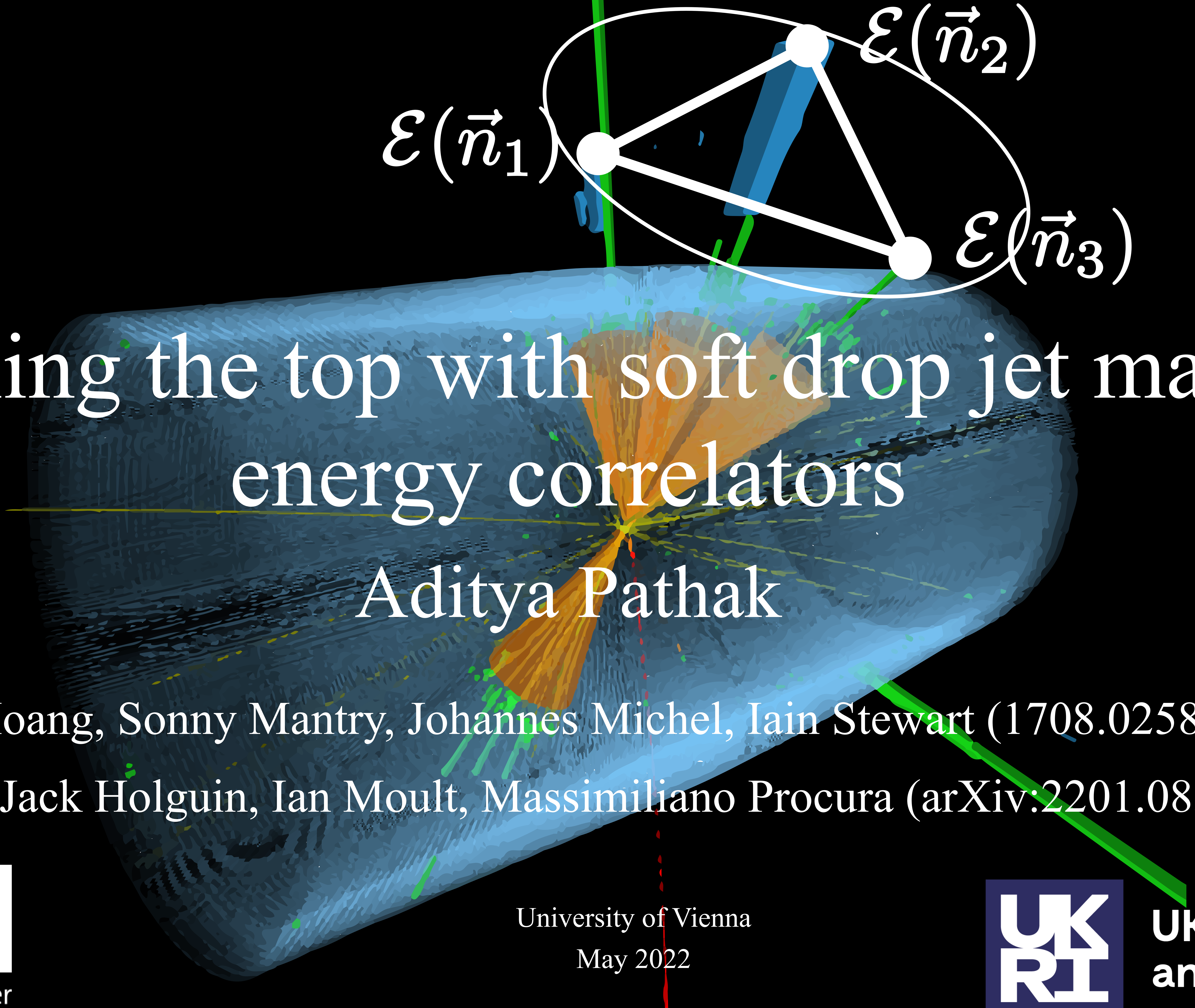

Weighing the top with soft drop jet mass & energy correlators

Aditya Pathak

SD: Andre Hoang, Sonny Mantry, Johannes Michel, Iain Stewart (1708.02586 + soon)

EEEC: Jack Holguin, Ian Moult, Massimiliano Procura (arXiv:2201.08393)

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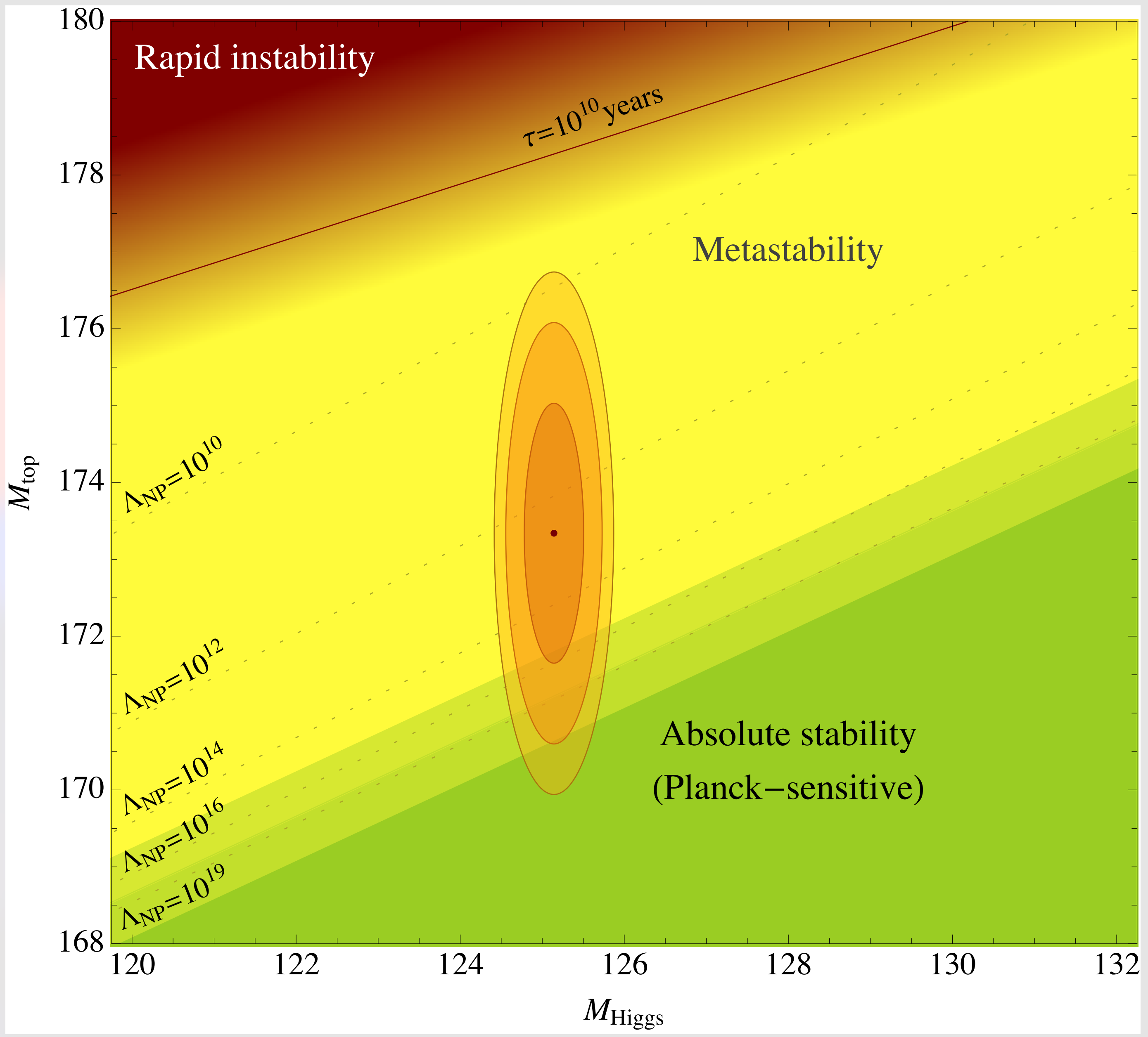
May 2022

UK Research and Innovation

Why the top mass?

Buttazzo, et al., 2013; Andreassen, et al. 2014

- Top quark Yukawa: $y_t = 0.94 \rightarrow$ plays an important role in electroweak vacuum stability



- Current world average (HL-LHC projection ~ 200 MeV)

$$m_t = 172.76 \pm 0.3\,\text{GeV}$$ PDG

Some of the numbers that enter this world average:

$$m_t^{\text{MC}} = 172.69 \pm 0.48\,\text{GeV}$$

ATLAS, 1810.01772

$$m_t^{\text{MC}} = 172.26 \pm 0.61\,\text{GeV}$$

CMS, 1812.06489

Compare with Tevatron:

$$m_t^{\text{MC}} = 174.34 \pm 0.64\,\text{GeV}$$

Tevatron, 1407.2682

A recent CMS analysis yielded:

$$m_t^{\text{pole}} = 170.5 \pm 0.8\,\text{GeV}$$

CMS, 1904.05237

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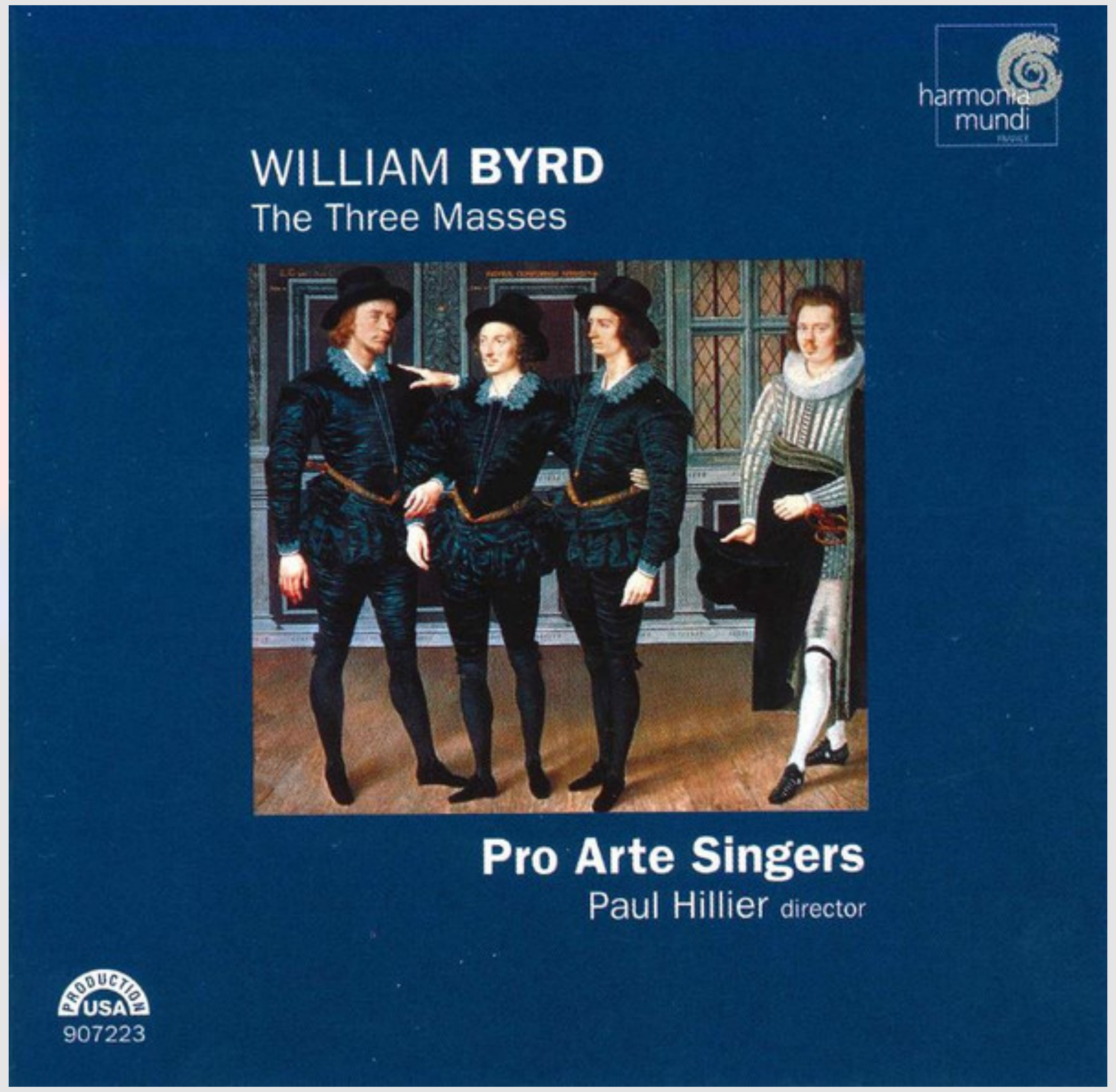


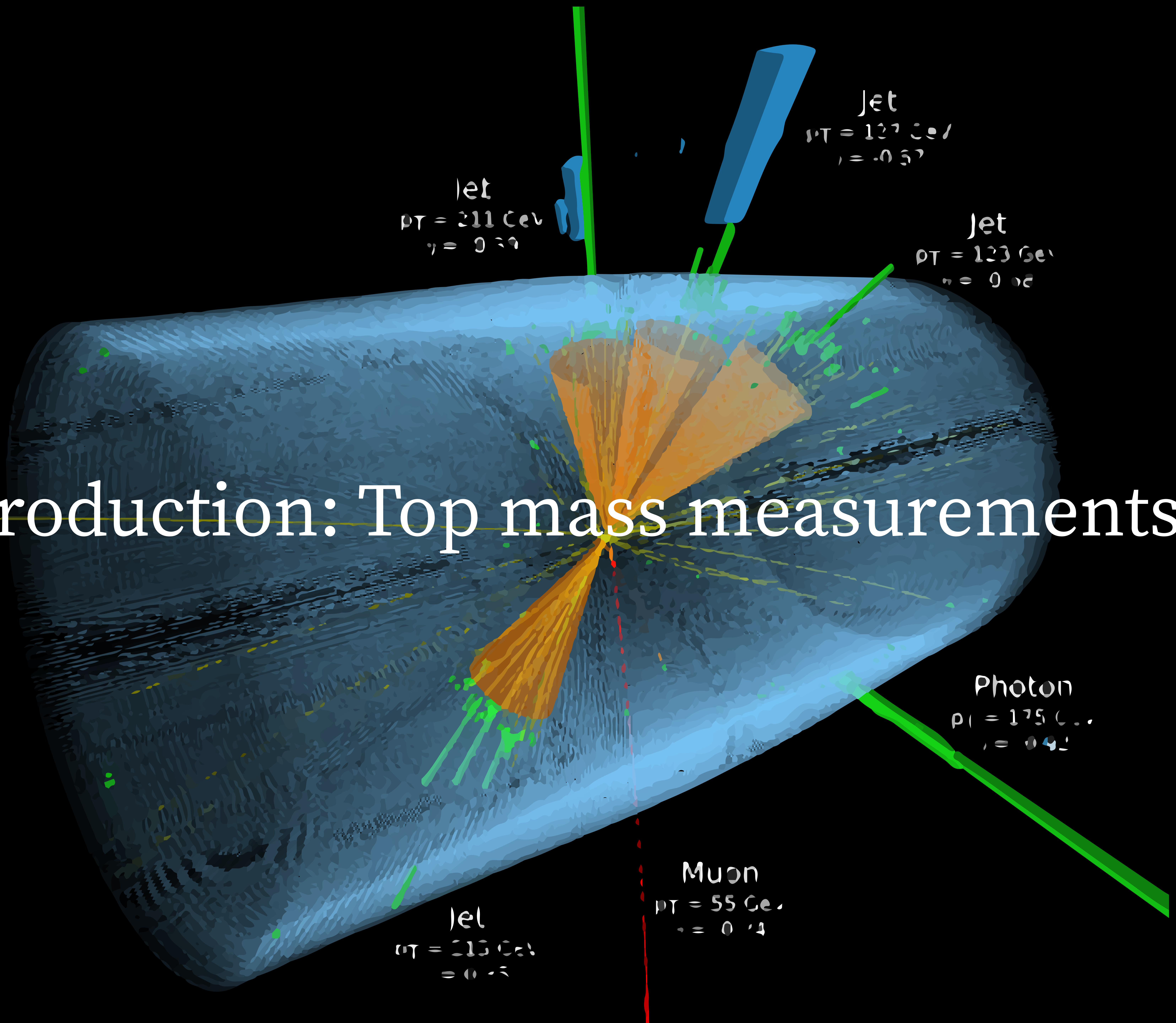
The only quark with three masses in PDG:

Mass (direct measurements) $m = 172.76 \pm 0.30$ GeV [a,b] (S = 1.2)
Mass (from cross-section measurements) $m = 162.5^{+2.1}_{-1.5}$ GeV [a]
Mass (Pole from cross-section measurements) $m = 172.5 \pm 0.7$ GeV

Outline

1. A brief introduction
2. Top mass using soft drop jet mass
3. Top mass using energy correlators

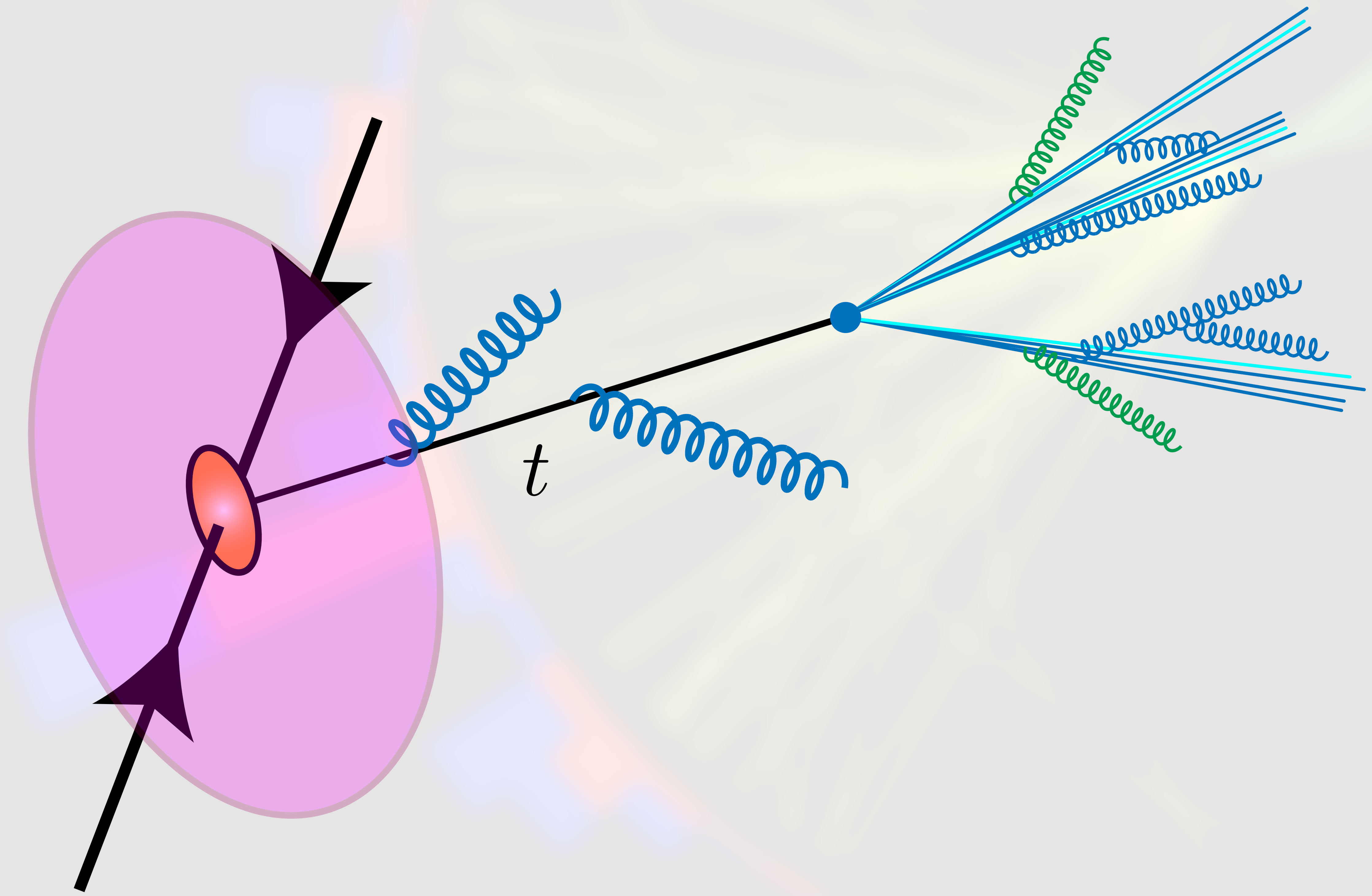
Introduction: Top mass measurements



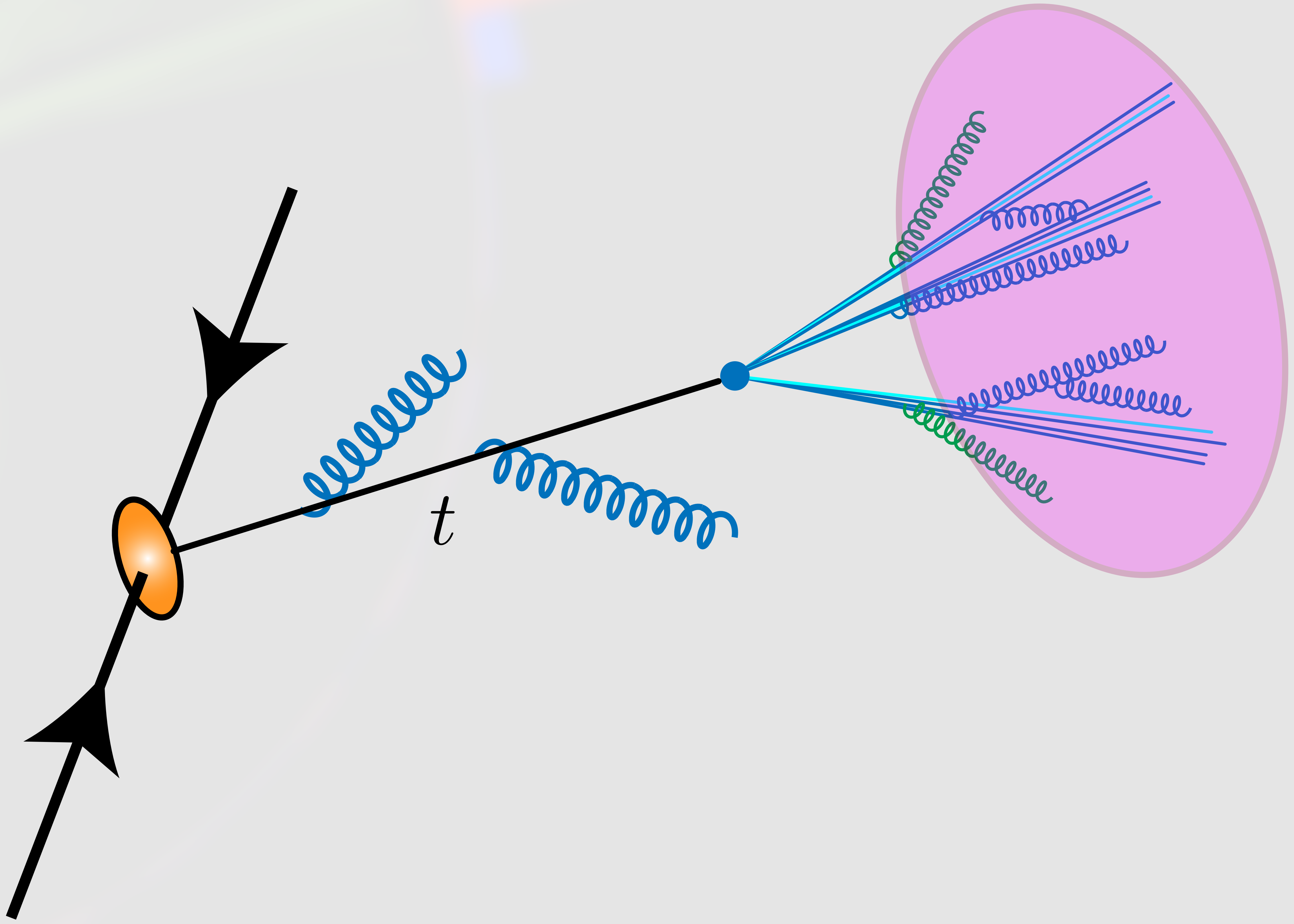
How to measure the top mass?

1. Exploit the production mechanism

2. Exploit the final state decay products



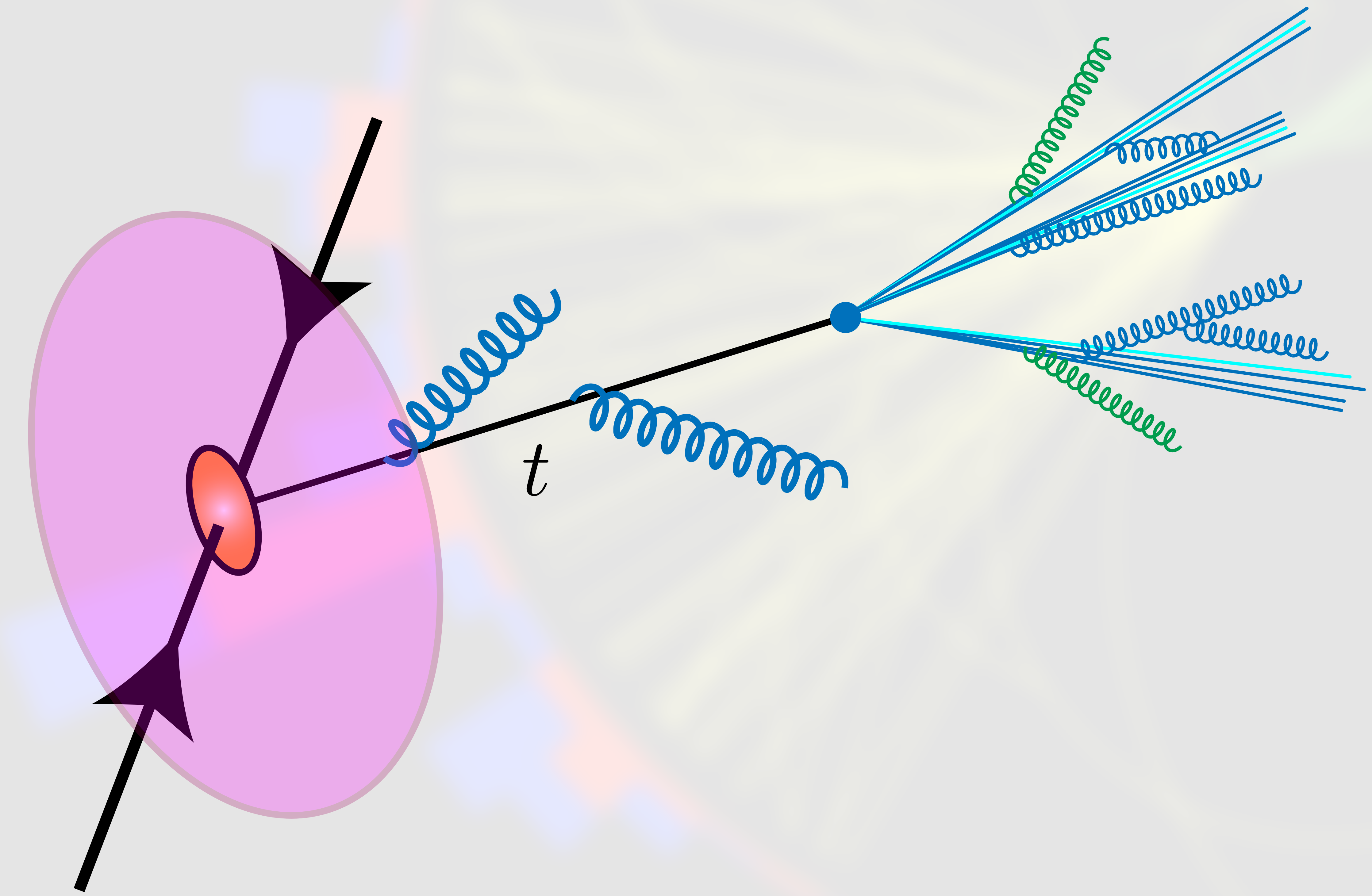
(inclusive over final state)



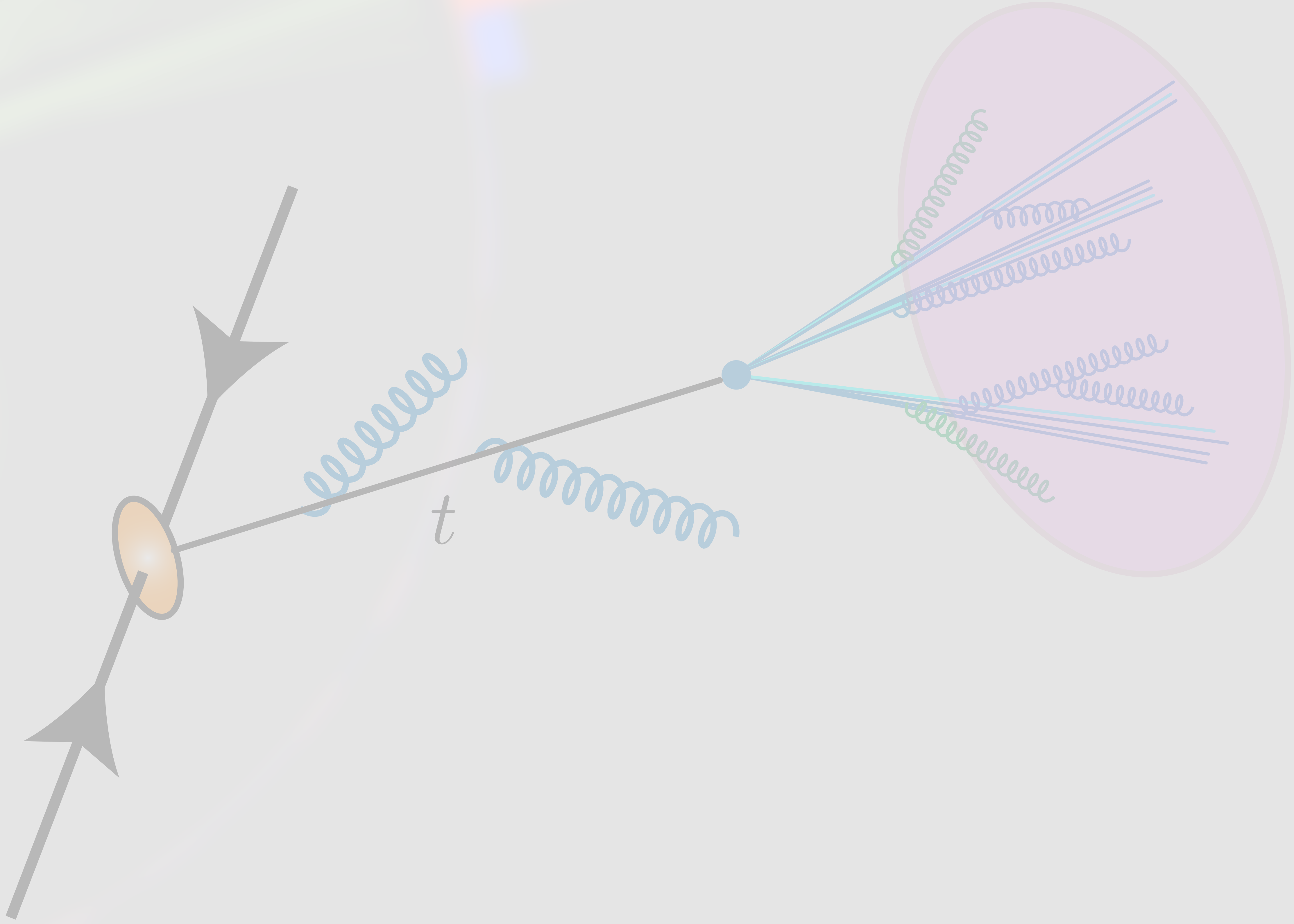
(insensitive to the production)

How to measure the top mass?

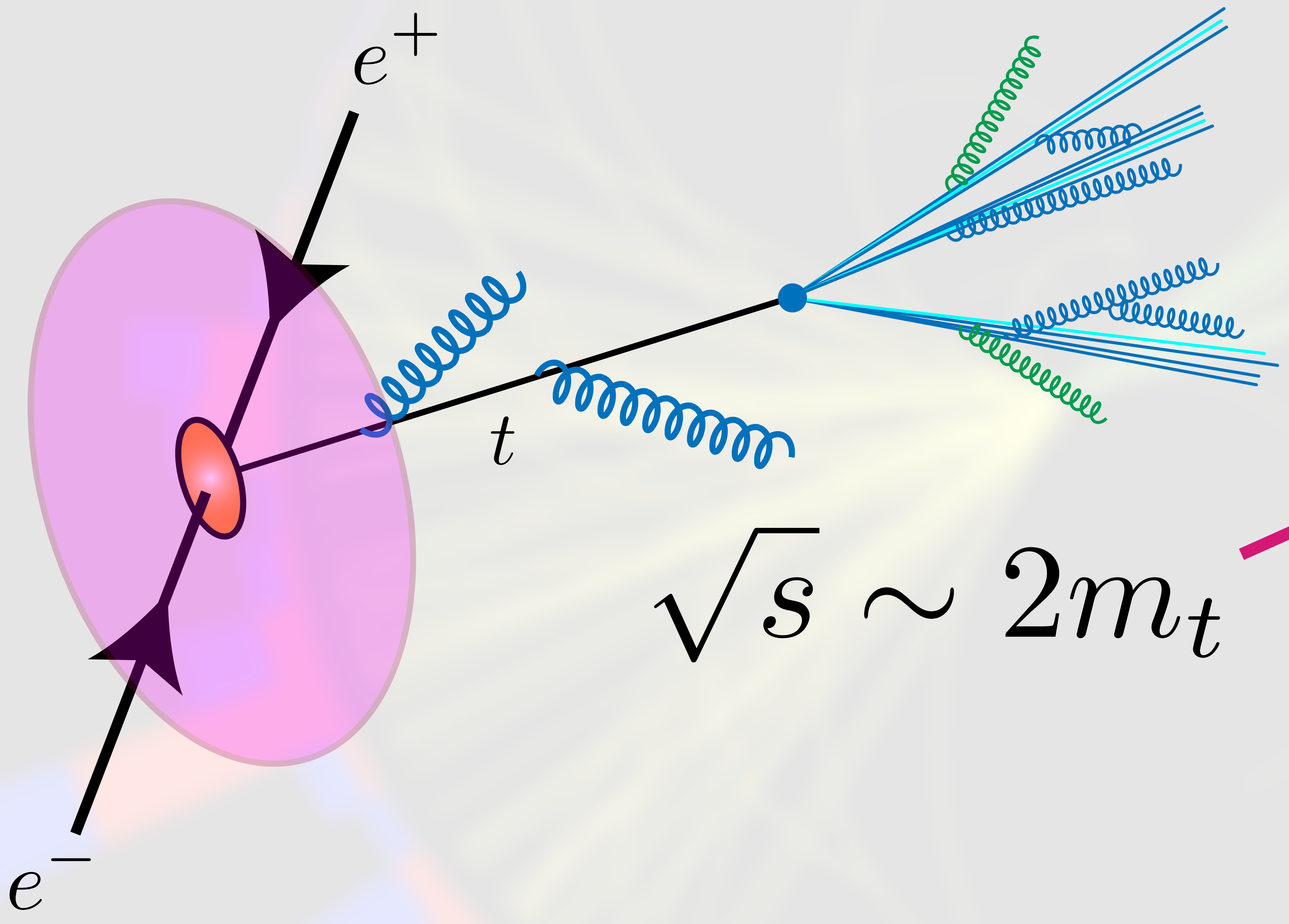
1. Exploit the production mechanism



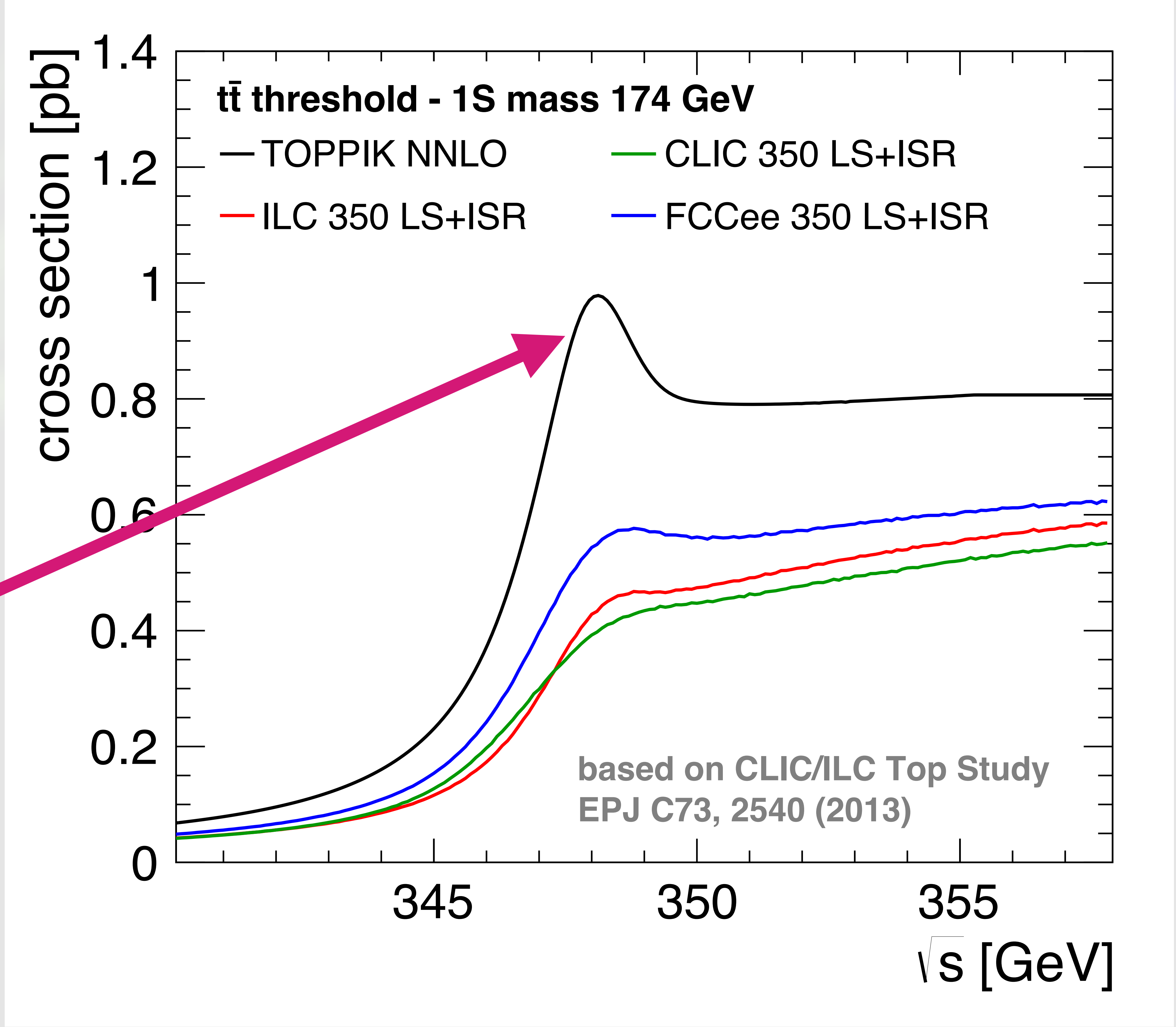
2. Exploit the final state decay products



1. Exploit the production mechanism



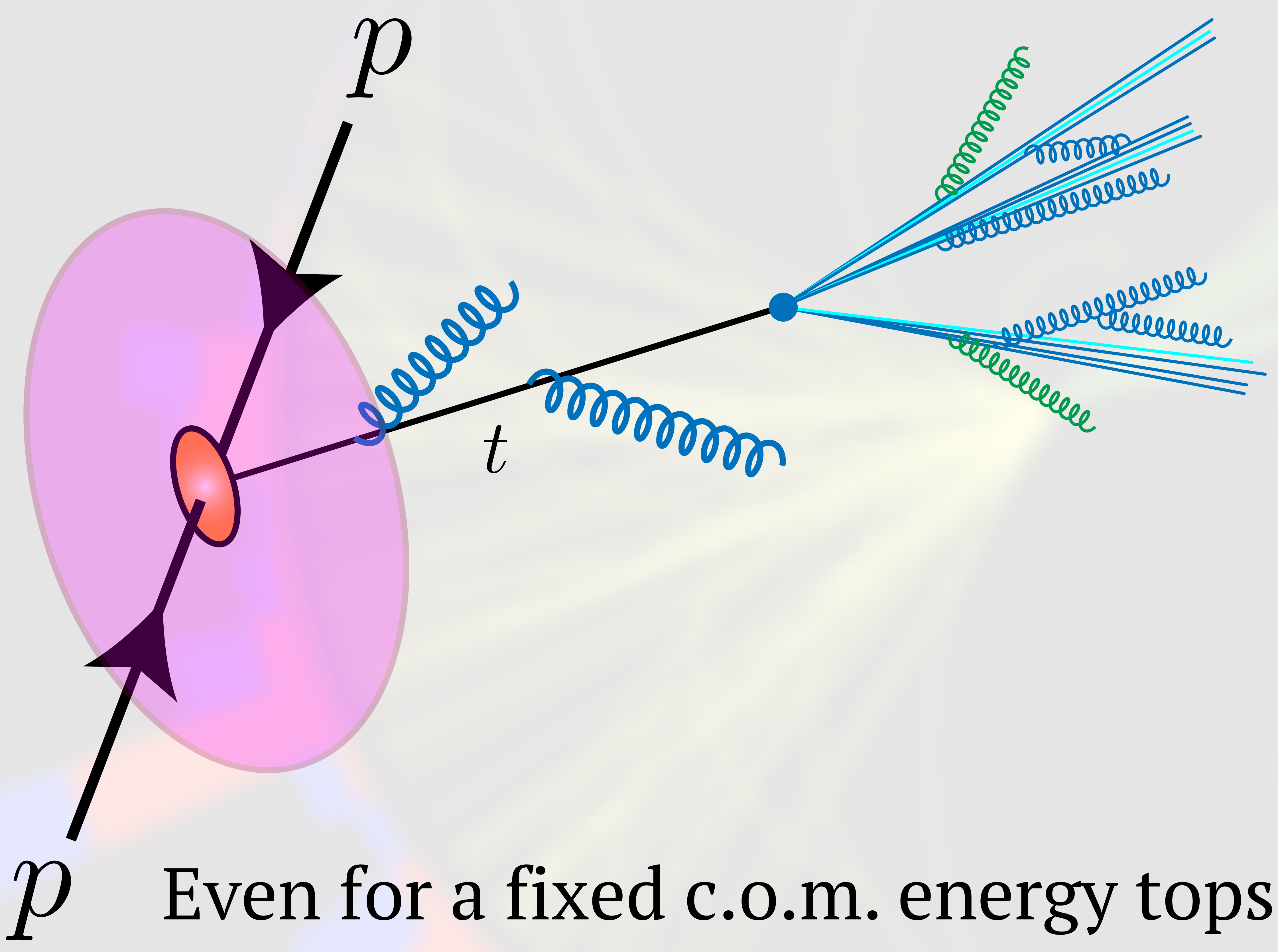
$$\sqrt{s} \sim 2m_t$$

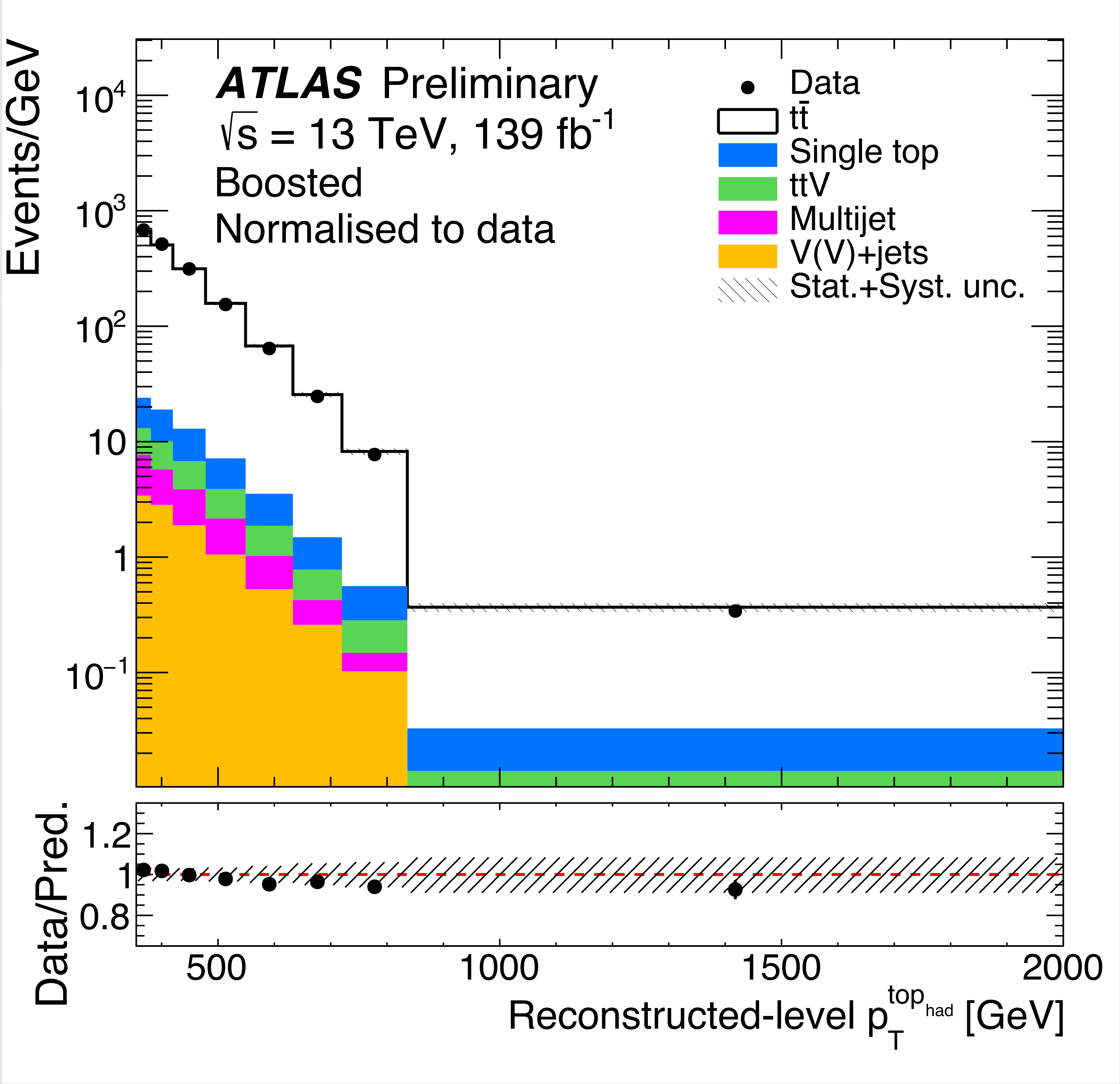


Threshold scan in e+e- colliders: < 50 MeV precision

1. Exploit the production mechanism

Challenging to exploit in *pp* collisions



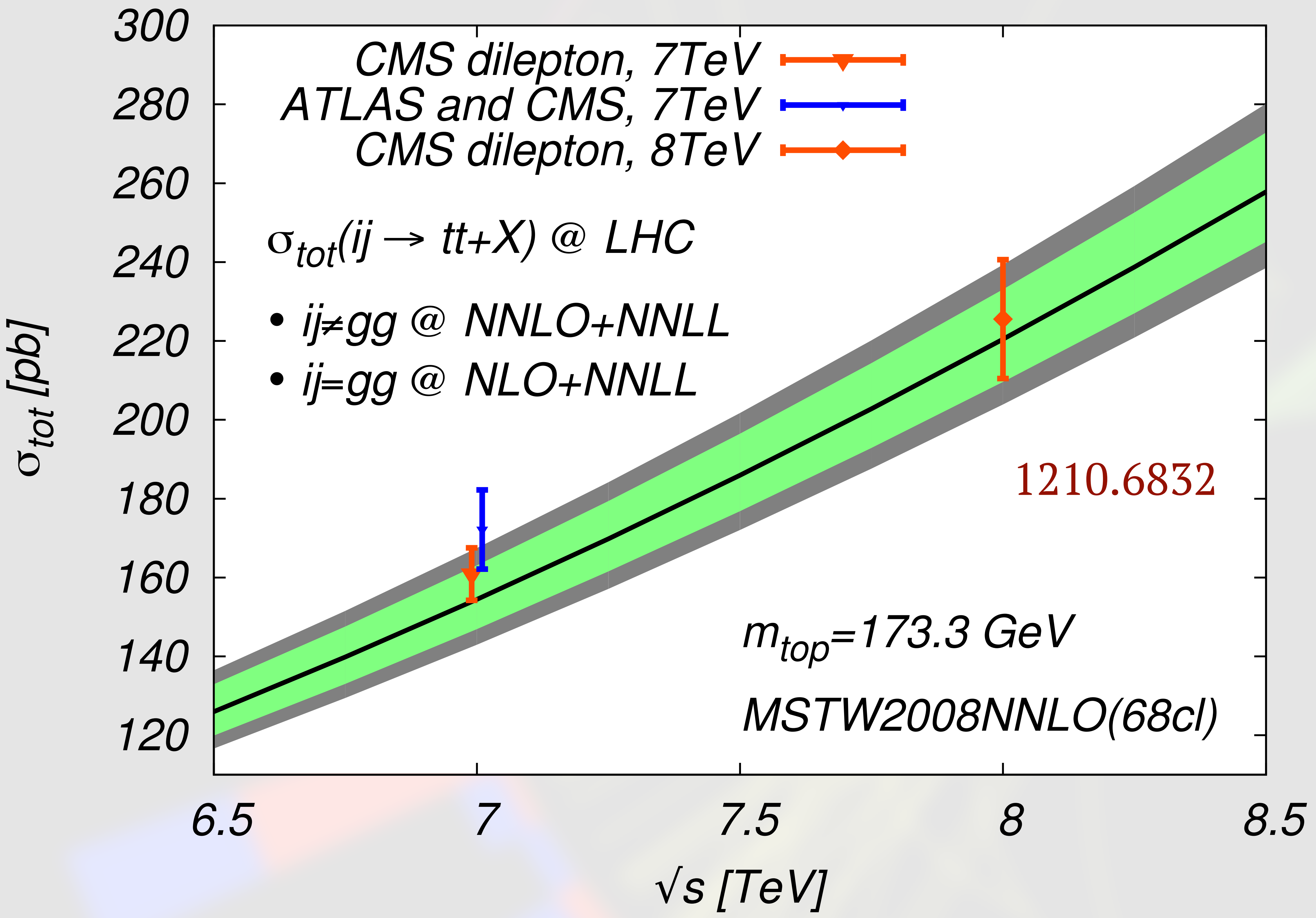


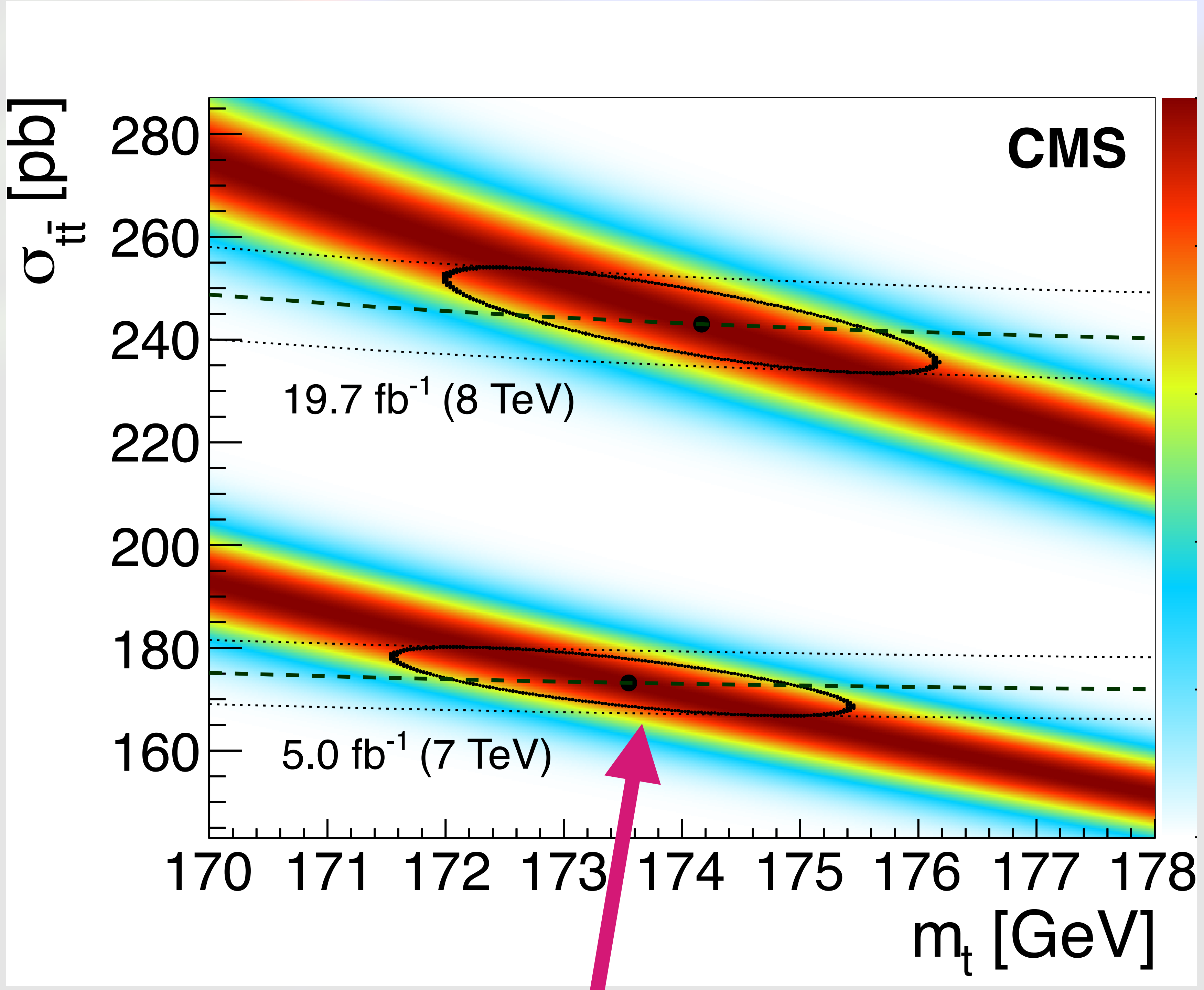
Even for a fixed c.o.m. energy tops are produced with a distribution of p_T

