A Markov Chain Monte Carlo Revision of the Universal Texture Zero of Flavor

[2211.15700]

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[1710.01741]

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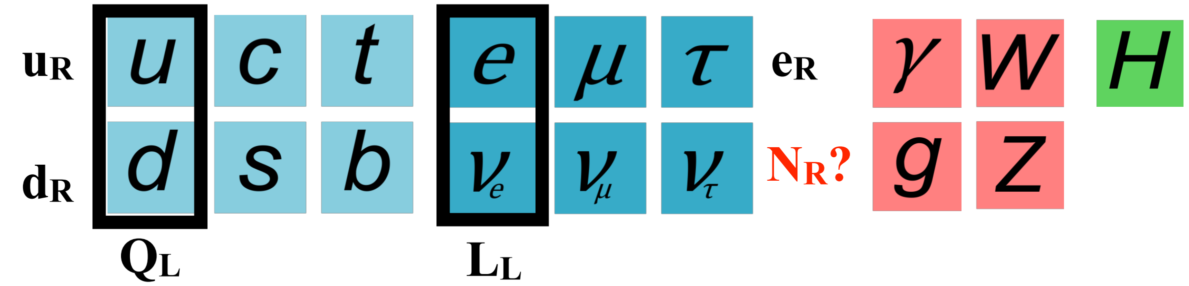
24 January 2023 || U. Vienna || Theory Seminar

Flavor in the SM



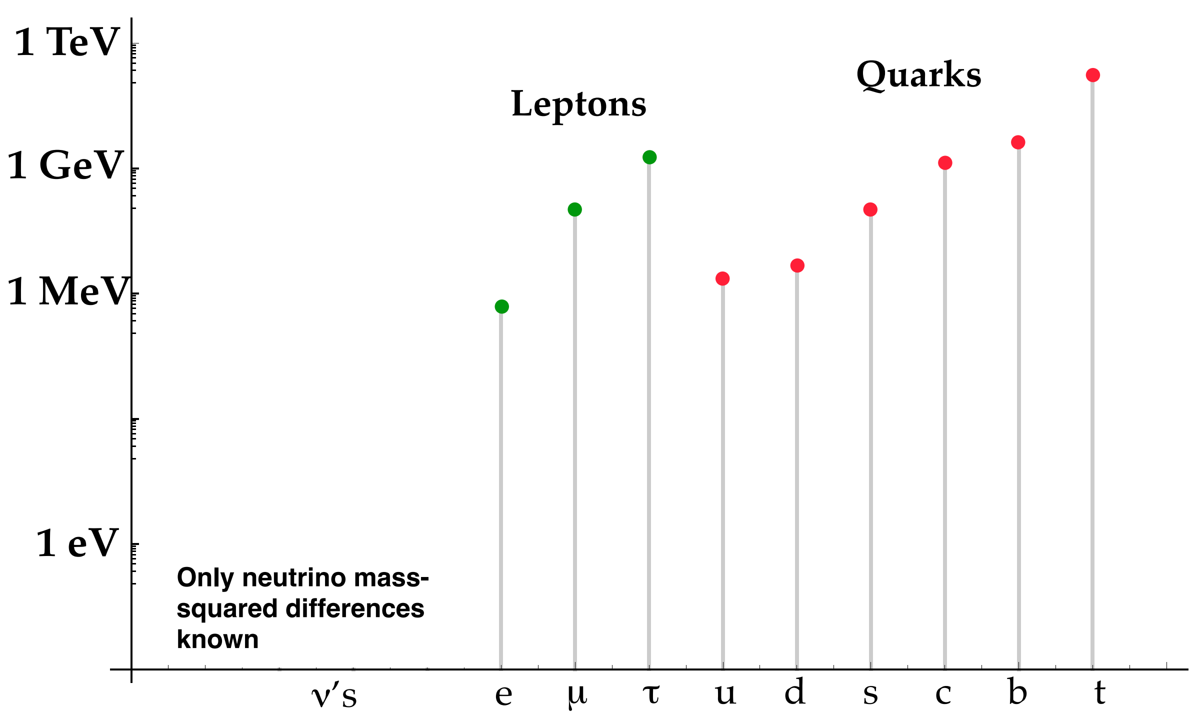
$$\mathcal{L}_{SM}^{Y} \supset Y_{pr}^{u}\, \overline{Q}_{L,p}\, \tilde{H}\, u_{R,r} + Y_{pr}^{d}\, \overline{Q}_{L,p}\, H\, d_{R,r} + Y_{pr}^{e}\, \overline{L}_{L,p}\, H\, e_{R,r} + \text{h.c.}$$

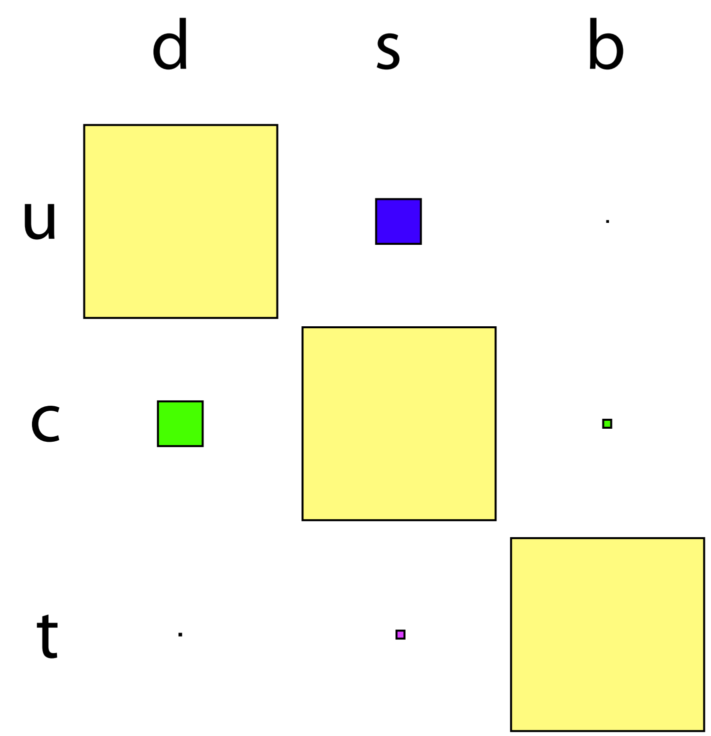
U(3)⁵

- From these Lagrangian terms one can use field redefinitions to show that only 9 masses, 3 mixing angles, and one CP-violating phase are needed for physical description.

$$[U_{\psi L}^{\dagger}]_{ir}\, [\mathcal{Y}^{\psi}]_{rp}\, [U_{\psi R}]_{pj} \equiv [D_{\psi}]_{ij} = \text{diag}\,(y_{\psi 1}, y_{\psi 2}, y_{\psi 3})$$

$$V_{CKM} \equiv U_u^{\dagger} U_d \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





[1212.6374]

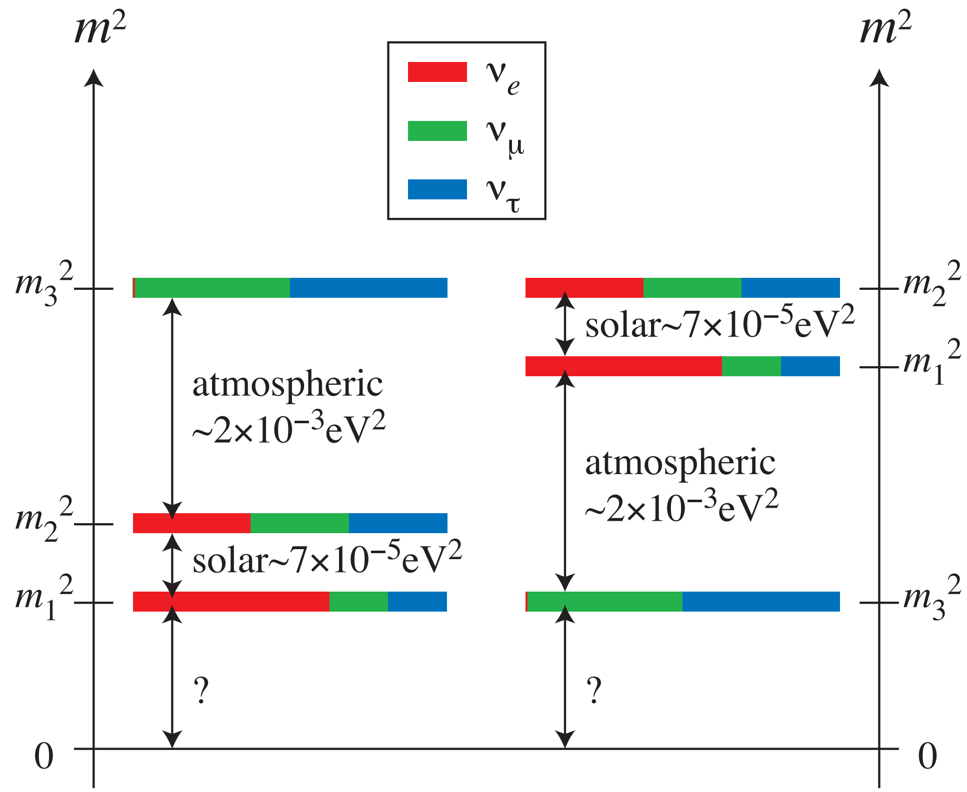
[1405.5495]

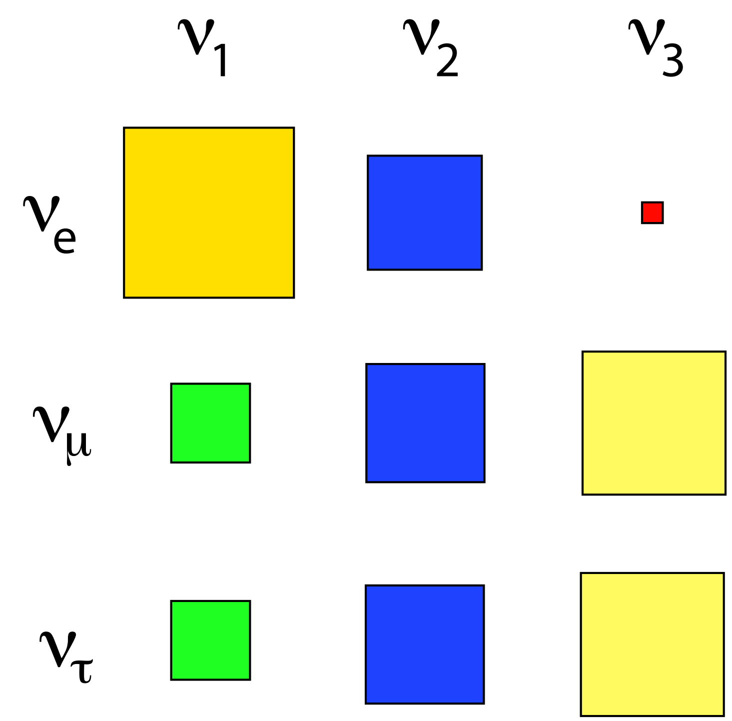
13 free and unexplained parameters exist in SM Yukawa sector

Neutrino flavor structure

- Neutrino mass/mixing is an experimental fact, and it represents a departure from the SM

hep-ph/1301.1340





hep-ph/1212.6374

- Additional **7-9** free and unexplained parameters. However, origin(s) of mass scale unknown. Type-I Seesaw model (Minkowski '77, et al.) most popular natural explanation:

$$\mathcal{L}_N = \frac{1}{2}\left(\bar{N}_p i\partial\!\!\!/ N_p - \bar{N}_p M_{pr} N_r\right) - \left[\bar{N}_p\, \omega_{p\beta}\, \tilde{H}^\dagger l_\beta + \mathrm{h.c.}\right]$$

Match to dim-5 SMEFT

$$\mathcal{L}^{(5)} \supset -\frac{m_{\nu,k}}{2}\, \overline{\nu_L'^{c,k}}\, \nu_L'^{k} + \mathrm{h.c.} \qquad m_{\nu,k} = -\frac{v^2}{2}(U^T)_{k\alpha}\, c_{5,\alpha\beta}\, U_{\beta k}$$

EWSB -> light neutrinos!

$$\mathcal{L}^{(5)} = \frac{c^{(5)}_{\alpha\beta}}{2}\left(l^T_\alpha \tilde{H}^*\right) C \left(\tilde{H}^\dagger l_\beta\right) + \mathrm{h.c.} \qquad c^{(5)}_{\alpha\beta} = \left(\omega^T M^{-1} \omega\right)_{\alpha\beta}$$

Symmetric solutions to the Flavor Puzzle

- **9** charged fermion masses + **3** active neutrino masses
- **6** mixing angles and **2 - 4** CP violating phases

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

20-22 free and unexplained parameters exist in the (ν)SM

- Can we address this Flavor Puzzle by appending the SM with a new symmetry?

$$BSM\ theory \quad \sim \quad \mathcal{G}_{BSM} \times SM$$

- Such a symmetry would presumedly relate fermions in a given family— i.e. a \`horizontal' or \`family' or \`flavor' symmetry

$$\mathcal{R}\left(\mathcal{G}_{BSM}\right) \sim 3, \bar{3}, 2, 1, \ldots$$

u *c* *t*

- New dynamical scalar sector to realize its breaking patterns?

