A Markov Chain Monte Carlo Revision of the Universal Texture Zero of Flavor

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Flavor in the SM

\[ \mathcal{L}_{SM}^Y \supset Y_{pr}^u \overline{Q}_{L,p} \tilde{H} u_{R,r} + Y_{pr}^d \overline{Q}_{L,p} H d_{R,r} + Y_{pr}^e \overline{L}_{L,p} H e_{R,r} + \text{h.c.} \]

- From these Lagrangian terms one can use field redefinitions to show that only 9 masses, 3 mixing angles, and one CP-violating phase are needed for physical description.

\[ [U^\dagger_{\psi L}]_{ir} [Y^\psi]_{rp} [U_{\psi R}]_{pj} \equiv [D_\psi]_{ij} = \text{diag}(y_{\psi 1}, y_{\psi 2}, y_{\psi 3}) \]

\[ V_{CKM} \equiv U_u \dagger U_d \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

13 free and unexplained parameters exist in SM Yukawa sector.
**Neutrino flavor structure**

- Neutrino mass/mixing is an experimental fact, and it represents a departure from the SM

![Diagram showing neutrino mass squared](image)

- Additional 7-9 free and unexplained parameters. However, origin(s) of mass scale unknown. Type-I Seesaw model (Minkowski ’77, et al.) most popular natural explanation:

\[
L_N = \frac{1}{2} \left( \bar{N}_p i \slashed{D} N_p - \bar{N}_p M_{p\tau} N_{\tau} \right) - \left[ \bar{N}_p \omega_{p\beta} \tilde{H}^\dagger l_\beta + \text{h.c.} \right]
\]

\[
L^{(5)} \supset -\frac{m_{\nu,k}}{2} \nu^{c,k}_{L} \nu^{k}_{L} + \text{h.c.} \quad m_{\nu,k} = -\frac{v^2}{2} (U^T)_{k\alpha} c_{5,\alpha\beta} U_{\beta k}:
\]

\[
L^{(5)} = \frac{c^{(5)}_{\alpha\beta}}{2} \left( l^{T}_\alpha \tilde{H}^* \right) C \left( \tilde{H}^\dagger l_\beta \right) + \text{h.c.} \quad c^{(5)}_{\alpha\beta} = (\omega^T M^{-1} \omega)_{\alpha\beta}
\]

\[\text{Match to dim-5 SMEFT} \]

\[\text{EWSB -&gt; light neutrinos!}\]
Symmetric solutions to the Flavor Puzzle

- 9 charged fermion masses + 3 active neutrino masses
- 6 mixing angles and 2 - 4 CP violating phases

\[
\begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \cdot \begin{pmatrix}
  1 & 0 & 0 \\
  0 & e^{i\alpha_1} & 0 \\
  0 & 0 & e^{i\alpha_2}
\end{pmatrix}
\]

20-22 free and unexplained parameters exist in the (ν)SM

- Can we address this Flavor Puzzle by appending the SM with a new symmetry?

\[\text{BSM theory} \sim \mathcal{G}_{BSM} \times SM\]

- Such a symmetry would presumably relate fermions in a given family— i.e. a ‘horizontal’ or ‘family’ or ‘flavor’ symmetry

\[\mathcal{R}(\mathcal{G}_{BSM}) \sim 3, \bar{3}, 2, 1, \ldots\]

- New dynamical scalar sector to realize its breaking patterns?

- Also, what are the mathematical properties of the required symmetry?
A simple example: U(1) Froggatt-Nielsen

- The Froggatt-Nielsen Mechanism is the most famous example of a family symmetry. It implements an Abelian U(1) symmetry with charge Q.

- Standard Yukawa couplings are forbidden if the Higgs is charged under U(1)$_{FN}$. New ‘flavon’ fields necessary:

  \[
  \mathcal{L}_Y \sim y_{ij} \bar{\psi}_i \Phi \psi_j \rightarrow c_{ij} \bar{\psi}_i \Phi \psi_j \left( \frac{\theta}{\Lambda} \right)^{x_{ij}} \quad \text{Integer chosen to cancel charges}
  \]

  Underdetermined O(1) coefficients

  \[
  \quad \text{Mass scale of new physics — must be dynamically realized by integrating out new fields}
  \]

- Let’s consider the down quark sector:

