Top-quark hadro-production and the top-quark mass

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Top-quark mass on social media

- Top-quark mass already known with unprecedented precision from world combination ATLAS, CMS, CDF, D0 coll. ‘14
  \[ m_t = 173.34 \pm 0.27\text{(stat)} \pm 0.71\text{(syst)} \text{ GeV} \]

Top-quark mass in twitter scheme

First LHC/Tevatron joint result on the mass of the top quark announced at #Moriond.
home.web.cern.ch/about/updates/…
Discovery of charged photons

- Spectacular achievements of the 2022 Nobel laureates (according to tagesschau.de)

"[Clauser] then used a filter to check the charge of the photons".
Based on work done in collaboration with:

- One-loop soft anomalous dimension matrices for $t\bar{t}j$ hadroproduction
  B. Chargeishvili, M. V. Garzelli, and S. M. [arXiv:2206.10977]

- Phenomenology of $t\bar{t}j + X$ production at the LHC

- Cross-sections for $t\bar{t}H$ production with the top quark $\overline{MS}$ mass

- Heavy-flavor hadro-production with heavy-quark masses renormalized in the $\overline{MS}$, MSR and on-shell schemes

- [...]
Why top-quark physics?

**Experiment**

- Top-quark hadro-production processes measured at the LHC with high precision
  - $t\bar{t}$
  - $t\bar{t}V$ with $V = \gamma, W, Z$
  - $t\bar{t} + n$ jets with $n = 1, 2, 3, 4$
  - $t\bar{c}c$ and $t\bar{b}b$
  - $t\bar{t}t\bar{t}$
  - $t\bar{t}H$
  - single-$t$, $t\gamma$, $tW$, $tH$, $tZq$

**Theory**

- Challenge for theory predictions
- Dependence on fundamental parameters of the Standard Model
  - top-quark mass $m_t$
  - strong coupling $\alpha_s$
- Constraints on new physics
Standard Model cross sections

- Standard Model cross sections and predictions at the LHC CMS coll. '22

Overview of CMS cross section results

<table>
<thead>
<tr>
<th>Process</th>
<th>m_t (13 TeV)</th>
<th>m_t (2 TeV)</th>
<th>m_t (500 GeV)</th>
<th>m_t (100 GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t\bar{t}$</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
</tr>
<tr>
<td>$Wt$</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
</tr>
<tr>
<td>$Zt$</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
</tr>
<tr>
<td>$tt$</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
<td>25.0 pb</td>
</tr>
</tbody>
</table>

Figure showing cross sections for different processes at the LHC with m_t values ranging from 13 TeV to 100 GeV.

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Top-quark hadro-production and the top-quark mass – p.6
Cross sections and predictions for top-quark production

CMS Preliminary

May 2021

All results at: http://cern.ch/go/pNj7

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Top-quark hadro-production and the top-quark mass – p.7
Top-quark mass

- Top-quark is the heaviest elementary particle
- Masses and couplings are formal parameters of the theory
  - $m_t$ and $\alpha_s = g_s^2 / (4\pi)$ are no observables
- Classical part of QCD Lagrangian
  \[ \mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{\text{flavors}} \bar{q}_i (i\gamma_\mu - m_q)_{i,j} q_j \]
  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,i,j} = \partial_\mu \delta_{i,j} + ig_s (t_a)_{i,j} A^a_\mu$
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

- Suitable observables for measurements of $\alpha_s, m_q, \ldots$
  - comparison of theory predictions and experimental data
Coupling constant renormalization

- Running coupling constant $\alpha_s$ from radiative corrections, e.g. one loop
  - screening (like in QED)
  - anti-screening (color charge of $g$)

- QCD beta function
  $$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$$

- perturbative expansion to five loops
  Baikov, Chetyrkin, Kühn '16
  Herzog, Ruijl, Ueda, Vermaseren, Vogt '17
  Luthe, Maier, Marquard, Schröder '17

- very good convergence of perturbative series even at low scales

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Quark mass renormalization

- Heavy-quark self-energy \( \Sigma(p, m_q) \)

\[ \Sigma \quad + \quad \Sigma \quad + \quad \Sigma \quad + \ldots = \frac{i}{\not{p} - m_q - \Sigma(p, m_q)} \]

