Top-quark hadro-production and the top-quark mass

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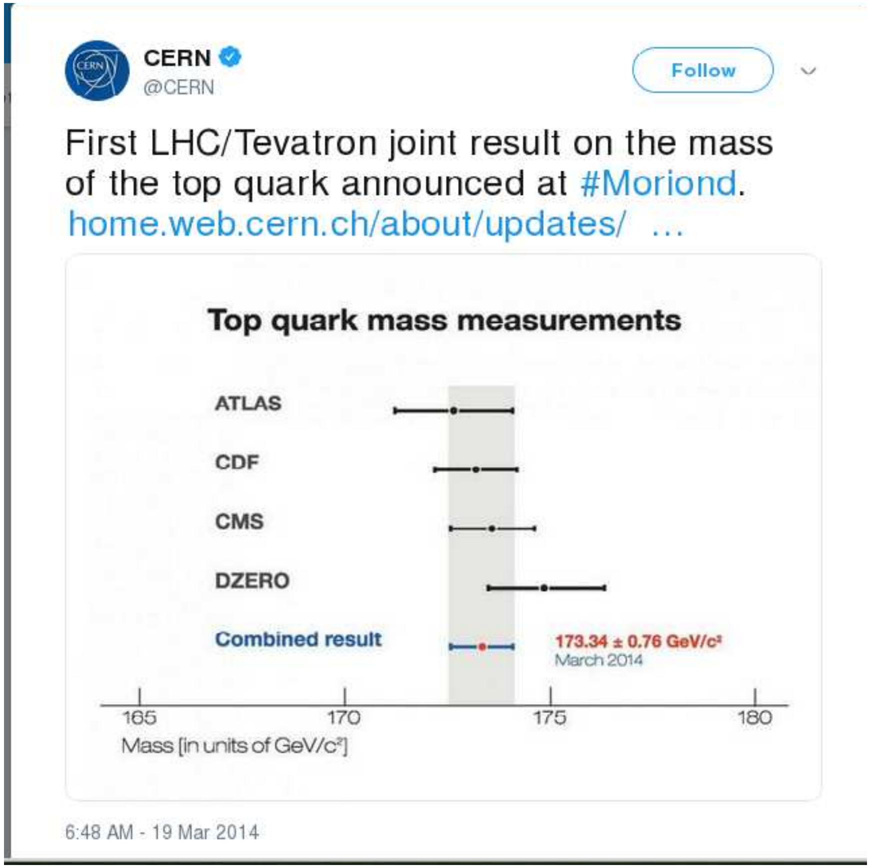
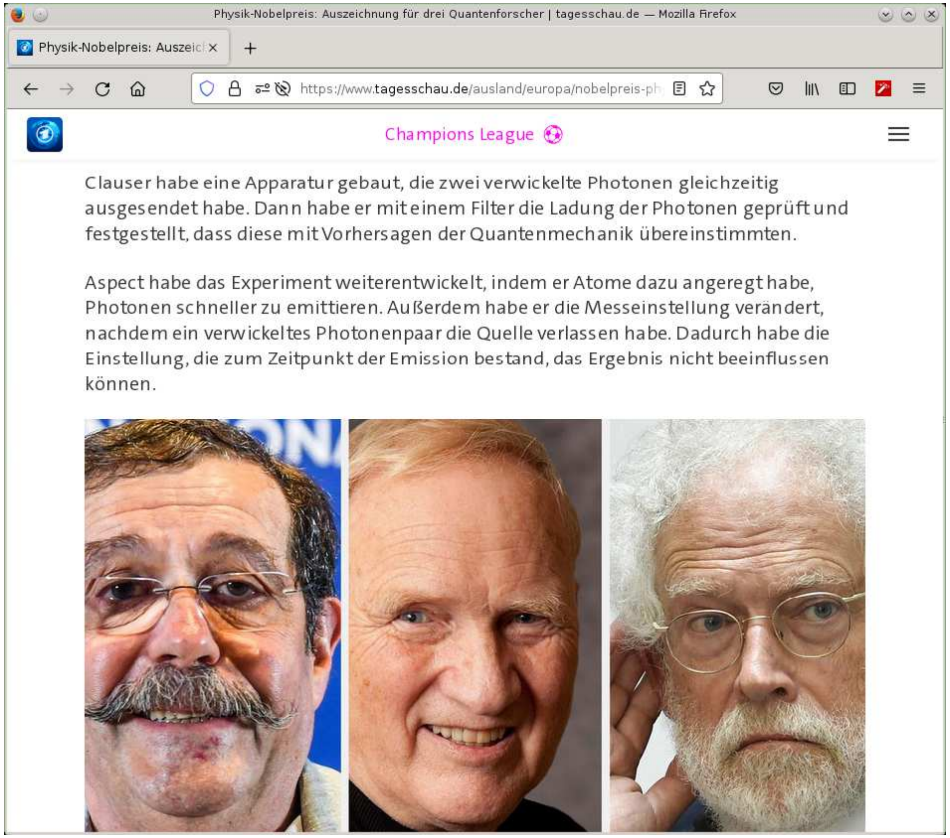
Top-quark mass on social media

- Top-quark mass already known with unprecedented precision from world combination ATLAS, CMS, CDF, D0 coll. ‘14
  $m_t = 173.34 \pm 0.27(\text{stat}) \pm 0.71(\text{syst})$ GeV

Top-quark mass in twitter scheme



Discovery of charged photons



- Spectacular achievements of the 2022 Nobel laureates (according to tagesschau.de)

  "[Clauser] then used a filter to check the charge of the photons".

Based on work done in collaboration with:

- *One-loop soft anomalous dimension matrices for $t\bar{t}j$ hadroproduction*
  B. Chargeishvili, M. V. Garzelli, and S. M. arXiv:2206.10977
- *Phenomenology of $t\bar{t}j + X$ production at the LHC*
  S. Alioli, J. Fuster, M. V. Garzelli, A. Gavardi, A. Irles, D. Melini, S. M. P. Uwer and K. Voß arXiv:2202.07975
- *Cross-sections for $t\bar{t}H$ production with the top quark $\overline{MS}$ mass*
  A. Saibel, S. M. and M. Aldaya Martin arXiv:2111.12505
- *Heavy-flavor hadro-production with heavy-quark masses renormalized in the $\overline{MS}$, MSR and on-shell schemes*
  M. V. Garzelli, L. Kemmler S. M. and O. Zenaiev arXiv:2009.07763
- [. . .]
- *Parton distribution functions, α_s, and heavy-quark masses for LHC Run II*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė arXiv:1701.05838
- [. . .]

Why top-quark physics?

Experiment

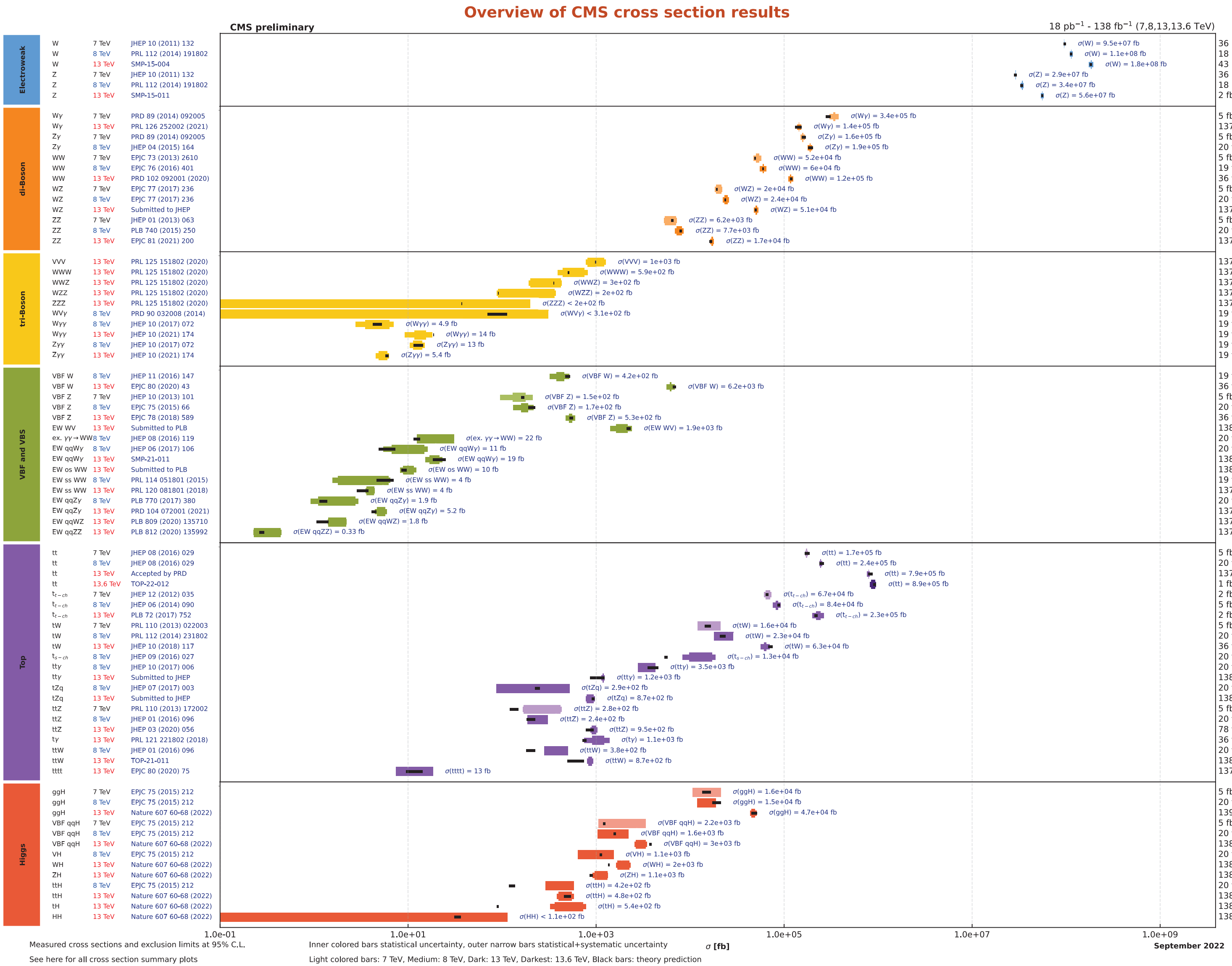
- Top-quark hadro-production processes measured at the LHC with high precision
  - $t\bar{t}$
  - $t\bar{t}V$ with $V = \gamma, W, Z$
  - $t\bar{t}$+n jets with $n = 1, 2, 3, 4$
  - $t\bar{t}c\bar{c}$ and $t\bar{t}b\bar{b}$
  - $t\bar{t}t\bar{t}$
  - $t\bar{t}H$
  - single-t, $t\gamma$, tW, tH, tZq

Theory

- Challenge for theory predictions
- Dependence on fundamental parameters of the Standard Model
  - top-quark mass m_t
  - strong coupling α_s
- Constraints on new physics

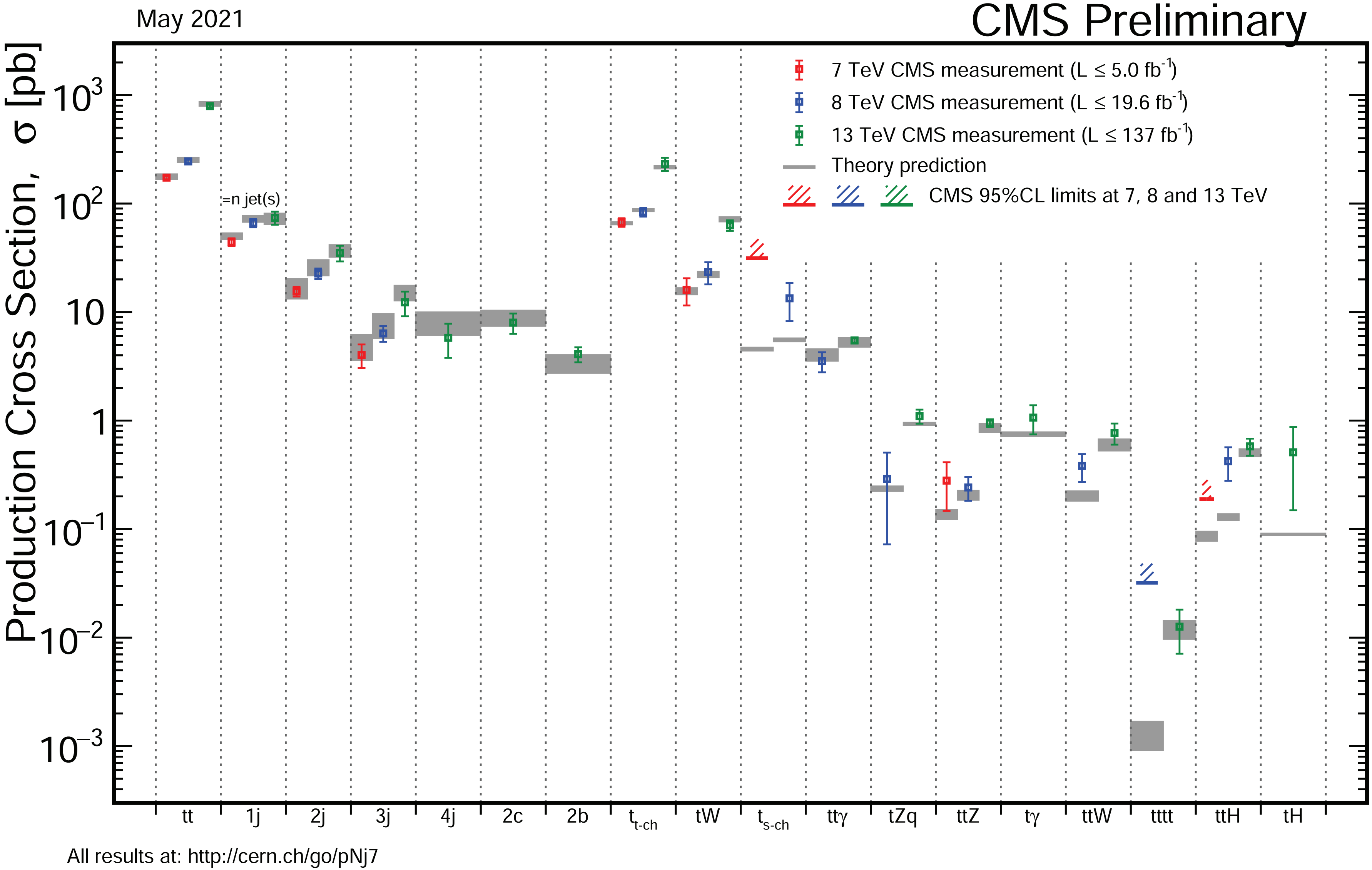
Standard Model cross sections

- Standard Model cross sections and predictions at the LHC CMS coll. '22



Top-quark cross sections

- Cross sections and predictions for top-quark production CMS coll. '21



Top-quark mass

- Top-quark is the heaviest elementary particle
- Masses and couplings are formal parameters of the theory
  - m_t and $\alpha_s = g_s^2/(4\pi)$ are no observables
- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F_{\mu\nu}^a F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i} \not{D} - m_q \right)_{ij} q_j$$

  - field strength tensor $F_{\mu\nu}^a$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a \right)_{ij} A_\mu^a$
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