? ? ?

- Also, what are the mathematical properties of the required symmetry?

Froggatt, Nielsen : NPB 147 (1979)
See also C. Luhn talk at FLASY 2014

A simple example: U(1) Froggatt-Nielsen

- The Froggatt-Nielsen Mechanism is the most famous example of a family symmetry. It implements an Abelian U(1) symmetry with charge Q.
- Standard Yukawa couplings are forbidden if the Higgs is charged under $U(1)_{FN}$. New `flavon' fields necessary:

$$\mathcal{L}_Y \sim y_{ij}\,\bar{\psi}_i\,\Phi\,\psi_j \;\longrightarrow\; c_{ij}\,\bar{\psi}_i\,\Phi\,\psi_j \left(\frac{\theta}{\Lambda}\right)^{x_{ij}}$$

Integer chosen to cancel charges

Underdetermined O(1) coefficients

Mass scale of new physics — must be dynamically realized by integrating out new fields

- Let's consider the down quark sector:

Field	$\bar{q}_1$	$\bar{q}_2$	$\bar{q}_3$	$d_{R,1}$	$d_{R,2}$	$d_{R,3}$	Φ	θ
Q	6	4	0	5	3	3	-3	-2

$$c_{ij}\,\bar{q}_i\,\Phi\,d_R\left(\frac{\theta}{\Lambda}\right)^{x_{ij}} \;\Rightarrow\; c_{ij}\,\bar{q}_i\,\Phi\,d_R\,\lambda^{x_{ij}} \;\Rightarrow\; y_d = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^3 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

SSB for the flavon $\frac{\langle\theta\rangle}{\Lambda} \sim \lambda$

- For the quarks this mechanism works quite well, but introduces a large number of free parameters….

Outline

Discrete EFTs for Flavor

The Universal Texture Zero
1710.01741

MCMC phenomenology,
& the future of BSM flavor

Guided by data

- Let's focus on the leptonic sector for the moment, and notice an approximate symmetry in the associated PMNS mixing matrix:

Nu-fit Collaboration

$$U_{PMNS} \equiv |U_l^\dagger U_\nu| \in \begin{pmatrix} \begin{pmatrix} 0.801 \\ 0.845 \end{pmatrix} & \begin{pmatrix} 0.513 \\ 0.579 \end{pmatrix} & \begin{pmatrix} 0.144 \\ 0.156 \end{pmatrix} \\ \begin{pmatrix} 0.244 \\ 0.499 \end{pmatrix} & \begin{pmatrix} 0.505 \\ 0.693 \end{pmatrix} & \begin{pmatrix} 0.631 \\ 0.768 \end{pmatrix} \\ \begin{pmatrix} 0.272 \\ 0.518 \end{pmatrix} & \begin{pmatrix} 0.471 \\ 0.669 \end{pmatrix} & \begin{pmatrix} 0.623 \\ 0.761 \end{pmatrix} \end{pmatrix} \quad |U_{\mu i}| \simeq |U_{\tau i}| \quad \longleftrightarrow \quad \begin{cases} \theta_{23}^l = \frac{\pi}{4} \;,\; \theta_{13}^l = 0 \\ \quad\quad or \\ \theta_{23}^l = \frac{\pi}{4} \;,\; \delta^l = \pm\frac{\pi}{2} \end{cases}$$

- This $\mu - \tau$ symmetry implies specific patterns of mass and mixing:

$$U_{PMNS}^{\mu\tau} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}c_{12}^l & \sqrt{2}s_{12}^l & 0 \\ -s_{12}^l & c_{12}^l & \mp 1 \\ \mp s_{12}^l & \pm c_{12}^l & 1 \end{pmatrix} \quad \longleftrightarrow \quad M_{\nu U}^{\mu\tau} = \begin{pmatrix} M_{ee} & M_{e\mu} & \pm M_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ \pm M_{e\mu} & M_{\mu\tau} & M_{\mu\mu} \end{pmatrix}$$

Clearly now just a toy model!

Nice review at hep-ph/1512.04207

- What does this mean at the level of the Lagrangian?

Hidden symmetries

- Let's assume neutrino masses generated by Type-I See-saw:

$$\mathcal{L}_{\nu,\text{mass}} \sim \frac{1}{2}\overline{\nu}^c_L M_{\nu U}\nu_L + h.c. \qquad \nu_L = (\nu_e, \nu_\mu, \nu_\tau) \qquad M_{\nu U} = U^\star_{PMNS} m_\nu U^\dagger_{PMNS} = M^T_{\nu U}$$

- We can promote the phenomenological observation to a physical symmetry:

$$\nu \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \to \begin{pmatrix} \nu_e \\ \pm\nu_\tau \\ \pm\nu_\mu \end{pmatrix} = S^{\mu\tau}\nu \qquad S^{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \pm 1 \\ 0 & \pm 1 & 0 \end{pmatrix} \qquad M^{\mu\tau}_{\nu U} = S^{\mu\tau} M_{\nu U} S^{\mu\tau}$$

- The group generated by the $\mu - \tau$ operator mediates the simplest type of flavor symmetry: an Abelian discrete symmetry:

$$\mathcal{G}_\nu = \mathbb{Z}_2$$

- This action is actually a specific instance of the rather generic statement that *the maximal 'residual' symmetry of a Majorana mass term is the Klein four group,* $\mathbb{Z}_2 \times \mathbb{Z}_2$, cf. **Lam 2007**.

Tri-bimaximal neutrino mixing

- Before 2012, the leptonic mixing data was also consistent with other famous relationships:

$$\sin^2 2\theta^l_{12} = \frac{8}{9} \quad \longleftrightarrow \quad U^{\mu\tau}_{TBM} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{pmatrix}$$

Harrison, Perkins, Scott (PLB 530 2002)

- This is consistent with another symmetry operation:

$$M^{\mu\tau}_{TBM} = S^{TBM} M^{\mu\tau}_{TBM} S^{TBM} \qquad \left[S^{TBM}, S^{\mu\tau}\right] = 0 \qquad S^{TBM} = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

- Hence $S^{TBM} \times S^{\mu\tau} \sim \mathbb{Z}_2 \times \mathbb{Z}_2$ as expected. The charged leptons respect $U(1)^3$ rotations…

$$\mathcal{L}_{l,\text{mass}} \sim \overline{E}_R m_l l_L + h.c. \qquad m_l^\dagger m_l = T^\dagger m_l^\dagger m_l T \qquad T = diag\left(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau}\right)$$

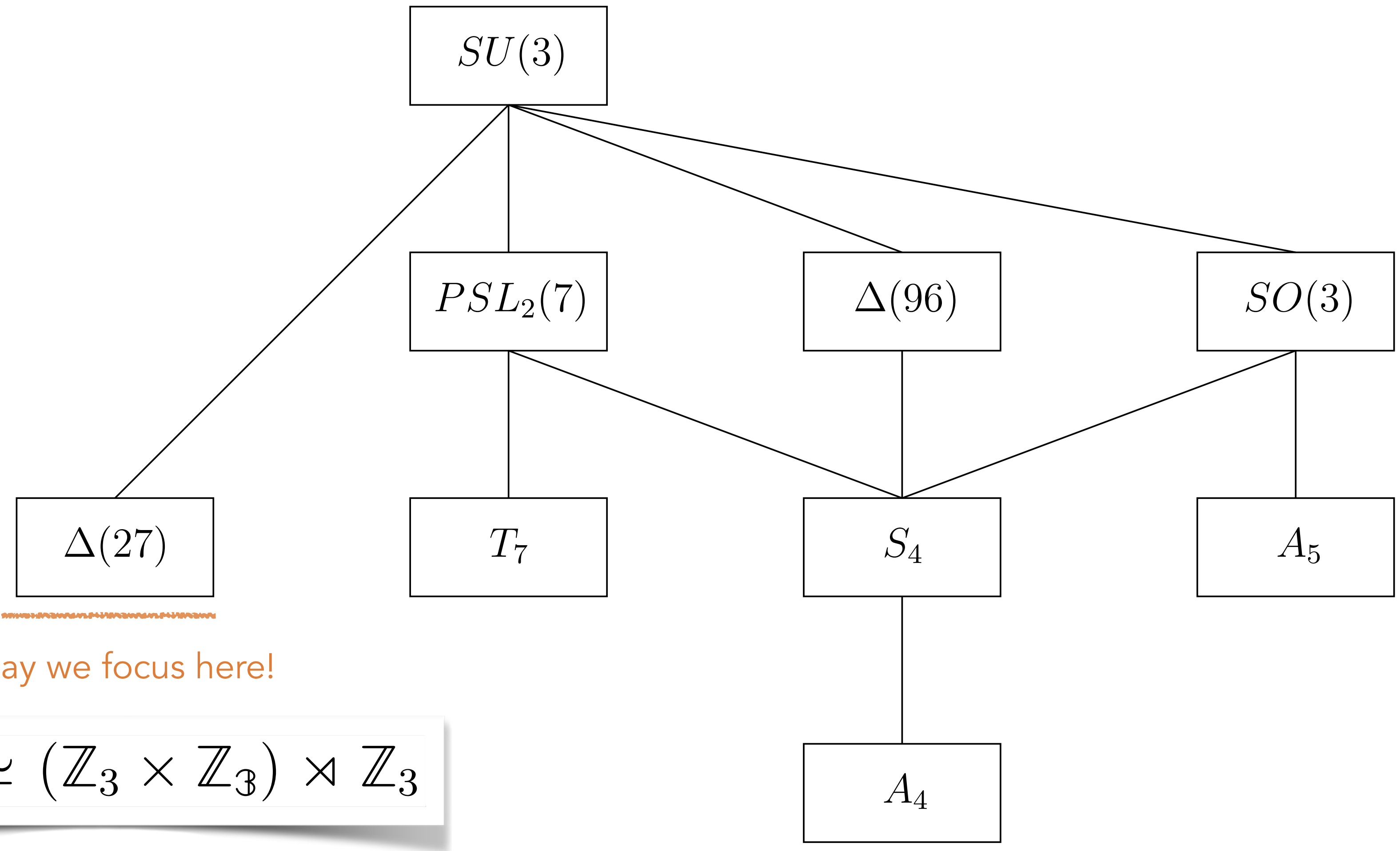
- The group closed by $\{S^{\mu\tau}, S^{TBM}, T\}$ is **S4** (cubic group), by $\{S^{TBM}, T\}$ is **A4** (tetrahedral group)…

- The most famous models to my knowledge implementing A_4 are **Babu, Ma, Valle:** hep-ph/0206292 and **Altarelli, Feruglio:** hep-ph/0512103, 0504165.