I will come back to this

Theoretically cleanest way: Count the tops

Czakon, Mitov 2012, 2013; Aliev et al. 1007.1327





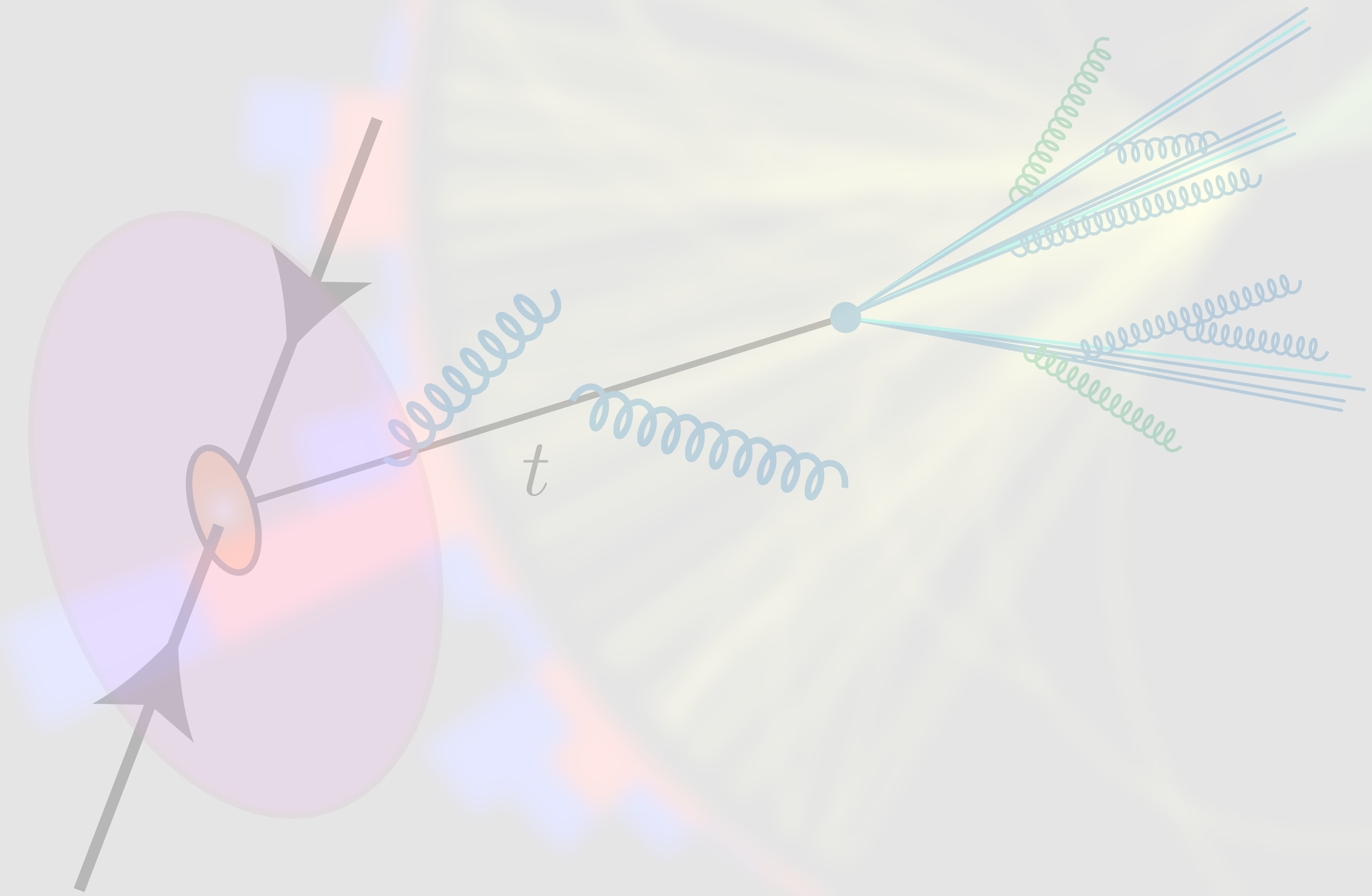
$$m_t^{\text{pole}} = 172.9^{+2.5}_{-2.6}\ \text{GeV}$$ ATLAS, 1406.5375

$$m_t^{\text{pole}} = 172.7^{+2.4}_{-2.7}\ \text{GeV}$$ CMS, 1701.06228

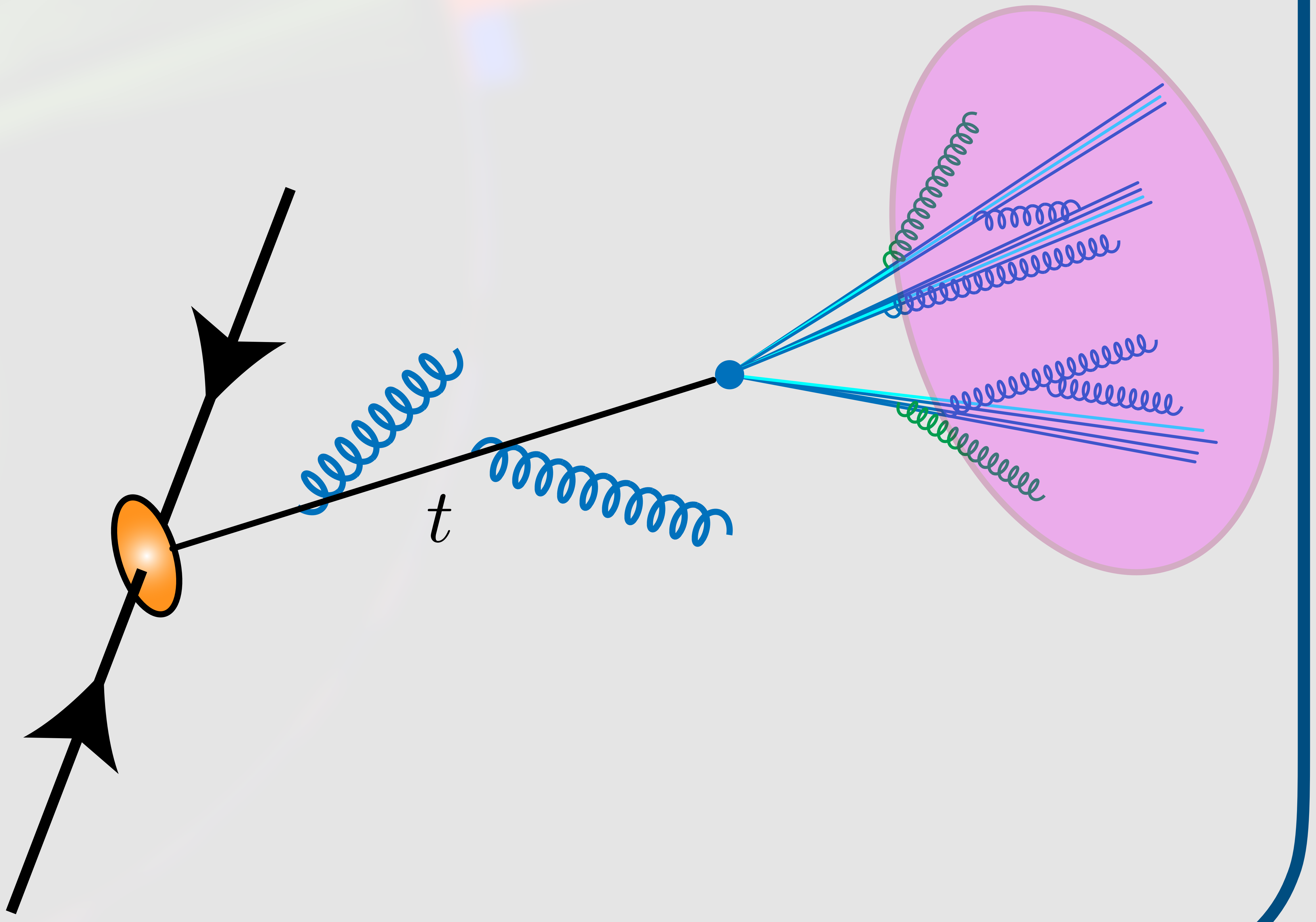
The large error results from normalization uncertainty

How to measure the top mass?

1. Exploit the production mechanism



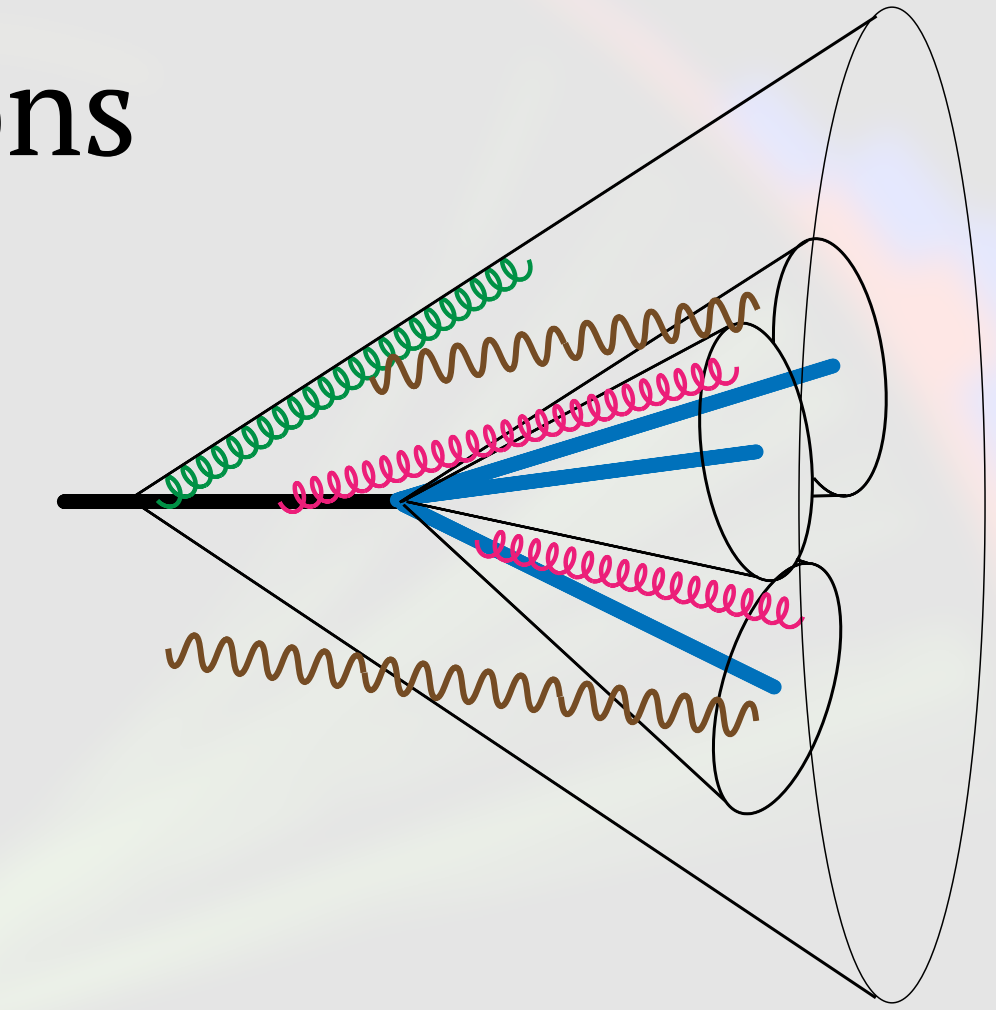
2. Exploit the final state decay products



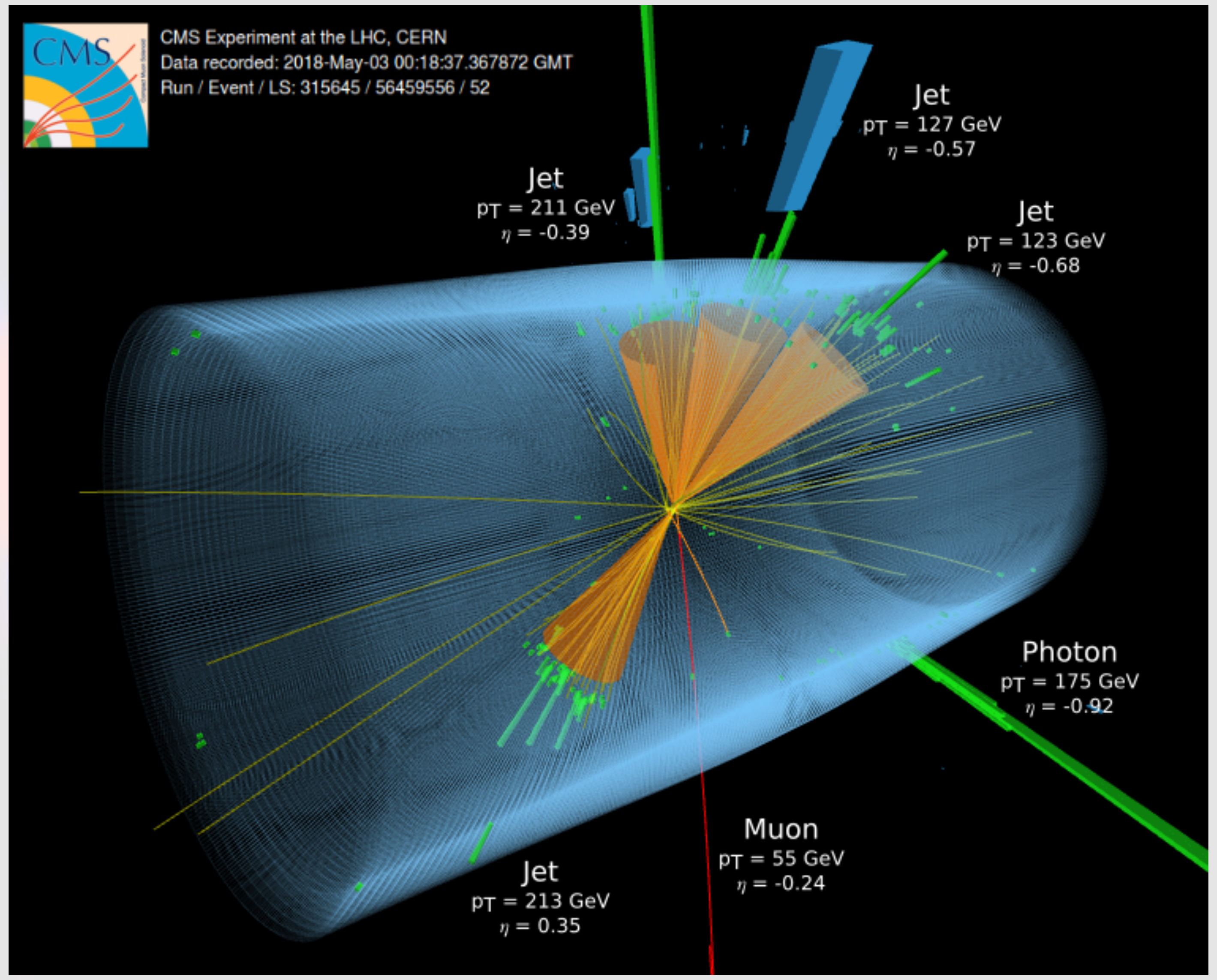
2. Exploit the final state decay products

Measure all sorts of differential distributions on top decay products:

$$\frac{d\sigma}{dm_t^{\text{reco}}}, \quad \frac{d\sigma}{dM_{bl}}, \quad \frac{d\sigma}{dM_{t\bar{t}}}, \quad \frac{d\sigma}{dM_{t\bar{t}j}}$$



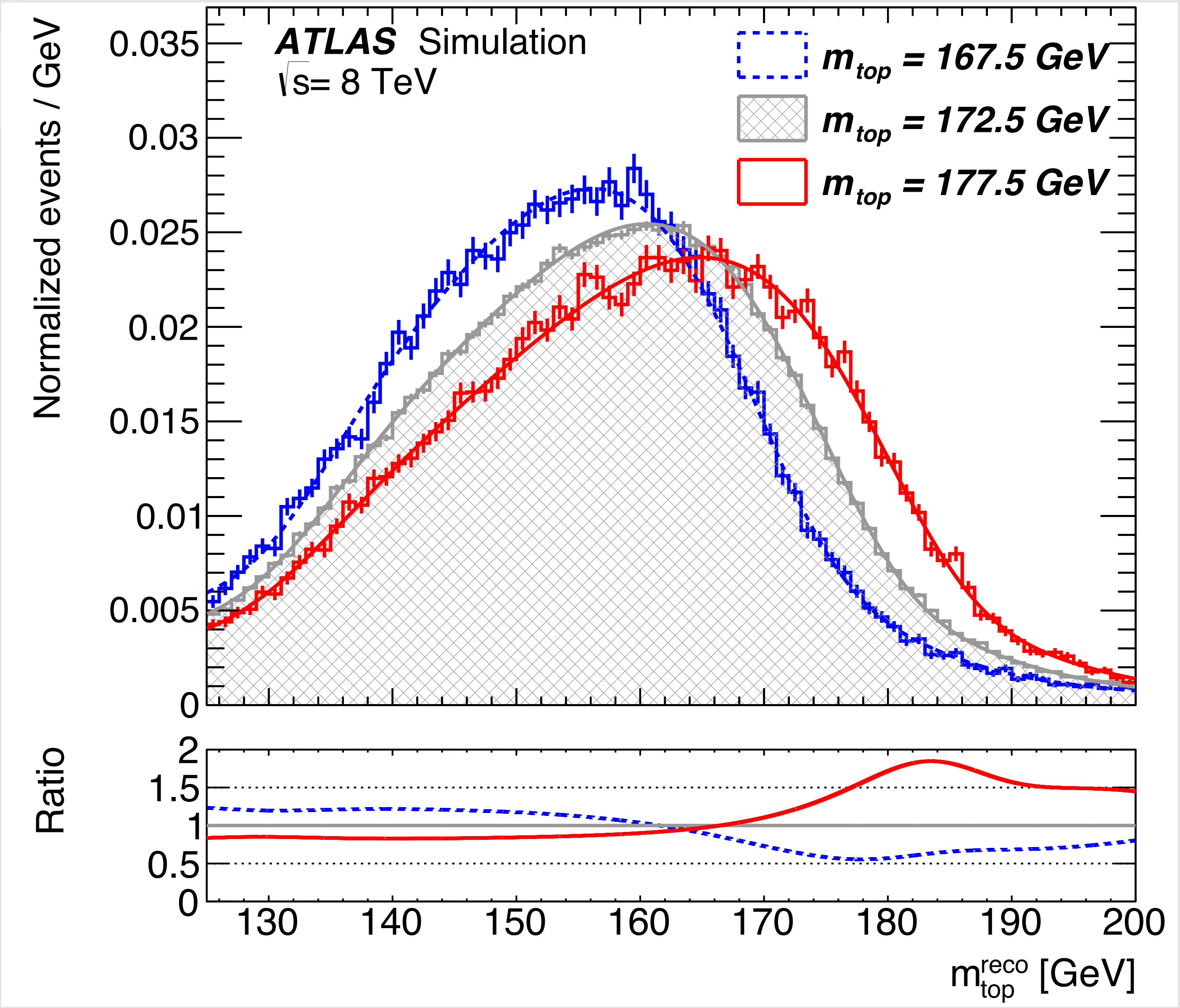
Use $E = mc^2$



This approach has yielded the most precise measurements:

$$m_t^{\text{MC}} = 172.69 \pm 0.48\,\text{GeV}$$ ATLAS, 1810.01772

$$m_t^{\text{MC}} = 172.26 \pm 0.61\,\text{GeV}$$ CMS, 1812.06489



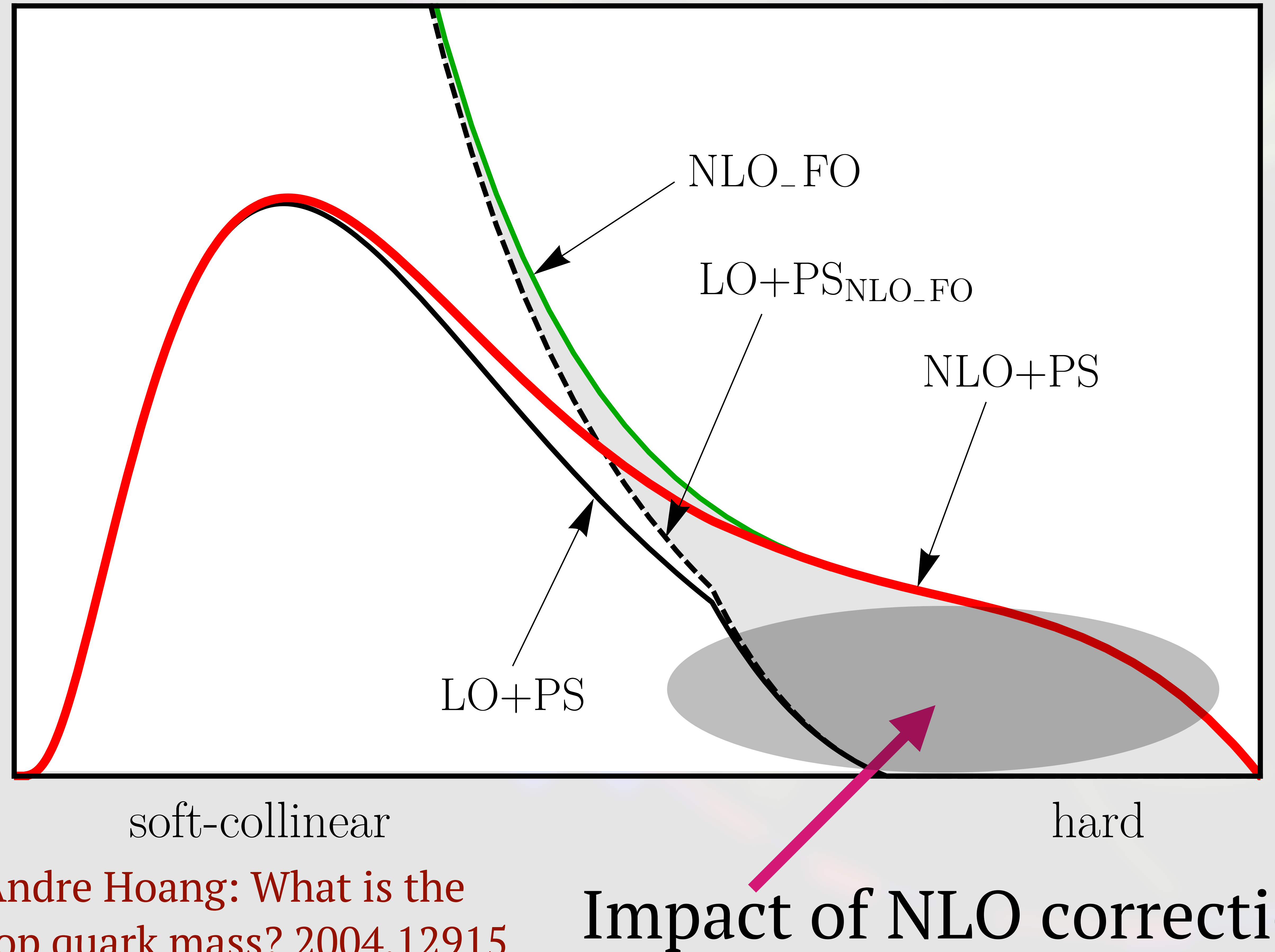
Conceptual problem: ***What mass is m_t^{MC}?***

Simulating the top as a particle with a definite mass ignores $\mathcal{O}(1\,\text{GeV})$ long-distance effects

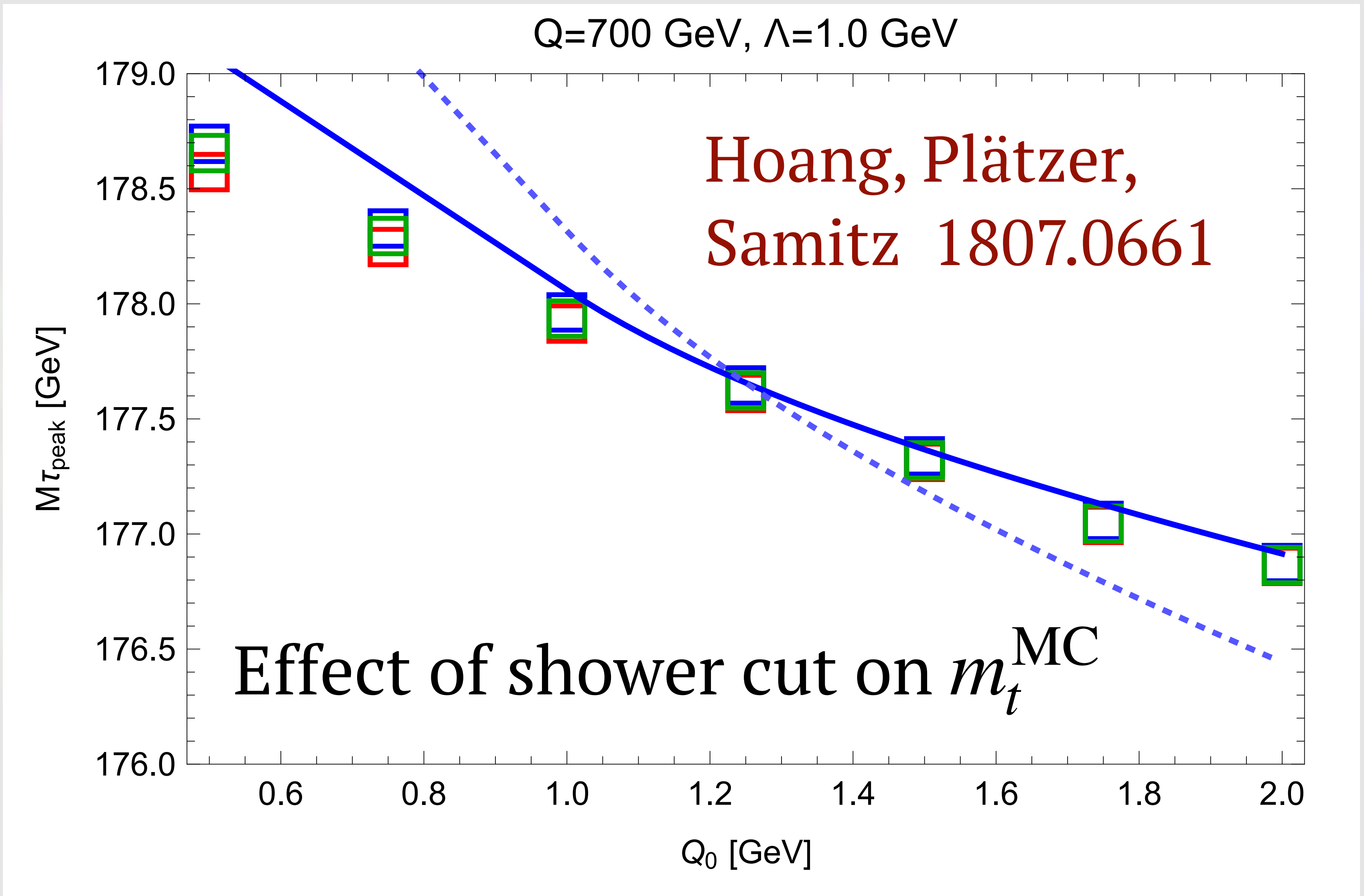
Why top mass interpretation problem?

Observations:

1. Threshold structure appears in the soft-collinear region
2. NLO corrections make an impact only in the tail



Andre Hoang: What is the top quark mass? 2004.12915



*Implications for **direct measurements:***

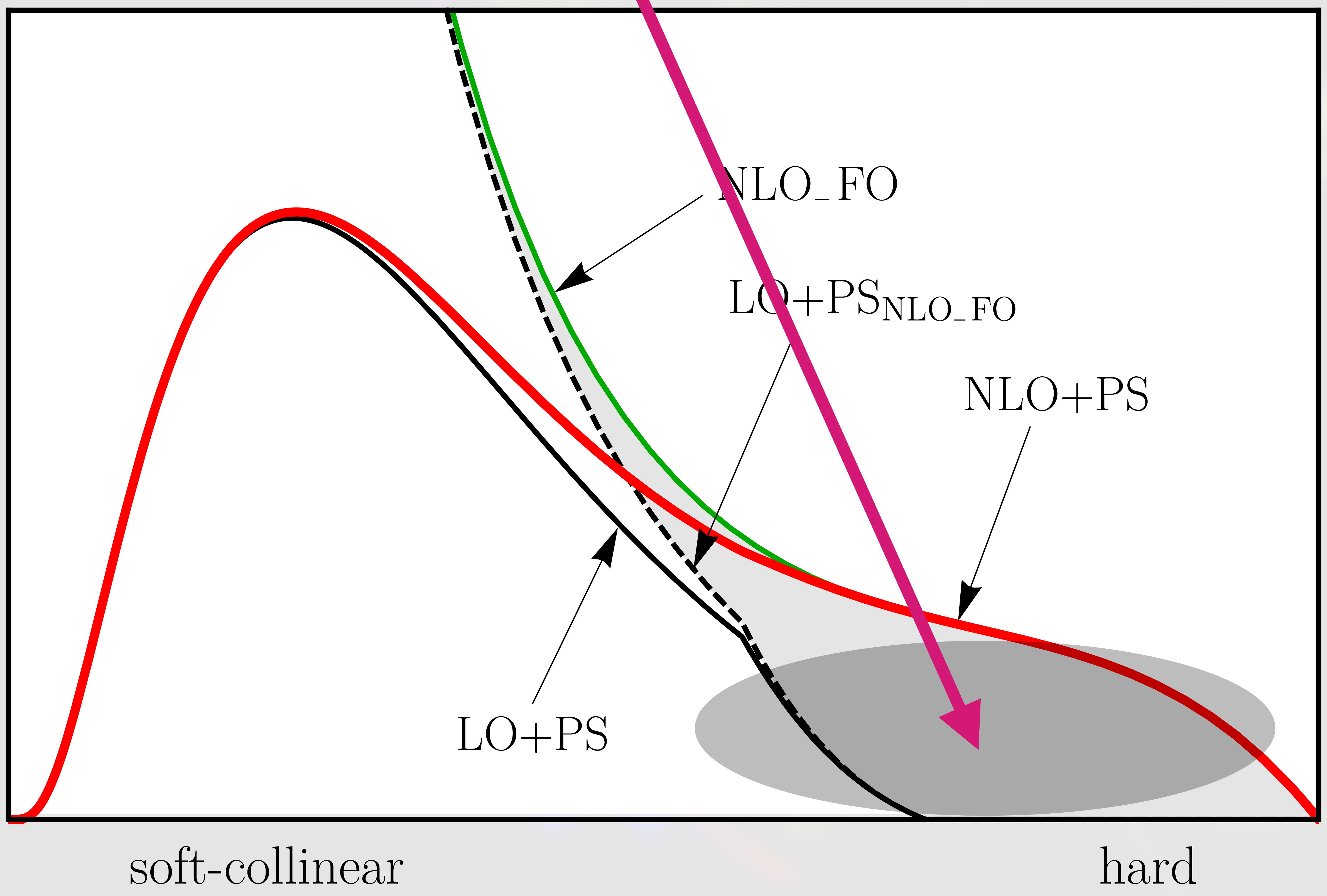
1. Very challenging to improve PS beyond NLL. Hadronization models make up for inadequacies of the PS: poor theoretical control.
2. PS impacts the meaning of the MC top mass parameter: effects as large as 0.5 GeV.

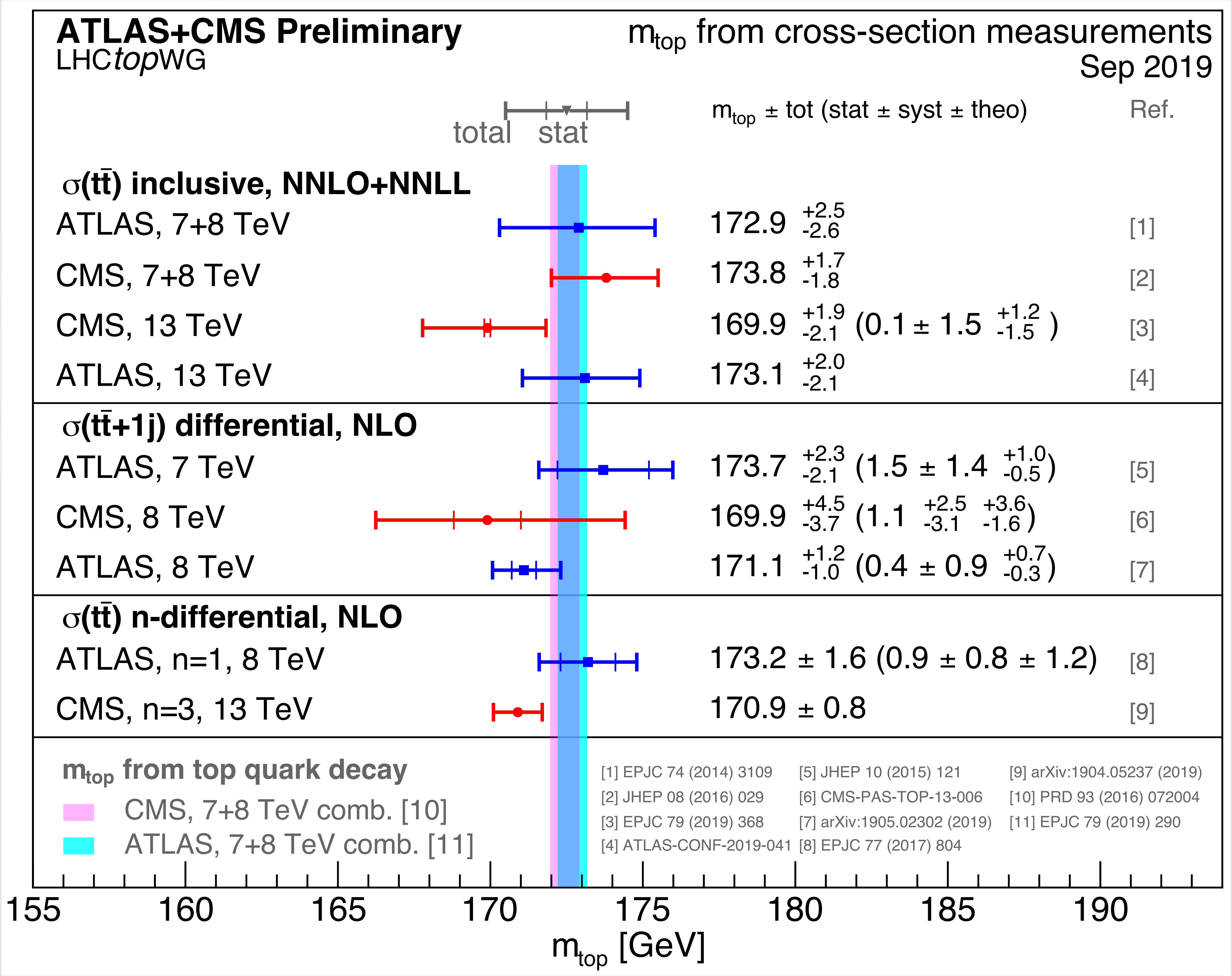
Hoang, Plätzer, Samitz 1807.0661

Avoid threshold region: indirect measurements

Away from threshold NLO matched MC are reliable

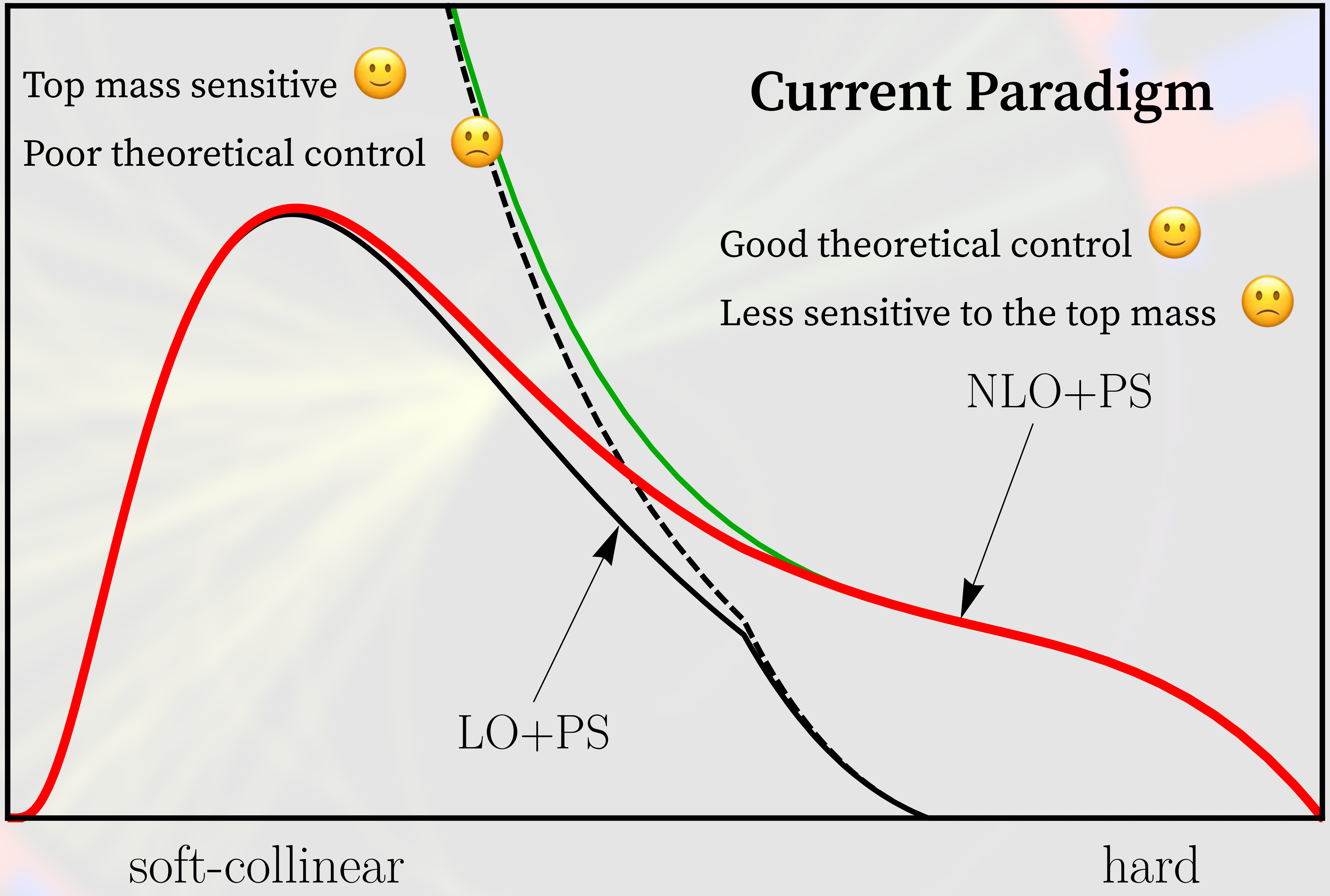
Focus here for good theoretical control



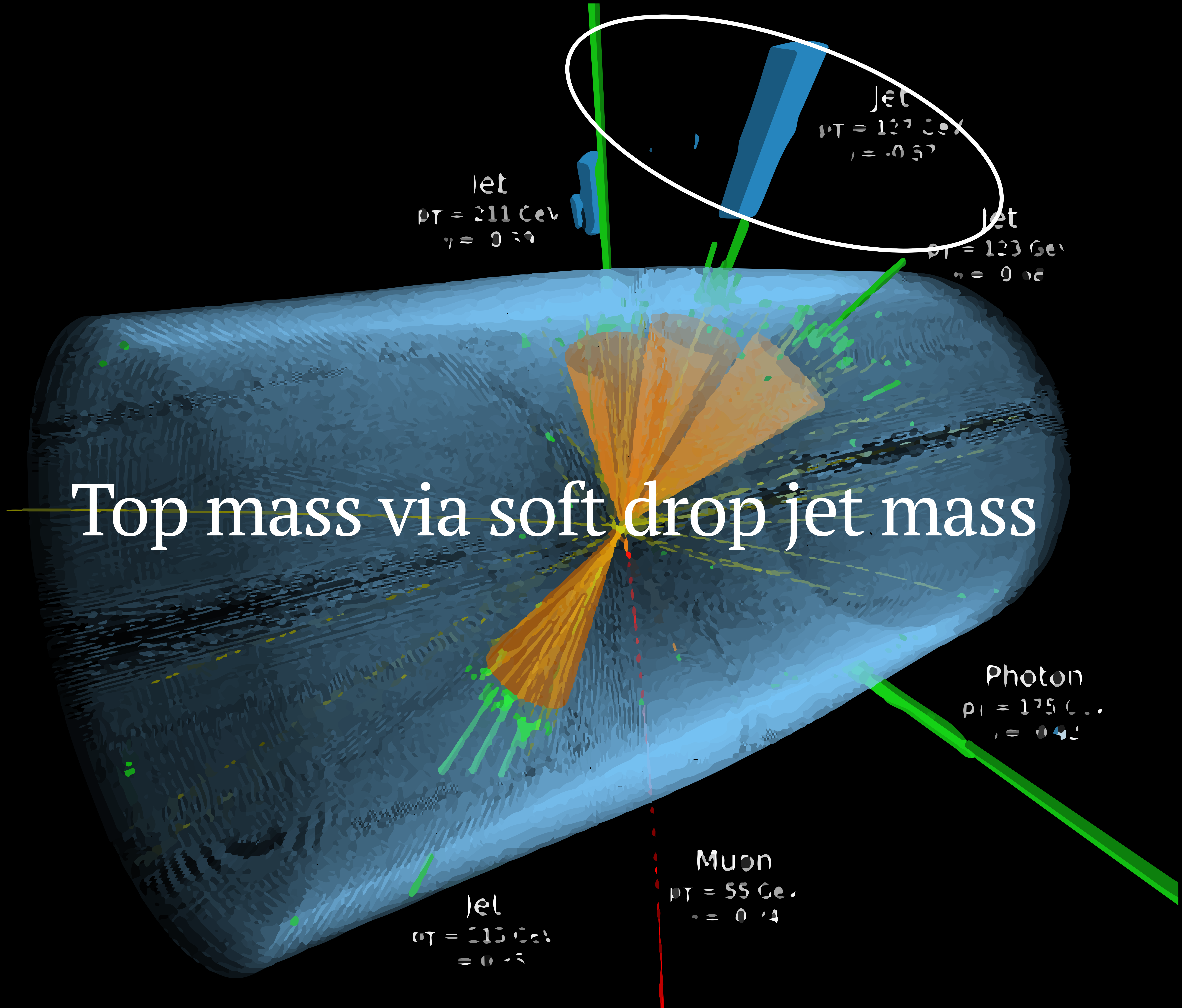


Unfortunately, poor sensitivity when not leveraging the threshold

Summary of challenges in the current paradigm



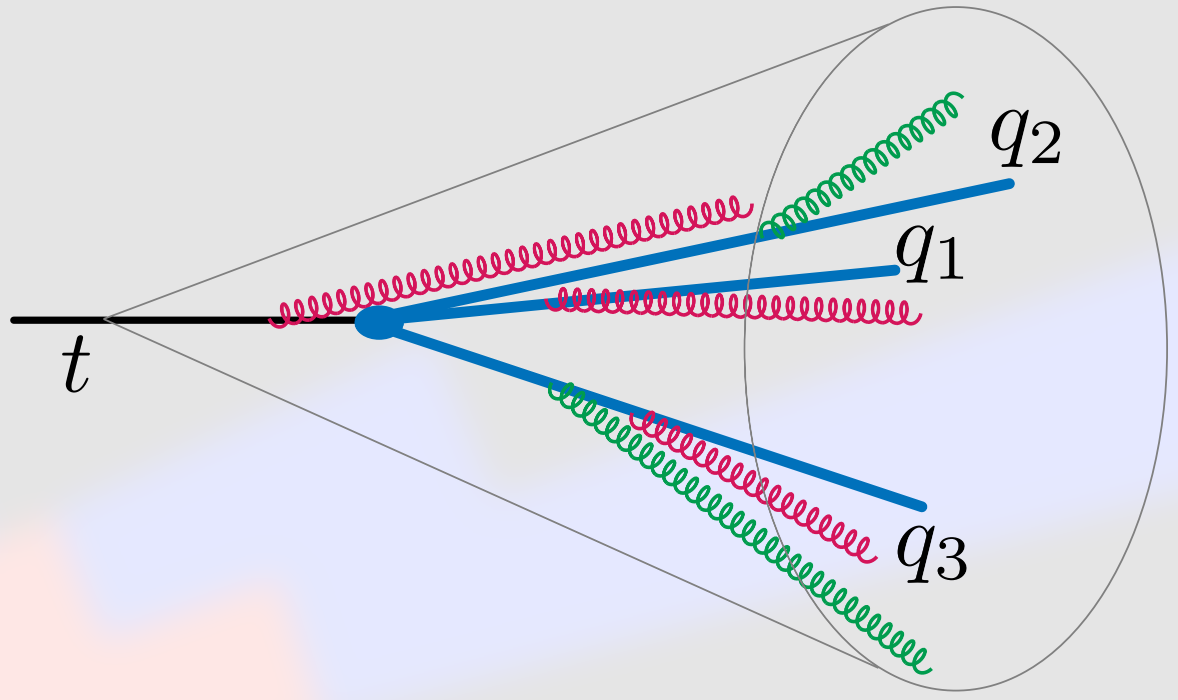
Top mass via soft drop jet mass



Overcome challenges via analytical resummation

Analytical resummation

Consider the jet mass:

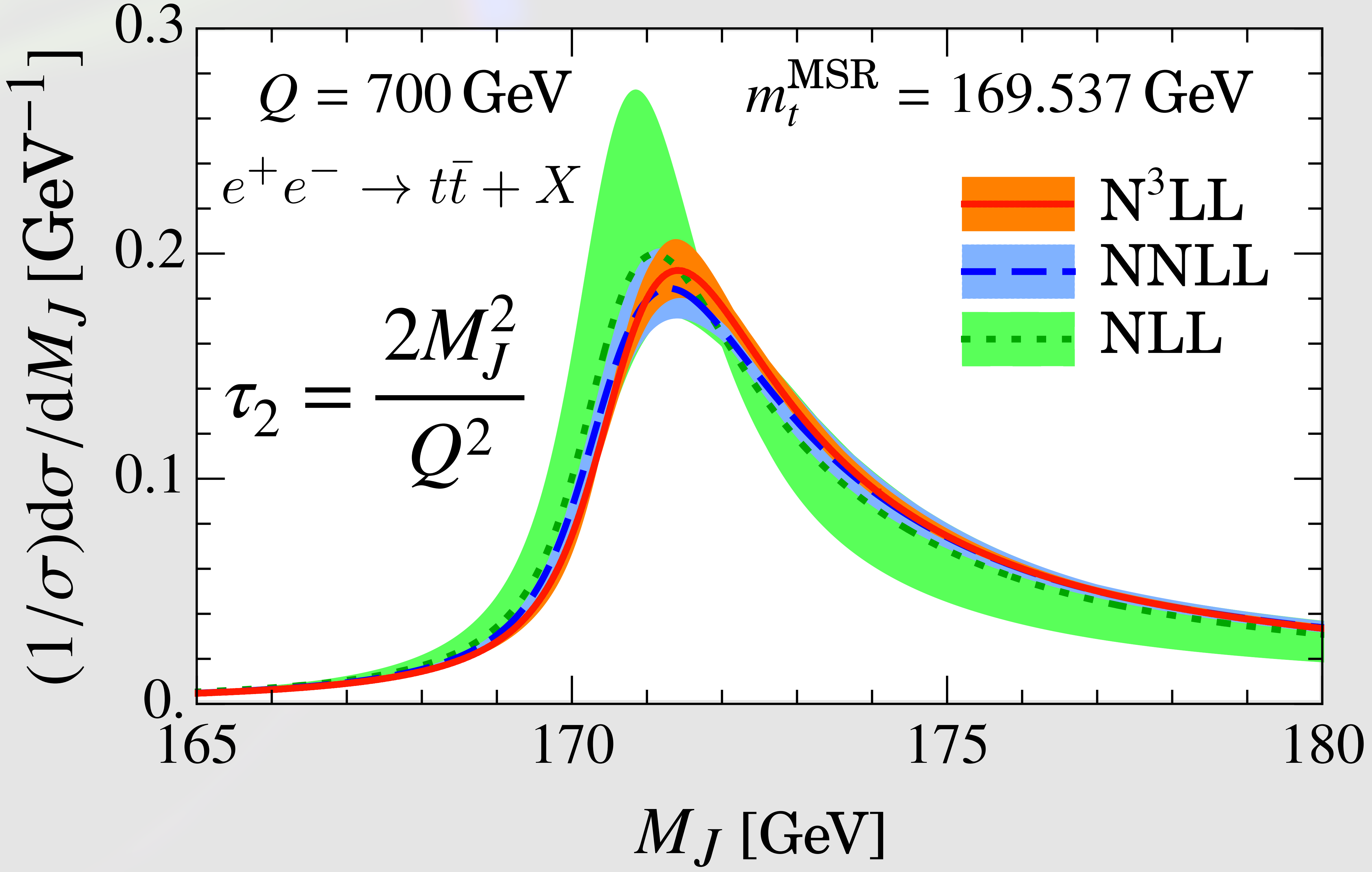


$$M_J^2 = \Big(\sum_{i\in J} p_i^\mu\Big)^2 \simeq m_t^2 + \Gamma_t m_t + \dots$$

Fleming et al. hep-ph/0703207, 0711.2079

Resummation using SCET and HQET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_2} = m_t\, Q^2 H^{(5,6)}_{\rm evol}(Q, m_t, \varrho, \mu; \mu_H, \mu_m)$$
$$\times \int \mathrm{d}\ell\, \mathrm{d}\hat s\, U_B^{(5)}(\hat s_\tau - \varrho\ell - \hat s, \mu, \mu_B)\, J^{(5)}_{B,\tau_2}(\hat s, \Gamma_t, \delta m, \mu_B)$$
$$\times \int \mathrm{d}\ell' \mathrm{d}k\, U_S^{(5)}(\ell - \ell', \mu, \mu_S) \hat S^{(5)}_{\tau_2}(\ell' - k, \bar\delta, \mu_S) F(k - 2\Delta)\,,$$