**QCD**

- QCD corrections to self-energy \( \Sigma(p, m_q) \)
  - dimensional regularization \( D = 4 - 2\epsilon \)
  - one-loop: UV divergence \( 1/\epsilon \) (Laurent expansion)

\[
\Sigma^{(1),\text{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left( \frac{\mu^2}{m_q^2} \right)^\epsilon \left\{ (\not{p} - m_q) \left( -C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left( 3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}
\]

- Relate bare and renormalized mass parameter \( m_q^{\text{bare}} = m_q^{\text{ren}} + \delta m_q \)

\[
\Sigma^{\text{ren}}(p, m_q) = \quad + \quad \quad + \quad \quad + \ldots \quad (Z_\psi - 1)\not{p} - (Z_m - 1)m_\zeta
\]
Mass renormalization scheme

Pole mass

• Based on (unphysical) concept of top-quark being a free parton
  • $m_q^{\text{ren}}$ coincides with pole of propagator at each order

$$\psi - m_q - \Sigma(p, m_q) \bigg|_{\psi = m_q} \rightarrow \psi - m_q^{\text{pole}}$$

• Definition of pole mass ambiguous up to corrections $O(\Lambda_{QCD})$
  • heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $O(\Lambda_{QCD})$

\underline{MS} scheme

• \underline{MS} mass definition
  • one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left( \frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

• \underline{MS} scheme induces scale dependence: $m(\mu)$
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension $\gamma$ known to five loops
    
    Baikov, Chetyrkin, Kühn '14, Luthe, Maier, Marquard, Schröder '17
    
    \[
    \left( \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) m(\mu) = \gamma(\alpha_s) m(\mu)
    \]

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$
Scheme transformations

• Conversion between different renormalization schemes possible in perturbation theory

• Relation for pole mass and $\overline{\text{MS}}$ mass
  • known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher ‘90; Chetyrkin, Steinhauser ‘99; Melnikov, v. Ritbergen ‘99; Marquard, Smirnov, Smirnov, Steinhauser ‘15
  • example: one-loop QCD

\[
m^{\text{pole}} = m(\mu) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} \left( \frac{4}{3} + \ln \left( \frac{\mu^2}{m(\mu)^2} \right) \right) + \ldots \right\}
\]

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Top-quark hadro-production and the top-quark mass – p.13
Meta-stability of the universe

- Large top-quark mass implies large Higgs-Yukawa coupling $y_t$
- Renormalization group for the Higgs self-coupling $\lambda(\mu)$ dependent on $y_t$
  - limit of small Higgs mass $m_H$ implies $\lambda(\mu)$ decreases with $\mu$
- Implications on stability of electroweak vacuum
  - Higgs potential unbounded from below for $\lambda(\mu) < 0$
- Renormalization group evolution of $\lambda$ with uncertainties in $m_H, m_t$ and $\alpha_s$
  up to $\mu_r = M_{\text{Planck}}$ (using program mr Kniehl, Pikelnert, Veretin ‘16)

![Graph showing metastability and stability of the electroweak vacuum](image-url)
QCD factorization
QCD factorization

\[ \sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to X} \left( \alpha_s(\mu^2), Q^2, \mu^2, m_X^2 \right) \]

- Factorization at scale $\mu$
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij \to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij \to k}$ for parton types $i, j$ and hadronic final state $X$
- Non-perturbative parameters: parton distribution functions $f_i$
  - strong coupling $\alpha_s$, particle masses $m_X$
  - known from global fits to exp. data, lattice computations, . . .
Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij \to k}$ calculable pertubatively in powers of $\alpha_s$
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few\%})$ theory uncertainty)

Accuracy of perturbative predictions

- LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
- NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
- NNLO (next-to-next-to-leading order) ($\lesssim \mathcal{O}(10\%)$ unc.)
- $N^3$LO (next-to-next-to-next-to-leading order)
- ...
Parton luminosity

- Long distance dynamics due to proton structure

\[
s_\text{pp} \rightarrow X = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \ldots
\]

- Cross section depends on parton distributions \( f_i \)

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage
Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage

\[ x_{1,2} = \left( \frac{M}{7 \text{ TeV}} \right) \exp(\pm y) \]

- LHC run at \( \sqrt{s} = 7/8 \text{ TeV} \)
  - parton kinematics well covered by HERA and fixed target experiments

