Inclusive semileptonic B-meson decays and CKM matrix determinations: challenges and theoretical framework(s).

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Motivation: Standard Model parameters

Standard Model (*input*) parameters

The **Standard Model** (SM) Lagrangian

$$\mathcal{L}_{\text{SM}} = i\left(\overline{\ell}_L \not{D} \ell_L + \overline{e}_R \not{D} e_R + \overline{Q}_L \not{D} Q_L + \overline{u}_R \not{D} u_R + \overline{d}_R \not{D} d_R\right)$$
$$- \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$
$$- \left(\overline{\ell}_L Y^\ell \phi e_R + \overline{Q}_L Y^u \epsilon \phi^* u_R + \overline{Q}_L Y^d \phi d_R + \text{h.c.}\right)$$

with

$$(D_\mu \psi)^{\alpha j} = \left[\delta_{\alpha\beta}\delta_{jk}\left(\partial_\mu + i g' Y_q B_\mu\right) + i g \delta_{\alpha\beta} S^I_{jk} W^I_\mu + i g_s \delta_{jk} T^A_{\alpha\beta} G^A_\mu\right] \psi^{\beta k}$$

depends on a (reduced) set of fundamental (**input**) parameters:

- ⇒ Three **gauge couplings**: g', g and g_s
- ⇒ **Higgs vev** and **Higgs mass**: $v = \sqrt{-\mu^2/\lambda}$ and $m_H = \sqrt{2\lambda}\, v$
- ⇒ Three **lepton masses**: $m^\ell_i = \lambda^\ell_i (v/\sqrt{2})$ with $Y^\ell_{ij} = \lambda^\ell_i \delta_{ij}$
- ⇒ Six **quark masses**: $m^q_i = V^q_L Y^q V^{q\dagger}_R (v/\sqrt{2})$ $(q = u, d)$ with $V^{u,d}_{L,R}$ unitary matrices
- ⇒ Four **CKM matrix parameters**: $V_{\text{CKM}} = V^u_L V^{d\dagger}_L$
- ⇒ + one **strong CP angle**: $\Delta\mathcal{L} \sim \bar{\theta} G^A_{\mu\nu} \tilde{G}^{A\mu\nu}$
- ⇒ + three **neutrino masses** + four (or five) **PMNS matrix elements**

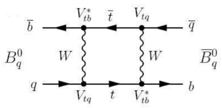
The importance of the CKM matrix

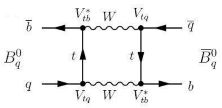
- CKM matrix parametrisation ⇒ $3\Re + 1\Im$ parameters:

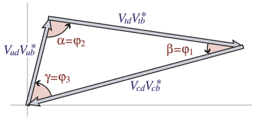
$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- The CKM matrix sets the strength of quark-level transitions
  $J_L \sim V_{ij} W^+_\mu \bar{q}_i \gamma^\mu (1-\gamma_5) q_i$
  ⇒ Hadronic lifetimes
  ⇒ Branching ratios

- Imaginary phase of the CKM matrix ⇒ only CP-violation (CPV) source in the SM
  ⇒ CKM triangles
  ⇒ CP-violation observables (including possible CPV New Physics):
  $K \to \pi\pi,\ K^0 - \bar{K}^0,\ B^0 - \bar{B}^0,\ B^0_s - \bar{B}^0_s$, etc







The structure of the CKM matrix: the *Flavour Puzzle*

[Fuentes-Martín]

- 13 parameters characterise the Yukawa sector of the $\mathcal{L}_{\mathrm{SM}}$: 3 lepton masses, 6 quark masses, 4 CKM parameters
- Strong hierarchical structure between quark masses and CKM parameters
  - ⇒ Why this hierarchy? SM does **not** provide an answer
  - ⇒ **Flavour Puzzle**
- Satisfactory NP must provide a mechanism:
  - ⇒ Hierarchy dynamically generated through local interactions (w/ symmetry breaking)
  - ⇒ Quark generations as excitations of more fundamental constituents
- First, we need **precise determinations** of V_{CKM}

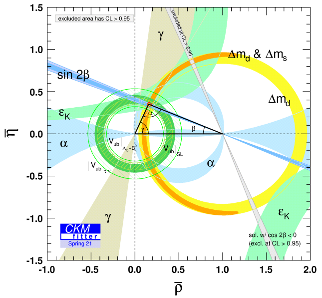
$|V_{ub}|$ and $|V_{cb}|$

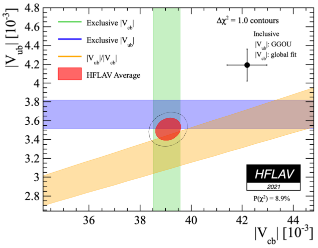
- $|V_{cb}|$ (and $|V_{ub}|$) crucial to CKM unitarity tests:
  - $\Rightarrow$ Both sides of the UT are constrained by $|V_{ub}|$ and $|V_{cb}|$

$$R_t \approx \sin\gamma \sim \left|\frac{V_{td}}{V_{cb}}\right|$$

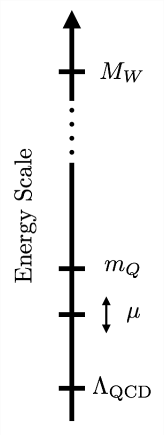
$$R_t \approx \sin\beta \sim \left|\frac{V_{ub}}{V_{cb}}\right|$$

  - $\Rightarrow$ Another important observable for the UT $\epsilon_K \sim |V_{cb}|^4 + \ldots$
- Tensions between inclusive and exclusive B-meson decays:
  - $\Rightarrow$ $\sim 1 - 3\sigma$
  - $\Rightarrow$ Depending on the theo. and exp. approaches
  - $\Rightarrow$ NP explanations not competitive in accommodating the data [Crivellin, Pokorski; Jung, Straub]
- Sets our precision on FCNC:
  - $\Rightarrow$ $\left|V_{tb}V_{ts}^*\right|^2 \simeq |V_{cb}|^2 \left[1 + O(\lambda^2)\right]$
  - $\Rightarrow$ Significantly limits NP searches





Theoretical framework for inclusive semileptonic B-decays



- Heavy modes $E_H \sim M$ cannot be resolved at low energies $E \sim \Lambda$ ($M > \Lambda$). We can **integrate them out** of the theory:

$$\mathcal{Z} = \int D\phi_L D\phi_H \mathrm{e}^{iS(\phi_L,\phi_H)+i\int d^D x\, J_L(x)\phi_L(x)}$$

$$\Rightarrow \mathcal{Z}_\Lambda = \int D\phi_L \mathrm{e}^{iS_\Lambda(\phi_L)+i\int d^D x\, J_L(x)\phi_L(x)}$$

- The **effective action** S_Λ is **nonlocal**
- **Local effective action** from an operator product expansion (OPE) of S_Λ with $1/M$ expansion parameter:

$$\mathcal{L}_{\text{eff}} \sim \frac{1}{M}\sum_i \mathcal{C}_i(\mu)\,\mathcal{O}_i + O\left(\frac{1}{M^2}\right)$$

- Short-distance dynamics incorporated by **matching** the full theory onto the effective theory at a high scale μ_0

$$\mathcal{A}(M_1 \to M_2) = \langle M_2|\,\mathcal{L}_{\text{full}}\,|M_1\rangle$$
$$= \frac{1}{M}\sum_i \mathcal{C}_i(\mu_0)\,\langle M_2|\,\mathcal{O}_i\,|M_1\rangle\,(\mu_0)$$