Implemented to N^{3}LL in SCETlib

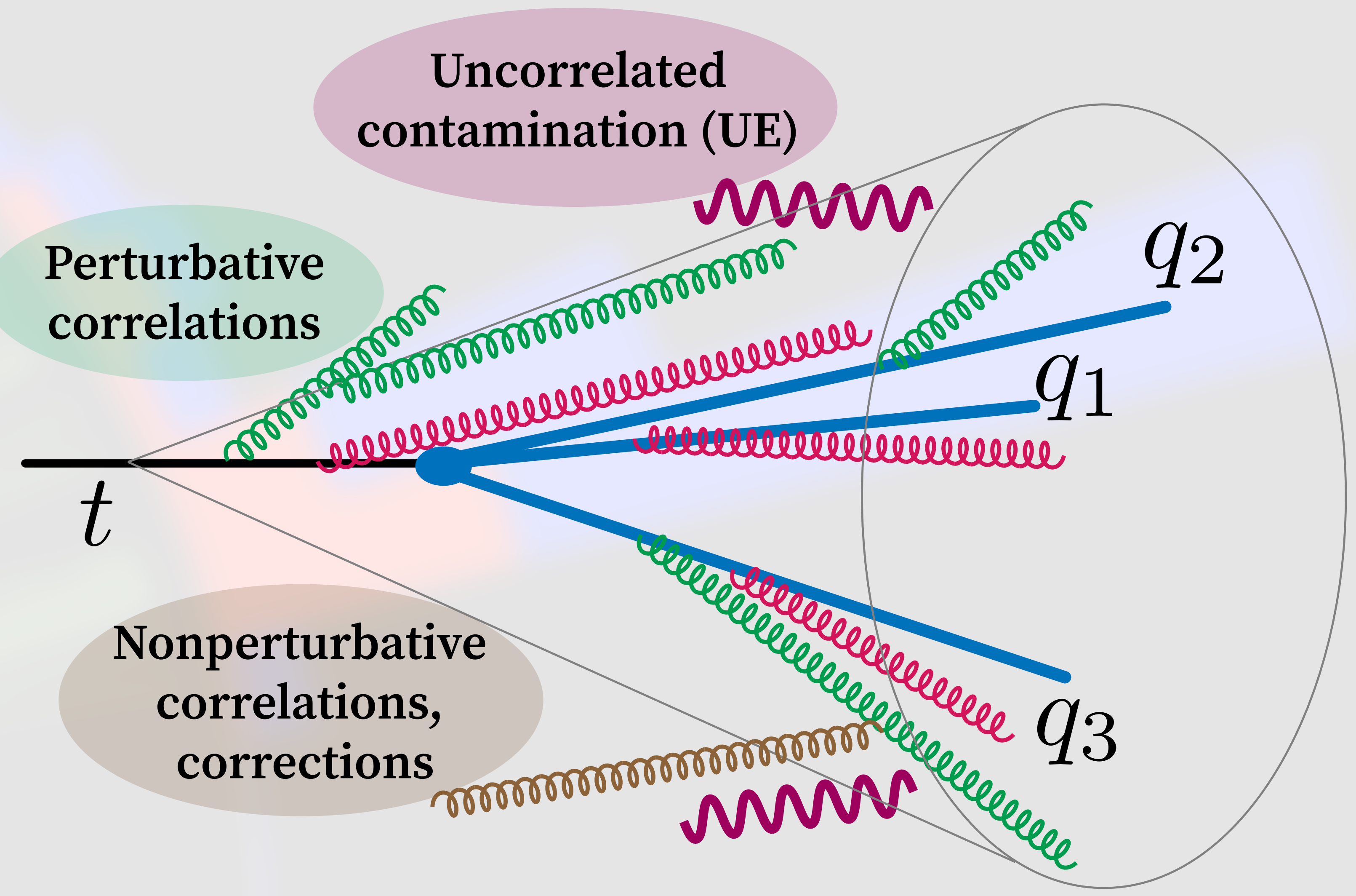
Ebert, Michel, Tackmann

Bachu, Hoang, AP, Mateu, Stewart 2012.12304

Analytical resummation

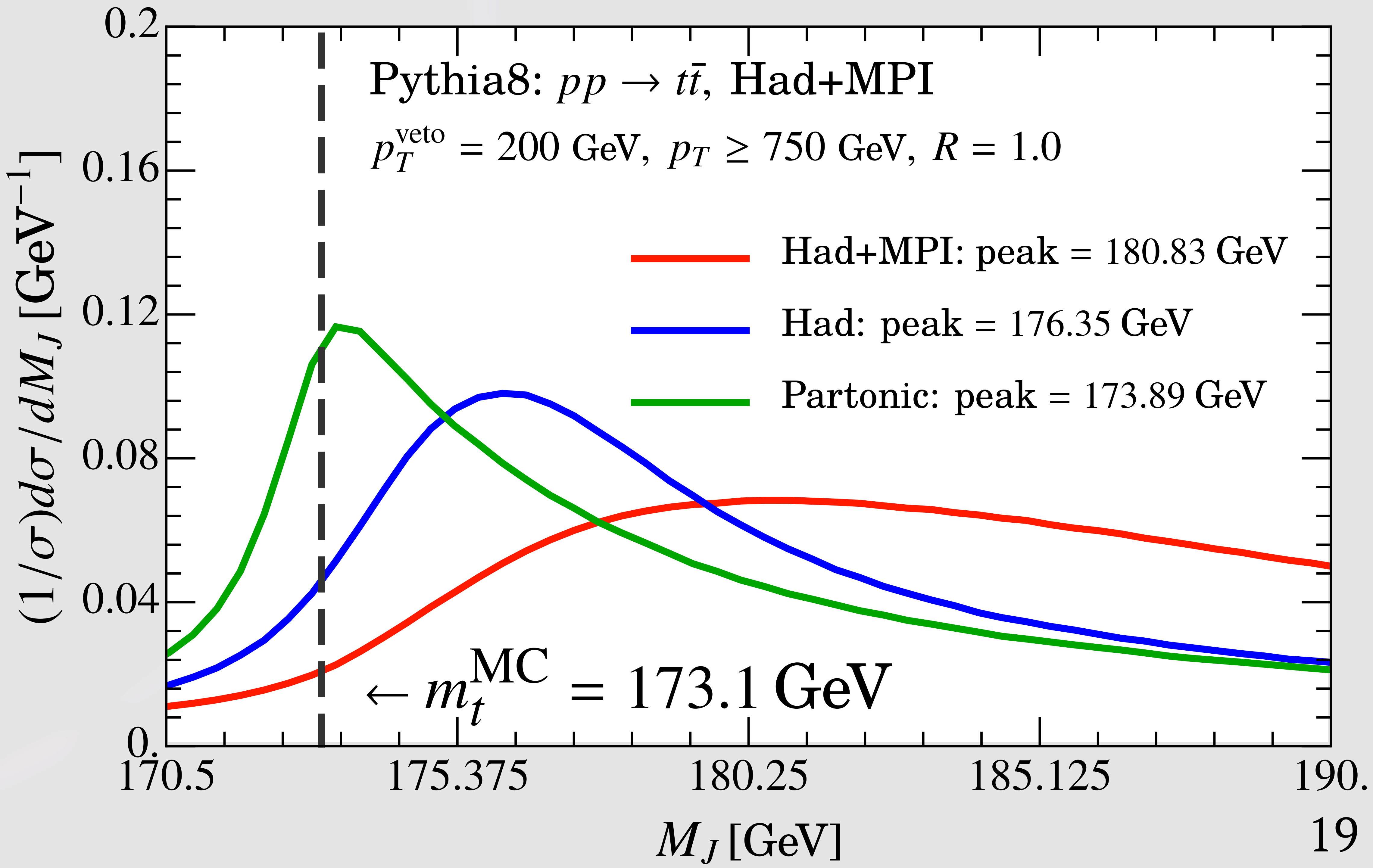
Consider the jet mass:

$$M_J^2 = \Big(\sum_{i\in J} p_i^\mu\Big)^2 \simeq m_t^2 + \Gamma_t m_t + \ldots$$



Challenges in pp

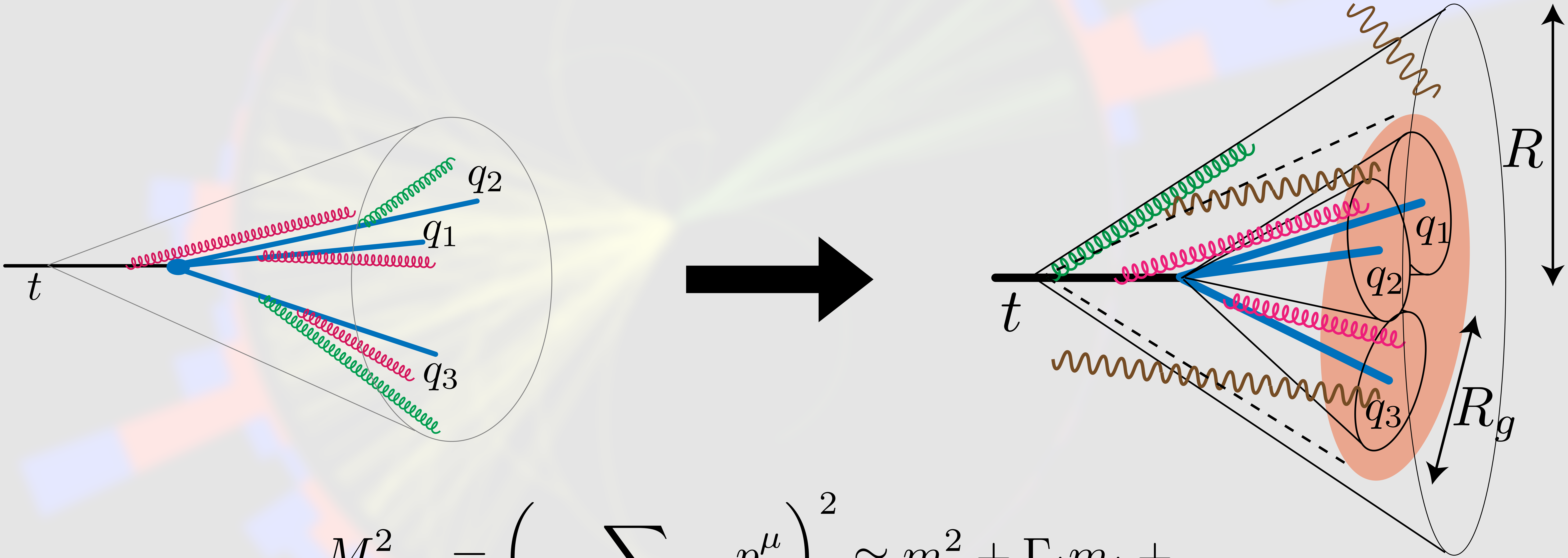
- Strongly correlated with outside radiation
- Precision spoiled by uncorrelated contamination



Soft drop jet mass

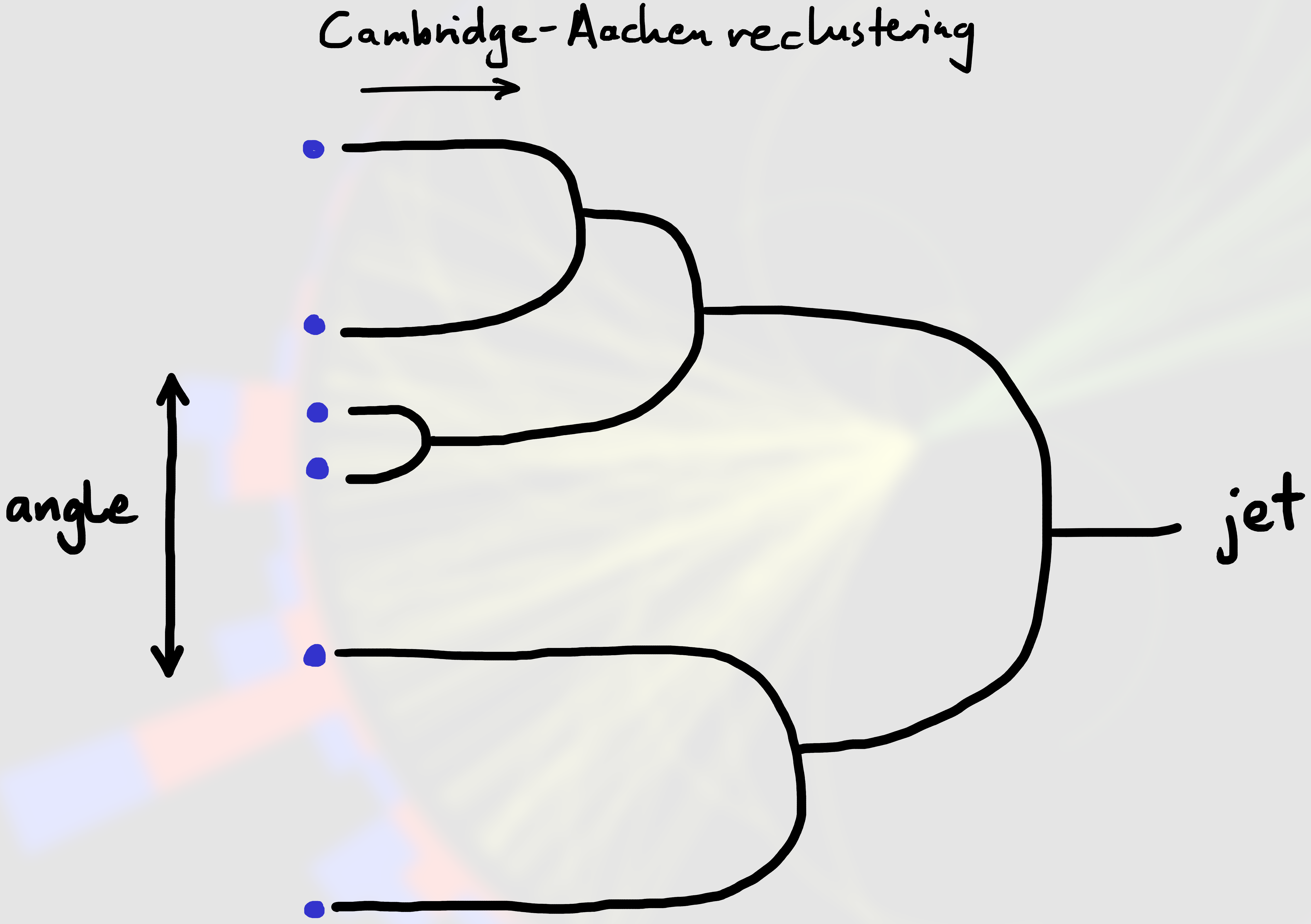
Dasgupta et al. 1307.0007; Larkoski, Marzani, Soyez, Thaler 1402.2657

Improve robustness for the LHC by considering the soft drop jet mass



$$M_{J,\text{sd}}^2 = \Big(\sum_{i \in J^{\text{groomed}}} p_i^\mu \Big)^2 \simeq m_t^2 + \Gamma_t m_t + \ldots$$

Soft drop algorithm

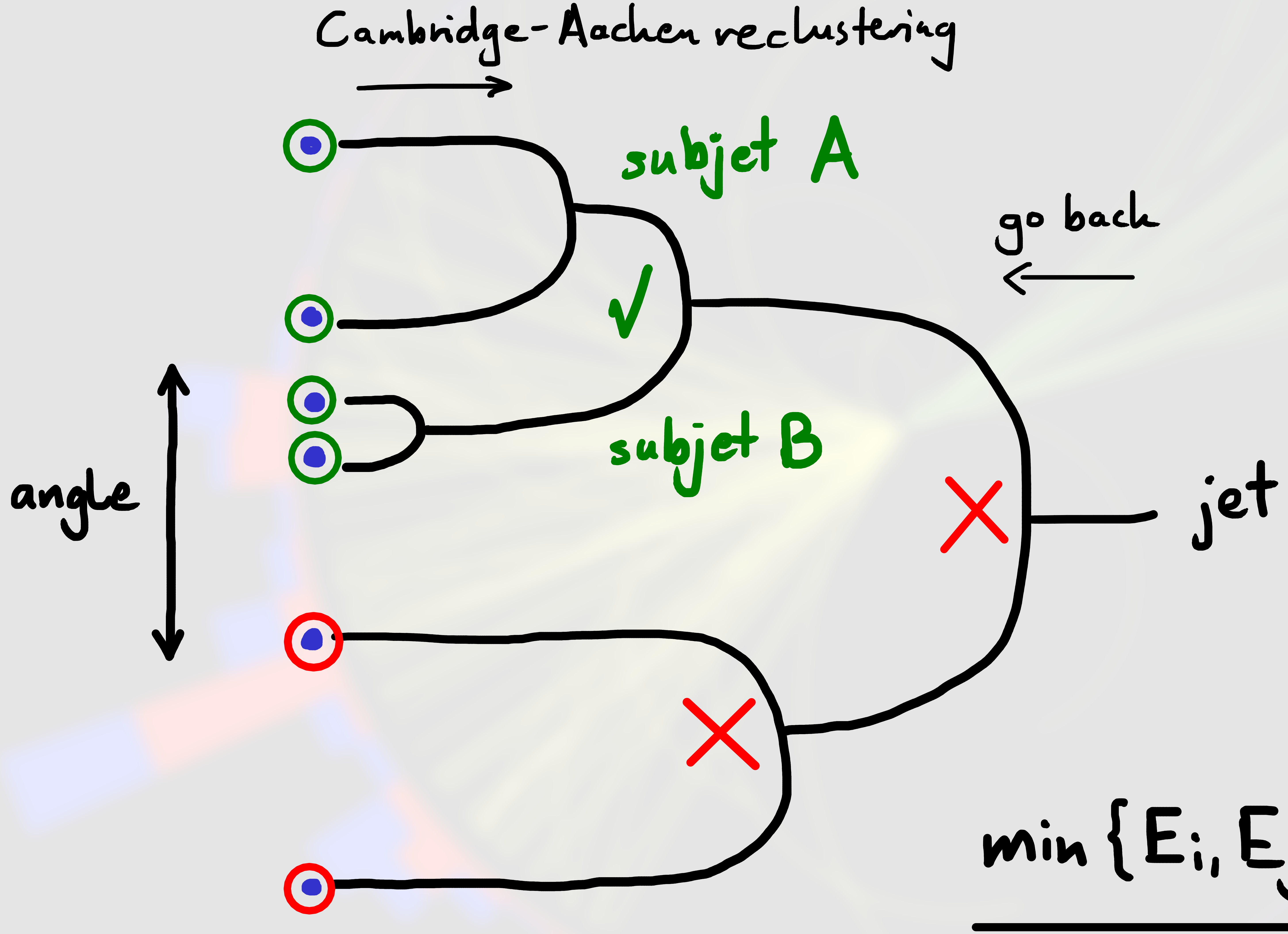


N.B.: Clustering history is mostly a tool to give structure to event record.

↪ Only tells you what "really" happened in the strongly ordered limit ↔ ignore QM.

Sketch credits: Johannes Michel

Soft drop algorithm



N.B.: Clustering history is mostly a tool to give structure to event record.
↳ Only tells you what "really" happened in the strongly ordered limit ↔ ignore QM.

$$\frac{\min\{E_i, E_j\}}{E_i + E_j} > \tilde{z}_{cut}\, \vartheta_{ij}^{\beta}$$

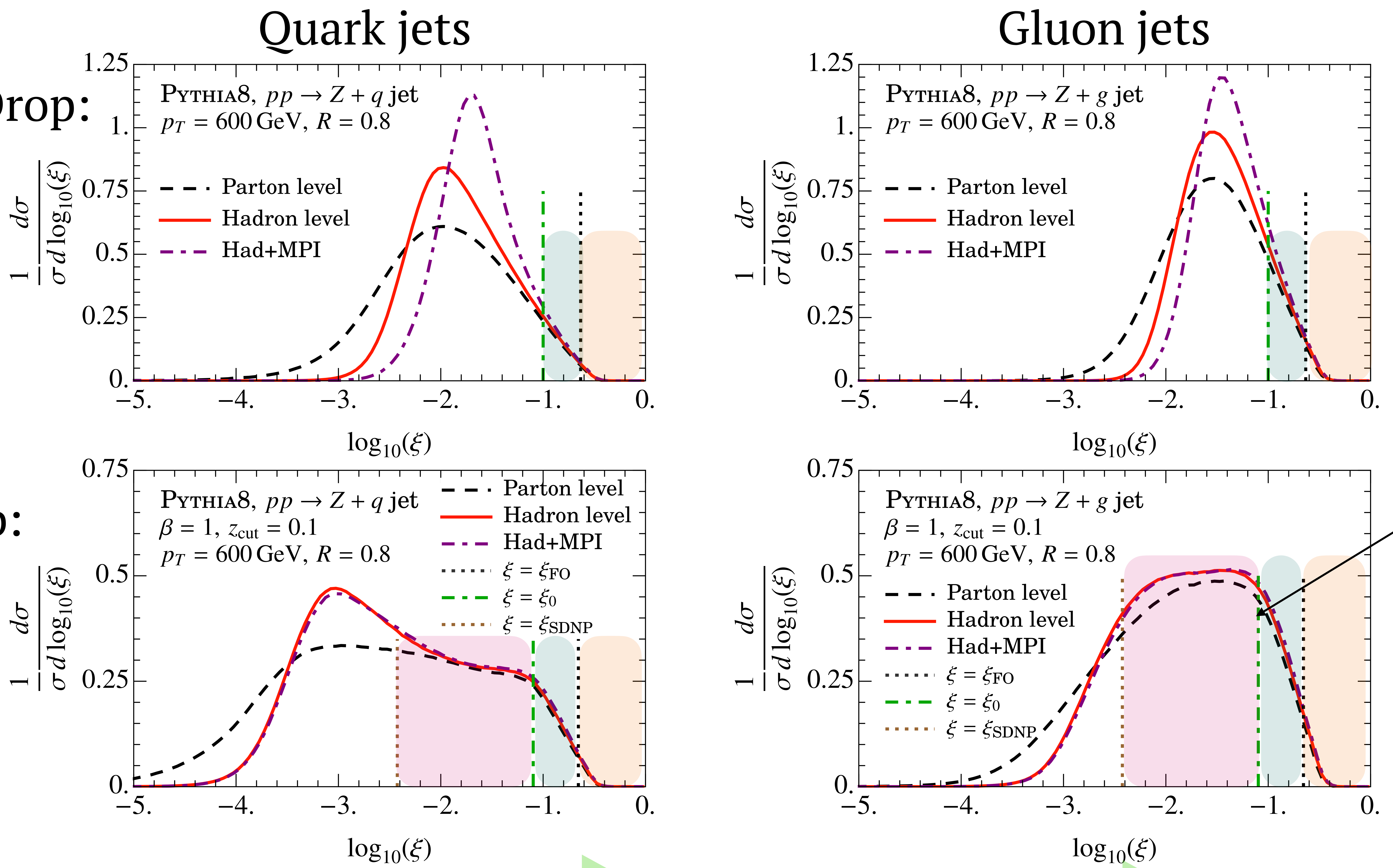
✓ / ✗

Sketch credits: Johannes Michel

Soft drop jet mass

No Soft Drop:

Soft Drop:



$$\xi = \frac{m_J^2}{p_T^2 R^2}$$

$$\xi_0 = z_{\text{cut}} R^\beta$$

Soft drop resummation region → Ungroomed region → Fixed order region

Inclusive jets

For *pp* collisions work with inclusive jets: $pp \to j_\kappa + X$

Semi inclusive jet function

$$\frac{d\sigma}{d\eta_J dp_T d\tau} = \sum_{abc} \int \frac{\mathrm{d}x_a \mathrm{d}x_b \mathrm{d}z}{x_a x_b z} f_a(x_a,\mu) f_b(x_b,\mu) H^c_{ab}\left(x_a, x_b, \eta, \frac{p_T}{z}, \mu\right) \mathcal{G}_c(z, \tau, p_T R, \mu, \ldots)$$

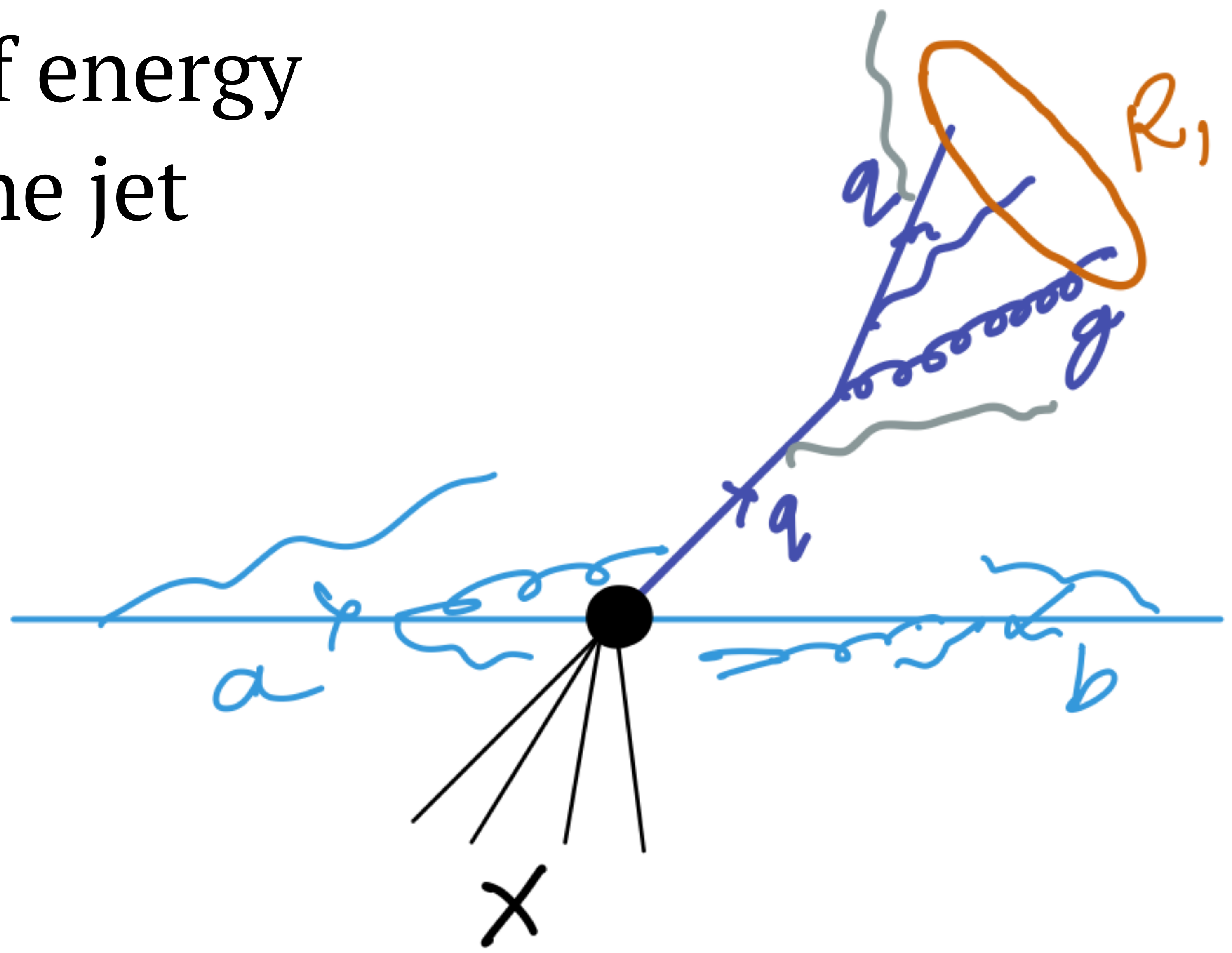
Kang, Ringer, Vitev 1606.07063

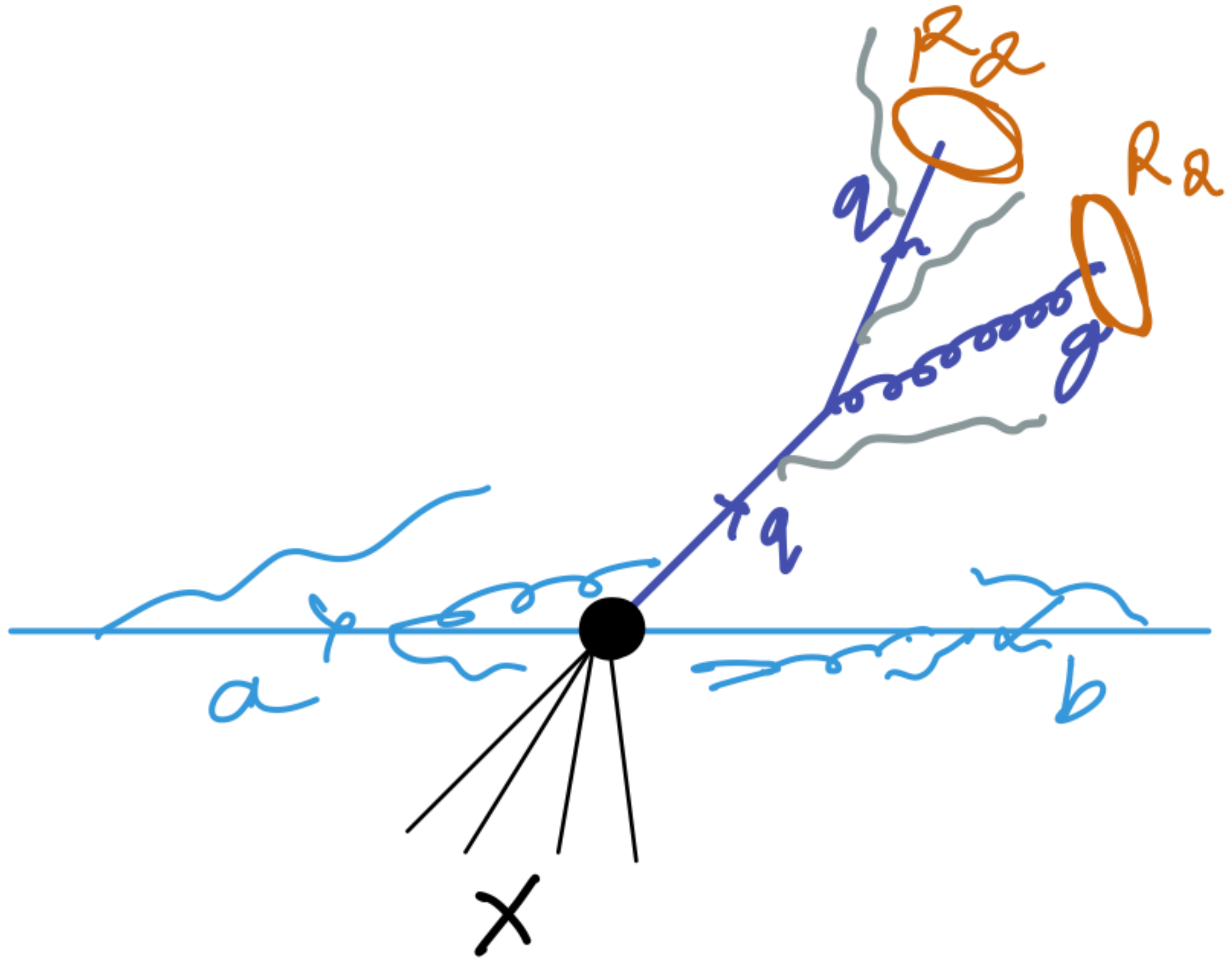
DGLAP splitting

Measurement of τ on the jet

$$\mathcal{G}_c\left(z, \tau, p_T R, \mu, \ldots\right) = \sum_\kappa J_{c\kappa}(z, p_T R, \mu) \mathcal{G}_\kappa\left(\tau, \alpha_s(\mu), \ldots\right) + \mathcal{O}(\alpha_s^2)$$

z = fraction of energy retained by the jet





Ungroomed resummation region

Ungroomed jet mass factorization (ignoring NGLs) for $\xi \ll 1$

$$\tilde{\mathcal{G}}_\kappa^{\text{no sd}}(\xi, \alpha_s(\mu)) = N_{\text{incl}}^\kappa(Q_R, \mu) \times \int_0^\xi \mathrm{d}y\, J_\kappa\big(Q_R^2(\xi - y), \mu\big) S_{\text{plain}}^\kappa\big(Q_R y, \mu\big) \times \Big[1 + \mathcal{O}(\xi)\Big]$$

$$\zeta = \frac{R}{2\cosh \eta_J}$$

$$Q_R = p_T R$$

$$\xi = \frac{m_J^2}{p_T^2 R^2} = \frac{m_J^2}{Q_R^2}$$

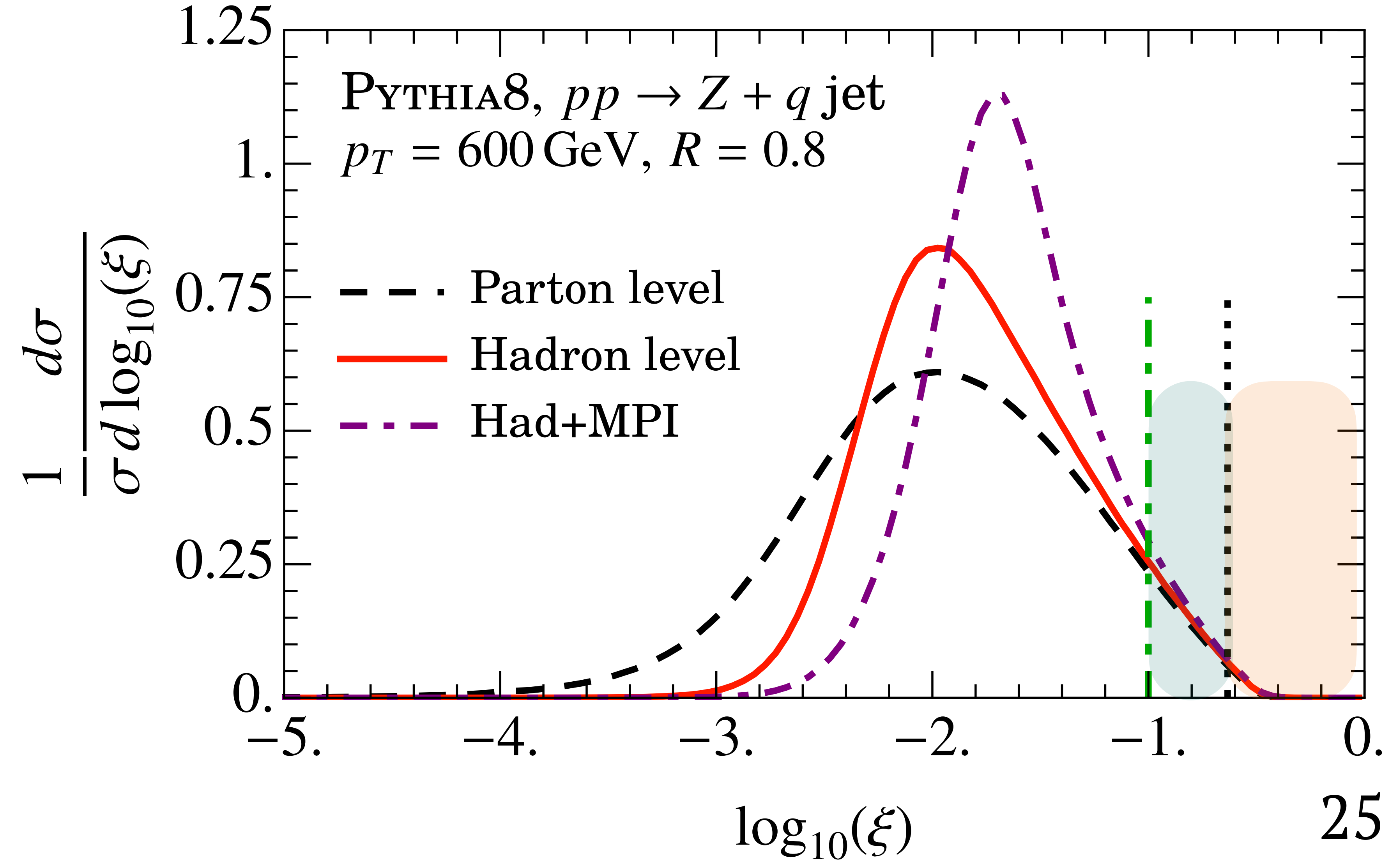
Include factors of jet radius in light cone decomposition:

$$q^\mu = q^+ \zeta \frac{\bar{n}^\mu}{2} + \frac{q^-}{\zeta}\frac{n^\mu}{2} + q_\perp^\mu$$

$$N_\kappa : \qquad \mu \sim Q_R$$

$$J_\kappa : \qquad \mu \sim Q_R\sqrt{\xi}$$

$$S_{\text{plain}}^\kappa : \qquad \mu \sim Q_R \xi$$



Soft drop resummation region

Groomed jet mass factorization for $\xi \ll \xi_0 \ll 1$

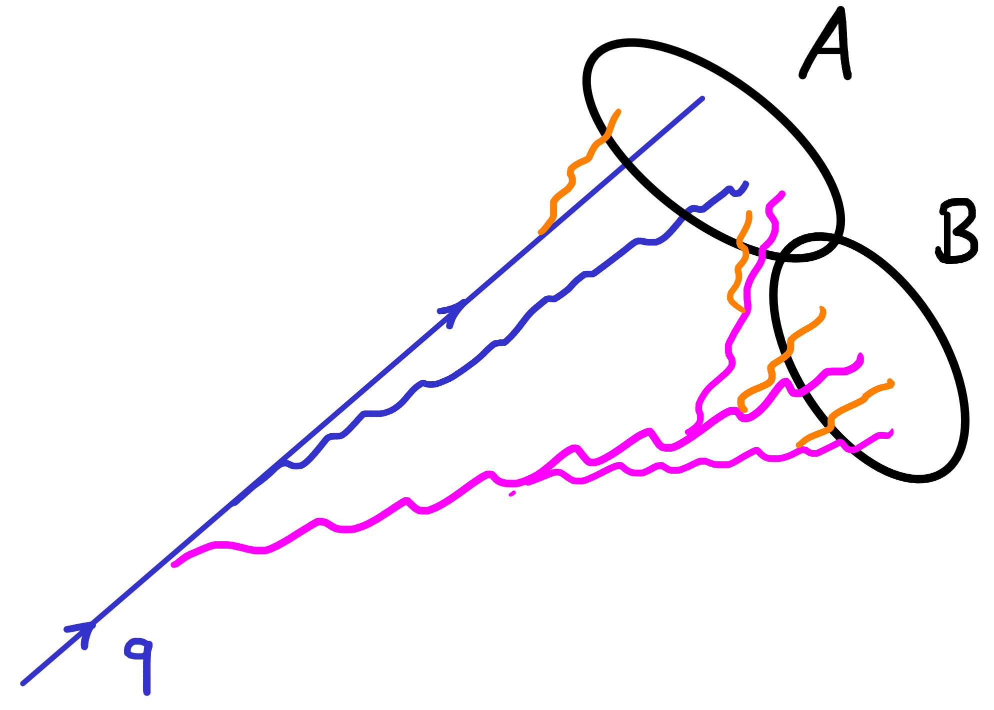
$$\tilde{\mathcal{G}}^{\rm sd}_\kappa(\xi,\alpha_s(\mu)) = N^\kappa_{\rm incl}(Q_R,\mu) S^\kappa_G\big(Q_R\xi_0,\beta,\zeta,\mu\big)\mathcal{S}^\kappa_{\rm NGL}(t_{gs}) \times \int_0^\xi {\rm d}y\, J_\kappa\big(Q_R^2(\xi-y),\mu\big)\, S^\kappa_c\Big(yQ_R(Q_R\xi_0)^{\frac{1}{1+\beta}},\beta,\mu\Big) \times \left[1+\mathcal{O}\left(\zeta^2\left(\frac{\xi}{\xi_0}\right)^{\frac{2}{2+\beta}}\right)\right]$$

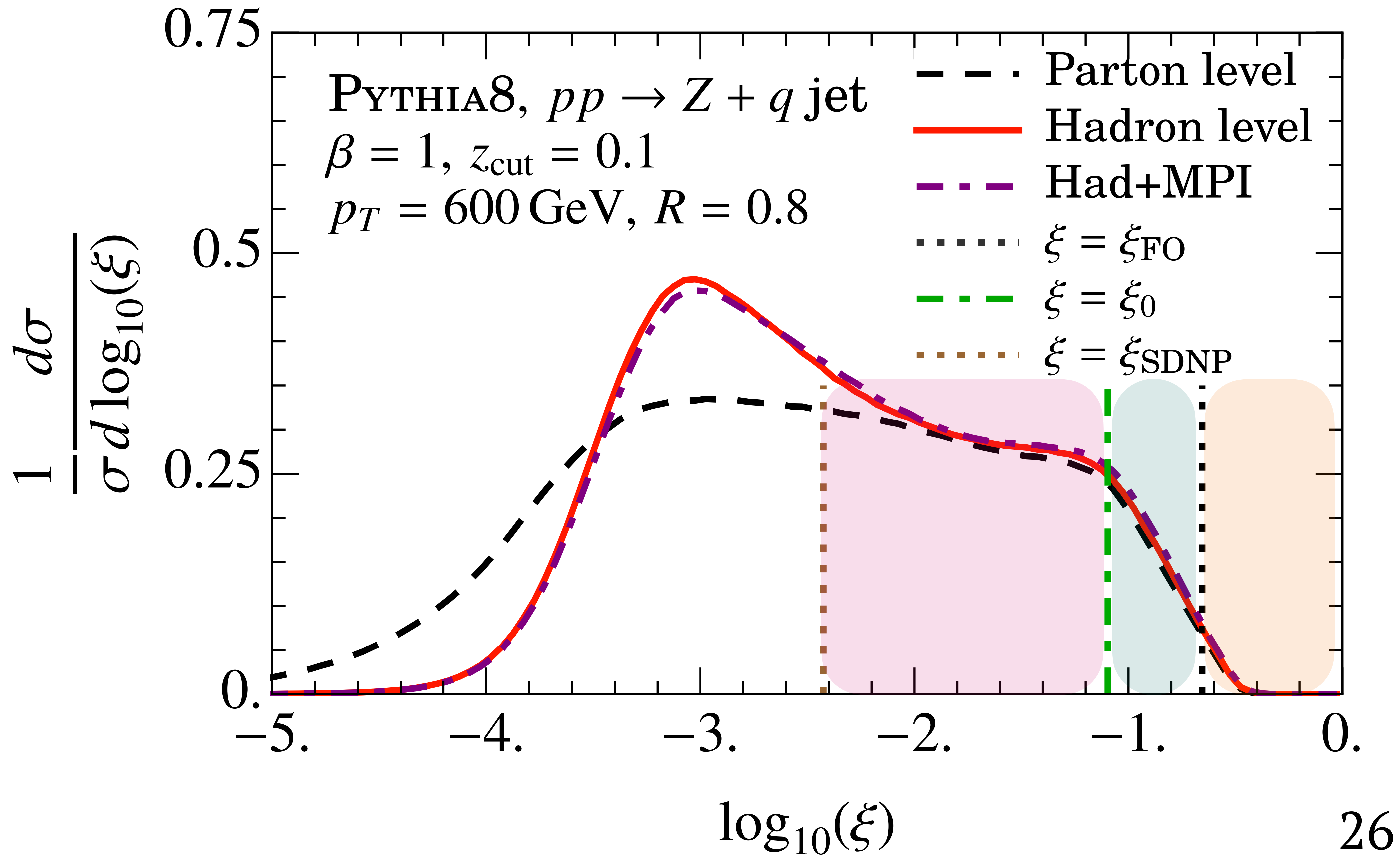
N_κ : $\mu \sim Q_R$

S^κ_G : $\mu \sim Q_R\xi_0$

J_κ : $\mu \sim Q_R\sqrt{\xi}$

S^κ_c : $\mu \sim Q_R \xi^{\frac{1+\beta}{2+\beta}} \xi_0^{\frac{1}{2+\beta}}$





$$\xi = \frac{m_J^2}{p_T^2 R^2} \qquad Q_R = p_T R \qquad \xi_0 = z_{\rm cut} R^\beta \qquad \zeta = \frac{R}{2\cosh\eta_J}$$

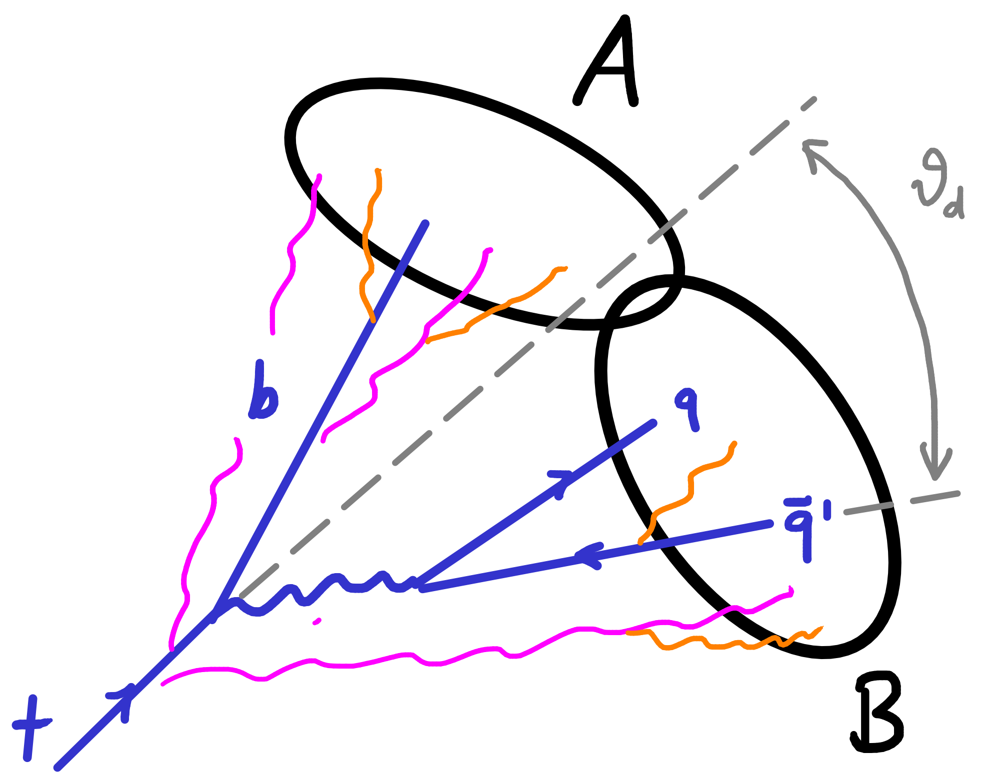
Factorization for top jets

Factorization for top jets, $\xi \ll \xi_0 \ll 1$

$$\xi = \frac{M_J^2 - m_t^2}{Q_R^2} = \frac{m_t \hat{s}_t}{Q_R^2}$$

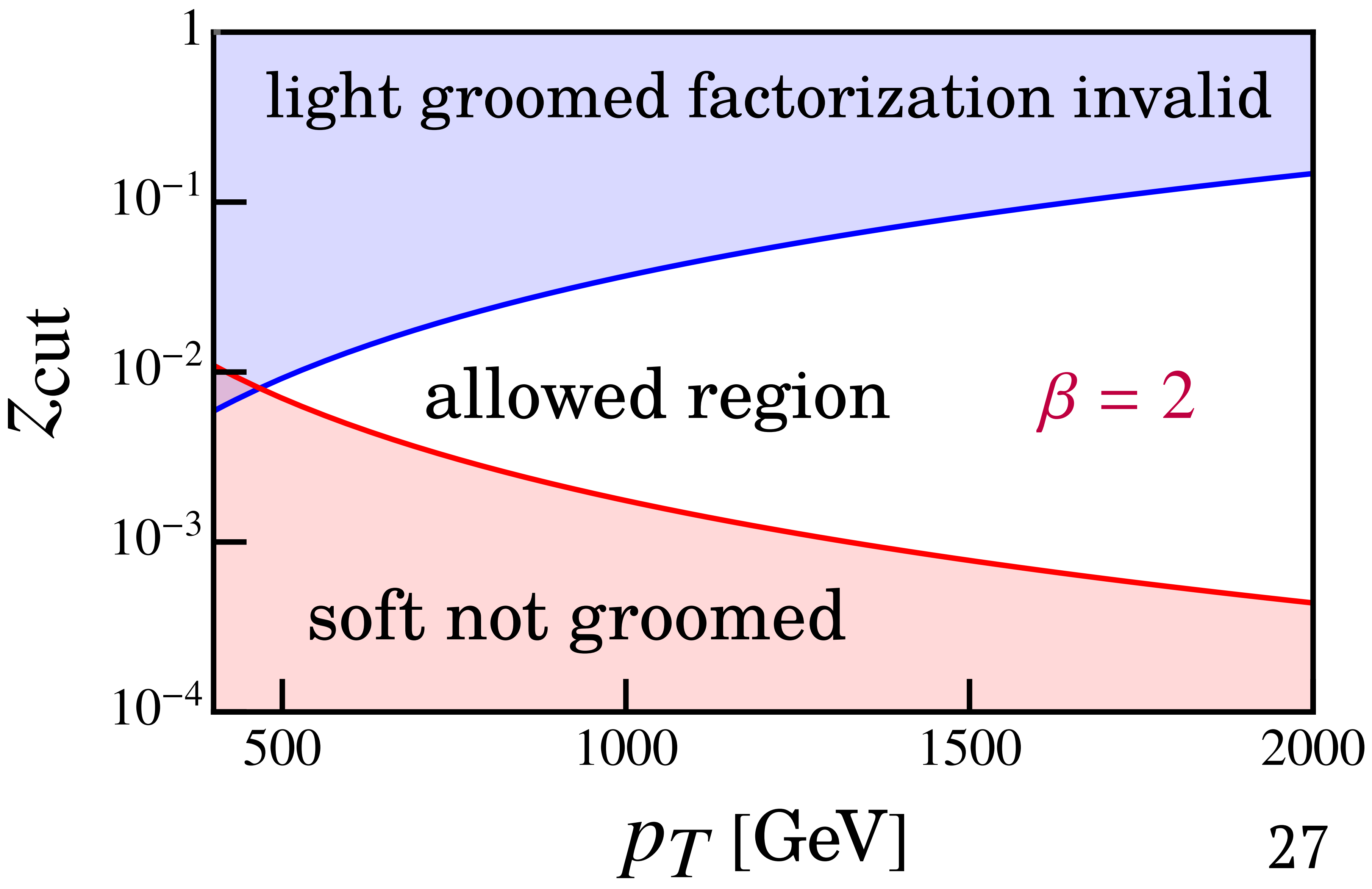
$$\tilde{\mathcal{G}}_t^{\rm sd}(\xi, \alpha_s(\mu)) = N_{\rm incl}^q(Q_R, \mu) H_m^{\frac{1}{2}}(m_t, \mu) S_G^\kappa\big(Q_R\xi_0, \beta, \zeta, \mu\big) \mathcal{S}_{\rm NGL}^\kappa\big([Q_R\xi_0, Q_R]\big)$$
$$\times \int_0^\xi {\rm d}y\, J_B\left(\frac{Q_R^2(\xi - y)}{m_t}, \Gamma_t, \mu\right) S_c^{(d)}\left(yQ_R(Q_R\xi_0)^{\frac{1}{1+\beta}}, \theta_d, \beta, \mu\right)$$

Need decay products collimated enough to treat top as a single Wilson line



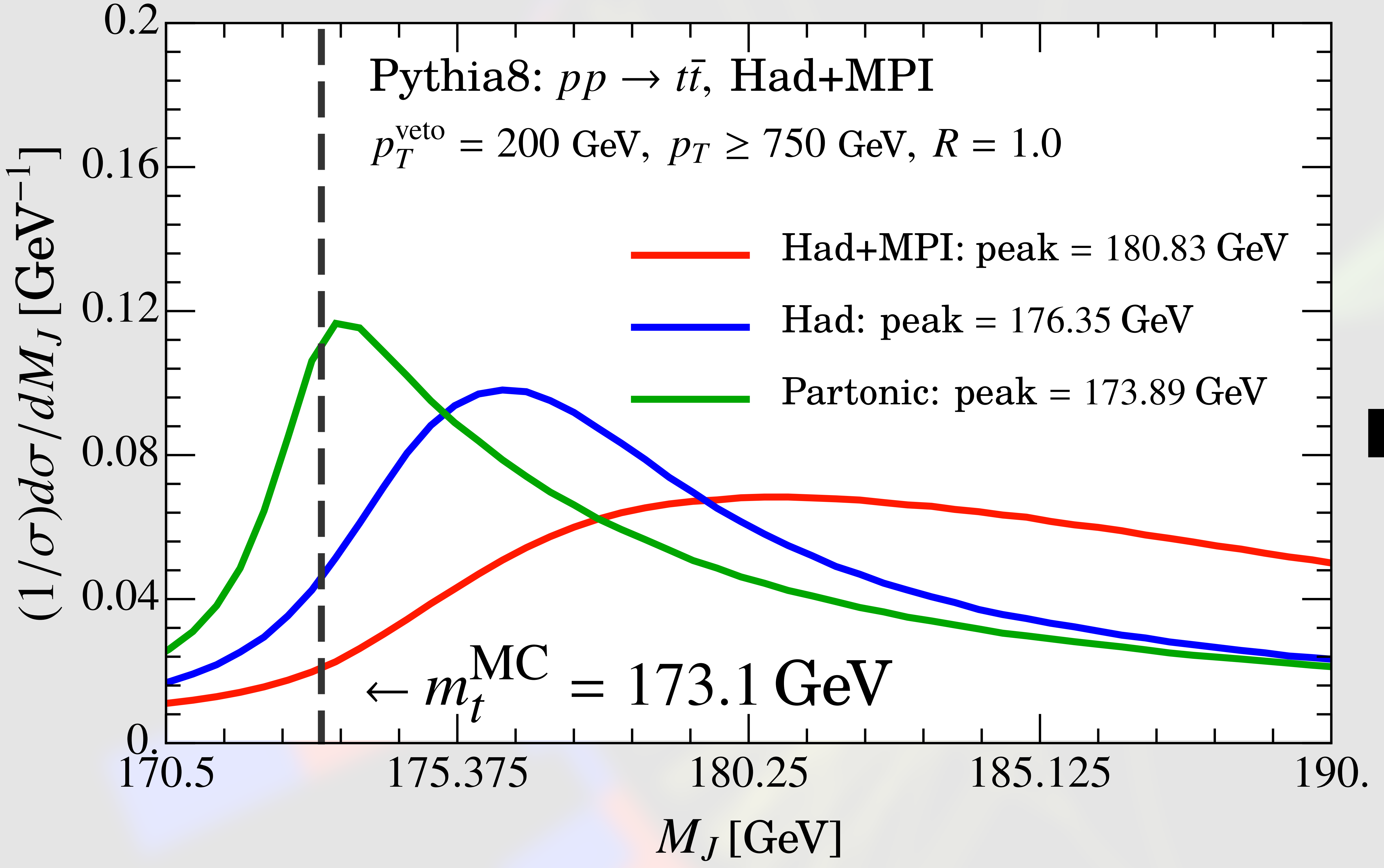
$$\frac{R_d}{R} = \frac{m_t}{Q_R} h \lesssim \frac{\langle R_g \rangle(\xi)}{R} \simeq \left(\frac{\xi}{\xi_0}\right)^{\frac{1}{2+\beta}} \ll 1$$