- Parton kinematics with \( x_{1,2} = \frac{M}{\sqrt{S}} e^{\pm y} \)
  - forward rapidities sensitive to small-\( x \)

- Cross section depends on convolution of parton distributions
  - small-\( x \) part of \( f_i \) and large-\( x \) PDFs \( f_j \)

\[
\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \ldots
\]
Top-quark theory status
**Total cross section**

**Exact result at NNLO in QCD**

- **NNLO perturbative corrections** (e.g. at LHC with $\sqrt{s} = 8$ TeV)
- $\overline{\text{MS}}$ renormalization scheme for $\alpha_s$, on-shell scheme for $m_t$
  - $K$-factors: $K_{\text{LO} \rightarrow \text{NLO}} = 1.46$ and $K_{\text{NLO} \rightarrow \text{NNLO}} = 1.12$
  - scale stability at NNLO of $O(\pm 5\%)$
  - point of minimal sensitivity at low scales $\mu \sim O(m_t/4) \sim O(45) \text{ GeV}$
**Total cross section with running mass**

**Comparison pole mass vs. \( \overline{\text{MS}} \) mass**

\[
\sigma_{pp \rightarrow tt} \text{ [pb]} \text{ at LHC8}
\]

- \( m_t^{\text{pole}} = 173 \text{ GeV} \)
- \( m(m) = 163 \text{ GeV} \)

- NNLO cross section with \( \overline{\text{MS}} \) renormalization scheme for \( \alpha_s \) and \( m_t \)
- running mass with better apparent perturbative convergence
- \( K \)-factors: \( K_{\text{LO} \rightarrow \text{NLO}} = 1.26 \) and \( K_{\text{NLO} \rightarrow \text{NNLO}} = 1.03 \)
- point of minimal sensitivity at natural hard scales
  \[
  \mu \sim \mathcal{O}(m_t(m_t)) \sim \mathcal{O}(160) \text{ GeV}
  \]
Top-quark mass from total cross section

- Cross section for $t\bar{t}$-production with parametric dependence

$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)$$

$$= \hat{\sigma}_{ij \to X}^{(0)} + \alpha_s \hat{\sigma}_{ij \to X}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij \to X}^{(2)} + \ldots$$

- PDFs $f_i$, strong coupling $\alpha_s$, masses $m_X$
- PDFs and $\alpha_s(M_Z)$ already well constrained by global fit
- effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$

Top-quark mass determination

- Choice of renormalization scheme for treatment of heavy quarks
  - $\overline{\text{MS}}$-scheme for quark masses and $\alpha_s$
- Intrinsic limitation of sensitivity in total cross section

$$\left| \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \approx 5 \times \left| \frac{\Delta m_t}{m_t} \right|$$
Data on top-quark cross sections (2017)

- Pulls for $t\bar{t}$-inclusive cross sections in ABMP16

\[ \sigma(t\bar{t}X) \]

- Tevatron
  - LHC $\sqrt{s}=5\text{ TeV}$
  - LHC $\sqrt{s}=7\text{ TeV}$
  - LHC $\sqrt{s}=8\text{ TeV}$
  - LHC $\sqrt{s}=13\text{ TeV}$
Fit quality

- Goodness-of-fit estimator $\chi^2$ for extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values
- $\chi^2$ of global fit with $NDP = 2834$
- Data on top-quark production with $NDP = 36$ D0, ATLAS, CMS, LHCb

![Graph showing $\chi^2$ vs. $\alpha_s(n_f=5,M_Z)$ and $m_t(m_t)$ (GeV)]
Correlations

- Correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and $m_t(m_t)$

- Fits with fixed values of $m_t$ and $\alpha_s(M_Z)$ carry significant bias
Data on top-quark cross sections (2022)

- $t\bar{t}$-inclusive cross sections from ATLAS and CMS at $\sqrt{s} = 7$, 8 and 13 TeV
- high precision data with small experimental uncertainties
- cross section combinations at $\sqrt{s} = 7$ and 8 TeV with accuracy of $\mathcal{O}(\pm 2 - 3\%)$

\[ \sigma_{\text{top}} \]

\[ \sigma_{\text{LHC}} \]

\[ \sigma_{\text{NNLO+NNLL}} \]

\[ \alpha_s(M_Z) = 0.118 \pm 0.001 \]

\[ \sigma_{\text{NNLO+NNLL}} = 0.113 \pm 0.001 \]