The discrete approach

Reviews: King, Luhn: hep-ph/1301.1340, **Grimus**, Ludl: hep-ph/1110.6376, Altarelli, Feruglio: hep-ph/1002.0211
Encyclopedia: Ishimori et al.: hep-ph/1003.3552

All of these symmetries have been explored in models…



Today we focus here!

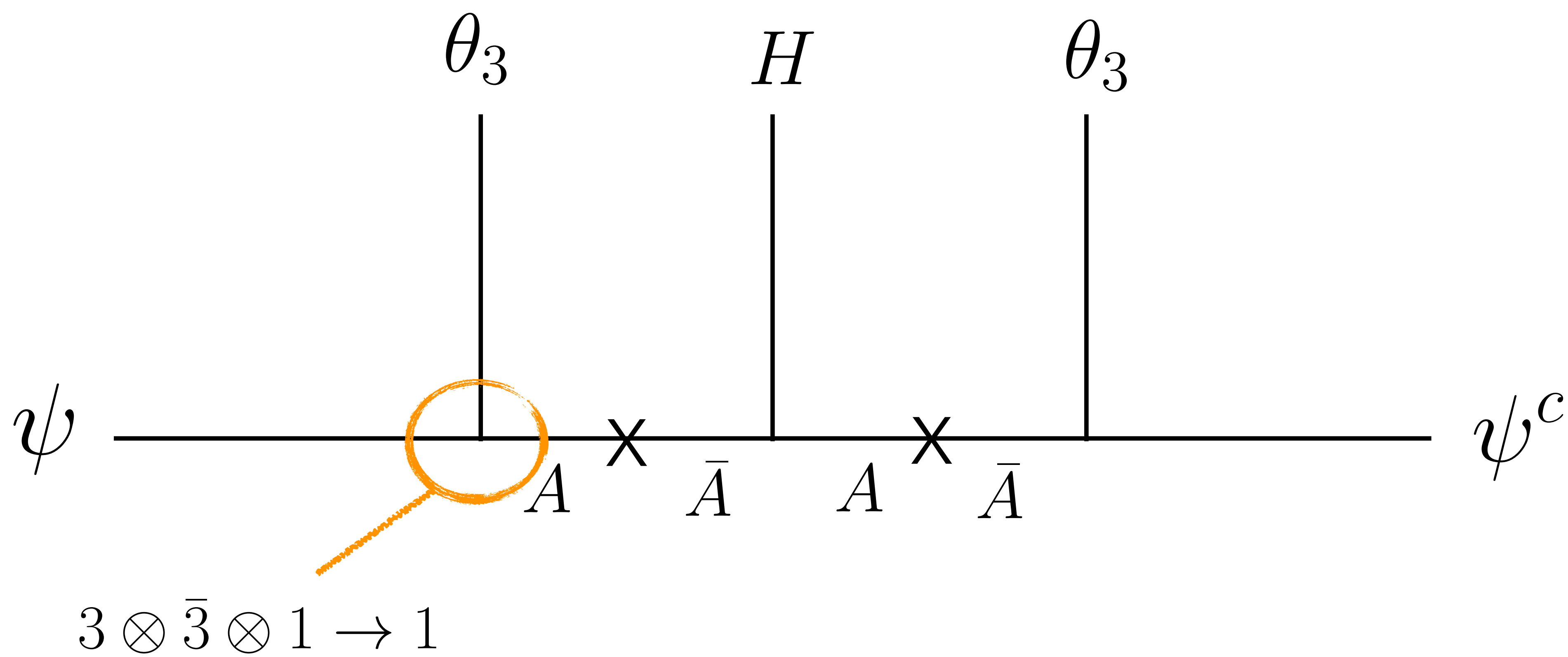
$$\Delta(27) \simeq (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_3$$

Symmetry breaking can also be studied in a 'model-independent' way…cf. work from Grimus et al. here in Vienna, e.g.!

- Discrete symmetries avoid Goldstone modes that could spoil phenomenology, easily embedded in SUSY GUTs, extra dimensional theories.
- Easier facilitation of vacuum alignment than with continuous symmetries
- Huge literature: Pakvasa, Sugawara (1977) use S3 for Cabibbo angle. Deshpande uses S4 for full CKM and Pakvasa applies S4 to neutrino mass and mixing (1984). Early 90s discussion (Kaplan, Schmaltz; Frampton, Kephart), TBM and GUT models established early-mid 00s (Ma, Rajasekaran; Altarelli, Feruglio, de M. Varzielas, King, Ross +), new flood in 2012/13 after reactor angle…

Effective operators

- In the IR, we can build effective mass matrices with higher dimensional operators:



- In the UV, each vertex is part of the full Lagrangian (messengers A integrated out):

$$\mathcal{L}_{UV} \sim \psi\, \theta_3\, A + \bar{A}\, H\, A + \ldots \qquad \mathcal{L}_{IR} \sim \psi\, \theta_3\, H\, \theta_3\, \psi^c$$

- Hence by assigning the messengers to trivial singlets, one can form family symmetry invariants:

$$\mathcal{L} \sim \psi_i\, \theta_3^i\, \theta_3^j\, \psi_j^c\, H \sim \mathbf{1}$$

Mass matrices from flavons

- Flavons acquire vacuum expectation values along specific directions in flavour space:

$$\langle \theta_3 \rangle = v_3 \cdot (0,0,1)$$

Note: alignment is generally non-trivial to achieve, but facilitated with the NADS

- Mass matrices then follow from the form of the effective operator:

$$\mathcal{L}_Y \left(\psi, \psi^c, H, \theta_i\right) \Leftrightarrow \mathcal{M}\left(\psi, \psi^c, \langle H, \theta_i \rangle\right)$$

$$\mathcal{L} \sim \psi_i \, \theta_3^i \, \theta_3^j \, \psi_j^c \, H \quad \Rightarrow \quad \mathcal{M} \propto v_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Family symmetries shape the Yukawa sector, align VEVs, and thereby control fermionic mass and mixing matrices

- Let's use phenomenologically successful mass patterns to guide the construction of our model…

See e.g Roberts, Romanino, Ross, Velasco-Sevilla: hep-ph/0104088
See talk from Steve King from Corfu 2022 for history of this matrix…

A universal texture zero (UTZ) for fermions

- A (1,1) texture zero can accurately reproduce the phenomenology of the charged fermions.

$$M_a^D \approx m_3 \begin{pmatrix} 0 & \varepsilon_a^3 & \varepsilon_a^3 \\ \varepsilon_a^3 & r_a\varepsilon_a^2 & r_a\varepsilon_a^2 \\ \varepsilon_a^3 & r_a\varepsilon_a^3 & 1 \end{pmatrix}, \quad r_{u,d} = 1, \quad r_l = -3$$

$$\epsilon_u \simeq 0.05, \qquad \epsilon_{d,l} \simeq 0.15$$

- It implements the well known **Georgi-Jarlskog (PLB 86 1979)** mass relation and also the successful **Gatto-Sartori-Tonin (PLB 28 1968)** relation:

$$\sin\theta_c = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \right|$$

$$m_b \approx 3\, m_\tau$$
$$m_s \approx 3 \times \frac{1}{3}\, m_\mu$$
$$m_d \approx 3 \times 3\, m_e$$

Red indicates RGE to IR

IDEA: Can this successful texture be extended to the neutrino sector?

- If so, can we realize this phenomenology in a concrete model?

The UTZ model

See de Medeiros Varzielas, Ross, JT:
UTZ: [1710.01741]

Fields	$\psi_{q,e,\nu}$	$\psi^c_{q,e,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	3	3	1_{00}	1_{00}	1_{00}	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	3
Z_N	0	0	0	2	-1	0	-1	2	0	x

$$\mathcal{G}_F = \Delta(27) \times \mathbb{Z}_N$$

- The UTZ can appear from the same Lagrangian for all Dirac sectors!

$$\mathcal{L}^{LO}_{D,f} = \psi_i \left(\frac{c_3^{(6)}}{M^2_{3,f}} \theta^i_3 \theta^j_3 + \frac{c_{23}^{(7)}}{M^3_{23,f}} \theta^i_{23}\theta^j_{23}\Sigma + \frac{c_{123}^{(7)}}{M^3_{123,f}} (\theta^i_{123}\theta^j_{23} + \theta^i_{23}\theta^j_{123}) S \right) \psi^c_j H$$

$f \in \{u, d, e, \nu\}$

Messenger masses distinguish fermion species

VEV implements Georgi-Jarlskog

Needed for shaping symmetry

$$\mathcal{L}^{\nu}_{\mathcal{M}} = \psi^c_i \left(\frac{c_M^{(5)}}{M} \theta^i \theta^j + \frac{1}{M^4} [c^{(8)}_{M,1}\, \theta^i_{23}\theta^j_{23} (\theta^k \theta^k \theta^k_{123}) + c^{(8)}_{M,2}\, (\theta^i_{23}\theta^j_{123} + \theta^i_{123}\theta^j_{23})(\theta^k\theta^k\theta^k_{23})] \right) \psi^c_j$$

- A Type-I See-saw generates active light neutrinos:

$$M^D_\nu \cdot M^{M,-1}_\nu \cdot M^{D,T}_\nu \Longrightarrow M_\nu$$

Vacuum alignment (proof in principle)

- We want to achieve the 3, 123, and 23 alignments.

$$\langle\theta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mathrm{v}_3, \quad \langle\theta_{123}\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\beta} \\ e^{i\alpha} \\ -1 \end{pmatrix} \mathrm{v}_{123}, \quad \langle\theta_{23}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\alpha} \\ 1 \end{pmatrix} \mathrm{v}_{23}, \quad \left\langle\theta_X^\dagger\right\rangle = \frac{1}{\sqrt{6}} \begin{pmatrix} 2e^{i\beta} \\ -e^{i\alpha} \\ 1 \end{pmatrix} \mathrm{v}_X$$

$$V_1(\theta_i) = m_i^2 |\theta_i|^2 \qquad V_2(\theta_i) = h_i (\theta_i)^2 \left(\theta^{\dagger i}\right)^2.$$

Sets scale of familon fields and breaks family symmetry upon negative m^2

Completes alignment of $\theta_{3,123}$, depending on sign of h (h<0 -> θ_3)!

$$V_3 = k_1 \theta_{X,i} \theta_{123}^{\dagger i} \theta_{123,j} \theta_X^{\dagger j}, \quad k_1 > 0.$$

Orthogonal to 123, but does not distinguish between (0,1,1) and (2,-1,1)

$$V_4 = k_2 m_0 \theta_X^1 \theta_X^2 \theta_X^3 \qquad V_5 = k_3 \theta_{23,i} \theta_X^i \theta_{23}^{\dagger j} \theta_X^{\dagger j} + k_4 \theta_{23,i} \theta_3^{\dagger i} \theta_{3,i} \theta_{23}^{\dagger i}, \quad \text{with } k_3 > 0 \text{ and } k_4 < 0$$

Respectively select (2,-1,1), (0,1,1)!

$$\boxed{V = \sum_{i=3,123} (V_1(\theta_i) + V_2(\theta_i)) + V_3 + V_4 + V_5}$$

- Alignment of LNV family discussed in paper…

Discrete anomaly freedom

Krauss, Wilczek : PRL 62 (1989)
Ibanez, Ross : PLB 260 (1991)
Banks, Dine : PRD 45 (1992)
Araki et al. : NPB 805 (2008)
Talbert : PLB 786 (2018)

- Some argue that discrete symmetries should/must be gauged in the UV. This means anomaly cancellation must be enforced.

$$D-G-G, \quad D-g-g, \quad Z-G-G, \quad Z-g-g$$

$Z/D-G-G:$ $\boxed{\sum_{\mathbf{r}^{(f)},\mathbf{d}^{(f)}} tr\left[\tau(\mathbf{d}^{(f)})\right]\cdot l(\mathbf{r}^{(f)}) \overset{!}{=} 0 \bmod \frac{N}{2}}$ $\quad tr\left[\tau(\mathbf{d}^{(f)})\right] = N\frac{\ln\det U(\mathbf{d}^{(f)})}{2\pi i}$

