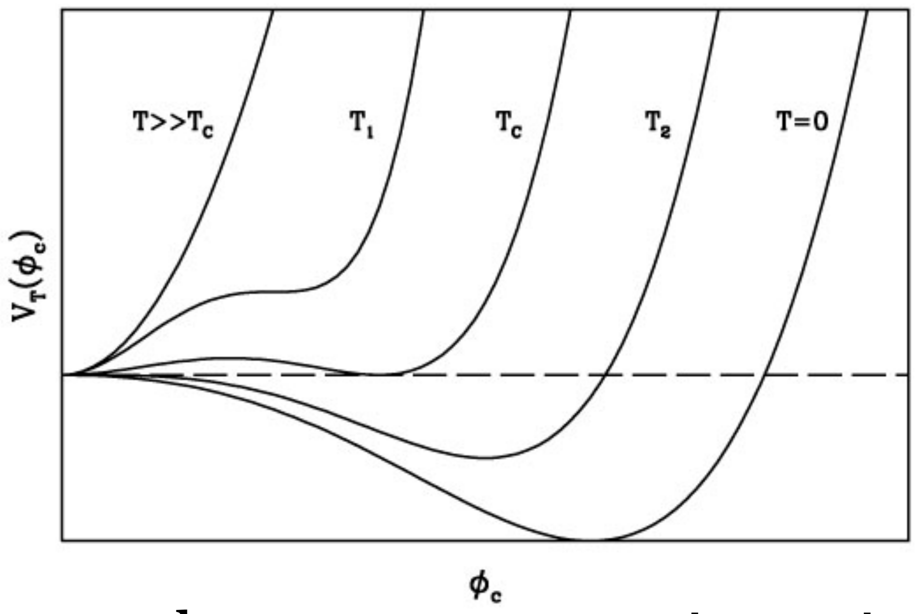
Self-organised localisation

G. F. Giudice

Erwin Schrödinger Guest Professor Lecture, 27 April 2023

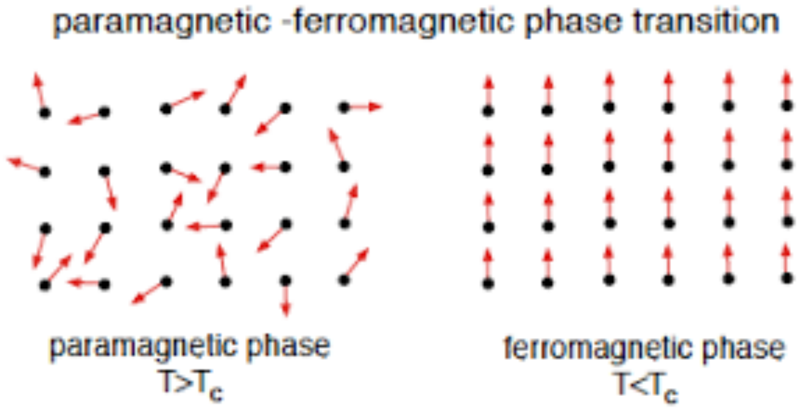
Critical phenomena

Phase transitions in the early universe
(QCD, EW, inflation?, baryogenesis?)



Classical phase transitions: the phase changes as the temperature is varied.

Ferromagnet:

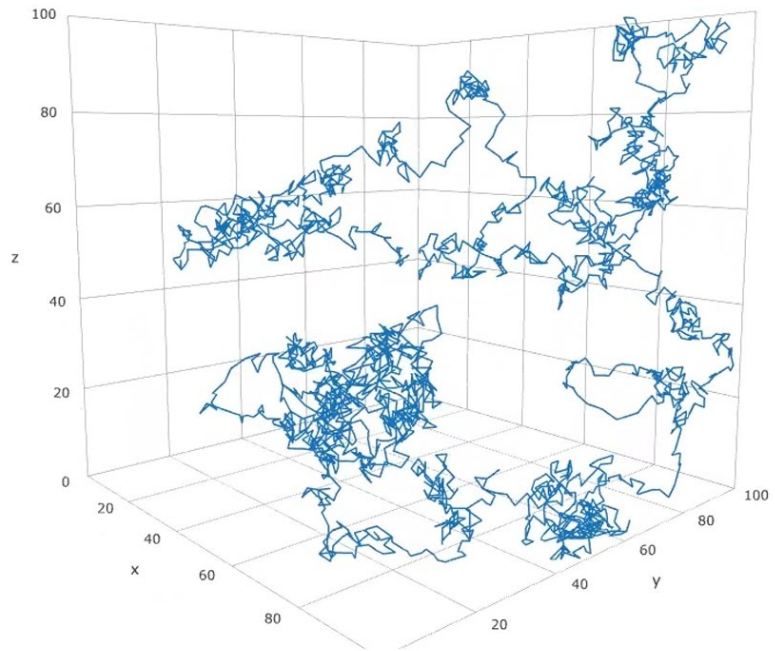


Quantum phase transitions: the phase changes as an external field is varied.

$$V(\phi) = V_\phi + (\phi - \phi_c)\,\mathcal{O} \qquad \begin{cases} \langle\mathcal{O}\rangle = 0 & \phi > \phi_c \\ \langle\mathcal{O}\rangle \neq 0 & \phi < \phi_c \end{cases} \quad \Rightarrow \quad \begin{array}{c} V'(\phi) = V'_\phi + \langle\mathcal{O}\rangle \\ \text{discontinuous at } \phi = \phi_c \end{array}$$

Ingredient 1:

Some parameters of the microscopic theory are promoted to functions of one or more scalar fields.



$$\mu \longrightarrow \mu(\phi)$$

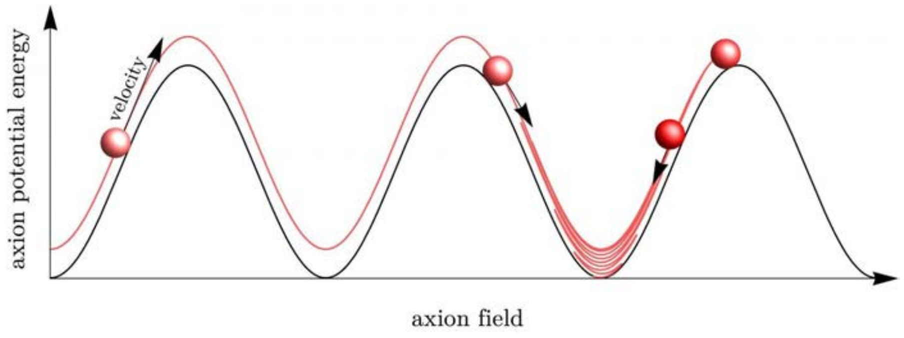
Axion: $\mathcal{L}_{\text{dim}=4} = \frac{g_s^2}{32\pi^2}\,\bar{\theta} G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$ $\qquad \bar{\theta} \longrightarrow a$

Cosmological constant: Abbott, Brown-Teitelboim, etc.

Higgs mass: relaxion, etc.

Ingredient 2:

Selection mechanism in the multiverse.



Axion: symmetry

Cosmological constant: anthropics (Weinberg)

Higgs mass: back-reaction from EW breaking (relaxion)

Self-Organised Criticality (SOL): criticality

(GFG, M. McCullough and T. You, JHEP 10, 093)

Ingredient 2:

Selection mechanism in the multiverse.

Self-Organised Criticality (SOL): criticality

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Stochastic approach (Vilenkin, Starobinsky, Linde, ...)

$P(\phi,t)$ distribution of volume occupied by ϕ at time t

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P)}{\partial\phi} + \frac{V' P}{3H}\right] + 3HP = \frac{\partial P}{\partial t} \qquad \text{(FPV)}$$

↑ classical term

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↑ quantum term

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↑ volume term

Stochastic approach (Vilenkin, Starobinsky, Linde, ...)