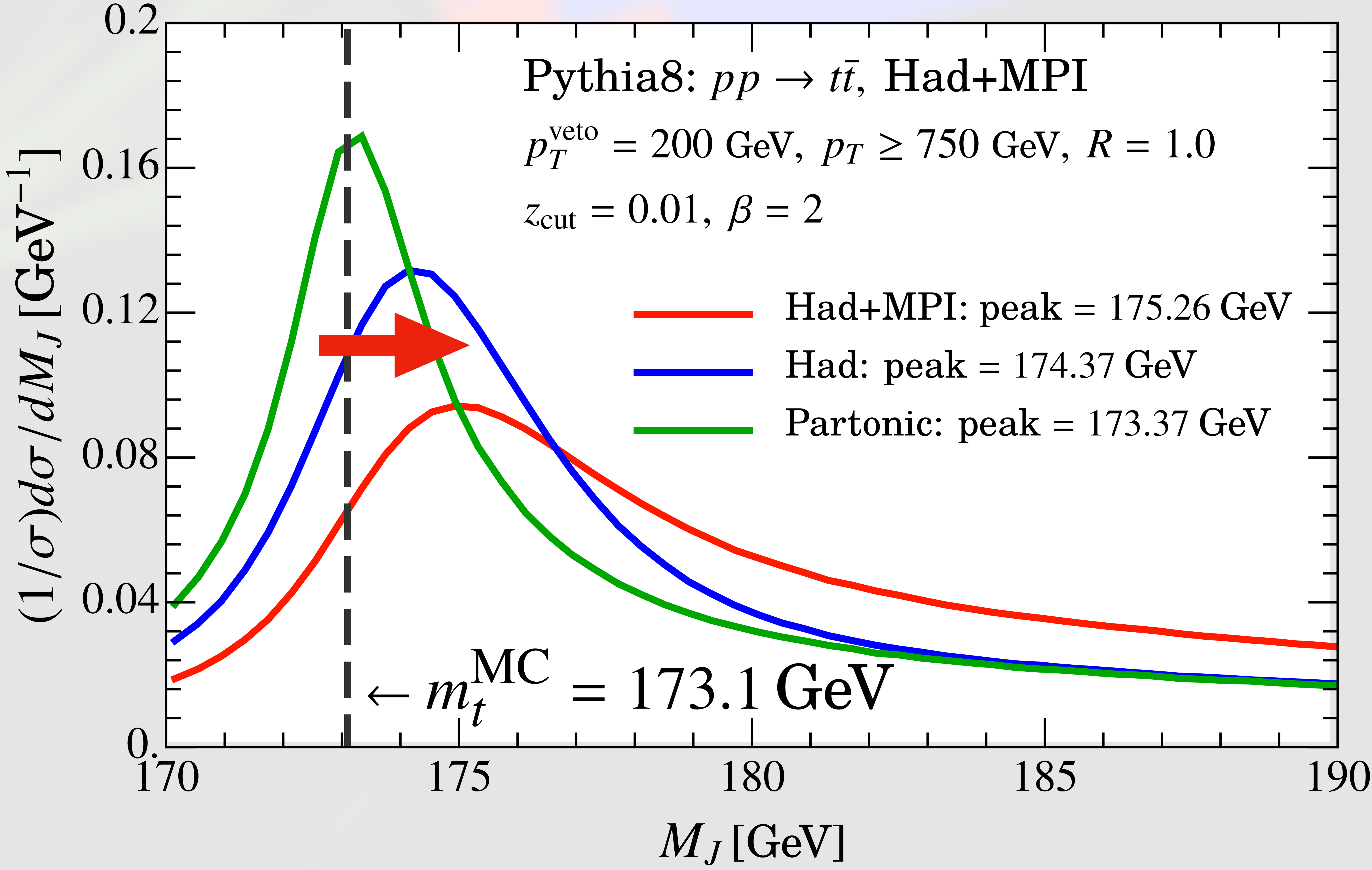
$h \sim 2$



Soft drop jet mass

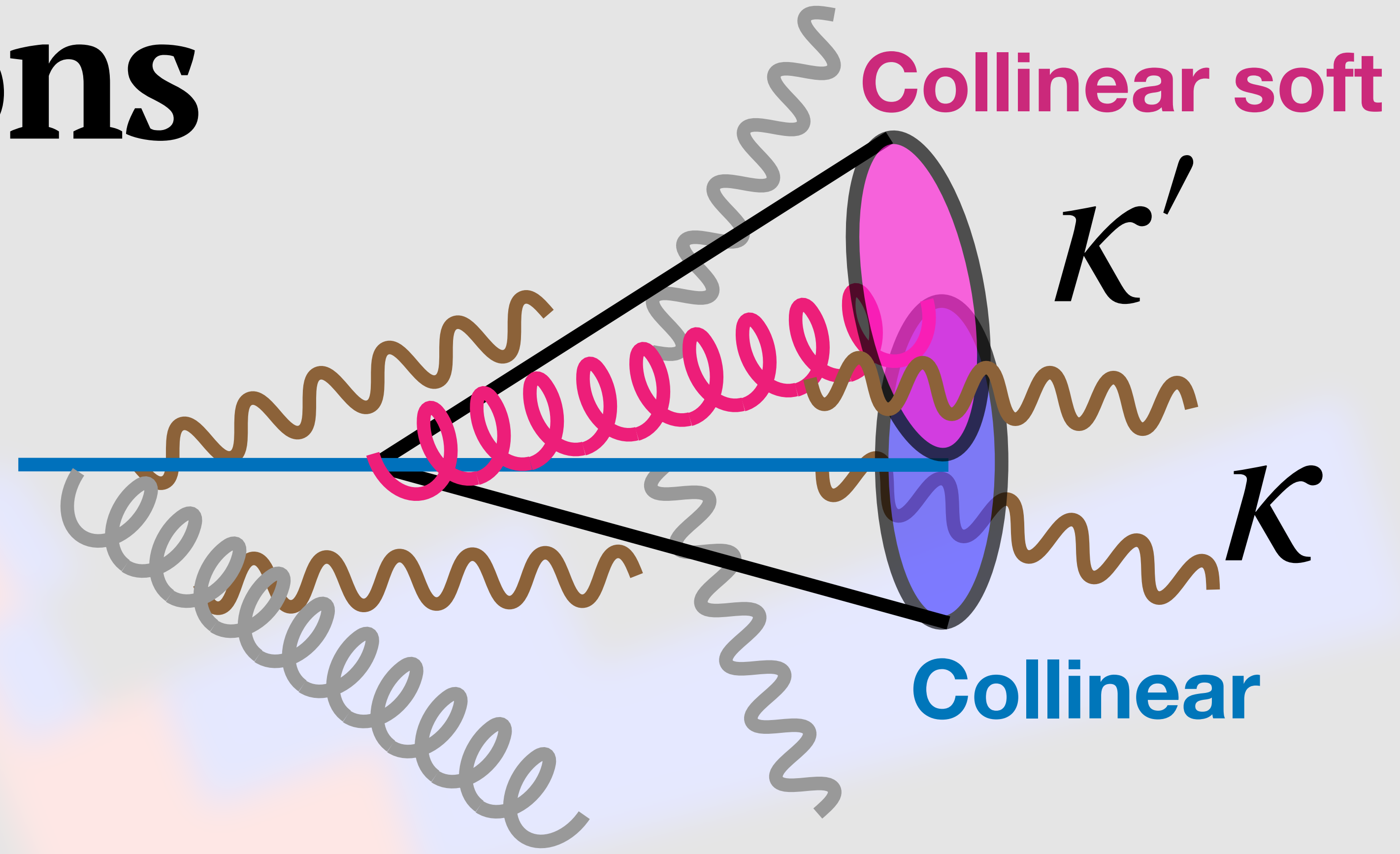
Soft drop improves resilience of the peak against hadronization and the UE



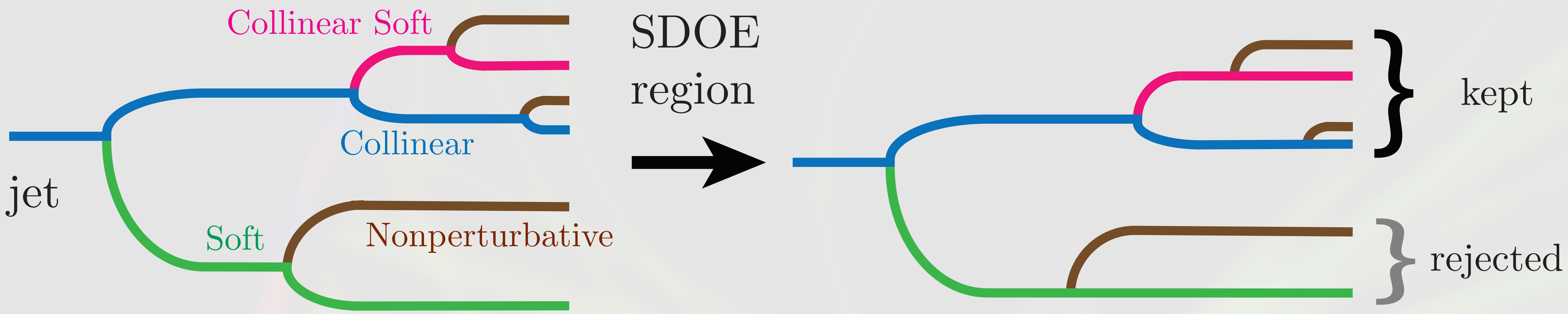


Still need to account for residual shifts

Hadronization corrections



To describe the hadronization corrections we looked closely into the effects of clustering and two-pronged geometry of the groomed jet



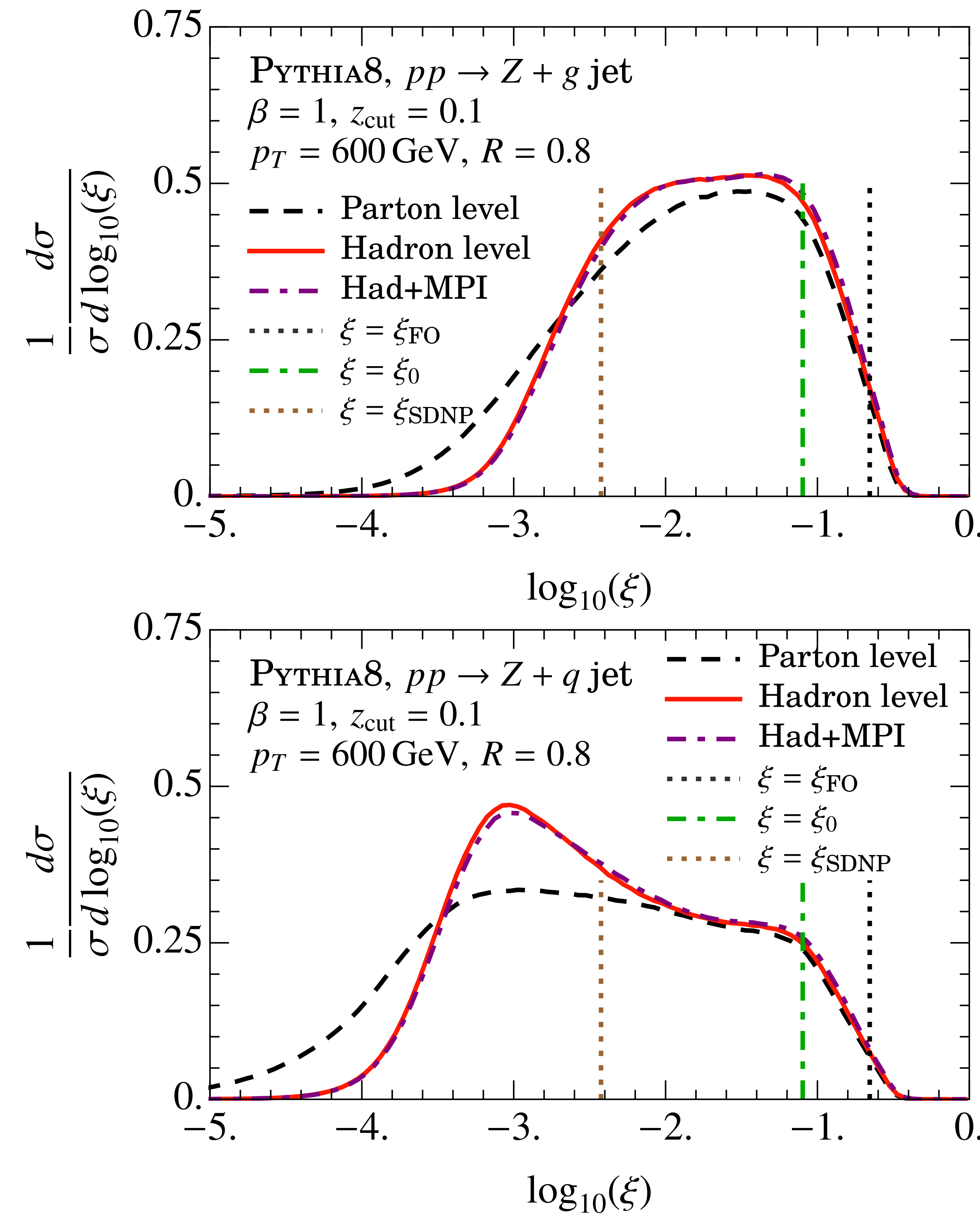
Hoang, AP, Mantry, Stewart 1906.11843; AP, Stewart, Vaidya, Zoppi 2012.15568

Factorization of NP corrections:

$$\frac{d\sigma_\kappa^{\rm had}}{d\xi} = \frac{d\hat\sigma_\kappa}{d\xi} - \frac{\Omega^{\infty}_{1\kappa\kappa'}}{Q_R}\frac{d}{d\xi}\left(C_1^\kappa(\xi)\frac{d\hat\sigma_\kappa}{d\xi}\right) + \frac{\Upsilon^{\kappa\kappa'}_{1,0} + \beta\Upsilon^{\kappa\kappa'}_{1,1}}{Q_R}\frac{C_2^\kappa(\xi)}{\xi}\frac{d\hat\sigma_\kappa}{d\xi}$$

NP corrections governed by 3 universal constants and 2 perturbative coefficients

$$C_1^\kappa(\xi) = \frac{1}{\langle 1\rangle(\xi)}\left\langle \frac{R_g}{R}\right\rangle, \qquad C_2^\kappa(\xi) = \frac{\xi}{\langle 1\rangle(\xi)}\left\langle \frac{R}{R_g}\delta(z_g - z'_{\rm cut}R_g^\beta)\right\rangle,$$



Hadronization corrections

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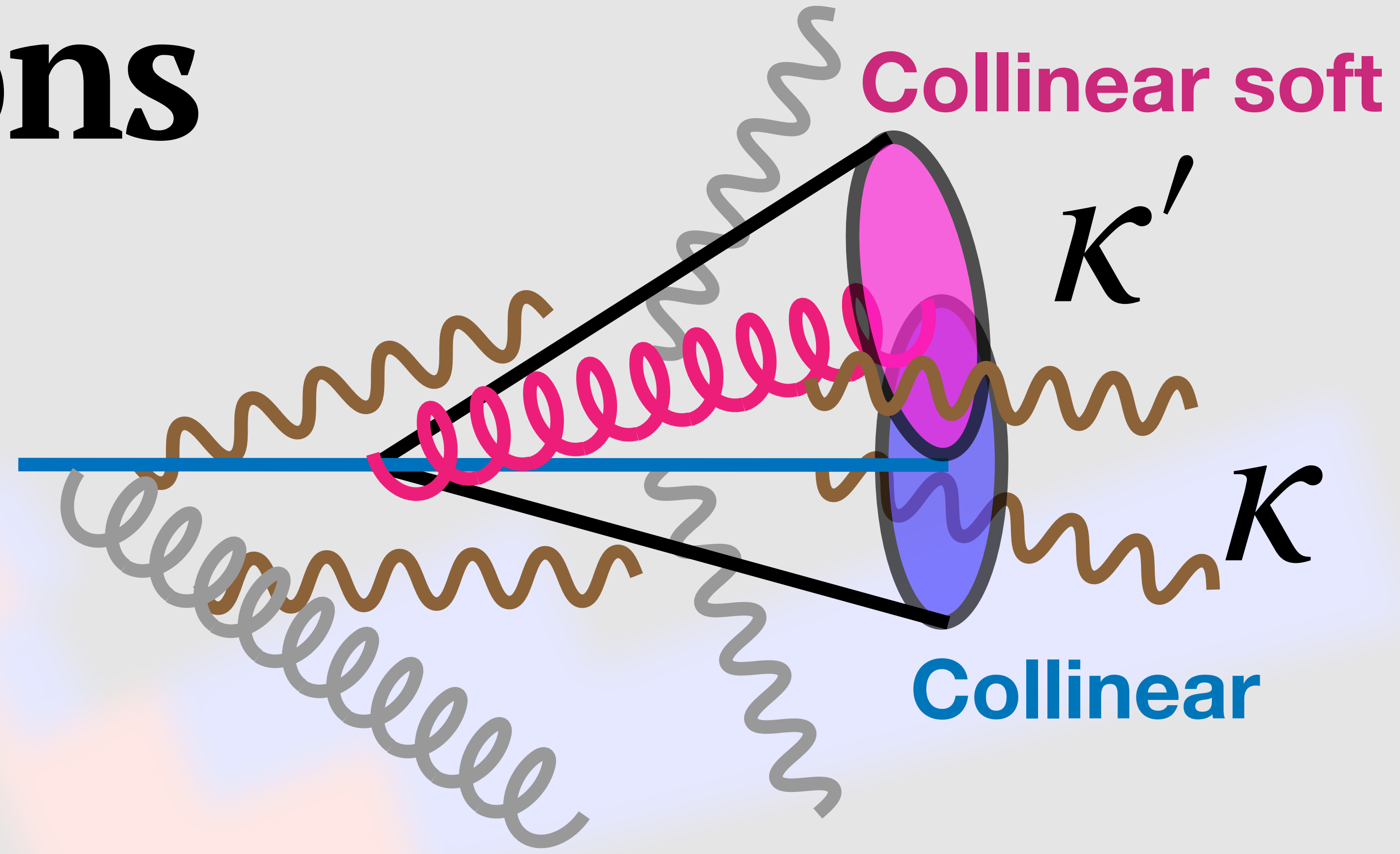
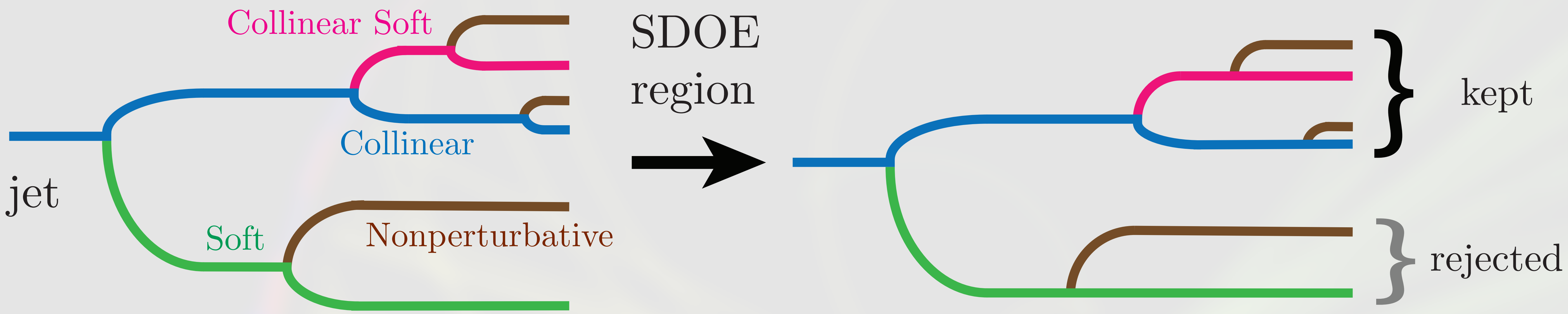
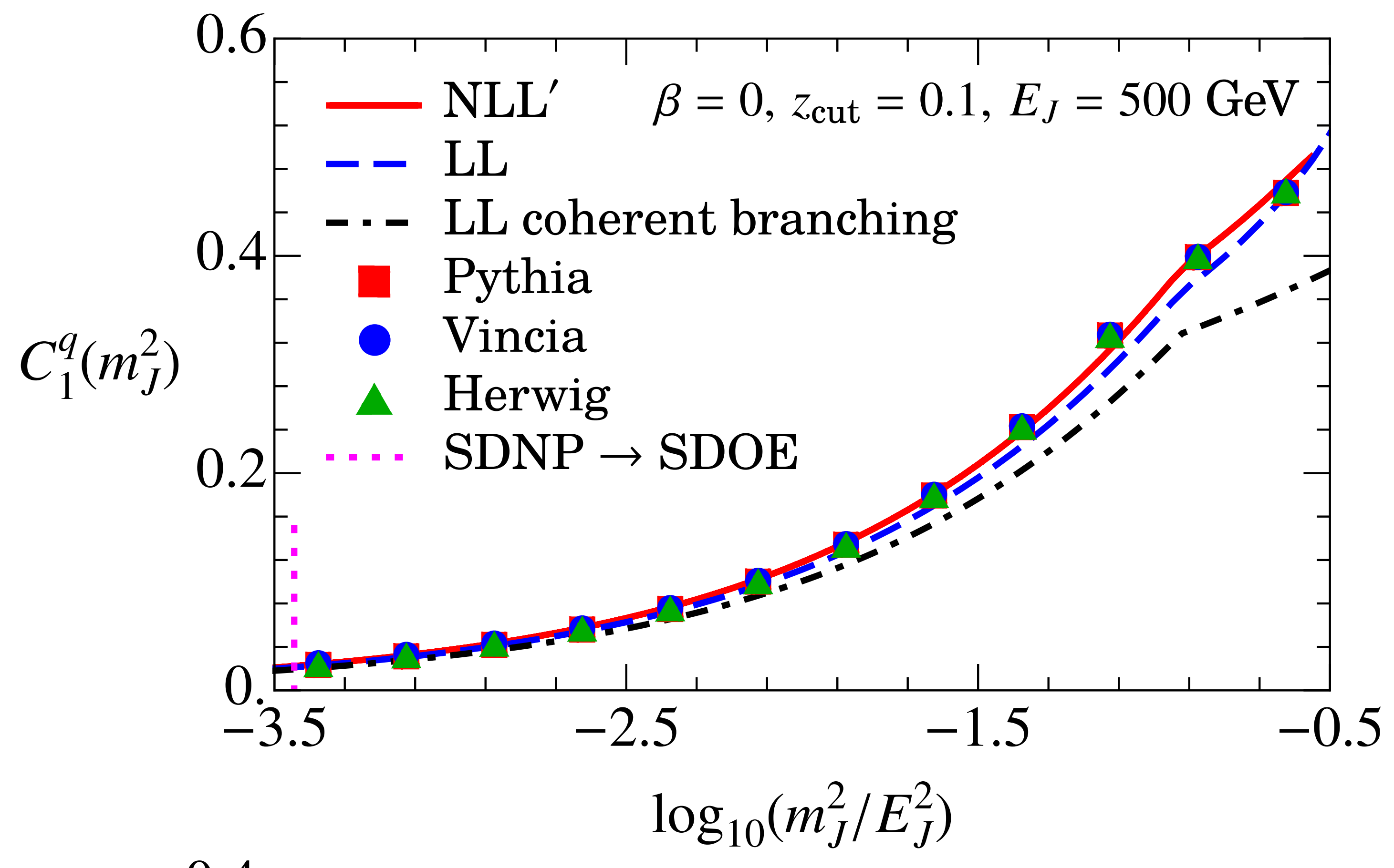




Hoang, AP, Mantry, Stewart 1906.11843; AP, Stewart, Vaidya, Zoppi 2012.15568



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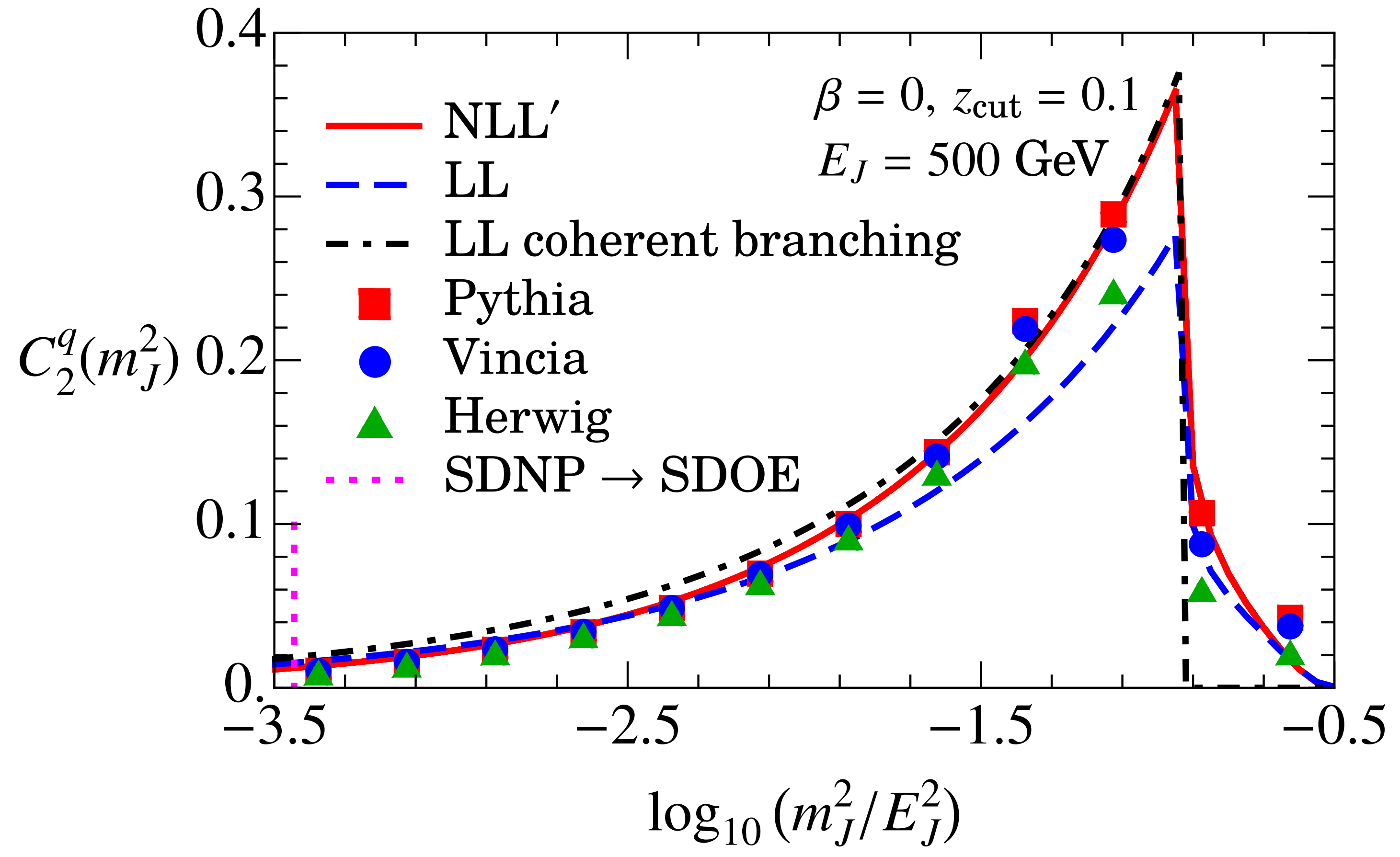
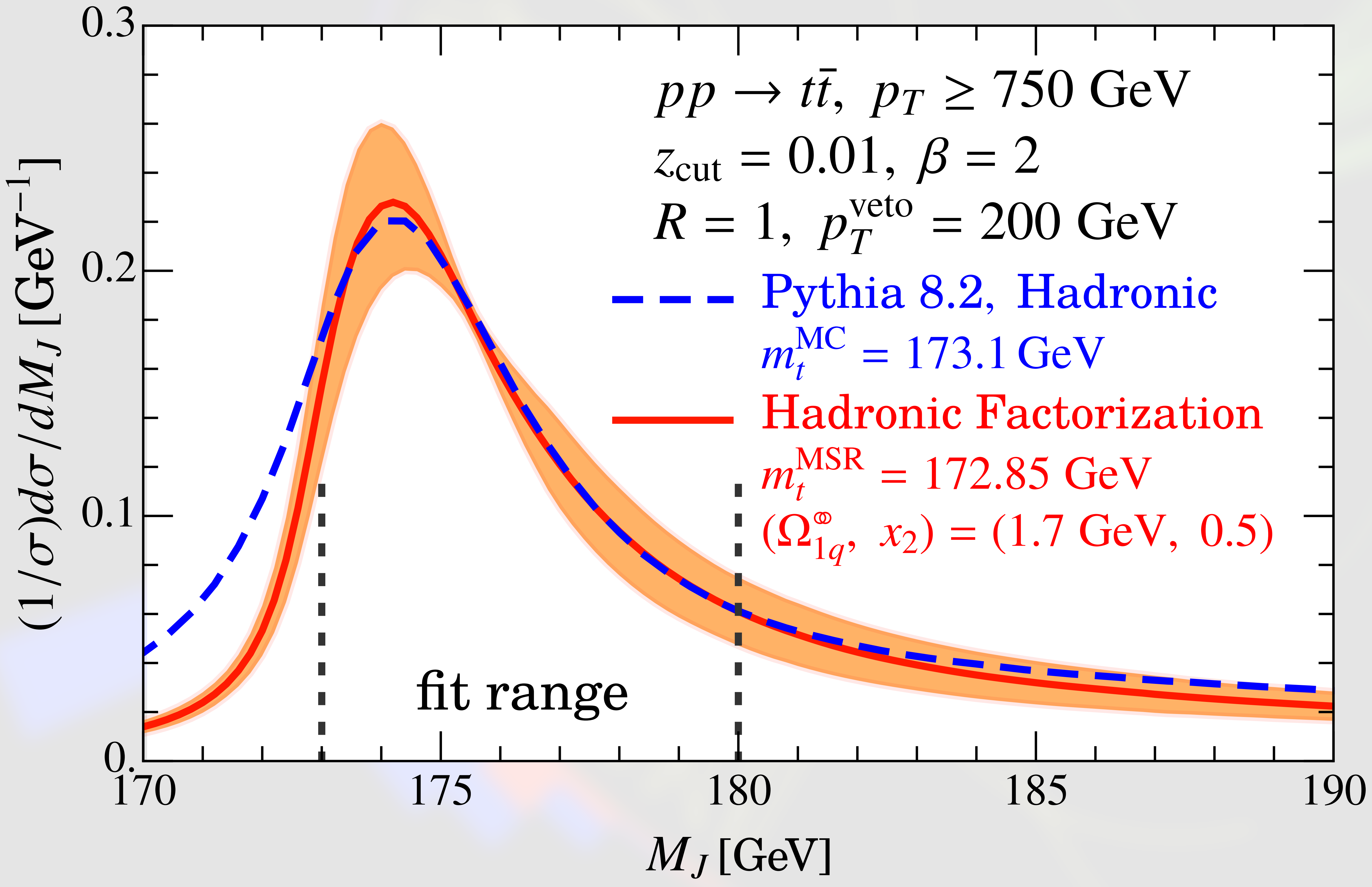


To calculate $C_1^\kappa(\xi)$ and $C_2^\kappa(\xi)$ one considers the cross section doubly differential in jet mass ξ and groomed jet radius R_g

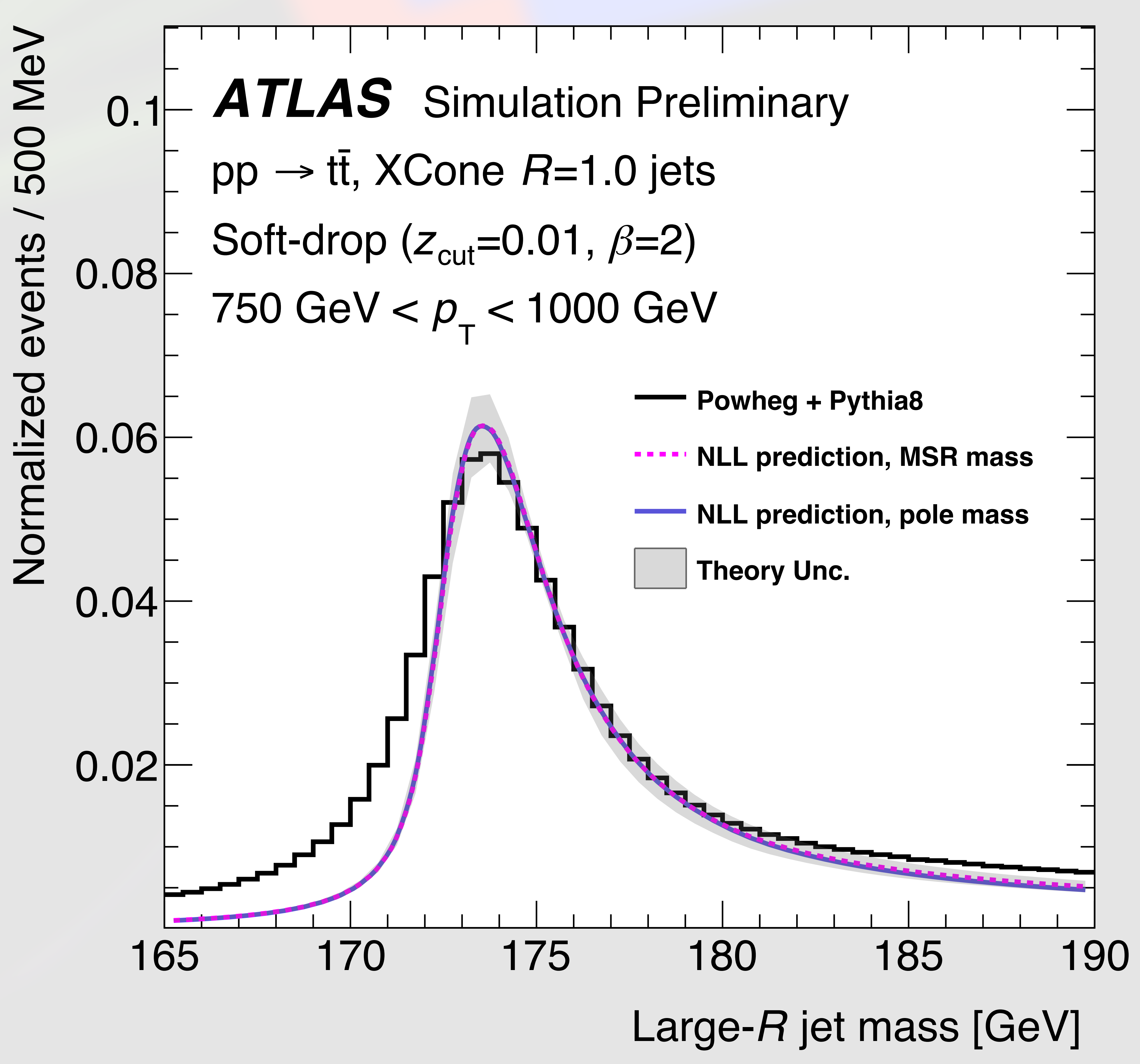
Calibration of Monte Carlo Top Mass

Comparing theory prediction with MC simulations enable m_t^{MC} calibration

Hoang, Mantry, AP, Stewart 1708.02586;
Hoang, Mantry, Michel, AP, Stewart (soon)



Hoang, Mantry, AP, Stewart and
ATLAS, ATL-PHYS-PUB-2021-034



Simultaneously fit for m_t and Ω_1°

Calibration of Monte Carlo Top Mass

Uncertainty breakdown:

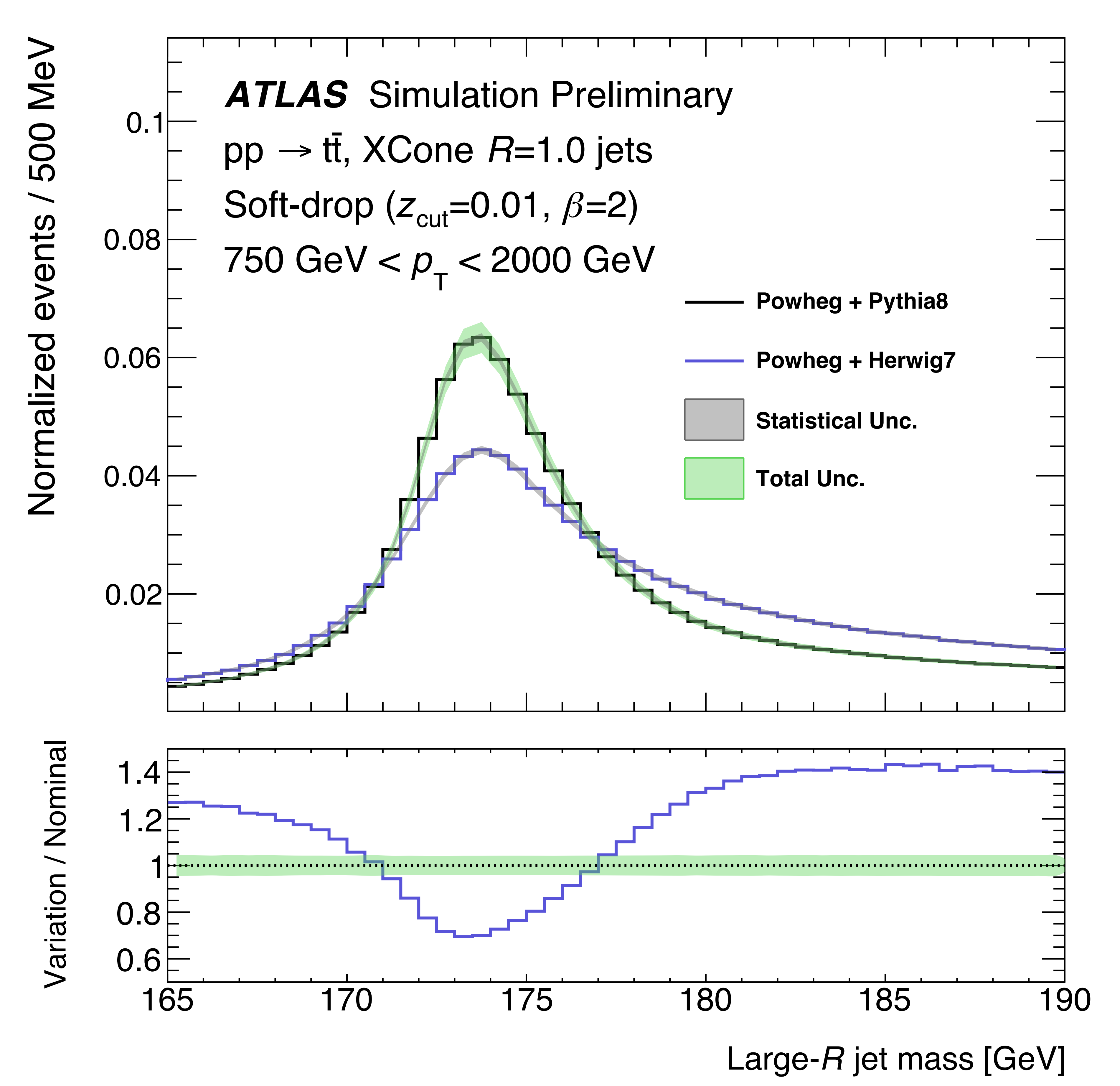
Source of Uncertainty	size [MeV]	comment
Theory	+ 230/-310	Envelope of NLL scale variations
Fit methodology	± 190	fit range, p_T bins
UE model	± 155	A14 eigentune variations, CR models
Observable definition	± 200	$z_{\rm cut} = 0.01, 0.005, 0.02,\ \beta = 1, 2,$ Anti-k_t / XCone jets

$$m_t^{\rm MSR,P8}(R = 1\,{\rm GeV}) = 172.42 \pm 0.1\,{\rm GeV}$$

$$m_t^{\rm MSR,H7}(R = 1{\rm GeV}) = 172.27 \pm 0.09\,{\rm GeV}$$

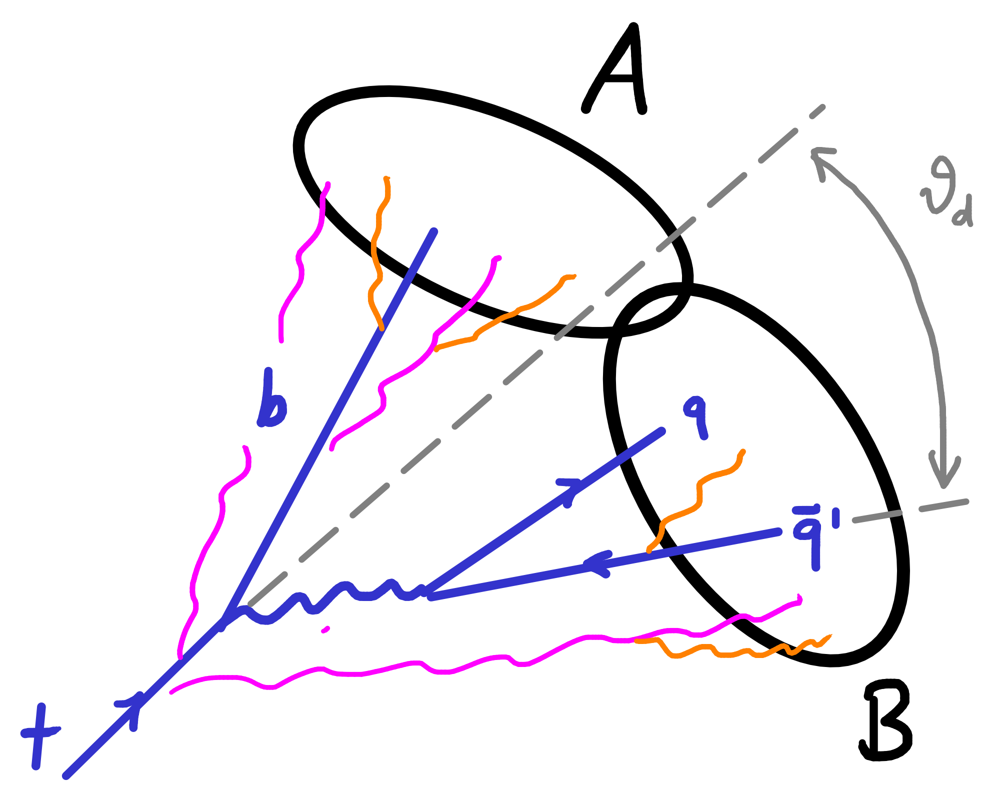
$$\Omega_{1q}^{\infty,\,\rm P8} = 1.49 \pm 0.03\,{\rm GeV}\,, \qquad x_2^{\rm P8} = 0.52 \pm 0.09$$

$$\Omega_{1q}^{\infty,\rm H7} = 1.9 \pm 0.07\,{\rm GeV}\,, \qquad x_2^{\rm H7} = 0.98 \pm 0.12$$



Calibration for Herwig consistent with Pythia despite very different shapes

Extending to NLL$'$



Factorization for tops with soft drop involves careful consideration of the presence of the decay products.

$$\tilde{\mathcal{G}}_t^{\rm sd}(\xi,\alpha_s(\mu)) = N^q_{\rm incl}(Q_R,\mu) H_m^{\frac{1}{2}}(m_t,\mu) S_G^\kappa(Q_R\xi_0,\beta,\zeta,\mu)\mathcal{S}^\kappa_{\rm NGL}([Q_R\xi_0,Q_R]) \times \int_0^\xi {\rm d}y\, J_B\left(\frac{Q_R^2(\xi-y)}{m_t},\Gamma_t,\mu\right) S_c^{(d)}\left(yQ_R(Q_R\xi_0)^{\frac{1}{1+\beta}},\theta_d,\beta,\mu\right)$$

Collinear-Soft function now modified due to the presence of the decay products:

$$\xi = \frac{M_J^2 - m_t^2}{Q_R^2} = \frac{m_t\hat{s}_t}{Q_R^2}$$

$$S_c^{(d)}\left(\xi Q_R(Q_R\xi_0)^{\frac{1}{1+\beta}},\theta_d,\beta,\mu\right) = S_c^q\left(\xi Q_R(Q_R\xi_0)^{\frac{1}{1+\beta}},\beta,\mu\right) + \Delta S_c^{(d)}$$

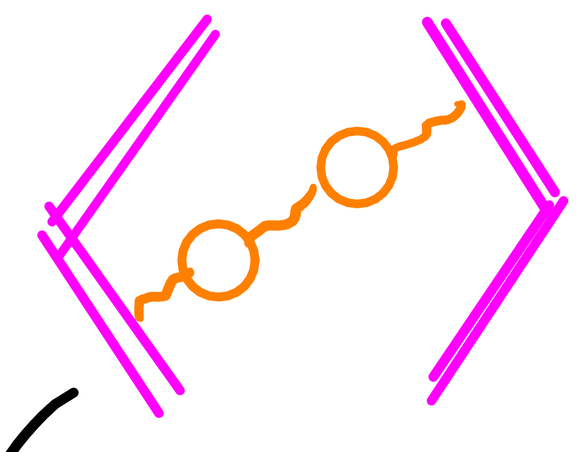
Massless jets — Correction piece

$$\Delta S_c^{(d)} \approx \frac{2\alpha_s C_F}{\pi}\left[\frac{\Theta\left(\psi_d - \psi_g^\star(\xi)\right)}{\xi}\ln\left(\frac{\psi_d}{\psi_g^\star(\xi)}\right)\right]_+^{[\xi_0\psi_d^{2+\beta}]} \qquad \psi_d = \frac{R_d}{R}\,, \qquad \psi_g^\star(\xi) = \left(\frac{\xi}{\xi_0}\right)^{\frac{1}{2+\beta}}$$

This piece corrects for radiation at angles smaller than decay product subjets where the radiation is protected from soft drop grooming.

Renormalon subtraction for massless jets

Renormalon analysis allows us to probe scaling of the power corrections and stabilize perturbative expansion.



csoft function

↳ absorb into shape function by so-called gap subtraction

Moritz Preißer, Ph.D. thesis

$$\frac{\alpha_s}{\pi}\frac{\mathrm{i}}{-k^2-\mathrm{i}0} \mapsto \frac{4}{\beta_0}\frac{\mathrm{i}\tilde{\mu}^{2u}}{(-k^2-\mathrm{i}0)^{1+u}}$$

Massless jets

$$S_{c,\text{bare}}^{\kappa\,(1)}\Big(\ell^+, Q_{\text{cut}}\Big) = \frac{\alpha_s C_\kappa}{\pi}\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}\left[+\mathcal{L}^{-2\epsilon\frac{1+\beta}{2+\beta}}\Big(\ell^+, \mu^{\frac{2+\beta}{1+\beta}}Q_{\text{cut}}^{-\frac{1}{1+\beta}}\Big) - \frac{2+\beta}{2\epsilon(1+\beta)}\delta(\ell^+)\right]$$

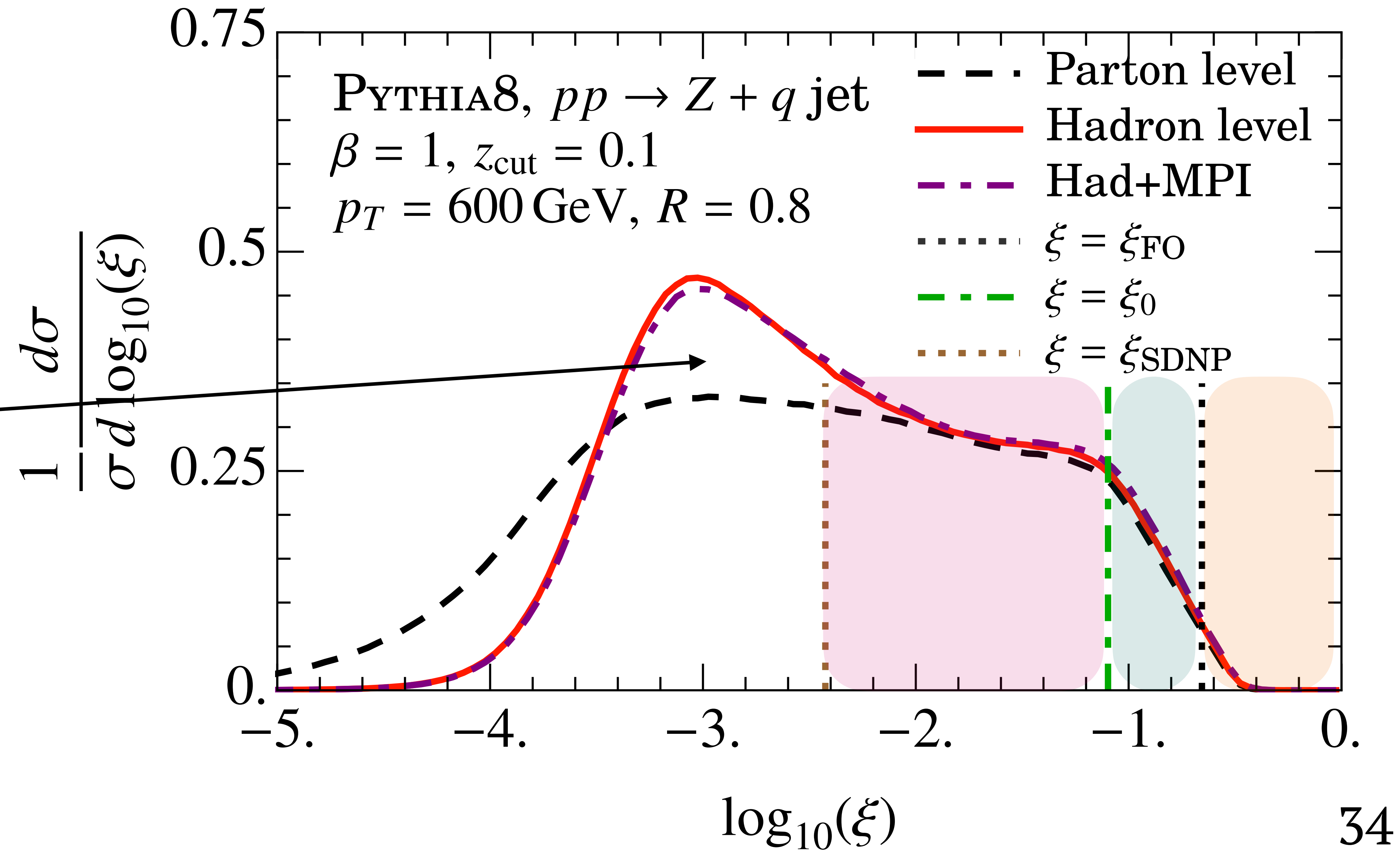
$$B\left[S_c^{\kappa,\text{pert}}\Big(\ell^+, Q_{\text{cut}}, \mu\Big)\right](u)$$

$$= \frac{4C_\kappa}{\beta_0}\frac{\text{sinc}(\pi u)}{u}\left[\mathcal{L}^{-2u\frac{1+\beta}{2+\beta}}\Big(\ell^+, \tilde{\mu}^{\frac{2+\beta}{1+\beta}}Q_{\text{cut}}^{-\frac{1}{1+\beta}}\Big) - \frac{2+\beta}{2u(1+\beta)}\delta(\ell^+)\right]$$

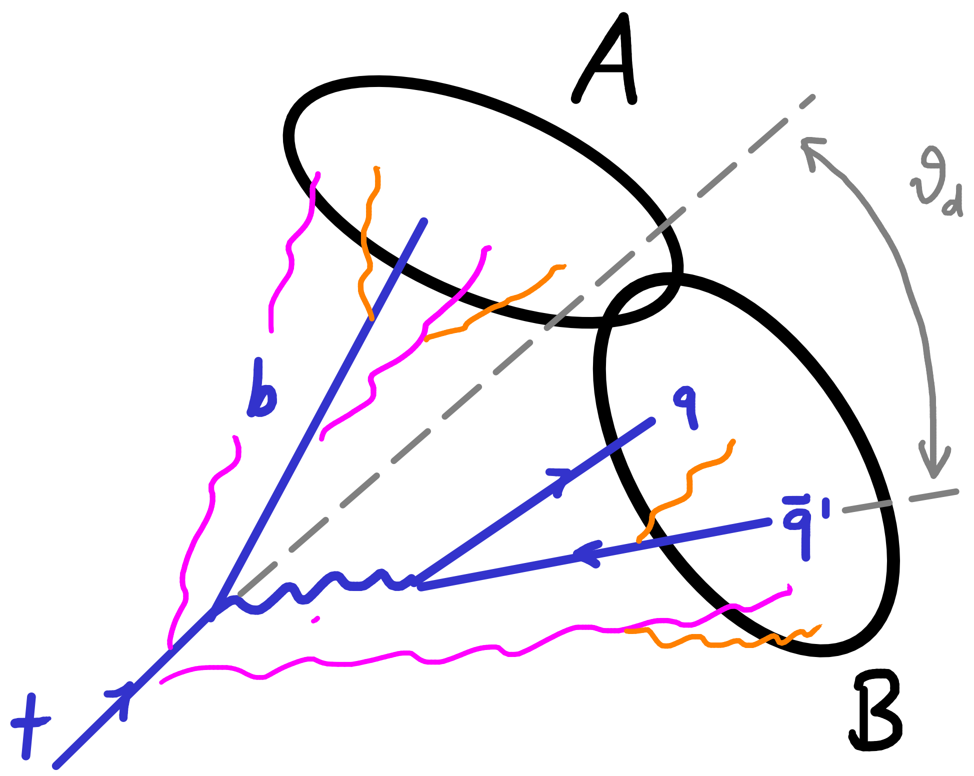
$$a \equiv -2u\frac{1+\beta}{2+\beta} = -1 \quad \Leftrightarrow \quad u = \frac{2+\beta}{2(1+\beta)}$$

This tells us the nature of nonperturbative power corrections in the SDNP region:

$$\Delta\xi_{\text{NP}}^{\text{sd}} \sim \frac{\Lambda_{\text{QCD}}}{Q_R}\left(\frac{\Lambda_{\text{QCD}}}{Q_R\xi_0}\right)^{\frac{1}{1+\beta}}$$



Renormalon subtraction in the $S_c^{(d)}$ function



Presence of the decay products screens the SDNP renormalon!

$$S_c^{(d)}\big(\xi Q_R (Q_R\xi_0)^{\frac{1}{1+\beta}}, \theta_d, \beta, \mu\big) = S_c^q\big(\xi Q_R (Q_R\xi_0)^{\frac{1}{1+\beta}}, \beta, \mu\big) + \Delta S_c^{(d)}$$

Massless jets

Correction piece

The correction piece modifies the NP structure:

$$B\Big[S_c^{(d),\text{pert}}(\ell^+, Q_{\text{cut}}, \beta, \theta_d, \mu)\Big](u) = B\Big[S_{\text{plain}}^{q,\text{pert}}(\ell^+, \theta_d, \mu)\Big](u) + (\text{finite at } u > 0)\,,$$

$$B\Big[S_{\text{plain}}^{q,\text{pert}}(\ell^+, \theta_d, \mu)\Big](u) = \frac{4C_F}{\beta_0}\frac{\text{sinc}(\pi u)}{u}\left[\mathcal{L}^{-2u}\Big(\ell^+, \mu\frac{\theta_d}{2}\Big) - \frac{\delta(\ell^+)}{2u}\right]$$

The $S_c^{(d)}$ has the same $u = 1/2$ renormalon as the ungroomed soft function $S_{\text{plain}}^{(q)}$!