<table>
<thead>
<tr>
<th>Field</th>
<th>$\bar{q}_1$</th>
<th>$\bar{q}_2$</th>
<th>$\bar{q}_3$</th>
<th>$d_{R,1}$</th>
<th>$d_{R,2}$</th>
<th>$d_{R,3}$</th>
<th>$\Phi$</th>
<th>$\theta$</th>
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<td>0</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>-3</td>
<td>-2</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  c_{ij} \bar{q}_i \Phi d_R \left( \frac{\theta}{\Lambda} \right)^{x_{ij}} & \quad \Rightarrow \\
  c_{ij} \bar{q}_i \Phi d_R \lambda^{x_{ij}} & \quad \Rightarrow \\
  y_d & = \begin{pmatrix}
    \lambda^4 & \lambda^3 & \lambda^3 \\
    \lambda^3 & \lambda^3 & \lambda^2 \\
    \lambda & 1 & 1
  \end{pmatrix}
\]

- SSB for the flavon

\[
\frac{\langle \theta \rangle}{\Lambda} \sim \lambda
\]

- For the quarks this mechanism works quite well, but introduces a large number of free parameters….
Outline

Discrete EFTs for Flavor

The Universal Texture Zero
1710.01741

MCMC phenomenology, & the future of BSM flavor
Guided by data

- Let’s focus on the leptonic sector for the moment, and notice an approximate symmetry in the associated PMNS mixing matrix:

  \[ U_{PMNS} \equiv |u_l^T u_\nu| \in \begin{pmatrix} (0.801, 0.513, 0.144) \\ (0.845, 0.579, 0.156) \\ (0.244, 0.505, 0.631) \\ (0.499, 0.693, 0.768) \\ (0.272, 0.471, 0.623) \\ (0.518, 0.669, 0.761) \end{pmatrix} \]

  \[ |U_{\mu i}| \simeq |U_{\tau i}| \quad \longleftrightarrow \quad \begin{cases} \theta_{23}^l = \frac{\pi}{4}, \ \theta_{13}^l = 0 \\ \text{or} \\ \theta_{23}^l = \frac{\pi}{4}, \ \delta^l = \pm \frac{\pi}{2} \end{cases} \]

- This \( \mu - \tau \) symmetry implies specific patterns of mass and mixing:

  \[ U_{PMNS}^{\mu\tau} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}c_{12}^l & \sqrt{2}s_{12}^l & 0 \\ -s_{12}^l & c_{12}^l & \mp 1 \\ \mp s_{12}^l & \pm c_{12}^l & 1 \end{pmatrix} \quad \leftrightarrow \quad M_{\nu U}^{\mu\tau} = \begin{pmatrix} M_{ee} & M_{e\mu} & \pm M_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ \pm M_{e\mu} & M_{\mu\tau} & M_{\mu\mu} \end{pmatrix} \]

  Clearly now just a toy model!

- What does this mean at the level of the Lagrangian?
Hidden symmetries

- Let's assume neutrino masses generated by Type-I See-saw:

\[
\mathcal{L}_{\nu,\text{mass}} \sim \frac{1}{2} \overline{\nu}_L M_{\nu U} \nu_L + \text{h.c.} \quad \nu_L = (\nu_e, \nu_\mu, \nu_\tau) \quad M_{\nu U} = U_{\text{PMNS}}^* m_\nu U_{\text{PMNS}}^\dagger = M_{\nu U}^T
\]

- We can promote the phenomenological observation to a physical symmetry:

\[
\nu \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ \pm \nu_\tau \\ \pm \nu_\mu \end{pmatrix} = S^{\mu\tau} \nu \quad S^{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \pm 1 \\ 0 & \pm 1 & 0 \end{pmatrix} \quad M_{\nu U}^{\mu\tau} = S^{\mu\tau} M_{\nu U} S^{\mu\tau}
\]

- The group generated by the \( \mu - \tau \) operator mediates the simplest type of flavor symmetry: an Abelian discrete symmetry:

\[ \mathcal{G}_\nu = \mathbb{Z}_2 \]

- This action is actually a specific instance of the rather generic statement that the maximal 'residual' symmetry of a Majorana mass term is the Klein four group, \( \mathbb{Z}_2 \times \mathbb{Z}_2 \), cf. Lam 2007.
Tri-bimaximal neutrino mixing

- Before 2012, the leptonic mixing data was also consistent with other famous relationships:

\[ \sin^2 2\theta_{12} = \frac{8}{9} \quad \leftrightarrow \quad U_{TBM}^{\mu\tau} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{pmatrix} \]

- This is consistent with another symmetry operation:

\[ M_{TBM}^{\mu\tau} = S_{TBM}^{TBM} M_{TBM}^{\mu\tau} S_{TBM}^{TBM} \quad \left[ S_{TBM}^{TBM}, S_{\mu\tau} \right] = 0 \quad S_{TBM}^{TBM} = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \]

- Hence \( S_{TBM}^{TBM} \times S_{\mu\tau} \sim \mathbb{Z}_2 \times \mathbb{Z}_2 \) as expected. The charged leptons respect \( U(1)^3 \) rotations...

\[ \mathcal{L}_{l,\text{mass}} \sim \overline{E}_R m_l l_L + h.c. \quad m_l \dagger m_l = T \dagger m_l \dagger m_l T \quad T = \text{diag} \left( e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau} \right) \]

- The group closed by \( \{ S_{\mu\tau}, S_{TBM}^{TBM}, T \} \) is \( S_4 \) (cubic group), by \( \{ S_{TBM}^{TBM}, T \} \) is \( A_4 \) (tetrahedral group)...

