HLbL contribution to muon g-2 at short distances

Antonio Rodríguez Sánchez

Seminar on Particle Physics. University of Vienna

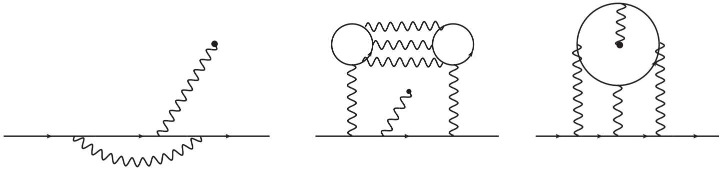
29/3/2022

Based on works with Johan Bijnens, Nils Hermansson-Truedsson and Laetitia Laub

Phys.Lett.B 798 134994
JHEP 10 (2020) 203
JHEP 04 (2021) 240

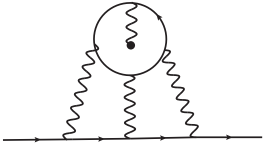
Muon g-2

- Muon response to weak magnetic field?
- Linear term in (soft) photon momentum of $\langle \mu^-_{p',\sigma'} | J^\mu_{\rm EM} e^{iS_{\rm int}} | \mu^-_{p,\sigma} \rangle$
- $S_{\rm int} \approx 0 \to g_\mu \approx 2$ $\qquad g_{\mu,\rm exp} \approx 2.002$
- $a_\mu^{\rm exp} \equiv (g_\mu^{\rm exp} - 2)/2 = 0.00116592061(41)$ PRL 126, 141801



- $a_\mu^{\rm SM} = 0.00116591810(43)$ Phys.Rept. 887 (2020) 1-166

Data-driven HLbL: a multiscale problem

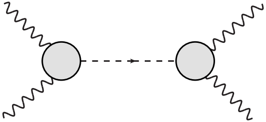


$$\Pi^q \sim \langle 0 | T(\prod_j^4 \int dx_j e^{-iq_j x_j} J^q(x_j)) e^{iS_{\rm int}} | 0 \rangle$$

$$J_q^\mu = Q_q \, \bar{q} \gamma^\mu q$$

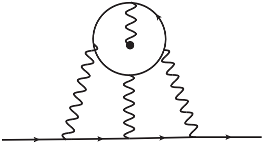
$$a_\mu^{\rm HLbL} \sim \int_0^\infty dQ_{1,2,3} \sum_i T_i' \, \overline{\Pi}_i$$

- Π^q? Weights T_i' enhance low-energy contributions
- A nonperturbative problem



- One and two mesons as intermediate states: leading and precisely known, dispersion relations Colangelo et al.
- Subleading intermediate and high-energy contributions. Resonance models and SD constraints Melnikov-Vainshtein, Brodsky-Lepage

Data-driven HLbL: quark loop?



$$\Pi^q \sim \langle 0 | T\left(\prod_j^4 \int dx_j e^{-iq_j x_j} J^q(x_j)\right) e^{iS_{\mathrm{int}}} |0\rangle$$

$$J_q^\mu = Q_q \, \bar{q}\gamma^\mu q$$

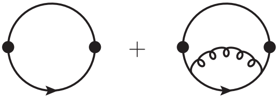
$$a_\mu^{\mathrm{HLbL}} \sim \int_0^\infty dQ_{1,2,3} \sum_i T_i' \, \overline{\Pi}_i$$

- Quark loop contribution ($\alpha_s = 0$) makes sense, but how?
- Technically finite, but $\ln \frac{m_\mu}{m_q}$
- Constituent quark mass as infrared regulator? Not satisfactory within a model-independent paradigm
- Maybe $Q_1 \sim Q_2 \sim Q_3 \gg \Lambda_{\mathrm{QCD}}$?

