O}(\alpha_s^2) + \bar z_\chi^0 + \mathcal{O}\Big(\frac{1}{\rho}\Big)$$

Light-quark EEC

NEW



[Cf. Craft, Lee, Meçaj, Moult, 2210.09311 for NLO mass effects in the collinear limit of the EEC.]

[Cf. Lepenik, Mateu, 1912.08211 for analytic SCETI massive event shapes at one loop.]

Overview of today's talk

1. NNLL Resummation of Sudakov Shoulder Logarithms in the **Heavy** Jet Mass Distribution

2. Transverse Momentum-Dependent Fragmentation Functions of **Heavy** Quarks & Hadrons

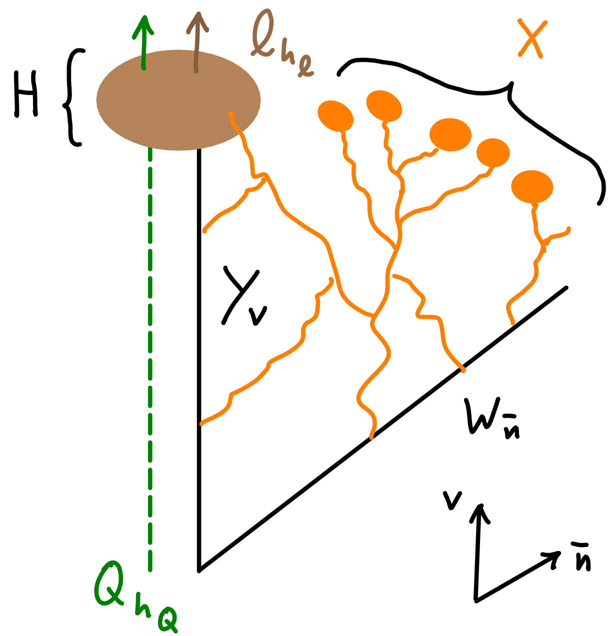
- JHEP 09 (2023) 205, 2305.15461, with Rebecca von Kuk (DESY) and Zhiquan Sun (MIT)
- 2404.08622 with R. von Kuk and Z. Sun
- Work in preparation with R. von Kuk, Z. Sun, and Kyle Lee (MIT)



Polarized hadrons: Back to the light-spin density matrix

$$\rho_{\ell, h_\ell h'_\ell}(b_\perp, z^\mu) \equiv \frac{1}{N_c} \mathrm{Tr} \sum\!\!\!\!\!\!\int_X \langle 0 | W^\dagger(b_\perp)\, Y_v(b_\perp) | s_\ell, h_\ell, f_\ell; X \rangle \langle s_\ell, h'_\ell, f_\ell; X | Y_v^\dagger(0)\, W(0) | 0 \rangle$$

⇒ Also predicts all remaining TMD FFs for *polarized* final-state hadrons!



Useful unit vector choices:

$$x^\mu = \frac{b_\perp^\mu}{b_T}$$

$$z^\mu = v^\mu - \frac{\bar{n}^\mu}{\bar{n} \cdot v}$$

$$y^\mu = x_\nu \epsilon_\perp^{\mu\nu}$$

Building a basis

$$\rho_{\ell,h_\ell h'_\ell}(b_\perp, z^\mu) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\int_X \langle 0|W^\dagger(b_\perp)\,Y_v(b_\perp)|s_\ell, h_\ell, f_\ell; X\rangle\langle s_\ell, h'_\ell, f_\ell; X|Y_v^\dagger(0)\,W(0)|0\rangle$$

- Hermiticity: $\rho_\ell^\dagger(b_\perp, z) = \rho_\ell(-b_\perp, z)$
- Parity: $\rho_\ell(b_\perp, z) = \rho_\ell(-b_\perp, -z)$
- Rotations: $\rho_\ell(R_{\vec\alpha} b_\perp, R_{\vec\alpha} z) = e^{+\mathrm{i}\alpha\cdot\Sigma_\ell}\rho_\ell(b_\perp, z)e^{-\mathrm{i}\alpha\cdot\Sigma_\ell}$

$$x^\mu = \frac{b_\perp^\mu}{b_T}$$

$$z^\mu = v^\mu - \frac{\bar n^\mu}{\bar n\cdot v}$$

$$y^\mu = x_\nu \epsilon_\perp^{\mu\nu}$$

$\Sigma_\ell^{\mu_1\cdots\mu_N} \equiv \left\{\Sigma_\ell^{\mu_1}, \ldots, \Sigma_\ell^{\mu_N}\right\}$ – Gram-Schmidt $\quad N \le 2s_\ell$