$D-g-g:$ $\boxed{\sum_{\mathbf{d}^{(f)}} tr\left[\tau(\mathbf{d}^{(f)})\right] \overset{!}{=} 0 \bmod \frac{N}{2}}$ $\quad Z-g-g:$ $\boxed{\sum_f q^{(f)} = \sum_m q^{(m)}\cdot\dim\mathbf{R}^{(m)} \overset{!}{=} 0 \bmod \frac{N}{2}}$

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Z_N	0	0	0	2	-1	0	-1	2	0	x

$\Delta(3N^2)$	$\mathbf{1}_{\mathbf{k},\mathbf{l}}$	$\mathbf{3}_{[\mathbf{k}][\mathbf{l}]}$
$\det(b)$	ω^k	1
$\det(a)$	ω^l	1
$\det(a')$	ω^l	1

Anomalies trivially satisfied at relevant scale of EFT

Neutrino mixing, qualitatively

- Sequential Dominance limit (King: 1998-2002): the third generation RH Majorana neutrino mass is large:

$$M_{R,3} \gg M_{R,2} \gg M_{R,1}$$

- This means that the LH neutrino mass matrix via the see-saw is effectively 2D:

$$M_{Majorana} \propto \begin{pmatrix} 0 & c_2 \\ c_2 & c_1 + 2\,c_2 \end{pmatrix}, \quad M_{Dirac} \propto \begin{pmatrix} 0 & \sqrt{3/2} \\ 1 & 1+s \end{pmatrix} \underbrace{\Longrightarrow}_{\text{see-saw}} M_\nu \propto \begin{pmatrix} 0 & -\sqrt{3/2}\,c_2 \\ -\sqrt{3/2}\,c_2 & c_1' \end{pmatrix}$$

$$c_1 \gg c_2 \qquad s \propto \langle\Sigma\rangle\langle\theta_{23}\rangle/(\langle S\rangle\langle\theta_{123}\rangle) \qquad c_1' \equiv c_1 - 2\,c_2\,s$$

- From which we can derive relationships for the mass ratios and neutrino mass eigenstates:

$$\frac{m_2}{m_1} \approx \frac{3}{2}\frac{c_2^2}{c_1^{\prime,2}}, \quad \frac{c_2}{c_1'} \equiv \left|\frac{c_2}{c_1'}\right| e^{i\eta}, \qquad \nu_1 \propto \nu_a - e^{i\eta}\sqrt{\frac{m_2}{m_1}}\,\nu_b$$

- Which then also generates simple equalities for neutrino mixing angles:

$$\sin\theta_{13}^\nu \approx \sqrt{\frac{m_2}{3m_1}} \qquad \sin\theta_{23}^\nu \approx \left|\frac{1}{\sqrt{2}} - e^{i\eta}\sin\theta_{13}^\nu\right| \qquad \sin\theta_{12}^\nu \approx \frac{1}{\sqrt{3}}$$

- Note the clear departure from TBM mixing. Also, corrections from the charged lepton sector drive the reactor angle to acceptable values…

Complete UTZ mass matrices

- The UTZ Lagrangian then generates LO Dirac matrices of the following form:

$$\mathcal{M}_i^D \equiv \frac{M_i^D}{c} \simeq \begin{pmatrix} 0 & a\,e^{i(\alpha+\beta+\gamma)} & a\,e^{i(\beta+\gamma)} \\ a\,e^{i(\alpha+\beta+\gamma)} & (b\,e^{-i\gamma} + 2a\,e^{-i\delta})\,e^{i(2\alpha+\gamma+\delta)} & b\,e^{i(\alpha+\delta)} \\ a\,e^{i(\beta+\gamma)} & b\,e^{i(\alpha+\delta)} & 1 - 2a\,e^{i\gamma} + b\,e^{i\delta} \end{pmatrix}$$

$$a_f' = \frac{c_{123}^{(7)}\mathrm{v}_{123}\mathrm{v}_{23}\,\langle S\rangle}{\sqrt{6}M_{123,f}^3}, \quad b_f' = \frac{c_{23}^{(7)} r_f \mathrm{v}_{23}^2\,\langle\Sigma\rangle}{2M_{23,f}^3}, \quad c_f' = \frac{c_{123}^{(8)}\mathrm{v}_{123}\mathrm{v}_3\langle \mathrm{S}\rangle^2}{\sqrt{3}M_{123,f}^4}, \quad d_f' = \frac{c_{23}^{(8)} r_f \mathrm{v}_{23}\mathrm{v}_3\langle\Sigma\rangle\langle \mathrm{S}\rangle}{\sqrt{2}M_{23,f}^4}$$

Can 9 low-energy parameters successfully describe 18+ 'observables'?

- Also: corrections come from HO operators in UTZ Lagrangian in principle exist…

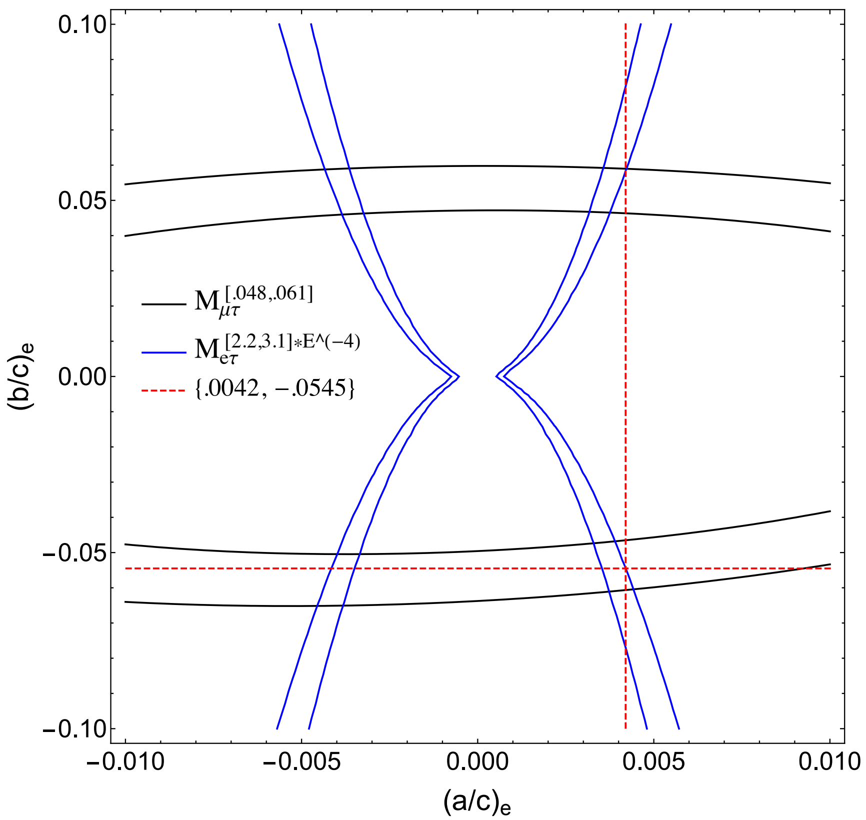
$$\mathcal{L}_{D,f}^{HO} = \psi_i \left(\frac{c_{23}^{(8)}}{M_{23,f}^4}(\theta_{23}^i\theta_3^j + \theta_3^i\theta_{23}^j)\Sigma S + \frac{c_{123}^{(8)}}{M_{123,f}^4}(\theta_{123}^i\theta_3^j + \theta_3^i\theta_{123}^j)S^2 \right) \psi_j^c H$$

$$\mathcal{M}_f^{\mathcal{D}} \simeq \begin{pmatrix} 0 & a\,e^{i(\alpha+\beta+\gamma)} & a\,e^{i(\beta+\gamma)} + c\,e^{i(\beta+\zeta)} \\ a\,e^{i(\alpha+\beta+\gamma)} & (b\,e^{-i\gamma} + 2a\,e^{-i\delta})\,e^{i(2\alpha+\gamma+\delta)} & b\,e^{i(\alpha+\delta)} + c\,e^{i(\alpha+\zeta)} + d\,e^{i(\alpha+\psi)} \\ a\,e^{i(\beta+\gamma)} + c\,e^{i(\beta+\zeta)} & b\,e^{i(\alpha+\delta)} + c\,e^{i(\alpha+\zeta)} + d\,e^{i(\alpha+\psi)} & 1 - 2a\,e^{i\gamma} + b\,e^{i\delta} - 2c\,e^{i\zeta} + 2d\,e^{i\psi} \end{pmatrix}$$

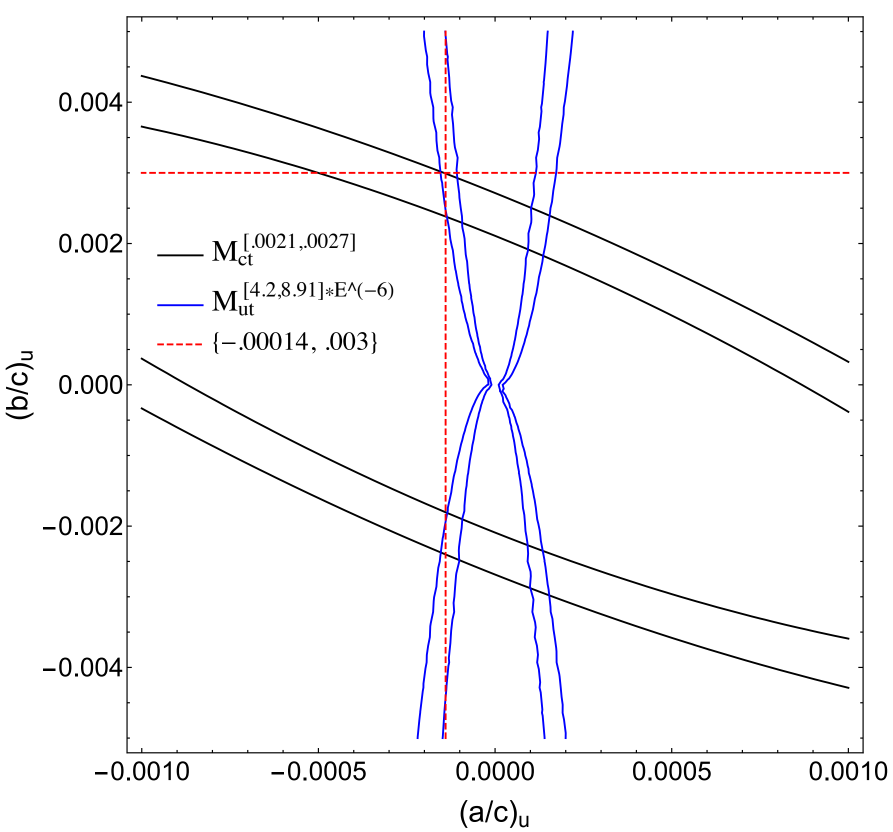
A naive fit to masses and mixings

UTZ: [1710.01741]

Charged Leptons



Up Quarks



2017 Pheno

Basic 'contour' analysis, looking for theory correlations in a semi-analytic fashion.