- Use **renormalisation group techniques** to run down the value of the **effective coefficients** from μ_0 to Λ

Heavy Quark Effective Theory

"The heavy quark effective theory (HQET) is constructed to provide a simplified description of processes where a heavy quark interacts with light degrees of freedom by the exchange of soft gluons." [Neubert]

$\Rightarrow$ m_Q sets the high-scale

$\Rightarrow$ Λ_{QCD} scale of hadronic physics we want to describe

- **Soft dominance**: a heavy quark inside a hadron moves with nearly the hadron's velocity v and is almost on-shell
  - $\Rightarrow$ $p_Q = m_Q v + k$, with $|k| \lll |m_Q v|$
  - $\Rightarrow$ Heavy quark interactions with light degrees of freedom change its momentum by $\Delta k \sim \Lambda_{\text{QCD}}$ $\Rightarrow$ $\Delta v \sim \Lambda_{\text{QCD}}/m_Q \to 0$
- HQET describes properties of heavy hadrons
  - $\Rightarrow$ Heavy quark fields cannot be fully integrated out
  - $\Rightarrow$ Only the "small components" of the heavy quark fields are removed

The HQET Lagrangian

- Let $Q(x)$ be a heavy quark field. We project out its "large component" and "small component" fields:

$$h_v(x) = \mathrm{e}^{im_Q v\cdot x} P_+ \, Q(x),$$
$$H_v(x) = \mathrm{e}^{im_Q v\cdot x} P_- \, Q(x)$$

  with the projectors $P_\pm = (1 \pm \not{v})/2$

- The HQET Lagrangian takes the form of an OPE in $1/m_Q$:

$$\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v \, iv\cdot D_s \, h_v + \frac{\mathcal{C}_{\mathrm{kin}}}{2m_Q} \bar{h}_v \, (iD_{s\perp})^2 \, h_v + \frac{\mathcal{C}_{\mathrm{mag}} \, g_s}{4m_Q} \bar{h}_v \sigma_{\mu\nu} \, G_s^{\mu\nu} \, h_v + O(1/m_Q^2)$$

  with $D_\perp^\mu = D^\mu - v^\mu (v\cdot D)$

  ⇒ Leading term $\mathcal{L}_{\mathrm{HQET}}^{m_Q\to\infty} = \bar{h}_v \, iv\cdot D_s \, h_v$ invariant under $SU(2N_H)$ (N_H number of heavy flavours)

  ⇒ **Heavy quark symmetry**: form factor constraints, Isgur-Wise functions, relations between hadron masses, etc

  ⇒ Wilson coefficients $\mathcal{C}_{\mathrm{kin}} = \mathcal{C}_{\mathrm{mag}} = 1 + O(\alpha_s)$

- Connection between QCD and HQET fields at $O(1/m_b)$ but LO in α_s:

$$Q(x) = \mathrm{e}^{-im_Q v\cdot x} \left(1 + i \frac{\not{D}_\perp}{2m_Q} + \ldots \right) h_v(x)$$

Power corrections

- **Power corrections** of $O(1/m_b)$ parametrised in terms of the operators:

$$\mathcal{O}_{\text{kin}} = -\bar{h}_v \, (iD_{s\perp})^2 \, h_v, \quad \mathcal{O}_{\text{mag}} = \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G_s^{\mu\nu} h_v$$

⇒ $\mathcal{O}_{\text{kin}}$ kinetic energy of the heavy quark inside the hadron (**Fermi motion**)

⇒ $\mathcal{O}_{\text{mag}}$ chromomagnetic interaction of the heavy quark spin with the gluon field

- Forward B-meson matrix elements:

$$\lambda_1 = \frac{1}{2m_B} \langle \bar{B}(v) | \mathcal{O}_{\text{kin}} | \bar{B}(v) \rangle, \quad \lambda_2 = -\frac{1}{6m_B} \langle \bar{B}(v) | \mathcal{O}_{\text{mag}} | \bar{B}(v) \rangle$$

⇒ QCD corrections: $\lambda_2 = \lambda_2(\mu)$, such that $\mathcal{C}_{\text{mag}}(\mu)\lambda_2(\mu)$ is scale independent

⇒ Instead, λ_1 is protected by **reparametrisation invariance** and, hence, scale-independent: $\mathcal{C}_{\text{kin}}(\mu) = 1$ to all orders

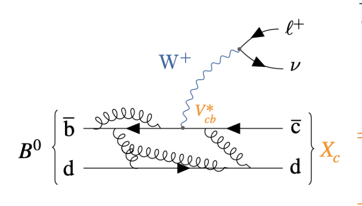
⇒ $|\bar{B}(v)\rangle$ in $\lambda_{1,2}$ eigenstates of $\mathcal{L}_{\text{HQET}}^{m_Q \to \infty}$. In terms of QCD states:

$$\mu_\pi^2 = -\lambda_1 + O(1/m_b), \quad \mu_G^2 = 3\lambda_2 + O(1/m_b)$$

⇒ $\lambda_{1,2}$ (or $\mu^2_{\pi,G}$) constrained from B-meson dynamics and hadron spectroscopy:

$$K_b = -\frac{\lambda_1}{2m_b}, \quad m_{B*}^2 - m_B^2 = 4\lambda_2$$

The Hadronic Tensor and the Local OPE



- Decay distribution ($B \to X_c \ell \nu$):

$$\frac{d\Gamma}{dq^2\, dE_\ell\, dE_\nu} \sim \sum_{X_c} \sum_{\text{pols.}} \frac{|\langle X_c \ell\nu | \mathcal{H}_{\text{eff}} | B\rangle|^2}{2m_B} \delta^4(p_B - p_{X_c} - q)$$

$$= \frac{G_F^2\, |V_{ub}|^2}{8\pi^3} L_{\mu\nu}\, W^{\mu\nu}$$

⇒ **Inclusive decays**: inclusive quantities do not depend on the hadronic final state

⇒ **Optical Theorem**: $d\Gamma \sim B$-meson forward scattering amplitude

$$W^{\mu\nu} \sim \text{Im} \int d^4x\, \mathrm{e}^{-iq\cdot x} \langle \bar{B}|\, T\left\{ \bar{b}(x)\gamma_\mu(1-\gamma_5)c(x)\bar{c}\gamma^\nu(1-\gamma_5)b \right\} |\bar{B}\rangle$$

⇒ **Form factors**: $m_b\, W^{\mu\nu} = -W_1\, g^{\mu\nu} + W_2\, v^\mu v^\nu + iW_3\, \epsilon^{\mu\nu\rho\sigma} v_\rho \hat{q}_\sigma + \ldots$

⇒ **Heavy Quark Expansion** (HQE): OPE in $1/m_b$

$$W_i = W_i^{(0)} + W_i^{(\pi)} \frac{\mu_\pi^2}{m_b^2} + W_i^{(G)} \frac{\mu_G^2}{m_b^2} + W_i^{(D)} \frac{\rho_D^3}{m_b^3} + W_i^{(LS)} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

$$W_i^{(j)} = \sum_n W_i^{(j,n)} \left(\frac{\alpha_s}{\pi}\right)^n$$

⇒ $W_i^{(j)}$ are perturbatively calculable coefficients

⇒ $W_i^{(0)}$ up to $O(\beta_0 \alpha_s^2)$ (b decay), $W_i^{(\pi,G)}$ up to $O(\alpha_s)$, $W_i^{(D,LS)}$ at tree-level,...