- The most famous models to my knowledge implementing \( A_4 \) are Babu, Ma, Valle: hep-ph/0206292 and Altarelli, Feruglio: hep-ph/0512103, 0504165.
The discrete approach

All of these symmetries have been explored in models...

Symmetry breaking can also be studied in a ‘model-independent’ way...cf. work from Grimus et al. here in Vienna, e.g.!

- Discrete symmetries avoid Goldstone modes that could spoil phenomenology, easily embedded in SUSY GUTs, extra dimensional theories.
- Easier facilitation of vacuum alignment than with continuous symmetries
- Huge literature: Pakvasa, Sugawara (1977) use S3 for Cabibbo angle. Deshpande uses S4 for full CKM and Pakvasa applies S4 to neutrino mass and mixing (1984). Early 90s discussion (Kaplan, Schmaltz; Frampton, Kephart), TBM and GUT models established early-mid 00s (Ma, Rajasekaran; Altarelli, Feruglio, de M. Varzielas, King, Ross +), new flood in 2012/13 after reactor angle...

\[ \Delta(27) \simeq (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_3 \]

Today we focus here!

Effective operators

- In the IR, we can build effective mass matrices with higher dimensional operators:

\[ \psi Y \theta_3 A \times A \times A \rightarrow \psi^c \]

- In the UV, each vertex is part of the full Lagrangian (messengers A integrated out):

\[ \mathcal{L}_{UV} \sim \psi \theta_3 A + \bar{A} H A + \ldots \quad \mathcal{L}_{IR} \sim \psi \theta_3 H \theta_3 \psi^c \]

- Hence by assigning the messengers to trivial singlets, one can form family symmetry invariants:

\[ \mathcal{L} \sim \psi_i \theta_3^i \bar{\theta}_3^j \psi^c_j H \sim 1 \]
Mass matrices from flavons

- Flavons acquire vacuum expectation values along specific directions in flavour space:
  \[
  \langle \theta_3 \rangle = v_3 \cdot (0, 0, 1)
  \]

- Mass matrices then follow from the form of the effective operator:
  \[
  \mathcal{L}_Y (\psi, \psi^c, H, \theta_i) \leftrightarrow \mathcal{M} (\psi, \psi^c, \langle H, \theta_i \rangle)
  \]
  \[
  \mathcal{L} \sim \psi_i \theta_3^i \theta_3^j \psi_j^c H \quad \Rightarrow \quad \mathcal{M} \propto v_3^2 \begin{pmatrix}
  0 & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & 1
\end{pmatrix},
  \]

Note: alignment is generally non-trivial to achieve, but facilitated with the NADS

Family symmetries shape the Yukawa sector, align VEVs, and thereby control fermionic mass and mixing matrices

- Let’s use phenomenologically successful mass patterns to guide the construction of our model…
A universal texture zero (UTZ) for fermions

- A (1,1) texture zero can accurately reproduce the phenomenology of the charged fermions.

\[ M_a^D \approx m_3 \begin{pmatrix} 0 & \varepsilon_a^3 & \varepsilon_a^3 \\ \varepsilon_a^3 & r_a \varepsilon_a^2 & r_a \varepsilon_a^2 \\ \varepsilon_a^3 & r_a \varepsilon_a^3 & 1 \end{pmatrix}, \quad r_{u,d} = 1, \quad r_l = -3 \]

\[ \varepsilon_u \approx 0.05, \quad \varepsilon_{d,l} \approx 0.15 \]

- It implements the well known Georgi-Jarlskog (PLB 86 1979) mass relation and also the successful Gatto-Sartori-Tonin (PLB 28 1968) relation:

\[ \sin \theta_c = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right| \]

\[ m_b \approx 3 m_\tau \]
\[ m_s \approx 3 \times \frac{1}{3} m_\mu \]
\[ m_d \approx 3 \times 3 m_e \]

Red indicates RGE to IR

IDEA: Can this successful texture be extended to the neutrino sector?