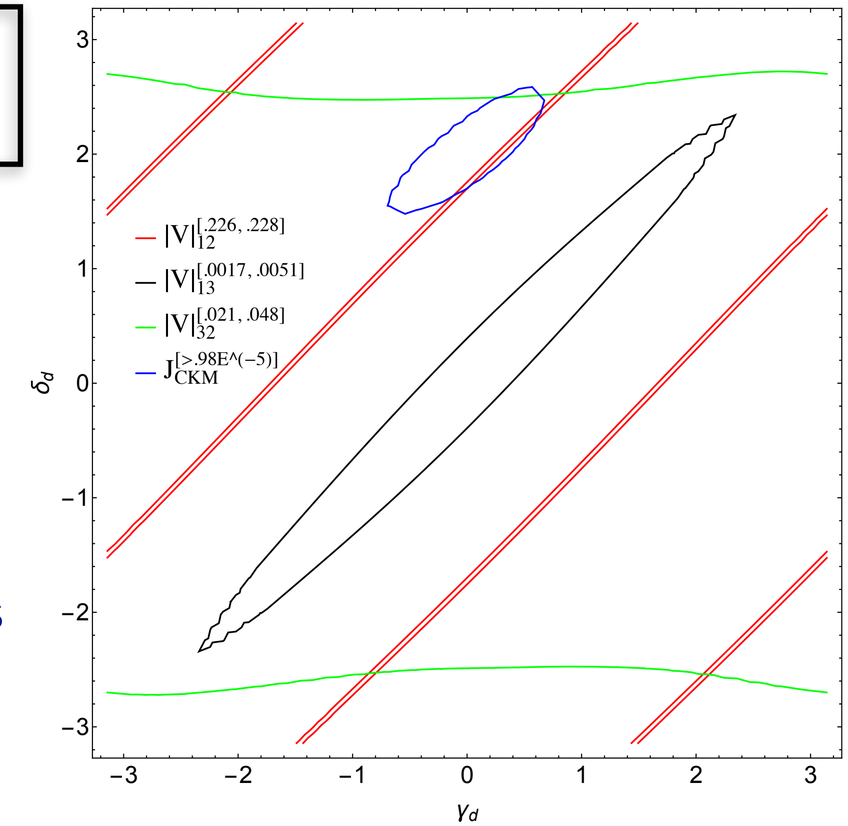
Successful, 'proof-in-principle' fit. Mass ratios + PMNS looked great.

Problem: Looks fine-tuned, + minor discrepancies in exterior off diagonal CKM elements @ LO.

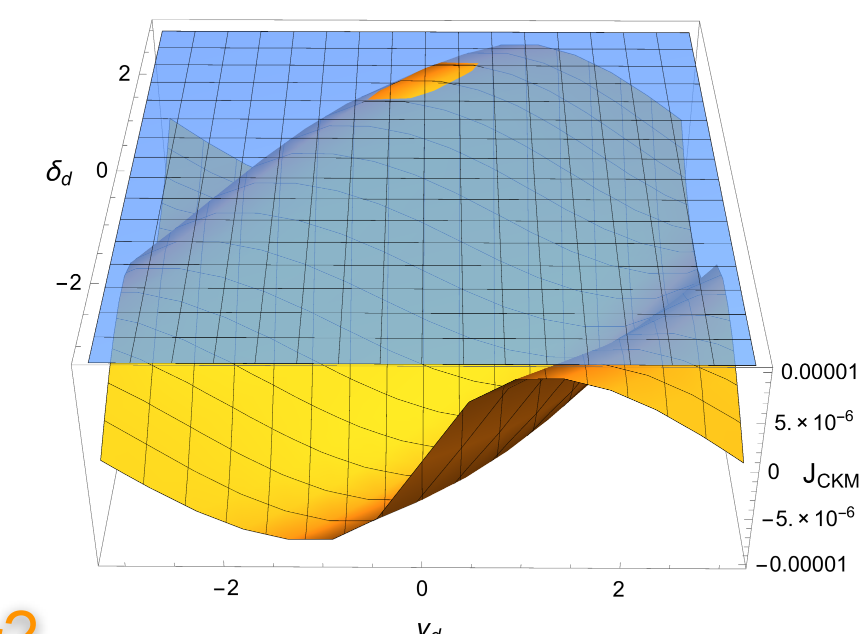
Proposal: Include two additional parameters from HO operators in OPE, by hand…

Can we do better?

CKM Elements



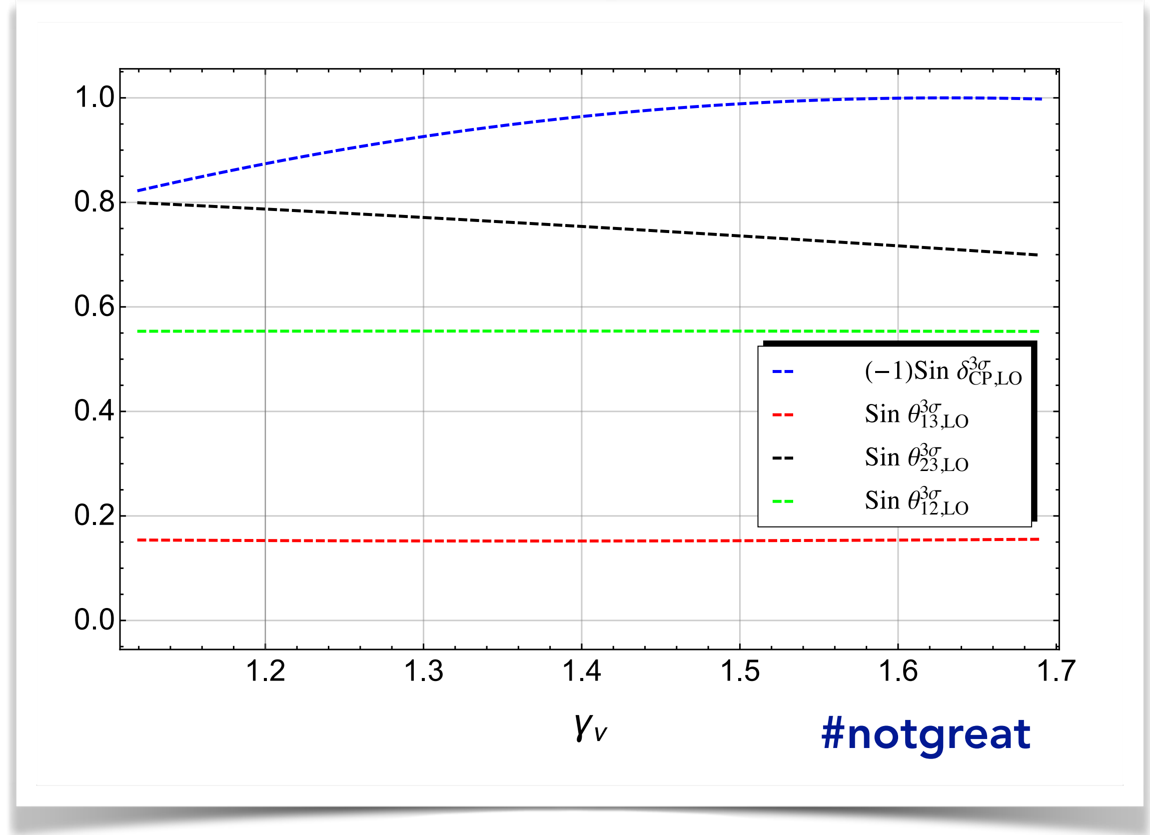
Jarlskog Invariant



Leptonic CP violation

Nu-Fit Collaboration (2020)

	Normal Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$
$\delta_{\rm CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$



UTZ: [1710.01741]

- Experiment has not yet put fully robust bounds on the PMNS' Dirac CP-violating phase δ_{CP}, while Majorana phases ϕ_i are fully unconstrained.
- This represents an opportunity for flavor models to actually predict, rather than retrodict, fundamental flavor structure.
- In 2017 we did not have the numerical tools necessary to do so reliably.

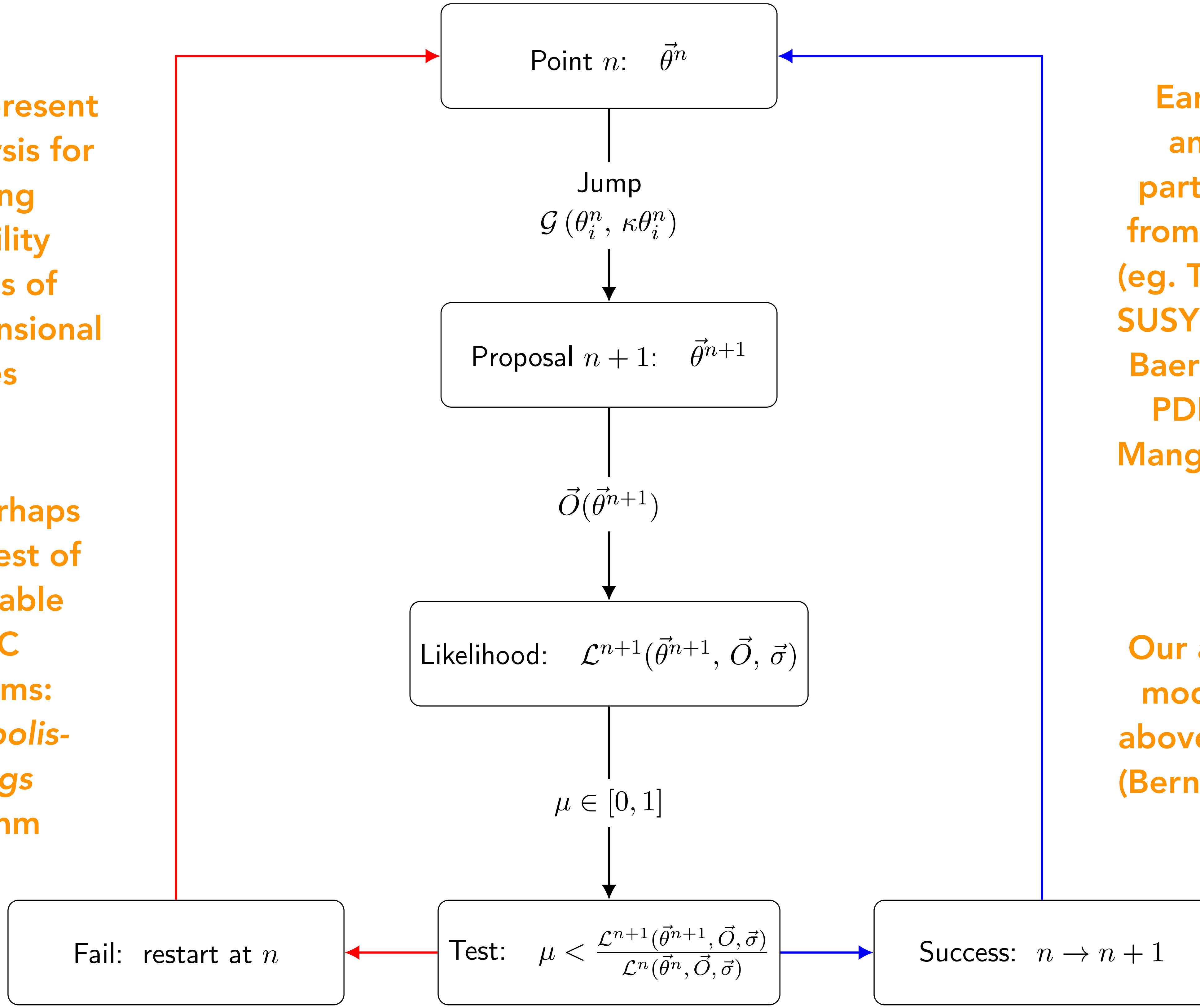
ME FROM 2017/18: ***"…the intricate interdependence between model parameters and physical observables in our GUT construction makes obtaining concrete error bands very difficult (future work)"***

A simple Markov Chain Monte Carlo

[2107.04613]
[2111.10199]

MCMC represent ideal analysis for sampling probability functions of multi-dimensional spaces

This is perhaps the simplest of the available MCMC algorithms: a *Metropolis-Hastings* algorithm



Early MCMC analyses in particle theory from cosmology (eg. Trotta et al.), SUSY pheno (e.g. Baer et al.), and PDF fits (e.g. Mangin-Brenet at al.) …

Our approach is modified from above references (Bernigaud et al.)