$P(\phi,t)$ distribution of volume occupied by ϕ at time t

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P)}{\partial\phi}+\frac{V' P}{3H}\right]+3HP=\frac{\partial P}{\partial t} \qquad \text{(FPV)}$$

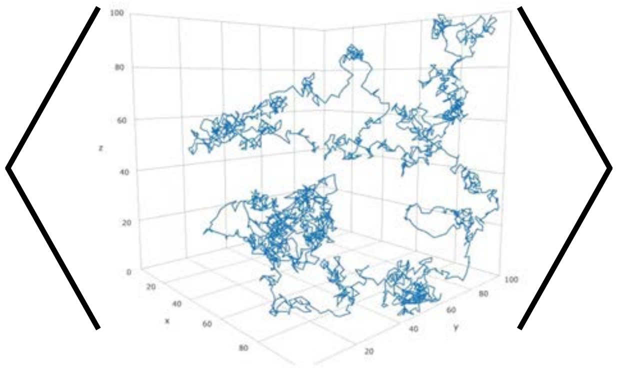
quantum term	classical term	volume term
$\hbar$	1	$1/M_P^2$

classical mechanics ⇩

quantum mechanics ⇨ SOL ⇦ general relativity

⇧ critical phenomena

Fokker-Planck: $$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P_{\rm FP})}{\partial\phi}+\frac{V' P_{\rm FP}}{3H}\right]=\frac{\partial P_{\rm FP}}{\partial t}$$



Langevin: $$\frac{d\phi}{dt}+\frac{V'(\phi)}{3H}=\eta(t)\ ,\quad \langle\eta(t)\eta(t')\rangle=\frac{H^3}{4\pi^2}\delta(t-t')$$

Volume-weighted Fokker-Planck (FPV): $$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P)}{\partial\phi}+\frac{V' P}{3H}\right]+3HP=\frac{\partial P}{\partial t}$$

each trajectory is weighed by e^{3Ht}

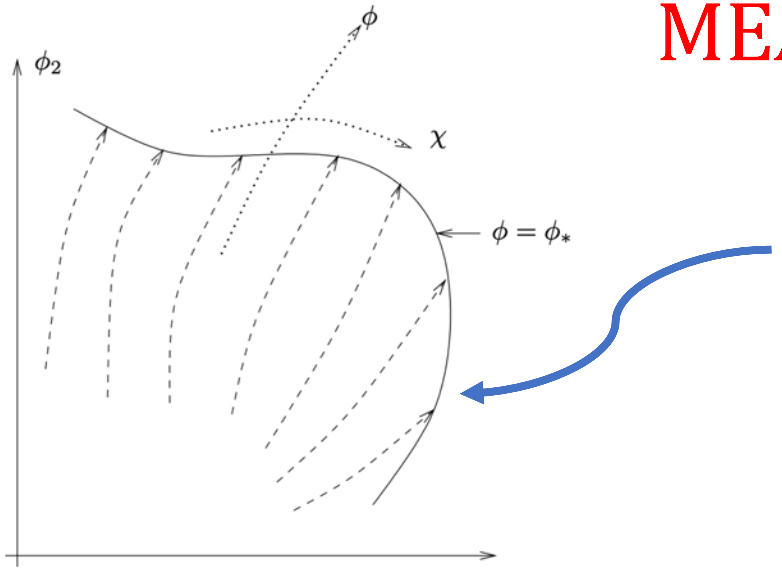
Probabilistic predictions in the multiverse?

GAUGE DEPENDENCE

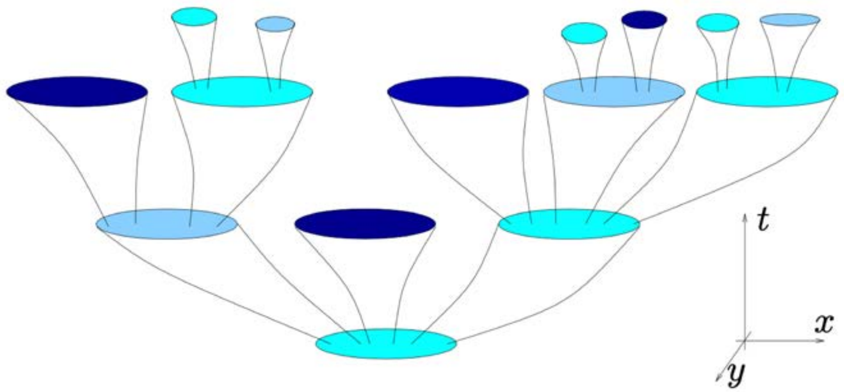
$$t \to t_\xi \qquad \frac{dt_\xi}{dt} = \left(\frac{H}{H_0}\right)^{1-\xi} \qquad 0 \leq \xi \leq 1$$

$$\begin{cases} \xi = 1 & \text{proper-time gauge} \\ \xi = 0 & e\text{-folding gauge} \end{cases}$$

MEASURE PROBLEM



Reheating surface: 3-volume hypersurface of all reheating events in spacetime.



Eternal inflation: the reheating surface is infinite and non-compact.

Steady-state solutions: $P(\phi, t) \xrightarrow{t \gg t_R} e^{K(t)}\, p(\phi)$

VALIDITY OF THE SEMICLASSICAL APPROXIMATION

$$N < S_{\rm dS} = \frac{8\pi^2 M_P^2}{\hbar\, H^2}$$

Arkani-Hamed *et al*, 0704.1814
Creminelli *et al*, 0802.1067
Dubovsky *et al*, 0812.2246; 1111.1725

Does the semiclassical approach break down after this time?

Dvali *et al*, 1312.4795; 1701.08776

SWAMPLAND CONJECTURES

Do super-Planckian field excursions, slow-roll inflation and eternal inflation live in the swampland?

PROBABILISTIC INTERPRETATION OF THE FPV EQUATION

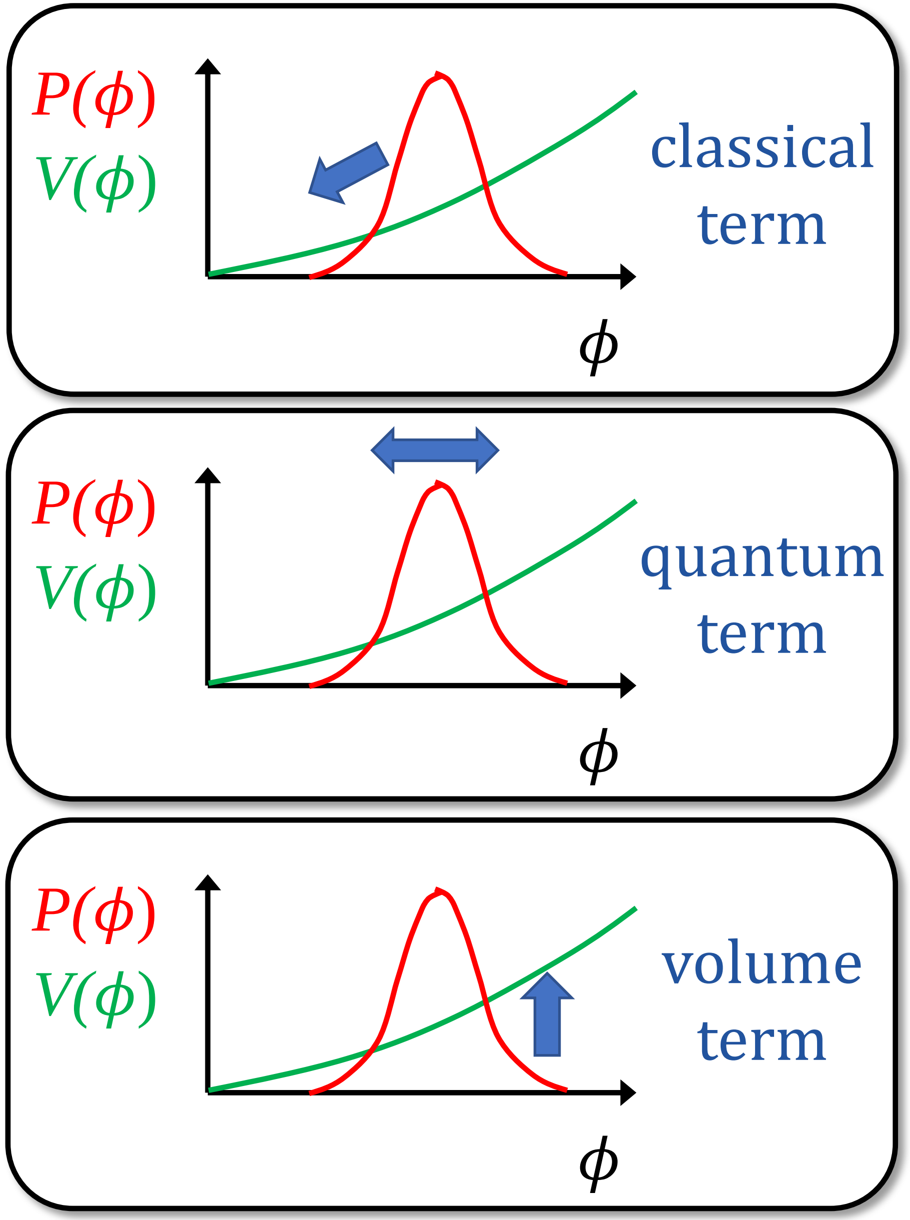
FPV

$$\frac{\partial}{\partial \phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H}\right] + 3HP = \frac{\partial P}{\partial t}$$