\[ \sigma_{\text{NNPDF3.0}} = 0.112 \pm 0.001 \]

\[ \sigma_{\text{CT14}} = 0.111 \pm 0.001 \]

\[ \sigma_{\text{MMHT14}} = 0.111 \pm 0.001 \]

\[ \sigma_{\text{ABM12}} = 0.112 \pm 0.001 \]

\[ \sigma_{\text{NNPDF3.0}} = 0.110 \pm 0.001 \]

\[ \sigma_{\text{CT14}} = 0.109 \pm 0.001 \]

\[ \sigma_{\text{MMHT14}} = 0.108 \pm 0.001 \]

\[ \sigma_{\text{ABM12}} = 0.107 \pm 0.001 \]

\[ \sigma_{\text{NNLO-NLL}} \]
Theory status 2022

- NNLO QCD differential predictions for top-quark pairs at the LHC
  Czakon, Heymes, Mitov ‘15

- Top-quark pair hadroproduction at NNLO in QCD
  Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan ‘19
  - to be implemented in future public release of MATRIX code
    Catani, Devoto, Grazzini, Kallweit, Mazzitelli ‘19

- NNLO event generation for top-quark pair production
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi ‘20

- Top-pair production at the LHC with MiNNLO_PS
  Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi ‘21

- Narrow-width-approximation at NNLO
  - NNLO QCD corrections to leptonic observables in top-quark pair
    production and decay
    - implemented in private STRIPPER code
      Czakon, Mitov, Poncelet ‘20
Differential cross sections (I)

CMS Preliminary 138 fb\(^{-1}\) (13 TeV)

**dilepton, parton level**

- Data, dof=6
- POW+PYT, \(\chi^2=15\)
- aNLO, \(\chi^2=35\)
- MATRIX (NNLO), \(\chi^2=3\)
- STRIPPER (NNLO), \(\chi^2=8\)
- MiNNLOPS (NNLOPS), \(\chi^2=4\)

\[\frac{1}{\sigma} \frac{d\sigma}{dp_t}(t) [\text{GeV}]\]

**dilepton, particle level**

- Data, dof=6
- POW+PYT, \(\chi^2=17\)
- STRIPPER (NNLO), \(\chi^2=11\)

\[\frac{1}{\sigma} \frac{d\sigma}{dp_t}(t) [\text{GeV}]\]

- Total unc.
- Stat unc.

**NNLO QCD predictions in fiducial phase space**
- dynamic scales \(\mu_R = \mu_F = H_T/4\) with
  \[H_T = \sqrt{p(t)^2 + m_t^2} + \sqrt{\bar{p}(t)^2 + m_t^2}\]
- top quark mass is set to \(m_t = 172.5\) GeV with NNPDF31 NNLO PDFs

- Parton level with stable top-quarks (left)
- Particle level with decay leptons (right)

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Top-quark hadro-production and the top-quark mass – p.29
Differential cross sections (II)

Summary

• The beyond-NLO theoretical predictions provide descriptions of the data that are of similar or improved quality, compared to POW+PYT, except for kinematic spectra where the theory scale uncertainties are large.

CMS TOP-20-006-PAS

Challenges

• NNLO codes not publicly accessible
• Very long run times (few CPU years) for distributions with fixed input parameters ($m_t$, PDFs, . . .)
• Accuracy of NNLO subtraction schemes
  • local sector subtraction (STRIPTER)
  • phase space slicing with $q_T^{\text{cut}}$ (MATRIX)

Needs

• NNLO QCD predictions for range of $m_t$ values
• Variation of PDFs (complete set of eigenvectors)
Phenomenological studies at NLO

- Top-quark $p_T$ distribution at NLO
- Different top-quark mass renormalization schemes
  - on-shell scheme for $m_t$
  - $\overline{\text{MS}}$ mass renormalization scheme $m_t(\mu_m)$
- MSR mass $m^{\text{MSR}}_t(R)$

Differential cross sections (III)

$\frac{d\sigma}{dp_T} [\text{fb/GeV}]$

$pp \to t, 0 < y < 1$

$\mu^2 = 4m^2 + p_T^2 \ (\mu \text{ unc.})$

- $m_{\text{pole}} = 169.6 \text{ GeV}$
- $m(m) = 162.1 \text{ GeV}$
- $m^{\text{MSR(3GeV)}} = 170.7 \text{ GeV}$
Scale uncertainties for running mass (I)