UTZ MCMC specifics

Parameter ranges obtained via
1) trial and error, and
2) with basic physics assumptions regarding the magnitudes of suppressed EFT coefficients

Note that the MCMC distributions obtained are of course sensitive to experimental constraints contributing to the overall Likelihood functions driving the Markov Chain…

LO UTZ Model Parameter MCMC Ranges & Global Best Fits

	$(a,b)_d \cdot 10^3$	$(a,b)_u \cdot 10^5$	$(a,b)_\nu \cdot 10^1$	$(x,y) \cdot 10^3$
Range	$([2,6],[10,20])$	$(\mp 30, \mp 800)$	∓ 5	∓ 5
LO	$(3.579, 15.924)$	$(6.720, -192.922)$	$(-1.166, 1.818)$	$(-0.146, -4.641)$
HO	$(3.415, 15.416)$	$(7.604, -200.279)$	$(-1.819, 2.440)$	$(3.728, 3.501)$
	$(\gamma,\delta)_d$	$(\gamma,\delta)_\nu$	(ρ,ϕ)	$M_\theta \cdot 10^{-11}$ [GeV]
Range	$[0, 2\pi]$	$[0, 2\pi]$	$[0, 2\pi]$	$[0.1, 10]$
LO	$(3.910, 5.782)$	$(3.163, 4.553)$	$(2.964, 4.784)$	3.084
HO	$(4.228, 6.134)$	$(0.464, 2.293)$	$(3.636, 3.976)$	9.918

HO UTZ Model Parameter MCMC Ranges

	$(c,d)_d \cdot 10^5$	$(c,d)_u \cdot 10^6$	$(c,d)_\nu \cdot 10^3$
Range	$(\mp 5, \mp 50)$	$(\mp 5, \mp 50)$	$(\mp 5, \mp 50)$
HO	$(0.640, 10.811)$	$(0.916, -37.298)$	$(-0.896, -1.565)$

$$\textbf{Constraints}: \quad \{R_{f_i f_3}\,(f \in u,d,e),\ \sin\theta_{ij}^{q,l}\,,\sin\delta^{q,l}\,,\Delta m^2_{sol,atm},\ m_{\beta(\beta)},\ m_\Sigma,\ \xi,\ \text{n.h.}\}$$

$$\textbf{Predictions}: \quad \{R_{\nu_i\nu_3},\ m_{\beta\beta}/m_\Sigma,\ m_\beta/m_\Sigma,\ m_{\beta\beta}/m_\beta,\ \sin\phi_1,\ \sin\phi_2\}$$

$$\textbf{Quasi-Predictions}: \quad \{\sin\delta^{q,l}\,,\ \xi\}$$

MCMC model parameter convergence

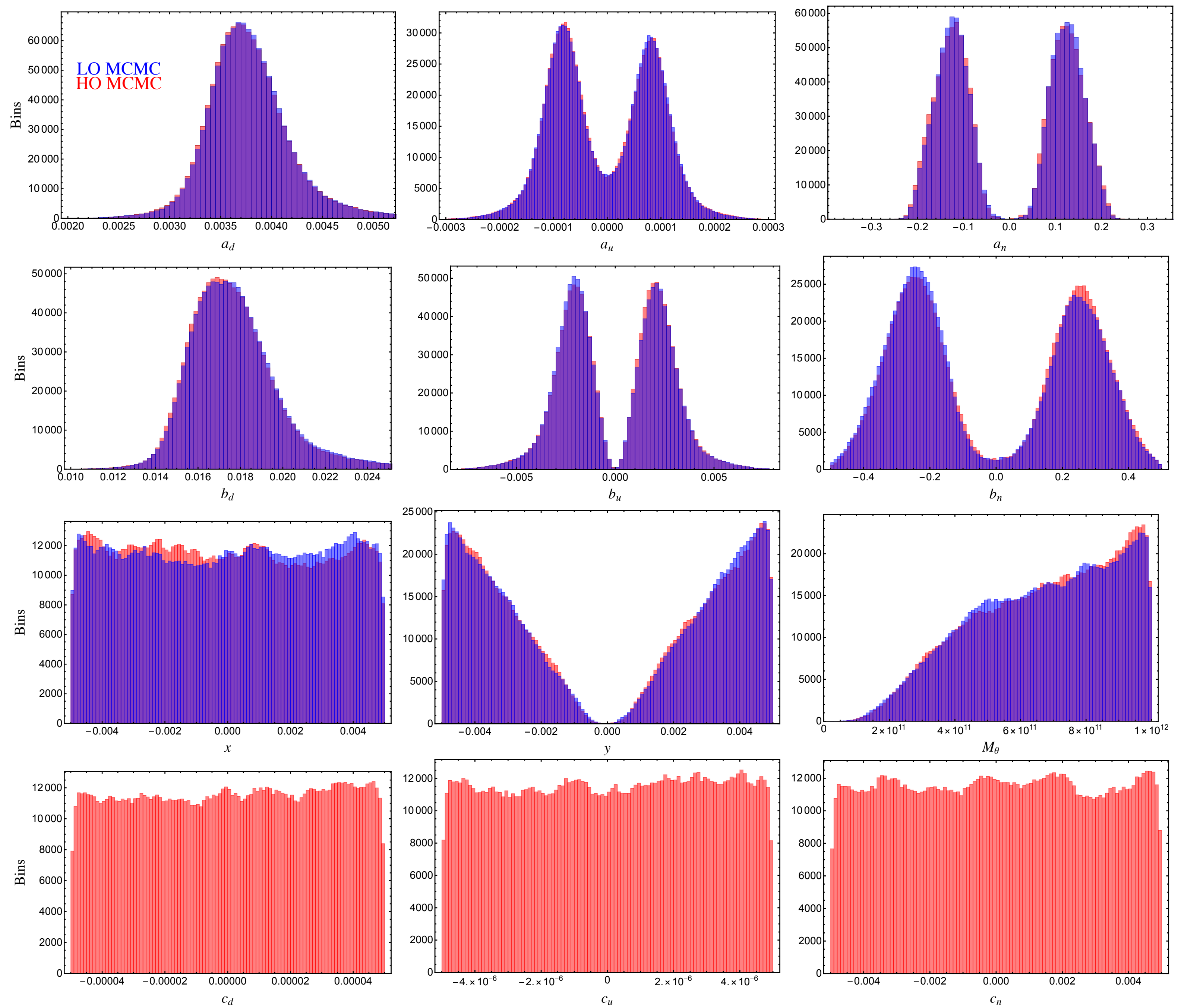
$$N_{\text{chains}} = 2500, \quad L_{\text{chains}} = 500, \quad N_{\text{burn}} = 40, \quad \kappa_0 = 0.01, \quad \epsilon = 0.00005$$

Our simple *Metropolis-Hastings* MCMC exhibits exceptional convergence at LO and HO.

Notice that including HO corrections has negligible impact on preference for LO parameters.

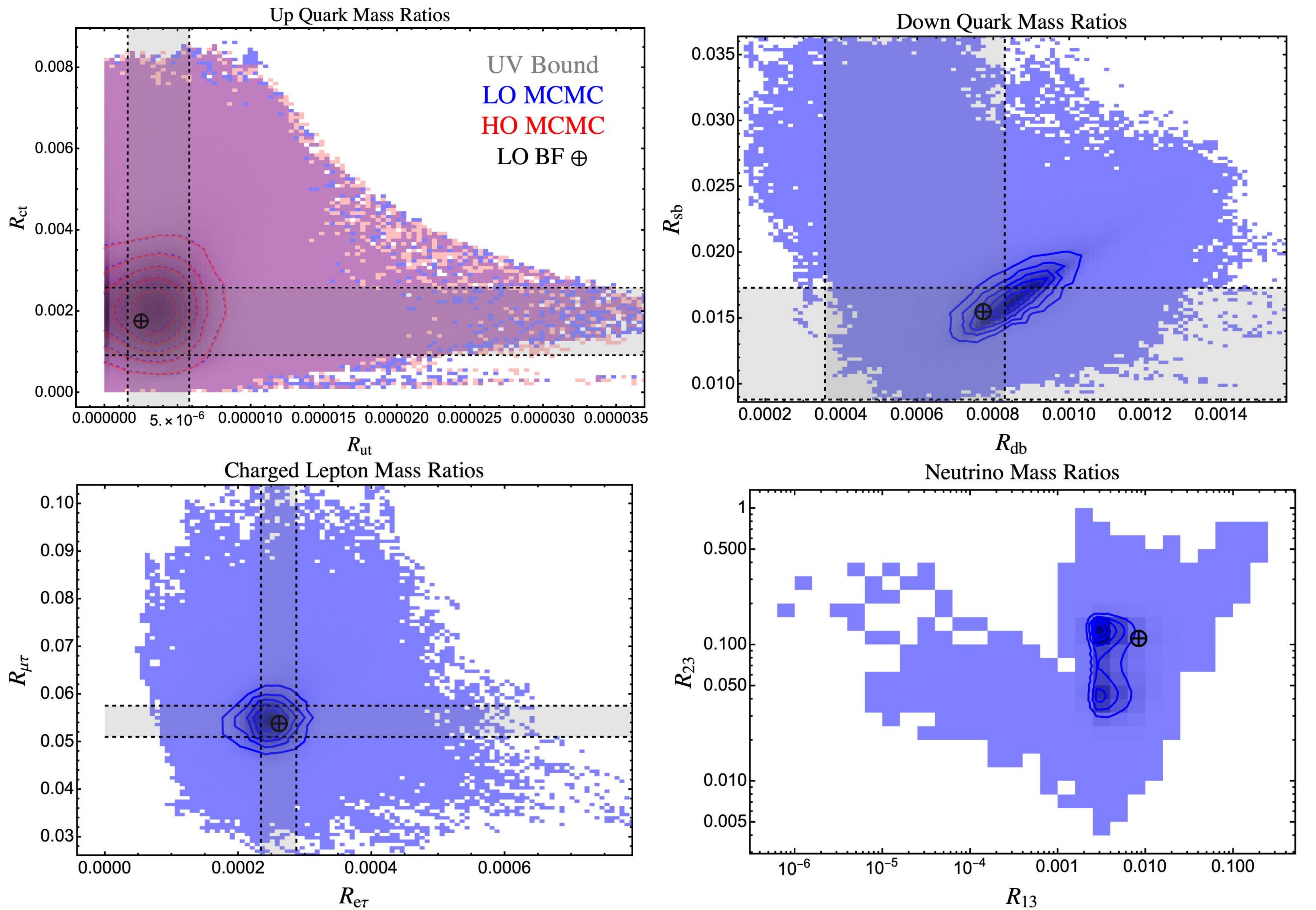
Also notice that there is virtually no preference for the purely HO parameter space (orange).

We did multiple consistency tests with our MCMC scripts, including stability of MCMC parameter variations…