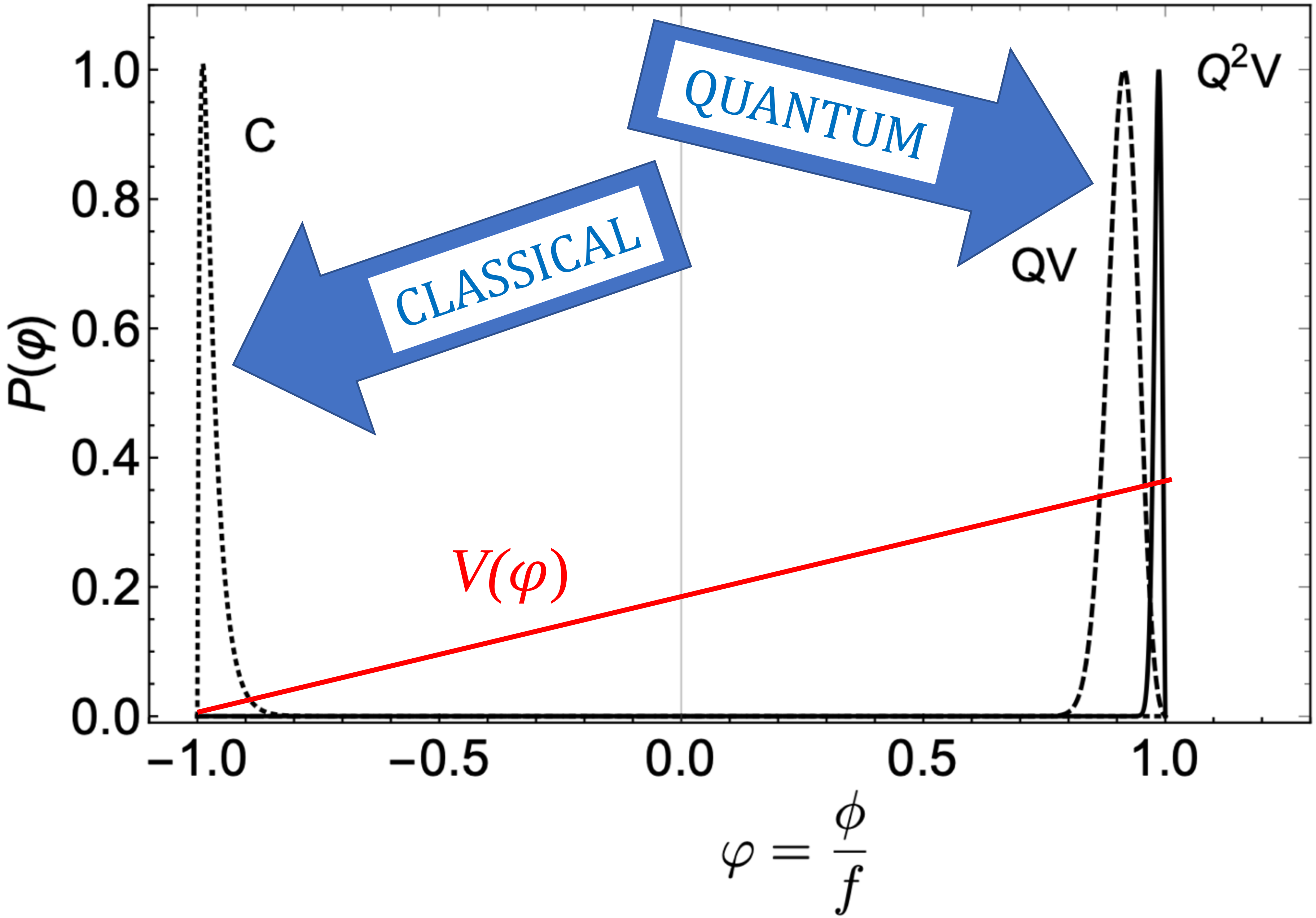
quantum term

classical term

volume term

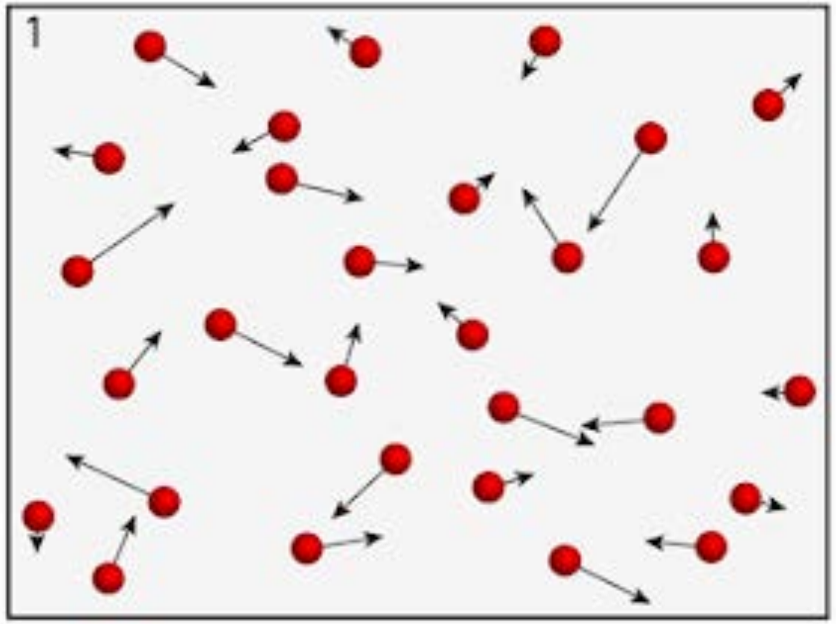


LINEAR POTENTIAL

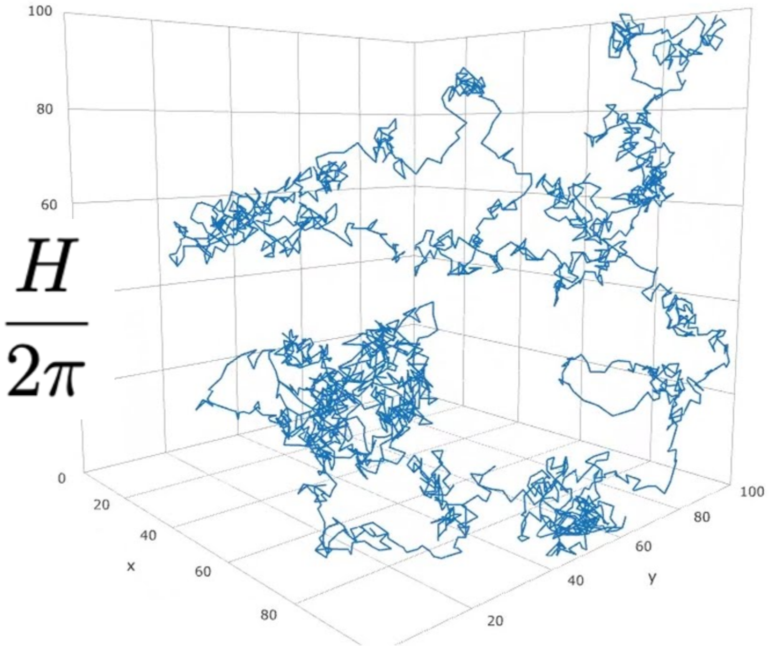


GAS

MULTIVERSE



$$T_{\mathrm{dS}} = \frac{H}{2\pi}$$

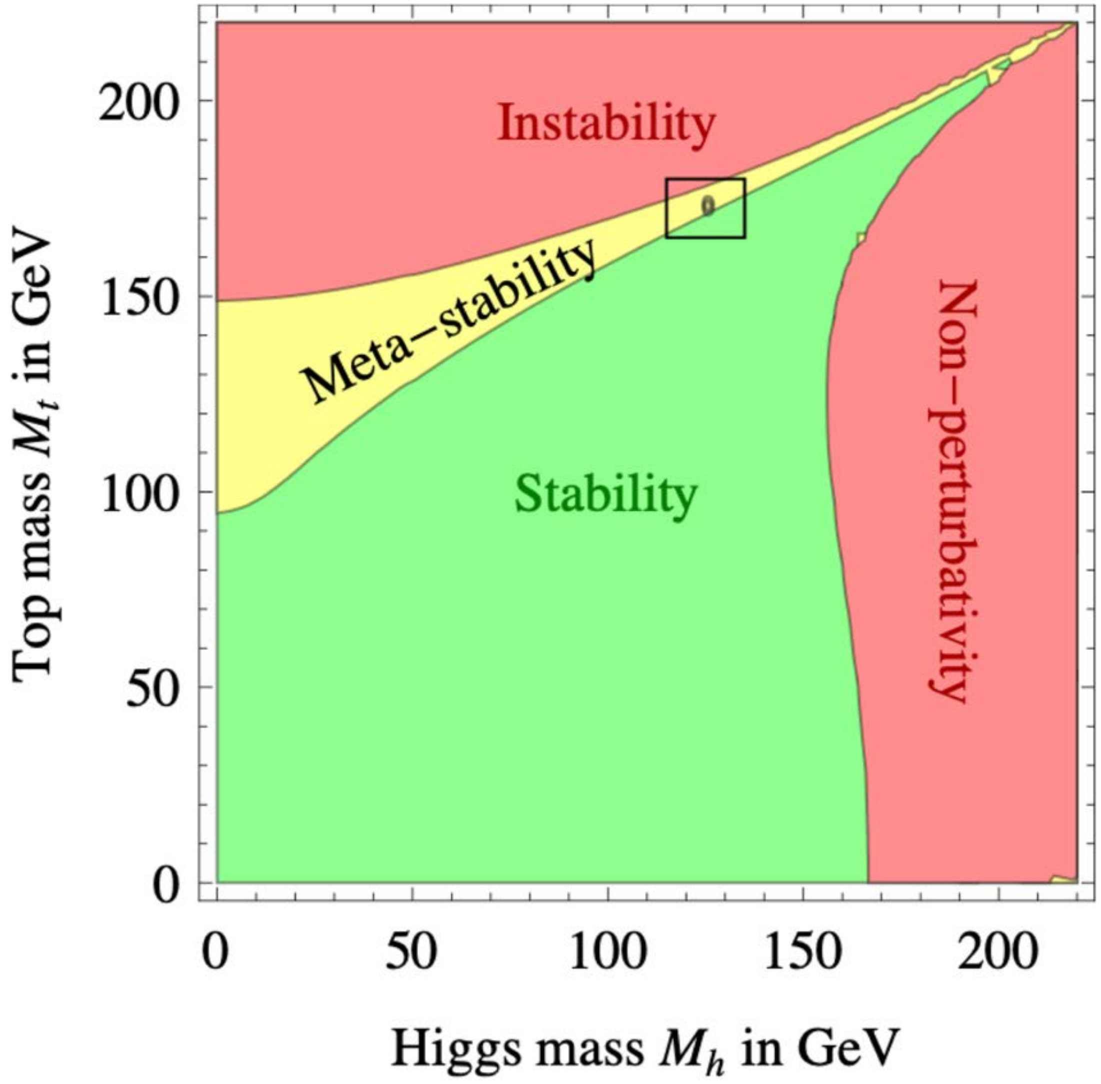


T = constant

Steady-state solutions:

$$P(\phi, t) \xrightarrow{t \gg t_R} e^{K(t)}\, p(\phi)$$

NEAR-CRITICALITY OF THE HIGGS SELF-COUPLING



$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2$$