- If so, can we realize this phenomenology in a concrete model?
The UTZ model

<table>
<thead>
<tr>
<th>Fields</th>
<th>$\psi_{q,e,\nu}$</th>
<th>$\psi_{q,e,\nu}^c$</th>
<th>$H_5$</th>
<th>$\Sigma$</th>
<th>$S$</th>
<th>$\theta_3$</th>
<th>$\theta_{23}$</th>
<th>$\theta_{123}$</th>
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<tr>
<td>$Z_N$</td>
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<td>2</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>2</td>
<td>0</td>
<td>$x$</td>
</tr>
</tbody>
</table>

- The UTZ can appear from the same Lagrangian for all Dirac sectors!

$$\mathcal{L}_{D,f}^{LO} = \psi_i \left( \frac{c_3^{(6)}}{M_{3,f}^2} \theta_3^i \theta_3^j + \frac{c_{23}^{(7)}}{M_{23,f}^3} \theta_{23}^i \theta_{23}^j \Sigma + \frac{c_{123}^{(7)}}{M_{123,f}^3} (\theta_{123}^i \theta_{23}^j + \theta_{23}^i \theta_{123}^j) S \right) \psi_j^c H$$

$f \in \{u,d,e,\nu\}$

**Messenger masses distinguish fermion species**

**VEV implements Georgi-Jarlskog**

**Needed for shaping symmetry**

$$\mathcal{L}_{\mathcal{M}}^\nu = \psi_i^c \left( \frac{c_M^{(5)}}{M} \theta^i \theta^j + \frac{1}{M^4} [c_{M,1}^{(8)} \theta_{23}^i \theta_{23}^j (\theta^k \theta^k_{123}) + c_{M,2}^{(8)} (\theta_{123}^i \theta_{23}^j + \theta_{123}^j \theta_{23}^i) (\theta^k \theta^k_{23})] \right) \psi_j^c$$

- A Type-I See-saw generates active light neutrinos:

$$M_{\nu}^D \cdot M_{\nu}^{M,-1} \cdot M_{\nu}^{D,T} \rightarrow M_{\nu}$$
Vacuum alignment (proof in principle)

- We want to achieve the 3, 123, and 23 alignments.

\[
\langle \theta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v_3, \quad \langle \theta_{123} \rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\beta} \\ e^{i\alpha} \\ -1 \end{pmatrix} v_{123}, \quad \langle \theta_{23} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\alpha} \\ 1 \end{pmatrix} v_{23}, \quad \langle \theta_X \rangle = \frac{1}{\sqrt{6}} \begin{pmatrix} 2e^{i\beta} \\ -e^{i\alpha} \\ 1 \end{pmatrix} v_X
\]

\[V_1(\theta_i) = m_i^2 |\theta_i|^2\]

Sets scale of familon fields and breaks family symmetry upon negative \(m^2\)

\[V_2(\theta_i) = h_i(\theta_i)^2 \left( \theta_{i}^\dagger \right)^2.
\]

Completes alignment of \(\theta_{3,123}\), depending on sign of \(h\) (h<0 -> \(\theta_3\))!

\[V_3 = k_1 \theta_X, i \theta_{123}^{i} \theta_{123,j} \theta_{X}^{j}, \quad k_1 > 0.\]

Orthogonal to 123, but does not distinguish between (0,1,1) and (2,-1,1)

\[V_4 = k_2 m_0 \theta_{X}^{1} \theta_{X}^{2} \theta_{X}^{3} \]

\[V_5 = k_3 \theta_{23}^{i} \theta_{23,j}^{i} \theta_{X}^{j} + k_4 \theta_{23}^{i} \theta_{3}^{i} \theta_{23}^{j}, \quad \text{with } k_3 > 0 \text{ and } k_4 < 0\]

Respectively select (2,-1,1), (0,1,1)!

\[V = \sum_{i=3,123} (V_1(\theta_i) + V_2(\theta_i)) + V_3 + V_4 + V_5\]

- Alignment of LNV family discussed in paper…
Discrete anomaly freedom

- Some argue that discrete symmetries should/must be gauged in the UV. This means anomaly cancellation must be enforced.

\[ D - G - G, \quad D - g - g, \quad Z - G - G, \quad Z - g - g \]

\[
\begin{align*}
Z/D - G - G : & \quad \sum_{r^{(f)},d^{(f)}} tr \left[ \tau(d^{(f)}) \right] \cdot l(r^{(f)}) \cdot \frac{1}{i} \equiv 0 \mod \frac{N}{2} \\
D - g - g : & \quad \sum_{d^{(f)}} tr \left[ \tau(d^{(f)}) \right] \equiv 0 \mod \frac{N}{2} \\
Z - g - g : & \quad \sum_{f} q^{(f)} = \sum_{m} q^{(m)} \cdot \text{dim } R^{(m)} \cdot \frac{1}{i} \equiv 0 \mod \frac{N}{2}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Fields</th>
<th>( \psi_{q,e,\nu} )</th>
<th>( \psi_{q,e,\nu}^c )</th>
<th>( H_5 )</th>
<th>( \Sigma )</th>
<th>( S )</th>
<th>( \theta_{3} )</th>
<th>( \theta_{23} )</th>
<th>( \theta_{123} )</th>
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\[ \Delta(3N^2) \]