- NLO QCD predictions with $\overline{\text{MS}}$ mass: fixed $m_t(m_t)$
  - dynamic scale $\mu_R = \mu_F = \sqrt{p_T^2 + 4m_t^2(m_t)}$
  - $\mu_R = \mu_F$ (left)
  - $\mu_R = \kappa \mu_F$ with $1/2 < \kappa < 2$ (right)
- top quark mass $m_t(m_t) = 163.0$ GeV with ABMP16 NLO PDFs
Scale uncertainties for running mass (II)

- NLO QCD predictions with \( \overline{\text{MS}} \) mass: running \( m_t(\mu_m) \)
  - dynamic scale \( \mu_R = \mu_F = \sqrt{p_T^2 + 4m_t^2(\mu_m)} \)
  - \( \mu_R = \mu_F \) (left)
  - \( \mu_R = \kappa \mu_F \) with \( 1/2 < \kappa < 2 \) (right)
- top quark mass \( m_t(m_t) = 163.0 \) GeV with ABMP16 NLO PDFs
Correlated determination of PDFs, $\alpha_s(M_Z)$ and $m_t(m_t)$ using HERA DIS data and $t\bar{t}$ cross sections by CMS collaboration

Garzelli, Kemmler, S. M., Zenaiev ‘20

ansatz from HERAPDF for PDFs

<table>
<thead>
<tr>
<th>Settings</th>
<th>Fit results</th>
</tr>
</thead>
<tbody>
<tr>
<td>pole mass</td>
<td>$\chi^2$/dof = 1364/1151, $\chi_{tt}^2$/dof = 20/23</td>
</tr>
<tr>
<td>$\mu_R = \mu_F = H'$</td>
<td>$m_t^{\text{pole}} = 170.5 \pm 0.7 \text{(fit)} \pm 0.1 \text{(mod)}^{+0.0}_{-0.1} \text{(par)} \pm 0.3 (\mu) \text{ GeV}$</td>
</tr>
<tr>
<td>CMS, arXiv:1904.05237</td>
<td>$\alpha_S(M_Z) = 0.1135 \pm 0.0016 \text{(fit)}^{+0.0002}<em>{-0.0001} \text{(mod)}^{+0.0008}</em>{-0.0001} \text{(par)}^{+0.0011}_{-0.0005} (\mu)$</td>
</tr>
<tr>
<td>pole mass</td>
<td>$\chi^2$/dof = 1363/1151, $\chi_{tt}^2$/dof = 19/23</td>
</tr>
<tr>
<td>$\mu_R = \mu_F = m_t^{\text{pole}}$</td>
<td>$m_t^{\text{pole}} = 169.9 \pm 0.7 \text{(fit)} \pm 0.1 \text{(mod)}^{+0.0}<em>{-0.0} \text{(par)}^{+0.3}</em>{-0.3} (\mu) \text{ GeV}$</td>
</tr>
<tr>
<td>this work</td>
<td>$\alpha_S(M_Z) = 0.1132 \pm 0.0016 \text{(fit)}^{+0.0003}<em>{-0.0002} \text{(mod)}^{+0.0003}</em>{-0.0001} \text{(par)}^{+0.0016}_{-0.0008} (\mu)$</td>
</tr>
<tr>
<td>MS mass</td>
<td>$\chi^2$/dof = 1363/1151, $\chi_{tt}^2$/dof = 19/23</td>
</tr>
<tr>
<td>$\mu_R = \mu_F = m_t(m_t)$</td>
<td>$m_t(m_t) = 161.0 \pm 0.6 \text{(fit)} \pm 0.1 \text{(mod)}^{+0.0}<em>{-0.0} \text{(par)}^{+0.4}</em>{-0.8} (\mu) \text{ GeV}$</td>
</tr>
<tr>
<td>this work</td>
<td>$\alpha_S(M_Z) = 0.1136 \pm 0.0016 \text{(fit)}^{+0.0002}<em>{-0.0001} \text{(mod)}^{+0.0002}</em>{-0.0001} \text{(par)}^{+0.0015}_{-0.0005} (\mu)$</td>
</tr>
<tr>
<td>MSR mass, $R = 3$ GeV</td>
<td>$\chi^2$/dof = 1363/1151, $\chi_{tt}^2$/dof = 19/23</td>
</tr>
<tr>
<td>$\mu_R = \mu_F = m_t^{\text{MSR}}$</td>
<td>$m_t^{\text{MSR}} = 169.6 \pm 0.7 \text{(fit)} \pm 0.1 \text{(mod)}^{+0.0}<em>{-0.0} \text{(par)}^{+0.3}</em>{-0.3} (\mu) \text{ GeV}$</td>
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</tr>
</tbody>
</table>