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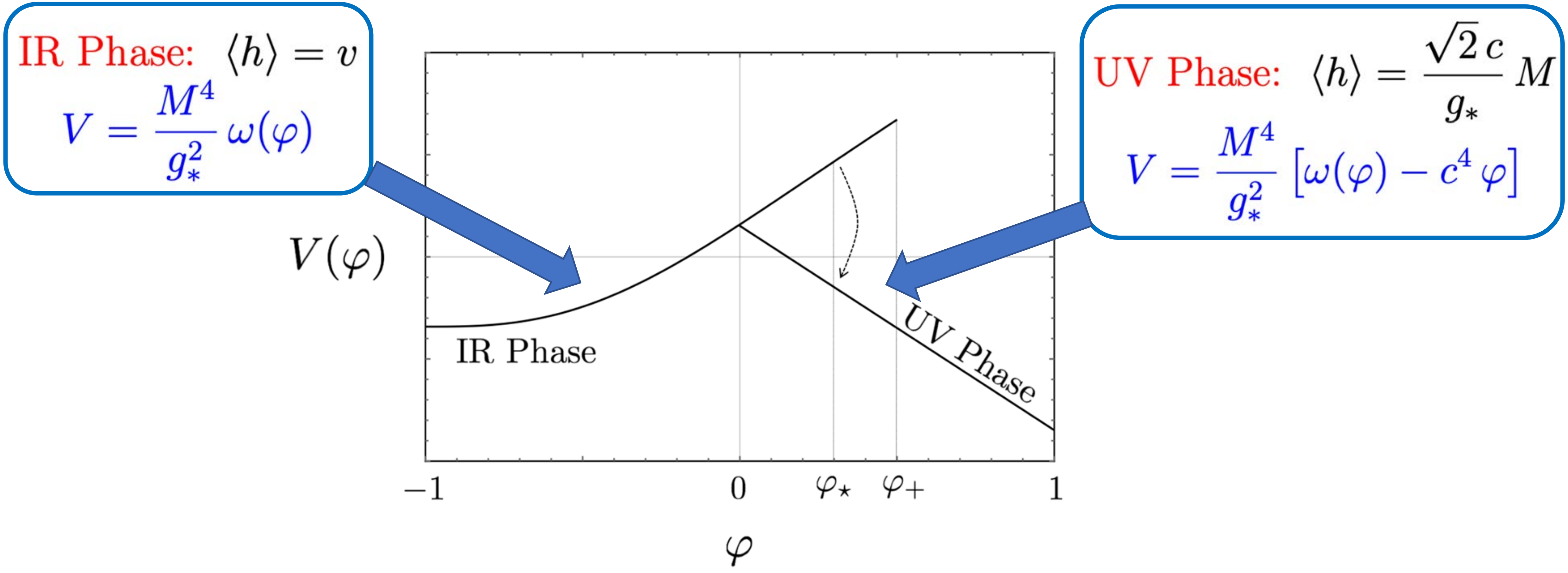
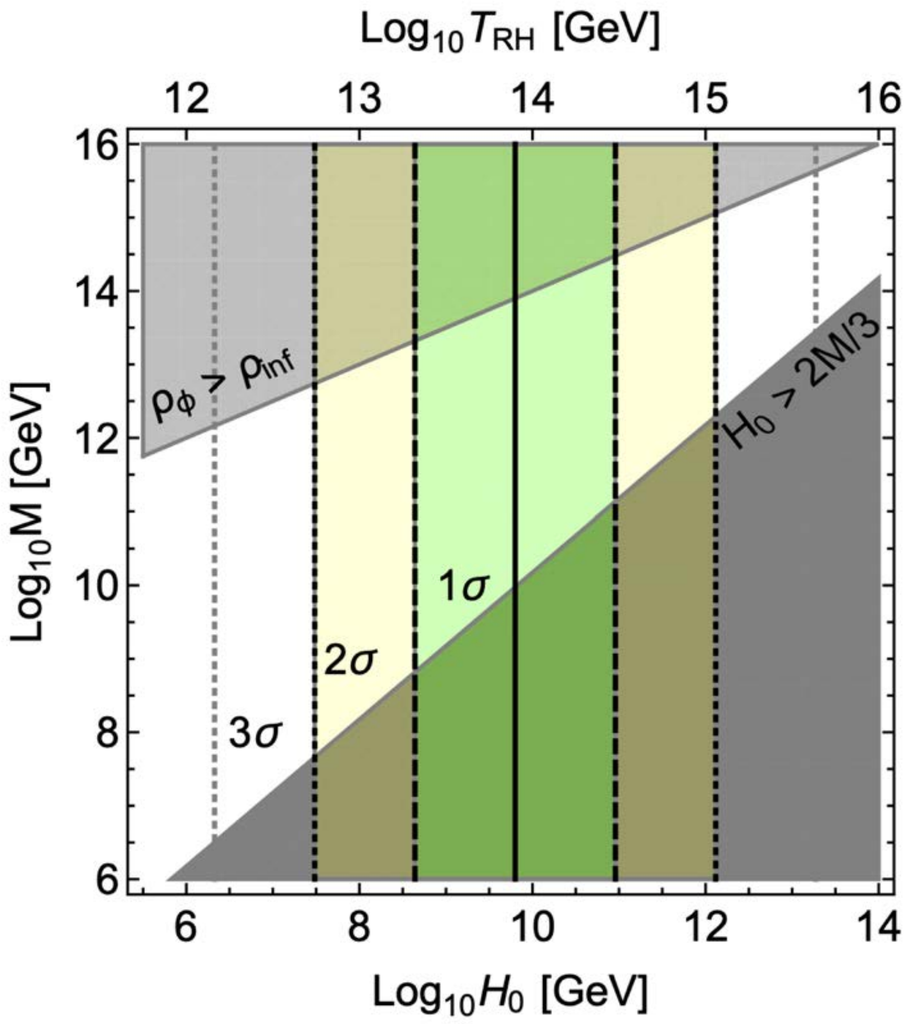
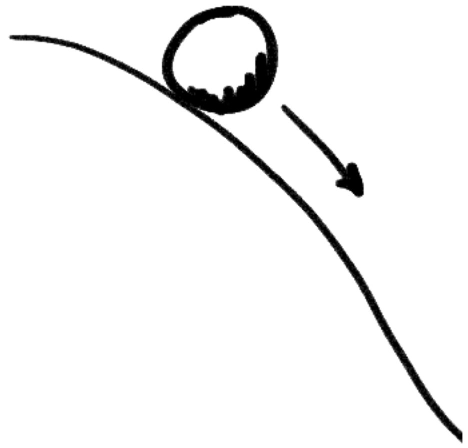
$$\lambda(\varphi, M/g_*) = -g_*^2\,\varphi\ , \qquad \frac{d\,\lambda(\varphi, h)}{d\ln h^2} = \beta_\lambda(h)$$



SOL: at the end of inflation, there is a strong probabilistic preference for patches of the Universe where the Higgs self-coupling is near its critical value.