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<th>( 3_{[k][l]} )</th>
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<td>( \omega^k )</td>
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<td>( \det(a) )</td>
<td>( \omega^l )</td>
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</tr>
<tr>
<td>( \det(a') )</td>
<td>( \omega^l )</td>
<td>1</td>
</tr>
</tbody>
</table>

Anomalies trivially satisfied at relevant scale of EFT
Neutrino mixing, qualitatively

- Sequential Dominance limit (King: 1998-2002): the third generation RH Majorana neutrino mass is large:

\[ M_{R,3} \gg M_{R,2} \gg M_{R,1} \]

- This means that the LH neutrino mass matrix via the see-saw is effectively 2D:

\[
M_{\text{Majorana}} \propto \begin{pmatrix} 0 & c_2 \\ c_2 & c_1 + 2c_2 \end{pmatrix}, \quad M_{\text{Dirac}} \propto \begin{pmatrix} 0 & \sqrt{3/2} \\ 1 & 1 + s \end{pmatrix} \quad \Rightarrow \quad M_\nu \propto \begin{pmatrix} 0 & -\sqrt{3/2}c_2 \\ -\sqrt{3/2}c_2 & c'_1 \end{pmatrix}
\]

\[ c_1 \gg c_2 \quad s \propto \langle \Sigma \rangle/\langle \theta_{23} \rangle/((S)/\langle \theta_{123} \rangle) \quad c'_1 \equiv c_1 - 2c_2 s \]

- From which we can derive relationships for the mass ratios and neutrino mass eigenstates:

\[
\frac{m_2}{m_1} \approx 3 \frac{c_2^2}{2 c'_1}, \quad \frac{c_2}{c'_1} \equiv |\frac{c_2}{c'_1}| e^{i\eta}, \quad \nu_1 \propto \nu_a - e^{i\eta} \sqrt{\frac{m_2}{m_1}} \nu_b
\]

- Which then also generates simple equalities for neutrino mixing angles:

\[
\sin \theta_{13}^\nu \approx \sqrt{\frac{m_2}{3m_1}} \quad \sin \theta_{23}^\nu \approx \frac{1}{\sqrt{2}} - e^{i\eta} \sin \theta_{13}^\nu \quad \sin \theta_{12}^\nu \approx \frac{1}{\sqrt{3}}
\]

- Note the clear departure from TBM mixing. Also, corrections from the charged lepton sector drive the reactor angle to acceptable values...
Complete UTZ mass matrices

- The UTZ Lagrangian then generates LO Dirac matrices of the following form:

\[
\mathcal{M}_i^D \equiv \frac{M_i^D}{c} \approx \begin{pmatrix}
0 & a e^{i(\alpha+\beta+\gamma)} & a e^{i(\beta+\gamma)} \\
\alpha e^{i(\alpha+\beta+\gamma)} & (b e^{-i\gamma} + 2a e^{-i\delta}) e^{i(2\alpha+\gamma+\delta)} & b e^{i(\alpha+\delta)} \\
\alpha e^{i(\beta+\gamma)} & b e^{i(\alpha+\delta)} & 1 - 2a e^{i\gamma} + b e^{i\delta}
\end{pmatrix}
\]

\[
a_f' = \frac{c_{123}^{(7)} v_{123} v_{23} \langle S \rangle}{\sqrt{6} M_{123,f}}, \quad b_f' = \frac{c_{23}^{(7)} r_f v_{23} \langle \Sigma \rangle}{2 M_{23,f}}, \quad c_f' = \frac{c_{123}^{(8)} v_{123} v_{3} \langle S \rangle^2}{\sqrt{3} M_{123,f}^4}, \quad d_f' = \frac{c_{23}^{(8)} r_f v_{23} v_{3} \langle \Sigma \rangle \langle S \rangle}{\sqrt{2} M_{23,f}^4}
\]