**Table 1.** The values for $\alpha_S(M_Z)$ and the top-quark mass in different mass schemes obtained in CMS, arXiv:1904.05237 and in this work by fitting the CMS data on $t\bar{t}$ production and the HERA DIS data arXiv:1506.06042 to theoretical predictions. The fit, model (mod), parametrisation (par) and scale variation (µ) uncertainties are reported. Also the values of $\chi^2$ are reported, as well as the partial $\chi^2$ values per number of degrees of freedom (dof) for the $t\bar{t}$ data ($\chi^2_{tt}$) for 23 $t\bar{t}$ cross-section bins in the fit. The scale $H'$ is defined in the text.
Determination of top-quark mass (II)

- Extraction of $m_t(m_t)$ at NLO from differential $t\bar{t}$ cross-sections using data of CMS collaboration

  Garzelli, Kemmler, S. M., Zenaiev '20

  - value of $m_t(m_t)$ compared to other determinations
  - world average labelled as PDG2018, appr. NNLO is based on a single determination of D0 collaboration
Top-quark pairs with one jet
**Top-quark pairs with one jet**

- Large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1$jet Dittmaier, Uwer, Weinzierl ’07-'08
- Scale dependence greatly reduced at NLO
  - left: corrections for total rate at scale $\mu_R = \mu_F = m_t$ are almost zero
  - right: dynamic scale $\mu_R = \mu_F = H_T/2$ shows better scale stability
    with $H_T = \sqrt{p(t)^2_T + m_t^2} + \sqrt{p(\bar{t})^2_T + m_t^2} + p(j)_T$

Bevilacqua, Hartanto, Kraus, Worek ‘15
Top-quark mass from $t\bar{t} + \text{jet}$-samples

- Differential $t\bar{t} + \text{jet}$ cross section as function of invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$ offers possibility for top-quark mass determination
  - additional jet raises kinematical threshold
- Normalized-differential $t\bar{t} + \text{jet}$ cross section
  \[
  \mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{jet}}} \frac{d\sigma_{t\bar{t}+1\text{jet}}}{d\rho_s}(m_t, \rho_s)
  \]
  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\text{jet}}}}$ with invariant mass of $t\bar{t} + 1\text{jet}$ system and fixed scale $m_0 = 170$ GeV
- Normalization with $1/\sigma_{t\bar{t}+1\text{jet}}$ cancels many (experimental) uncertainties

Sven-Olaf Moch
Mass sensitivity of $t\bar{t} + \text{jet}$-samples

- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, ...

- Increased sensitivity for system $t\bar{t} + \text{jet}$ compared

$$\left| \frac{\Delta \mathcal{R}}{\mathcal{R}} \right| \simeq (m_t S) \times \left| \frac{\Delta m_t}{m_t} \right|$$

- Significant mass sensitivity for $\rho_s \geq 0.5$
Theory status for $t\bar{t}j$ production

- Theory predictions at NLO QCD for $t\bar{t}j$ production with different scale choices
  Alioli, Fuster, Garzelli, Gavardi, Irles, Melini, S. M., Uwer, Voß ‘22
- Dynamical scale with better apparent perturbative convergence

\[ p_T^j > 30 \text{ GeV}, \ |\eta_j| < 2.4, \ R = 0.4, \ N_j \geq 1 \]
\[ \mu_0 = m_T^{\text{pole}} \quad \mu_0 = m_{t\bar{t}j}^B/2 \quad \mu_0 = H_T^B/2 \quad \mu_0 = H_T^B/4 \]
**Dependence on parton distributions**

- Predictions for $m_{t\bar{t}j}$ (left) and $\rho_s$ (right) distributions at LO computation with $\mu_0 = H_T^B/4$
- PDF uncertainties of ABMP16, CT18, MSHT20 and NNPDF3.1 NLO sets
Gluon distribution