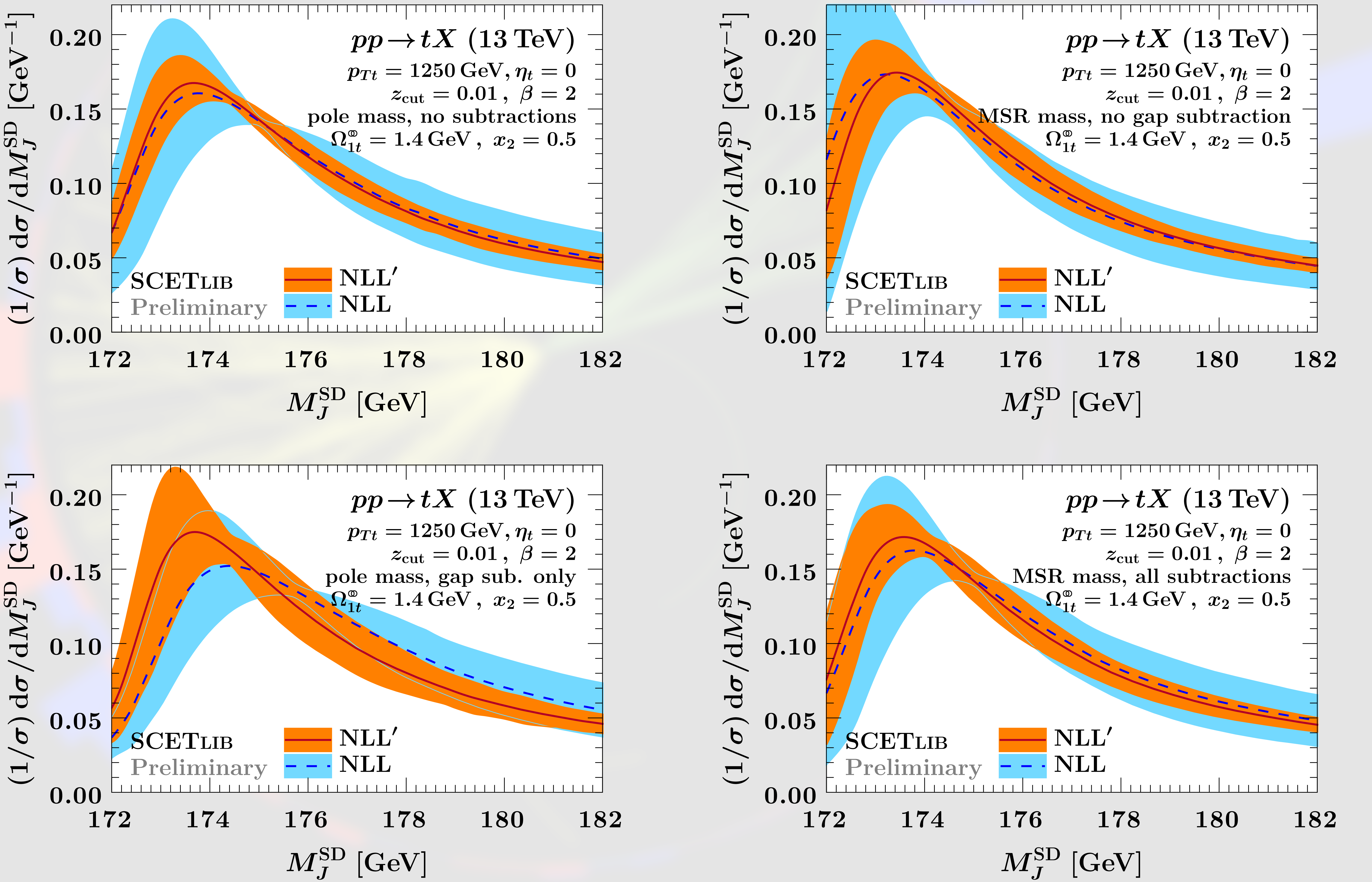
$$\Delta\xi_{\text{NP}}^{\text{sd}} \sim \frac{\Lambda_{\text{QCD}}}{Q}\left(\frac{\Lambda_{\text{QCD}}}{Q_R\xi_0}\right)^{\frac{1}{1+\beta}}$$

$$\Delta\xi_{\text{NP}}^{t} \sim \frac{\Lambda_{\text{QCD}}}{Q_R}$$

$$R_s(\hat{s}_t) = \frac{\hat{s}_t}{h} \qquad \theta_d = \frac{m_t}{Q}h$$

Renormalon subtraction scale

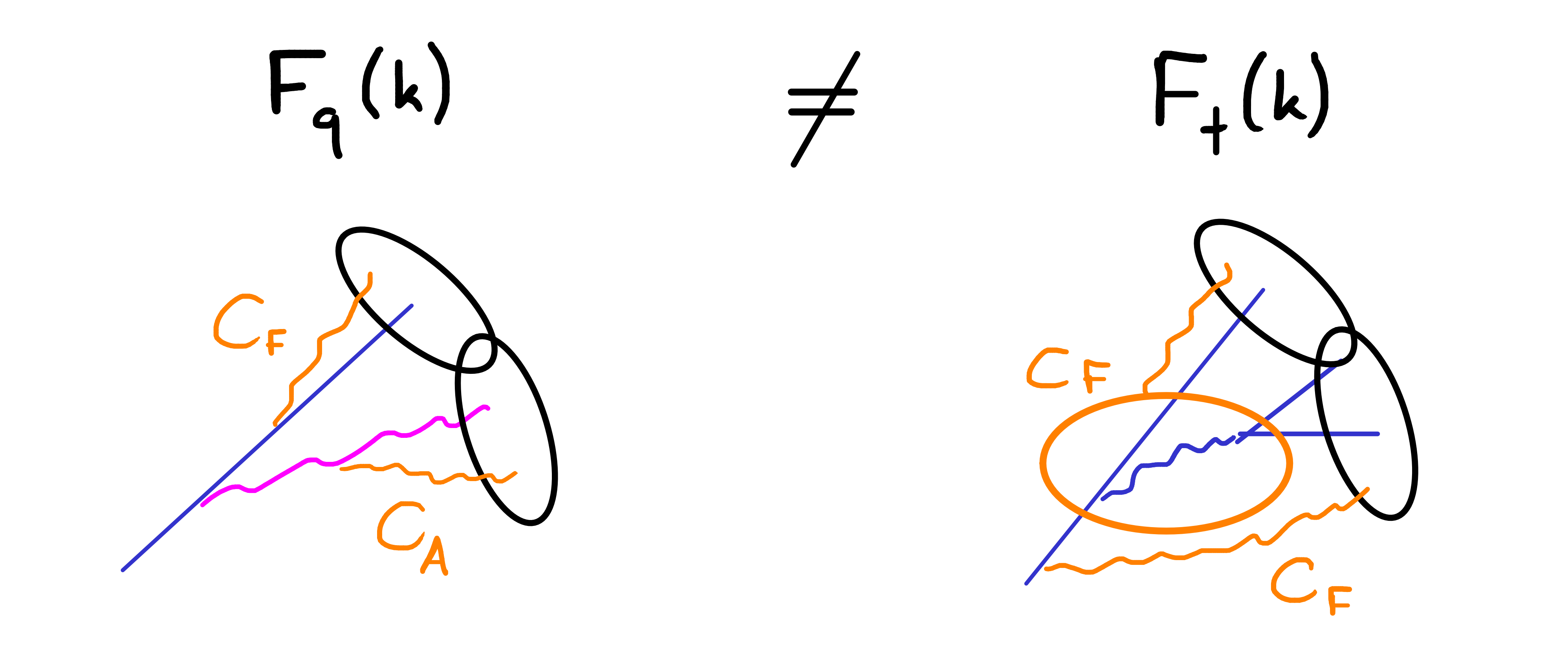
Results: NLL$'$ + renormalon subtractions



Outlook

Future work:

1. Breaking of nonperturbative universality: Hoang, Mantry, Michel, AP, Stewart (soon)



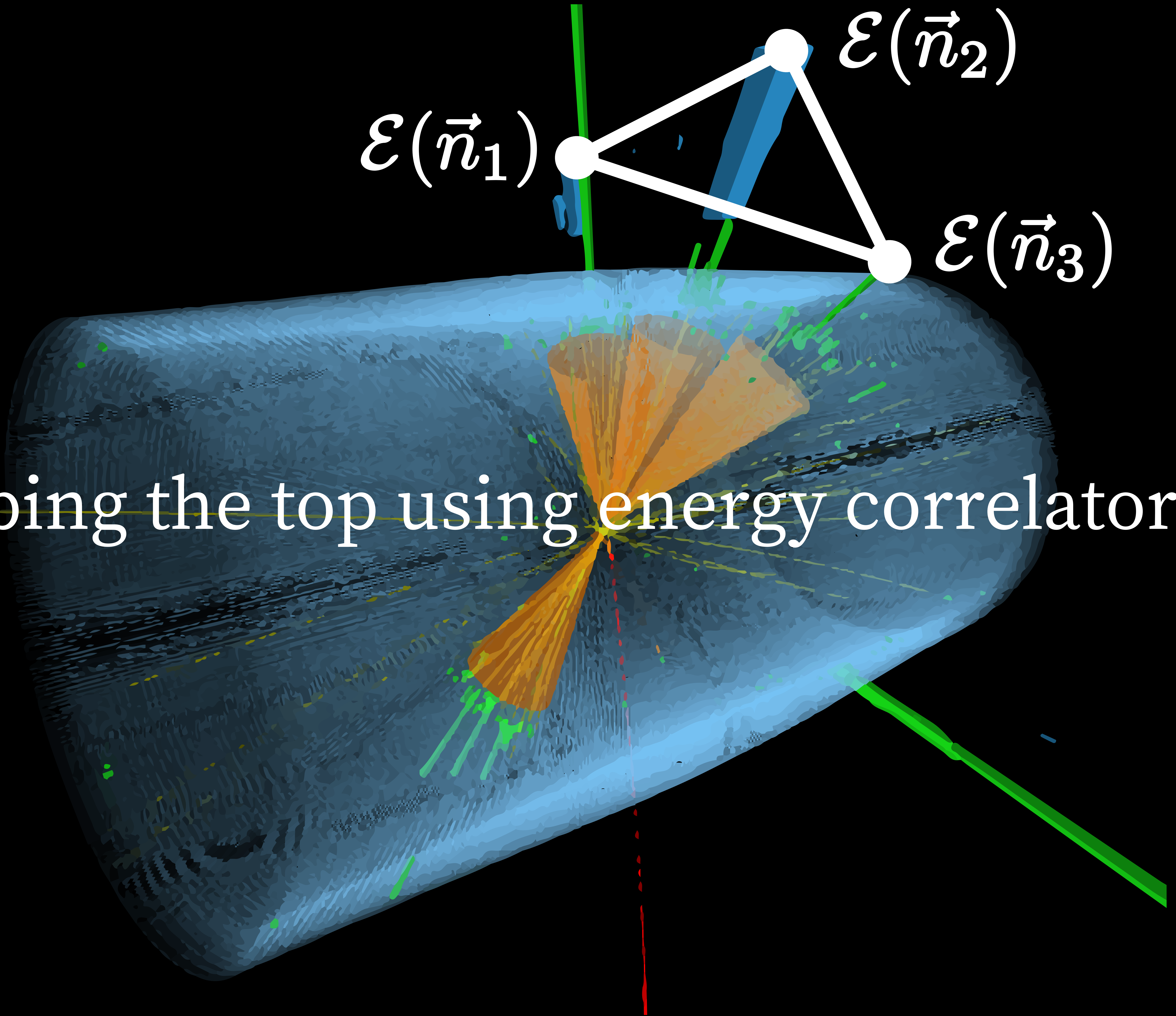
2. Effects of underlying event:

$$\frac{d\sigma_\kappa^{\rm UE}}{d\xi} = \frac{d\sigma_\kappa^{\rm had}}{d\xi} - \frac{\Omega_{1\rm UE}^{\varpi}}{Q_R}\frac{d}{d\xi}\left(C_1^{(2)\kappa}(\xi)\frac{d\hat\sigma}{d\xi}\right) + (1+\beta)\frac{\Upsilon_{1\rm UE}^{-}}{Q_R}\frac{C_2^{(2)\kappa}(\xi)}{\xi}\frac{d\hat\sigma}{d\xi} - \beta\frac{\Upsilon_{1\rm UE}^{\perp}}{Q_R}\frac{C_2^{(1)\kappa}(\xi)}{\xi}\frac{d\hat\sigma}{d\xi}$$

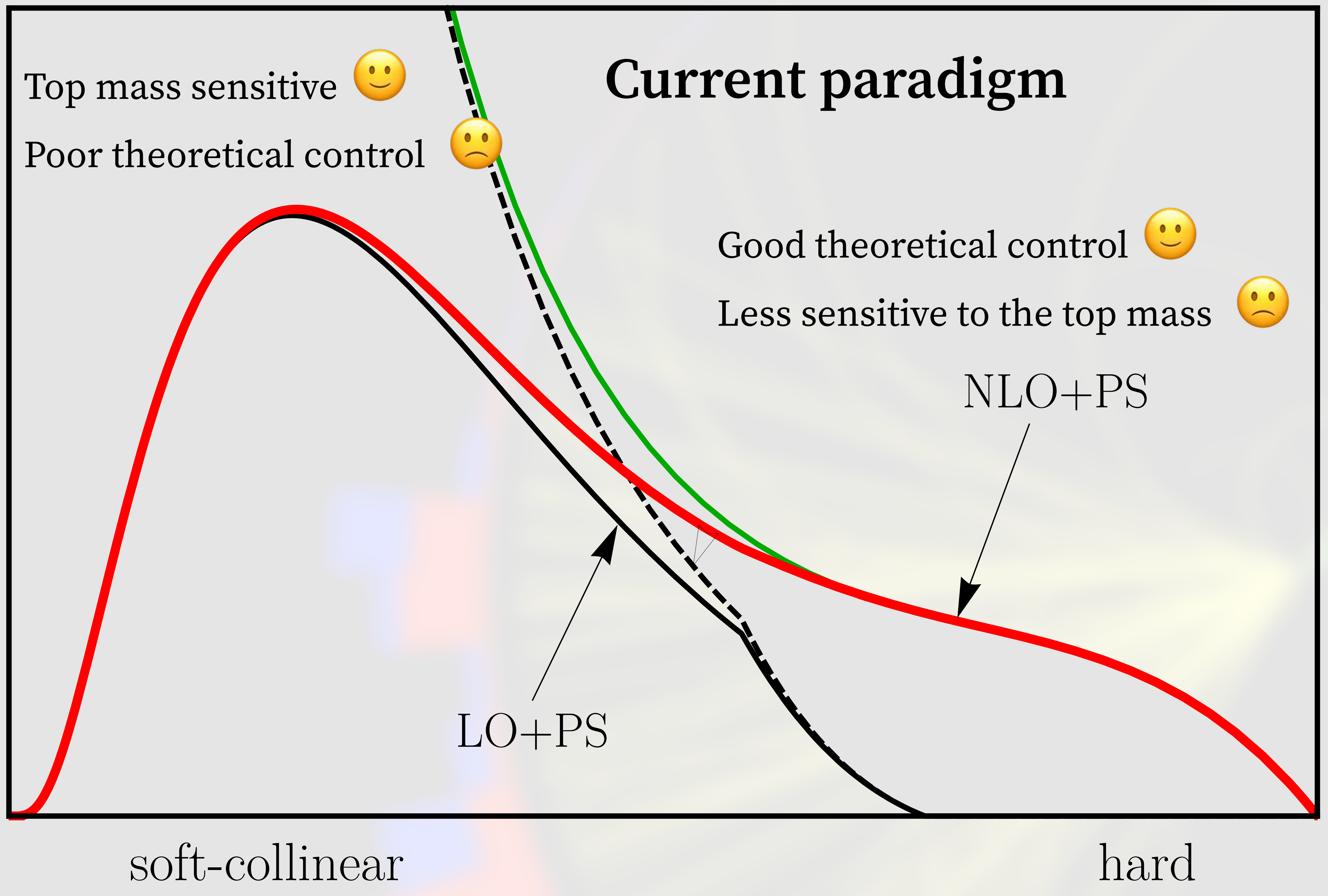
$$C_1^{\kappa(n)}(\xi, z_{\rm cut}, \beta) \sim \big\langle R_g^n / R^n \big\rangle\,,$$
$$C_2^{\kappa(n)}(\xi, z_{\rm cut}, \beta) \sim \big\langle R_g^n / R^n \delta(z_g - \tilde z_{\rm cut}\theta_g^\beta)\big\rangle(\xi)$$

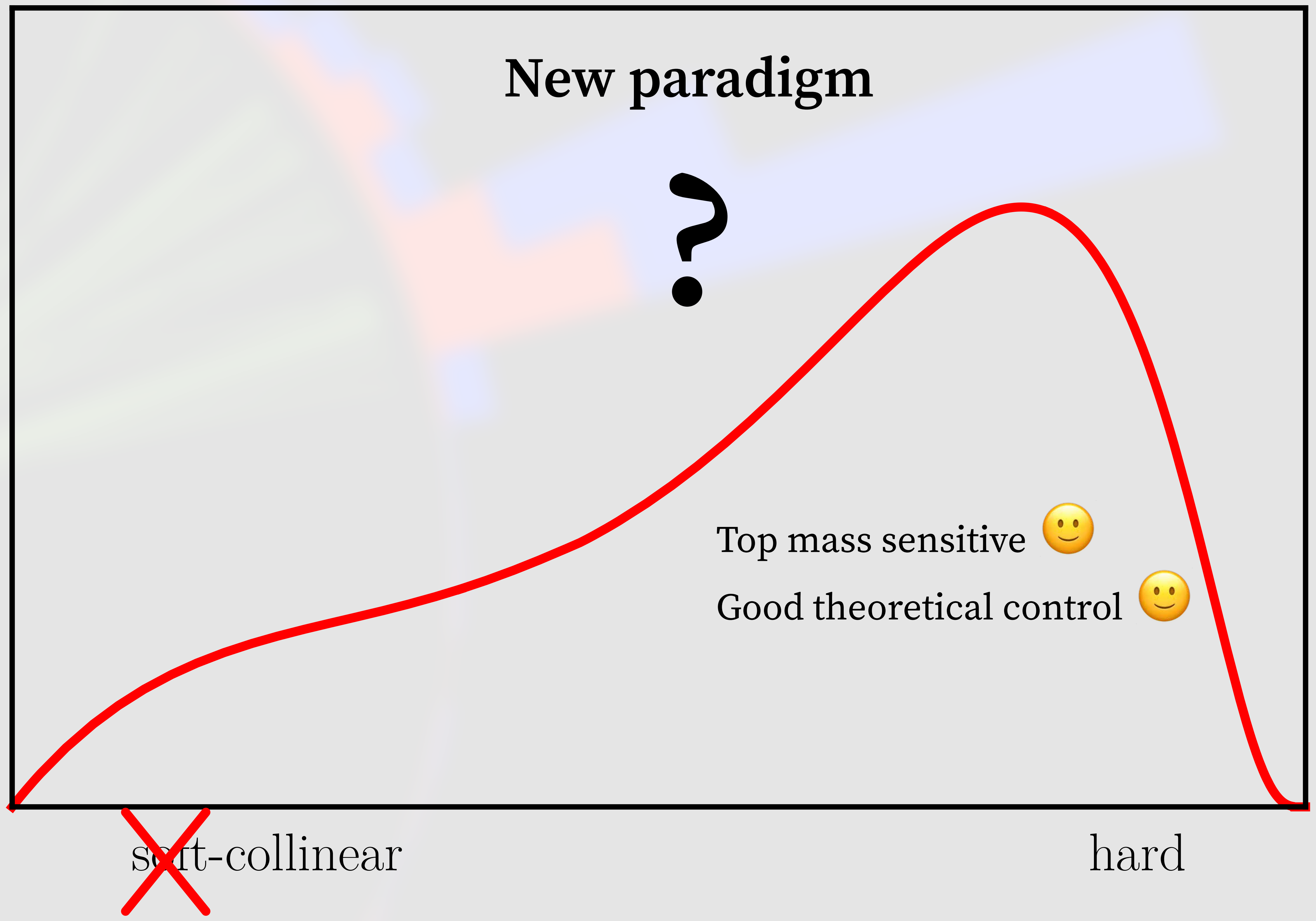
Ferdinand, Lee, AP (soon)

Probing the top using energy correlators



Overcome challenges in the new paradigm





To overcome these challenges we need:

1. Top mass sensitivity in the hard region
2. Insensitivity to soft physics and contamination from the underlying event

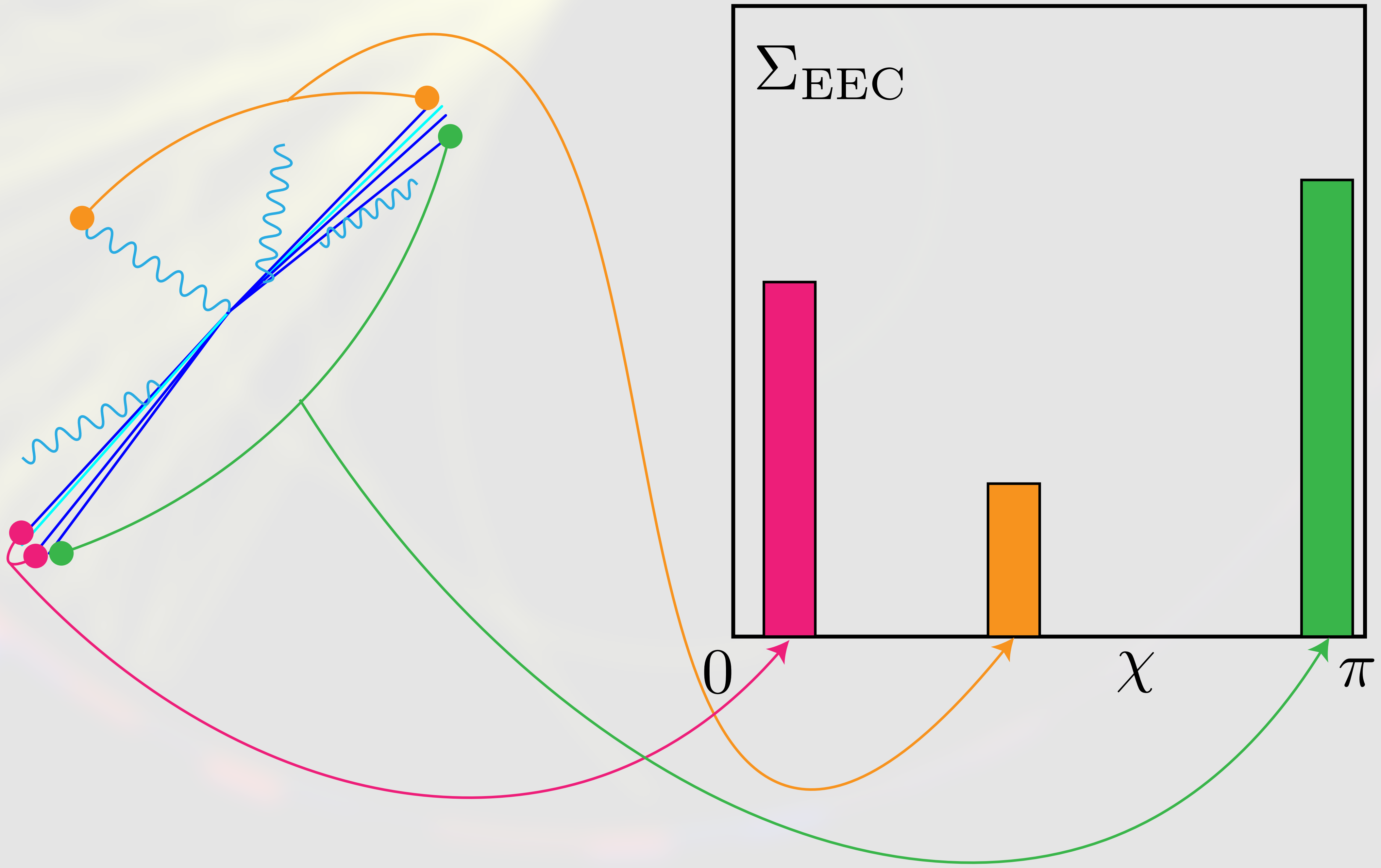
The 2-point correlator

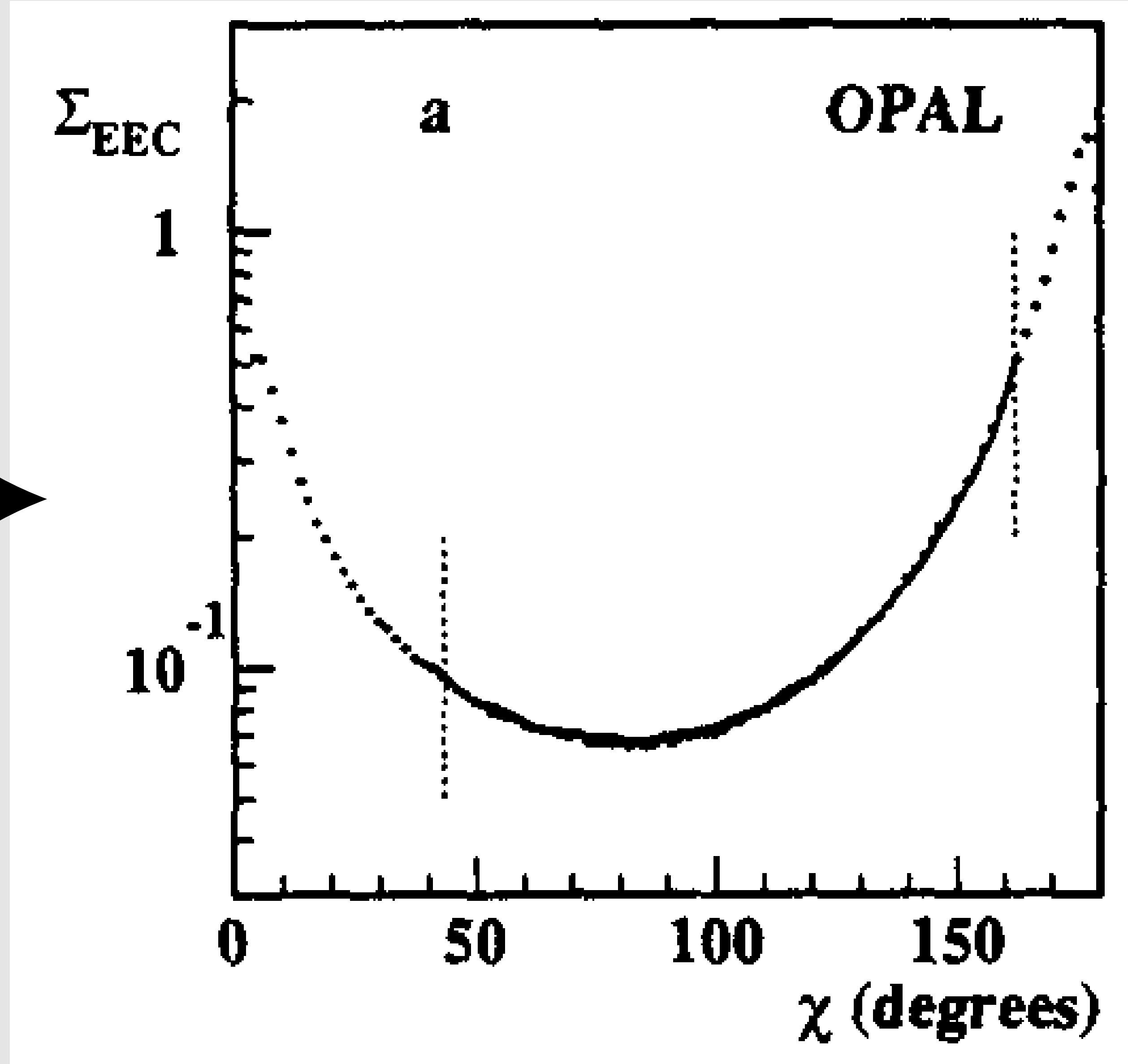
$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \sum_{ij}\int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}^2\vec{n}_i \mathrm{d}^2\vec{n}_j} E_i E_j \delta^2(\vec{n}_1 - \vec{n}_i)\delta^2(\vec{n}_2 - \vec{n}_j)$$

Inclusive cross section to produce particles *i* and *j* + anything else!

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\chi} = \int \mathrm{d}^2 n_1 \mathrm{d}^2 n_2\, \delta(\vec{n}_1 \cdot \vec{n}_2 - \cos\chi)\frac{\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle}{Q^2}$$

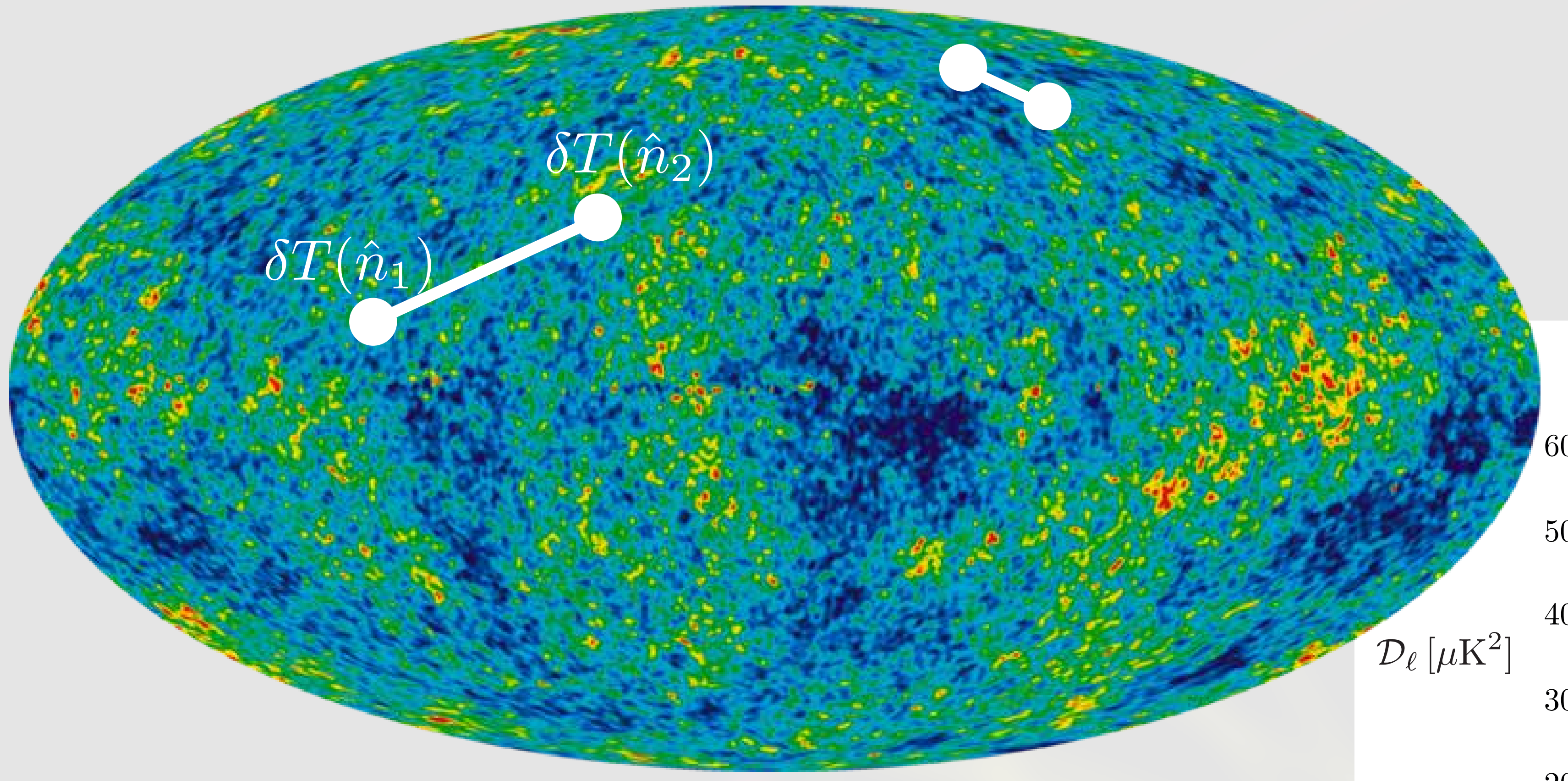
Not event by event:

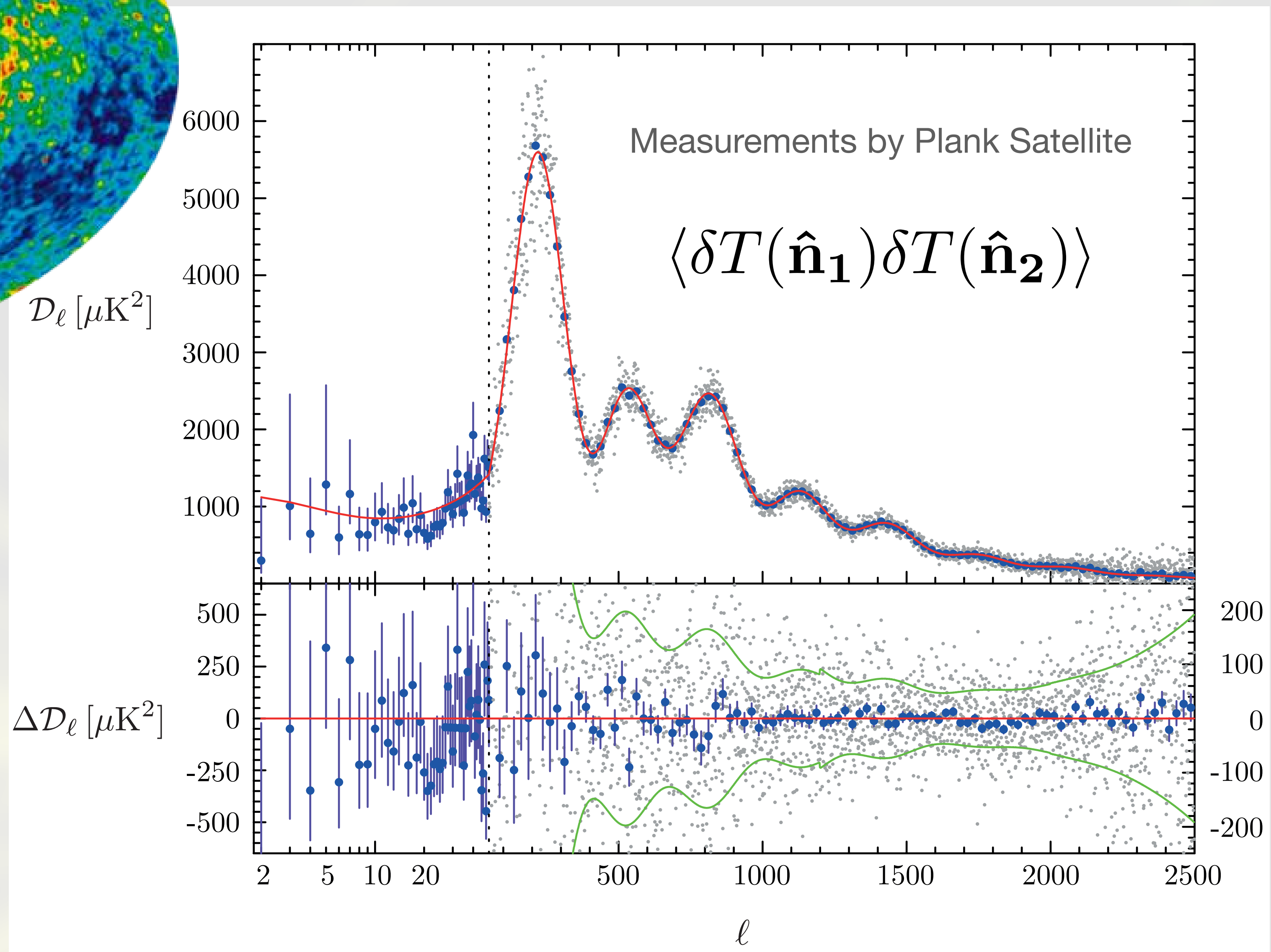


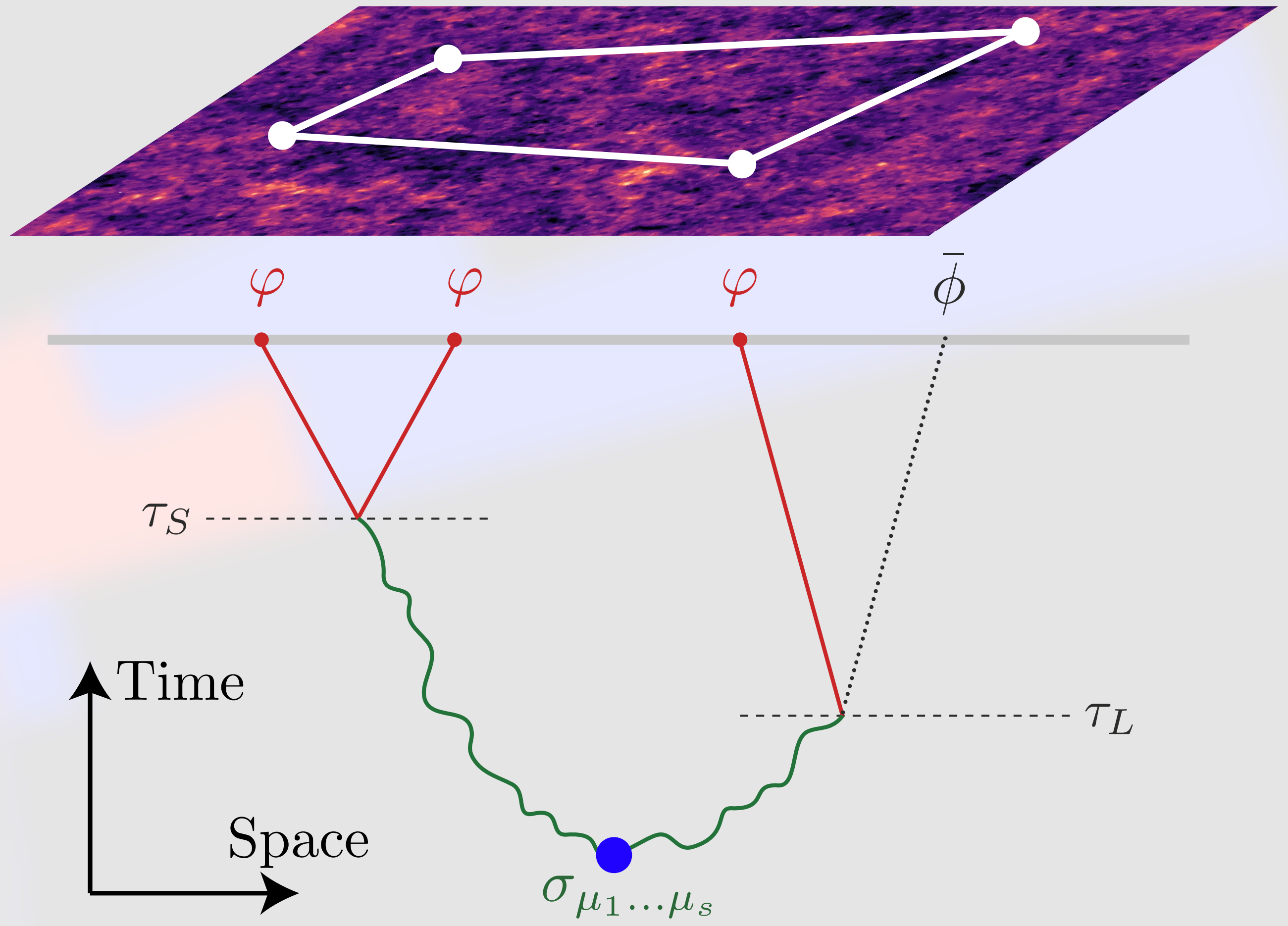


[Opal collboration, Z. Phys. C59 (1993) 21]

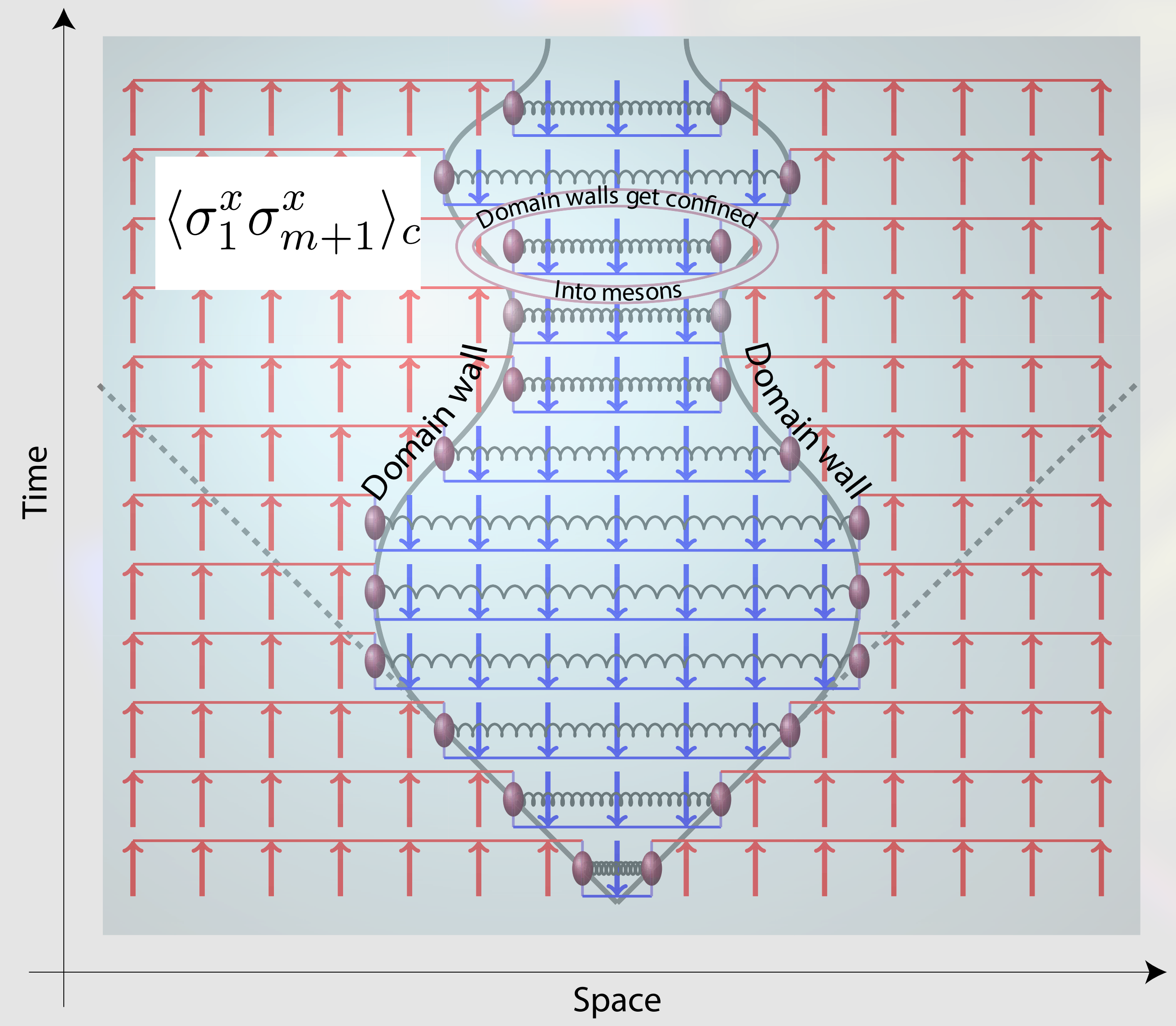
Correlation functions in other sciences



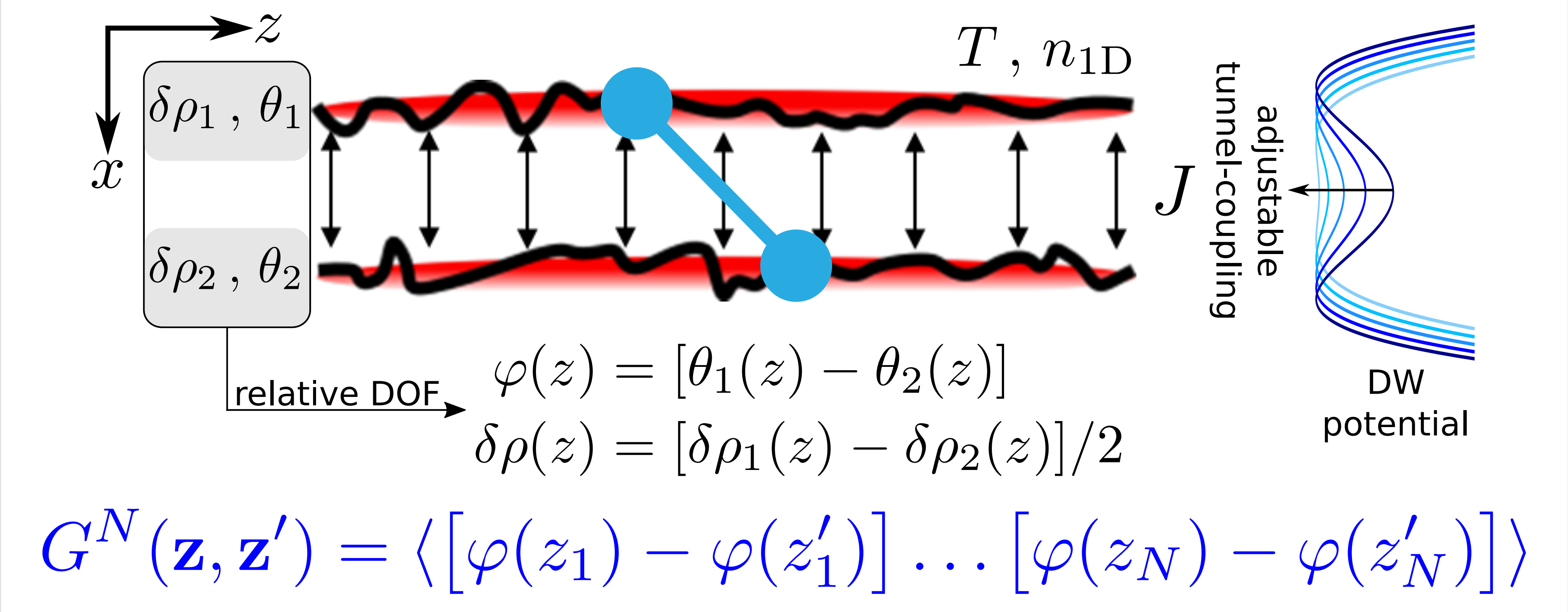




Arkani-Hamed, Maldacena 1503.08043;
Arkani-Hamed, Baumann, et al.;1811.00024



Correlation functions are extremely powerful!