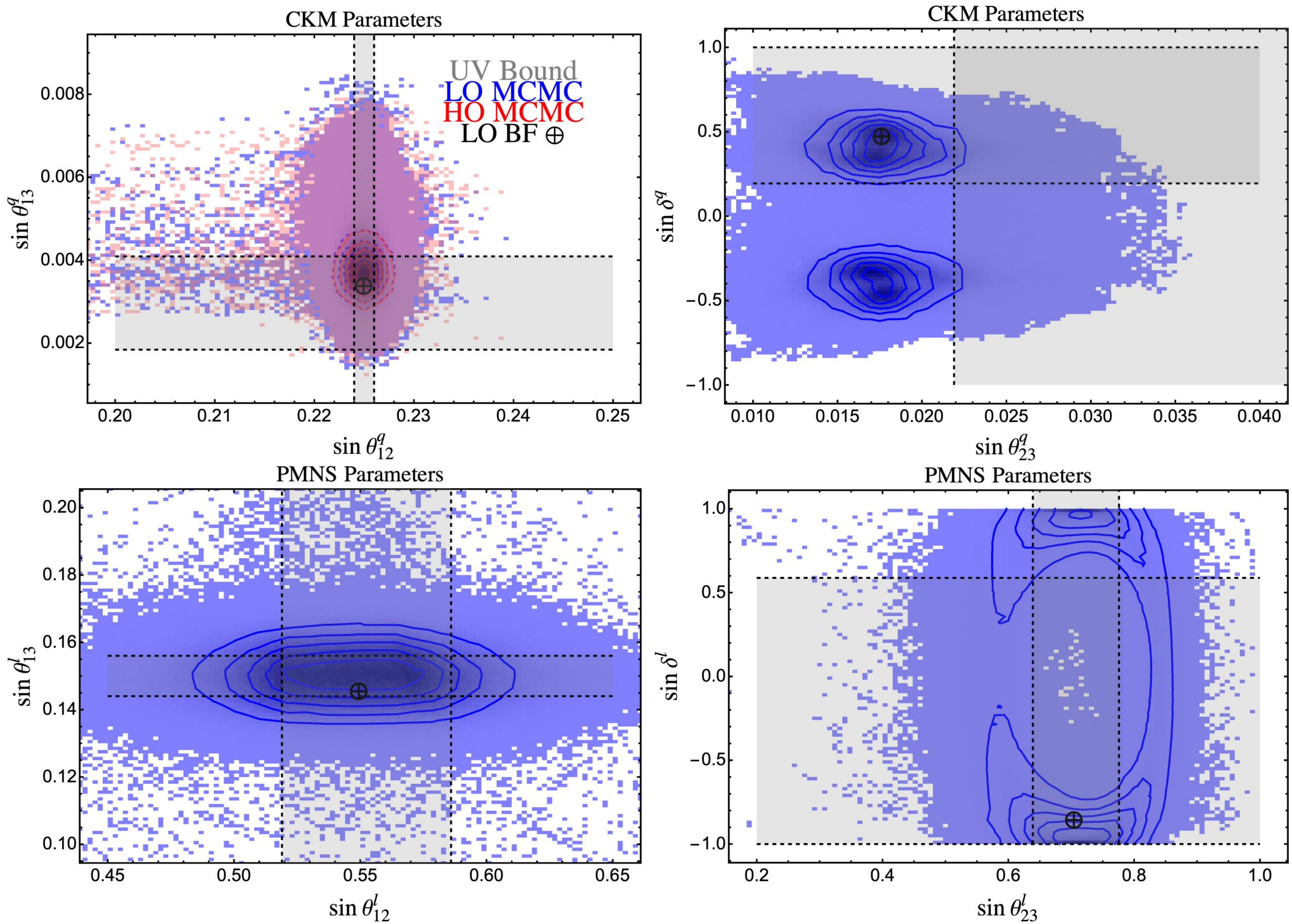


MCMC results: fermion mass ratios

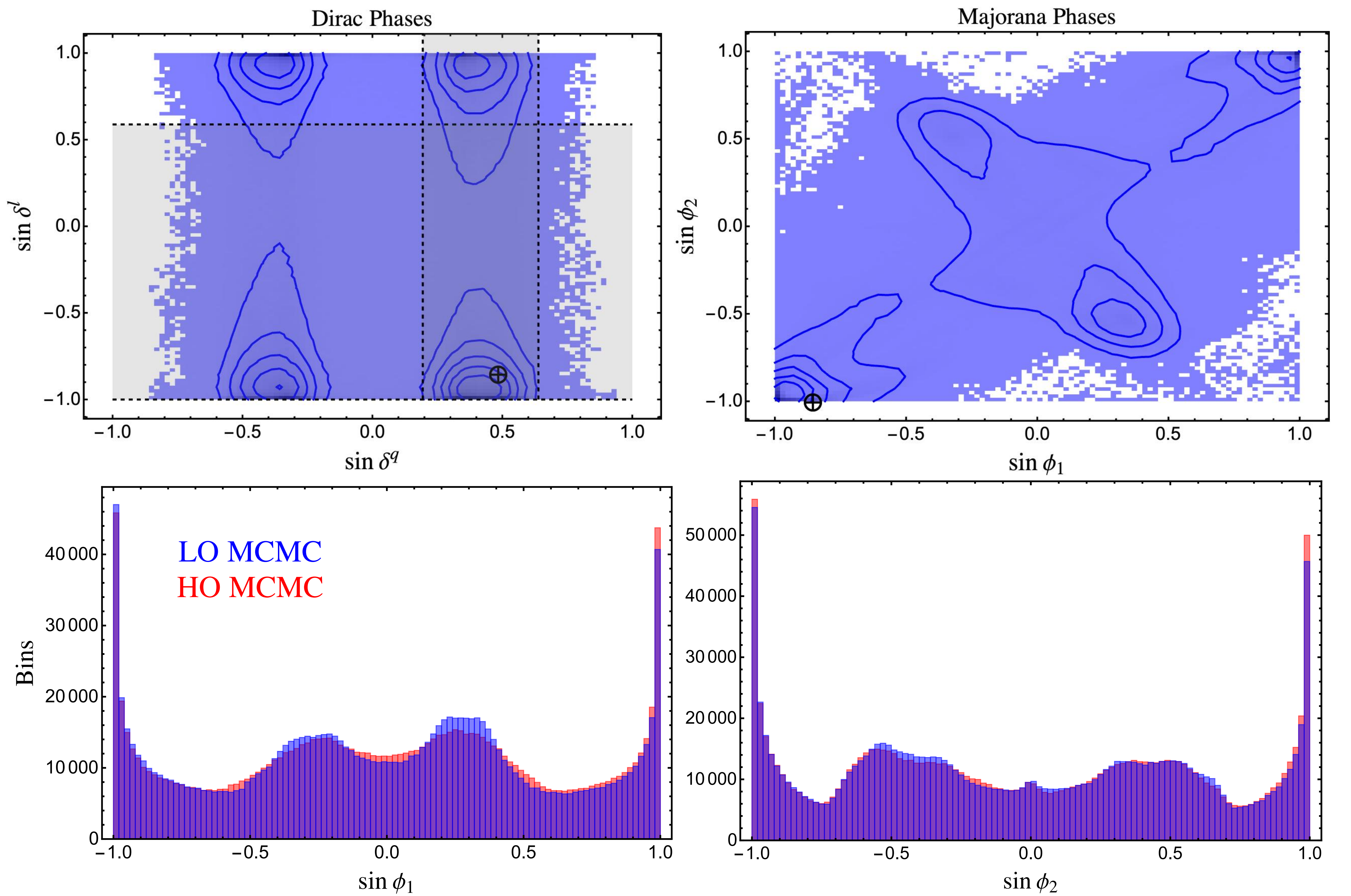
$R_{ij} = m_i / m_j$



MCMC results: fermion mixings



MCMC results: CP-violation



Precision probes of neutrino mass

- Cosmological constraints on the overall sum of neutrino masses exist:

$$m_\Sigma \equiv \sum_i m_{\nu_i} < 0.26 \;\; \mathrm{eV}$$

Planck

- As do non-trivial neutrino flavor constraints from β-decay probes:

$$\langle m_{\beta\beta}\rangle \equiv \Big|\sum_i V_{ei}^2\, m_i\Big| \; < \; (61 - 165)\cdot 10^{-3}\ \mathrm{eV}\,,$$