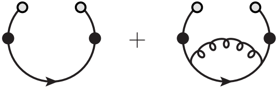
Operator Product Expansion (OPE)

Asymptotic behaviour of two-point correlation functions

$$\Pi(q) = \int dx\, e^{-iqx} \langle 0 | T(J_1(x) J_2(0)|0\rangle; \;\; J_i \sim \bar{q}\Gamma_i q$$



$+ \ldots = c(\text{pert})$

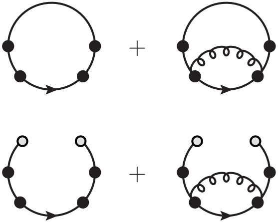


$+ \ldots = c_{qq}\langle m_q \bar{q} q\rangle$

$$\Pi(Q) = \sum_{i,D} \frac{c_{i,D}(Q^2,\mu)\langle \mathcal{O}_{i,D}(\mu)\rangle}{Q^D}$$ Nucl.Phys.B 147 385-447

HLbL for $g-2$. Same procedure?

$$\Pi^{\mu_1\mu_2\mu_3\mu_4} = -i\int \frac{d^4q_3}{(2\pi)^4}\left(\prod_i^4 \int d^4x_i\, e^{-iq_ix_i}\right)\langle 0|\, T\left(\prod_j^4 J^{\mu_j}(x_j)\right)|0\rangle$$



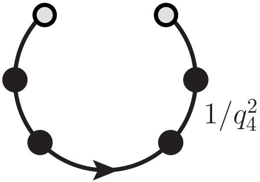
$$+\ldots = c(\mathrm{pert})$$

$$+\ldots = c_{qq}\langle m_q\bar{q}q\rangle$$

$$\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle\mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1}\ldots Q_4^{d_4}} \qquad \sum_i d_i = D$$

HLbL for g-2

- $\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle\mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1}\dots Q_4^{d_4}}$ $\qquad \sum_i d_i = D$
- External photon: static limit $\rightarrow \lim_{q_4\to 0} \frac{\partial \Pi^{\mu_1\mu_2\mu_3\nu_4}}{\partial q_{4,\,\mu_4}}$
- $\lim_{q_4\to 0} \Pi^{\mathrm{OPE}}$?



Phys.Lett.B 798 134994

- OPE only supposed to work at large Euclidean Momenta

Rethinking the problem: soft static photon

$$\langle 0|e^{iS}|\gamma_1^*\gamma_2^*\gamma_3^*\gamma_4\rangle \to \Pi^{\mu_1\mu_2\mu_3\mu_4}$$

One step backwards

$$\Pi^{\mu_1\mu_2\mu_3} \sim \int \frac{d^4q_3}{(2\pi)^4}\left(\prod_i^3 \int d^4x_i\, e^{-iq_ix_i}\right)\langle 0|\,T\left(\prod_j^3 J^{\mu_j}(x_j)\right) e^{iS_{\rm int}}|\gamma_E(q_4)\rangle$$

- $Q_{1,2,3} \gg \Lambda_{\rm QCD} \to$ OPE valid for the tensor
- We are looking for a static (soft) photon contribution: $F^{\mu\nu}$
- From the OPE keep those operator contributions with the same quantum numbers as the static photon, $F^{\mu\nu}$

  Nucl.Phys.B 232 109-142, Phys.Lett.B 129 328-334, Phys.Rev.D 67 073006

Operators

$$S_{1,\,\mu\nu} \equiv e\, e_q F_{\mu\nu}$$
$$S_{2,\,\mu\nu} \equiv \bar{q}\sigma_{\mu\nu} q$$
$$S_{3,\,\mu\nu} \equiv i\ \bar{q}\, G_{\mu\nu} q$$
$$S_{4,\,\mu\nu} \equiv i\, \bar{q}\, \bar{G}_{\mu\nu}\gamma_5 q$$
$$S_{5,\,\mu\nu} \equiv \bar{q}q\ e\, e_q F_{\mu\nu}$$
$$S_{6,\,\mu\nu} \equiv \frac{\alpha_s}{\pi}\, G_a^{\alpha\beta} G^a_{\alpha\beta}\ e\, e_q F_{\mu\nu}$$
$$S_{7,\,\mu\nu} \equiv \bar{q}(G_{\mu\lambda} D_\nu + D_\nu G_{\mu\lambda})\gamma^\lambda q - (\mu \leftrightarrow \nu)$$
$$S_{\{8\},\,\mu\nu} \equiv \alpha_s\, (\bar{q}\,\Gamma\, q\ \bar{q}\Gamma\, q)_{\mu\nu}$$

$$\Pi^{\mu_1\mu_2\mu_3}(q_1, q_2) = \frac{1}{e}\vec{C}^{\,T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1, q_2)\langle \vec{S}_{\mu_4\nu_4}\rangle; \quad \langle S_{i,\mu\nu}\rangle = e e_q X_S^i \langle F_{\mu\nu}\rangle$$