Inclusive V_{cb}: the HQE fit

The kinetic scheme

- The double series in Λ/m_b and α_s has a strong dependence on m_b
- If the **pole mass scheme** is used:
  - ⇒ **Renormalon** ambiguity
  - ⇒ Leads to a factorially divergent perturbative series for the width

$$\Gamma_{B\to X_c \ell\nu} \sim \sum_k k! \left(\frac{\beta_0}{2}\frac{\alpha_s}{\pi}\right)^k$$

- **Kinetic scheme** can be used to “resum” the divergent behaviour

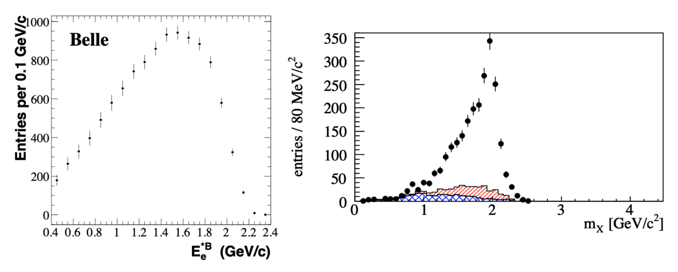
$$m_b^{kin}(\mu) = m_b^{\rm OS} - \left[\bar\Lambda(\mu)\right]_{pert} - \frac{\left[\mu_\pi^2(\mu)\right]_{pert}}{2m_b^{kin}(\mu)}$$

$$\mu_\pi^2(0) = \mu_\pi^2(\mu) - \left[\mu_\pi^2(\mu)\right]_{pert}$$

$$\rho_D^3(0) = \rho_D^3(\mu) - \left[\rho_D^3(\mu)\right]_{pert}$$

  - ⇒ Short-distance, **renormalon free** definition of heavy quark mass and OPE parameters
  - ⇒ A **Wilsonian cutoff** $\mu \sim 1$ GeV is introduced to factor out IR physics
  - ⇒ Beyond 1-loop, kinetic scheme conversion formulae usually realised via Small Velocity (SV) sum rules
  - ⇒ Other renormalon subtracted masses available (PS, MRS, ...). The kinetic scheme is the only tailored on the HQE

[Bigi, Shifman, Uraltsev, Vainshtein; Czarnecki, Melnikov, Uraltsev]



[Belle PRD 75, 032001 (2007); BaBar PRD 81, 032003]

- Inclusive observables M_i are **double series** in Λ/m_b and α_s:

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right)\frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right)\frac{\mu_G^2}{m_b^2} + M_i^{(D,0)}\frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)}\frac{\rho_{LS}^3}{m_b^3} + \dots$$

$\Rightarrow$ **Moments of the kinematic distributions** with exp. cuts, i.e.

$$\langle E_\ell^n \rangle_{E_\ell > E_{\ell,\text{cut}}} = \frac{\int_{E_\ell > E_{\ell,\text{cut}}} dE_\ell \, E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\text{cut}}}}, \quad R^*(E_{\ell,\text{cut}}) = \frac{\int_{E_{\ell,\text{cut}}}^{E_{\ell,max}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int_0^{E_{\ell,max}} dE_\ell \frac{d\Gamma}{dE_\ell}}, \quad \dots$$

$\Rightarrow$ **total rate** $\Gamma_{B\to X_c \ell\nu}$

[BABAR PRL 97, 171803 (2006); Buchmüller, Flächer; Gambino, Schwanda]

- χ^2-fit to the experimental data on the moments

$$\chi^2 = \sum_{ij} (M_{\rm HQE} - M_{\rm exp})_i C^{\rm tot}_{ij} (M_{\rm HQE} - M_{\rm exp})_j$$

with $C^{\rm tot} = C^{\rm HQE} + C^{\rm exp}$

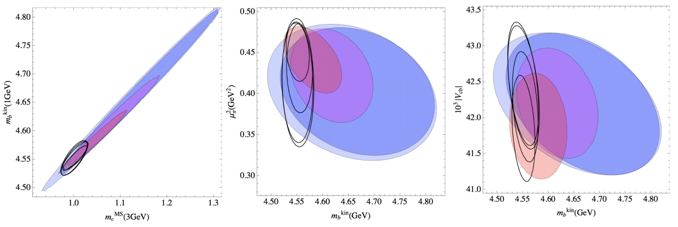
- Total semileptonic branching fraction extracted in the fit by including available data on **partial fractions**

$$\mathrm{BR}_{c\ell\nu\, E_{\ell,\rm cut}} = \mathrm{BR}_{c\ell\nu}\, R^*(E_{\ell,\rm cut})$$

- **Total rate** $\Gamma_{B\to X_c\ell\nu}$ used to extract $|V_{cb}|$

$$\Rightarrow \quad |V_{cb}| = \sqrt{\frac{|V_{cb}|^2\, \mathrm{BR}_{c\ell\nu}}{\tau_B \Gamma_{B\to X_c\ell\nu}}}$$

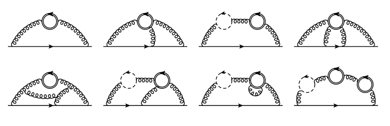
[BABAR PRL 97, 171803 (2006); Buchmüller, Flächer; Gambino, Schwanda]



[Gambino, Schwanda]

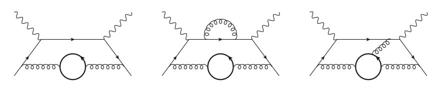
Three-loop calculations

- Newly available $O(\alpha_s^3)$ QCD corrections
  - ⇒ Charm mass effects to the $\overline{\text{MS}}$ – OS scheme relation



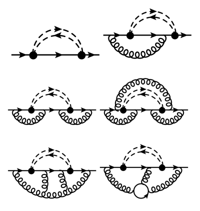
[Fael, Schoenwald, Steinhauser; JHEP 10 (2020) 087]

  - ⇒ *kinetic* – $\overline{\text{MS}}$ scheme relation for heavy quark masses



[Fael, Schoenwald, Steinhauser; PRD 103 (2021) 1, 014005]

  - ⇒ Total decay rate $\Gamma_{B\to X_c\ell\nu}$, including finite charm mass effects



[Fael, Schoenwald, Steinhauser; PRD 104 (2021) 1, 016003]

Heavy quark mass relations: $\overline{\text{MS}}$-*kinetic* schemes at $O(\alpha_s^3)$

- Using $\overline{m}_b(\overline{m}_b) = 4.163$ GeV and $\overline{m}_c(3\,\text{GeV}) = 1.279$ GeV

  ⇒ $\overline{m}_b - m_b^{kin}$ in terms of $\alpha_s^{(3)}$. c quark mass effects only from $\overline{m}_b - m_b^{\text{OS}}$

$$m_b^{kin}(1\,\text{GeV}) = \left[4.163 + 0.248_{\alpha_s} + 0.080_{\alpha_s^2} + 0.030_{\alpha_s^3}\right]\text{GeV}$$
$$= 4.520(15)\,\text{GeV}$$

  [Fael, Schoenwald, Steinhauser]

  ⇒ $\overline{m}_b - m_b^{kin}$ in terms of $\alpha_s^{(4)}$. c quark mass effects from $\overline{m}_b - m_b^{\text{OS}}$ and $m_b^{kin} - m_b^{\text{OS}}$ due to recoupling

$$m_b^{kin}(1\,\text{GeV}) = \left[4.163 + 0.259_{\alpha_s} + 0.078_{\alpha_s^2} + 0.026_{\alpha_s^3}\right]\text{GeV}$$
$$= 4.163(15)\,\text{GeV}$$

  [Fael, Schoenwald, Steinhauser; Bordone, BC, Gambino]