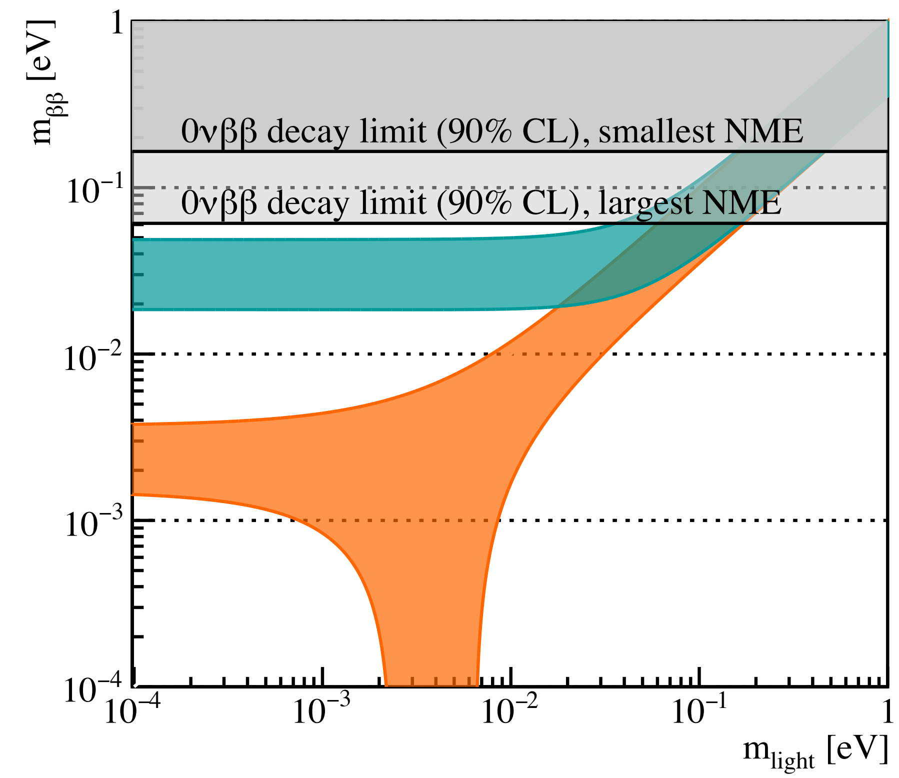
KamLand-ZEN

$$m_\beta \equiv \sqrt{\sum_i |V_{ei}|^2\, m_i^2} \; < \; 0.8\ \mathrm{eV}\,,$$

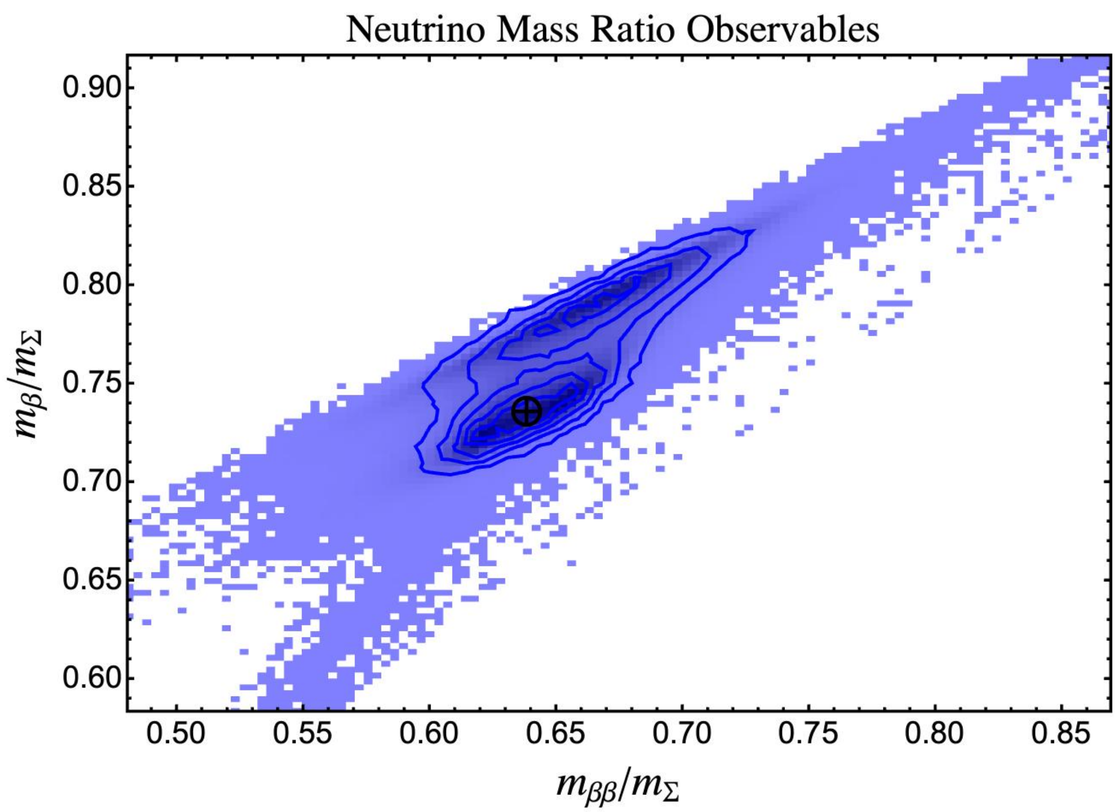
KATRIN

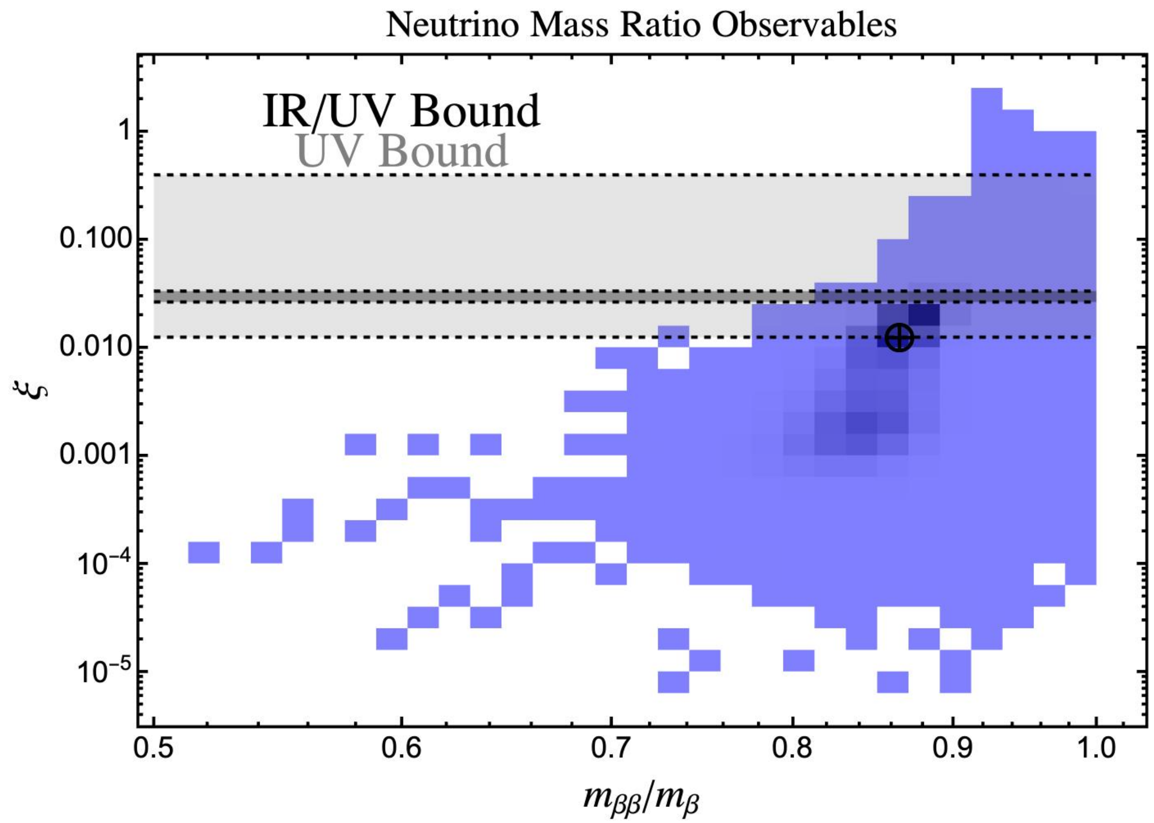
- Active and future experiment represents major area of low-energy research:

Figure from Snowmass 2022 report, borrowed from 2202.01787

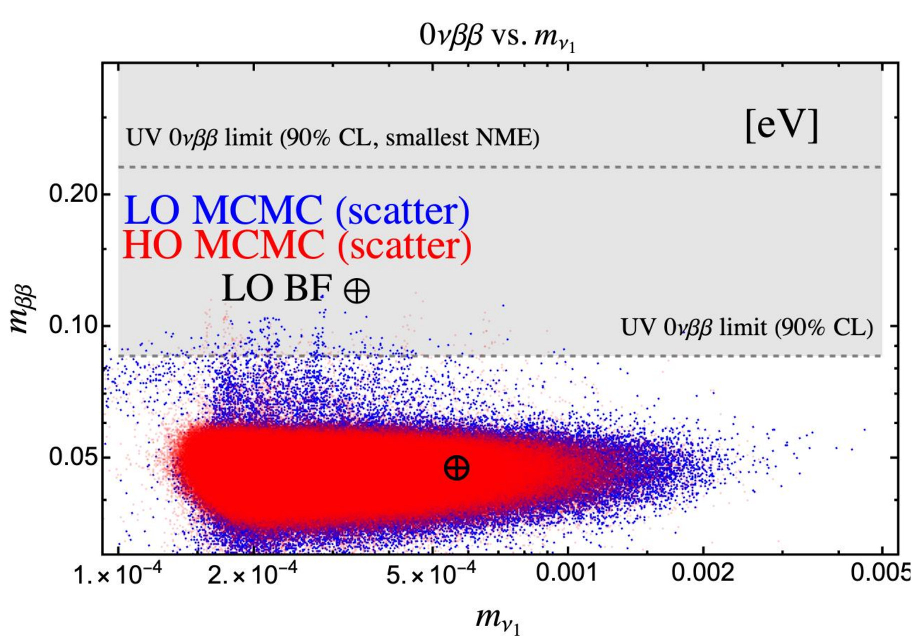


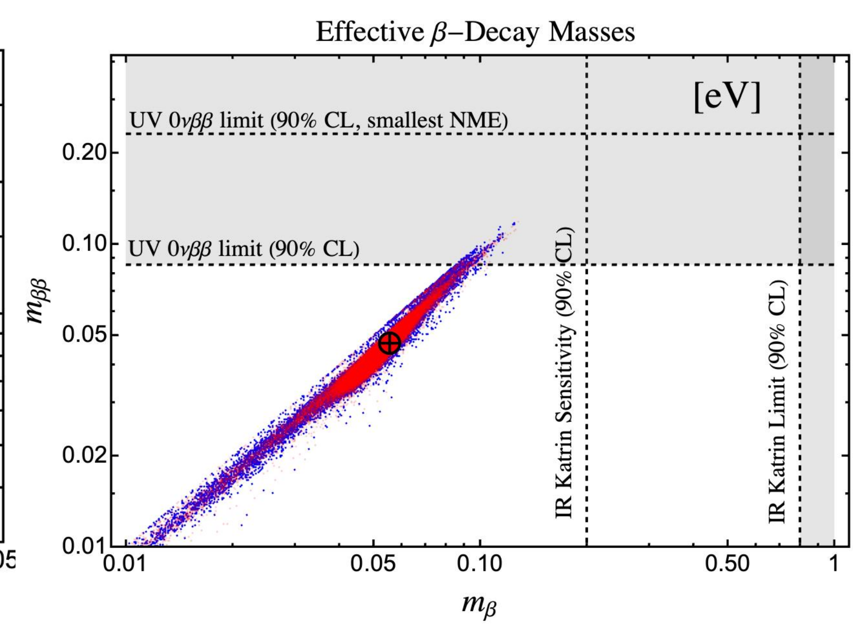
MCMC results: neutrino masses

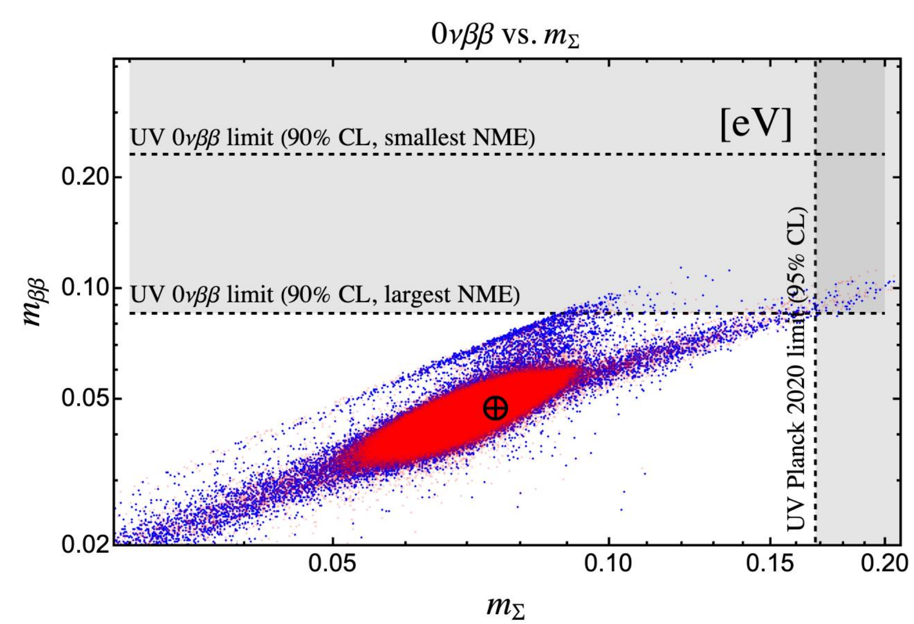




- Ratios unambiguous predictions of model, while individual observables extremely sensitive to model scale-setting parameter.







Summary of our MCMC Analysis of the UTZ

Our goals at the outset of this 'revision of the UTZ':

1. exhaustively explore the available UTZ model space, robustly accounting for all theory correlations amongst its Lagrangian parameters and therefore conclusively determine whether the LO UTZ effective Lagrangian adequately describes nature;

2. explore the complete set of corrections coming from NLO effective operators as discussed above Only the largest corrections identified in the Dirac Lagrangian were briefly considered in [1], and only in the down-quark sector (the corrections parameterized by d_d and ψ_d).

3. identify sufficiently generic *predictions* for (e.g.) the CP-violating phases δ^l and $\phi_{1,2}$ or PMNS atmospheric angle θ^l_{23}, when all other (well-measured) flavour parameters were simultaneously resolved by the UTZ;

4. consider in any way the experimental constraints from, nor predictions for, neutrino-sector observables like $0\nu\beta\beta$, single β-decay rates, or the sum of neutrino masses m_Σ.

MCMC = way to study high-dimensional (read, flavored) parameter spaces

Towards a more model-independent approach

- Model building in an area where falsifiable predictions are scarce is of debatable value.
- Tools should be, are in the process of, and will continue to be developed to study the (e.g.) flavored (ν)**SMEFT**, which can presumedly encode the IR effects of UV flavor constructions in a model-independent way.
- For example, Mike Trott and I (**2107.03941**) recently derived the exact expressions for the Yukawa contributions to Dirac mass and mixing at all orders in the geoSMEFT,

$$s_{13} = \left[\frac{-\hat{I}_{10} - y_b^2\left(\hat{I}_7 - \Delta^+_{ds}\Delta^+_{uc}\Delta^+_{ut}\right) - y_u^2\left(\hat{I}_9 + y_b^2\left(\hat{I}_5 - y_b^2\Delta^+_{ct}\right) - y_d^2\, y_s^2\,\Delta^+_{ct}\right)}{\Delta^-_{bd}\Delta^-_{bs}\Delta^-_{cu}\Delta^-_{ut}}\right]^{1/2} \qquad \Delta^{\pm}_{ij} \equiv y_i^2 \pm y_j^2$$

Big Questions:

How do EFTs/models of flavor match to the SMEFT, LEFT?

How does one control the extreme growth of flavored EFT operators with mass dimension? cf. recent progress in (geo)(ν)SMEFT

How can we use available flavor data to constrain flavored operators? cf. Dawson et al., Falkowski et al., Westhoff et al., + ...

Status of discrete flavor models?

Bad

- Symmetry landscape underdetermined: multiple symmetries can predict the same mixing mixing patterns, and the same symmetry can predict multiple patterns.
- Shaping symmetries still required to constrain the form of Lagrangians (Yukawa and alignment)
- Making concrete predictions from the UV is difficult without additional input — guideposts from RGE, SUSY, anomaly constraints, higher dimensions? Also, what to predict?!
- In the absence of a proper global covariance matrix, all model 'fits' should be scrutinized…

Good

- NADS are well-motivated by data and can be easily incorporated into UV theories.
- They are also naturally pumped out of dimensional compactifications -> More to come!
- They are more powerful than conventional local symmetries at aligning flavored vacua.
- We have shown that the **UTZ** can economically model both quarks and leptons, no easy task.
- Numerical tools (e.g. MCMC) to properly study the parameter space of these models are improving.
- Proper bottom-up EFT technologies for studying the space of BSM flavor are developing and, in my opinion, represent the most promising route to insight in this exciting area of physics!

Thanks!