Quark loop

$$\Pi^{\mu_1\mu_2\mu_3} = -\frac{1}{e}\int \frac{d^4 q_3}{(2\pi)^4}\left(\prod_{i=1}^{3}\int d^4 x_i\, e^{-iq_i x_i}\right)\langle 0|\, T\left(\prod_{j=1}^{3} J^{\mu_j}(x_j)\right)|\gamma(q_4)\rangle$$

- Direct $S_1^{\mu\nu} = e e_q F_{\mu\nu}$ contribution
- Take one extra $\mathcal{L}_{\mathrm{EM}} \sim A^{\mu_4}(x_4) J_{\mu_4}(x_4)$ from $e^{iS_{int}}$
- $A^{\mu_4}(x_4) \sim -\frac{1}{2} x_{4\nu_4} F^{\mu_4\nu_4} \sim F^{\mu_4\nu_4} \lim_{q_4\to 0} \partial^{q_4}_{\nu_4} e^{-iq_4 x_4}$
- Massless quark loop is the leading term of the correct OPE

Quark loop

$$-\frac{N_c e_q^4}{2} \lim_{q_4\to 0} \frac{\partial}{\partial q_4^{\nu_4}} \left[\sum_{\sigma(1,2,4)} \mathrm{Tr}\left(\gamma^{\mu_3} S(p+q_1+q_2+q_4) \gamma^{\mu_4} S(p+q_1+q_2) \gamma^{\mu_1} S(p+q_2) \gamma^{\mu_2} S(p) \right) \right] ,$$

$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\, \vec{X}\, \langle e_q F_{\mu_4\nu_4} \rangle$$

$a_\mu^{\mathrm{HLbL}} \sim \int_0^\infty dQ_{1,2} \int_{-1}^{1} d\tau \sum_i T_i' \,\overline{\Pi}_i$ JHEP 09 (2015) 074, JHEP 04 (2017) 161

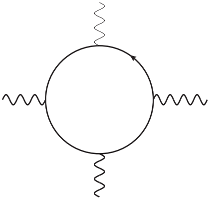
1. Build general projectors P: $P_{\mu_1\mu_2\mu_3\mu_4\nu_4} C^{\mu_1\mu_2\mu_3\mu_4\nu_4} = \overline{\Pi}$
2. Reduce scalar integrals KIRA, REDUZE
3. Perform the g-2 integral

For example $Q_1 = Q_2 = Q_3 = Q$:

$$\hat{\Pi}_4 = \frac{N_c e_q^4}{16\pi^2} \frac{2}{Q^4} \left(-\frac{352}{27} + \frac{128}{81} \Delta^{(1)} \right) , \quad \Delta^{(1)} = 7.03172\ldots$$

Perturbative mass correction?

$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \frac{1}{e}\vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\langle\vec{S}_{\mu_4\nu_4}\rangle; \quad \langle S_{i,\mu\nu}\rangle = e e_q X_S^i \langle F_{\mu\nu}\rangle$$



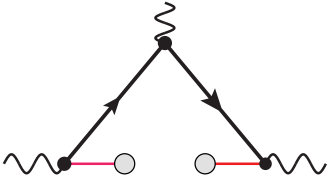
$$c_{m_q^2} \sim \alpha_s^n(Q^2)\log^m\left(\frac{Q^2}{m_q^2}\right)$$

- Separation of long and short distance not complete
- Tree-level operators should be dressed and renormalized $(m_q \to \mu)$

  Rev.Mod.Phys. 68 1125-1144, Z.Phys.C 60 569-578
- OPE renormalization program built in the $\overline{\mathrm{MS}}$ JHEP 10 (2020) 203
- Naive mass correction to quark loop is not the genuine quark mass correction

Leading power correction

$$Q_{2,\,\mu\nu} \equiv \bar{q}\sigma_{\mu\nu}q = ee_q X_2^i \langle F_{\mu\nu} \rangle$$



Leading power correction. For example

$$\hat{\Pi}_4 = m_q X_2 e_q^4 \, \frac{8}{Q_1^2 Q_2^2 Q_3^2}$$

and short-distance constraint to deviation from chiral limit. X_2?