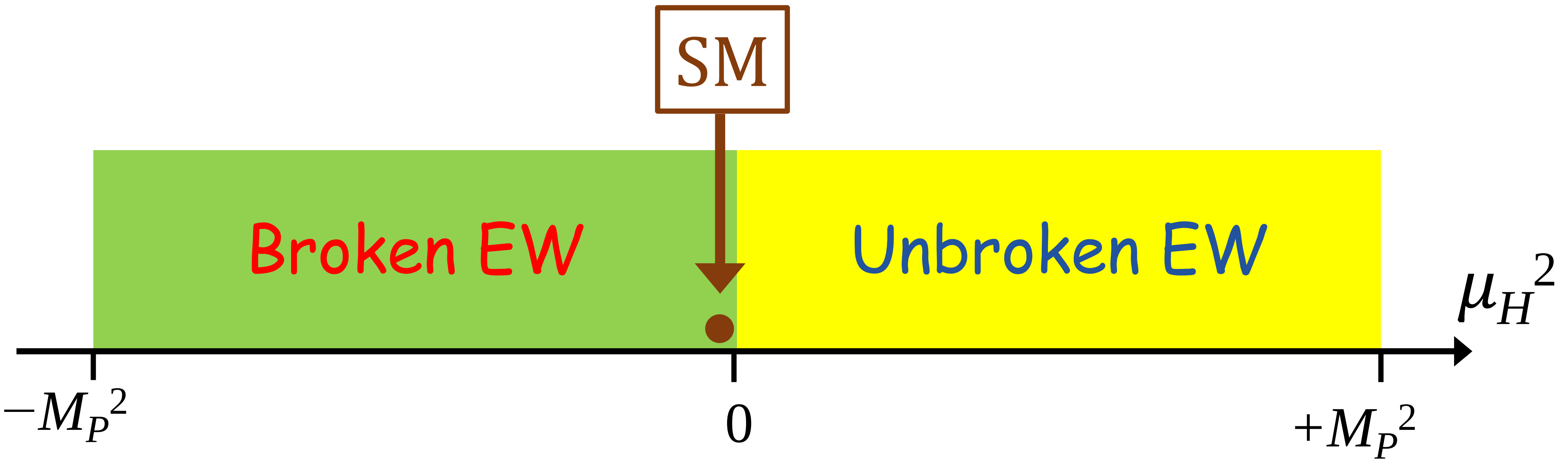
What happens to the SOL prediction during the thermal phase of the Universe?



$$\alpha^2\beta > \left(\frac{\hbar\, H_0^4}{M_P H_{\rm now}\Lambda^2}\right)^2 = \left(\frac{H_0}{2\times 10^{-3}\ {\rm eV}}\right)^8$$

$\Rightarrow$ Q^2V & eternal inflation

HIGGS NATURALNESS

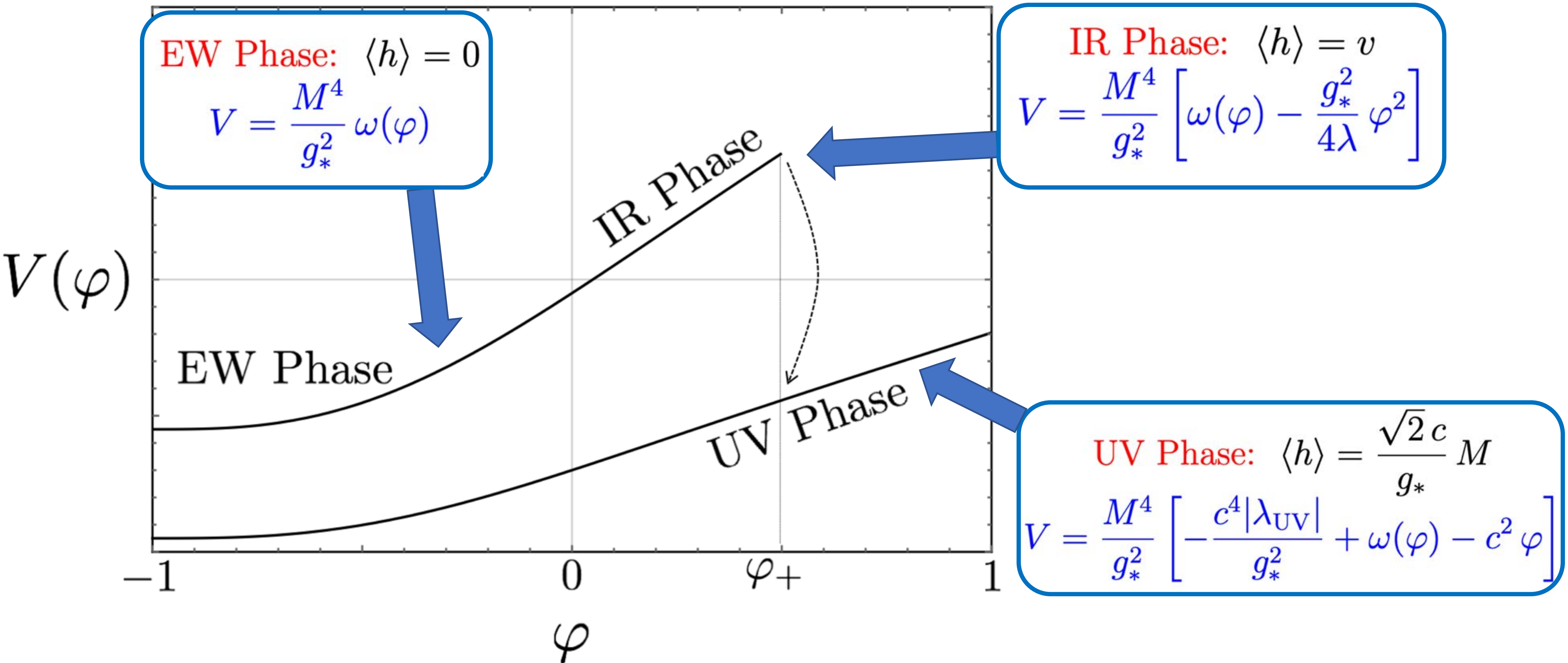


Higgs naturalness: why is nature so close to the critical point?