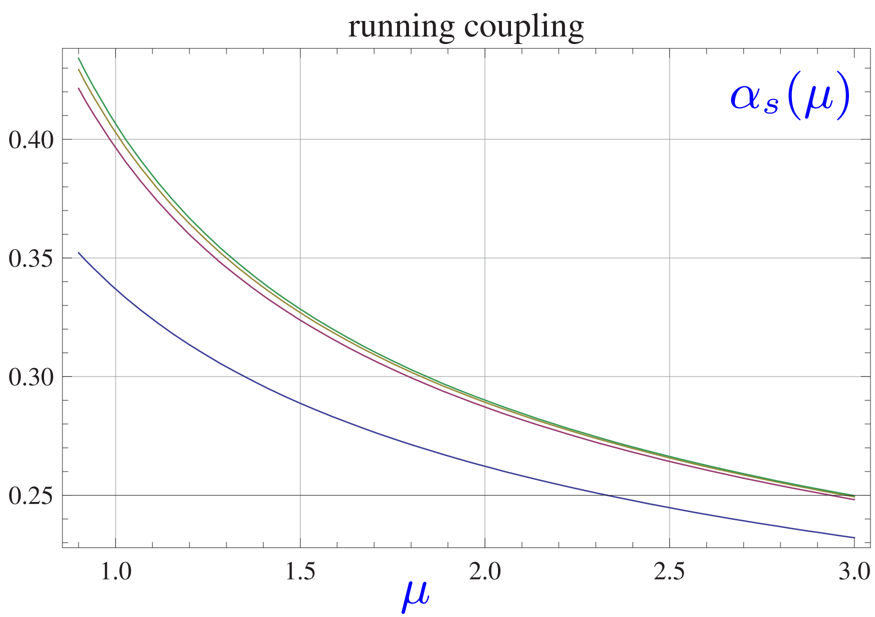
- Suitable observables for measurements of α_s, m_q, ...
  - comparison of theory predictions and experimental data

Coupling constant renormalization

- Running coupling constant α_s from radiative corrections, e.g. one loop

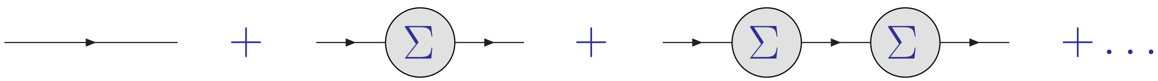
  - screening (like in QED)
  - anti-screening (color charge of g)

- QCD beta function $\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$
  - perturbative expansion to five loops
    Baikov, Chetyrkin, Kühn '16
    Herzog, Ruijl, Ueda, Vermaseren, Vogt '17
    Luthe, Maier, Marquard, Schröder '17
  - very good convergence of perturbative series even at low scales



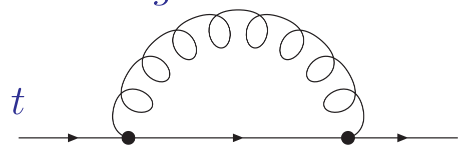
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

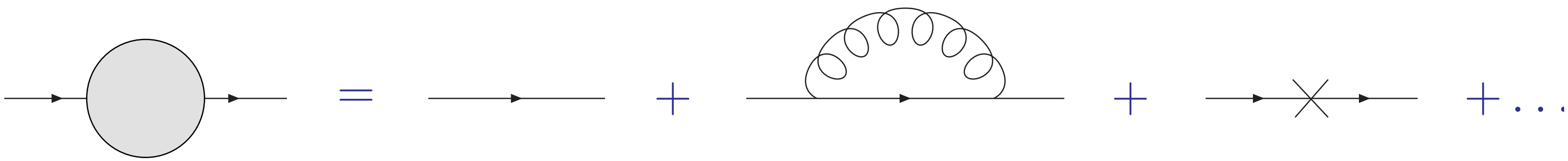
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) \;=\; \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_q^2}\right)^{\epsilon}\left\{(\not{p} - m_q)\left(-C_F\,\frac{1}{\epsilon} + \text{fin.}\right) + m_q\left(3C_F\,\frac{1}{\epsilon} + \text{fin.}\right)\right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



$\Sigma^{\mathrm{ren}}(p, m_q)$ $\qquad$ $(Z_\psi - 1)\not{p} - (Z_m - 1)m_q$

Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - m_q^{ren} coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \to \not{p} - m_q^{\mathrm{pole}}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$

$\overline{\mathrm{MS}}$ *scheme*

- $\overline{\mathrm{MS}}$ mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$$

- $\overline{\mathrm{MS}}$ scheme induces scale dependence: $m(\mu)$

Running quark mass

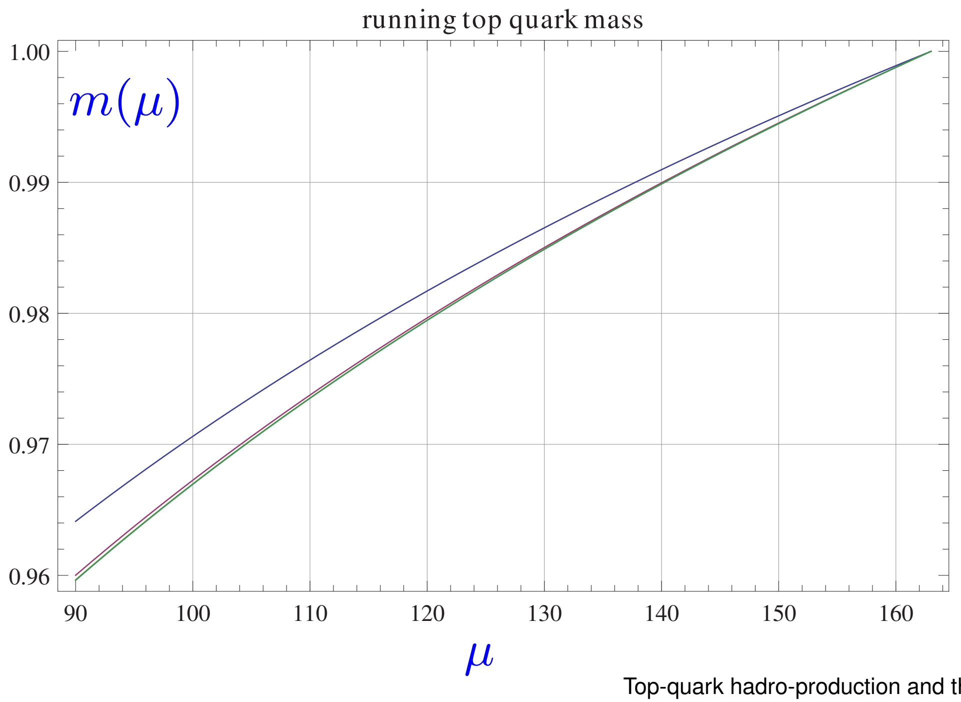
Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to five loops

    Baikov, Chetyrkin, Kühn '14, Luthe, Maier, Marquard, Schröder '17

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



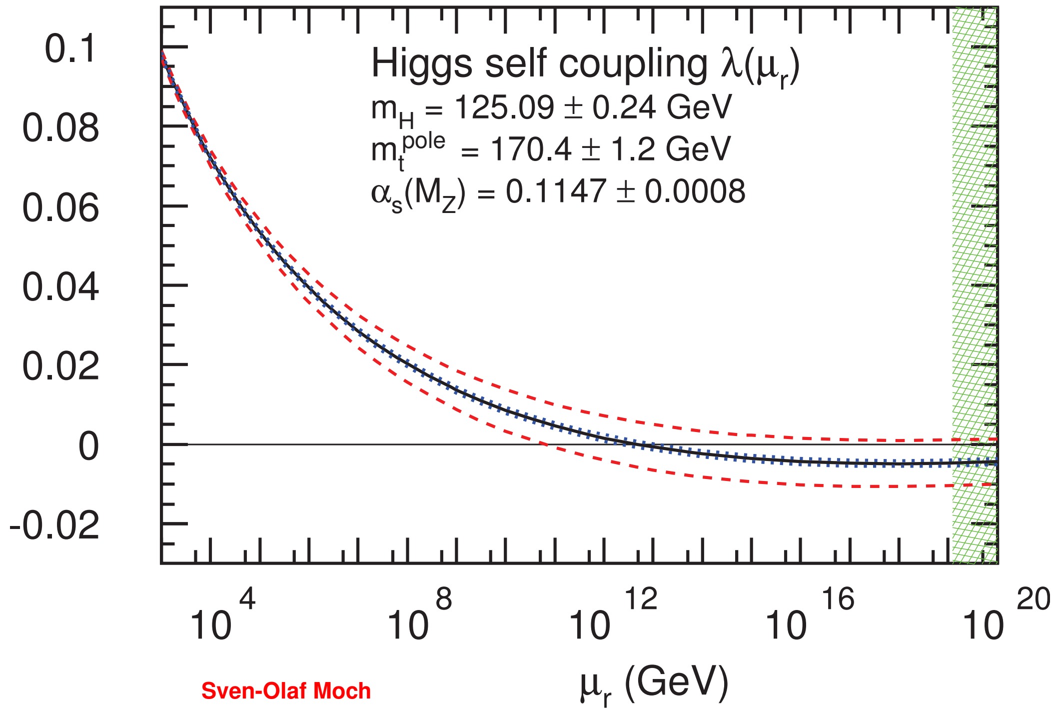
Scheme transformations

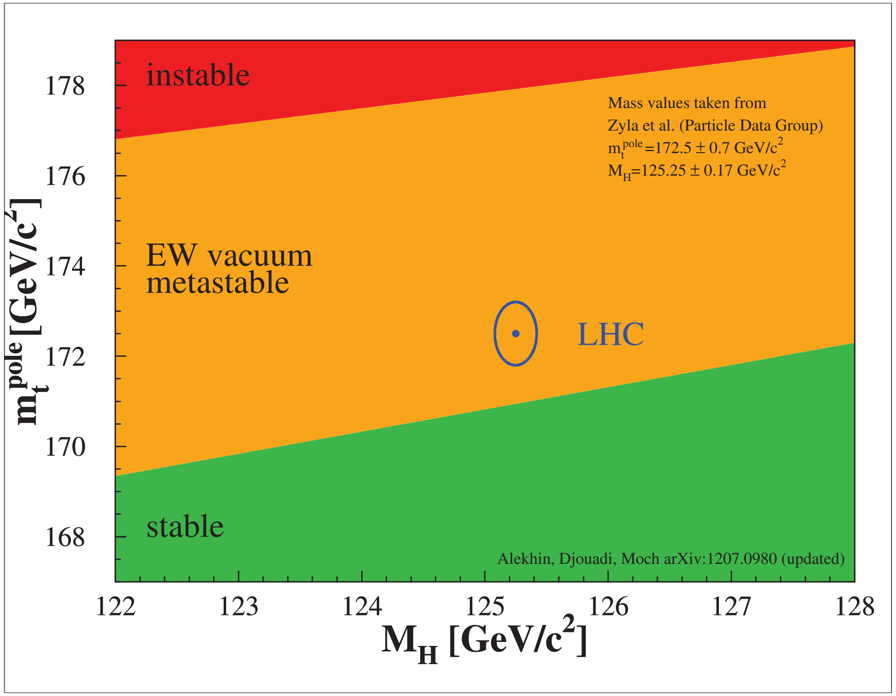
- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{\mathrm{MS}}$ mass
  - known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99; Marquard, Smirnov, Smirnov, Steinhauser '15
  - example: one-loop QCD

$$m^{\mathrm{pole}} = m(\mu) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} \left(\frac{4}{3} + \ln \left(\frac{\mu^2}{m(\mu)^2} \right) \right) + \dots \right\}$$

Meta-stability of the universe

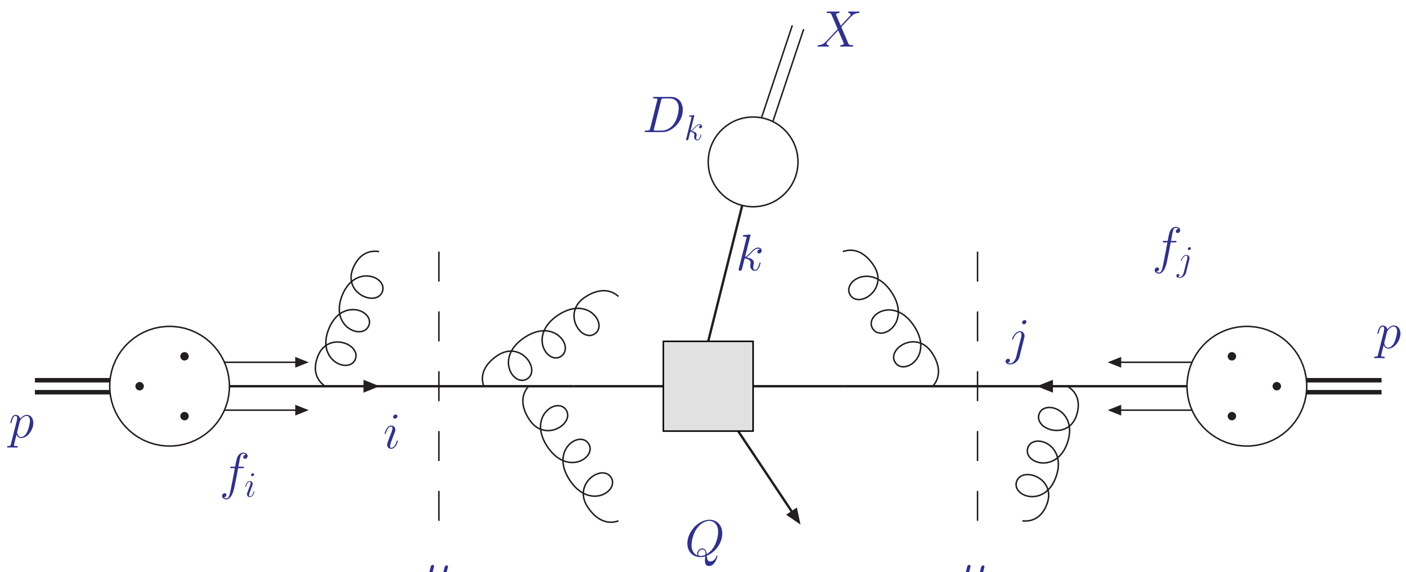
- Large top-quark mass implies large Higgs-Yukawa coupling y_t
- Renormalization group for the Higgs self-coupling $\lambda(\mu)$ dependent on y_t
  - limit of small Higgs mass m_H implies $\lambda(\mu)$ decreases with μ
- Implications on stability of electroweak vacuum
  - Higgs potential unbounded from below for $\lambda(\mu) < 0$
- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s up to $\mu_r = M_{\rm Planck}$ (using program mr Kniehl, Pikelner, Veretin '16)





QCD factorization

QCD factorization

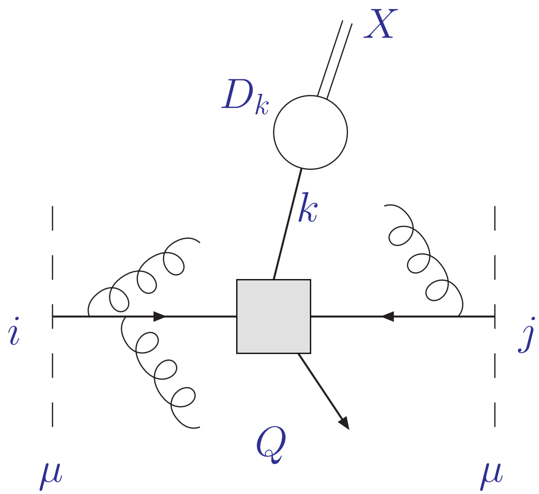


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij \to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^{3}LO (next-to-next-to-next-to-leading order)
  - . . .