  ⇒ $\overline{m}_b - m_b^{kin}$ in terms of $\alpha_s^{(4)}$ but $n_l = 4$ in $m_b^{kin} - m_b^{\text{OS}}$ (n_l = number of light quarks)

$$m_b^{kin}(1\,\text{GeV}) = \left[4.163 + 0.259_{\alpha_s} + 0.084_{\alpha_s^2} + 0.041_{\alpha_s^3}\right]\text{GeV}$$
$$= 4.547(20)\,\text{GeV}$$

  [Fael, Schoenwald, Steinhauser]

  ⇒ Infinitely heavy charm mass: $\alpha_s^{(3)}$ and $n_l = 3$. No charm quark mass effects

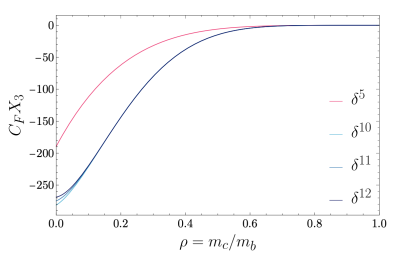
$$m_b^{kin}(1\,\text{GeV}) = \left[4.163 + 0.248_{\alpha_s} + 0.081_{\alpha_s^2} + 0.030_{\alpha_s^3}\right]\text{GeV}$$
$$= 4.521(15)\,\text{GeV}$$

  [Fael, Schoenwald, Steinhauser]

  ⇒ $O(\alpha_s^3)$ correction leads to a **50% reduction** of the error on m_b^{kin}

Total width at $O(\alpha_s^3)$

- Total width $\Gamma_{B\to X_c\ell\nu}$ at three loops
  - ⇒ Finite charm mass effects via an expansion in $\delta = 1 - \rho = 1 - \frac{m_c^{OS}}{m_b^{OS}}$ [Fael, Schoenwald, Steinhauser]
  - ⇒ Two loop correction converges well in this expansion down to $\rho \to 0$ [Czarnecki, Dowling, Piclum]



$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 \left[X_0 + C_F \sum_{n\geq 1} \frac{\alpha_s^{(5)}}{\pi} X_n \right]$$

with $\Gamma_0 = G_F^2 m_b^2 \left| V_{cb} \right|^2 / 192\pi^3$

⇒ **Kinetic sc.**: $\mu = 1$ GeV, $\alpha_s^{(4)}(\mu_b)$, $\mu_b = m_b^{kin}$ and $\overline{m}_c(3\,\text{GeV}) = 0.988$ GeV:

$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 f(\rho) \left[0.9255 - 0.1162_{\alpha_s} - 0.0350_{\alpha_s^2} - 0.0097_{\alpha_s^3} \right]$$

[Fael, Schoenwald, Steinhauser; Bordone, BC, Gambino]

⇒ **Kinetic sc.**: $\mu = 1$ GeV, $\alpha_s^{(4)}(\mu_b)$, $\mu_b = m_b^{kin}$ and $\overline{m}_c(2\,\text{GeV}) = 1.091$ GeV:

$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 f(\rho) \left[0.9258 - 0.0878_{\alpha_s} - 0.0179_{\alpha_s^2} - 0.0005_{\alpha_s^3} \right]$$

[Fael, Schoenwald, Steinhauser; Bordone, BC, Gambino]

⇒ Better convergence with $\mu_c = 2\,\text{GeV}$ both at $O(\alpha_s^2)$ and $O(\alpha_s^3)$

[Gambino, Schwanda; Fael, Schoenwald, Steinhauser; Bordone, BC, Gambino]

Theory inputs HQE default fit

- Quark masses: FLAG 2019 averages with $N_f = 2+1+1$ for $\overline{m}_b(\overline{m}_b) = 4.198(12)$ GeV and $\overline{m}_c(3\,\text{GeV}) = 0.988(7)$ GeV

  ⇒ b quark mass: kinetic scheme with $\alpha_s^{(4)}(\mu_b)$ (finite charm quark mass effects due to recoupling) and a Wilsonian cutoff $\mu = 1\,\text{GeV}$

  $$m_b^{kin}(1\,\text{GeV}) = \left[4.198 + 0.261_{\alpha_s} + 0.079_{\alpha_s^2} + 0.027_{\alpha_s^3}\right]\text{GeV} = 4.565(19)\,\text{GeV}$$

  [Bordone, BC, Gambino]

  ⇒ c quark mass: $\overline{\text{MS}}$ scheme (most precise determination of m_c). Avoid scales below ~ 2 GeV

  $$\overline{m}_c(2\,\text{GeV}) = 1.198(12)\,\text{GeV}$$

  [Bordone, BC, Gambino]

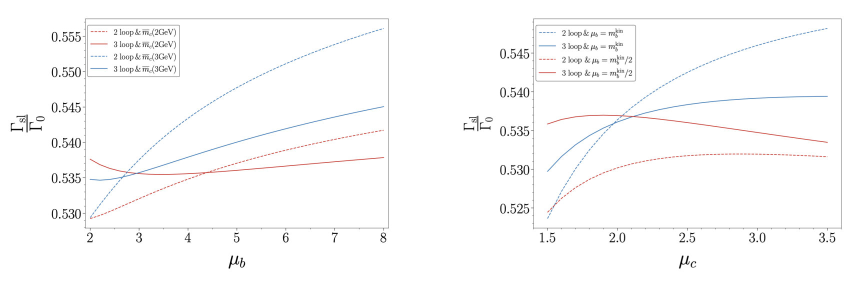
- Total rate: **kinetic scheme** with $\mu = 1$ GeV, $\mu_b = m_b^{kin}/2$ and $\mu_c = 2\,\text{GeV}$

  $$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 f(\rho)\left[0.9255 - 0.1140_{\alpha_s} - 0.0011_{\alpha_s^2} - 0.0103_{\alpha_s^3}\right]$$

  ⇒ + power corrections up to $O(1/m_b^3)$

  [Bordone, BC, Gambino]

Theory uncertainties



- Estimation of theoretical uncertainties is crucial for a reliable HQE fit and the extraction of V_{cb}
  - $\Rightarrow$ A fit without theoretical errors is a very poor fit $\chi^2/dof \sim 2$

[Bordone, BC, Gambino]

- **Perturbative** uncertainty
  - $\Rightarrow$ Residual scale dependence of $\Gamma_{B\to X_c \ell\nu}$ at $O(\alpha_s^3)$
  - $\Rightarrow$ Max. spread within variations of μ_b, μ_c and μ
  - $\Rightarrow$ Conservative estimate of 0.7%

[Bordone, BC, Gambino]

Theory uncertainties

- Uncertainty due to poorly known higher **power corrections**
  - ⇒ $O(\alpha_s \rho_D^3/m_b^3)$ correction is known but tiny
  - ⇒ $O(1/m_b^4)$ and $O(1/m_b^3 m_c^2)$ power corrections: estimated as loose constraints on the OPE parameters via the **Lowest Lying State Saturation Approx** (LLSA)

$$\langle B| \mathcal{O}_1 \mathcal{O}_2 |B\rangle = \sum_n \langle B| \mathcal{O}_1 |n\rangle \langle n| \mathcal{O}_2 |B\rangle$$

[Mannel,Turczyk,Uraltsev; Heinonen, Mannel]

where $n = B$, B^* (ground-state HQET multiplet) or excited states with suitable parity