- PDFs sets ABMP16, CT18, MSHT20 and NNPDF3.1 at NLO for $Q^2 = m_t^2 = (172 \text{GeV})^2$
- effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 5 \cdot 10^{-2} \ldots 10^{-1}$ for $m_{t\bar{t}j}$
### Top-quark mass determinations (I)

**Top-quark mass measurement from $t\bar{t}j$ production becoming competitive**

- $t\bar{t}j$ cross-sections use NLO (NLO+PS) predictions for mass determination
- Elevating accuracy of theory predictions beyond NLO improves $m_t$ values and decreases their uncertainties

---

**ATLAS+CMS Preliminary**

**Top-quark mass determinations**

<table>
<thead>
<tr>
<th>$m_{t\bar{t}}$ from cross-section measurements</th>
<th>$m_{t\bar{t}}\pm$ tot (stat ± syst ± theo)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLAS, 7+8 TeV</td>
<td>172.9 ± 2.5 (1.5 ± 1.4 ± 1.0)</td>
<td>[1]</td>
</tr>
<tr>
<td>CMS, 7+8 TeV</td>
<td>173.8 ± 1.7</td>
<td>[2]</td>
</tr>
<tr>
<td>CMS, 13 TeV</td>
<td>169.9 ± 1.9 (0.1 ± 1.5 ± 1.2)</td>
<td>[3]</td>
</tr>
<tr>
<td>ATLAS, 13 TeV</td>
<td>173.1 ± 2.0 (0.4 ± 0.9 ± 0.7)</td>
<td>[4]</td>
</tr>
</tbody>
</table>

**$\sigma(tt)$ inclusive, NNLO+NNLL**

<table>
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</tr>
<tr>
<td>CMS, 13 TeV</td>
</tr>
<tr>
<td>ATLAS, 13 TeV</td>
</tr>
</tbody>
</table>

**$\sigma(tt)$ n-differential, NLO**

<table>
<thead>
<tr>
<th>$\sigma(tt)$ n-differential, NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLAS, n=1, 8 TeV</td>
</tr>
<tr>
<td>CMS, n=3, 13 TeV</td>
</tr>
</tbody>
</table>

**$m_{t\bar{t}}$ from top quark decay**

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>CMS, 7+8 TeV comb. [10]</td>
</tr>
<tr>
<td>ATLAS, 7+8 TeV comb. [11]</td>
</tr>
</tbody>
</table>
Top-quark mass determinations (II)

- CMS measurement of $m_t$ (pole mass) from distributions for $t\bar{t} + 1\text{jet}$ samples accurate to $\sim 0.8\%$

\[ m_t = 172.94^{+1.37}_{-1.34} \text{ GeV} \]

PhD thesis S. Wuchterl
Beyond NLO

- Scale uncertainties dominate theoretical uncertainties; need NNLO computations (very difficult)
- Focus on kinematical limits
  - threshold logarithms from emission of soft and/or collinear gluons
  - high energy (boosted) regime from $t$-channel gluon exchange
  - Coulomb corrections
- Sample of Feynman diagrams at Born level
Progress in theory (II)

Soft and collinear singularities

- Soft/collinear regions of phase space
- Massless partons

\[ \frac{1}{(p + k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos \theta_{qg})} \]

\[ \alpha_s \int d^4k \frac{1}{(p + k)^2} \rightarrow \alpha_s \int dE_g d\theta_{qg} \frac{1}{2E_q E_g (1 - \cos \theta_{qg})} \]

\[ \rightarrow \alpha_s \frac{1}{\epsilon^2} \times (\ldots) \quad \text{in dim. reg.} \quad D = 4 - 2\epsilon \]

Threshold logarithms

- Sudakov logarithms in velocity \( \beta_{t\bar{t}} = \sqrt{1 - 4m^2/s} \) of heavy quarks or
  \( \beta_{t\bar{t}j} = \sqrt{1 - m_{t\bar{t}j}^2/s} \) for \( t\bar{t}j \)-system

- All order resummation of large logarithms \( \alpha_s^n \ln^{2n}(\beta) \leftarrow \alpha_s^n \ln^{2n}(N) \)
  in Mellin space (renormalization group equation) Kidonakis, Sterman '97;
  Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; \ldots
Factorization and resummation