Parton luminosity

- Long distance dynamics due to proton structure

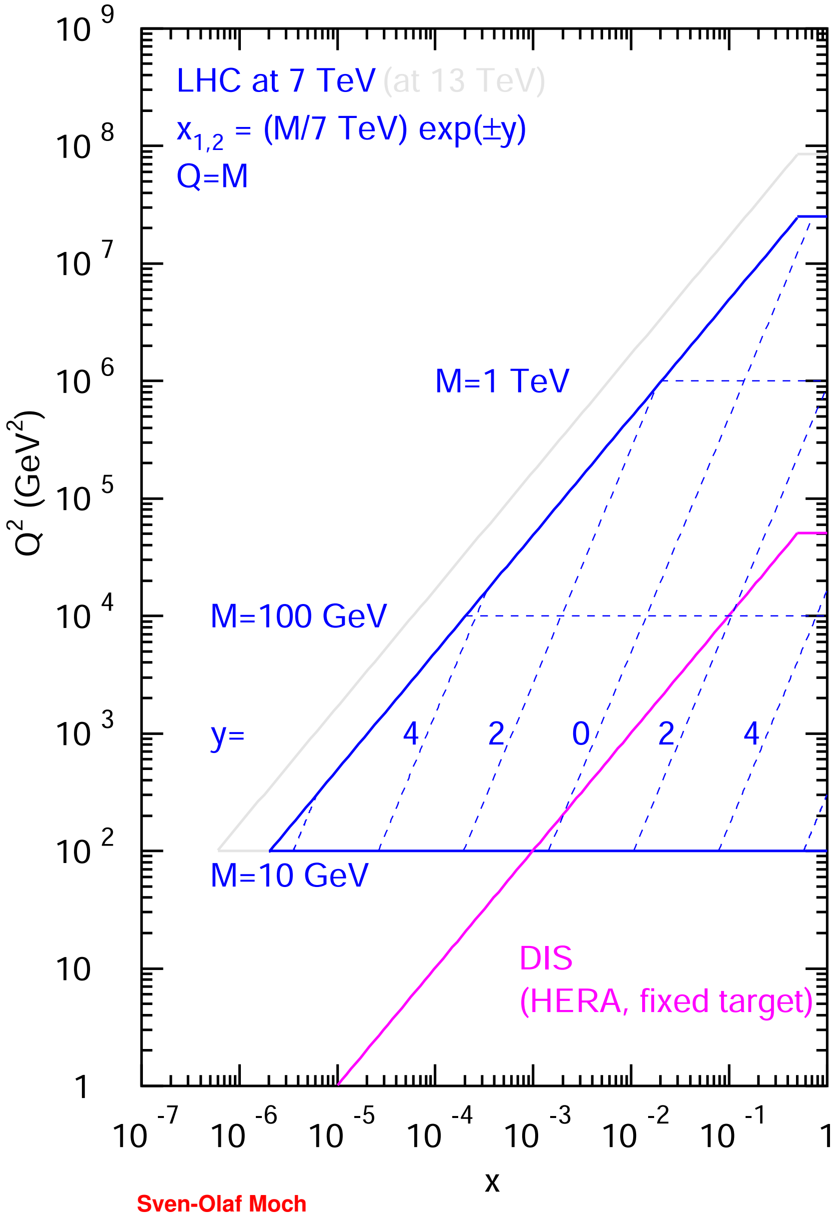
- Cross section depends on parton distributions f_i

$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots \right]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S} e^{\pm y}$
  - forward rapidities sensitive to small-x
- Cross section depends on convolution of parton distributions
  - small-x part of f_i and large-x PDFs f_j

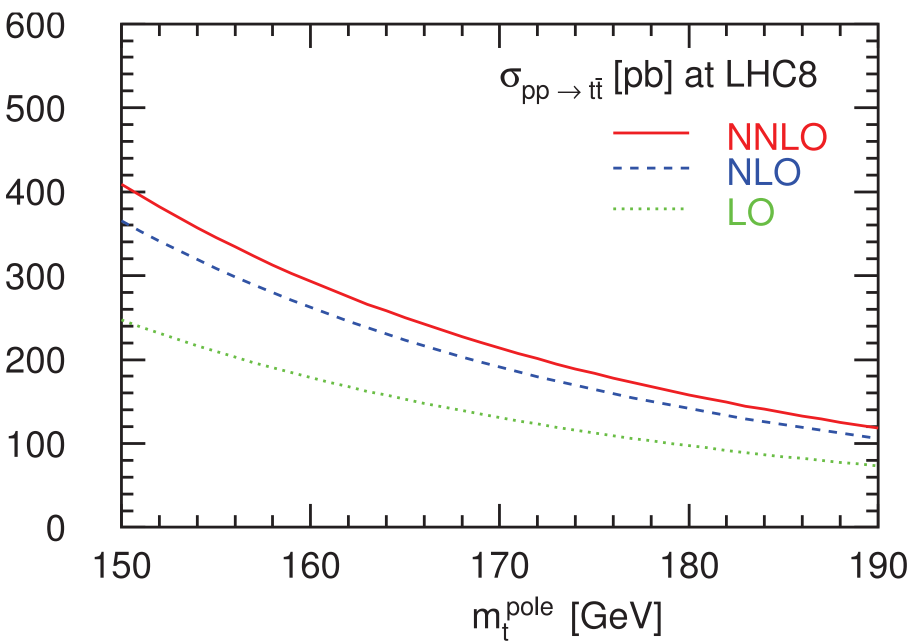
$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\dots \right]$$

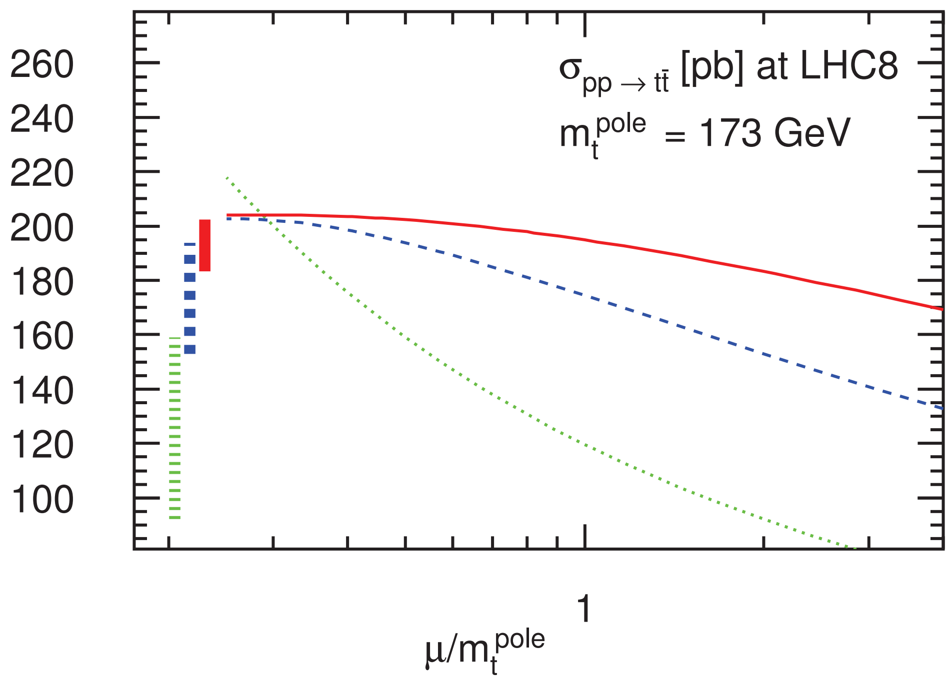
Top-quark theory status

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13



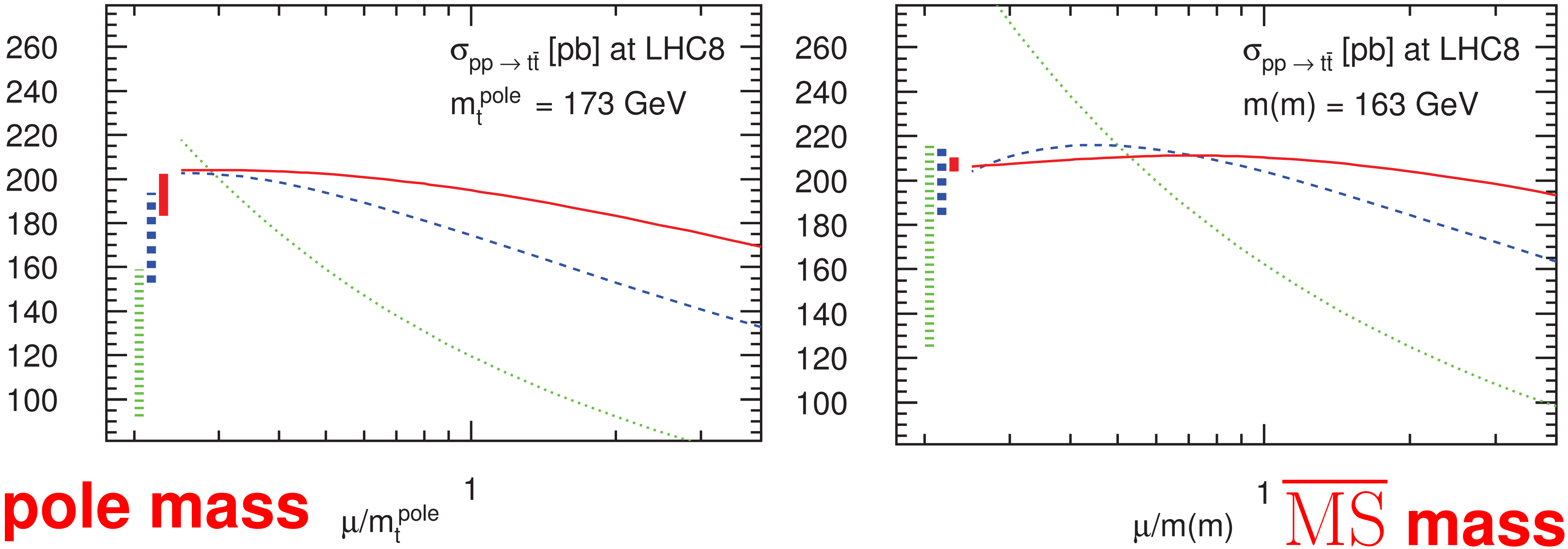


- NNLO perturbative corrections (e.g. at LHC with $\sqrt{s} = 8$ TeV)
- $\overline{\mathrm{MS}}$ renormalization scheme for α_s, on-shell scheme for m_t
  - K-factors: $K_{\mathrm{LO}\to\mathrm{NLO}} = 1.46$ and $K_{\mathrm{NLO}\to\mathrm{NNLO}} = 1.12$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$
  - point of minimal sensitivity at low scales $\mu \sim \mathcal{O}(m_t/4) \sim \mathcal{O}(45)$ GeV

Total cross section with running mass

Comparison pole mass vs. $\overline{\mathrm{MS}}$ mass

Dowling, S.M. '13



- NNLO cross section with $\overline{\mathrm{MS}}$ renormalization scheme for α_s and m_t
  - running mass with better apparent perturbative convergence
  - K-factors: $K_{\mathrm{LO} \to \mathrm{NLO}} = 1.26$ and $K_{\mathrm{NLO} \to \mathrm{NNLO}} = 1.03$
  - point of minimal sensitivity at natural hard scales
    $\mu \sim \mathcal{O}(m_t(m_t)) \sim \mathcal{O}(160)$ GeV

Top-quark mass from total cross section

- Cross section for $t\bar{t}$-production with parametric dependence

$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \underbrace{\hat{\sigma}_{ij \to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)}$$

$$= \hat{\sigma}^{(0)}_{ij \to X} + \alpha_s \, \hat{\sigma}^{(1)}_{ij \to X} + \alpha_s^2 \, \hat{\sigma}^{(2)}_{ij \to X} + \dots$$

  - PDFs f_i, strong coupling α_s, masses m_X
- PDFs and $\alpha_s(M_Z)$ already well constrained by global fit
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \dots 5 \cdot 10^{-2}$

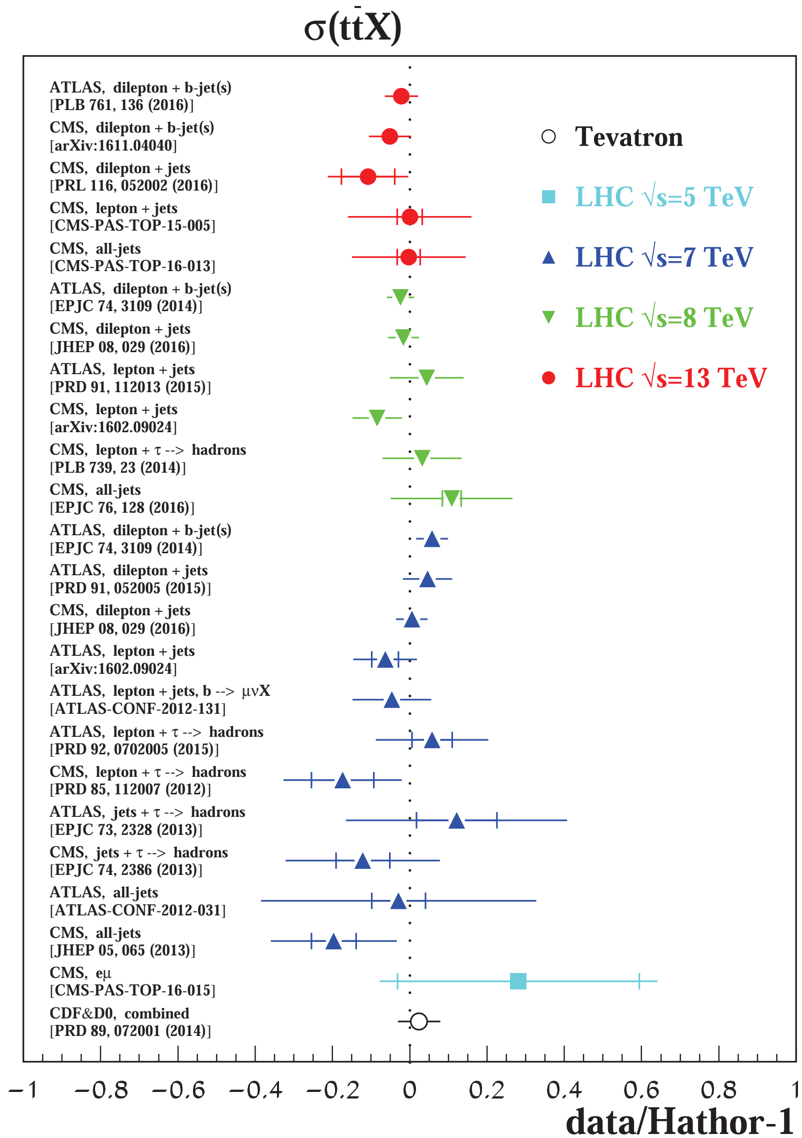
Top-quark mass determination

- Choice of renormalization scheme for treatment of heavy quarks
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Intrinsic limitation of sensitivity in total cross section

$$\left| \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq 5 \times \left| \frac{\Delta m_t}{m_t} \right|$$

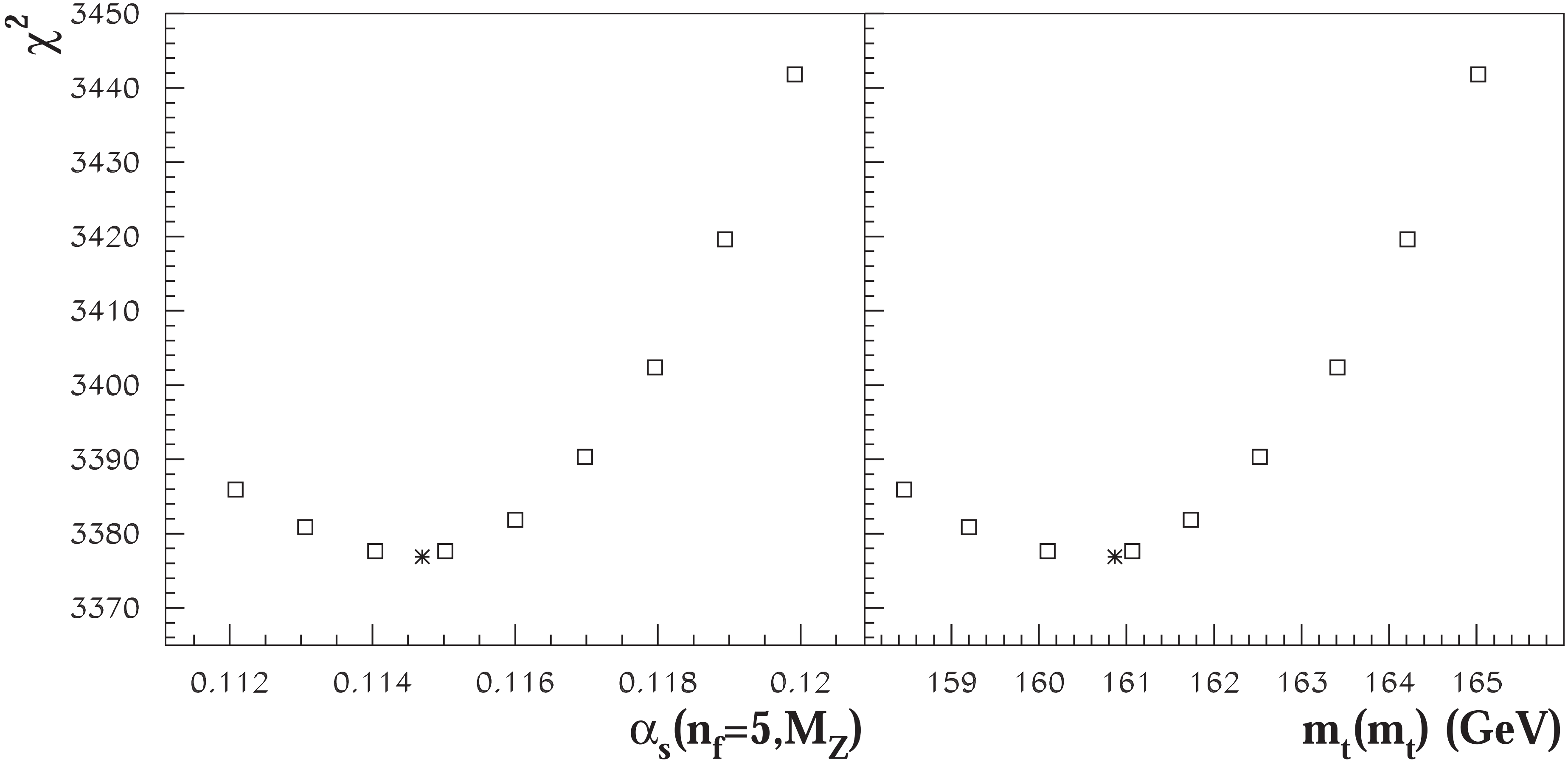
Data on top-quark cross sections (2017)