  - ⇒ $O(1/m_b^4)$ and $O(1/m_b^3 m_c^2)$ errors affect the fit to the moments: extra error budget
  - ⇒ Additive uncertainty due to quark-hadron duality violation
- **Final uncertainty** estimate
  - ⇒ 1.2% overall uncertainty on $\Gamma_{B\to X_c \ell\nu}$
  - ⇒ $\sim 50\%$ reduction w.r.t. $O(\alpha_s^2)$

[Bordone, BC, Gambino]

HQE fit: update 2021

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_g^2(m_b)$	ρ_{LS}^3	$\text{BR}_{B\to X_c\ell\nu}$	$10^3 \times \lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

[Bordone, BC, Gambino]

⇒ Exp. data: $\langle E_\ell^n \rangle$ and $\langle m_X^{2n} \rangle$ moments + partial fractions from Belle & BaBar
[Gambino, Schwanda]

⇒ Goodness-of-fit: $\chi^2/dof = 0.47$

⇒ Without b-quark mass constraint, we obtain $\overline{m}_b(\overline{m}_b) = 4.210(22)$ GeV (compatible with FLAG)

⇒ Determination of HQE params. with $\sim 15-20\%$ error

⇒ Robust determination of $|V_{cb}|$: 1.2% error and restricted sensitivity to theory errors and inputs

$$10^3 \times |V_{cb}| = 42.16(32)_{exp}(30)_{th}(25)_\Gamma$$

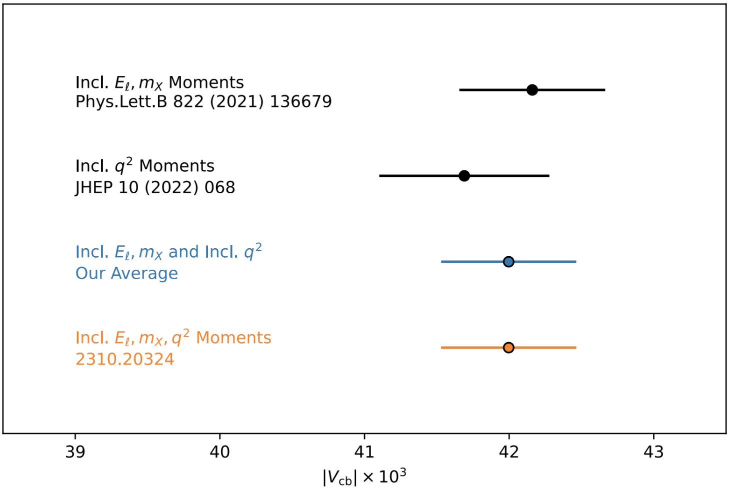
⇒ Updates 2014 result: $10^3 \times |V_{cb}| = 42.20(78)$ (same exp. data)
[Alberti, Gambino, Healey, Nandi]

⇒ Compatible with fit including higher power corrections through LLSA

$$10^3 \times |V_{cb}| = 42.00(53)$$

[Gambino, Healey, Turczyk; Bordone, BC, Gambino]

V_{cb} fits including q^2 moments (2022 & 2023)



[Fael, Prim, Vos]

Inclusive V_{ub}: challenges and the GGOU/NNVub approach

$B \to X_u \ell \nu$: theory status

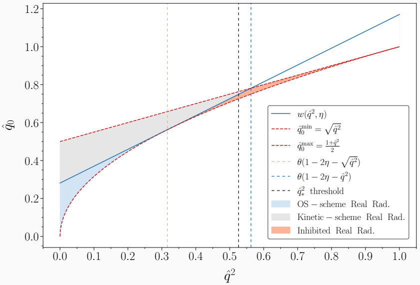
- $W_i(q_0, q^2, \mu)$ up to $O(\beta_0 \alpha_s^2)$ and $O(1/m_b^3)$

$$W_i^{pert}(q_0, q^2, \mu) = \left[W_i^{\rm tree}(\hat{q}^2) + C_F \frac{\alpha_s(m_b)}{\pi} V_i^{(1)}(\hat{q}^2, \eta) + C_F \frac{\alpha_s^2 \beta_0}{\pi^2} V_i^{(2)}(\hat{q}^2, \eta) \right] \delta(1 + \hat{q}^2 - 2\hat{q}_0)$$
$$+ C_F \frac{\alpha_s(m_b)}{\pi} \left[R_i^{(1)}(\hat{q}_0, \hat{q}^2, \eta) + \frac{\alpha_s \beta_0}{\pi} R_i^{(2)}(\hat{q}_0, \hat{q}^2, \eta) \right] \theta(1 + \hat{q}^2 - 2\hat{q}_0)$$
$$+ C_F \frac{\alpha_s(m_b)}{\pi} \left[B_i^{(1)}(\hat{q}^2, \eta) + \frac{\alpha_s \beta_0}{\pi} B_i^{(2)}(\hat{q}^2, \eta) \right] \delta'(1 + \hat{q}^2 - 2\hat{q}_0)$$

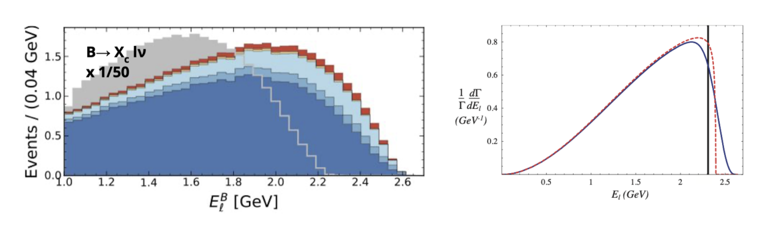
$$W_i^{pow}(q_0, q^2, \mu) = \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)} + \frac{\rho_D^3}{3m_b^3} W_i^{(D,0)} + \frac{\rho_{LS}^3}{3m_b^3} W_i^{(LS,0)}$$

- **Kinetic scheme**: OPE with a hard cutoff
  - ⇒ Induces modifications on the structure functions
  - ⇒ Real gluon emission spectrum

$$R_i^{(k)}(\hat{q}_0, \hat{q}^2, \eta) =$$
$$R_i^{{\rm OS},(k)}(\hat{q}_0, \hat{q}^2)\theta(w - \hat{q}_0)\, \theta\left(1 - 2\eta - \sqrt{\hat{q}^2}\right)$$
$$+ R_i^{cut,(k)}(\hat{q}_0, \hat{q}^2, \eta)\theta(\hat{q}_0 - w)$$



[Giordano, Gambino, Ossola, Uraltsev]



[Belle PRD 104 (2021) 1; Luke]

- $B \to X_c \ell \nu$ very CKM favoured w.r.t. $B \to X_u \ell \nu$ ($|V_{cb}/V_{ub}| \sim 10$)
  - $\Rightarrow$ Large charm backgrounds
  - $\Rightarrow$ $B \to X_u \ell \nu$ signal difficult to measure
  - $\Rightarrow$ Need to impose kinematic cuts:

$$\frac{m_b}{2} \sim E_\ell^{\max} \sim E_\ell > \frac{m_B^2 - m_D^2}{2m_B} \quad \text{and} \quad 0 \sim m_X^2 < m_D^2$$

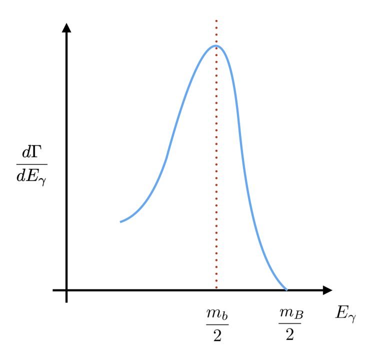
- Convergence of the local OPE is destroyed within the region allowed by the kinematic cuts
  - $\Rightarrow$ $(m_b v + k - q)^2 = (m_b v - q)^2 + O(m_b \Lambda_{\mathrm{QCD}}) + O(\Lambda_{\mathrm{QCD}}^2) \not\approx (m_b v - q)^2$ since $(m_b v - q)^2 \sim 0$
  - $\Rightarrow$ Region very sensitive to non-perturbative effects of $O(k) \sim O(\Lambda_{\mathrm{QCD}})$