HIGGS NATURALNESS

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\,h^4}{4}$$

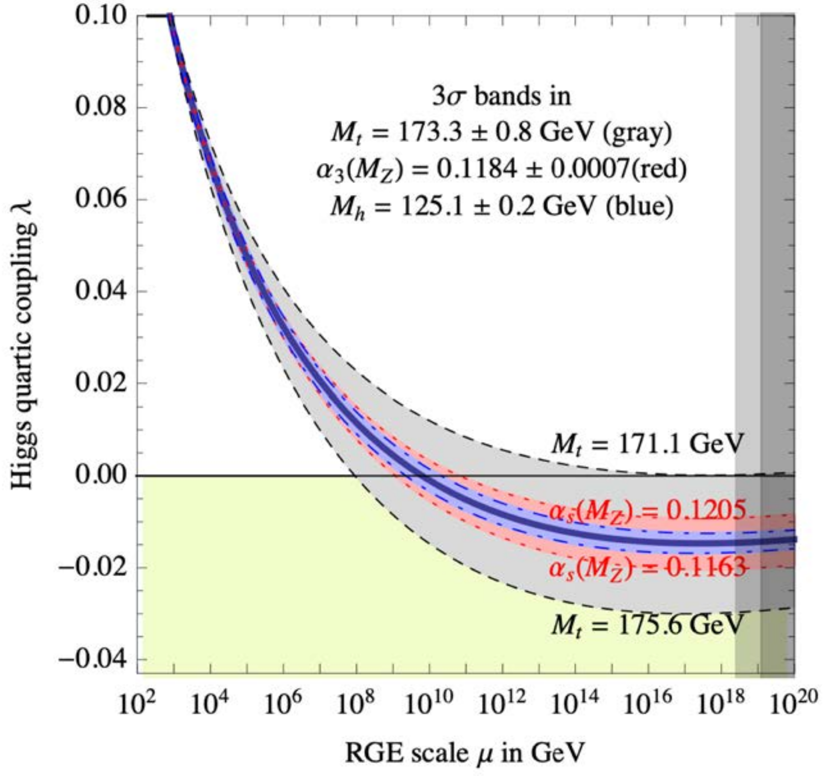
scanning mass term



SOL prediction: $v = e^{-\frac{3}{4}} \Lambda_I$

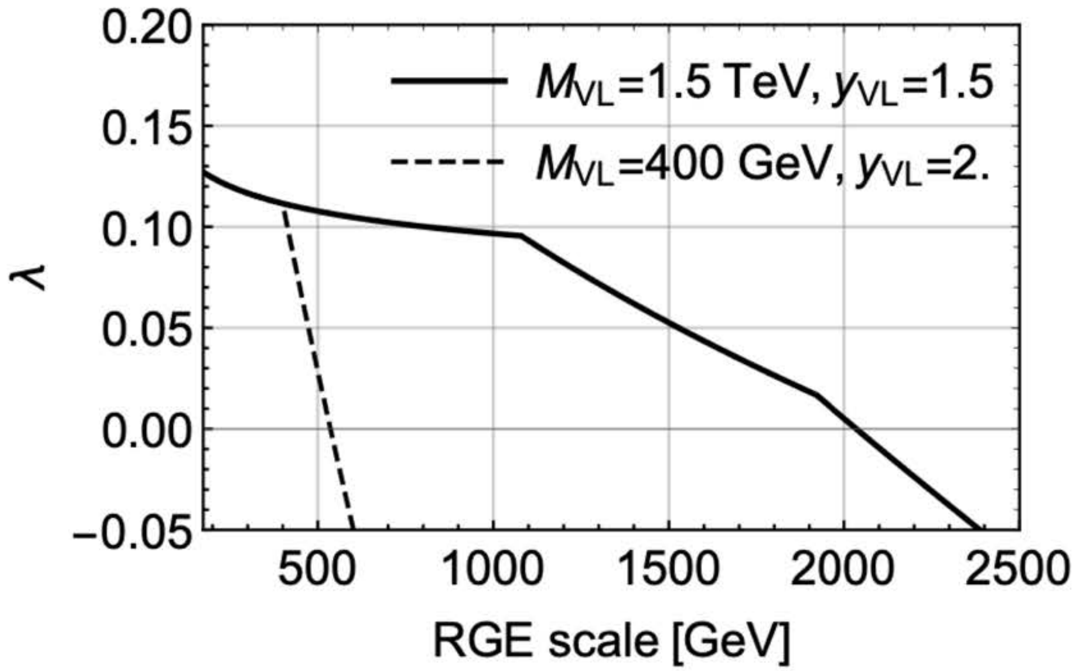
$v/M \sim \exp(-\lambda_{UV}/2\beta_\lambda)$ natural hierarchy from dimensional transmutation

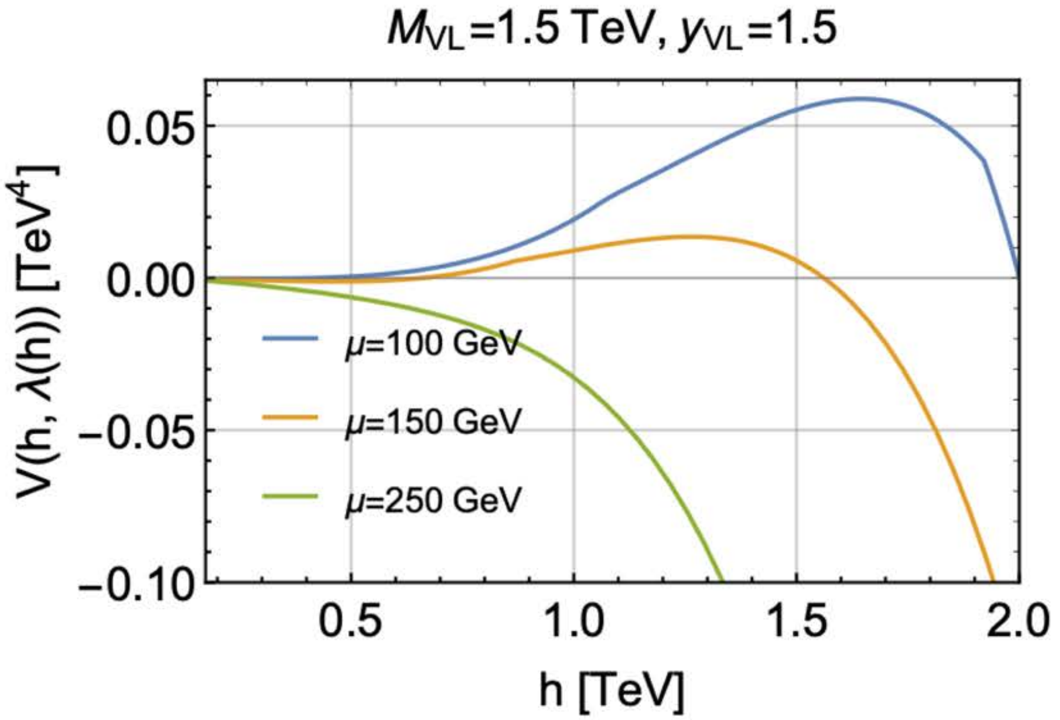
SM $\Rightarrow$



weak doublet χ and a SM singlet ψ

$$(a)\ \ \mathcal{L} = -y_{VL}\bar{\psi}\chi H_h + \text{h.c.} \qquad (b)\ \ \mathcal{L} = -y_{VL}\bar{\psi}LH_h + \text{h.c.}$$

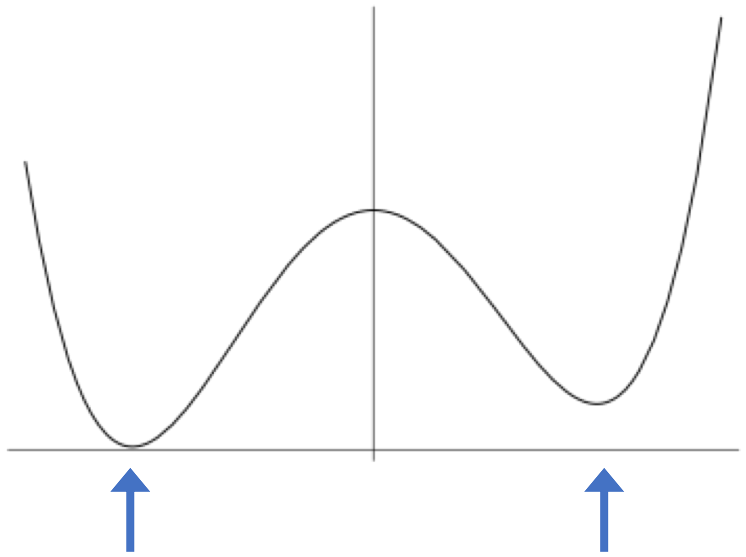




Phenomenological SOL prediction: new matter that modifies β_λ such that the theory is near-critical with respect to variations of the Higgs bilinear.