**Can 9 low-energy parameters successfully describe 18+ ‘observables’?**

- Also: corrections come from HO operators in UTZ Lagrangian in principle exist…

\[
\mathcal{L}_{D,f}^{HO} = \psi_i \left( \frac{c_{23}^{(8)}}{M_{23,f}^4} (\theta_{23}^i \theta_{3}^j + \theta_{3}^i \theta_{23}^j) \Sigma S + \frac{c_{123}^{(8)}}{M_{123,f}^4} (\theta_{123}^i \theta_{3}^j + \theta_{3}^i \theta_{123}^j) S^2 \right) \psi_j^c H
\]

\[
\mathcal{M}_f^D \approx \begin{pmatrix}
0 & a e^{i(\alpha+\beta+\gamma)} & a e^{i(\beta+\gamma)} + c e^{i(\beta+\zeta)} \\
a e^{i(\alpha+\beta+\gamma)} & (b e^{-i\gamma} + 2a e^{-i\delta}) e^{i(2\alpha+\gamma+\delta)} & b e^{i(\alpha+\delta)} + c e^{i(\alpha+\zeta)} + d e^{i(\alpha+\psi)} \\
a e^{i(\beta+\gamma)} + c e^{i(\beta+\zeta)} & b e^{i(\alpha+\delta)} + c e^{i(\alpha+\zeta)} + d e^{i(\alpha+\psi)} & 1 - 2a e^{i\gamma} + b e^{i\delta} - 2c e^{i\zeta} + 2d e^{i\psi}
\end{pmatrix}
\]
A naive fit to masses and mixings

2017 Pheno

Basic ‘contour’ analysis, looking for theory correlations in a semi-analytic fashion.


Problem: Looks fine-tuned, + minor discrepancies in exterior off diagonal CKM elements @ LO.

Proposal: Include two additional parameters from HO operators in OPE, by hand...

Can we do better?
Leptonic CP violation

Experiment has not yet put fully robust bounds on the PMNS’ Dirac CP-violating phase \( \delta_{CP} \), while Majorana phases \( \phi_i \) are fully unconstrained.

This represents an opportunity for flavor models to actually predict, rather than retrodict, fundamental flavor structure.

In 2017 we did not have the numerical tools necessary to do so reliably.

ME FROM 2017/18: “…the intricate interdependence between model parameters and physical observables in our GUT construction makes obtaining concrete error bands very difficult (future work)”
A simple Markov Chain Monte Carlo

MCMC represent ideal analysis for sampling probability functions of multi-dimensional spaces.

This is perhaps the simplest of the available MCMC algorithms: a Metropolis-Hastings algorithm.

Early MCMC analyses in particle theory from cosmology (e.g. Trotta et al.), SUSY pheno (e.g. Baer et al.), and PDF fits (e.g. Mangin-Brenet at al.) …

Our approach is modified from above references (Bernigaud et al.)

**Figure 1. Illustration of the MCMC algorithm utilisation.**

Point \( n: \tilde{\theta}^n \)

Jump \( G(\theta_i^n, \kappa \tilde{\theta}_i^n) \)

Proposal \( n + 1: \tilde{\theta}^{n+1} \)

\( \tilde{O}(\tilde{\theta}^{n+1}) \)

Likelihood: \( L^{n+1}(\tilde{\theta}^{n+1}, \tilde{O}, \tilde{\sigma}) \)

\( \mu \in [0, 1] \)

Fail: restart at \( n \)

Test: \( \mu < \frac{L^{n+1}(\tilde{\theta}^{n+1}, \tilde{O}, \tilde{\sigma})}{L^n(\tilde{\theta}^n, \tilde{O}, \tilde{\sigma})} \)

Success: \( n \to n + 1 \)
## UTZ MCMC specifics

Parameter ranges obtained via
1) trial and error, and
2) with basic physics assumptions regarding the magnitudes of suppressed EFT coefficients

Note that the MCMC distributions obtained are of course sensitive to experimental constraints contributing to the overall Likelihood functions driving the Markov Chain…

<table>
<thead>
<tr>
<th>LO UTZ Model Parameter MCMC Ranges &amp; Global Best Fits</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a, b)_d · 10^3</td>
</tr>
<tr>
<td>Range</td>
</tr>
<tr>
<td>LO</td>
</tr>
<tr>
<td>HO</td>
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<tr>
<td>(γ, δ)_d</td>
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<tr>
<td>Range</td>
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<tr>
<td>LO</td>
</tr>
<tr>
<td>HO</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HO UTZ Model Parameter MCMC Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c, d)_d · 10^5</td>
</tr>
<tr>
<td>Range</td>
</tr>
<tr>
<td>HO</td>
</tr>
</tbody>
</table>

Constraints: \{R_{f_i f_3} (f \in u, d, e), \sin \theta_{ij}^{q,l}, \sin \delta_{ij}^{q,l}, \Delta m_{sol, atm}^2, m_{\beta(\beta)}, m_\Sigma, \xi, \text{n.h.}\}

Predictions: \{R_{\nu i \nu 3}, m_{\beta\beta}/m_\Sigma, m_\beta/m_\Sigma, m_{\beta\beta}/m_\beta, \sin \phi_1, \sin \phi_2\}

Quasi-Predictions: \{\sin \delta_{ij}^{q,l}, \xi\}
Our simple Metropolis-Hastings MCMC exhibits exceptional convergence at LO and HO.