- Pulls for $t\bar{t}$-inclusive cross sections in ABMP16



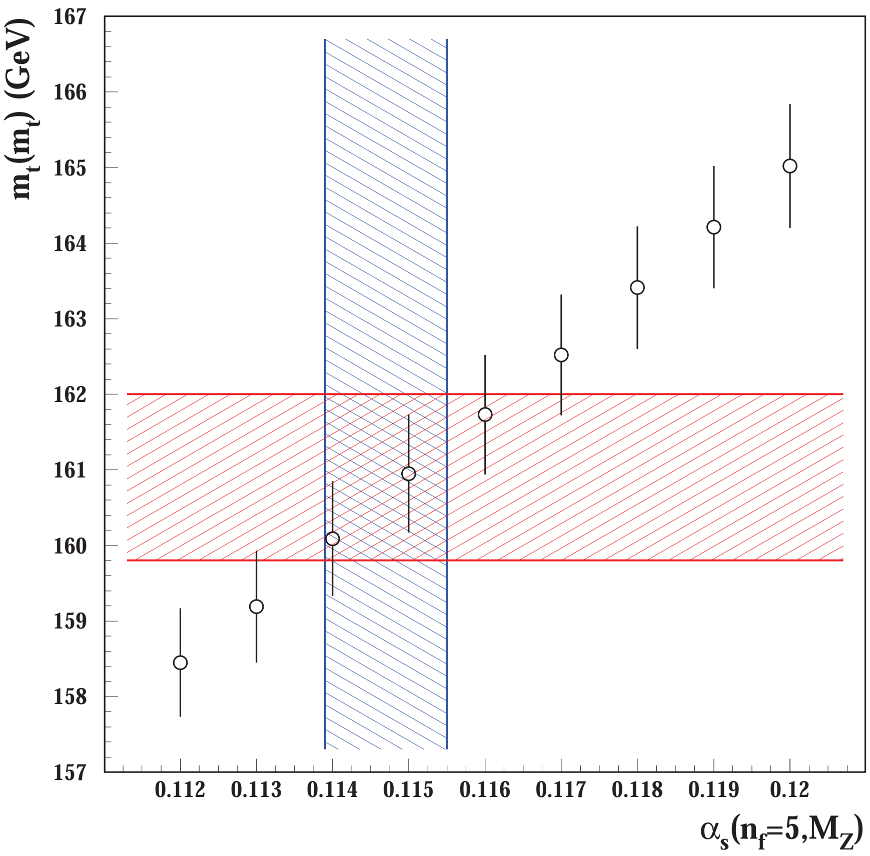
Fit quality

- Goodness-of-fit estimator χ^2 for extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values
  - χ^2 of global fit with $NDP = 2834$
  - data on top-quark production with $NDP = 36$ D0, ATLAS, CMS, LHCb



Correlations

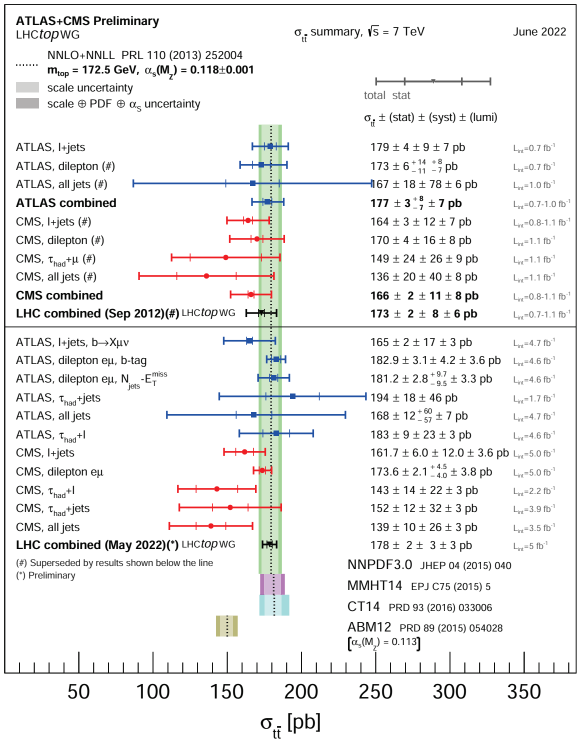
- Correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and $m_t(m_t)$

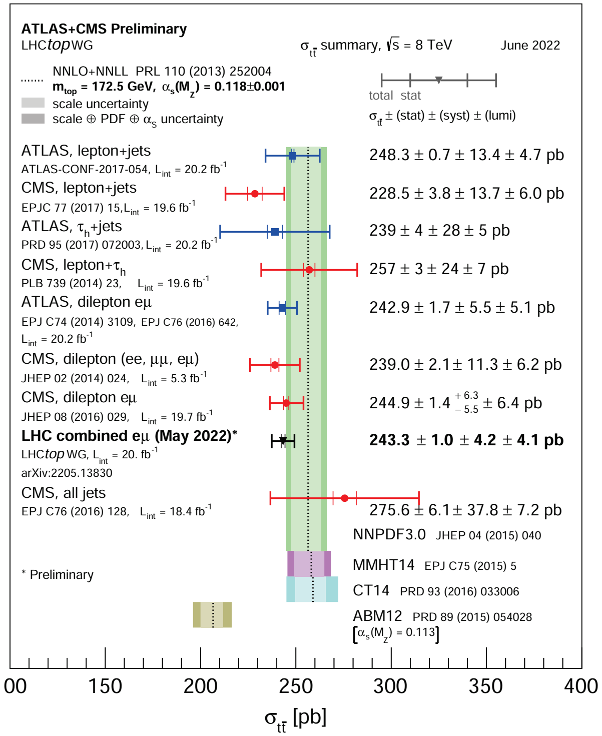


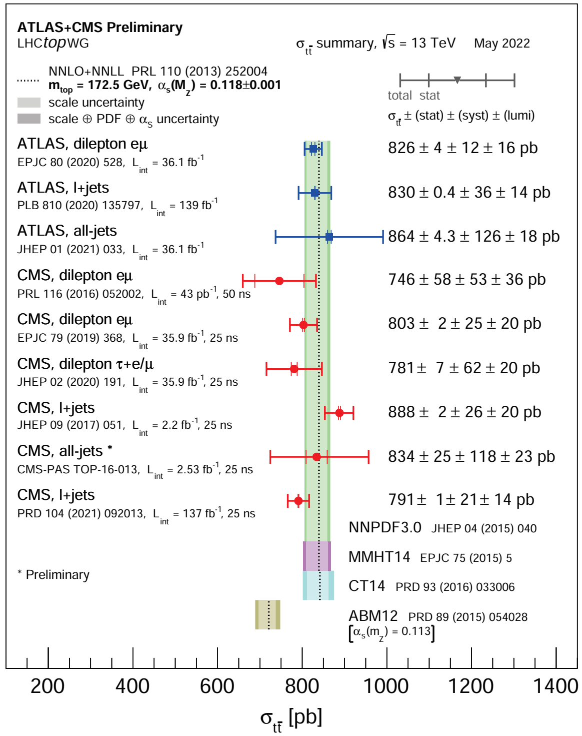
- Fits with fixed values of m_t and $\alpha_S(M_Z)$ carry significant bias

Data on top-quark cross sections (2022)

- $t\bar{t}$-inclusive cross sections from ATLAS and CMS at $\sqrt{s}$ =7, 8 and 13 TeV
- high precision data with small experimental uncertainties
  - cross section combinations at $\sqrt{s}$ =7 and 8 TeV with accuracy of $\mathcal{O}(\pm 2 - 3\%)$



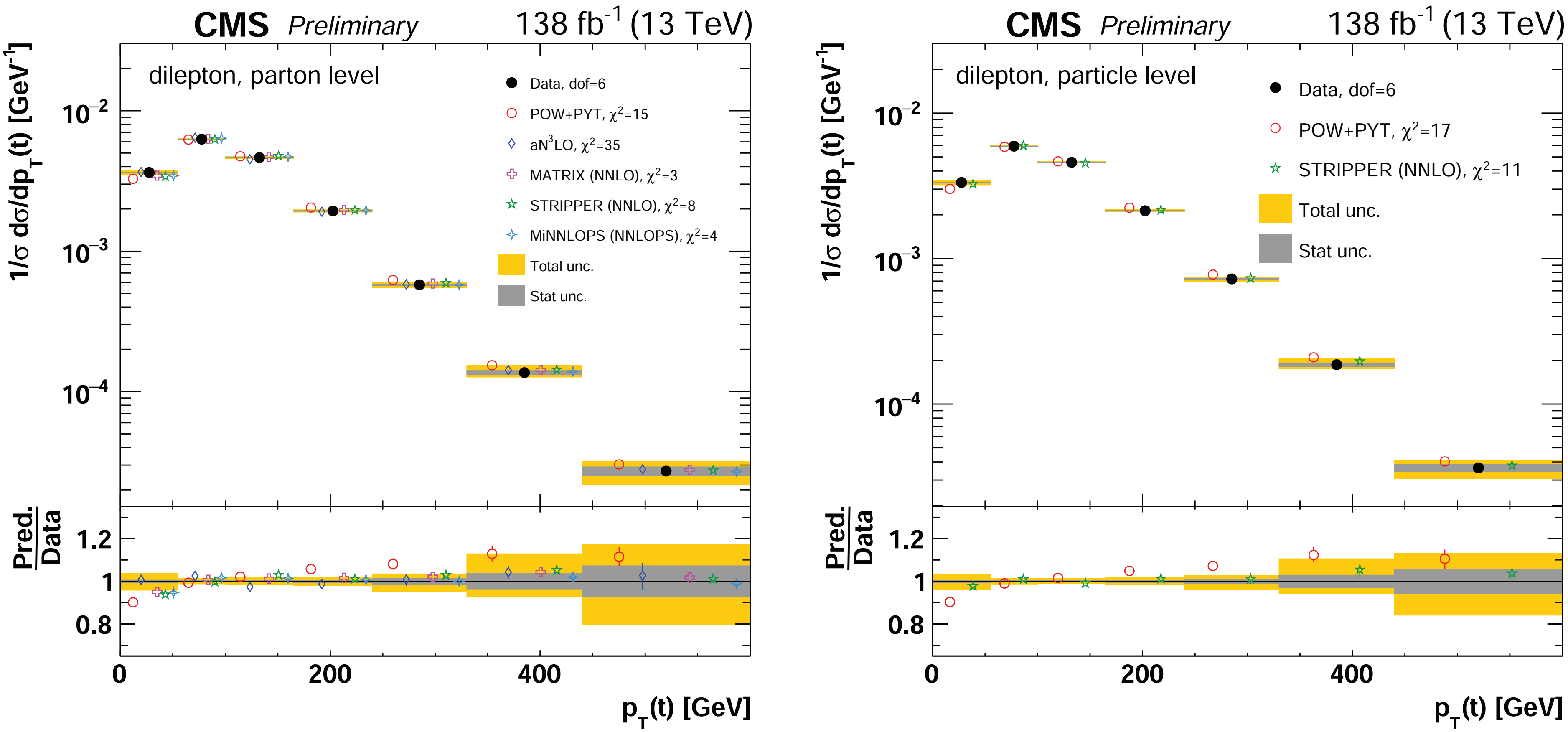




Theory status 2022

- NNLO QCD differential predictions for top-quark pairs at the LHC
  Czakon, Heymes, Mitov '15
- Top-quark pair hadroproduction at NNLO in QCD
  Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19
  - to be implemented in future public release of MATRIX code
    Catani, Devoto, Grazzini, Kallweit, Mazzitelli '19
- NNLO event generation for top-quark pair production
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi '20
- Top-pair production at the LHC with MiNNLO_PS
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi '21
- Narrow-width-approximation at NNLO
  - NNLO QCD corrections to leptonic observables in top-quark pair production and decay
    - implemented in private STRIPPER code
      Czakon, Mitov, Poncelet '20

Differential cross sections (I)



- NNLO QCD predictions in fiducial phase space
  - dynamic scales $\mu_R = \mu_F = H_T/4$ with $H_T = \sqrt{p(t)_T^2 + m_t^2} + \sqrt{p(\bar{t})_T^2 + m_t^2}$
  - top quark mass is set to $m_t = 172.5$ GeV with NNPDF31 NNLO PDFs
- Parton level with stable top-quarks (left)
- Particle level with decay leptons (right)

Differential cross sections (II)

Summary

- *The beyond-NLO theoretical predictions provide descriptions of the data that are of similar or improved quality, compared to POW+PYT, except for kinematic spectra where the theory scale uncertainties are large.*
  CMS TOP-20-006-PAS

Challenges

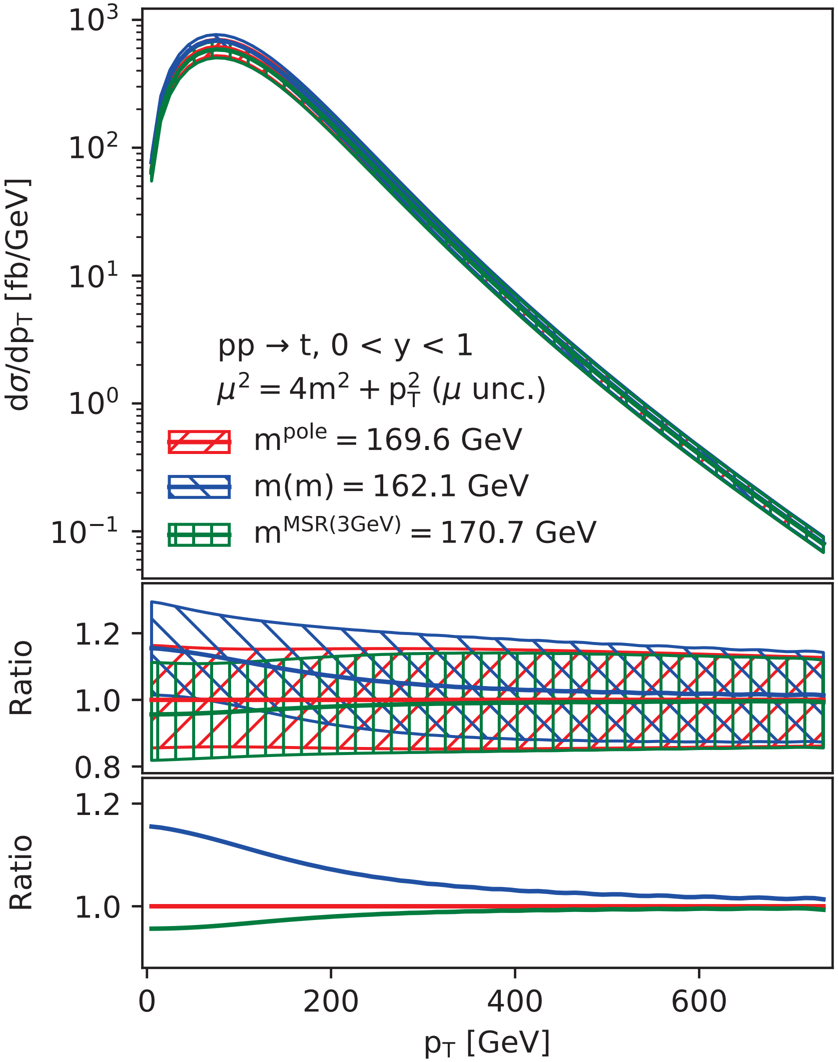
- NNLO codes not publicly accessible
- Very long run times (few CPU years) for distributions with fixed input parameters (m_t, PDFs, . . .)
- Accuracy of NNLO subtraction schemes
  - local sector subtraction (STRIPPER)
  - phase space slicing with q_T^{cut} (MATRIX)