NEW

$\Rightarrow$

$$\rho_\ell(b_\perp, z) = \sum_N \sum_n \chi_{1,\ell}^{(N,n)}(b_T)\,\Sigma^{\mu_1,\ldots,\mu_N} \times \begin{cases} x^{\mu_1}\ldots x^{\mu_n} z^{\mu_{n+1}\cdots z^{\mu_N}}, & N \in 2\mathbb{Z} \\ \mathrm{i}x^{\mu_1}\ldots x^{\mu_{n-1}} y^{\mu_n} z^{\mu_{n+1}\cdots z^{\mu_N}}, & N \notin 2\mathbb{Z} \end{cases}$$

So how many structure functions are there?

At light spin s_ℓ, rank $N = 2s_\ell$, add only $2\lfloor s_\ell \rfloor + 1$ structure functions!

Hadron multiplet	s_ℓ	# LP TMD FFs	# $\chi_{1,\ell}^{(N,n)}$'s	# $\chi_{1,\ell}^{(N,0)}$'s ($b_T \to 0$)
Λ_b	0	8	1	1
B, B^*	$1/2$	$2+12$	2	1
Σ_b, Σ_b^*	1	$8+16$	5	2
B_1, B_2^*	$3/2$	$12+20$	8	2

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B_1, B_2^*	$3/2$	$12+20$	8	2

$\Longrightarrow$ Can get complete spin ½ density matrix from unpolarized hadrons!

So how many structure functions are there?

At light spin s_ℓ, rank $N = 2s_\ell$, add only $2\lfloor s_\ell \rfloor + 1$ structure functions!

Hadron multiplet	s_ℓ	# LP TMD FFs	# $\chi_{1,\ell}^{(N,n)}$'s	# $\chi_{1,\ell}^{(N,0)}$'s ($b_T \to 0$)
Λ_b	0	8	1	1
B, B^*	$1/2$	$2+12$	2	1
Σ_b, Σ_b^*	1	$8+16$	5	2
B_1, B_2^*	$3/2$	$12+20$	8	2

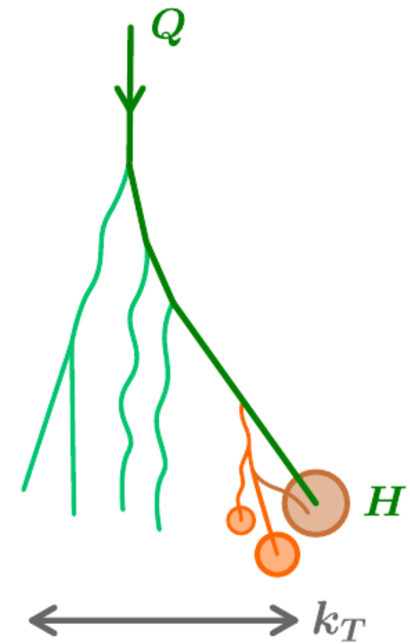
⇒ Can get complete spin ½ density matrix from unpolarized hadrons!

⇒ Nice target e.g. for an LHCb polarized hadron-in-jet measurement!

Stay tuned for final results for TMD FFs!

Summary: Heavy-Quark TMD FFs

- Initiated the study of heavy-quark TMD FFs.
  - Goal: Understand hadronization around a static source.
  - Rich phenomenology in our reach at B factories and EIC.
- Showed complete NLO results for unpolarized heavy-quark TMD FF, with nontrivial momentum-space singularities.
  - Checked vs. large-mass, small-mass, and new joint threshold/transverse momentum resummation regime
  - bHQET fragmentation factors feature rapidity evolution with respect to *dimensionless* boost parameter $\rho = \sqrt{\zeta}/m = \bar{n} \cdot v$.
- Derived powerful spin-symmetry relations for TMD FFs of *polarized* hadrons in terms of a small number of bHQET fragmentation factors.