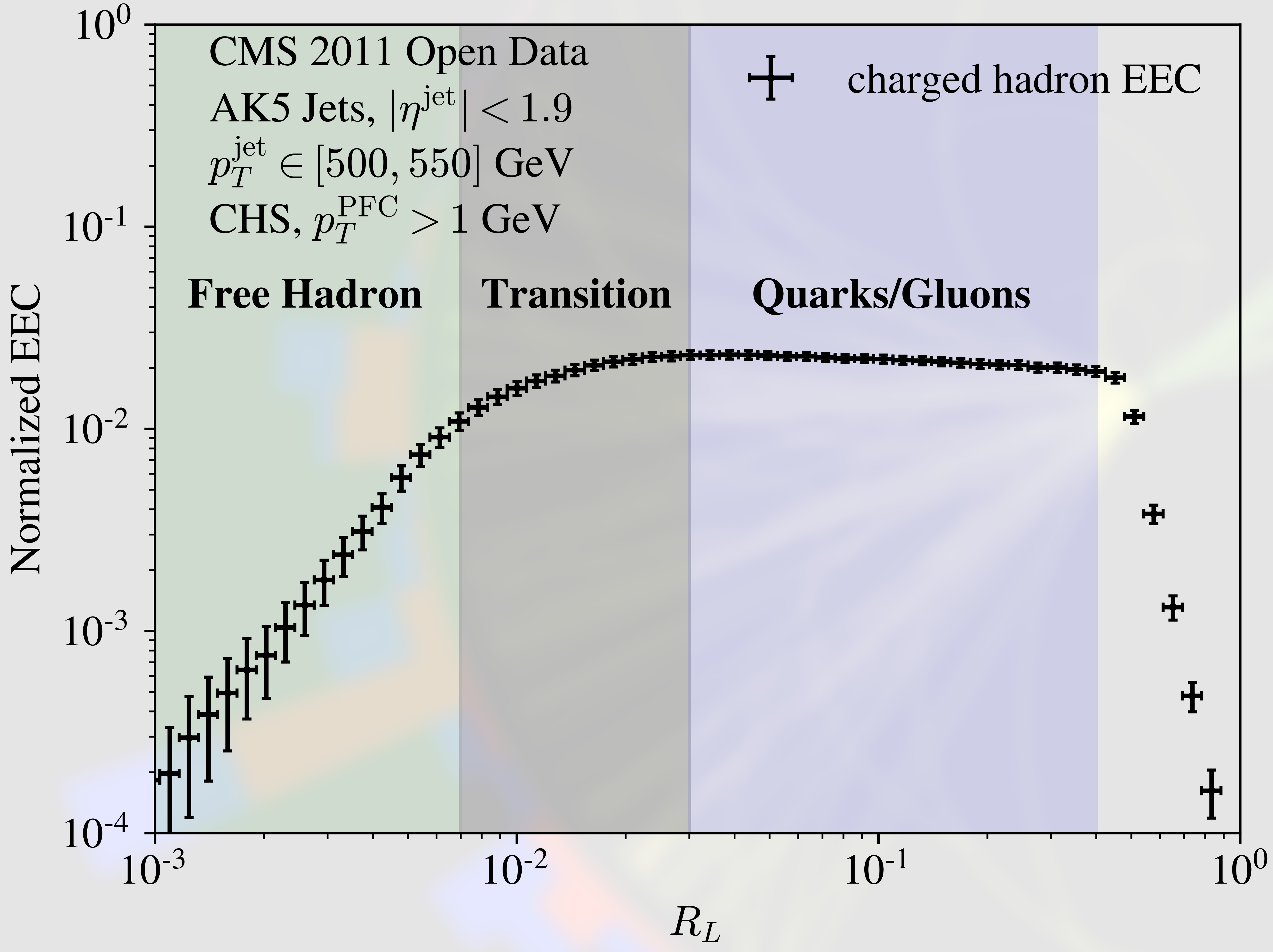
Kormos et al. Nature Phys. 13, 2017

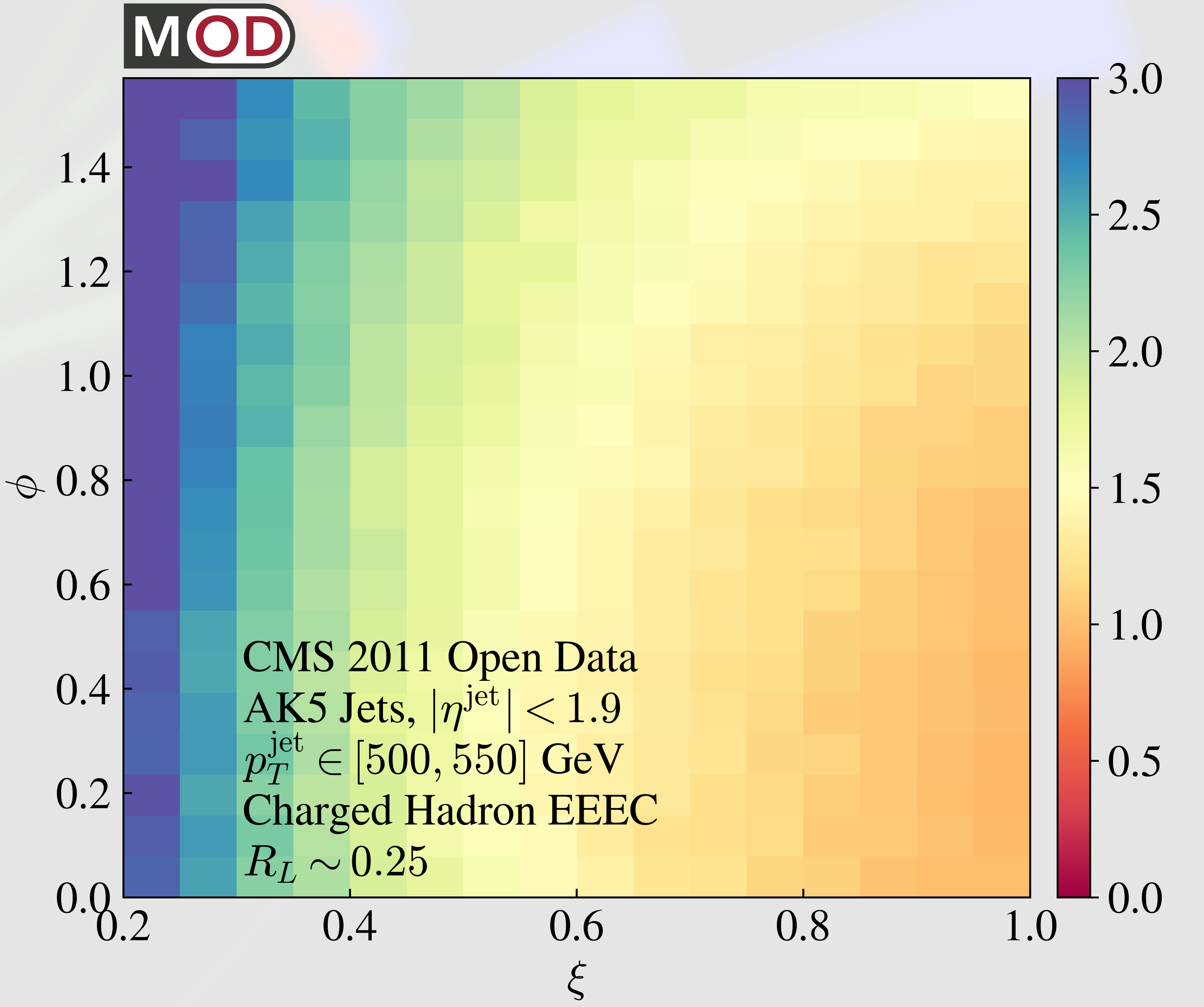
Schweigler et al. 1505.03126

Progress in recent years

Energy correlators map transition from perturbative to free hadron phase

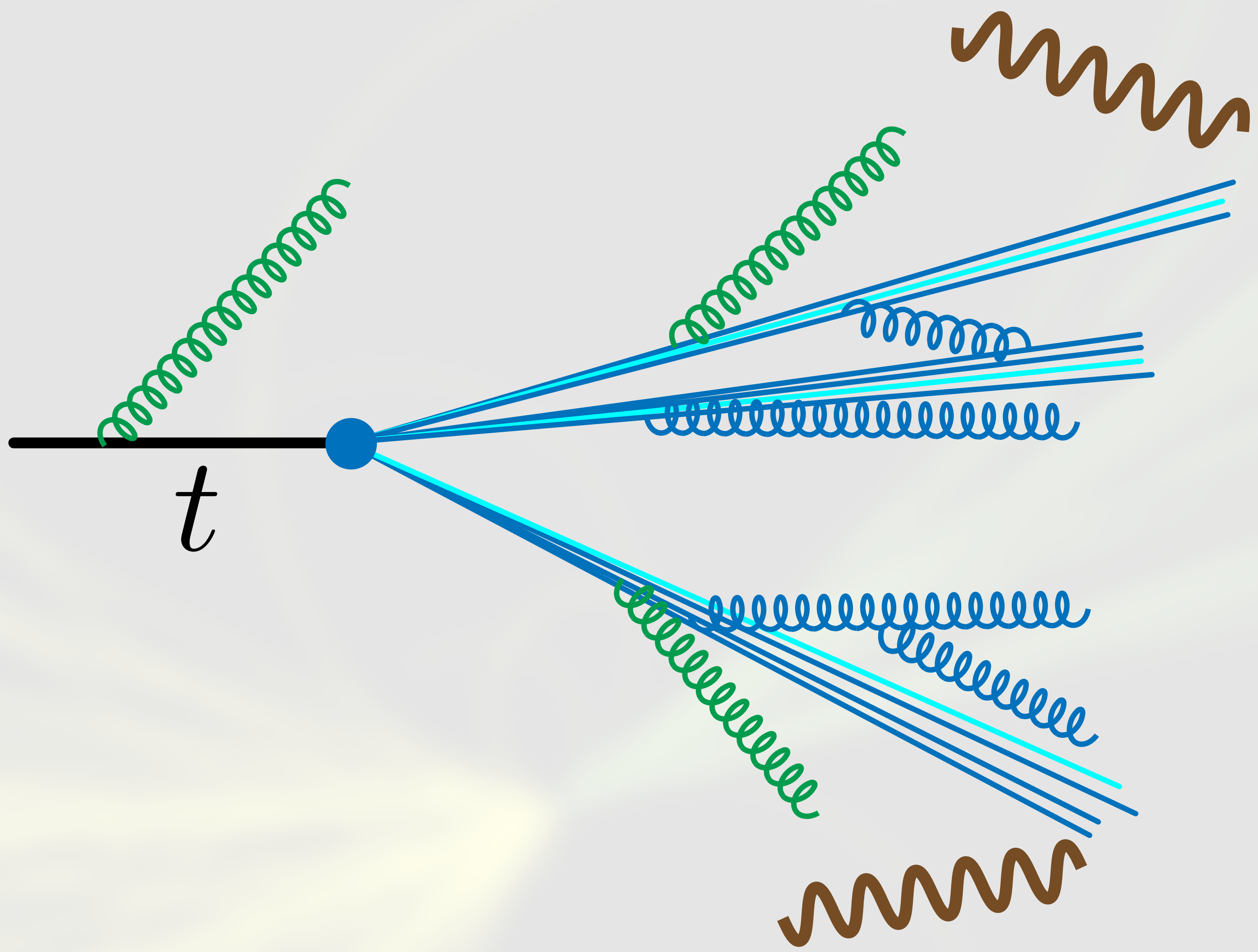
Komiske, Moult, Thaler, Zhu 2201.07800





This talk: first time applying them to top quarks

Which correlator will well characterize the top decay?



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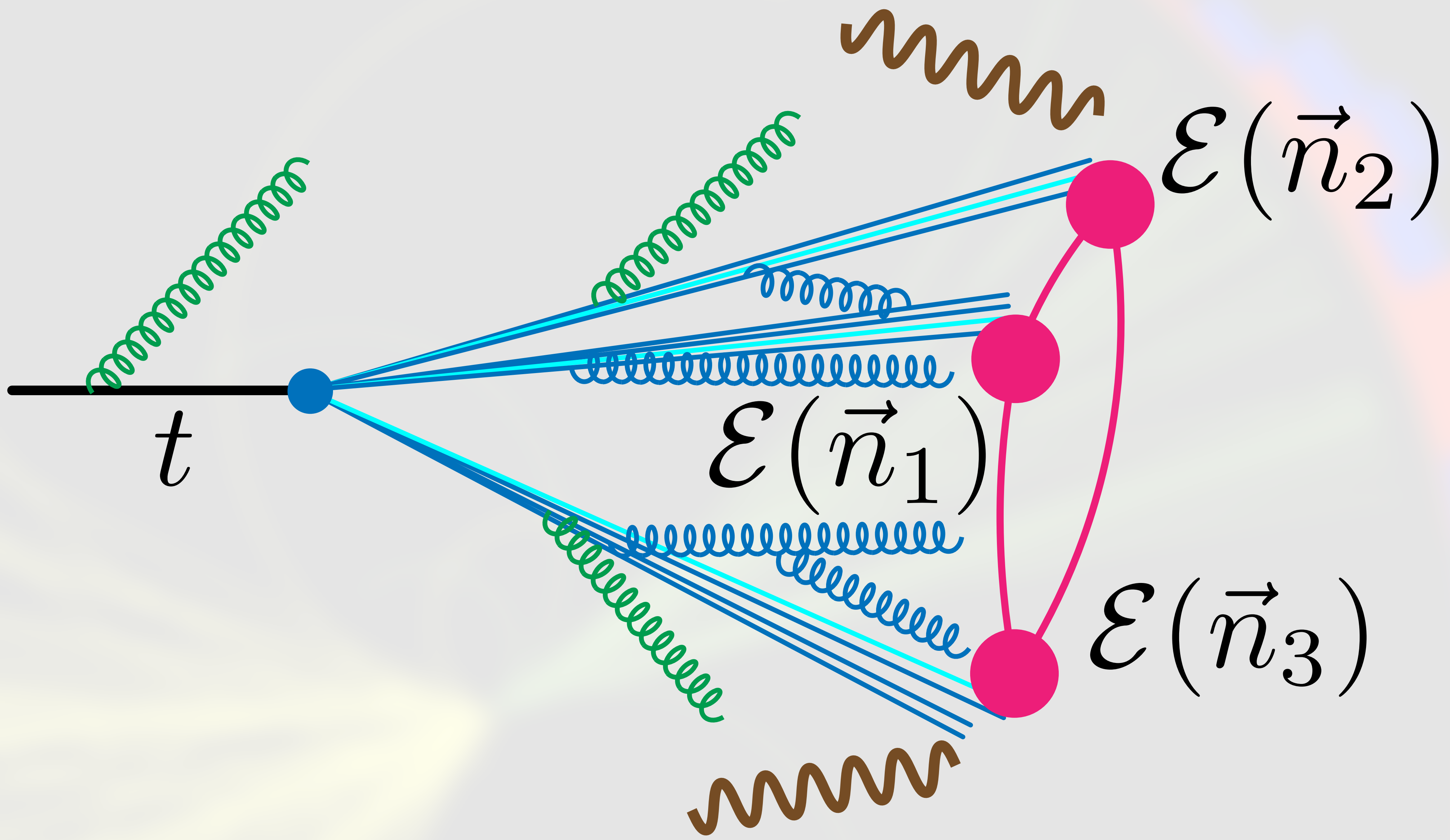
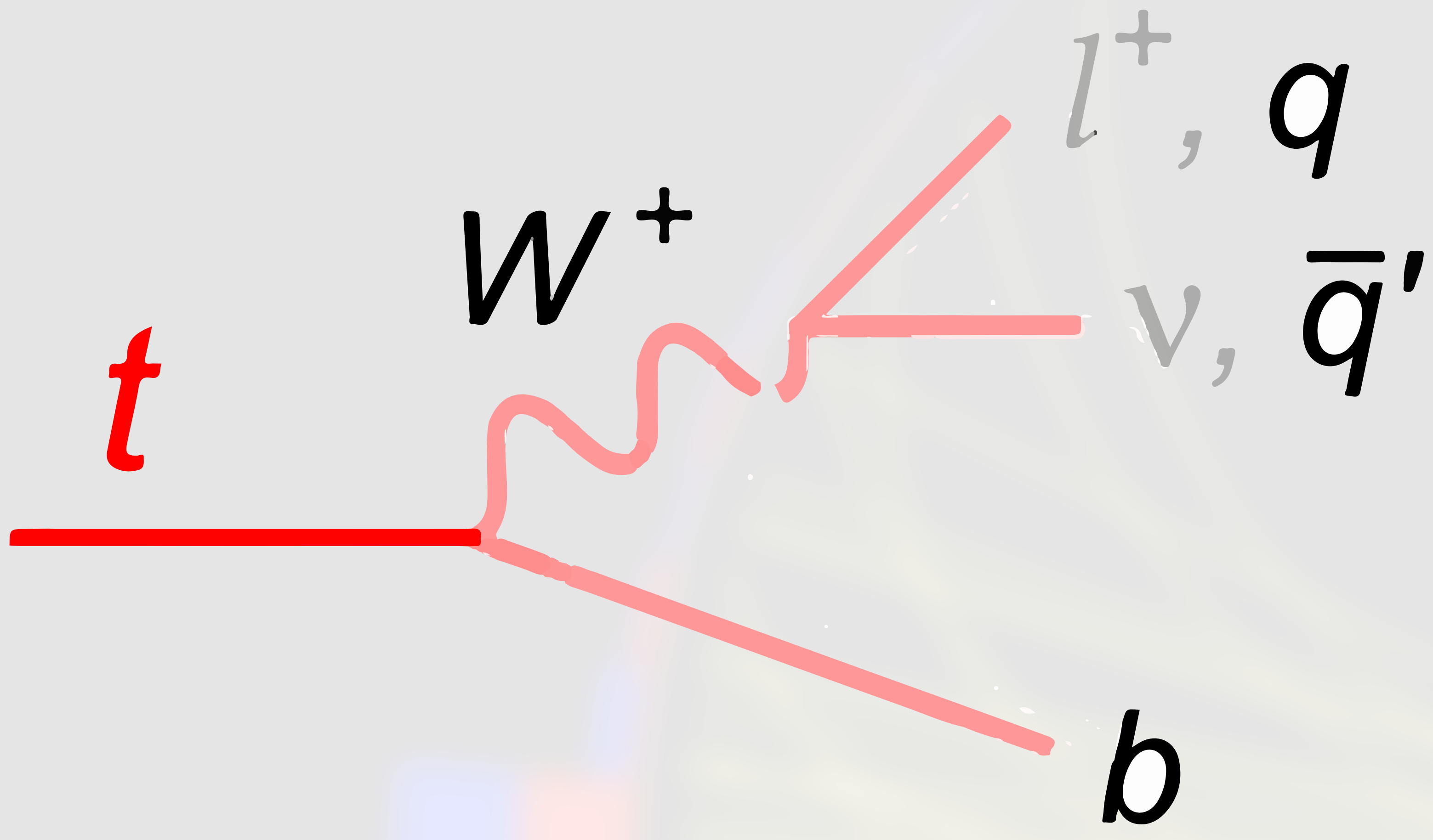


$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$$

$$= \sum_{ij} \int \frac{\mathrm{d}\sigma_{ijk}}{\mathrm{d}^2\vec{n}_i \mathrm{d}^2\vec{n}_j \mathrm{d}^2\vec{n}_k} E_i E_j E_k \, \delta^2(\vec{n}_1 - \vec{n}_i)\delta^2(\vec{n}_2 - \vec{n}_j)\delta^2(\vec{n}_3 - \vec{n}_k)$$

What do we expect to see at leading order?



The correlator is sensitive to angles between the decay products.

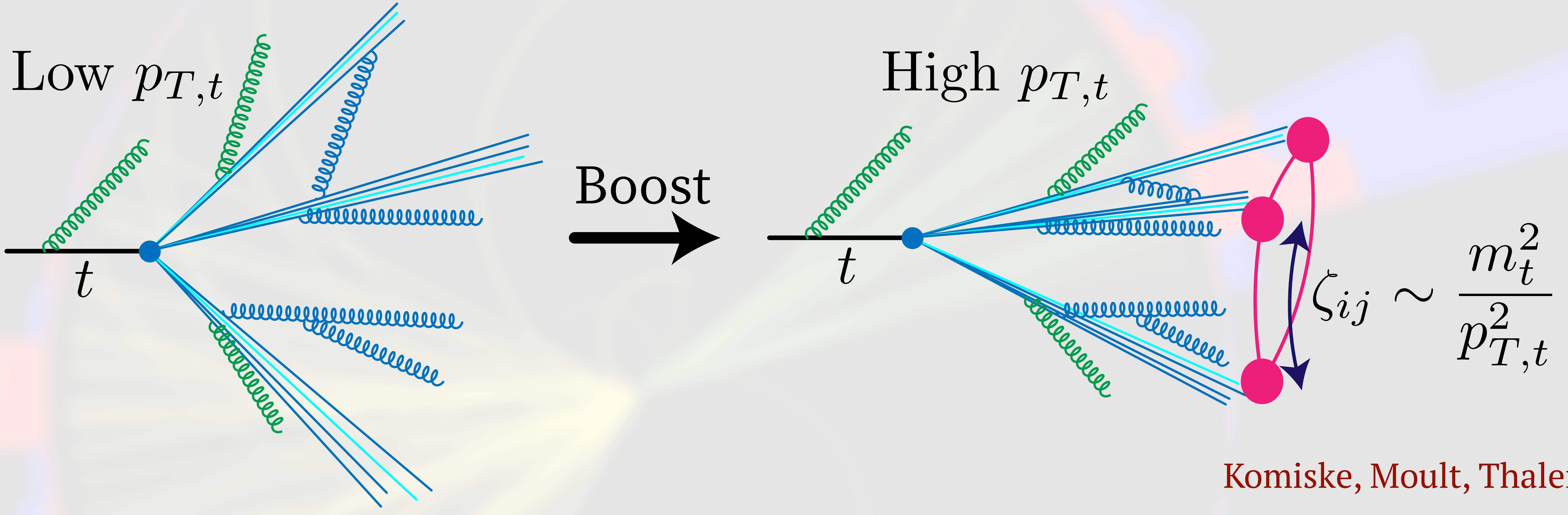
$$\zeta_{ij} = \frac{1-\cos\theta_{ij}}{2}$$

In the top rest frame: $\tilde{\zeta}_{12} + \tilde{\zeta}_{23} + \tilde{\zeta}_{31} \in [2, 2.25]$

Lab frame angles: $\zeta \equiv \sum_{i<j} \zeta_{ij} \approx \left(\frac{m_t}{Q}\right)^2 \sum_{i<j} \tilde{\zeta}_{ij}$

$$\langle \zeta \rangle \approx \frac{3m_t^2}{Q^2}$$

Top mass imprinted at a characteristic angle

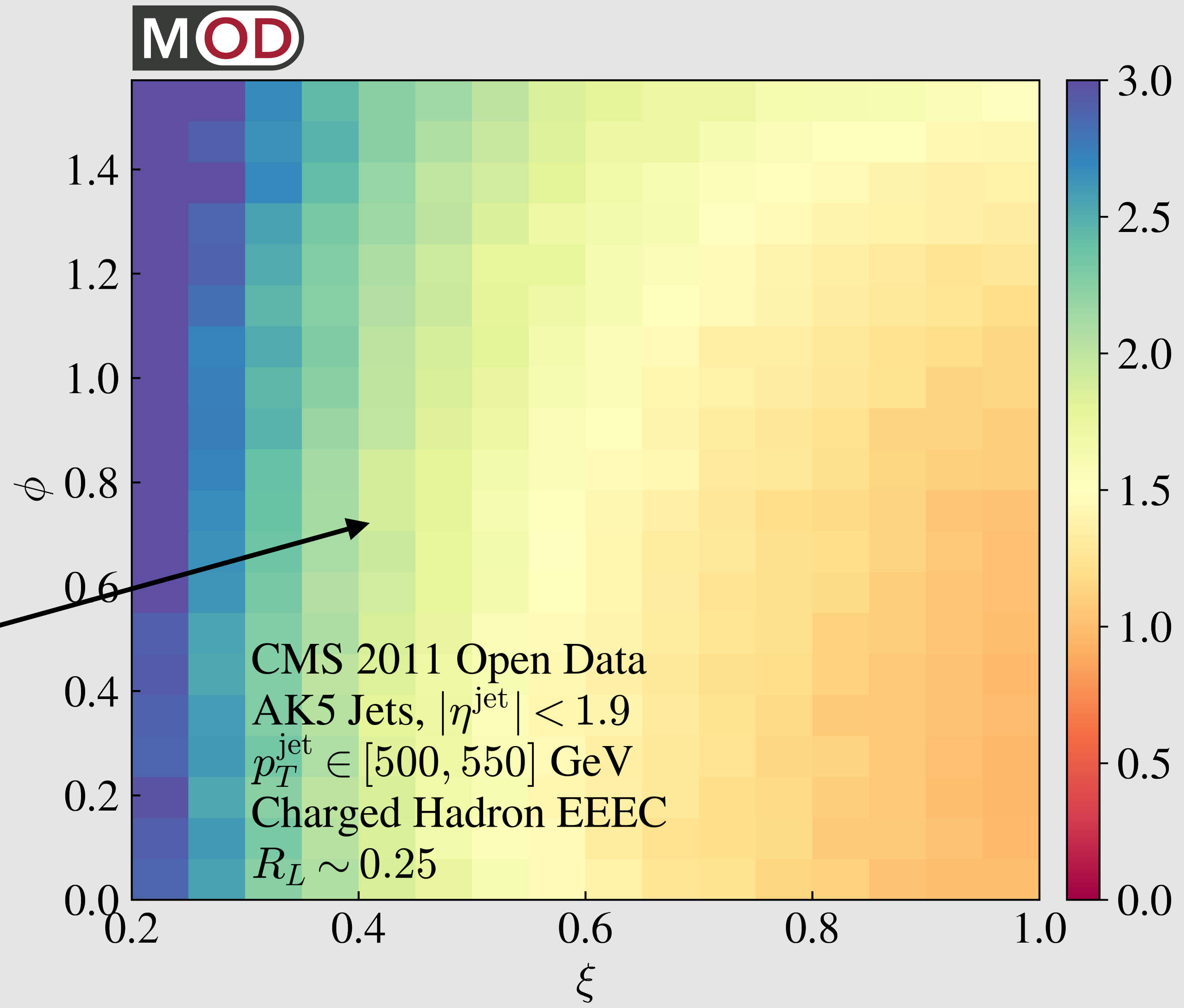


Komiske, Moult, Thaler, Zhu 2201.07800

In contrast, in the CFT limit EEEC exhibits a featureless power law:

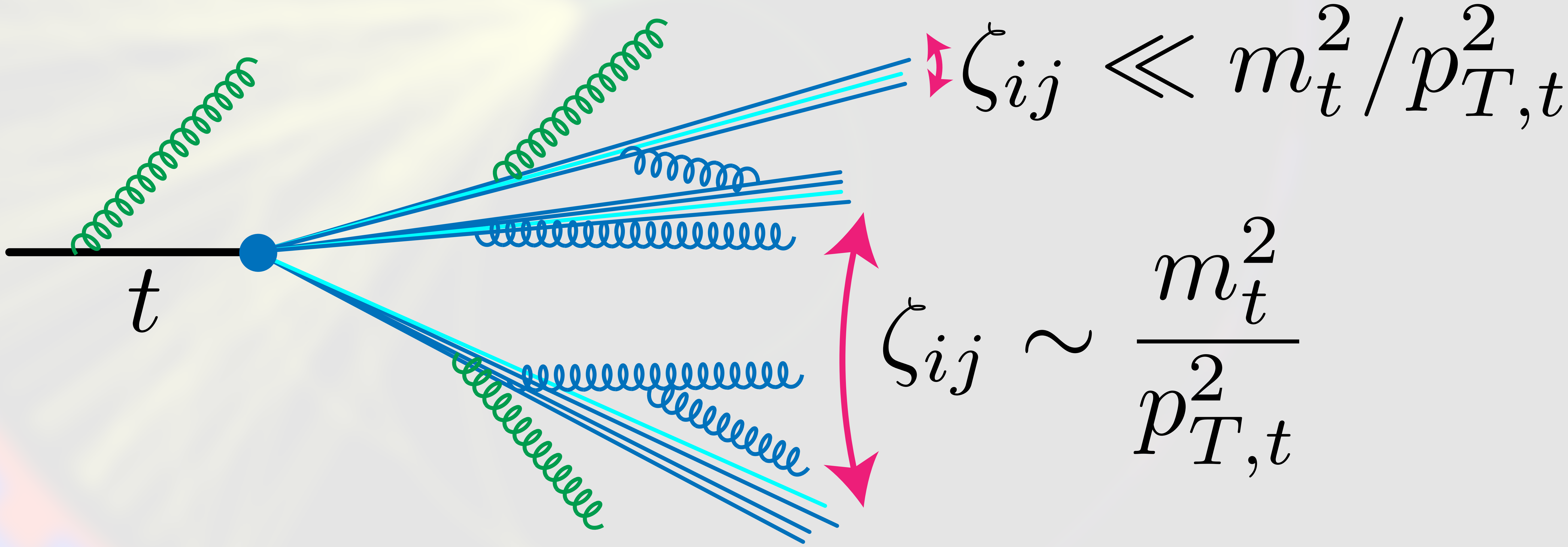
$$G^{(1)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) \xrightarrow{\text{CFT}} \zeta_{31}^{-1+\gamma(4)} G(z, \bar{z})$$

light quark/gluon jets



Suppress contribution from collinear splittings

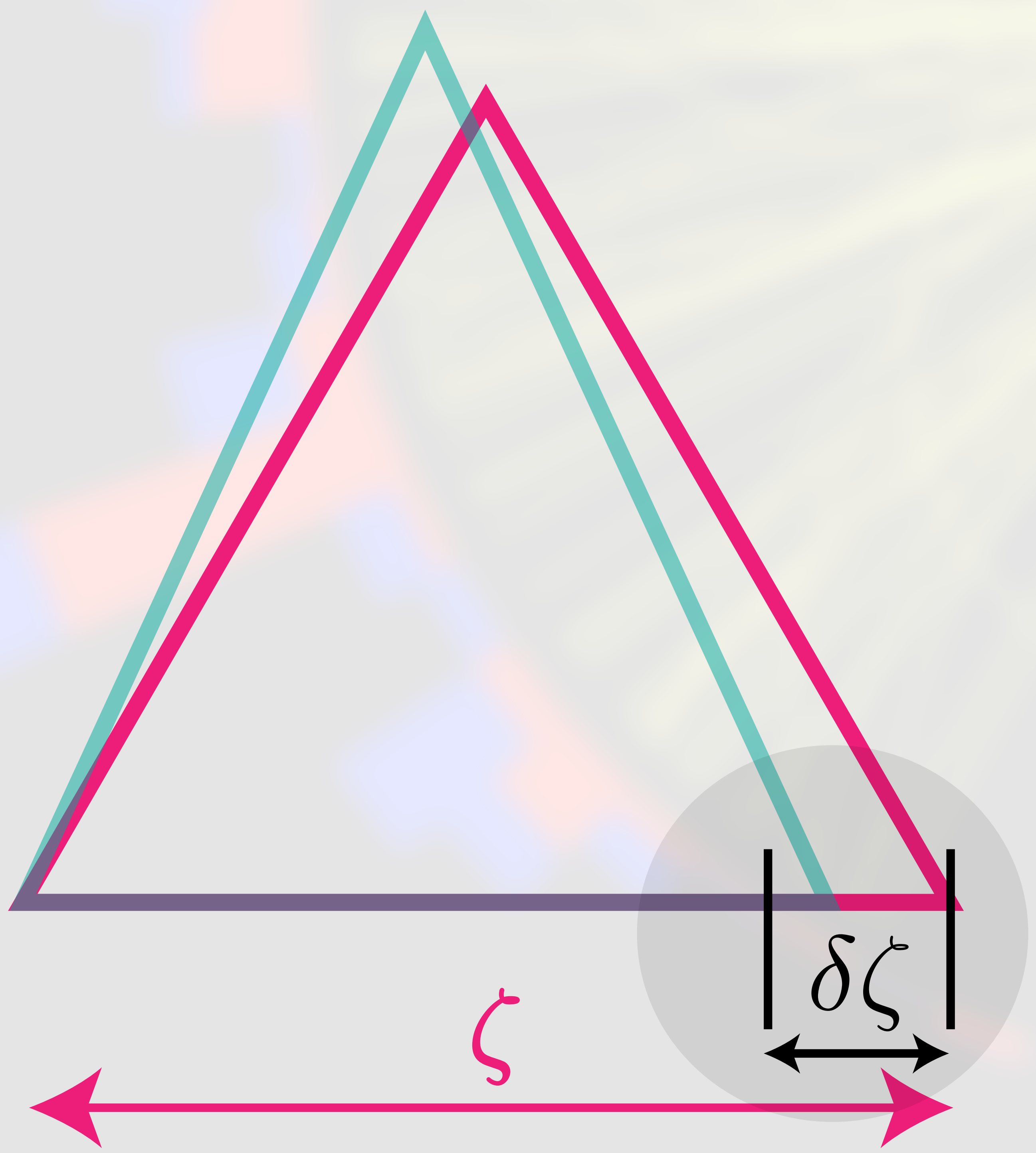
We want to preserve the $\langle\zeta\rangle \sim 3m_t^2/Q^2$ dependence but $\zeta = \sum_{i<j} \zeta_{ij}$ will also pick up collinear splittings

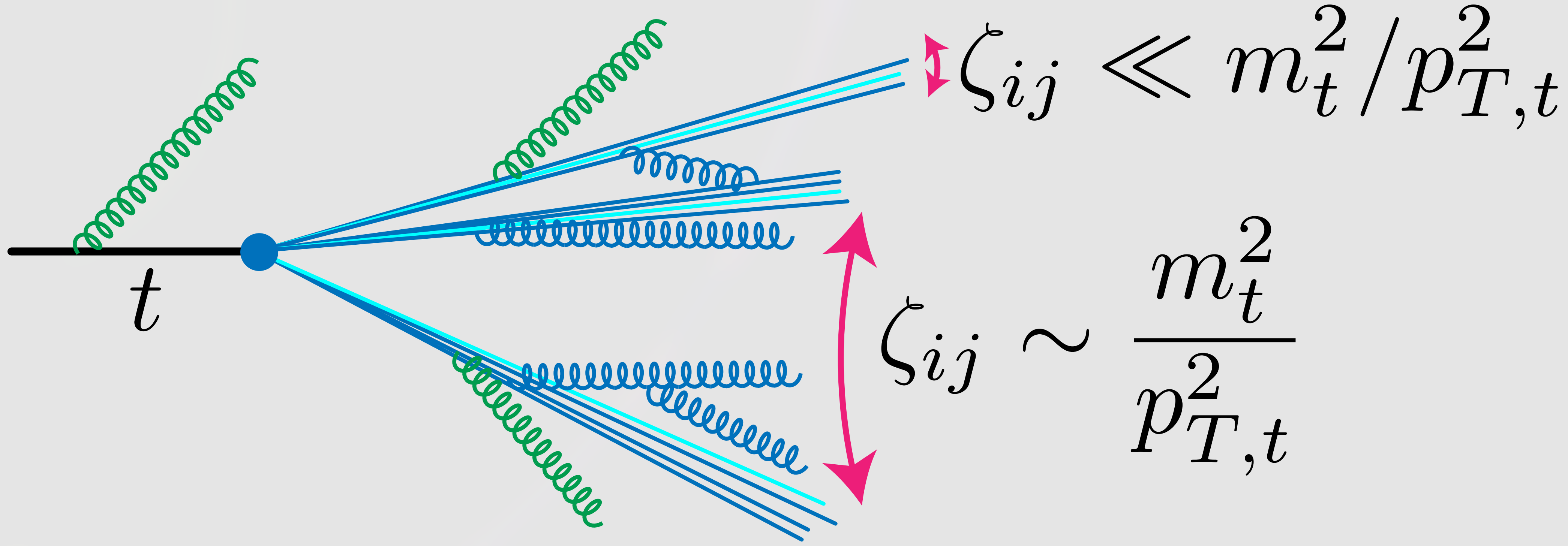


Constrain angles in equilateral configuration

$$\frac{d\Sigma(\delta\zeta)}{dQ\,d\zeta} = \int d\zeta_{12} d\zeta_{23} d\zeta_{31} \int d\sigma \widehat{\mathcal{M}}_{\triangle}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta)$$

$$\widehat{\mathcal{M}}_{\triangle}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta) = \sum_{i,j,k} \frac{E_i^n E_j^n E_k^n}{Q^{3n}} \delta\left(\zeta_{12} - \frac{\theta_{ij}^2}{4}\right) \delta\left(\zeta_{31} - \frac{\theta_{ik}^2}{4}\right) \delta\left(\zeta_{23} - \frac{\theta_{jk}^2}{4}\right)$$
$$\times\, \delta(3\zeta - \zeta_{12} - \zeta_{23} - \zeta_{31}) \prod_{l,m,n \in \{1,2,3\}} \Theta(\delta\zeta - |\zeta_{lm} - \zeta_{mn}|).$$

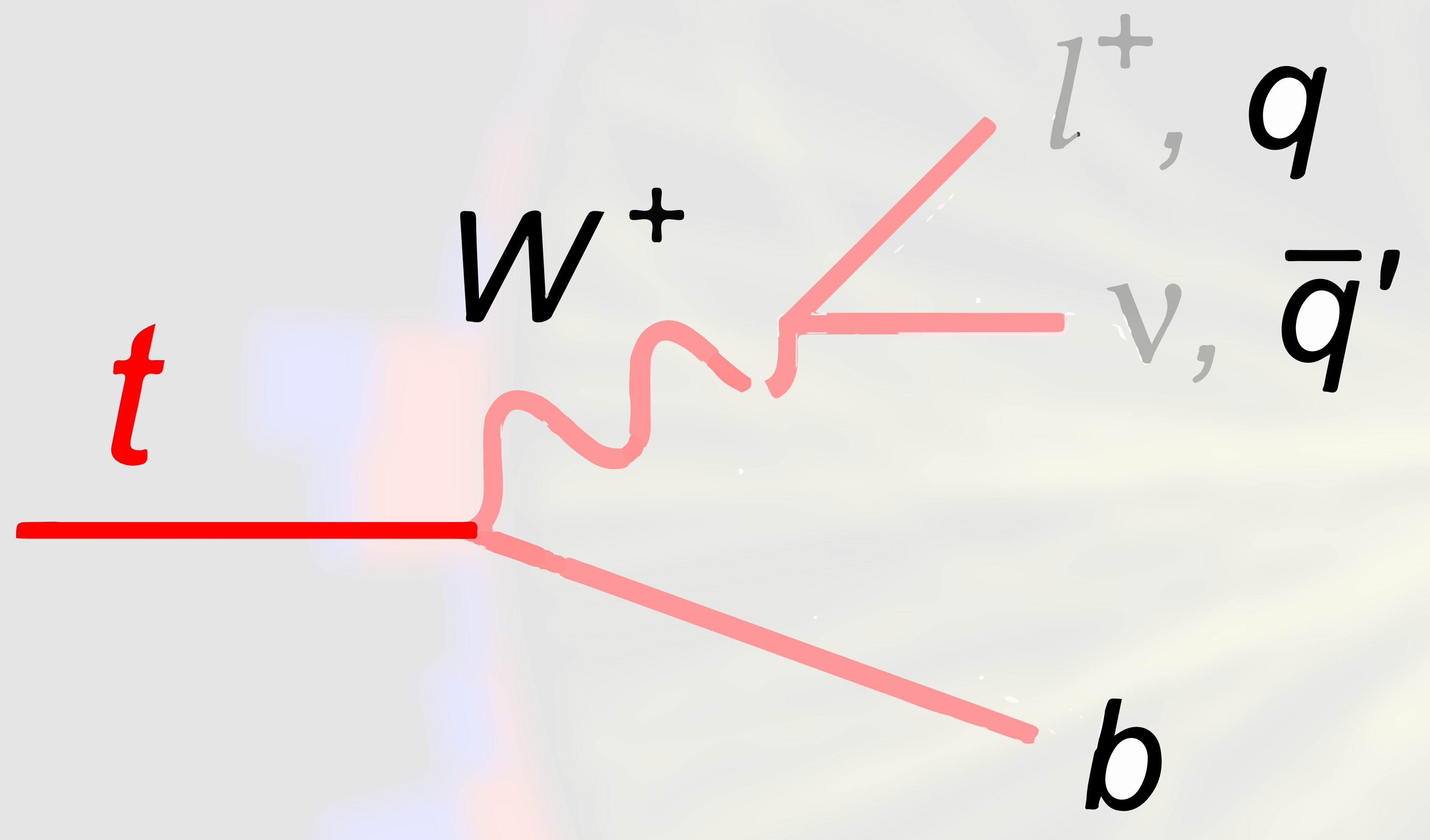


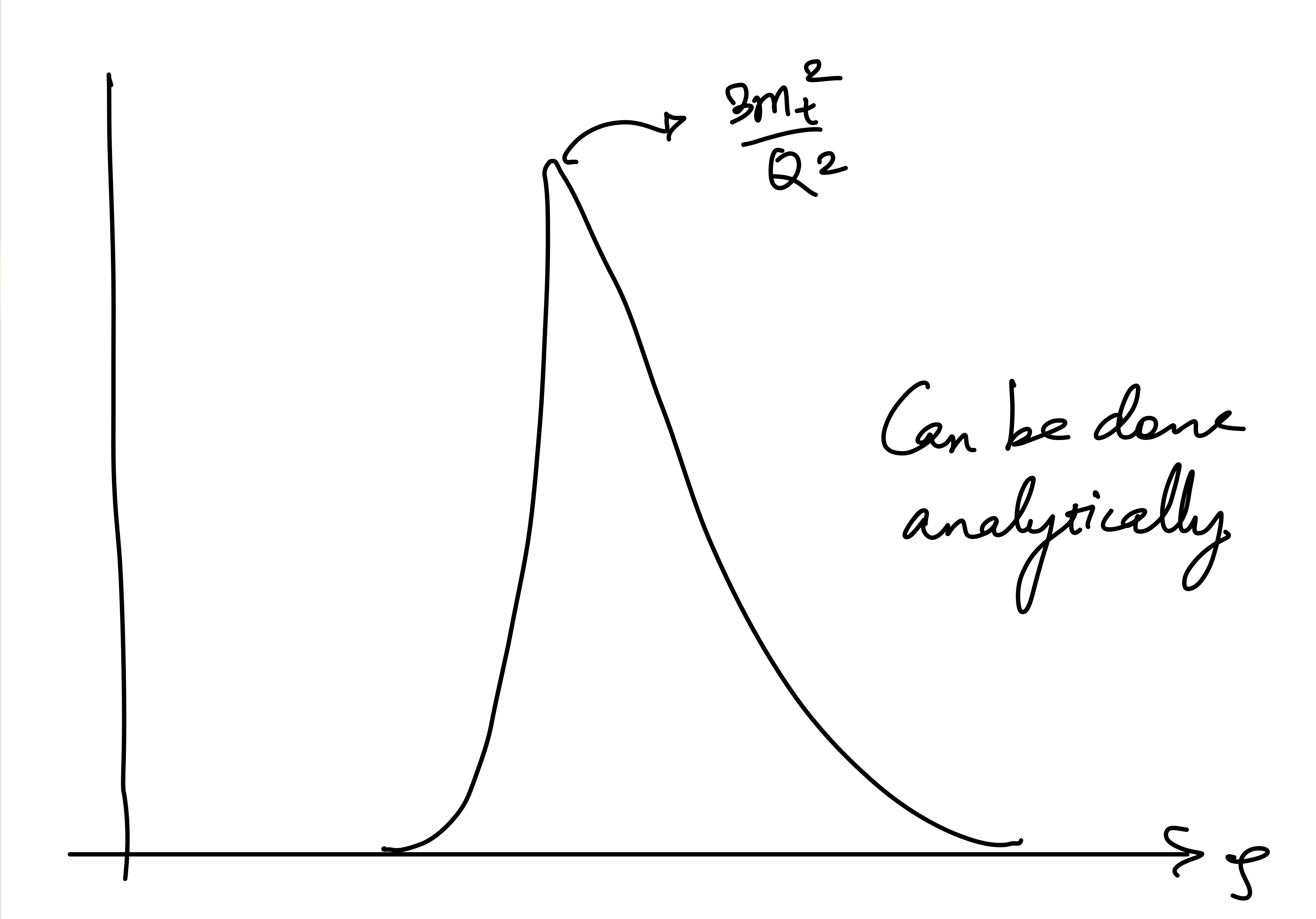


Allow for some small asymmetry

Understanding the distribution

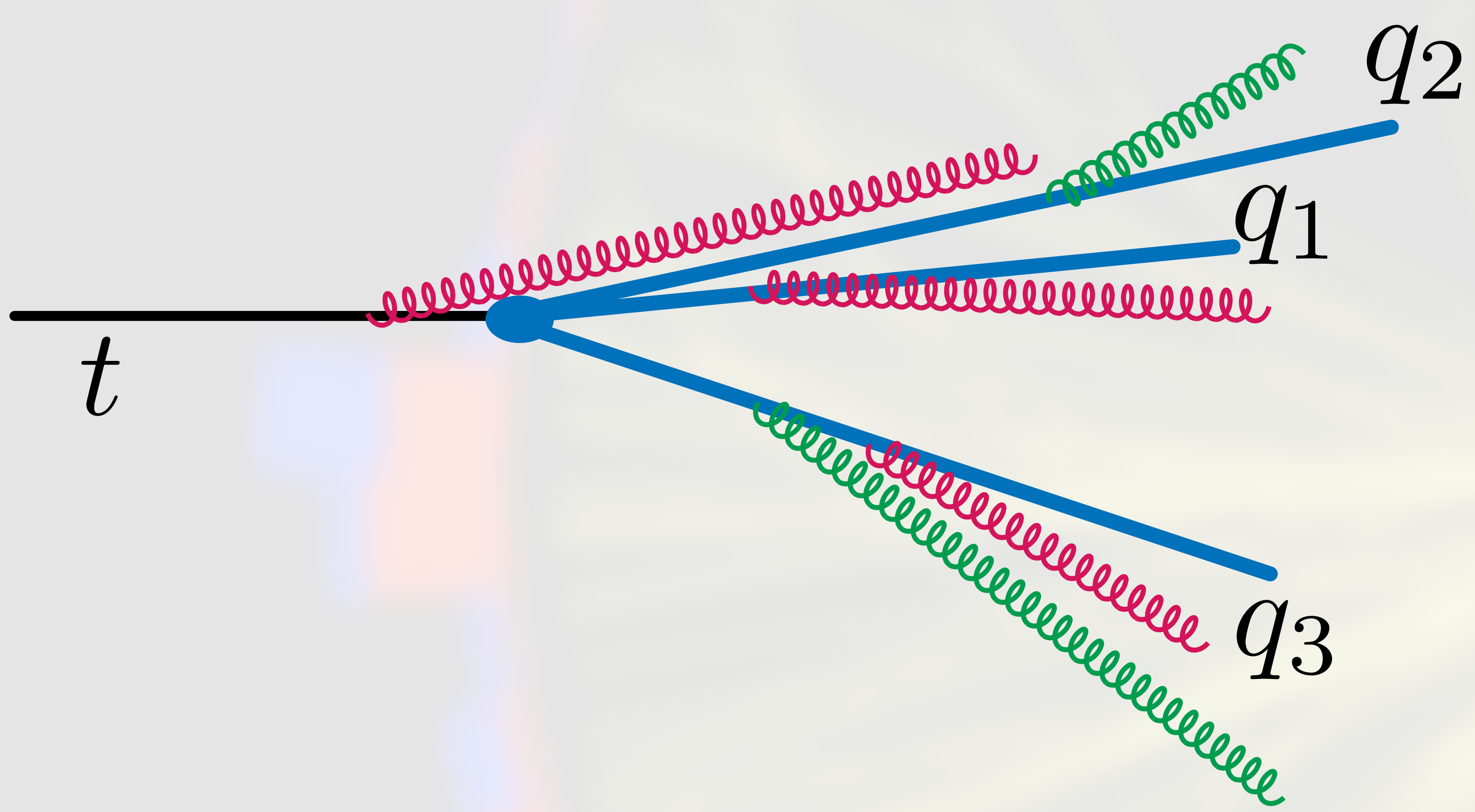
What does the distribution look like at leading order?

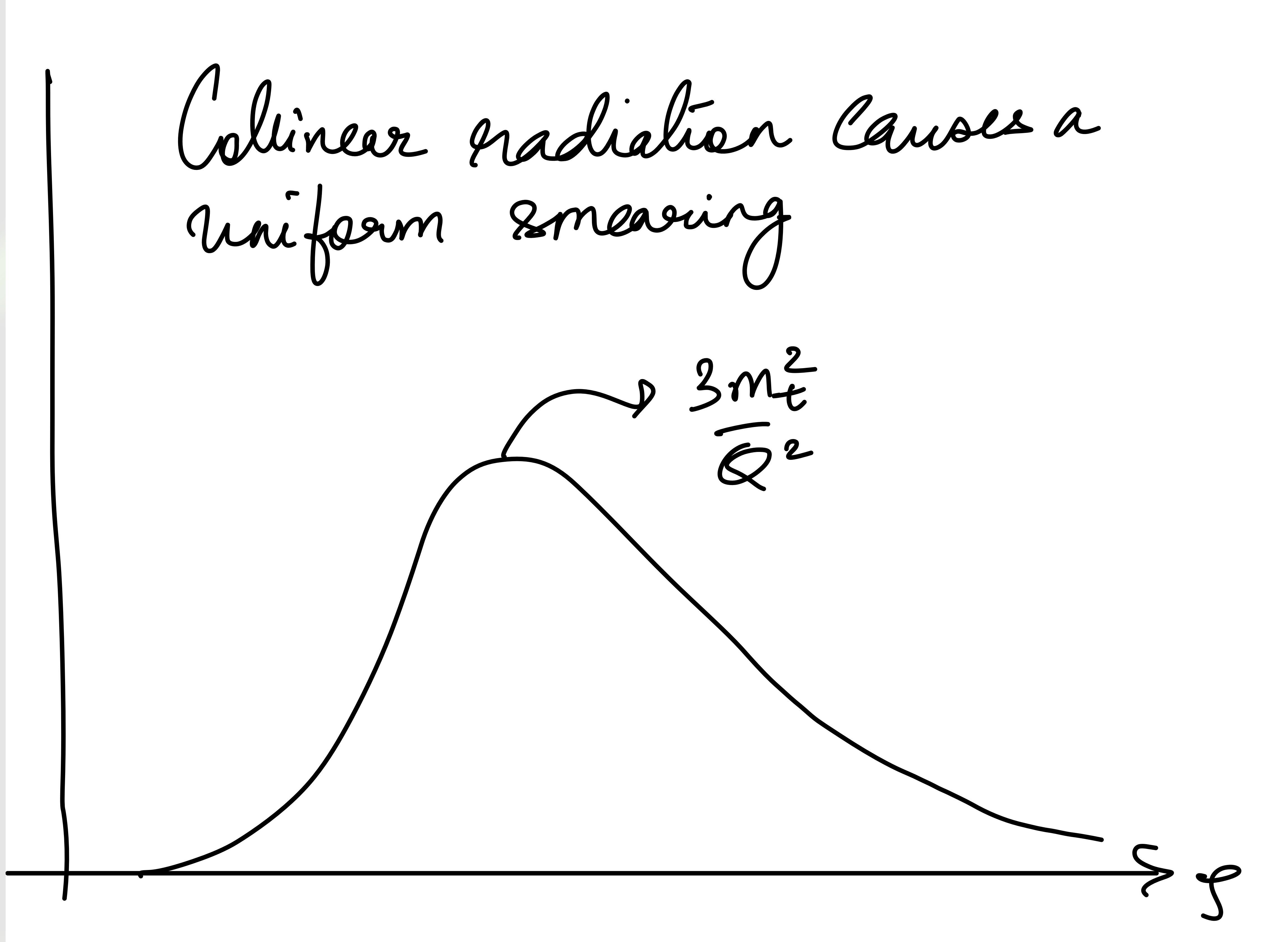




Understanding the distribution

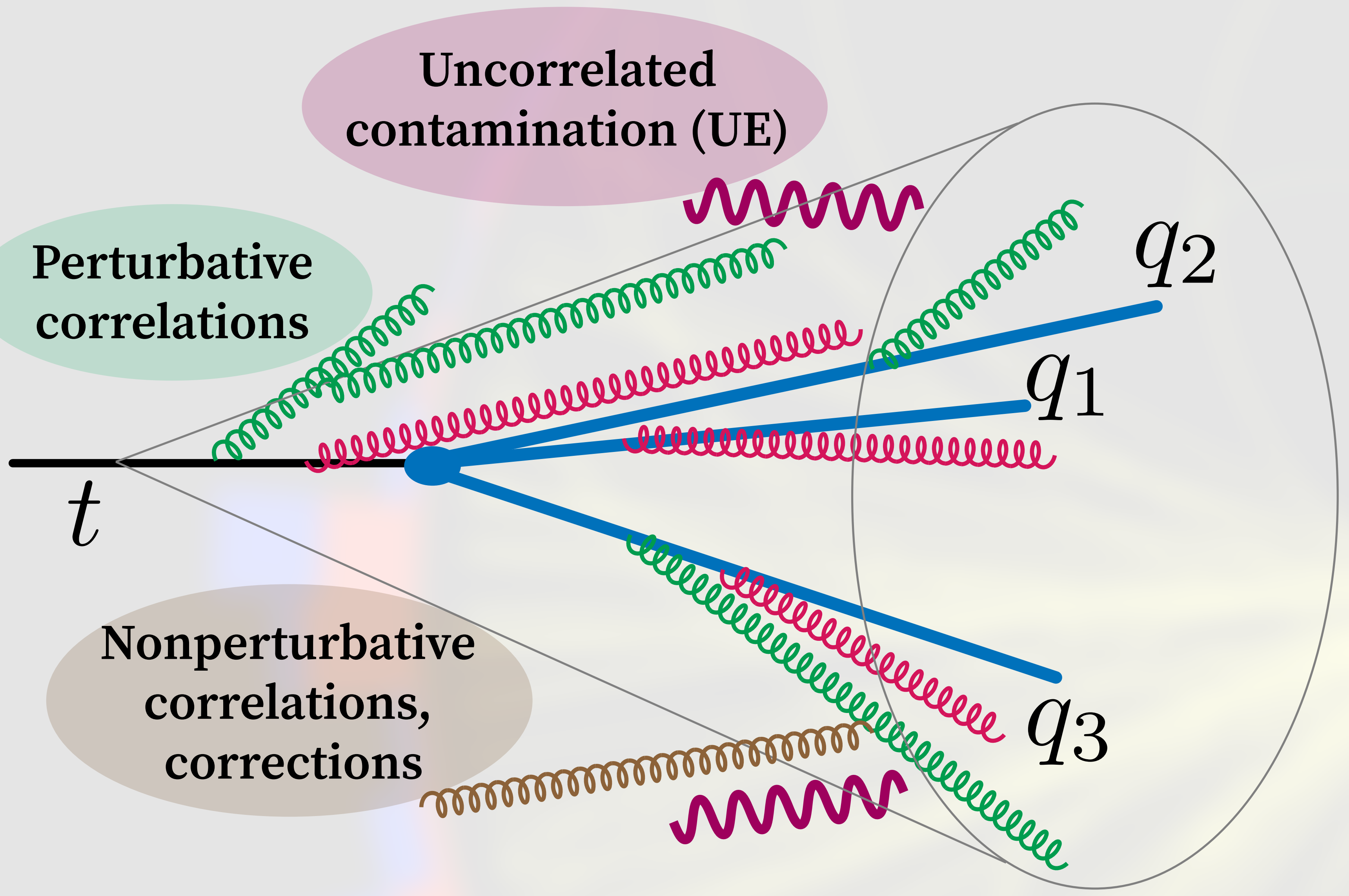
What does the distribution look like with higher order corrections?

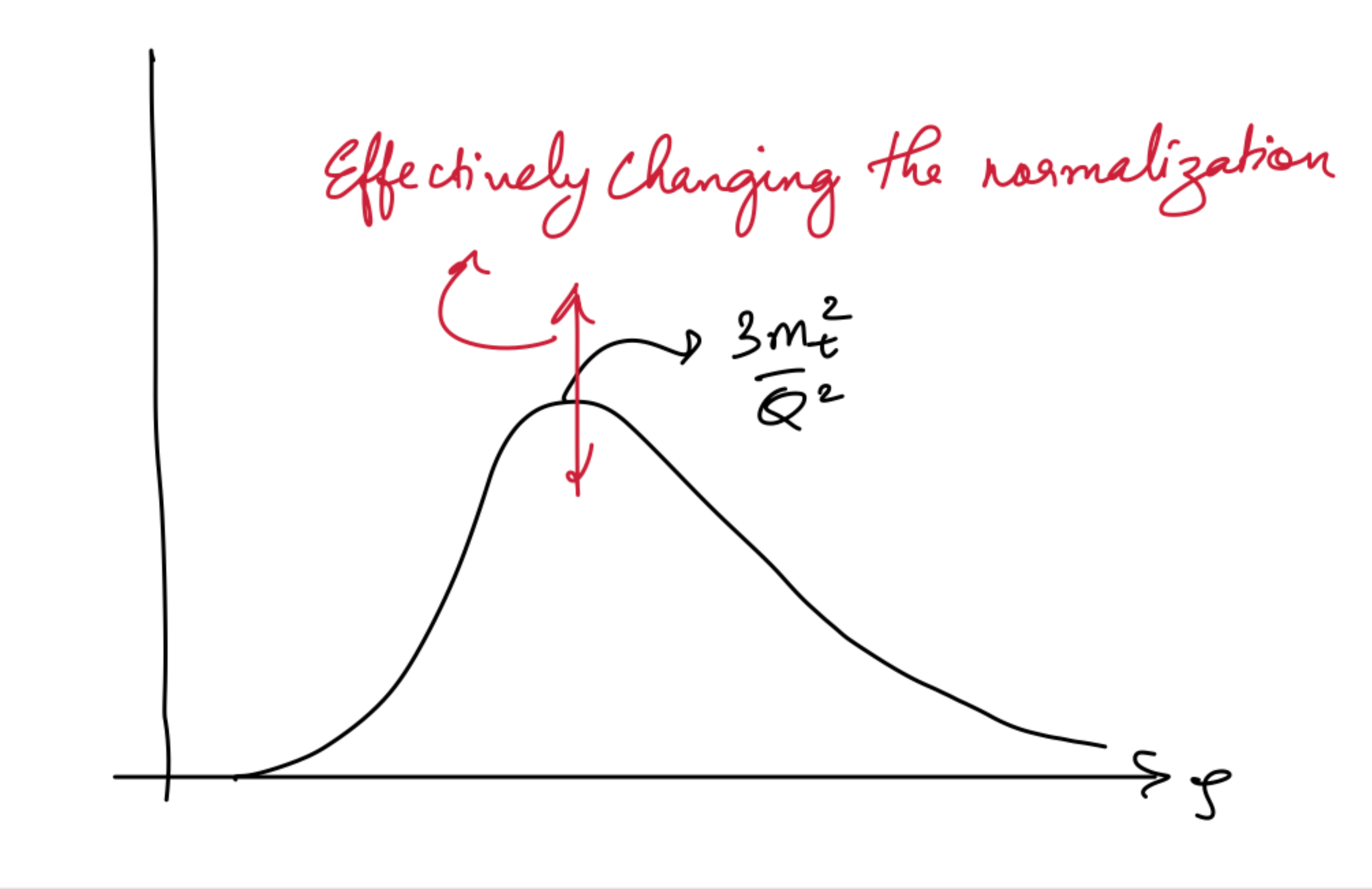




Understanding the distribution

What does the distribution look like with nonperturbative corrections?