Needs

- NNLO QCD predictions for range of m_t values
- Variation of PDFs (complete set of eigenvectors)

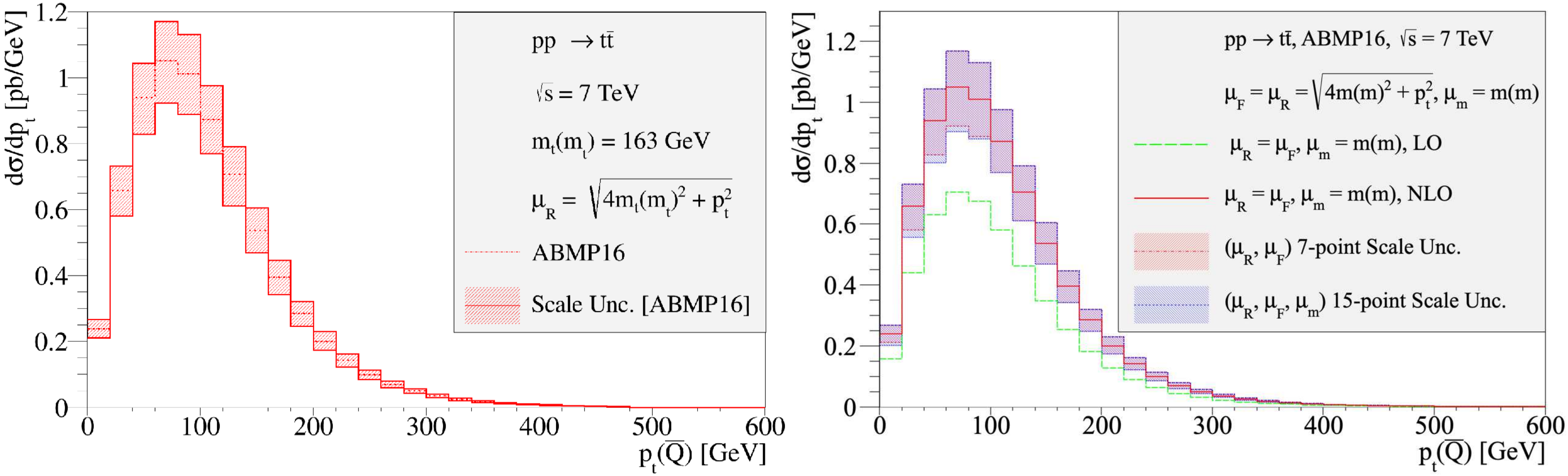
Differential cross sections (III)

Phenomenological studies at NLO



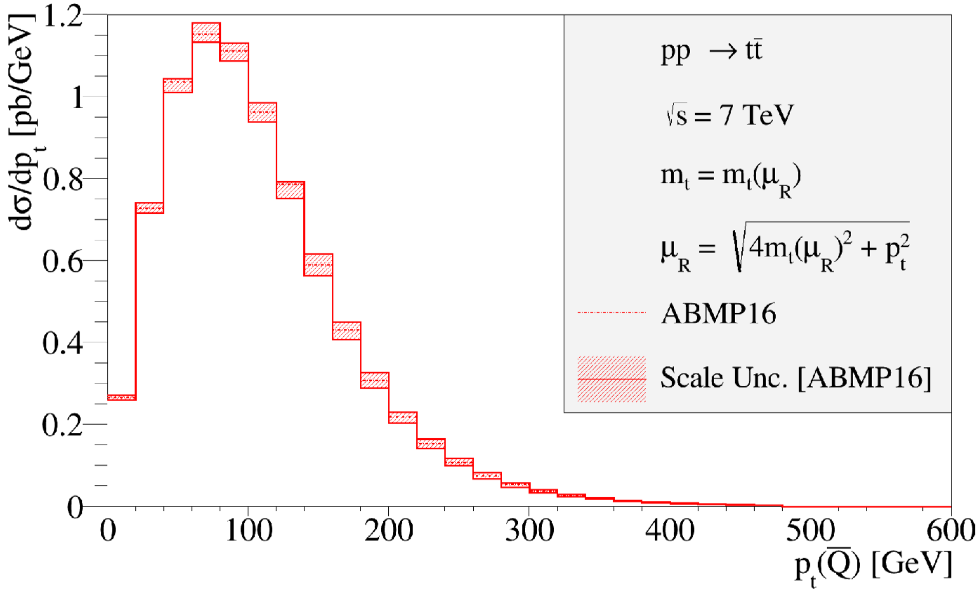
- Top-quark p_T distribution at NLO
- Different top-quark mass renormalization schemes
  - on-shell scheme for m_t
  - $\overline{\mathrm{MS}}$ mass renormalization scheme $m_t(\mu_m)$
  - MSR mass $m_t^{\mathrm{MSR}}(R)$

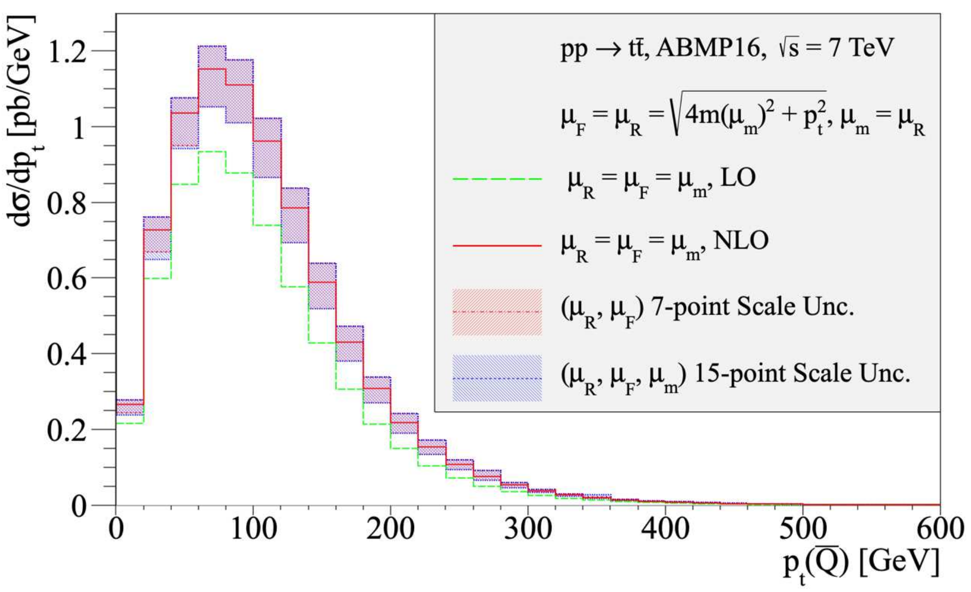
Scale uncertainties for running mass (I)



- NLO QCD predictions with $\overline{\mathrm{MS}}$ mass: fixed $m_t(m_t)$
  - dynamic scale $\mu_R = \mu_F = \sqrt{p_T^2 + 4m_t^2(m_t)}$
  - $\mu_R = \mu_F$ (left)
  - $\mu_R = \kappa\mu_F$ with $1/2 < \kappa < 2$ (right)
- top quark mass $m_t(m_t) = 163.0$ GeV with ABMP16 NLO PDFs

Scale uncertainties for running mass (II)





- NLO QCD predictions with $\overline{\text{MS}}$ mass: running $m_t(\mu_m)$
  - dynamic scale $\mu_R = \mu_F = \sqrt{p_T^2 + 4m_t^2(\mu_m)}$
  - $\mu_R = \mu_F$ (left)
  - $\mu_R = \kappa\mu_F$ with $1/2 < \kappa < 2$ (right)
- top quark mass $m_t(m_t) = 163.0$ GeV with ABMP16 NLO PDFs

Determination of top-quark mass (I)

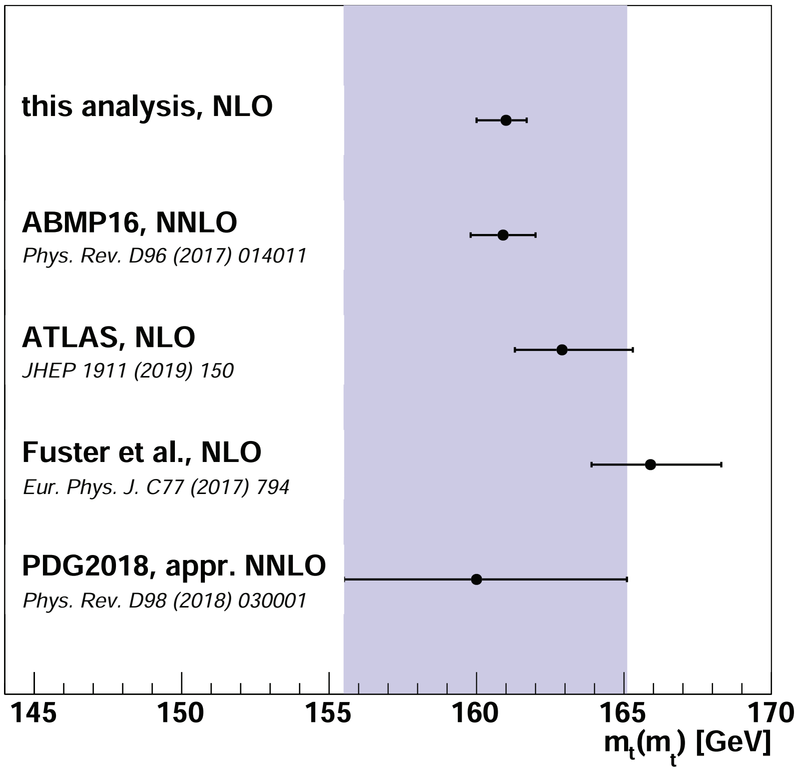
- Correlated determination of PDFs , $\alpha_s(M_Z)$ and $m_t(m_t)$ using HERA DIS data and $t\bar{t}$ cross sections by CMS collaboration
  Garzelli, Kemmler, S. M., Zenaiev '20
  - ansatz from HERAPDF for PDFs

Settings	Fit results
pole mass	$\chi^2/\text{dof} = 1364/1151$, $\chi^2_{t\bar{t}}/\text{dof} = 20/23$
$\mu_R = \mu_F = H'$	$m_t^{\text{pole}} = 170.5 \pm 0.7(\text{fit}) \pm 0.1(\text{mod})^{+0.0}_{-0.1}(\text{par}) \pm 0.3(\mu)$ GeV
CMS, arXiv:1904.05237	$\alpha_S(M_Z) = 0.1135 \pm 0.0016(\text{fit})^{+0.0002}_{-0.0004}(\text{mod})^{+0.0008}_{-0.0001}(\text{par})^{+0.0011}_{-0.0005}(\mu)$
pole mass	$\chi^2/\text{dof} = 1363/1151$, $\chi^2_{t\bar{t}}/\text{dof} = 19/23$
$\mu_R = \mu_F = m_t^{\text{pole}}$	$m_t^{\text{pole}} = 169.9 \pm 0.7(\text{fit}) \pm 0.1(\text{mod})^{+0.0}_{-0.0}(\text{par})^{+0.3}_{-0.9}(\mu)$ GeV
this work	$\alpha_S(M_Z) = 0.1132 \pm 0.0016(\text{fit})^{+0.0003}_{-0.0004}(\text{mod})^{+0.0003}_{-0.0000}(\text{par})^{+0.0016}_{-0.0008}(\mu)$
$\overline{\text{MS}}$ mass	$\chi^2/\text{dof} = 1363/1151$, $\chi^2_{t\bar{t}}/\text{dof} = 19/23$
$\mu_R = \mu_F = m_t(m_t)$	$m_t(m_t) = 161.0 \pm 0.6(\text{fit}) \pm 0.1(\text{mod})^{+0.0}_{-0.0}(\text{par})^{+0.4}_{-0.8}(\mu)$ GeV
this work	$\alpha_S(M_Z) = 0.1136 \pm 0.0016(\text{fit})^{+0.0002}_{-0.0005}(\text{mod})^{+0.0002}_{-0.0001}(\text{par})^{+0.0015}_{-0.0009}(\mu)$
MSR mass, $R = 3$ GeV	$\chi^2/\text{dof} = 1363/1151$, $\chi^2_{t\bar{t}}/\text{dof} = 19/23$
$\mu_R = \mu_F = m_t^{\text{MSR}}$	$m_t^{\text{MSR}} = 169.6 \pm 0.7(\text{fit}) \pm 0.1(\text{mod})^{+0.0}_{-0.0}(\text{par})^{+0.3}_{-0.9}(\mu)$ GeV
this work	$\alpha_S(M_Z) = 0.1132 \pm 0.0016(\text{fit})^{+0.0003}_{-0.0004}(\text{mod})^{+0.0002}_{-0.0000}(\text{par})^{+0.0016}_{-0.0008}(\mu)$

Table 1. The values for $\alpha_S(M_Z)$ and the top-quark mass in different mass schemes obtained in CMS, arXiv:1904.05237 and in this work by fitting the CMS data on $t\bar{t}$ production and the HERA DIS data arXiv:1506.06042 to theoretical predictions. The fit, model (mod), parametrisation (par) and scale variation (μ) uncertainties are reported. Also the values of χ^2 are reported, as well as the partial χ^2 values per number of degrees of freedom (dof) for the $t\bar{t}$ data ($\chi^2_{t\bar{t}}$) for 23 $t\bar{t}$ cross-section bins in the fit. The scale H' is defined in the text.