Matrix element estimates

$$\langle 0|Q_2^{\mu\nu}|\gamma(q_4)\rangle = X_2\langle 0|e\, e_q F^{\mu\nu}|\gamma(q_4)\rangle,\; Q_2^{\mu\nu} = \bar{q}\sigma^{\mu\nu}q$$

First try: ChPT with tensor sources JHEP 09 (2007) 078

$$X_2(\mu) \sim \Lambda_1(\mu) + \mathcal{O}(\frac{M_K^2}{\Lambda_\chi^2})\ ...\Lambda_1?$$

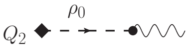
Second try: expand EM vertex

$$\langle 0|Q_{2,0}^{\mu\nu}|\gamma(q_4)\rangle \sim \langle 0|T(Q_{2,0(\mathrm{QCD})}^{\mu\nu}\int d^4x\, \mathcal{L}_{\mathrm{EM}}^{\mathrm{int}}(x))|\gamma(q_4)\rangle = -\langle 0|ee_qF^{\mu\nu}|\gamma(q_4)\rangle\Pi_{VT}^{\mathrm{QCD}}(0)$$

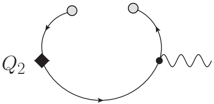
Matrix element estimate

$$X_2 = -\Pi_{VT}(0), \text{ where } \Pi_{VT}^{\text{QCD}}(q) \sim \int d^4x\, e^{-iqx}\, T(V(x)T_i(0))$$

- ChPT? Again, $\Pi(0) \sim \Lambda_1 + \cdots$, but Λ_1?
- Low-energy spectrum$\sim$ large N_c. High-energy spectrum flat



$\Pi = c\frac{1}{q^2 - M_\rho^2}F_V$



$\Pi \sim \frac{\langle \bar{q}q \rangle}{Q^2}$

Eur.Phys.J.C 52 325-338

$$X_2 \approx \frac{2}{M_\rho^2}\langle \bar{q}q \rangle, X_3 \approx -\frac{m_0^2}{6M_\rho^2}\langle \bar{q}q \rangle, X_4 \approx -\frac{m_0^2}{6M_\rho^2}\langle \bar{q}q \rangle$$

X_2 in very good agreement with the lattice JHEP 07 (2020) 183

Leading power suppression

Leading term $Q_1 = Q_2 = Q_3 = Q$:

$$\hat{\Pi}_1^{ql} = -\frac{N_c e_q^4}{16\pi^2}\frac{32}{3Q^4}, \hat{\Pi}_4^{ql} = \frac{N_c e_q^4}{16\pi^2}\frac{2}{Q^4}\left(-\frac{352}{27} + \frac{128}{81}\Delta^{(1)}\right) \approx -\frac{N_c e_q^4}{16\pi^2}\frac{4}{Q^4}$$

Leading power correction $Q_1 = Q_2 = Q_3 = Q$:

$$\hat{\Pi}_4^{X_2} = m_q X_2 e_q^4 \frac{8}{Q^6} \approx \frac{16 m_q \langle \bar{q} q \rangle e_q^4}{M_\rho^2 Q^6} \approx -\frac{\Lambda_\chi^2}{M_\rho^2}\frac{e_q^4}{16\pi^2}\frac{8 M_{PS}^2}{Q^6}$$

$$\hat{\Pi}_1^{X_2} \approx \frac{\Lambda_\chi^2}{M_\rho^2}\frac{e_q^4}{16\pi^2}\frac{4 M_{PS}^2}{Q^6}$$

- No signatures of constituent quark mass
- More similar to remnant of pseudoscalar propagators (hinted for EW corrections in Phys.Rev.D 67 073006)
- Possible applications

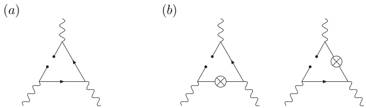
Rest of matrix element estimates

$$\langle 0|Q_i^{\mu\nu}|\gamma(q_4)\rangle = X_i \langle 0|e\, e_q F^{\mu\nu}|\gamma(q_4)\rangle$$