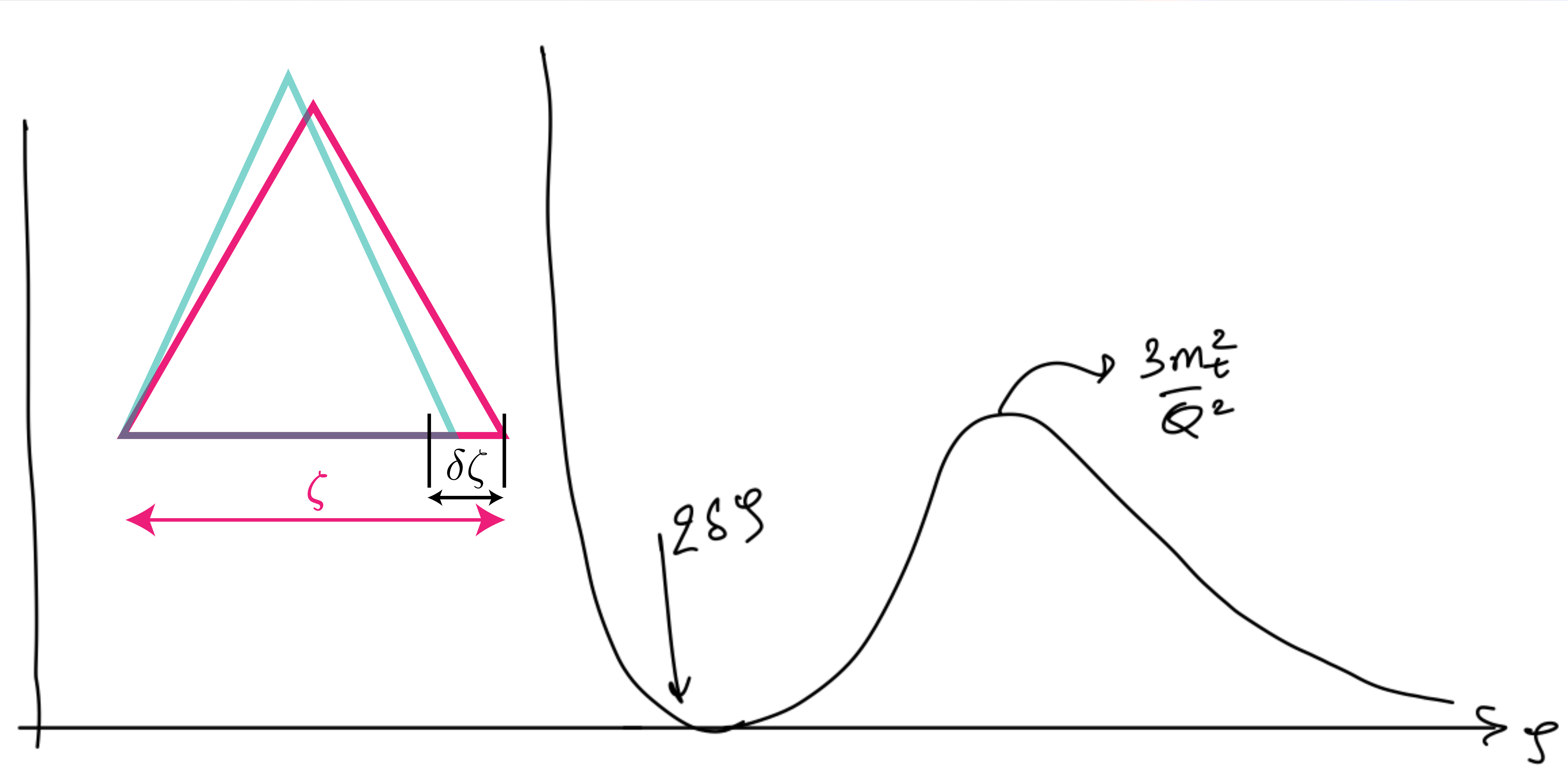


We know from other studies of energy correlators that NP corrections are an additive power law

hep-ph/9902341
hep-ph/9411211
hep-ph/9708346

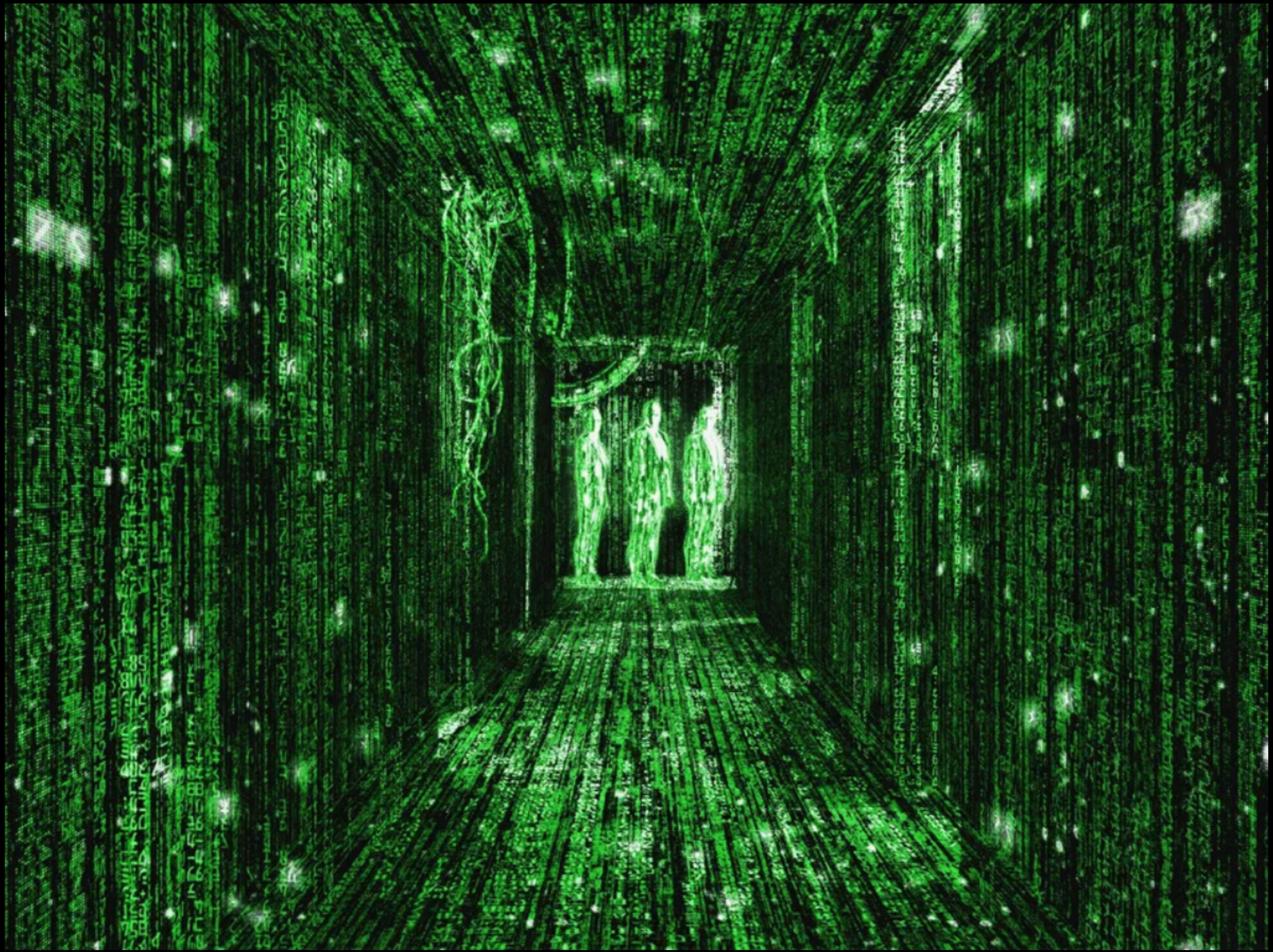
Understanding the distribution

What is the effect of asymmetry cut?

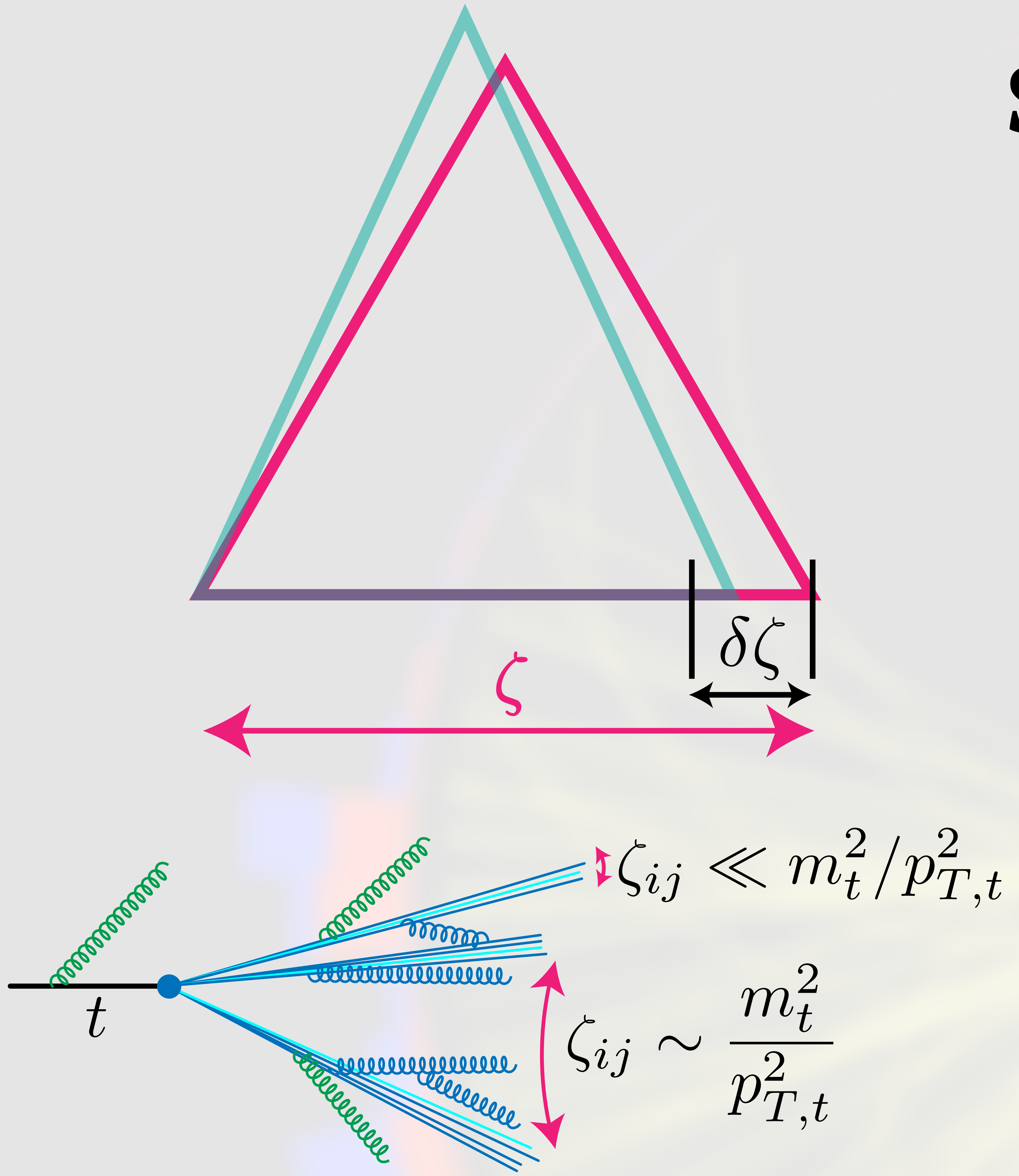


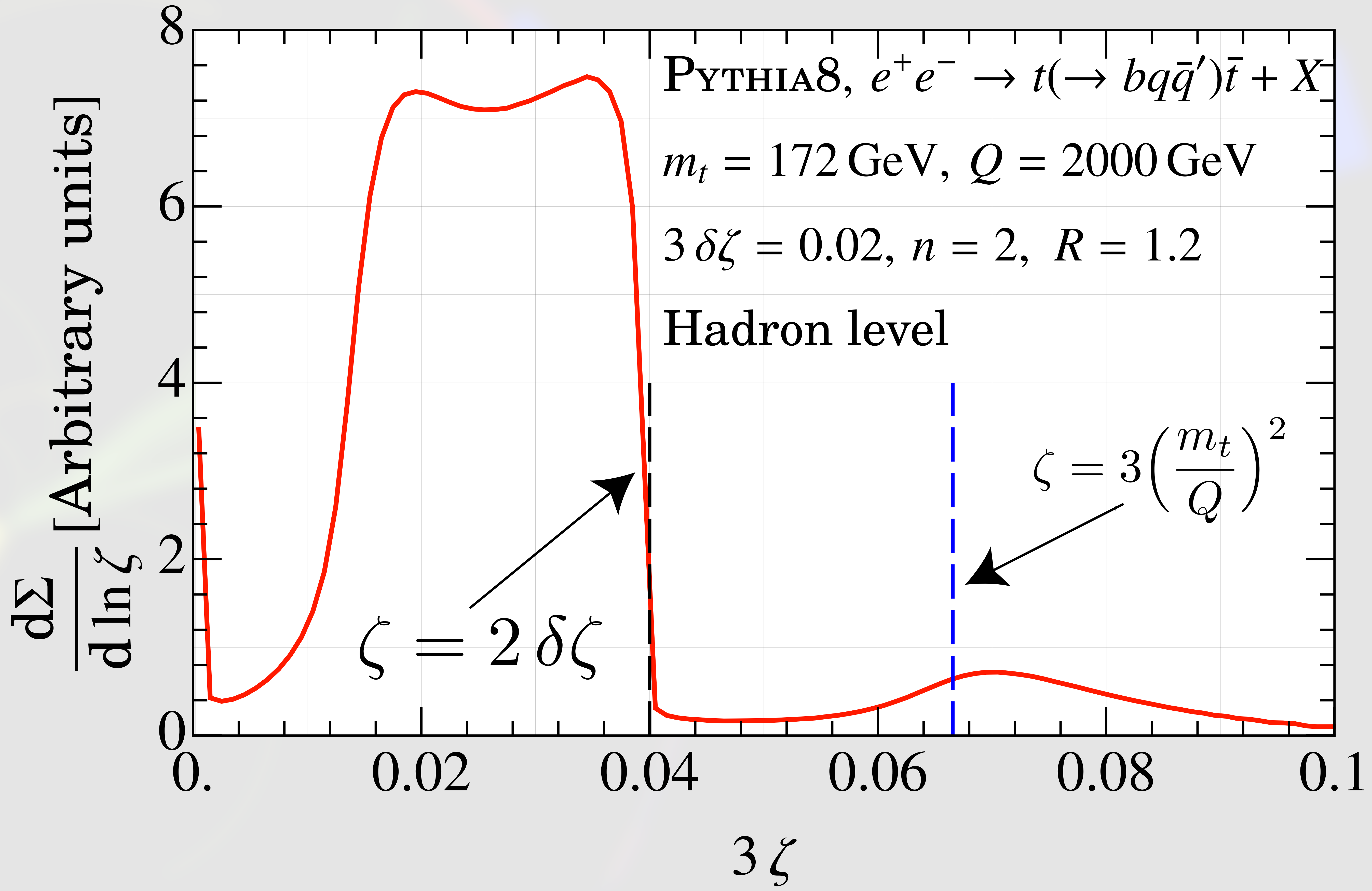
Enough sketching!

Let us simulate



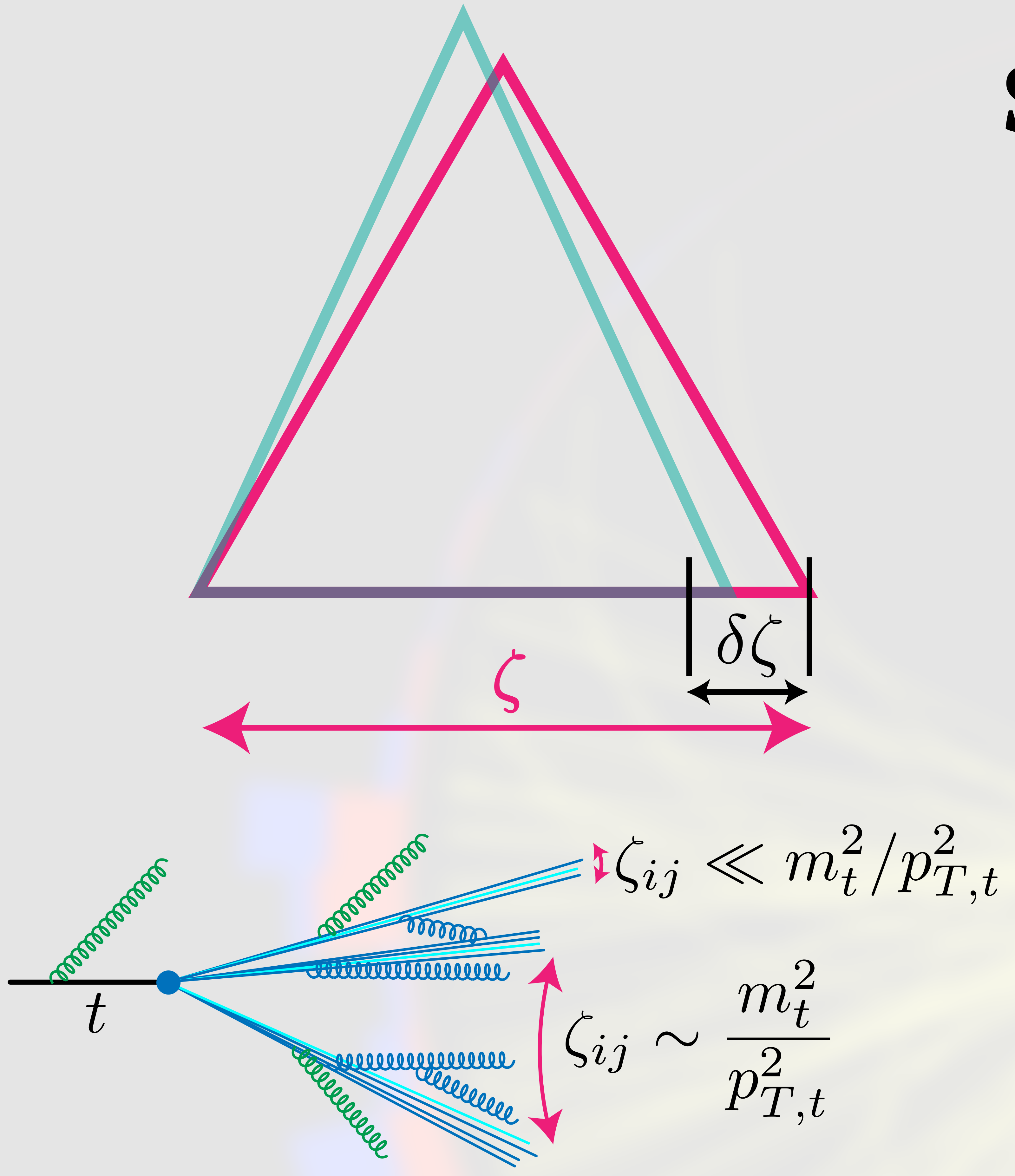
Simulation in PYTHIA8

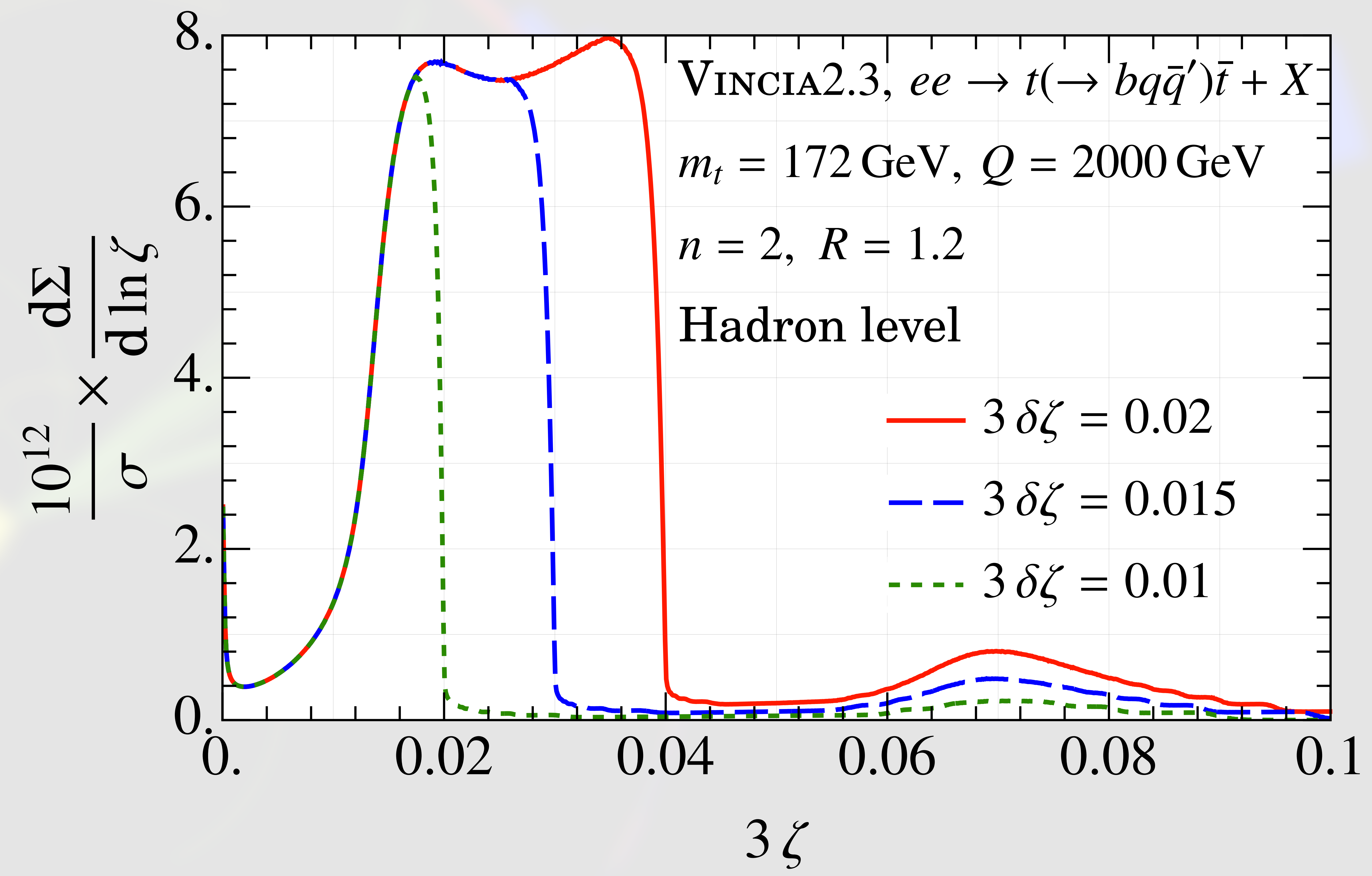




1. Distinct peak at $\zeta \sim 3(m_t/Q)^2$: peak dominated by hard decay of the top
2. Resilient to collinear radiation, $\alpha_s \ln \zeta_{\text{peak}} < 1$: fixed order perturbation theory sufficient

Simulation in PYTHIA8

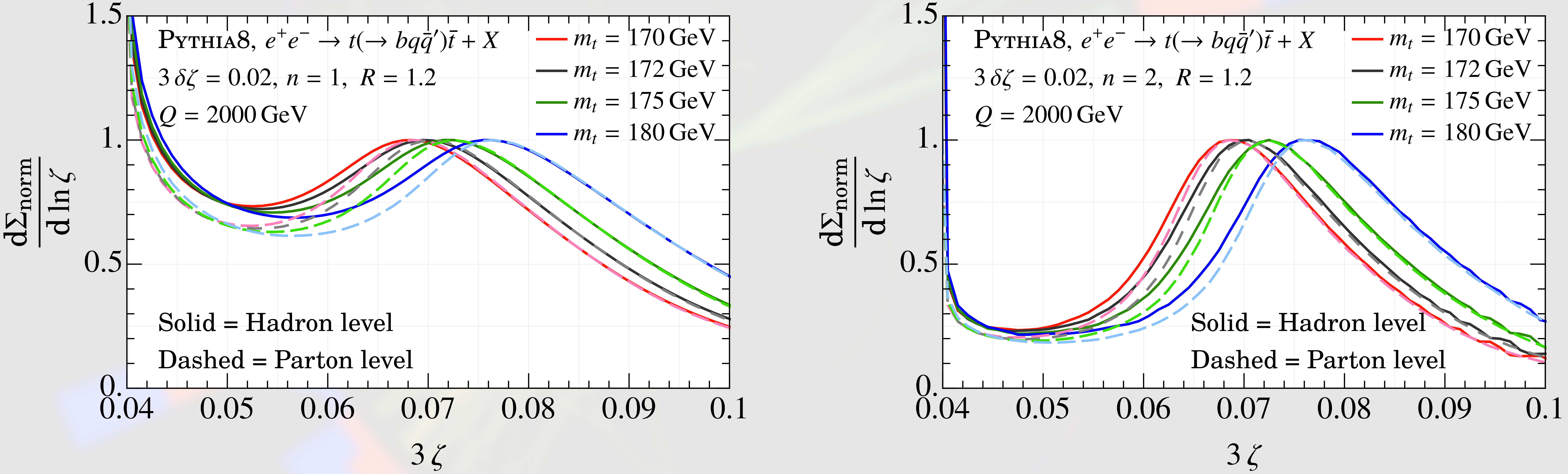




1. Asymmetry cut creates a sharp cutoff and makes the top peak visible. No hierarchy required.
2. Impact on statistics: $d\Sigma/d\zeta \approx 4(\delta\zeta)^2 G^{(n)}(\zeta,\zeta,\zeta;m_t)$

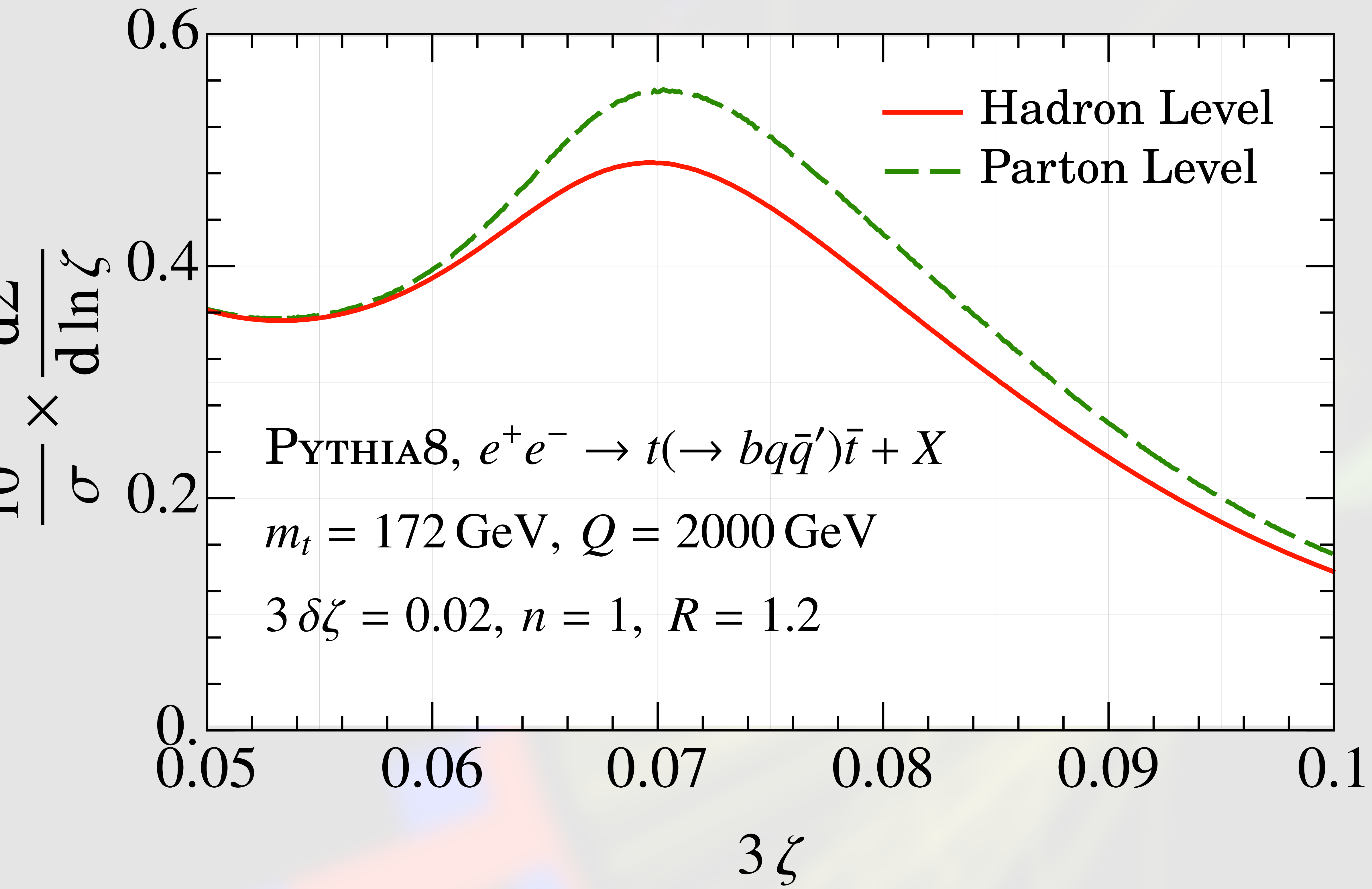
Excellent top mass sensitivity

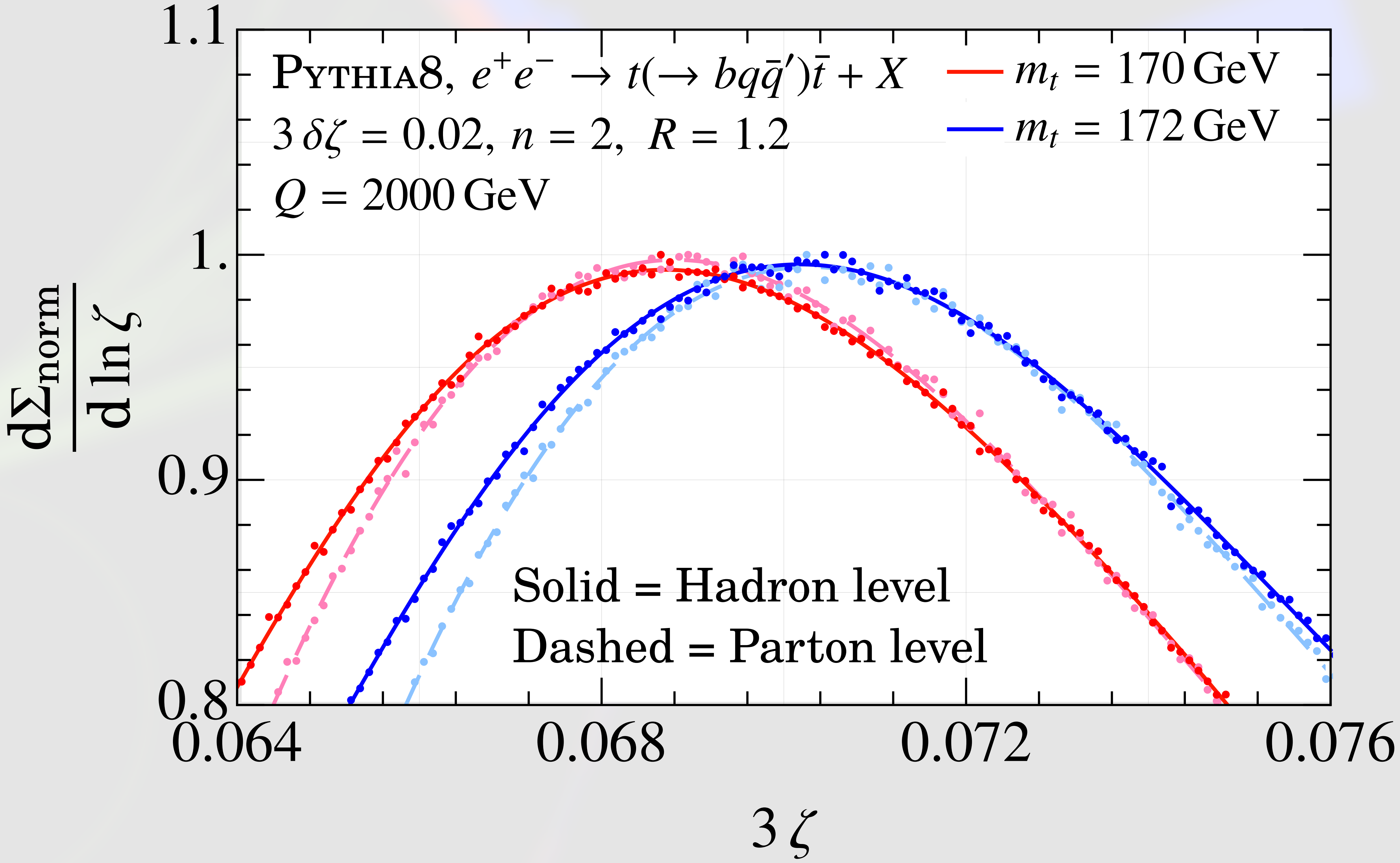
$$\widehat{\mathcal{M}}^{(n)}(\zeta_{12},\zeta_{23},\zeta_{31}) = \sum_{i,j,k} \frac{E_i^n E_j^n E_k^n}{Q^{3n}} \delta\left(\zeta_{12} - \hat{\zeta}_{ij}\right) \delta\left(\zeta_{23} - \hat{\zeta}_{ik}\right) \delta\left(\zeta_{31} - \hat{\zeta}_{jk}\right)$$



$n = 2$ is not IRC safe: absorb IRC sensitive pieces in moments of fragmentation function

Hadronization corrections



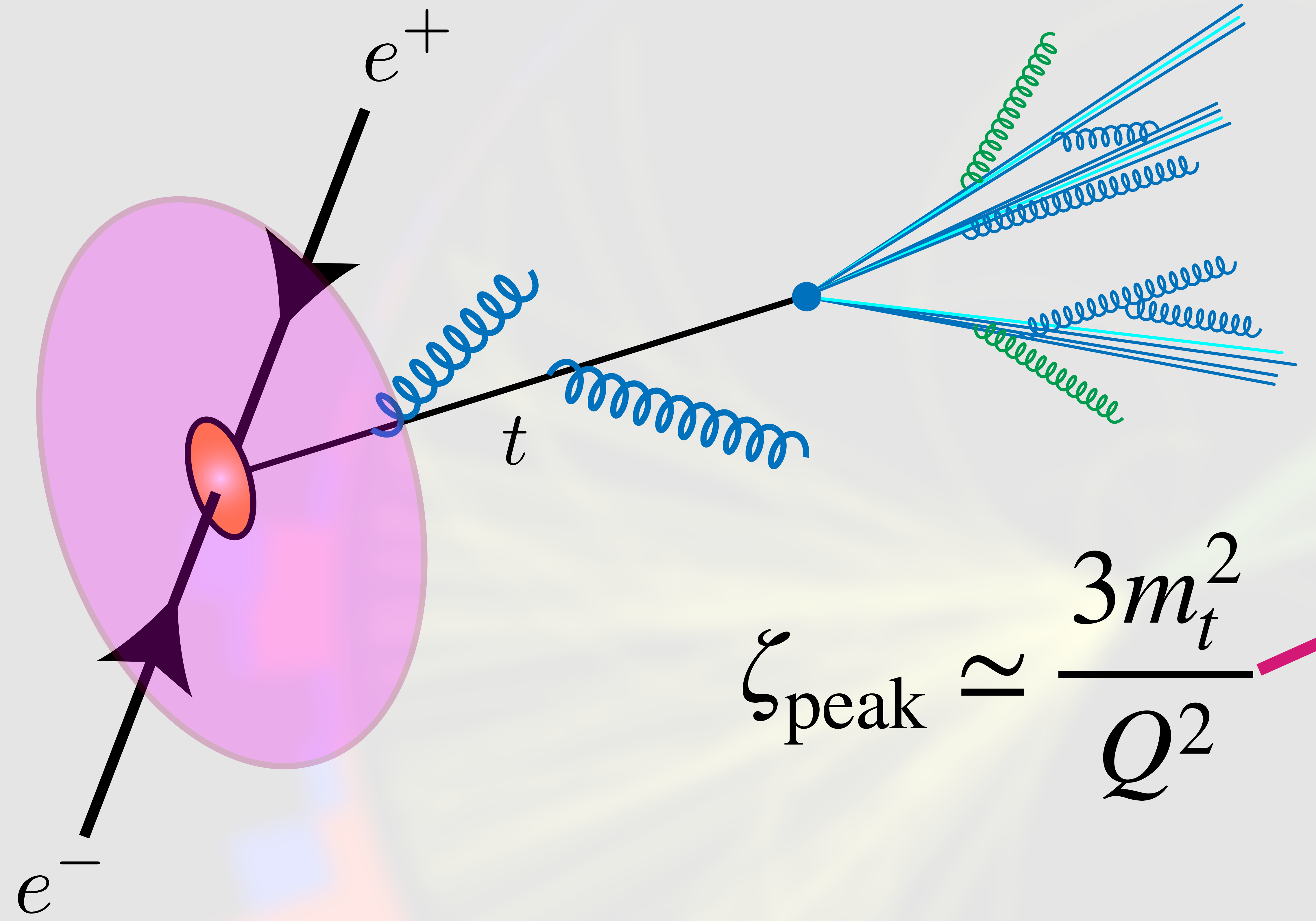


1. Nonperturbative effects enter as an additive power law: not a shift as in the case of jet mass

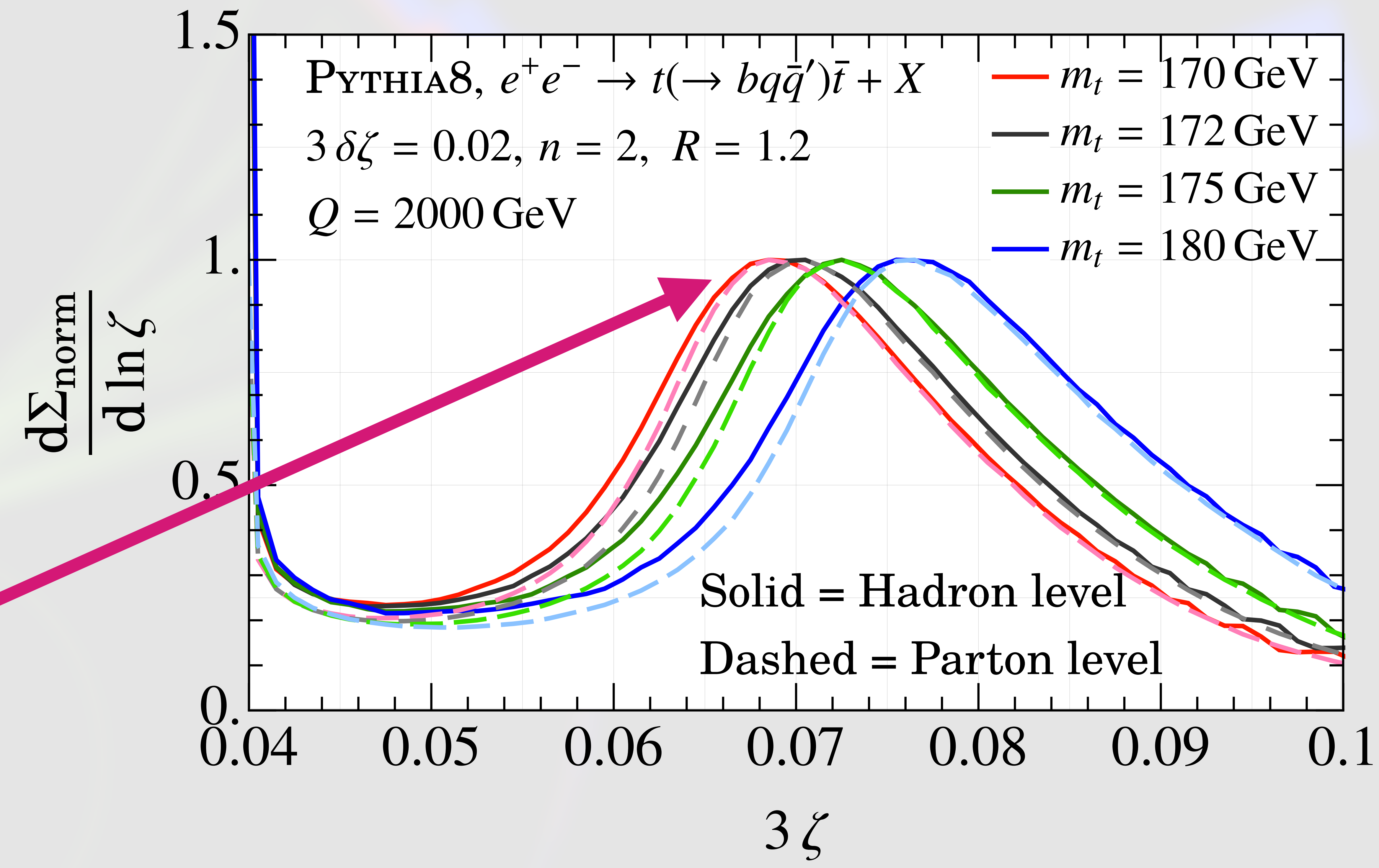
2. Normalized distribution: small effect on the peak, $\Delta m_t^{\rm Had} \approx 150 \pm 50\,\mathrm{MeV}$

EEEC on tops at the LHC

We are in fact sensitive to the production mechanism



$$\zeta_{\text{peak}} \simeq \frac{3m_t^2}{Q^2}$$

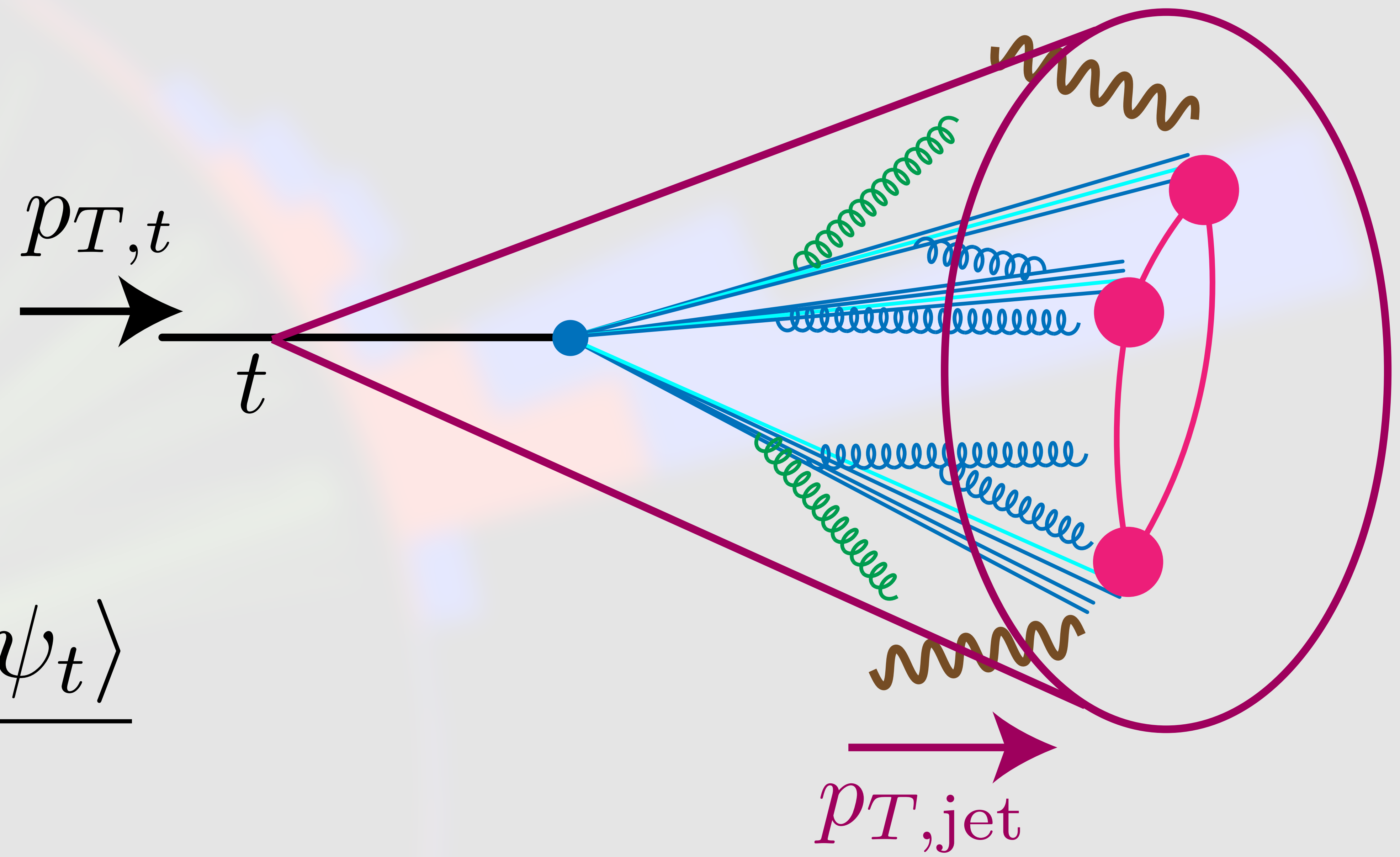


$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle_t \equiv \frac{\langle \psi_t|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)|\psi_t\rangle}{\langle \psi_t|\psi_t\rangle}$$

For e^+e^- collisions we can define a state via a local operator $\mathcal{O}$: $|\psi_t\rangle = \mathcal{O}\,|0\rangle$, and produce tops with definite velocity Q/m_t

Energy correlators at hadron colliders

Let us take a closer look at the definition of the correlator:



$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle_t \equiv \frac{\langle \psi_t|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)|\psi_t\rangle}{\langle \psi_t|\psi_t\rangle}$$

At hadron colliders we have something like:

$$|\psi_t\rangle_{pp} = \left|\text{An anti-}k_T\text{ jet with } R = 1.2 \text{ and } p_{T,\text{jet}} \in [600{,}650]\text{ GeV}\right\rangle$$

Here we need jets to specify the state

Implications for hadron colliders

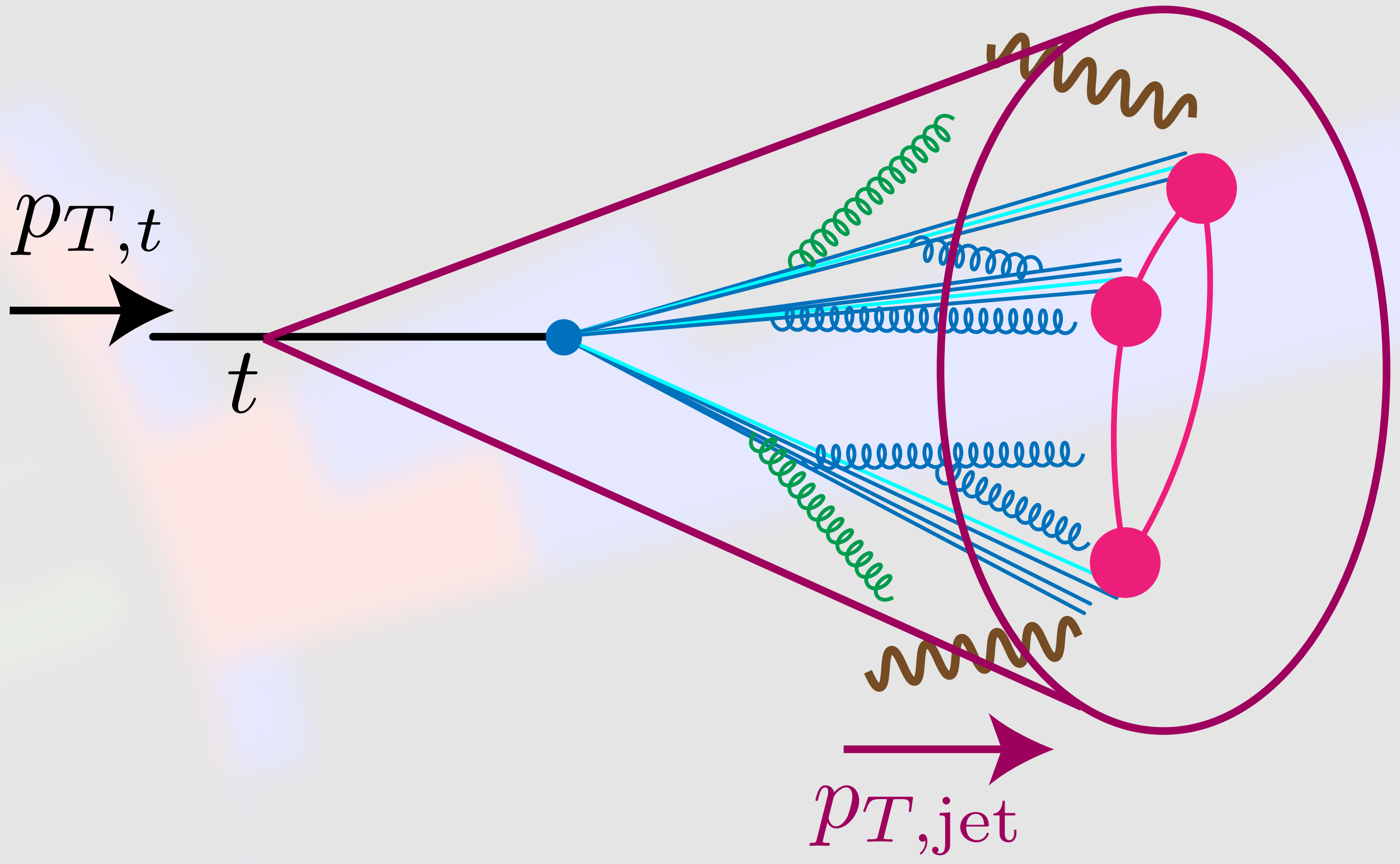
At the LHC we also have soft junk from the underlying event

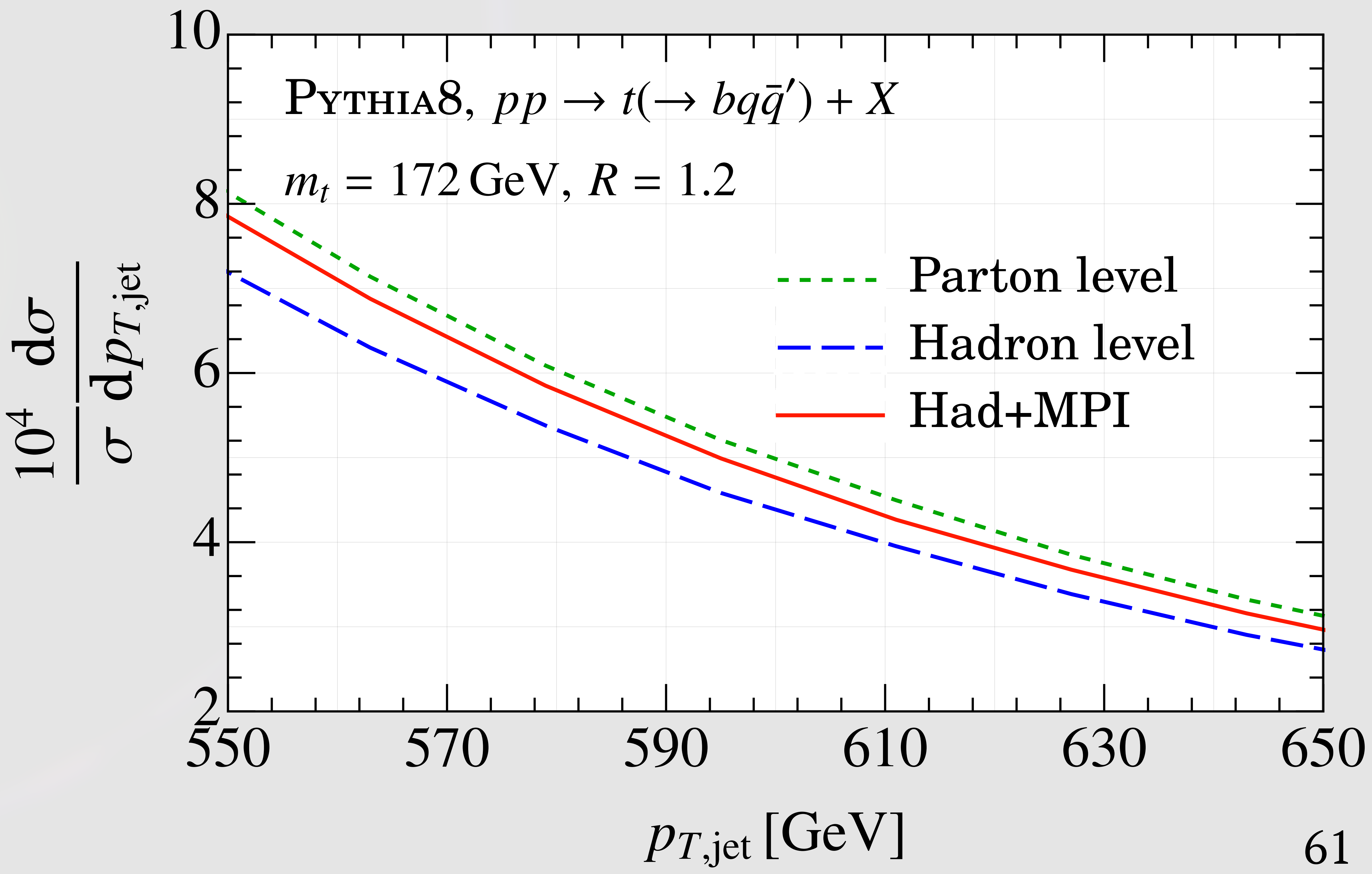
Q1. How does adding UE impact the observable?