Determination of top-quark mass (II)

- Extraction of $m_t(m_t)$ at NLO from differential $t\bar{t}$ cross-sections using data of CMS collaboration
  Garzelli, Kemmler, S. M., Zenaiev '20
  - value of $m_t(m_t)$ compared to other determinations
  - world average labelled as PDG2018, appr. NNLO is based on a single determination of D0 collaboration

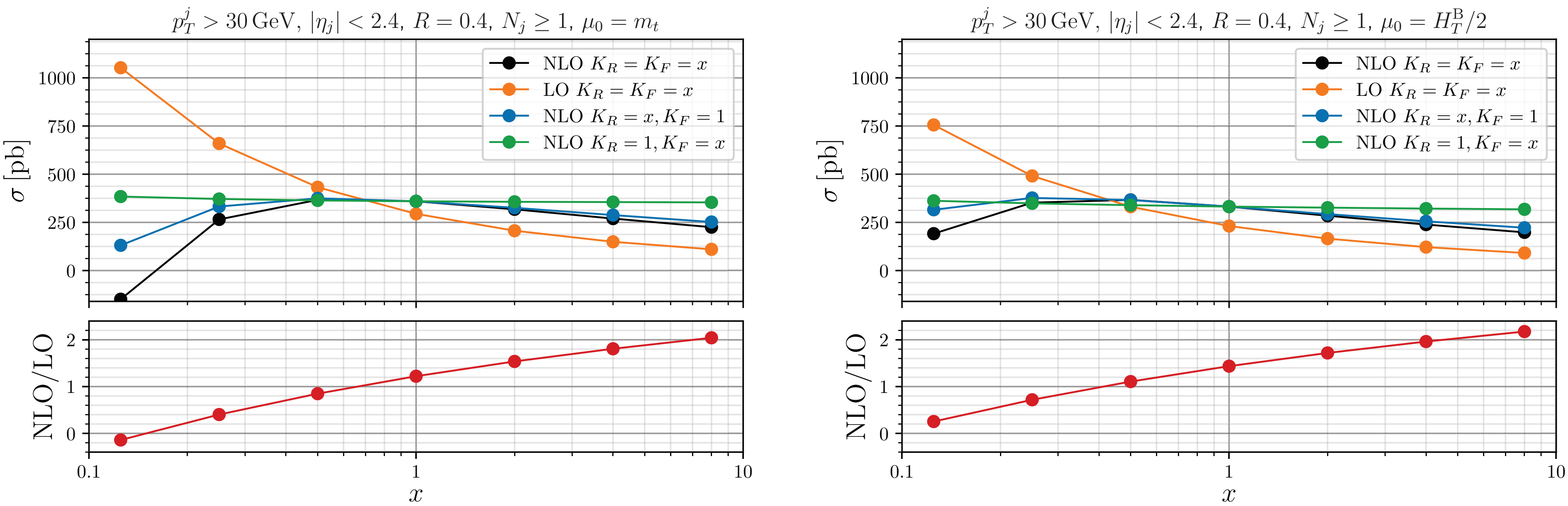


Top-quark pairs with one jet

Top-quark pairs with one jet

- Large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
- Scale dependence greatly reduced at NLO
  - left: corrections for total rate at scale $\mu_R = \mu_F = m_t$ are almost zero
  - right: dynamic scale $\mu_R = \mu_F = H_T/2$ shows better scale stability with $H_T = \sqrt{p(t)_T^2 + m_t^2} + \sqrt{p(\bar{t})_T^2 + m_t^2} + p(j)_T$

Bevilacqua, Hartanto, Kraus, Worek '15



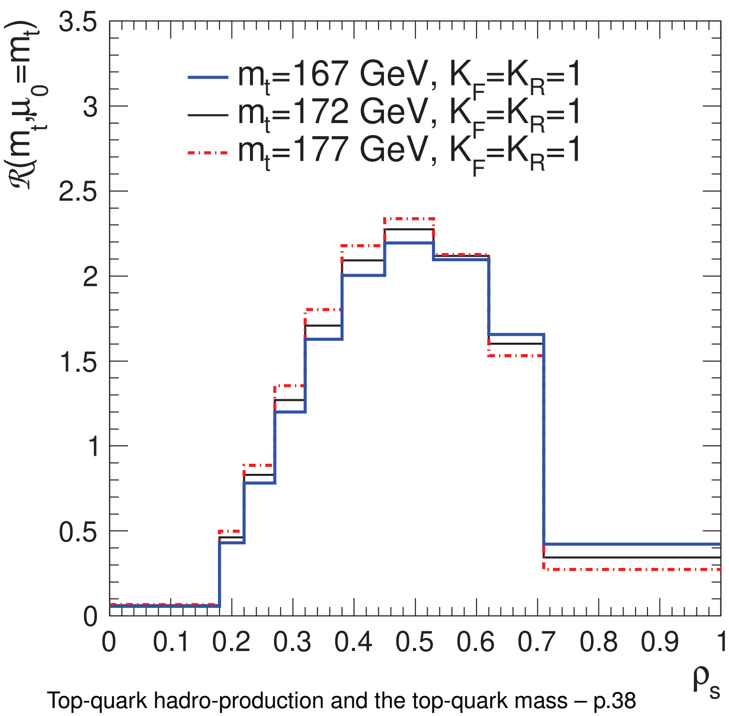
Top-quark mass from $t\bar{t}$ + jet-samples

- Differential $t\bar{t} + \mathrm{jet}$ cross section as function of invariant mass $\sqrt{s_{t\bar{t}+1\mathrm{jet}}}$ offers possibility for top-quark mass determination
  - additional jet raises kinematical threshold
- Normalized-differential $t\bar{t} + \mathrm{jet}$ cross section

Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos '13

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

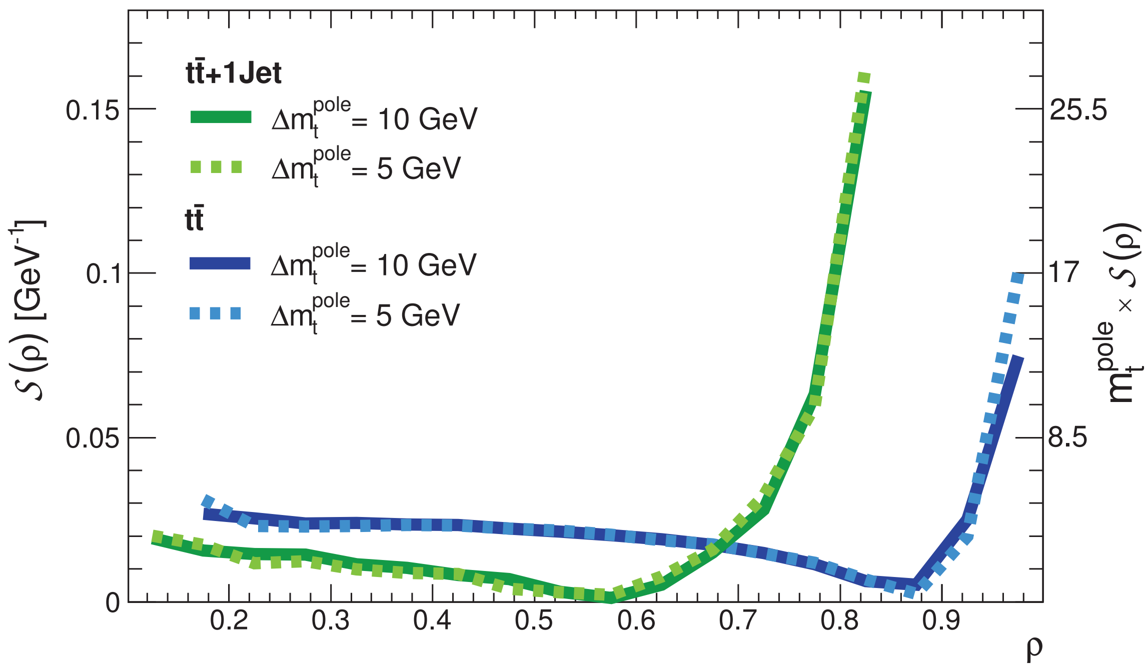
  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t} + 1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$
- Normalization with $1/\sigma_{t\bar{t}+1\mathrm{jet}}$ cancels many (experimental) uncertainties



Mass sensitivity of $t\bar{t} + \mathrm{jet}$*-samples*

- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .
- Increased sensitivity for system $t\bar{t} + \mathrm{jet}$ compared

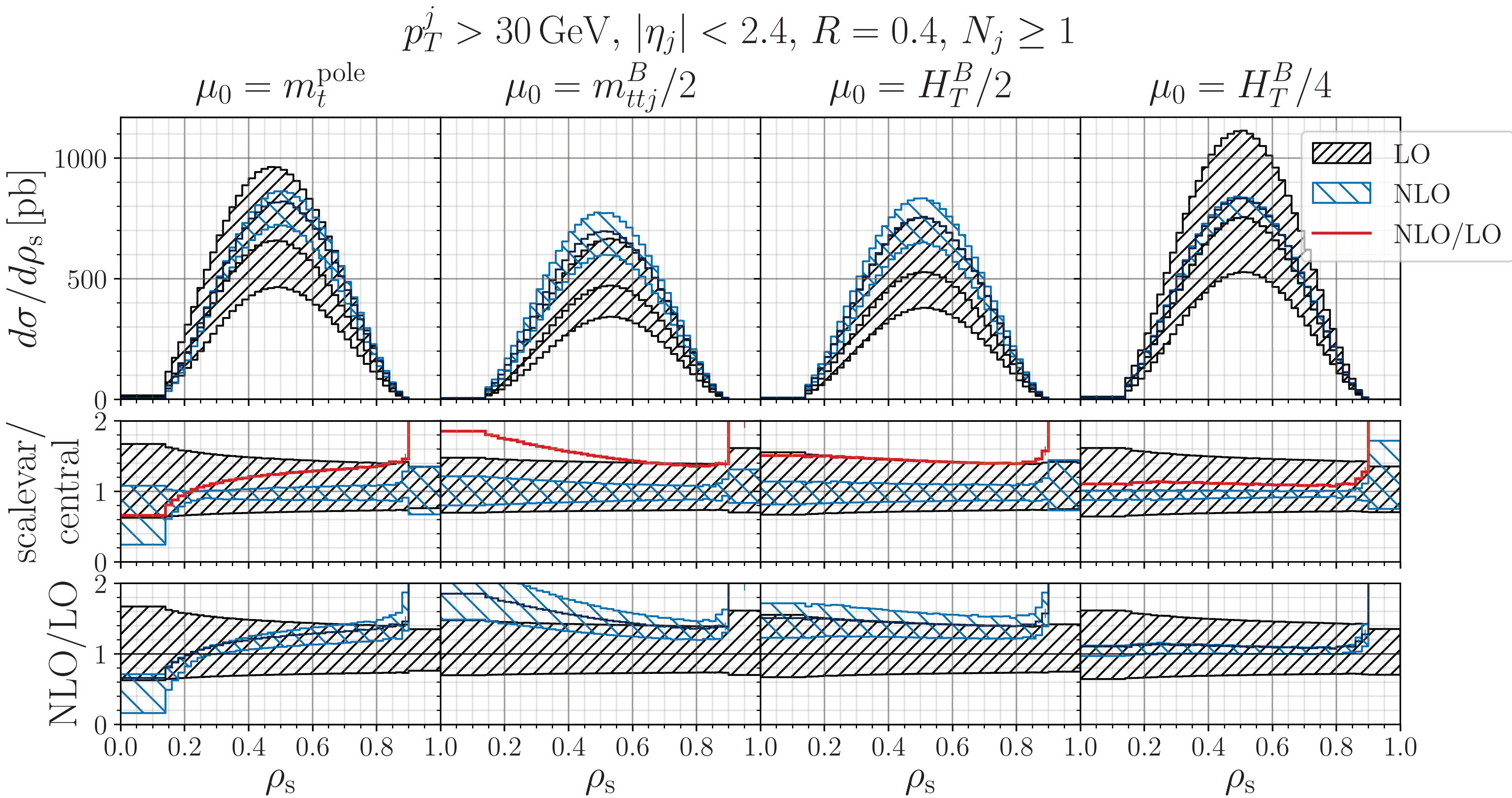
$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$



- Significant mass sensitivity for $\rho_s \geq 0.5$

Theory status for $t\bar{t}j$ production

- Theory predictions at NLO QCD for $t\bar{t}j$ production with different scale choices Alioli, Fuster, Garzelli, Gavardi, Irles, Melini, S. M., Uwer, Voß '22
  - dynamical scale with better apparent perturbative convergence

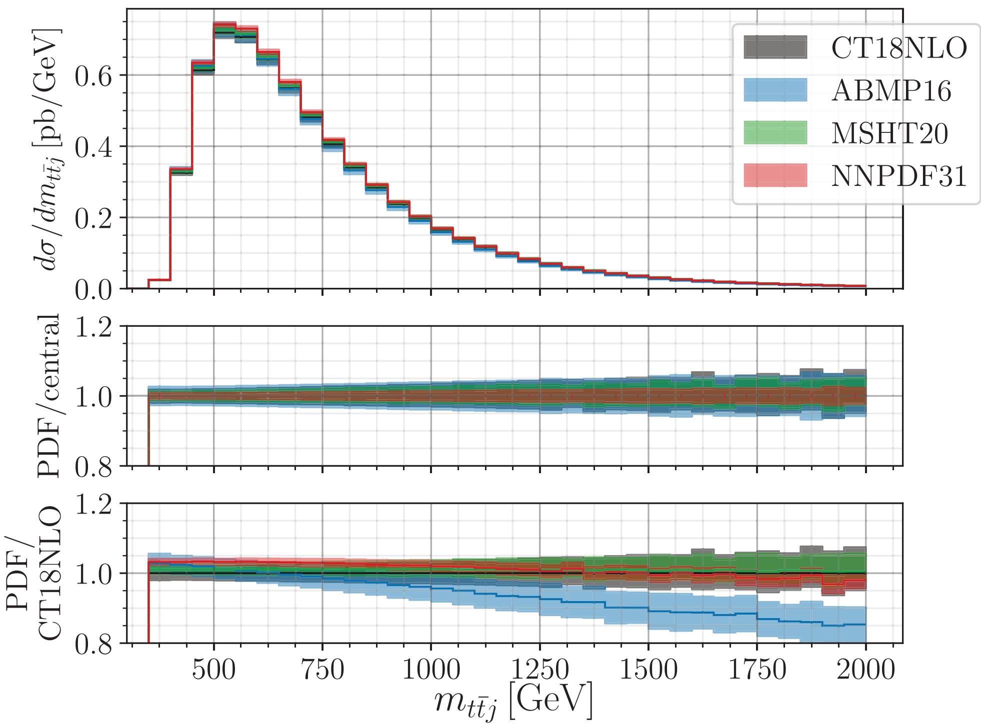


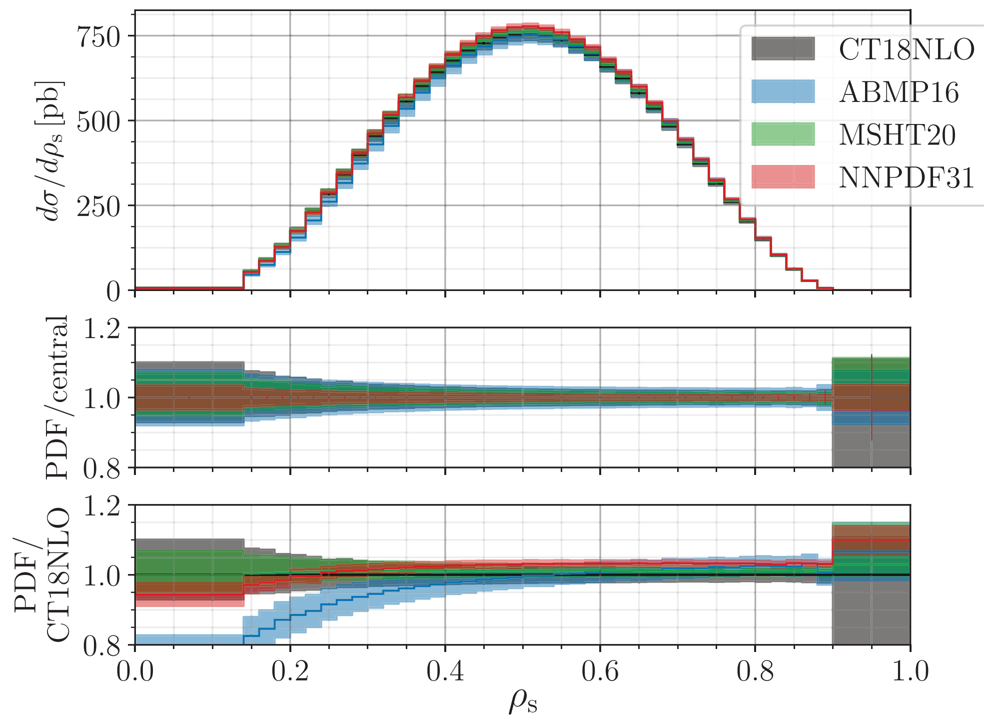
Dependence on parton distributions

- Predictions for $m_{t\bar{t}j}$ (left) and ρ_s (right) distributions at LO computation with $\mu_0 = H_T^B/4$