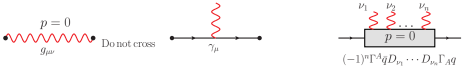
$$\begin{aligned}
Q_{5,\,\mu\nu} &\equiv \bar{q}q\; e\, e_q F_{\mu\nu} \\
Q_{6,\,\mu\nu} &\equiv \frac{\alpha_s}{\pi}\, G_a^{\alpha\beta} G^a_{\alpha\beta}\; e\, e_q F_{\mu\nu} \\
Q_{7,\,\mu\nu} &\equiv \bar{q}(G_{\mu\lambda} D_\nu + D_\nu G_{\mu\lambda})\gamma^\lambda q - (\mu \leftrightarrow \nu) \\
Q_{\{8\},\,\mu\nu} &\equiv \alpha_s\, (\bar{q}\,\Gamma\, q\; \bar{q}\Gamma\, q)_{\mu\nu}
\end{aligned}$$

- X_5 and X_6 are the usual vacuum condensates.
- Q_8. $SU(3)_{\mathrm{V}}$ Nucl.Phys.B 234 (1984) 173-188, P, $C \to 12$
- Large-$N_c \to X_8 \sim X_2 \langle \bar{q}q \rangle$
- Q_7? Dimensional guess

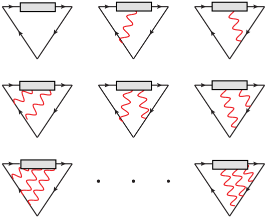
One cut topologies: the emerging pattern



- Three simple rules:



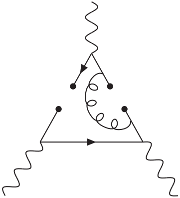
- Our case



$$\Pi^{\mu_1\mu_2\mu_3} \sim \sum (-1)^n \langle 0|\bar{q} D_{\nu_1} \dots D_{\nu_n} \Gamma^A q|\gamma(q_4)\rangle$$
$$\times \mathrm{Tr}\left\{\gamma^{\mu_3}\Gamma^A\gamma^{\mu_1}\, iS(-q_1)\gamma^{\nu_1}\, iS(-q_1)\dots\gamma^{\nu_p}\, iS(-q_1)\gamma^{\mu_2}\, iS(q_3)\gamma^{\nu_{p+1}}\, iS(q_3)\dots\gamma^{\nu_n}\, iS(q_3)\right\}$$

- $\langle 0|\bar{q} D_{\nu_1} \dots D_{\nu_n} \Gamma^A q|\gamma(q_4)\rangle$ can be decomposed into our basis

Four-quark operators

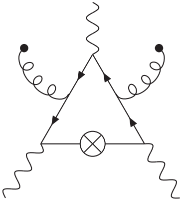


- Only contributions not associated to one flavor

$$\hat{\Pi}_1 = \hat{\Pi}_4 = 8\overline{X}_{8,2}\frac{Q_1^2+Q_2^2}{Q_1^4 Q_2^4 Q_3^2}$$
$$\hat{\Pi}_{54} = 8\overline{X}_{8,2}\frac{Q_2^4-Q_1^4}{Q_1^6 Q_2^6 Q_3^2}$$
$$\hat{\Pi}_7 = \hat{\Pi}_{17} = \hat{\Pi}_{39} = 0$$

- Probably subleading wrt perturbative two-gluon exchange. Maybe feasible at the symmetric point JHEP 06 (2021) 083

Gluon-Photon operator

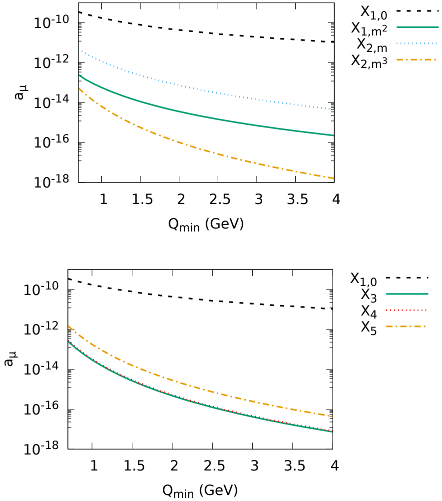


$$\left(\prod_{i=4}^{6} \lim_{q_i \to 0} \frac{\partial}{\partial q_i^{\nu_i}}\right) \times \sum_{\sigma(1,2,4,5,6)} \mathrm{Tr}\Big(\gamma^{\mu_3} S(p+q_1+q_2+q_4+q_5+q_6)\gamma^{\mu_1} S(p+q_2+q_4+q_5+q_6)$$

$$\times \gamma^{\mu_2} S(p+q_4+q_5+q_6)\gamma^{\mu_4} S(p+q_5+q_6)\gamma^{\mu_5} S(p+q_6)\gamma^{\mu_6} S(p)\Big)$$