Notice that including HO corrections has negligible impact on preference for LO parameters.

Also notice that there is virtually no preference for the purely HO parameter space (orange).

We did multiple consistency tests with our MCMC scripts, including stability of MCMC parameter variations…
MCMC results: fermion mass ratios \( R_{ij} = \frac{m_i}{m_j} \)

Figure 2: MCMC density plots for UTZ quark and lepton flavoured mass ratio predictions. Plots are generated with the hyper-parameter choices in (33) with model-parameter variations as given in Table 4. The blue (red) regions correspond to the LO (HO) MCMC scan results, with darker regions corresponding to places of higher density. Gray regions represent the UV bounds for the mass ratios as presented in Table 3, and the black target markers correspond to the global best-fit values shown numerically in Table 6.

Fermion Flavour Mass Ratios

We first examine the MCMC results for the UTZ’s predictions in the fermion mass sector. Figure 2 illustrates these for mass ratios in the up quark, down quark, charged lepton, and neutrino families. Both Figure 2 (and upcoming figures) and Table 6 give results for the LO and, when indicated, HO MCMC scans, with the former given in blue and the latter in red.

Note that these figures represent density plots, in that darker regions correspond to parameter domains where more Markov chains evolved. Also, the black ‘target’ marker in Figures 2-6 corresponds to the location of the overall (global) best-fit data-set \( \cdot \), which is also given numerically in Table 6.

The gray bands correspond to the global data available from the PDG (NuFit) collaborations for the charged fermion (neutrino) masses, corrected to the UV according to the 12

Although recall that in the sequentially-dominant IR limit only one neutrino phase, formed from a combination of said UV phases, dominates the phenomenology.

\[ R_{ij} = \frac{m_i}{m_j} \]
MCMC results: fermion mixings

Figure 3: The same as Figure 2, but for the CKM and PMNS mixing angles and Dirac CP-violating phases.

Comparing these to the blue and red regions, we see that the UTZ is capable of successfully resolving the entire charged fermion mass spectrum, for both quarks and leptons, up to the RGE and threshold correction uncertainties. Furthermore, the UTZ predictions for (currently unmeasured) neutrino mass ratios are shown in the bottom-right panel; given the model parameter ranges explored in Table 4, the ratio $R_{i3}$ is densely populated within $2.5 \cdot 10^2 < R_{23} < 2 \cdot 10^2$ for the heavier generations while the smaller mass ratio is densely populated between $1.5 \cdot 10^3 < R_{13} < 2 \cdot 10^2$. However, we see that, albeit less frequent, much larger neutrino mass hierarchies are also resolved, with $R_{13} (R_{23})$ falling below $10^6 (5 \cdot 10^3)$.

Finally, we notice from the up-family plot that the inclusion of the red HO corrections sourced from (13) do not qualitatively change the physics conclusions of the blue LO regions. We have in fact observed this quite generically across family and observable sectors, and hence for visual clarity we only display the dominant LO results in what follows, unless otherwise specified.
MCMC results: CP-violation

Figure 4: The same as Figure 3, but now comparing the quark and lepton Dirac CP-violating phases, and presenting the novel predictions for the Majorana CP-violating phases $\theta_1$, $\theta_2$.

Fermion Mixings and CP-Violation

In Figure 3 we have presented the MCMC UTZ predictions for the CKM (top two plots) and PMNS (bottom two plots) mixing angles $\theta_{ij}^{q,l}$ as well as the associated Dirac CP-violating phases $\theta_{ij}^{q,l}$. Here we again compare to (radiatively corrected) data from the PDG and NuFit given in gray, and notice that the blue (red) LO (HO) UTZ Lagrangian is again highly successful at resolving these parameters. Indeed, while we observe that the overlap with PMNS uncertainty bands is perhaps qualitatively more successful than that of the quarks, the regions overlap with the UV bounds for all fermion families.

Note that this conclusion differs from the naive analysis in [1], which found elements in the third row and column of the CKM to be outside of the UV uncertainty bands considering only the LO UTZ Lagrangian, a deviation sourced by the $\theta_23^{l}$ mixing angle. While we observe that the bulk of the MCMC sample points for $\sin\theta_{23}^{q}$ are indeed lower than the allowed uncertainty region, a significant number of LO points do overlap successfully. Studying [12], one concludes that lower values of $\theta_2^{q}$ tend to correspond to higher $\tan\beta$ RGE scenarios. Hence independent evidence that a background spectrum imitating this UV MSSM structure [13] is not physical would in principle also disfavor the UTZ theory of flavor, up to the extent the bounds on $\sin\theta_{23}^{q}$ drive our current MCMC likelihoods.