COSMOLOGICAL CONSTANT

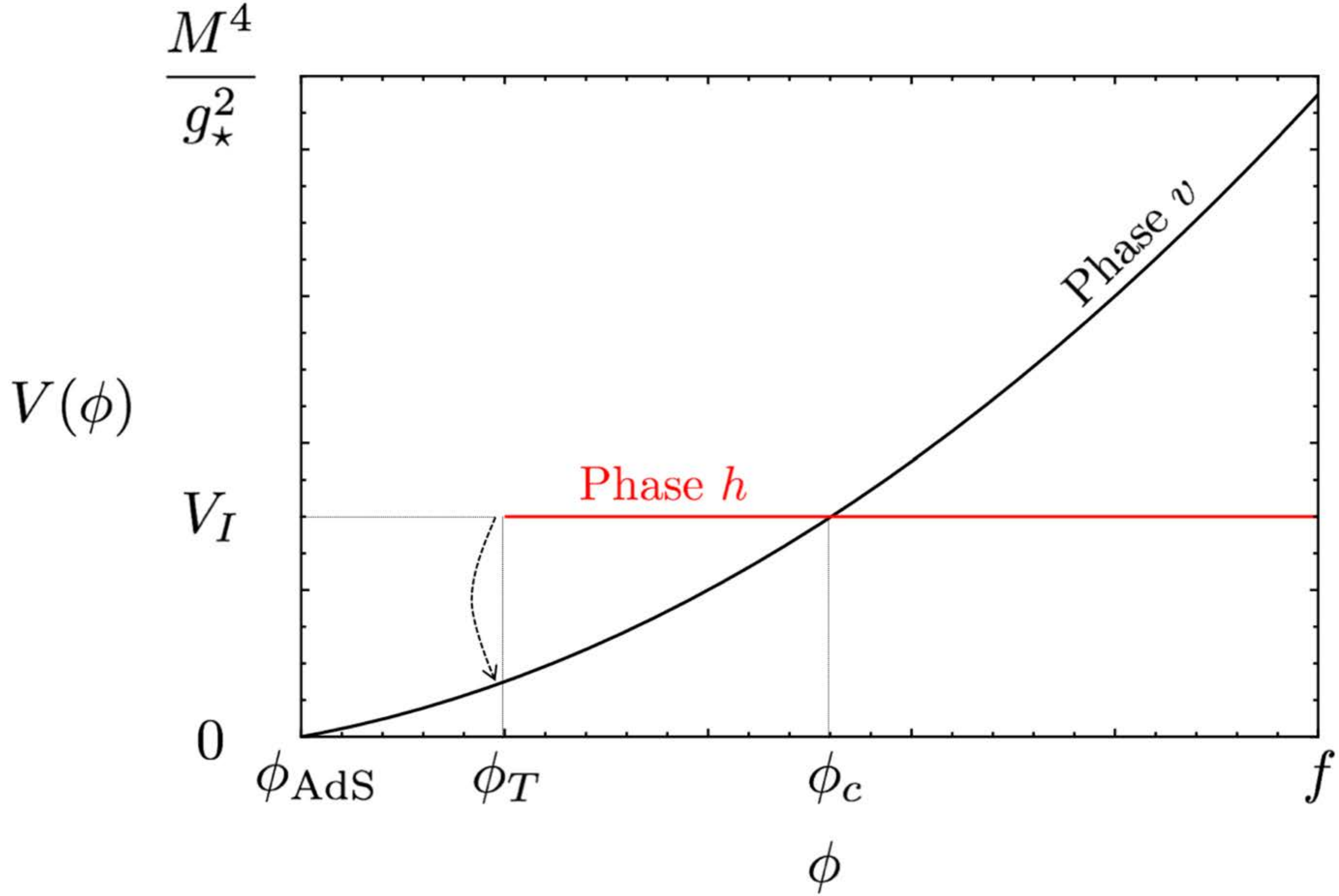
Parameters of a microscopic theory are functions of the apeiron.

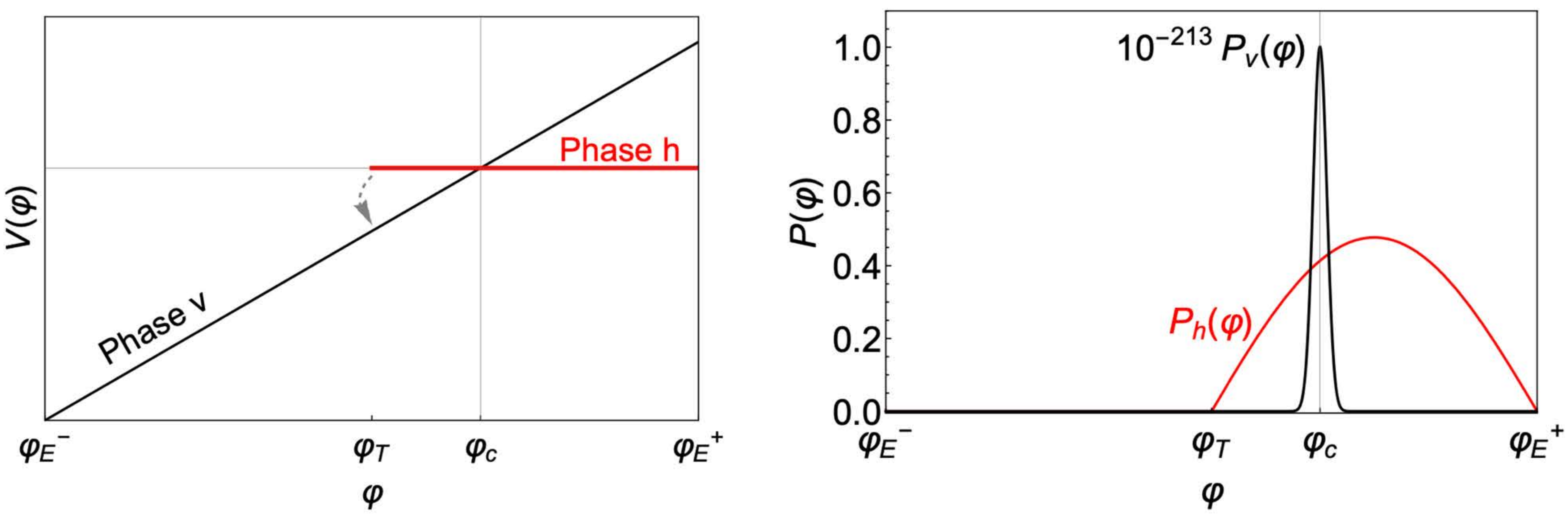


Inflaton

Phase *h*
(supersymmetry
& R-symmetry)

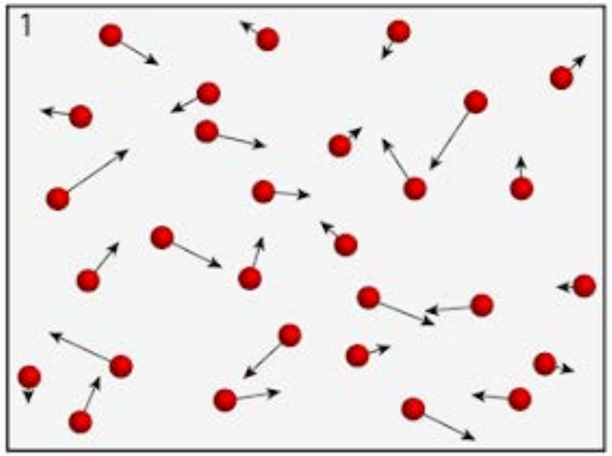
Phase *v*
(visible phase)

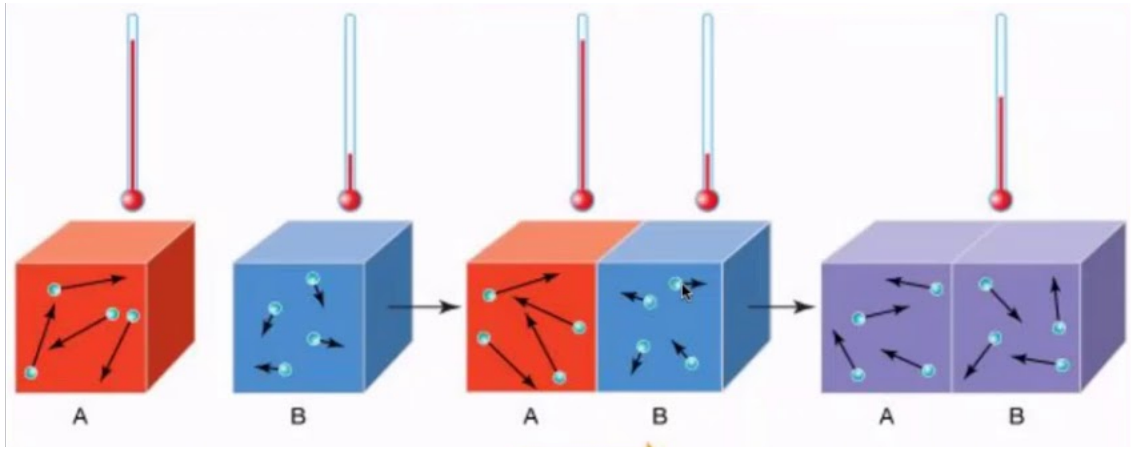

$\frac{M^4}{g_\star^2}$
$V(\phi)$
V_I
0
Phase v
Phase h
$\phi_{\rm AdS}$
ϕ_T
ϕ_c
f
ϕ



SOL prediction: the distribution is peaked on phase *v* at the point where the two phases are degenerate.

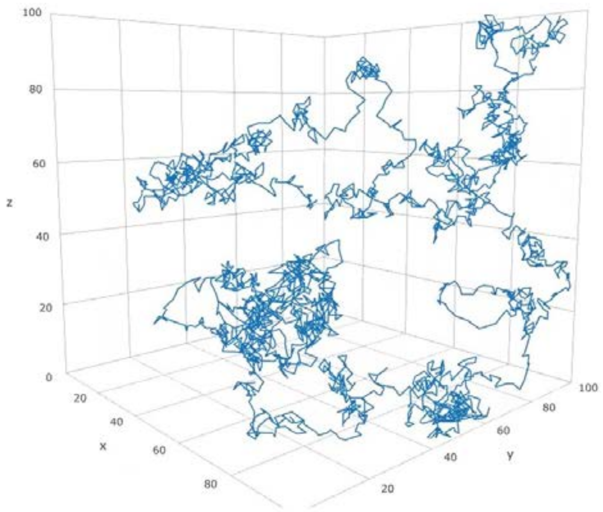
GAS

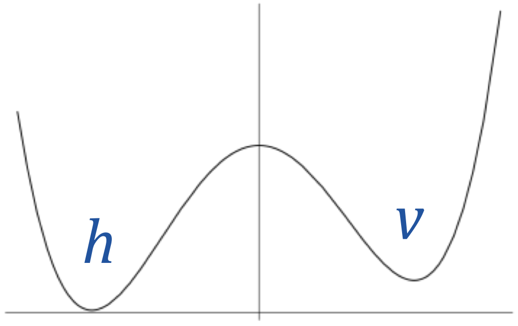




In equilibrium, T in box A and B become equal.