- Same procedure as in the quark loop
- Infrared divergences cancel after including mixing

Numerical results: quark loop vs power corrections



- Power corrections numerically very small above 1 GeV
- Gluonic corrections?...

The gluonic corrections

Gluonic corrections to

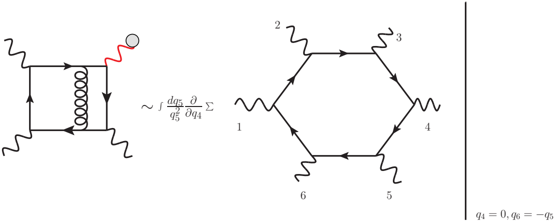
$$\lim_{q_4 \to 0} \frac{\partial \Pi^{\mu_1 \mu_2 \mu_3 \nu_4}}{\partial q_4^{\mu_4}}$$

Full information on five scalar functions

$$\tilde{\Pi}_i = P^{\tilde{\Pi}_i}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} \lim_{q_4 \to 0} \frac{\partial \Pi^{\mu_1 \mu_2 \mu_3 \nu_4}}{\partial q_4^{\mu_4}}$$

$$\begin{aligned}
P^{\tilde{\Pi}_1}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} &= g_{\mu_1 \mu_2}\, g_{\mu_3 \nu_4}\; q_{1,\mu_4} \\
P^{\tilde{\Pi}_7}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} &= g_{\mu_1 \nu_4}\, g_{\mu_2 \mu_4}\; q_{2,\mu_3} \\
P^{\tilde{\Pi}_{10}}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} &= g_{\mu_1 \mu_2}\, q_{1,\mu_3} q_{1,\nu_4}\; q_{2,\mu_4} \\
P^{\tilde{\Pi}_{13}}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} &= g_{\mu_1 \nu_4}\, q_{1,\mu_2} q_{2,\mu_3}\; q_{3,\mu_4} \\
P^{\tilde{\Pi}_{19}}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} &= q_{3,\mu_1} q_{1,\mu_2}\; q_{2,\mu_3} q_{1,\nu_4}\; q_{2,\mu_4}
\end{aligned}$$

The two loops: a symmetric sum of hexagons



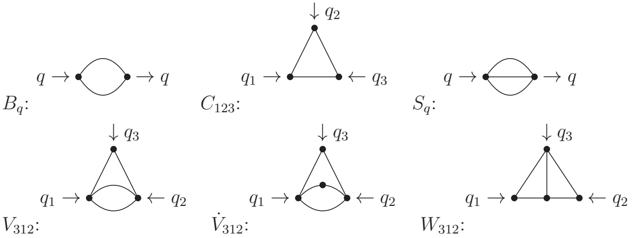
$$\tilde{\Pi}_i = -\frac{(N_c^2-1) g_s^2 e_q^4}{4} \int \frac{d^4 q_5}{(2\pi)^4} \frac{g_{\mu_5 \mu_6}}{q_5^2} \lim_{\substack{q_4 \to 0 \\ q_6 \to -q_5}} P^{\tilde{\Pi}_i}_{\mu_1 \mu_2 \mu_3 \mu_4 \nu_4} \frac{\partial}{\partial q_4^{\nu_4}} H^{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5 \mu_6}$$

$$H^{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5 \mu_6} \equiv \int \frac{d^4 p}{(2\pi)^4} \sum_{\sigma(1,2,4,5,6)} \mathrm{Tr} \Big(\gamma^{\mu_3} S(p+q_1+q_2+q_4+q_5+q_6) \gamma^{\mu_1} S(p+q_2+q_4+q_5+q_6)$$

$$\times \gamma^{\mu_2} S(p+q_4+q_5+q_6) \gamma^{\mu_4} S(p+q_5+q_6) \gamma^{\mu_5} S(p+q_6) \gamma^{\mu_6} S(p) \Big)$$