...assuming a certain threshold correction structure and SUSY breaking scale, of course...
Precision probes of neutrino mass

- Cosmological constraints on the overall sum of neutrino masses exist:

\[ m_\Sigma \equiv \sum_i m_{\nu_i} < 0.26 \text{ eV} \]

- As do non-trivial neutrino flavor constraints from \( \beta \)-decay probes:

\[ \langle m_{\beta\beta} \rangle \equiv \left| \sum_i V_{ei}^2 m_i \right| < (61 - 165) \cdot 10^{-3} \text{ eV} , \]

\[ m_\beta \equiv \sqrt{\sum_i |V_{ei}|^2 m_i^2} < 0.8 \text{ eV} , \]

- Active and future experiment represents major area of low-energy research:

Figure from Snowmass 2022 report, borrowed from 2202.01787

Planck

KamLand-ZEN

KATRIN
MCMC results: neutrino masses

- Ratios unambiguous predictions of model, while individual observables extremely sensitive to model scale-setting parameter.
Summary of our MCMC Analysis of the UTZ

Our goals at the outset of this ‘revision of the UTZ’:

1. exhaustively explore the available UTZ model space, robustly accounting for all theory correlations amongst its Lagrangian parameters and therefore conclusively determine whether the LO UTZ effective Lagrangian adequately describes nature;

2. explore the complete set of corrections coming from NLO effective operators as discussed above. Only the largest corrections identified in the Dirac Lagrangian were briefly considered in [1], and only in the down-quark sector (the corrections parameterized by $d_d$ and $\psi_d$).

3. identify sufficiently generic predictions for (e.g.) the CP-violating phases $\delta^l$ and $\phi_{1,2}$ or PMNS atmospheric angle $\theta_{23}^{l}$, when all other (well-measured) flavour parameters were simultaneously resolved by the UTZ;

4. consider in any way the experimental constraints from, nor predictions for, neutrino-sector observables like $0\nu\beta\beta$, single $\beta$-decay rates, or the sum of neutrino masses $m_{\Sigma}$.

MCMC = way to study high-dimensional (read, flavored) parameter spaces
Towards a more model-independent approach

- Model building in an area where falsifiable predictions are scarce is of debatable value.
- Tools should be, are in the process of, and will continue to be developed to study the (e.g.) flavored ($\nu$)SMEFT, which can presumably encode the IR effects of UV flavor constructions in a model-independent way.
- For example, Mike Trott and I (2107.03941) recently derived the exact expressions for the Yukawa contributions to Dirac mass and mixing at all orders in the geoSMEFT,

\[
s_{13} = \left[ \frac{-\hat{I}_{10} - y_b^2 \left( \hat{I}_7 - \Delta_{ds}^+ \Delta_{uc}^+ \Delta_{ut}^+ \right) - y_u^2 \left( \hat{I}_9 + y_b^2 \left( \hat{I}_5 - y_b^2 \Delta_{ct}^+ \right) - y_d^2 y_s^2 \Delta_{ct}^+ \right)}{\Delta_{bd}^- \Delta_{bs}^- \Delta_{cu}^- \Delta_{ut}^-} \right]^{1/2} \Delta_{ij}^\pm \equiv y_i^2 \pm y_j^2
\]

Big Questions:

How do EFTs/models of flavor match to the SMEFT, LEFT?

How does one control the extreme growth of flavored EFT operators with mass dimension? cf. recent progress in (geo)($\nu$)SMEFT

How can we use available flavor data to constrain flavored operators? cf. Dawson et al., Falkowski et al., Westhoff et al., + ....
Status of discrete flavor models?

**Bad**

- Symmetry landscape underdetermined: multiple symmetries can predict the same mixing mixing patterns, and the same symmetry can predict multiple patterns.
- Shaping symmetries still required to constrain the form of Lagrangians (Yukawa and alignment)
- Making concrete predictions from the UV is difficult without additional input — guideposts from RGE, SUSY, anomaly constraints, higher dimensions? Also, what to predict?!
- In the absence of a proper global covariance matrix, all model ‘fits’ should be scrutinized…

**Good**

- NADS are well-motivated by data and can be easily incorporated into UV theories.
- They are also naturally pumped out of dimensional compactifications -> More to come!
- They are more powerful than conventional local symmetries at aligning flavored vacua.
- We have shown that the UTZ can economically model both quarks and leptons, no easy task.
- Numerical tools (e.g. MCMC) to properly study the parameter space of these models are improving.
- Proper bottom-up EFT technologies for studying the space of BSM flavor are developing and, in my opinion, represent the most promising route to insight in this exciting area of physics!

**Thanks!**