- Partonic cross-section factorizes in threshold limit (Collins, Soper, Sterman '83)
  \[ \hat{\sigma} = \psi_i \otimes \psi_j \otimes H \otimes S \otimes J \]

- \( \psi_i \) - initial state jet functions, modeling the initial state collinear radiation
- \( S \) - soft gluon exchange
- \( H \) - hard matrix squared
- \( J \) - final state jet function

Strategy of calculation

- Evaluation of each functions perturbative
- Resummation of logarithms through renormalization group evolution
- Previously applied to \( t\bar{t} \) production processes with associated bosons \( t\bar{t} + W/Z/H \) (Kulesza, Motyka, Theeuwes et al ‘17, Broggio, Ferroglia, Pecjak et al. ‘17).
- \( t\bar{t}j \) production
  - complicated because of richer color structure
  - final state jet with non-trivial soft singularity structure
Soft function

- Renormalization group evolution for soft function

\[ \mu \frac{d}{d\mu} S_{LI}^{(f)} = \left( \mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) S_{LI}^{(f)} = - \left( \Gamma_S^{(f)} \right)^{\dagger}_{LB} S_{BI}^{(f)} - S_{LA}^{(f)} \left( \Gamma_S^{(f)} \right)_{AI} , \]

- soft anomalous dimension \( \left( \Gamma_S^{(f)} \right)_{LI} \) computed from UV divergence of eikonal amplitudes (Wilson lines)

- Integrals at one-loop level known Kidonakis, Sterman ’97.
- Color basis for Wilson lines

\[ S_{LI}^{\{f\},0} = \left\langle c_L^{\{f\}} | c_I^{\{f\}} \right\rangle . \]
Color basis

• **$gg$-channel color basis**

\[
\begin{align*}
    c_{abcd}^1 &= t^e_{cd}\delta_{ab} \\
    c_{abcd}^2 &= i f_{abe}\delta_{cd} \\
    c_{abcd}^3 &= i d_{abe}\delta_{cd} \\
    c_{abcd}^4 &= i f_{abn}\delta_{men}t^m_{cd} \\
    c_{abcd}^5 &= d_{abn}\delta_{men}t^m_{cd} \\
    c_{abcd}^6 &= i f_{abn}\delta_{men}t^m_{cd} \\
    c_{abcd}^7 &= d_{abn}\delta_{men}t^m_{cd} \\
    c_{abcd}^8 &= P_{abme}^{10+10}t^m_{cd} \\
    c_{abcd}^9 &= P_{abme}^{10-10}t^m_{cd} \\
    c_{abcd}^{10} &= -P_{abme}^{27}t^m_{cd} \\
    c_{abcd}^{11} &= P_{abme}^{0}t^m_{cd}
\end{align*}
\]

• **$q\bar{q}$-channel**

\[
\begin{align*}
    c_{abcd}^1 &= t^e_{cd}\delta_{ab} \\
    c_{abcd}^2 &= t^e_{ab}\delta_{cd} \\
    c_{abcd}^3 &= t^m_{ba}\delta_{cd}t^m_{mn}i f_{mne} \\
    c_{abcd}^4 &= t^m_{ba}\delta_{cd}t^m_{cd}dm_{me} \\
\end{align*}
\]

• $P^i$ are projectors:

\[
P^i_{ABmn}P^i_{mnCD} = \delta_{ij}P^i_{ABCD}
\]

• **Recent work on color evolution and infrared physics** Plätzer 22
Components of $(R^{(f)})_{LL}$ for $gg \rightarrow t\bar{t}j$-channel at one loop
Summary

Data analysis

• Top-quark mass extraction subject to correlations with $\alpha_s(M_Z)$ and PDFs
  • fixing of gluon PDF $g(x)$ and $\alpha_s(M_Z)$ may lead to bias
• Use of different mass schemes: $\overline{\text{MS}}$, MSR and on-shell schemes
  • preference for short distance mass schemes $\overline{\text{MS}}$, MSR

Theory improvements

• Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
• Need public NNLO QCD codes for hadro-production of top-quark pairs (incl. benchmarking)
• Need QCD perturbation theory for $t\bar{t}H$, $t\bar{t}j + X$ production at NNLO
  • generally very difficult: $2 \rightarrow 3$ processes with masses are beyond current state-of-the-art
  • progress in kinematic limits (threshold, high-energy, ...) feasible

Future tasks

• Joint effort theory and experiment