  Alioli, Fuster, Garzelli, Gavardi, Irles, Melini, S. M., Uwer, Voß '22

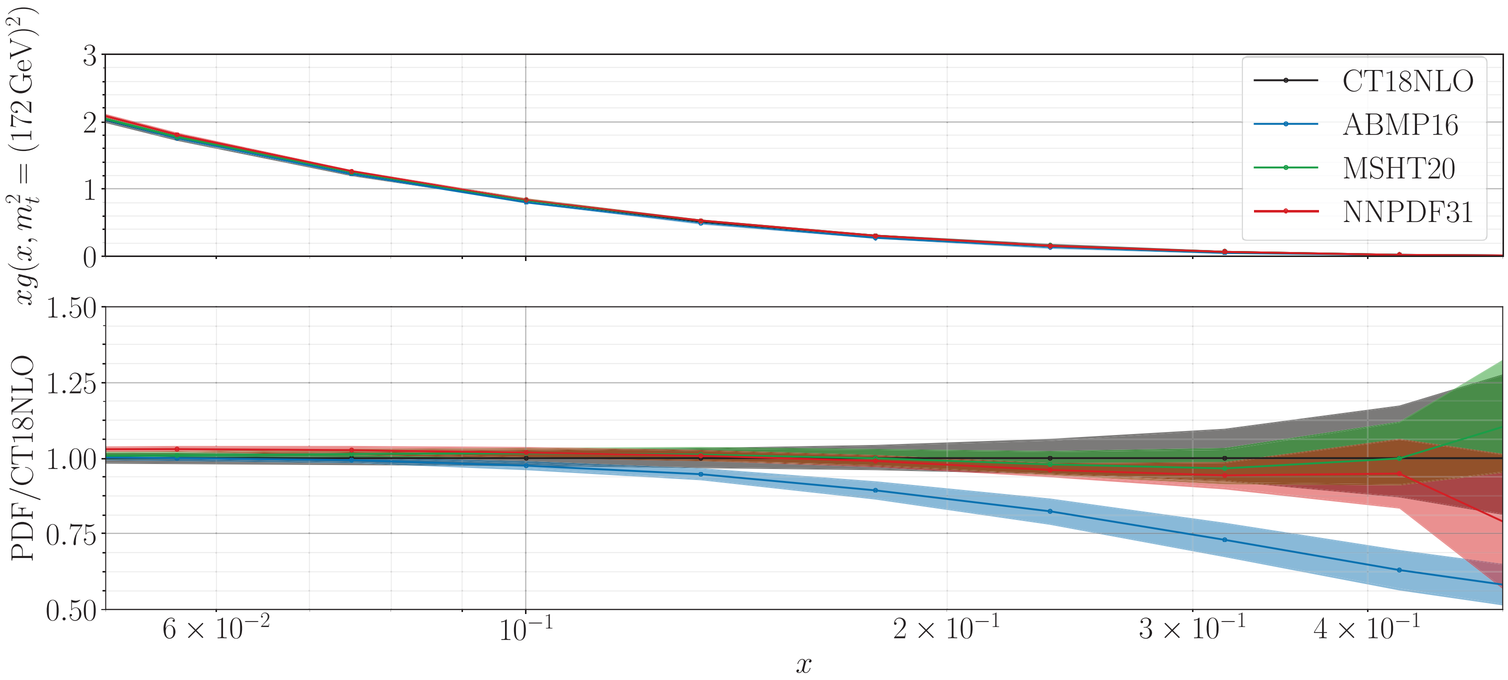
  - PDF uncertainties of ABMP16, CT18, MSHT20 and NNPDF3.1 NLO sets



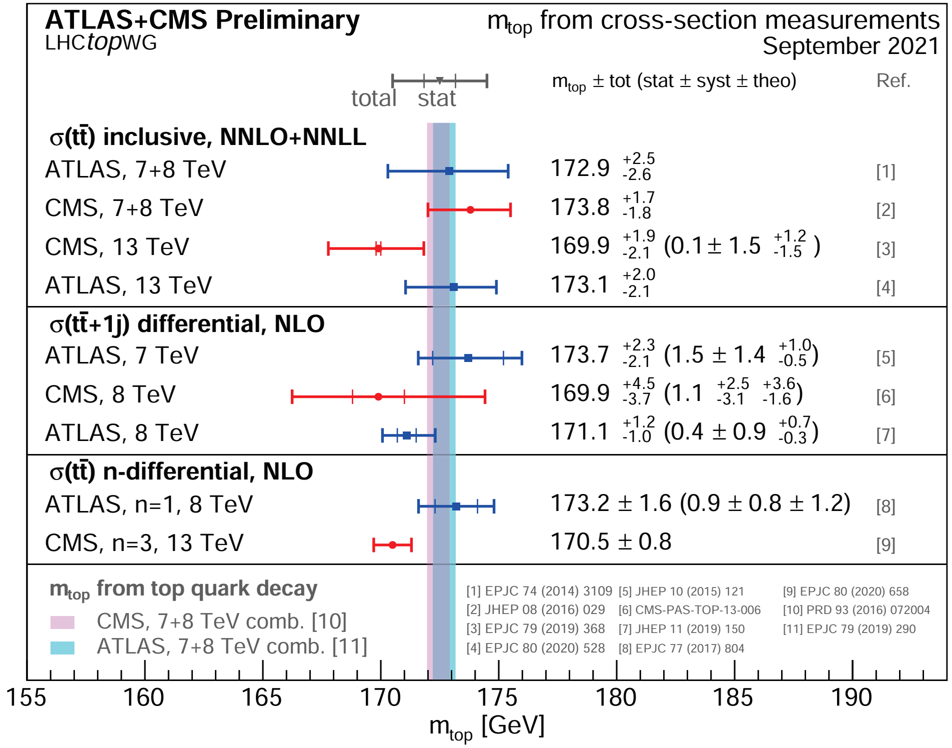


Gluon distribution

- PDFs sets ABMP16, CT18, MSHT20 and NNPDF3.1 at NLO for $Q^2 = m_t^2 = (172\,\text{GeV})^2$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 5 \cdot 10^{-2} \dots 10^{-1}$ for $m_{t\bar{t}j}$



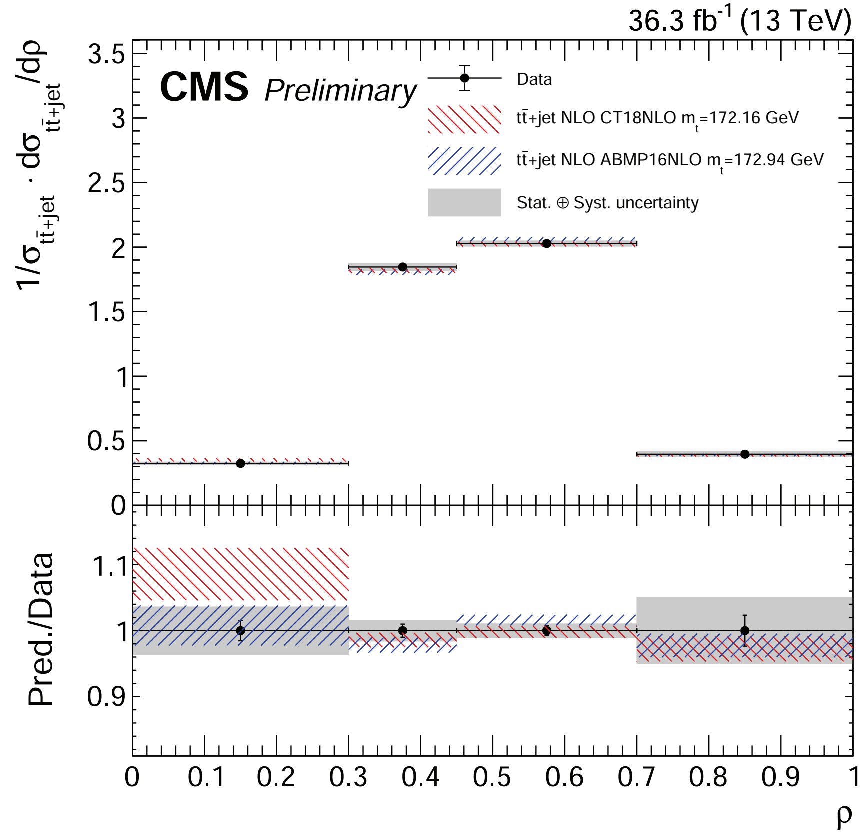
Top-quark mass determinations (I)



- Top-quark mass measurement from $t\bar{t}j$ production becoming competitive
- $t\bar{t}j$ cross-sections use NLO (NLO+PS) predictions for mass determination
- Elevating accuracy of theory predictions beyond NLO improves m_t values and decreases their uncertainties

Top-quark mass determinations (II)

- CMS measurement of m_t (pole mass) from distributions for $t\bar{t} + 1\text{jet}$ samples accurate to $\sim 0.8\%$ CMS coll. '22
  $m_t \; = \; 172.94\,^{+1.37}_{-1.34}$ GeV

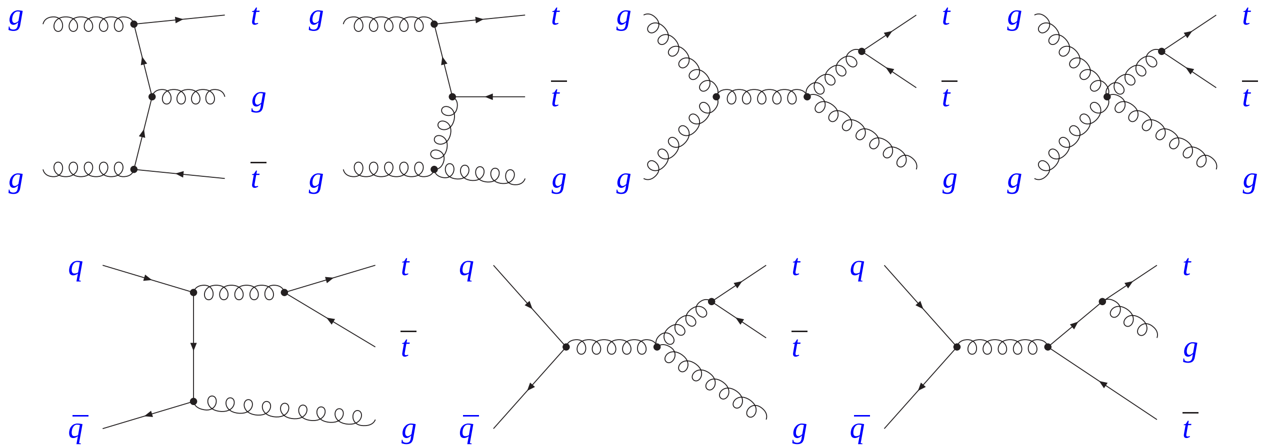


PhD thesis S. Wuchterl

Progress in theory (I)

Beyond NLO

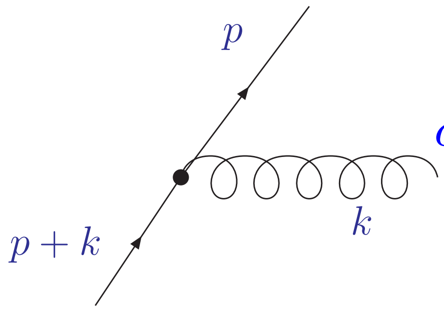
- Scale uncertainties dominate theoretical uncertainties; need NNLO computations (very difficult)
- Focus on kinematical limits
  - threshold logarithms from emission of soft and/or collinear gluons
  - high energy (boosted) regime from t-channel gluon exchange
  - Coulomb corrections
- Sample of Feynman diagrams at Born level



Progress in theory (II)

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg} \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

Threshold logarithms

- Sudakov logarithms in velocity $\beta_{t\bar{t}} = \sqrt{1 - 4m^2/s}$ of heavy quarks or $\beta_{t\bar{t}j} = \sqrt{1 - m^2_{t\bar{t}j}/s}$ for $t\bar{t}j$-system
  - all order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$ in Mellin space (renormalization group equation) Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; ...

Factorization and resummation

- Partonic cross-section factorizes in threshold limit Collins, Soper, Sterman '83

$$\hat{\sigma} = \psi_i \otimes \psi_j \otimes H \otimes S \otimes J$$

  - ψ_i - initial state jet functions, modeling the initial state collinear radiation
  - S - soft gluon exchange
  - H - hard matrix squared
  - J - final state jet function

Strategy of calculation

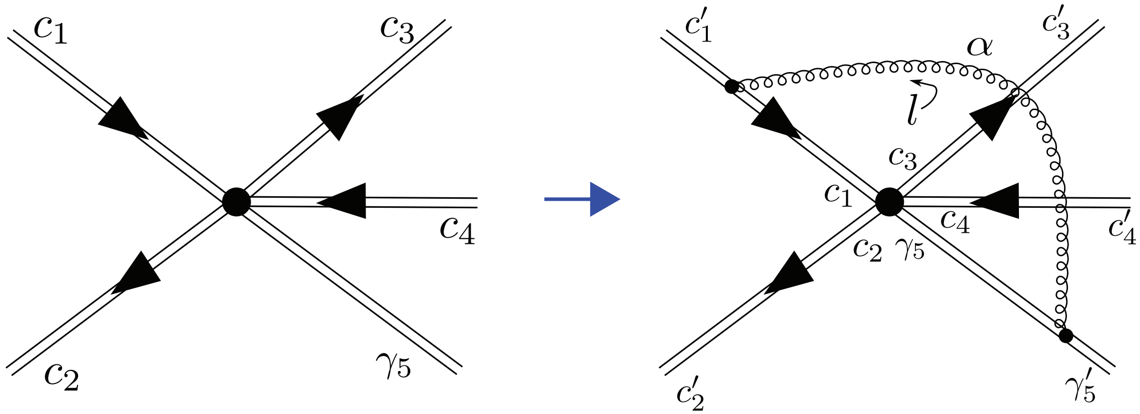
- Evaluation of each functions perturbative
- Resummation of logarithms through renormalization group evolution
- Previously applied to $t\bar{t}$ production processes with associated bosons $t\bar{t} + W/Z/H$ Kulesza, Motyka, Theeuwes et al '17, Broggio, Ferroglia, Pecjak et al. '17.
- $t\bar{t}j$ production
  - complicated because of richer color structure
  - final state jet with non-trivial soft singularity structure

Soft function

- Renormalization group evolution for soft function

$$\mu \frac{d}{d\mu} S_{LI}^{(f)} = \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) S_{LI}^{(f)} = - \left(\Gamma_S^{(f)} \right)^{\dagger}_{LB} S_{BI}^{(f)} - S_{LA}^{(f)} \left(\Gamma_S^{(f)} \right)_{AI} ,$$

- soft anomalous dimension $\left(\Gamma_S^{(f)} \right)_{LI}$ computed from UV divergence of eikonal amplitudes (Wilson lines)



- Integrals at one-loop level known Kidonakis, Sterman '97.
- Color basis for Wilson lines

$$S_{LI}^{\{f\},0} = \left\langle \mathbf{c}_L^{\{f\}} | \mathbf{c}_I^{\{f\}} \right\rangle .$$

Color basis

Sjödahl '08

- gg-channel color basis

$$\mathbf{c}^1_{abcde} = t^e{}_{cd}\delta_{ab}$$

$$\mathbf{c}^2_{abcde} = if_{abe}\delta_{cd}$$

$$\mathbf{c}^3_{abcde} = id_{abe}\delta_{cd}$$

$$\mathbf{c}^4_{abcde} = if_{abn}if_{men}t^m{}_{cd}$$

$$\mathbf{c}^5_{abcde} = d_{abn}if_{men}t^m{}_{cd}$$

$$\mathbf{c}^6_{abcde} = if_{abn}d_{men}t^m{}_{cd}$$

$$\mathbf{c}^7_{abcde} = d_{abn}d_{men}t^m{}_{cd}$$

$$\mathbf{c}^8_{abcde} = P^{10+\overline{10}}_{abme}t^m{}_{cd}$$

$$\mathbf{c}^9_{abcde} = P^{10-\overline{10}}_{abme}t^m{}_{cd}$$

$$\mathbf{c}^{10}_{abcde} = -P^{27}_{abme}t^m{}_{cd}$$

$$\mathbf{c}^{11}_{abcde} = P^{0}_{abme}t^m{}_{cd}$$

- $q\bar{q}$-channel

$$\mathbf{c}^1_{abcde} = t^e{}_{cd}\delta_{ab}$$

$$\mathbf{c}^2_{abcde} = t^e{}_{ab}\delta_{cd}$$

$$\mathbf{c}^3_{abcde} = t^m{}_{ba}t^n{}_{cd}if_{mne}$$

$$\mathbf{c}^4_{abcde} = t^m{}_{ba}t^n{}_{cd}d_{mne}$$

- P^i are projectors:
  $P^i_{ABmn}P^j_{mnCD} = \delta_{ij}P^i_{ABCD}$

- Recent work on color evolution and infrared physics Plätzer 22

Results for $t\bar{t}$+ jet

- Components of $\left(\Gamma_S^{(f)}\right)_{LI}$ for $g\bar{g} \to t\bar{t}g$-channel at one loop