[Neubert; Luke]

Shape function(s): $B \to X_s\gamma$

- The residual $\sim \Lambda_{\mathrm{QCD}}$ momentum of the b-quark in the B-meson cannot be encoded into the non-perturbative matrix elements of the OPE. Needs to be resumed into a non-perturbative **Shape Function**

[Neubert; Bigi, Shifman, Uraltsev, Vainshtein]



- Partonic decay (tree level): $b(p) \to s(p')\gamma(q)$ with $p = m_b v$
  - ⇒ Infinitely narrow photon line at $E_\gamma^{(0)} = \frac{m_b}{2}$
- Hadronic level: $B(p_B) \to X_s(p_{X_s})\gamma(q)$
  - ⇒ Hadronic kinematic boundary at $E_\gamma^{\max} = \frac{m_B}{2}$
  - ⇒ Partonic vs hadronic dynamics: $E_\gamma^{\max} - E_\gamma^{(0)} = \frac{m_B - m_b}{2} \sim \frac{\Lambda_{\mathrm{QCD}}}{2}$
  - ⇒ Partonic dynamics: $b(p) \to s(p')\gamma(q)$ with $p = m_b v + k$ and $k \sim \Lambda_{\mathrm{QCD}}$

- Decay distribution $d\Gamma/dE_\gamma$ is smeared due to purely non-perturbative effects:

$$\frac{d\Gamma}{dE_\gamma} = \int dk_+ \, F(k_+) \frac{d\Gamma^{\mathrm{pert}}}{dE_\gamma}\left(E_\gamma - \frac{k_+}{2}\right)$$

[Bigi, Shifman, Uraltsev, Vainshtein]

Shape function(s) in $B \to X_u \ell \nu$

- **Factorisation formula** for the W_i structure functions:

$$W_i(q_0, q^2) = \int dk_+ F(k_+) \, W_i^{pert} \left[q_0 - \frac{k_+}{2} \left(1 - \frac{q^2}{m_b M_B} \right), q^2 \right] + O(1/m_b)$$

  ⇒ The Shape Function (SF) is the parton distribution function for the b quark in the B meson

  ⇒ In the $m_b \to \infty$ limit, the Shape Function (SF) $F(k_+)$ is **universal**, i.e. shared by inclusive radiative and semileptonic decays

  ⇒ At finite m_b, non-universal subleading SFs emerge

  ⇒ SFs modelling is one of the **dominant uncertainties** in $|V_{ub}|$ determinations

- Different approaches for the estimation of the shape functions

  ⇒ OPE constrains on the SF moments + parametrisation with / without resummation (**GGOU**, **BLNP**)

  ⇒ Theory prediction based on resummed pQCD (**DGE**, **ADFR**)

  ⇒ Global fit radiative + $b \to u \ell \nu$ (**SIMBA**)

- Subleading $O(1/m_b)$ corrections absorbed into non-universal q^2-dependent SFs:

$$W_i(q_0, q^2) = \int dk_+ F_i(k_+, q^2)\, W_i^{pert}\left[q_0 - \frac{k_+}{2}\left(1 - \frac{q^2}{m_b M_B}\right), q^2\right]$$

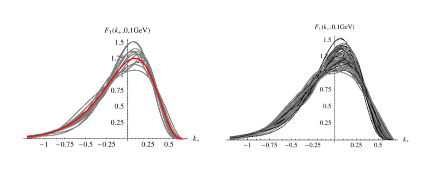
- SFs can be constrained by matching with the q_0-moments of the OPE for the structure functions:

$$\int dk_+ k_+^n F_i(k_+, q^2) = \left(\frac{2}{\Delta}\right)^n \left[\delta_{n0} + \frac{J_i^{(n,0)}}{I_i^{(0,0)}}\right]$$

⇒ Matching consistency implies W_i up to $O(1/m_b^3)$ and W_i^{pert} at tree-level in the convolution formula

⇒ $I_i^{(n,0)}$ and $J_i^{(n,0)}$ are the nth central q_0-moments of $W_i^{\rm tree}$ and $W_i^{\rm pow}$ (up to $O(1/m_b^3)$), respectively

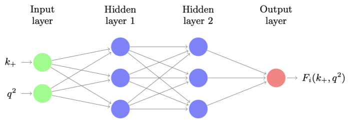
⇒ Different parametric families for $F_i(k_+, q^2)$ are used to estimate the theoretical errors

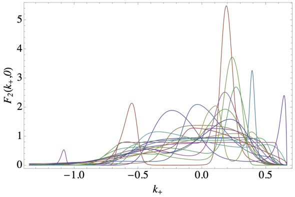


[Gambino, Giordano, Ossola, Uraltsev]

NNVub: GGOU + Neural Networks (update)

- Employ artificial **Neural Networks** as unbiased interpolants for the SFs, instead of relying on different particular parametrisations





[Gambino, Healey, Mondino]

- In GGOU $W_i(q_0, q^2)$ known through $O(\beta_0 \alpha_s^2)$ and $O(1/m_b^3)$

  $\Rightarrow$ + $O(\alpha_s \Lambda_{\rm QCD}^2/m_b^2)$ corrections (done!) [BC, Gambino, Nandi]

  $\Rightarrow$ + $O(\alpha_s^2)$ corrections (**in progress**) [Broggio, BC, Ferroglia, Gambino]

- Knowing the $O(\alpha_s \Lambda_{\rm QCD}^2/m_b^2)$ corrections to $W_i(q_0, q^2)$ allows to constrain the SFs moments up to $O(\alpha_s)$

$$\int dk_+ k_+^n F_i(k_+, q^2) = \left(\frac{2}{\Delta}\right)^n \left[\delta_{n0} + \frac{J_i^{(n,0)}}{I_i^{(0,0)}} + O(\alpha_s)\right]$$

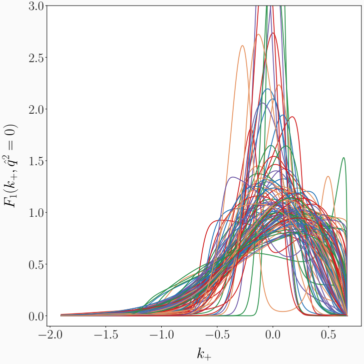
[BC, Gambino, Nandi]

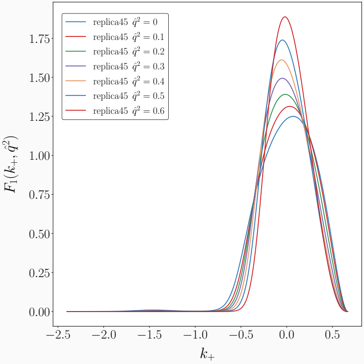
Update NNVub: first preliminary results

- Heuristic perspective: $F_i(k_+, q^2)$ must have a peak at $k_+ \sim (-\bar{\Lambda}, 0)$ and $F_i(k_+, q^2) \to 0$ rapidly as $k_+ \to -\infty$,
  $\Rightarrow$ SFs parametrisation

$$F_i(k_+, q^2) = (c_{i0} + c_{i1}\, q^2)\mathrm{e}^{-\left[(c_{i2}+c_{i3}\, q^2)(\bar{\Lambda}-k_+)\right]^2}(\bar{\Lambda} - k_+)^{(c_{i4}+c_{i5}\, q^2)} N_i(k_+, q^2)$$

  with $N_i(k_+, q^2)$ a **neural network**
- First results training **only** with the moments (not including $O(\alpha_s)$ corrections yet!)