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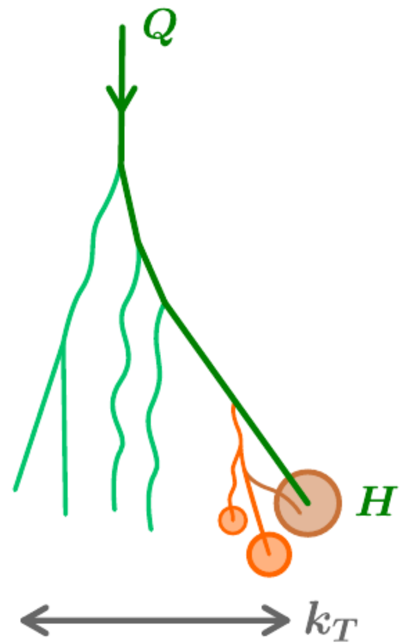

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Thank you for your attention!

Backup

Polarized heavy-quark TMD PDFs

- Heavy quarks have to be perturbatively produced from light partons within nucleon.

  …ignoring intrinsic charm – *TMD* intrinsic charm?

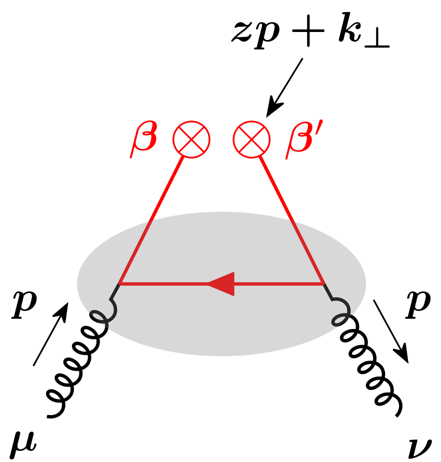
- Encoded in matching relations onto twist-2 collinear PDFs. All-order structure:

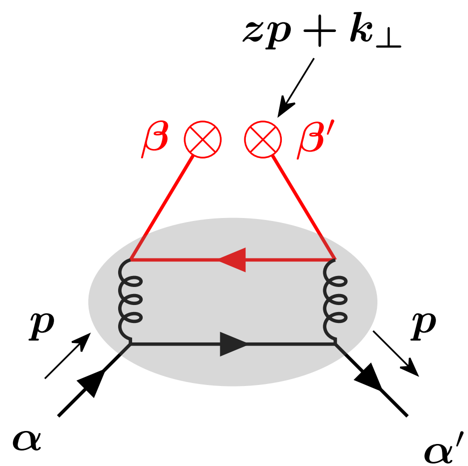
$$f_{1\,Q/N}(x, b_T) = \sum_j \int \frac{dz}{z}\, C_{Q/j}(z, b_T, m)\, f_{j/N}\left(\frac{x}{z}\right) \qquad \sim \quad \alpha_s \qquad \text{“unpolarized”}$$

$$h^{\perp}_{1\,Q/N}(x, b_T, \mu) = \sum_j \int \frac{dz}{z}\, C_{Q_\perp/j}(z, b_T, m)\, f_{j/N}\left(\frac{x}{z}\right) \qquad \sim \quad \alpha_s^2 \qquad \text{“Boer-Mulders”}$$

$$g_{1L\,Q/N}(x, b_T) = \sum_j \int \frac{dz}{z}\, C_{Q_\parallel/j_\parallel}(z, b_T, m)\, g_{j/N}\left(\frac{x}{z}\right) \qquad \sim \quad \alpha_s \qquad \text{“helicity”}$$

$$h^{\perp}_{1L\,Q/N}(x, b_T) = \sum_j \int \frac{dz}{z}\, C_{Q_\perp/j_\parallel}(z, b_T, m)\, g_{j/N}\left(\frac{x}{z}\right) \qquad \sim \quad \alpha_s \qquad \text{“Worm-Gear L”}$$