Reduction of integrals

Reduce $\sim \mathcal{O}(10^{3,4})$ scalar integrals (d dimensions) KIRA



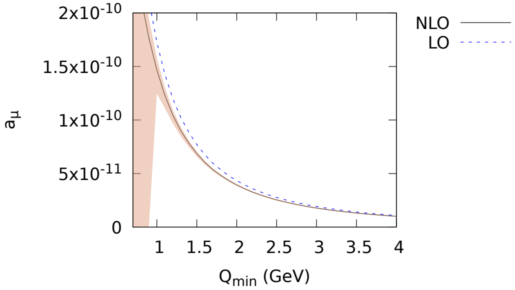
Master integrals known in terms of classical polylogs: analytic result for the HLbL tensor. Some numbers:

$(Q_1^2, Q_2^2, Q_3^2) = (1, 1.3, 1.7)$ GeV^2	$\tilde{\Pi}_1$	$\tilde{\Pi}_7$	$\tilde{\Pi}_{10}$	$\tilde{\Pi}_{13}$	$\tilde{\Pi}_{19}$
Quark loop	−0.0816	0.123	0.0363	0.0274	0.0263
Gluon corrections ($\times \pi/\alpha_s$)	0.0781	−0.136	−0.0376	−0.0398	−0.0411

Two loop: results

Integral from Q_{min}. Analytic expansions help in improving precision.

Check: Gluonic correction in M-V matches expectations for $\hat{\Pi}_1$ Eur.Phys.J.C 80 (2020) 12, 1108



Above $\sim 1 - 2\,\mathrm{GeV}$, gluonic corrections small and negative

Interplay with M-V region, preliminary exploration

Melnikov-Vainshtein regime. In essence, if $Q_{1,2} \gg \Lambda_{QCD}$, Q_3 start at

$$\Pi^{\mu_1\mu_2} = \frac{i}{e^2}\int \frac{d^4q_4}{(2\pi)^4}\int d^4x_1 \int d^4x_2 e^{-i(q_1x_1+q_2x_2)}\langle 0|T(J^{\mu_1}(x_1)J^{\mu_2}(x_2))|\gamma^*(q_3)\gamma(q_4)\rangle$$

Up to $D=4$:

$$\Pi^{\mu_1\mu_2}_{\bar{q}q} \approx \frac{e_q^2}{e^2}\left\langle \bar{q}(0)[\Gamma^{\mu_1\mu_2}(-\hat{q}) - \Gamma^{\mu_2\mu_1}(-\hat{q})]q(0)\right\rangle^{3,4} \quad \left(\Gamma^{\mu_1\mu_2}(k) = \gamma^{\mu_1}S(k)\gamma^{\mu_2}\,\text{M-V. Interplay with anomaly}\right)$$

$$-\frac{ie_q^2}{e^2\hat{q}^2}(g_{\mu_1\delta}g_{\mu_2\beta} + g_{\mu_2\delta}g_{\mu_1\beta} - g_{\mu_1\mu_2}g_{\delta\beta})\left(g^{\alpha\delta} - 2\frac{\hat{q}^\delta\hat{q}^\alpha}{\hat{q}^2}\right)\left\langle \bar{q}(0)(\overrightarrow{\partial}^\alpha - \overleftarrow{\partial}^\alpha)\gamma^\beta q(0)\right\rangle^{3,4}$$

$$\frac{\partial\Pi^{\mu_1\mu_2\mu_3\mu_4}}{\partial q_{4\nu_4}}\Big|_{\mathrm{FF}} \sim q_{3\nu_3}\lim_{q_3\to 0}\lim_{q_4\to 0}\partial_{q_3}^{\nu_3}\partial_{q_4}^{\mu_4}\Pi^{\mu_1\mu_2\mu_3\nu_4}$$

Check: Subleading logs from quark loop exactly recovered for all $\tilde{\Pi}$

Conclusions

- When the loop momenta are large, an OPE can be built
- The massless quark loop is the leading term
- Leading power correction linear in the quark mass
- Power corrections are found to be small
- Perturbative corrections small and negative
- Precise systematic expansion valid above $1 - 2\,\mathrm{GeV}$

Thank you