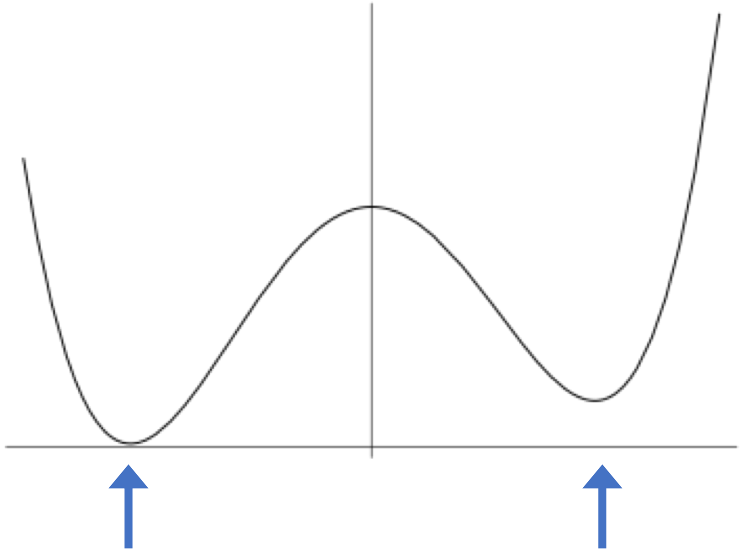
MULTIVERSE





In steady-state, the expansion rates in phase h and v become equal $\Rightarrow$ energy degeneracy.

A NEW WAY OF USING SUPERSYMMETRY



Phase *h*
(supersymmetry
& R-symmetry)

Phase *v*
(visible phase)

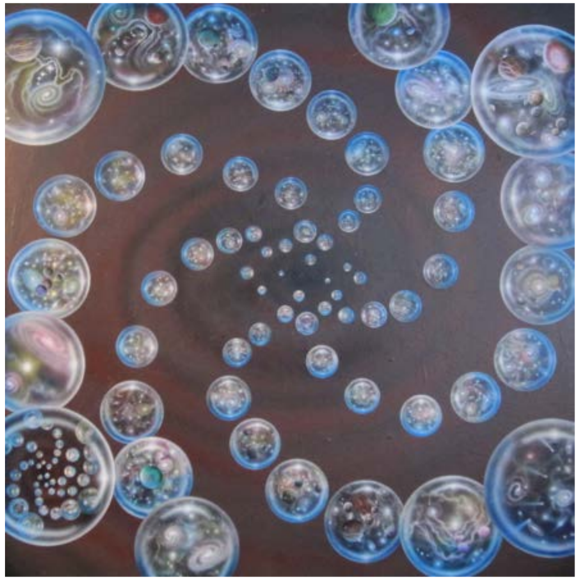
Supersymmetry is a hidden feature of the theory to any observer, like us, who lives in phase *v*, and yet it determines parameters measurable in our vacuum.

REHEATING TEMPERATURE

$$T_{\rm RH} < c_\xi^{1/6} (\Lambda_{\rm CC}^2 M_P)^{1/3} \approx c_\xi^{1/6}\, 25 \text{ MeV}$$

DARK-ENERGY EQUATION OF STATE

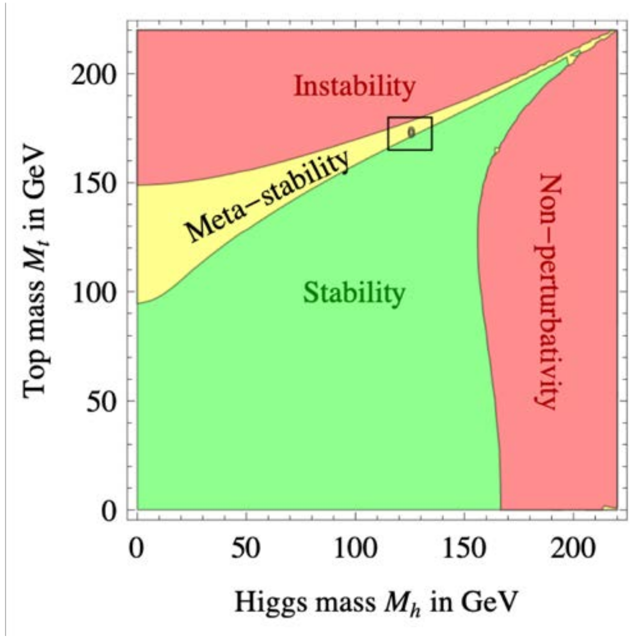
$$w = \frac{P_\phi}{\rho_\phi} = -1 + \left(\frac{V_v'(0)}{3 H_{\rm now} \Lambda_{\rm CC}^2} \right)^2 = -1 + \frac{c_\xi^2}{3}$$



TESTING SOL EXPERIMENTALLY?

SOL's smoking gun is phase coexistence.

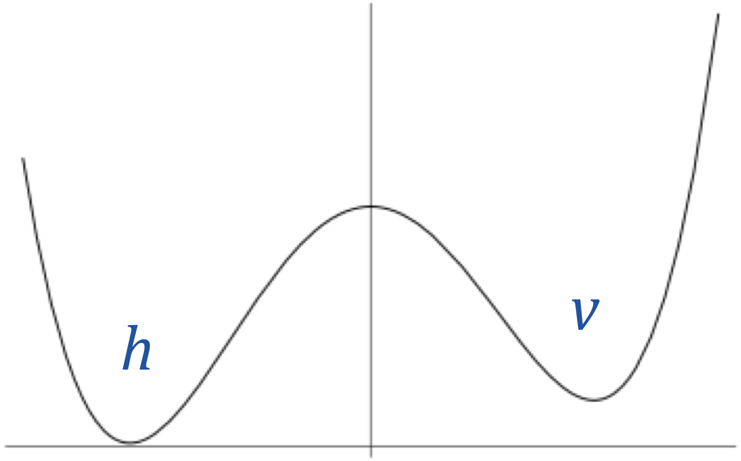
Near-criticality of the Higgs self-coupling



Higgs naturalness

New matter at the TeV makes the SM unstable under variations of the Higgs bilinear.

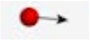
Cosmological constant



Dark energy EoS

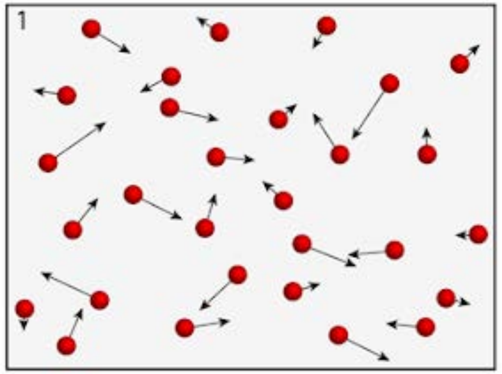
CONCLUSIONS

- SOL is an approach radically different from the symmetry paradigm: critical points can become dynamical attractors during inflation and determine low-energy parameters.

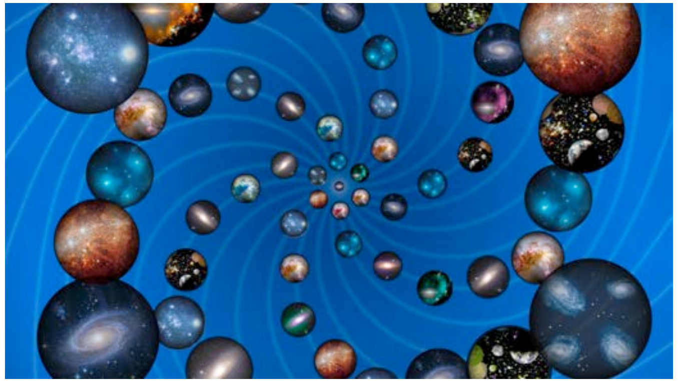


Single atom: energy?

Single Universe: SM parameters?



Gas in statistical equilibrium: probabilistic prediction.



Multiverse in steady-state: probabilistic prediction.

- SOL can address some of the classical open questions in particle physics.

Self-organised localisation

G. F. Giudice

Erwin Schrödinger Guest Professor Lecture, 27 April 2023