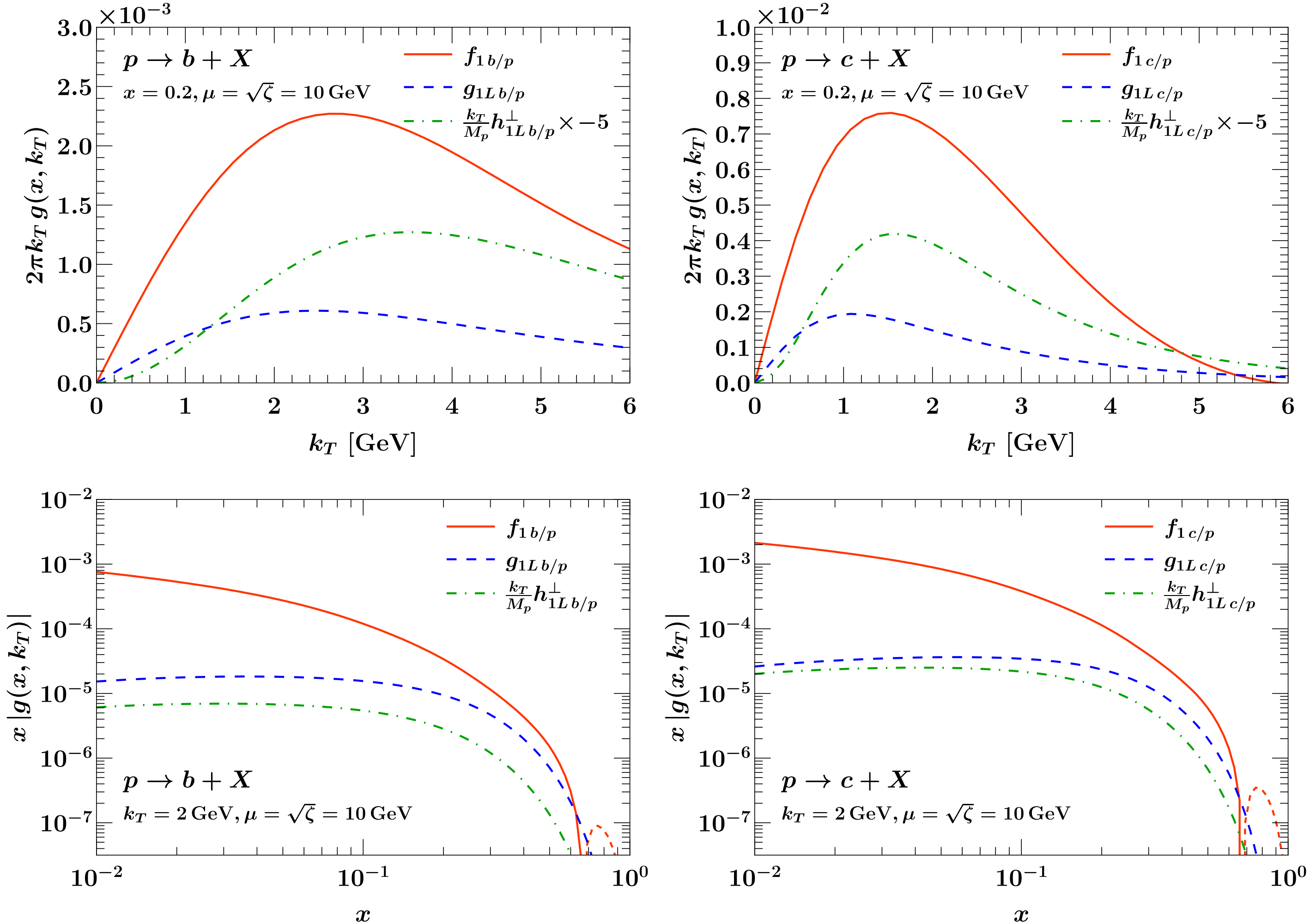




- ~~Transversity~~ by chirality & flavor conservation for light quarks.
- $C_{Q_\perp/j,j_\parallel} \sim m b_T$ are allowed b/c heavy quark mass breaks chirality.
- Calculated all matching coefficients onto gluons at leading order $\sim \alpha_s$.

  [Unpol. agrees with Nadolsky et al. '02]

Polarized heavy-quark TMD PDFs: Numerical results



[Gluon PDFs: NNPDF, 1406.5539, 1706.00428]