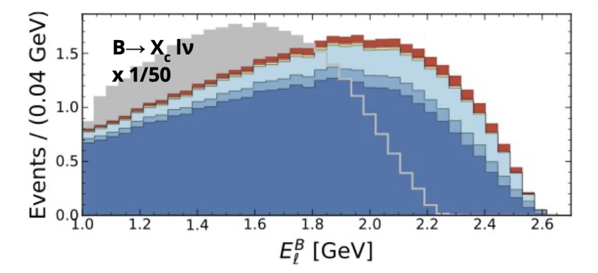




Update NNVub: computational challenge

- Available data on $B \to X_u \ell\nu$ kinematic distributions
  - ⇒ Training with data is computationally challenging: solve triple SF convolution integrals at each epoch of the training

$$\left\langle \frac{d\Gamma_{B\to X_u \ell\nu}}{dE_\ell} \right\rangle^{\rm bin}_{E_\ell > E_{\ell,\rm cut}} \sim \int_{\rm bin} dE_\ell \int d\hat{q}^2 \int_{\frac{2\hat{q}_0 - 1 - \hat{q}^2}{\Delta}}^{\hat{\Lambda}} d\kappa \; F_i(\kappa, \hat{q}^2, \mu) \left[R_i^{(1)}\left(\hat{q}_0 - \frac{\Delta}{2}\kappa, \hat{q}^2\right) + \frac{\alpha_s \beta_0}{\pi} R_i^{(2)}\left(\hat{q}_0 - \frac{\Delta}{2}\kappa, \hat{q}^2\right) \right]$$



[Belle PRD 104 (2021) 1]

Final remarks

Final remarks

- New $O(\alpha_s^3)$ calculations instrumental to further constrain theoretical errors. Theoretical uncertainties well under control
  - ⇒ V_{cb} from inclusive decays at $\sim 1\%$ accuracy
  - ⇒ Higher power corrections are becoming available, i.e. $O(\alpha_s \rho_D^3/m_b^3)$ for $\Gamma_{B\to X_c \ell\nu}$. Computations under way for the moments [Mannel, Pivovarov; Nandi, Gambino]
- New frameworks for the inclusive determination of V_{cb}
  - ⇒ *Reparametrisation invariance* (RI) to include higher power corrections but control proliferation of OPE parameters in the fit. Price to pay: only RI observables can be used. Larger errors on the extraction of V_{cb} [Fael, Mannel, Vos]
  - ⇒ Lattice computations to supplement the HQE fit [Gambino, Hashimoto]
  - ⇒ Measurements of new observables (i.e. FB asymmetry) already feasible in the B factories
- $\boldsymbol{V_{cb}}$ **puzzle remains**: inclusive vs exclusive tension not solved ($\sim 4\sigma$ with the latest FNAL/MILC form factors for $B \to D^* \ell\nu$)!

Final remarks

- Long standing discrepancy in the determination of $|V_{ub}|$ between $1-3\sigma$
  - ⇒ V_{ub} is important to understand the structure of CP-violation and to obtain high precision calculations for NP searches
  - ⇒ Work needs to be done to better understand the non-perturbative dynamics in $B \to X_u \ell \nu$, in particular the SFs
- Newly available $O(\alpha_s \Lambda^2_{\mathrm{QCD}}/m_b^2)$ corrections to $B \to X_u \ell \nu$:
  - ⇒ This will allow us to include $O(\alpha_s)$ contributions to constrain the moments of the SFs
- In progress: update of the NNVub framework
  - ⇒ Successful training of the Neural Network representation of the SFs with the q_0-moments
  - ⇒ Implementation of the kinematic distributions in the training: a computational challenge (work in progress as we speak!)
  - ⇒ Goal: training with both q_0-moments and data on the kinematic distributions (measured by Belle). Extraction of V_{ub} + $b \to u \ell \nu$-based HQE fit

Thank you!

Back-up slides

Tensions in V_{ub}: exclusive vs inclusive

- Inclusive ($B \to X_u \ell \nu$):

$$\Rightarrow |V_{ub}| = \sqrt{\frac{|V_{ub}|^2 \mathrm{BR}_{u\ell\nu}}{\tau_B \Gamma_{B\to X_u \ell\nu}}}.$$

$$|V_{ub}^{\mathrm{incl.}}| = \left(4.32 \pm 0.12^{+0.12}_{-0.13}\right) \times 10^{-3}.$$

[HFLAV 2019]

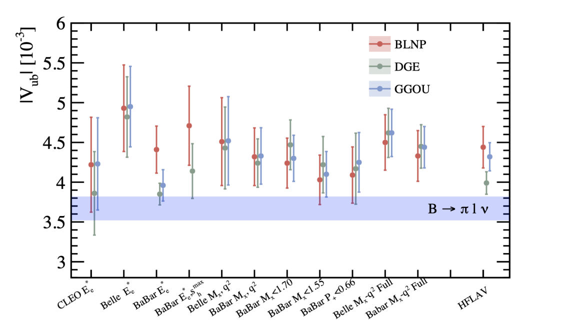
- Exclusive ($B \to \pi \ell^+ \nu$):

$$\Rightarrow \frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2.$$

$\Rightarrow$ Form factor $f_+(q^2)$ from Lattice and / or LCSRs.

$$|V_{ub}^{\mathrm{excl.}}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}.$$

[HFLAV 2019]



[Gambino, Kronfeld, Rotondo, Schwanda, Bernlochner, et al]

$\Rightarrow$ Tensions (inclusive vs exclusive) $\sim 1 - 3\sigma$ (depending on the theo. and exp. approaches).

Perturbative power suppressed effects in $B \to X_u \ell \nu$

$W_i^{(1)}$, $W_i^{(\pi,1)}$ and $W_i^{(G,1)}$ with massive charm

- Explicit calculation at $O(\alpha_s)$ available for $B \to X_c \ell \nu$.

  ⇒ Schematic structure:

$$W_i^{(1)} = w_i^{(0)} \left\{ S_i \, \delta(\hat{u}) - 2\,(1 - E_0 I_1) \left[\frac{1}{\hat{u}}\right]_+ + \frac{\theta(\hat{u})}{(\rho + \hat{u})} \right\} + R_i \, \theta(\hat{u}),$$

$$R_i = \frac{r_i^{(1)} \hat{u} + r_i^{(2)} \rho}{(\hat{u} + \rho)^2} + \frac{s_i}{\hat{u} + \rho} + t_i.$$

[Aquila, Gambino, Ridolfi, Uraltsev]

- Explicit calculation at $O(\alpha_s \Lambda_{\rm QCD}^2/m_b^2)$ available for $B \to X_c \ell \nu$.