Chargeishvili, Garzelli S.M.

$$\Gamma^{(1)}_{1,1} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \log(v_{35}) + \log(v_{45}) + \log\left(\frac{s}{m_t^2}\right) - \log(8) + 2 + 2i\pi\right) + 1\right]$$
$$\Gamma^{(1)}_{1,2} = \frac{2N_c}{N_c^2-1}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{1,3} = 0$$
$$\Gamma^{(1)}_{1,4} = \frac{N_c^2}{N_c^2-1}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{1,5} = 0$$
$$\Gamma^{(1)}_{1,6} = \frac{N_c^2-4}{N_c^2-1}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{1,7} = 0$$
$$\Gamma^{(1)}_{1,8} = 0$$
$$\Gamma^{(1)}_{1,9} = 0$$
$$\Gamma^{(1)}_{1,10} = 0$$
$$\Gamma^{(1)}_{1,11} = 0$$
$$\Gamma^{(1)}_{2,1} = \frac{1}{N_c}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{2,2} = \frac{1}{N_c}[-L_\beta(N_c^2-1) + N_c^2(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \log(v_{15}) + \log(v_{25}) - \log(8) + 2 + i\pi) + 1]$$
$$\Gamma^{(1)}_{2,3} = 0$$
$$\Gamma^{(1)}_{2,4} = -\tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{35}) - \log(v_{45})$$
$$\Gamma^{(1)}_{2,5} = 0$$
$$\Gamma^{(1)}_{2,6} = 0$$
$$\Gamma^{(1)}_{2,7} = \frac{N_c^2-4}{2N_c^2}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{2,8} = 0$$
$$\Gamma^{(1)}_{2,9} = 0$$
$$\Gamma^{(1)}_{2,10} = \frac{N_c+3}{4N_c+4}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{2,11} = \frac{N_c-3}{4N_c-4}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{3,1} = 0$$
$$\Gamma^{(1)}_{3,2} = 0$$
$$\Gamma^{(1)}_{3,3} = \frac{1}{N_c}[-L_\beta(N_c^2-1) + N_c^2(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \log(v_{15}) + \log(v_{25}) - \log(8) + 2 + i\pi) + 1]$$
$$\Gamma^{(1)}_{3,4} = 0$$
$$\Gamma^{(1)}_{3,5} = -\tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{35}) - \log(v_{45})$$
$$\Gamma^{(1)}_{3,6} = \tfrac{\log(v_{13})}{2} - \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2}$$
$$\Gamma^{(1)}_{3,7} = 0$$
$$\Gamma^{(1)}_{3,8} = 0$$
$$\Gamma^{(1)}_{3,9} = \tfrac{\log(v_{13})}{2} - \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2}$$
$$\Gamma^{(1)}_{3,10} = 0$$
$$\Gamma^{(1)}_{3,11} = 0$$
$$\Gamma^{(1)}_{4,1} = -\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})$$
$$\Gamma^{(1)}_{4,2} = -\log(v_{13}) + \log(v_{14}) - \log(v_{23}) + \log(v_{24}) + 2\log(v_{35}) - 2\log(v_{45})$$
$$\Gamma^{(1)}_{4,3} = 0$$
$$\Gamma^{(1)}_{4,4} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \tfrac{\log(v_{13})}{4} + \tfrac{\log(v_{14})}{4} + \tfrac{\log(v_{15})}{2} + \tfrac{\log(v_{23})}{4} + \tfrac{\log(v_{24})}{4} + \tfrac{\log(v_{25})}{2} + \tfrac{\log(v_{35})}{2} + \tfrac{\log(v_{45})}{2} + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(4096)}{4} + 2 + i\pi\right) + 1\right]$$
$$\Gamma^{(1)}_{4,5} = \frac{N_c^2-4}{4N_c}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{4,6} = \frac{N_c^2-4}{4N_c}[-\log(v_{13}) + \log(v_{14}) - \log(v_{23}) + \log(v_{24}) + 2\log(v_{35}) - 2\log(v_{45})]$$
$$\Gamma^{(1)}_{4,7} = \frac{N_c^2-4}{4N_c}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{4,8} = 0$$
$$\Gamma^{(1)}_{4,9} = 0$$
$$\Gamma^{(1)}_{4,10} = \frac{N_c+3}{4N_c+4}[\log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})]$$
$$\Gamma^{(1)}_{4,11} = \frac{N_c-3}{4N_c-4}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{5,1} = 0$$
$$\Gamma^{(1)}_{5,2} = 0$$
$$\Gamma^{(1)}_{5,3} = -\log(v_{13}) + \log(v_{14}) - \log(v_{23}) + \log(v_{24}) + 2\log(v_{35}) - 2\log(v_{45})$$
$$\Gamma^{(1)}_{5,4} = \frac{N_c}{4}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{5,5} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \tfrac{\log(v_{13})}{4} + \tfrac{\log(v_{14})}{4} + \tfrac{\log(v_{15})}{2} + \tfrac{\log(v_{23})}{4} + \tfrac{\log(v_{24})}{4} + \tfrac{\log(v_{25})}{2} + \tfrac{\log(v_{35})}{2} + \tfrac{\log(v_{45})}{2} + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(4096)}{4} + 2 + i\pi\right) + 1\right]$$
$$\Gamma^{(1)}_{5,6} = \frac{N_c}{4}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{5,7} = \frac{N_c^2-4}{4N_c}[-\log(v_{13}) + \log(v_{14}) - \log(v_{23}) + \log(v_{24}) + 2\log(v_{35}) - 2\log(v_{45})]$$
$$\Gamma^{(1)}_{5,8} = \tfrac{\log(v_{13})}{2} - \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2}$$
$$\Gamma^{(1)}_{5,9} = 0$$
$$\Gamma^{(1)}_{5,10} = 0$$
$$\Gamma^{(1)}_{5,11} = 0$$
$$\Gamma^{(1)}_{6,1} = \log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})$$
$$\Gamma^{(1)}_{6,2} = 0$$
$$\Gamma^{(1)}_{6,3} = \log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})$$
$$\Gamma^{(1)}_{6,4} = \frac{N_c}{4}[-\log(v_{13}) + \log(v_{14}) - \log(v_{23}) + \log(v_{24}) + 2\log(v_{35}) - 2\log(v_{45})]$$
$$\Gamma^{(1)}_{6,5} = \frac{N_c}{4}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{6,6} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \tfrac{\log(v_{13})}{4} + \tfrac{\log(v_{14})}{4} + \tfrac{\log(v_{15})}{2} + \tfrac{\log(v_{23})}{4} + \tfrac{\log(v_{24})}{4} + \tfrac{\log(v_{25})}{2} + \tfrac{\log(v_{35})}{2} + \tfrac{\log(v_{45})}{2} + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(4096)}{4} + 2 + i\pi\right) + 1\right]$$
$$\Gamma^{(1)}_{6,7} = \frac{N_c^2-12}{4N_c}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{6,8} = 0$$
$$\Gamma^{(1)}_{6,9} = 0$$
$$\Gamma^{(1)}_{6,10} = \frac{N_c(N_c+3)}{4N_c^2+12N_c+8}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{6,11} = \frac{N_c(N_c-3)}{4N_c^2-12N_c+8}[-\log(v_{13}) + \log(v_{14}) + \log(v_{23}) - \log(v_{24})]$$
$$\Gamma^{(1)}_{7,1} = 0$$
$$\Gamma^{(1)}_{7,2} = \frac{N_c^2}{N_c^2-4}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{7,3} = 0$$
$$\Gamma^{(1)}_{7,4} = \frac{N_c^3}{4N_c^2-16}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{7,5} = \frac{N_c}{4}[-\log(v_{13}) + \log(v_{14}) - \log(v_{23}) + \log(v_{24}) + 2\log(v_{35}) - 2\log(v_{45})]$$
$$\Gamma^{(1)}_{7,6} = \frac{N_c(N_c^2-12)}{4N_c^2-16}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{7,7} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \tfrac{\log(v_{13})}{4} + \tfrac{\log(v_{14})}{4} + \tfrac{\log(v_{15})}{2} + \tfrac{\log(v_{23})}{4} + \tfrac{\log(v_{24})}{4} + \tfrac{\log(v_{25})}{2} + \tfrac{\log(v_{35})}{2} + \tfrac{\log(v_{45})}{2} + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(4096)}{4} + 2 + i\pi\right) + 1\right]$$
$$\Gamma^{(1)}_{7,8} = \frac{N_c^2}{2N_c^2-8}[\log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})]$$
$$\Gamma^{(1)}_{7,9} = \frac{N_c}{N_c^2-4}[-\log(v_{13}) + \log(v_{14}) + \log(v_{23}) - \log(v_{24})]$$
$$\Gamma^{(1)}_{7,10} = 0$$
$$\Gamma^{(1)}_{7,11} = 0$$
$$\Gamma^{(1)}_{8,1} = 0$$
$$\Gamma^{(1)}_{8,2} = 0$$
$$\Gamma^{(1)}_{8,3} = 0$$
$$\Gamma^{(1)}_{8,4} = 0$$
$$\Gamma^{(1)}_{8,5} = \log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})$$
$$\Gamma^{(1)}_{8,6} = 0$$
$$\Gamma^{(1)}_{8,7} = \log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})$$
$$\Gamma^{(1)}_{8,8} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} + \log(v_{15}) + \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{25}) + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(64)}{2} + 2\right) + 1\right]$$
$$\Gamma^{(1)}_{8,9} = -\tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{35}) - \log(v_{45})$$
$$\Gamma^{(1)}_{8,10} = \frac{N_c(N_c+3)}{4N_c+8}[-\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})]$$
$$\Gamma^{(1)}_{8,11} = \frac{N_c(N_c-3)}{4N_c-8}[\log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})]$$
$$\Gamma^{(1)}_{9,1} = 0$$
$$\Gamma^{(1)}_{9,2} = 0$$
$$\Gamma^{(1)}_{9,3} = 2\log(v_{13}) - 2\log(v_{14}) - 2\log(v_{23}) + 2\log(v_{24})$$
$$\Gamma^{(1)}_{9,4} = 0$$
$$\Gamma^{(1)}_{9,5} = 0$$
$$\Gamma^{(1)}_{9,6} = 0$$
$$\Gamma^{(1)}_{9,7} = \frac{1}{N_c}[-2\log(v_{13}) + 2\log(v_{14}) + 2\log(v_{23}) - 2\log(v_{24})]$$
$$\Gamma^{(1)}_{9,8} = -\tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} - \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{35}) - \log(v_{45})$$
$$\Gamma^{(1)}_{9,9} = \frac{1}{N_c}\left[L_\beta + N_c^2\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} + \log(v_{15}) + \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{25}) + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(64)}{2} + 2\right) + 1\right]$$
$$\Gamma^{(1)}_{9,10} = \frac{N_c(N_c+3)}{4N_c+8}[-\log(v_{13}) + \log(v_{14}) + \log(v_{23}) - \log(v_{24})]$$
$$\Gamma^{(1)}_{9,11} = \frac{N_c(N_c-3)}{4N_c-8}[-\log(v_{13}) + \log(v_{14}) + \log(v_{23}) - \log(v_{24})]$$
$$\Gamma^{(1)}_{10,1} = 0$$
$$\Gamma^{(1)}_{10,2} = 2\log(v_{13}) - 2\log(v_{14}) - 2\log(v_{23}) + 2\log(v_{24})$$
$$\Gamma^{(1)}_{10,3} = 0$$
$$\Gamma^{(1)}_{10,4} = \log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})$$
$$\Gamma^{(1)}_{10,5} = 0$$
$$\Gamma^{(1)}_{10,6} = \frac{N_c-2}{N_c}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{10,7} = 0$$
$$\Gamma^{(1)}_{10,8} = \frac{-N_c(N_c-1)+2}{2N_c}[\log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})]$$
$$\Gamma^{(1)}_{10,9} = \frac{-N_c(N_c-1)+2}{2N_c}[\log(v_{13}) - \log(v_{14}) - \log(v_{23}) + \log(v_{24})]$$
$$\Gamma^{(1)}_{10,10} = \frac{1}{N_c}\left[L_\beta + N_c\left(N_c\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(64)}{2} + 2\right) + (N_c + 1)\left(\tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} + \log(v_{15}) + \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{25})\right) - \log(v_{35}) - \log(v_{45}) - 2i\pi\right) + 1\right]$$
$$\Gamma^{(1)}_{10,11} = 0$$
$$\Gamma^{(1)}_{11,1} = 0$$
$$\Gamma^{(1)}_{11,2} = 2\log(v_{13}) - 2\log(v_{14}) - 2\log(v_{23}) + 2\log(v_{24})$$
$$\Gamma^{(1)}_{11,3} = 0$$
$$\Gamma^{(1)}_{11,4} = -\log(v_{13}) - \log(v_{14}) + 2\log(v_{15}) + \log(v_{23}) + \log(v_{24}) - 2\log(v_{25})$$
$$\Gamma^{(1)}_{11,5} = 0$$
$$\Gamma^{(1)}_{11,6} = \frac{N_c+2}{N_c}[-\log(v_{13}) + \log(v_{14}) + \log(v_{23}) - \log(v_{24})]$$
$$\Gamma^{(1)}_{11,7} = 0$$
$$\Gamma^{(1)}_{11,8} = \frac{N_c(N_c+1)-2}{2N_c}[\log(v_{13}) + \log(v_{14}) - 2\log(v_{15}) - \log(v_{23}) - \log(v_{24}) + 2\log(v_{25})]$$
$$\Gamma^{(1)}_{11,9} = \frac{N_c(N_c+1)-2}{2N_c}[-\log(v_{13}) + \log(v_{14}) + \log(v_{23}) - \log(v_{24})]$$
$$\Gamma^{(1)}_{11,10} = 0$$
$$\Gamma^{(1)}_{11,11} = \frac{1}{N_c}\left[L_\beta + N_c\left(N_c\left(-\log(\nu_1) - \log(\nu_2) - \log(\nu_5) + \log\left(\tfrac{s}{m_t^2}\right) - \tfrac{\log(64)}{2} + 2\right) + (N_c - 1)\left(\tfrac{\log(v_{13})}{2} + \tfrac{\log(v_{14})}{2} + \log(v_{15}) + \tfrac{\log(v_{23})}{2} + \tfrac{\log(v_{24})}{2} + \log(v_{25})\right) + \log(v_{35}) + \log(v_{45}) + 2i\pi\right) + 1\right]$$

Summary

Data analysis

- Top-quark mass extraction subject to correlations with $\alpha_s(M_Z)$ and PDFs
  - fixing of gluon PDF $g(x)$ and $\alpha_s(M_Z)$ may lead to bias
- Use of different mass schemes: $\overline{\mathrm{MS}}$, MSR and on-shell schemes
  - preference for short distance mass schemes $\overline{\mathrm{MS}}$, MSR

Theory improvements

- Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
- Need public NNLO QCD codes for hadro-production of top-quark pairs (incl. benchmarking)
- Need QCD perturbation theory for $t\bar{t}H$, $t\bar{t}j + X$ production at NNLO
  - generally very difficult: $2 \to 3$ processes with masses are beyond current state-of-the-art
  - progress in kinematic limits (threshold, high-energy, . . .) feasible

Future tasks

- Joint effort theory and experiment