  ⇒ Schematic structure:

$$W_i^{(\pi,1)} = w_i^{(0)} \frac{\lambda_0}{3} \Big(S_i + 3(1 - E_0 I_{1,0}) \Big) \delta''(\hat{u}) + b_i \, \delta'(\hat{u}) + c_i \, \delta(\hat{u})$$

$$+ \; d_i \left[\frac{1}{\hat{u}^3}\right]_+ + e_i \left[\frac{1}{\hat{u}^2}\right]_+ + f_i \left[\frac{1}{\hat{u}}\right]_+ + R_i^{(\pi)} \, \theta(\hat{u}),$$

$$R_i^{(\pi)} = \frac{p_i^{(1)} \hat{u} + p_i^{(2)} \rho}{(\hat{u} + \rho)^4} + \frac{q_i}{(\hat{u} + \rho)^3} + \frac{r_i}{(\hat{u} + \rho)^2} + \frac{s_i}{\hat{u} + \rho} + t_i.$$

[Alberti, Ewerth, Gambino, Nandi; Alberti, Gambino, Nandi]

- The generalized plus distributions are defined by

$$\int d\hat{u} \left[\frac{\ln^n \hat{u}}{\hat{u}^m}\right]_+ f(\hat{u}) = \int_0^1 d\hat{u} \, \frac{\ln^n \hat{u}}{\hat{u}^m} \left[f(\hat{u}) - \sum_{p=0}^{m-1} \frac{\hat{u}^p}{p!} f^{(p)}(0) \right]$$

[Alberti, Ewerth, Gambino, Nandi; Alberti, Gambino, Nandi]

Cancellation of collinear divergences in the massless limit

- Compute the limit $\rho \to 0$ of the structures $W_i^{(1)}$, $W_i^{(\pi,1)}$ and $W_i^{(G,1)}$.
- **Collinear divergences** emerge under phase-space integration, together with $\rho \to 0$.
  - ⇒ $\hat{u}$ allowed range: $0 \le \hat{u} \le \hat{u}_+ = (1 - \sqrt{\hat{q}^2})^2 - \rho$.
  - ⇒ $\lim_{\rho\to 0} \int_0^1 d\hat{u} \, \frac{1}{(\hat{u}+\rho)^n} \to \infty$.
- Make collinear divergences apparent ($\sim \ln\rho$, $\sim \ln^2\rho$, $\sim 1/\rho$, $\sim 1/\rho^2$, etc.) by introducing the appropriate distributions.

$$\frac{1}{(\hat{u}+\rho)^3} \to \left[\frac{1}{\hat{u}^3}\right]_+ + \frac{1}{2}\left(\frac{1}{\rho^2} - 1\right)\delta(\hat{u}) - \left(\frac{1}{2\rho} - 1\right)\delta'(\hat{u}) - \frac{1}{4}(3 + 2\ln\rho)\,\delta''(\hat{u})$$

⇒ Collinear divergences in more complicated structures of the type $I_1 \left[\frac{1}{\hat{u}^n}\right]_+$ are more involved but can be extracted.

$$I_1 \left[\frac{1}{\hat{u}}\right]_+ \to -\frac{1}{6\hat{w}}\left(\pi^2 + 3\ln^2\rho\right)\delta(\hat{u}) + \frac{2\ln\hat{w}}{\hat{w}}\left[\frac{1}{\hat{u}}\right]_+ + \frac{1}{\hat{w}}\left[\frac{\ln\hat{u}}{\hat{u}}\right]_+ + \frac{1}{\hat{u}\hat{w}}\left(\ln\frac{\hat{u}}{\hat{w}^2} + \hat{w}\,\mathcal{I}_1\right)$$

⇒ $\hat{q}^2$ distr. and $\hat{E}_\ell$ distr. are observables: divergences cancel between virtual and real radiation structures.

[BC, Gambino, Nandi]

Applications: total width and $\hat{q}^2$ distribtion

- $O(\alpha_s \Lambda^2_{\mathrm{QCD}}/m_b^2)$ corrections to the total rate (pole mass scheme):

$$\Gamma_{B\to X_u \ell\nu} = \Gamma_0 \left[\left(1 - 2.41\,\frac{\alpha_s}{\pi}\right)\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.5 + 4.98\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

[BC, Gambino, Nandi]

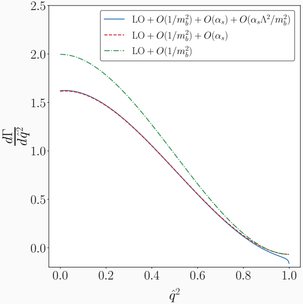
⇒ $\Gamma_0 = G_F^2 |V_{ub}|^2 m_b^5/192\pi^3$ is the lowest order result.

⇒ $O(\alpha_s \mu_\pi^2/m_b^2)$ comply with **Reparametrisation Invariance**.

⇒ $O(\alpha_s \mu_G^2/m_b^2)$ agrees with previous result in the literature.

[Mannel, Pivovarov, Rosenthal]

- $O(\alpha_s \Lambda^2_{\mathrm{QCD}}/m_b^2)$ corrections to the $\hat{q}^2$ distribution.



⇒ Total correction very small over the whole $\hat{q}^2$ range, except close to the endpoint (soft dynamics dominated region). [BC, Gambino, Nandi]

Applications: $\hat{q}_0$ moments

- $\hat{q}_0$ moments of the $O(\alpha_s \mu_\pi^2/m_b^2)$ and $O(\alpha_s \mu_G^2/m_b^2)$ structures will place further constraints on the SFs moments.

- Central moments of the perturbative and power suppressed contributions

$$J_{i,X}^{(n,j)}(\hat{q}^2) = \int_0^\infty (\hat{q}_0 - \hat{q}_0^{max})^n \, W_i^{(X,j)}(\hat{q}_0, \hat{q}^2) \, d\hat{q}_0,$$

  with $j = 0, 1$ and $X = \pi, G$ and $\hat{q}_0^{max} = \frac{1+\hat{q}^2}{2}$.

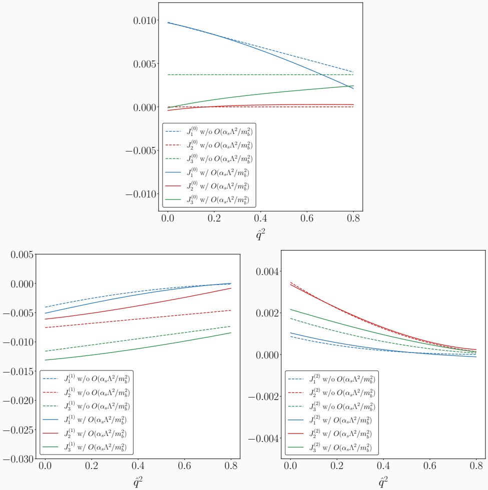
- We also define,

$$J_i^{(n)}(\hat{q}^2) = \frac{\mu_\pi^2}{2m_b^2} J_{i,\pi}^{(n,0)}(\hat{q}^2) + \frac{\mu_G^2}{2m_b^2} J_{i,G}^{(n,0)}(\hat{q}^2) + \frac{\alpha_s}{\pi}\left[\frac{\mu_\pi^2}{2m_b^2} C_F J_{i,\pi}^{(n,1)}(\hat{q}^2) + \frac{\mu_G^2}{2m_b^2} J_{i,G}^{(n,1)}(\hat{q}^2)\right].$$

[BC, Gambino, Nandi]

Applications: $\hat{q}_0$ moments

- Compare the moments $J_i^{(n)}$ with and without the $O(\alpha_s \Lambda^2/m_b^2)$ corrections.



$\Rightarrow$ $O(\alpha_s \Lambda^2/m_b^2)$ corrections to the zeroth moments are relatively small in most of the $\hat{q}^2$ range for $J_{1,2}^{(0)}$, and significant for $J_3^{(0)}$.

$\Rightarrow$ $O(\alpha_s \Lambda^2/m_b^2)$ corrections to the higher moments are generally moderate.

[BC, Gambino, Nandi]