We can only indirectly constrain top velocity through $p_{T,\mathrm{jet}}$

Q2. How do shifts in $p_{T,\mathrm{jet}}$ impact the state $|\psi_t\rangle$ and the EEEC measurement?

$$\frac{\langle\psi_t|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)|\psi_t\rangle}{\langle\psi_t|\psi_t\rangle}$$



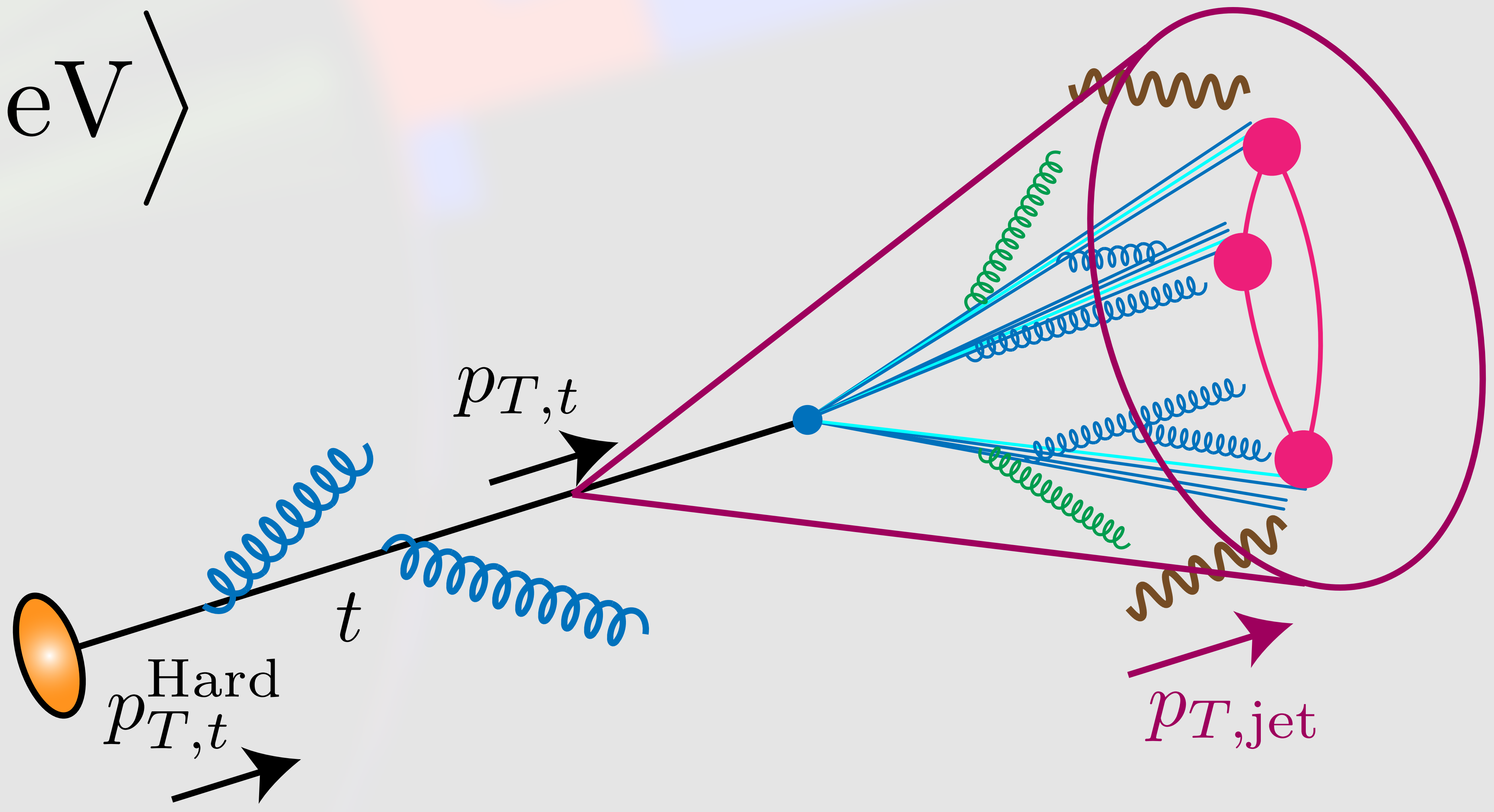


Q1: What is the impact of the underlying event?

For now fix the top quark velocity in *pp* but include underlying event and consider a *(unphysical)* state of hard tops with a definite velocity:

$$|\psi_t\rangle_{pp} = \left|\text{Tops produced with } p_{T,t}^{\text{hard}} = 600\text{ GeV}\right\rangle$$

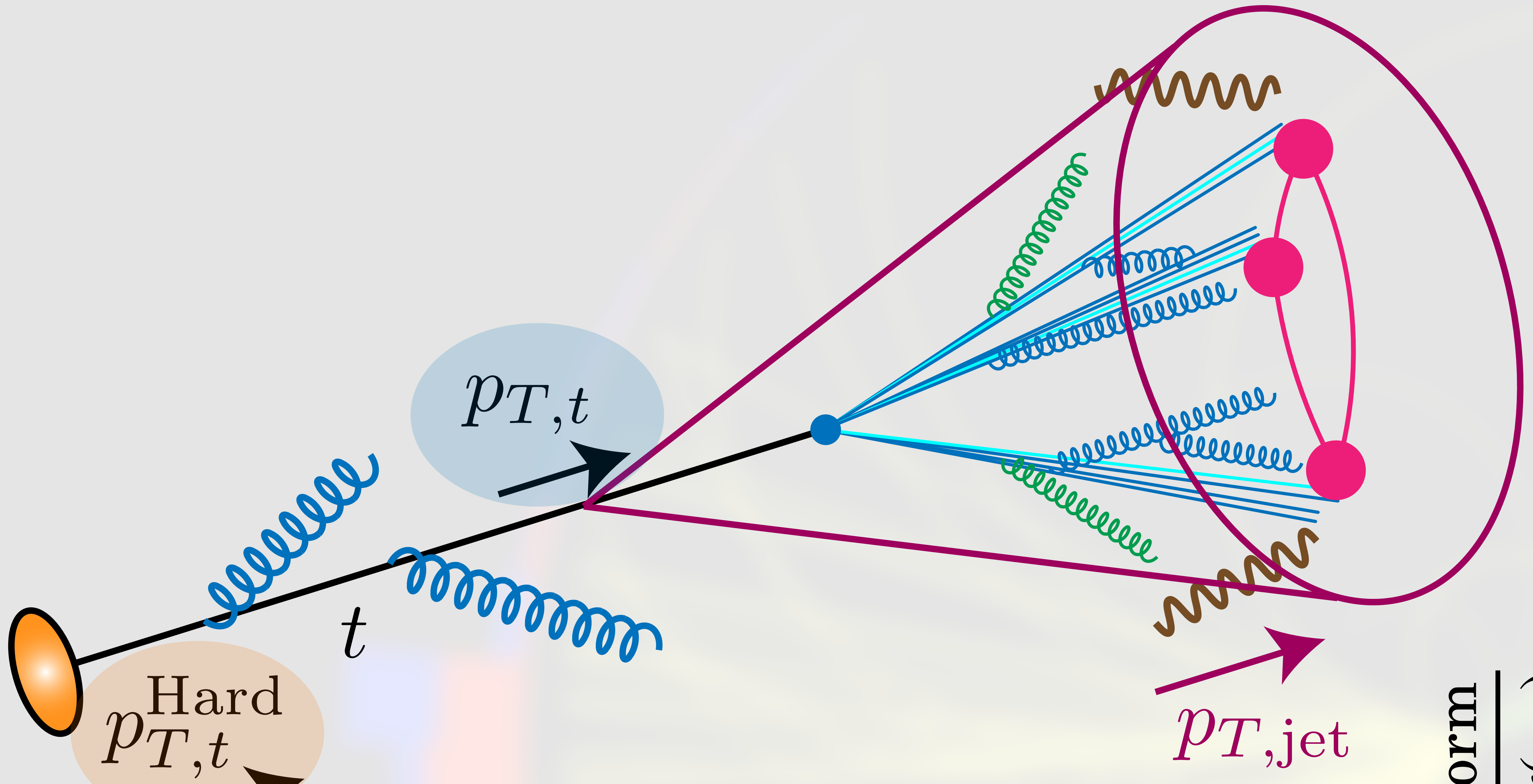
The underlying event still impacts the $p_{T,\text{jet}}$ and adds additional uncorrelated soft radiation to the measurement.



Use $p_{T,\text{jet}}$ in the energy weights

$$\widehat{\mathcal{M}}_{(pp)}^{(n)}(\zeta_{12},\zeta_{23},\zeta_{31}) = \sum_{i,j,k\,\in\,\text{jet}} \frac{(p_{T,i})^n (p_{T,j})^n (p_{T,k})^n}{(p_{T,\text{jet}})^{3n}} \delta\left(\zeta_{12} - \hat{\zeta}_{ij}^{(pp)}\right) \delta\left(\zeta_{23} - \hat{\zeta}_{ik}^{(pp)}\right) \delta\left(\zeta_{31} - \hat{\zeta}_{jk}^{(pp)}\right)$$

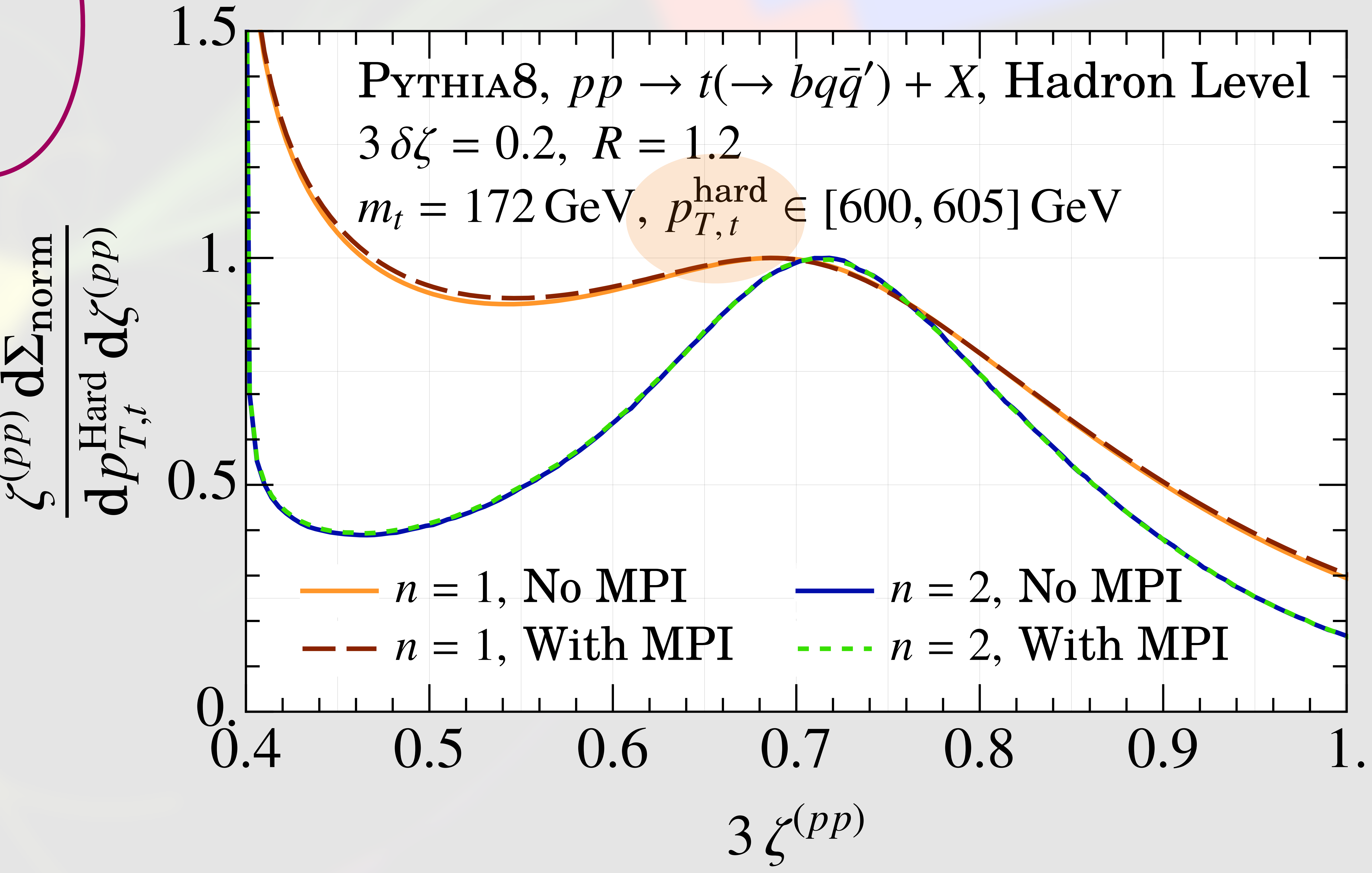
A: Correlators themselves are insensitive to the UE!



$$\zeta_{\text{peak}}^{(pp)} \approx 3\left(\frac{m_t}{p_{T,t}}\right)^2$$

Not $p_{T,\text{jet}}$!

$$|\psi_t\rangle_{pp} = \left|\text{Tops produced with } p_{T,t}^{\text{hard}} = 600 \text{ GeV}\right\rangle$$

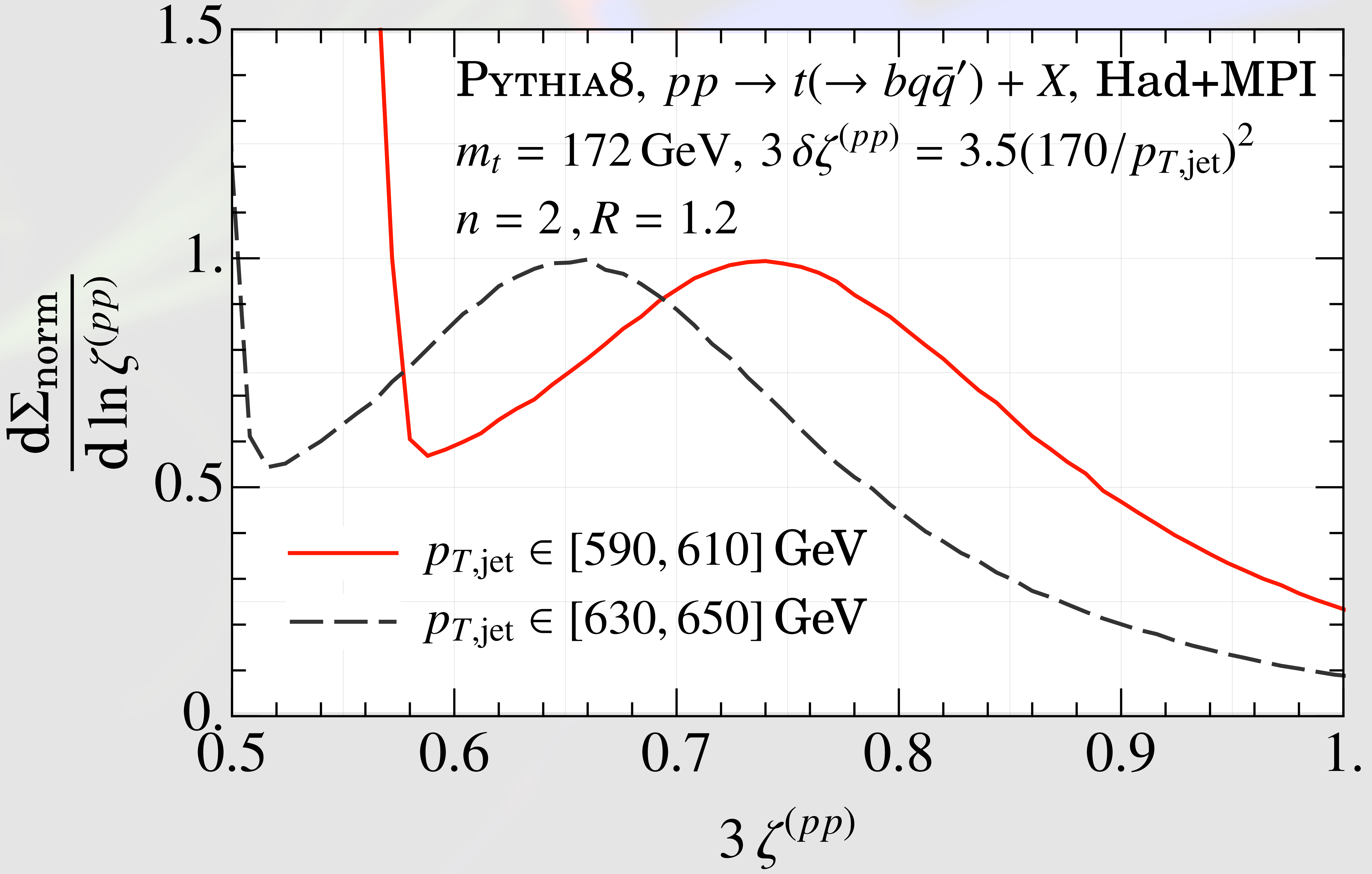


Q2. How to deal with shifts in jet p_T impacting $|\psi_t\rangle$?

Write the measurement as

$$\frac{d\Sigma(\delta\zeta)}{dp_{T,\text{jet}}\,d\zeta} = \boxed{\frac{d\Sigma(\delta\zeta)}{dp_{T,t}\,d\zeta}}\,\frac{dp_{T,t}}{dp_{T,\text{jet}}}$$

Completely insensitive to the underlying event



Only need to characterize the nonperturbative effects on the hard scale $p_{T,\text{jet}}$

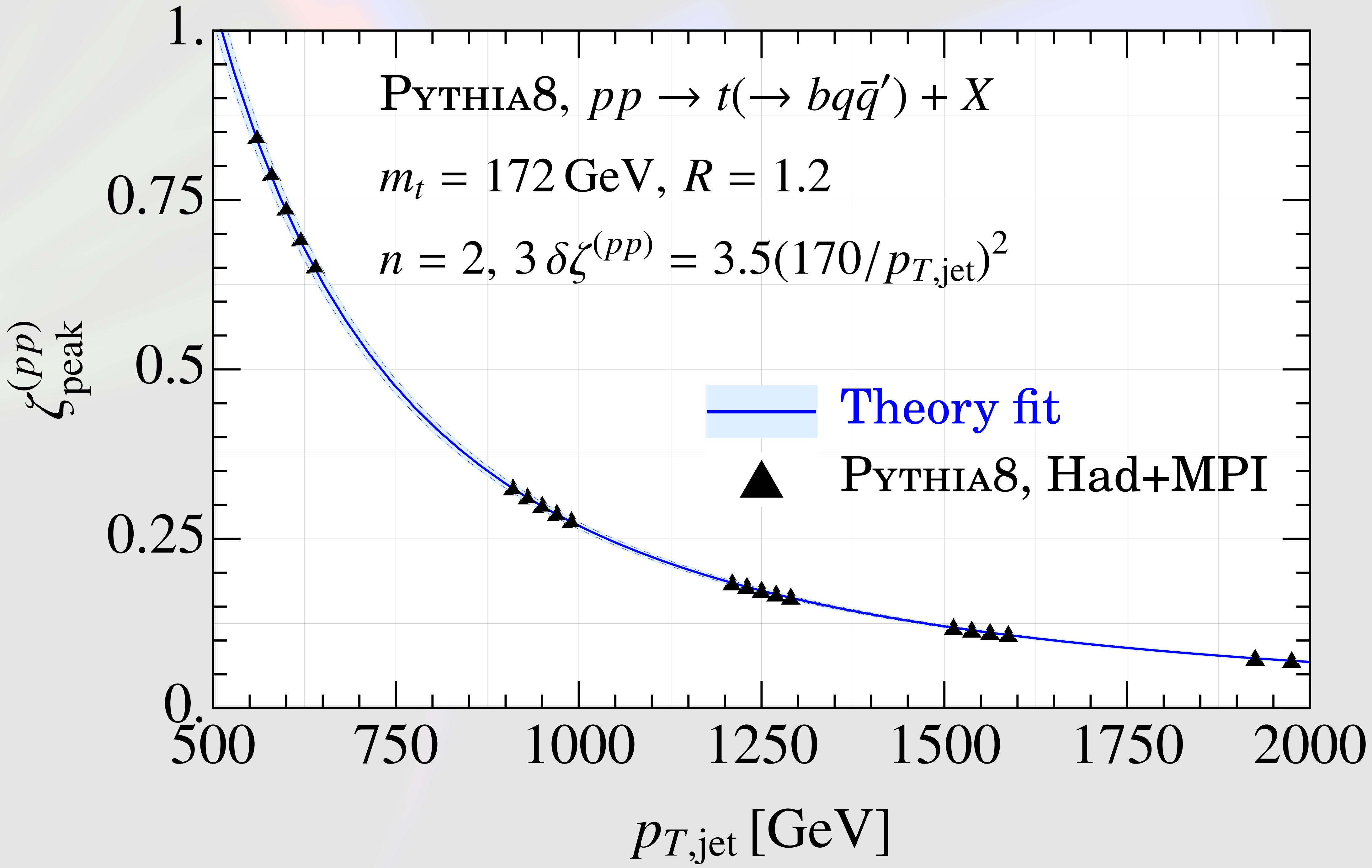
A: Disentangle by considering multiple p_T bins

Unlike jet mass, $p_{T,\mathrm{jet}}$ shifts impact the peak nonlinearly

$$\zeta_{\mathrm{peak}}^{(pp)} = \frac{3F_{\mathrm{pert}}(m_t, p_{T,\mathrm{jet}}, \alpha_s, R)}{\left(p_{T,\mathrm{jet}} + \Delta_{\mathrm{NP}}(R) + \Delta_{\mathrm{MPI}}(R)\right)^2}$$

At leading order $F_{\mathrm{pert}}^{\mathrm{LO}} = m_t^2$

Determine $\Delta_{\mathrm{NP}}(R)$ and $\Delta_{\mathrm{MPI}}(R)$ independently from the $p_{T,\mathrm{jet}}$ spectrum



Pythia8 m_t	Parton $\sqrt{F_{\mathrm{pert}}}$	Hadron + MPI $\sqrt{F_{\mathrm{pert}}}$
172 GeV	172.6 ± 0.3 GeV	$172.3 \pm 0.2 \pm 0.4$ GeV
173 GeV	173.5 ± 0.3 GeV	$173.6 \pm 0.2 \pm 0.4$ GeV
175 GeV	175.5 ± 0.4 GeV	$175.1 \pm 0.3 \pm 0.4$ GeV
173 – 172	0.9 ± 0.4 GeV	1.3 ± 0.6 GeV
175 – 172	2.9 ± 0.5 GeV	2.8 ± 0.6 GeV

A promising evidence for complete theoretical control of the top mass up to errors $\lesssim 1$ GeV!

Outlook

Future improvements:

1. Improve the MC analysis by optimizing for $\Delta\zeta$, binning of $p_{T,\mathrm{jet}}$ and exploring configurations other than equilateral triangle

2. A systematic study of statistical power including HL-LHC projections

Factorization theorem:

Mele, Nason 1990,1991;
Czakon et al 2102.08267

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_{T,\mathrm{jet}}\mathrm{d}\eta\,\mathrm{d}\zeta} = f_i \otimes f_j \otimes H_{i,j\to t}\left(z_J; p_{T,t} = \frac{p_{T,\mathrm{jet}}}{z_J}, \eta\right)$$

$$\otimes J_{t\to t}(z_J, z_h; R) \otimes J^{[\mathrm{tracks}]}_{\mathrm{EEEC}}(n, z_h, \zeta; m_t; \Gamma_t)$$

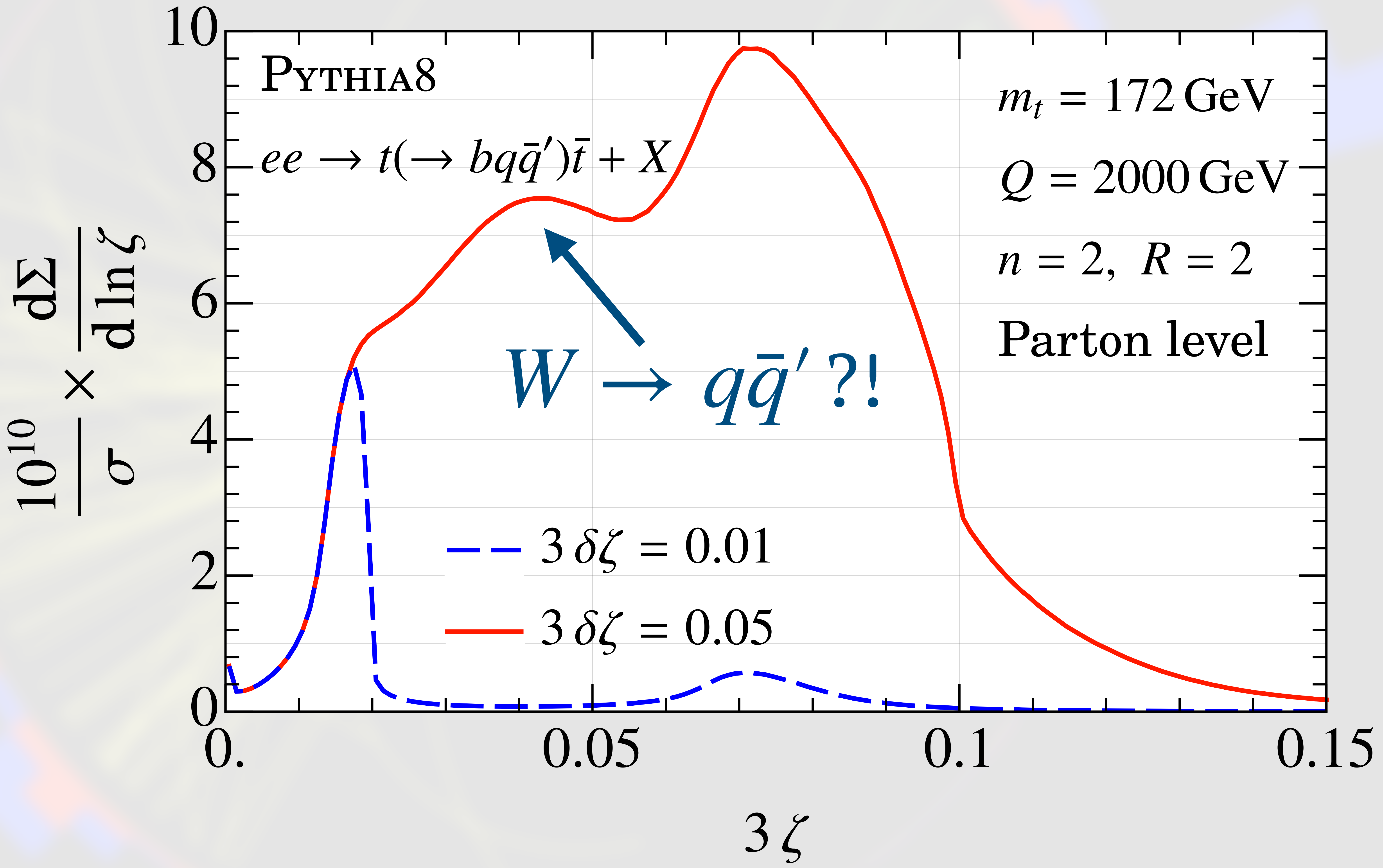
Kang, Ringer, Vitev 1606.07063

Energy correlator jet function

Ferdinand, Holguin, Moult, AP, Procura

Work in progress

Can we exploit the imprint of 2-body W decay in tops in the 3-point correlator?



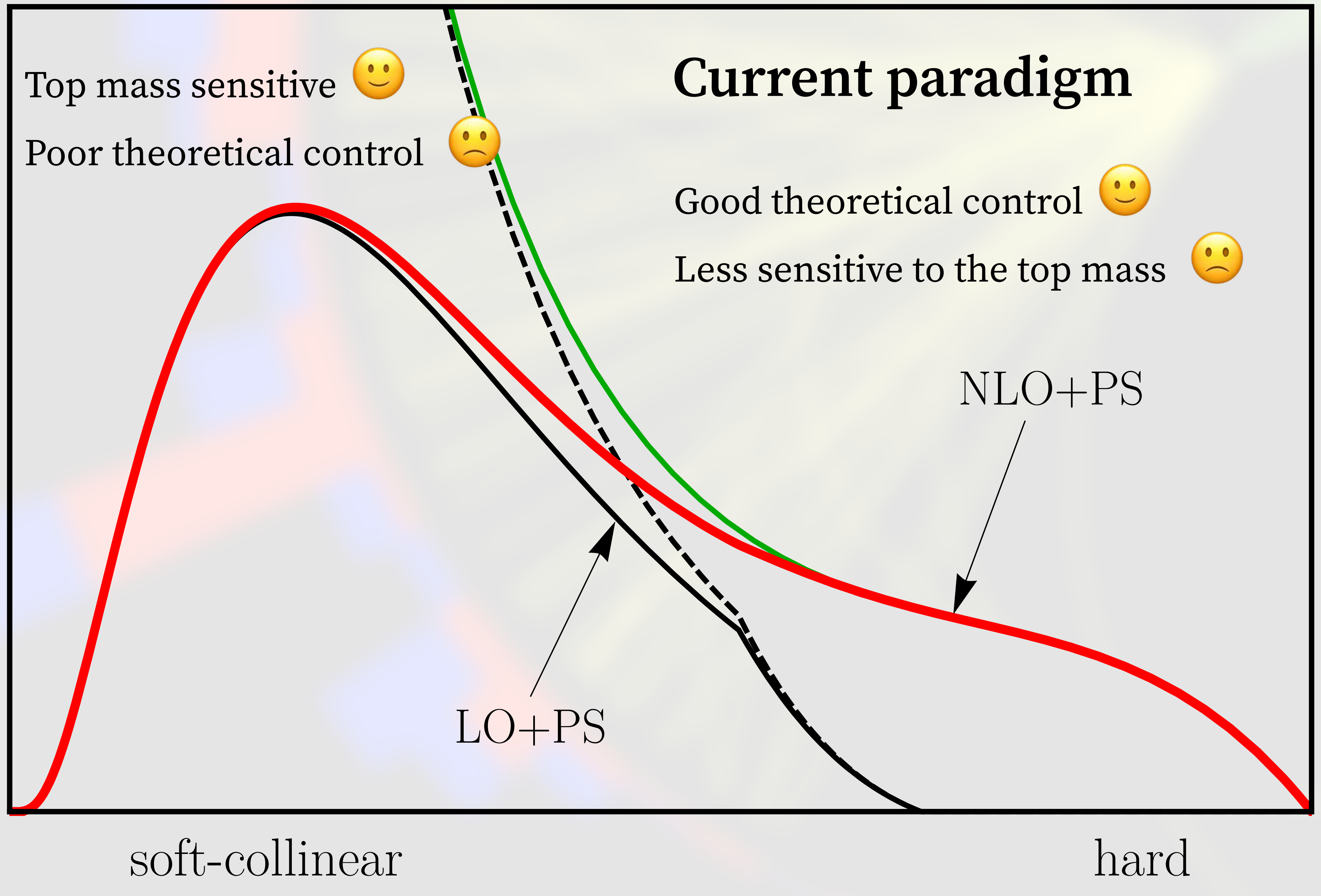
A promising way to overcome the systematics of $p_{T,\text{jet}}$ shifts due to the underlying event

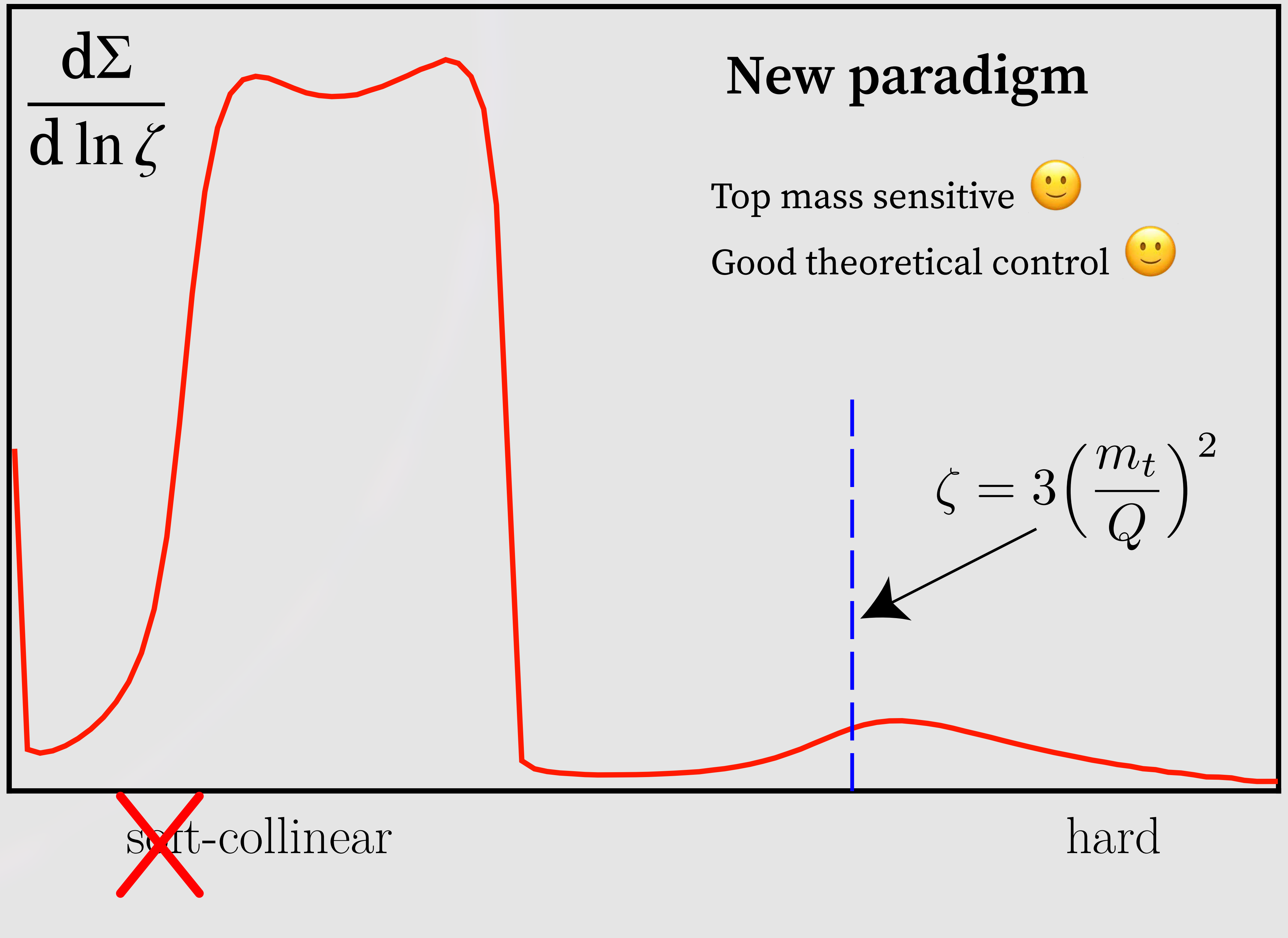
Conclusions

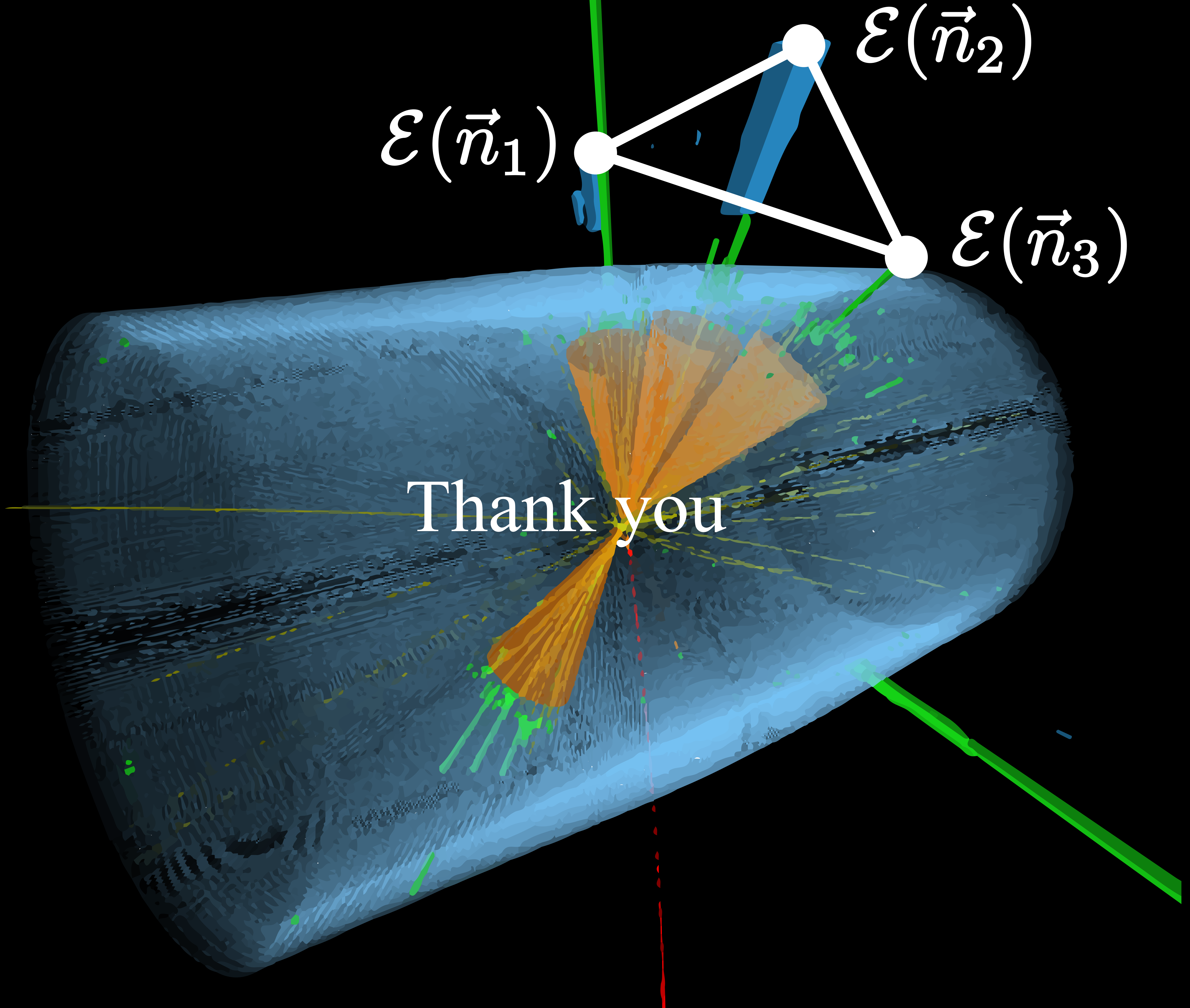
1. The 3-point correlation function gives a kinematic structure in the hard region

2. Very tiny hadronization corrections to the normalized spectrum

3. Impact on $p_{T,\mathrm{jet}}$ can be independently studied quantified.





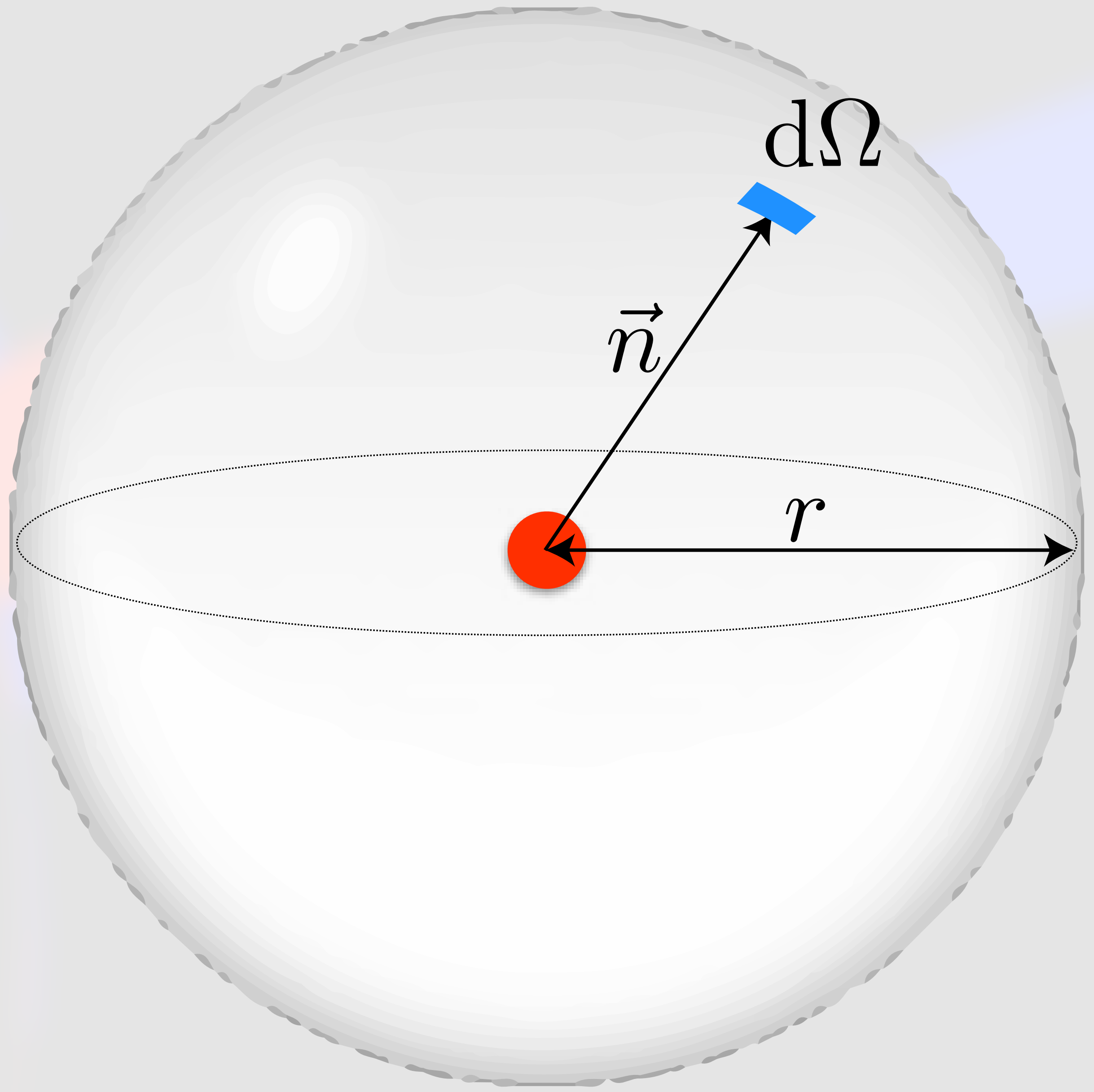

$\mathcal{E}(\vec{n}_2)$
$\mathcal{E}(\vec{n}_1)$
$\mathcal{E}(\vec{n}_3)$
Thank you

Supplementary slides

Light Ray Operators

Need light ray operators for Lorentzian signature

Sveshnikov, Tkachov hep-ph/9512370, Hofman, Maldacena; 0803.1467



$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n}) \simeq \int_0^\infty \mathrm{dt} \left(\text{Energy flux through d}\Omega\right)$$

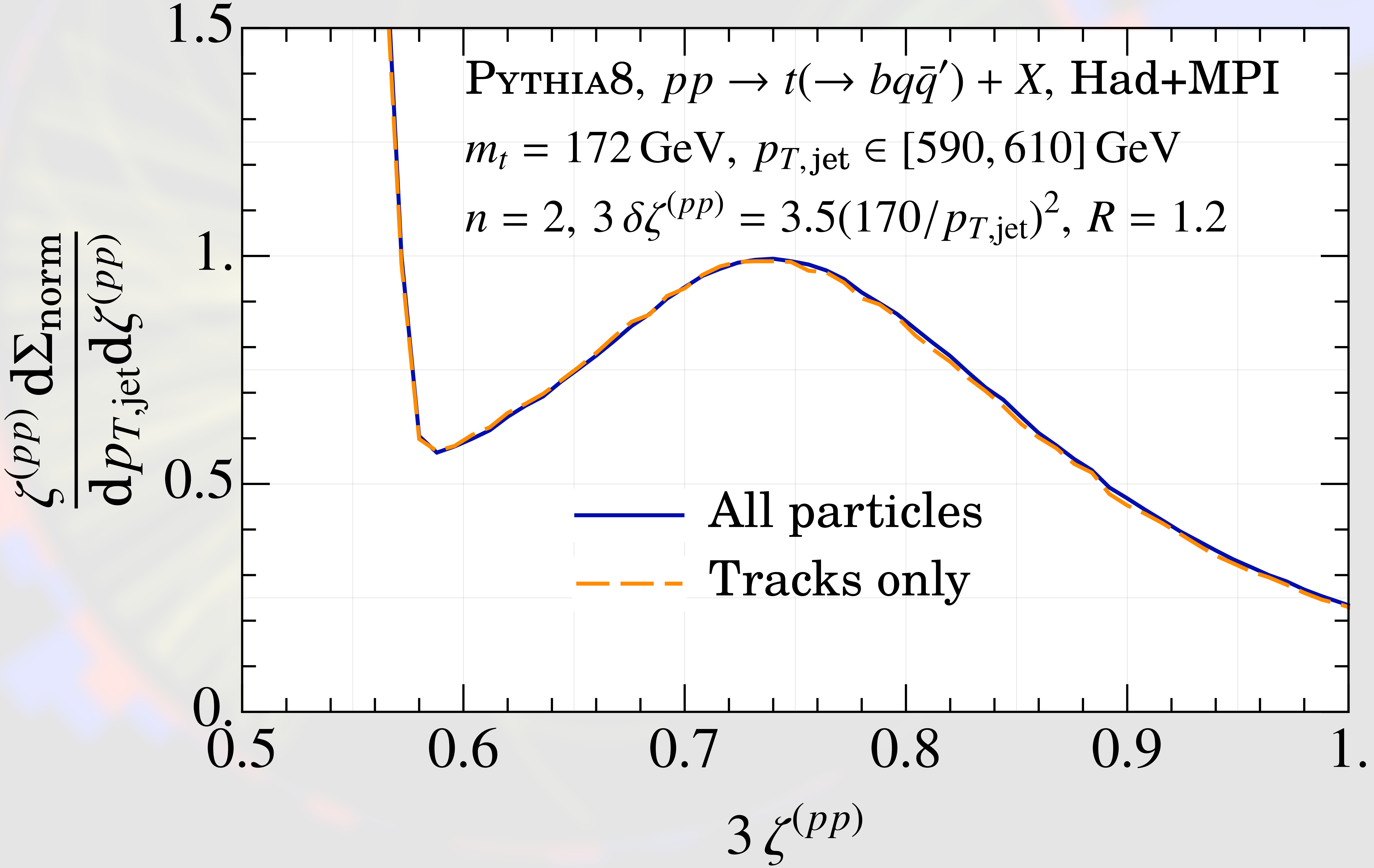
Consider correlation functions of energy flow operators:

$$\langle\psi|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots\mathcal{E}(\vec{n}_N)|\psi\rangle$$

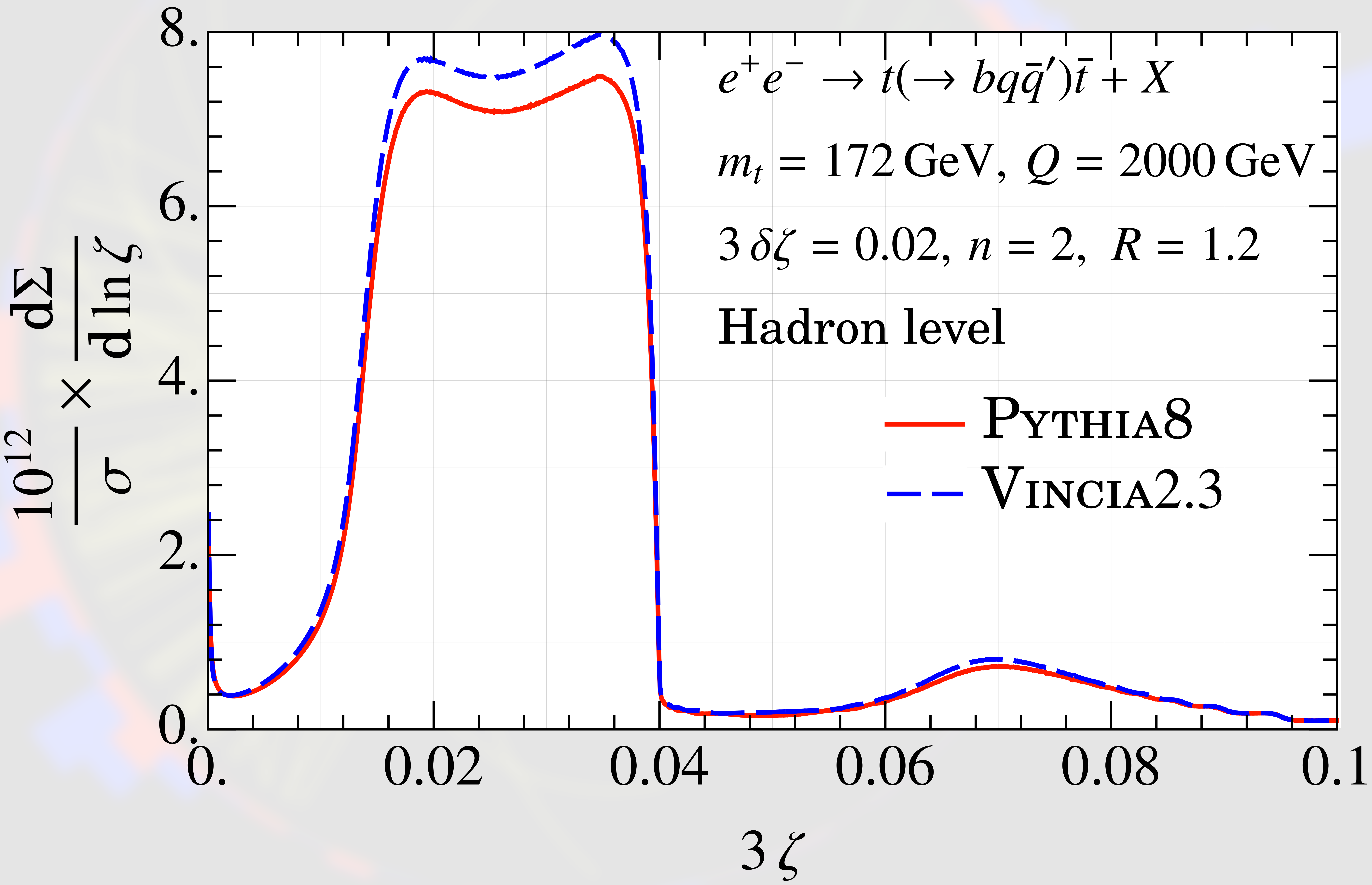
$|\psi\rangle$ specifies the state on which we measure the correlator

Measurement on tracks

The measurement is insensitive to the usage of tracks, allowing for high angular resolution.

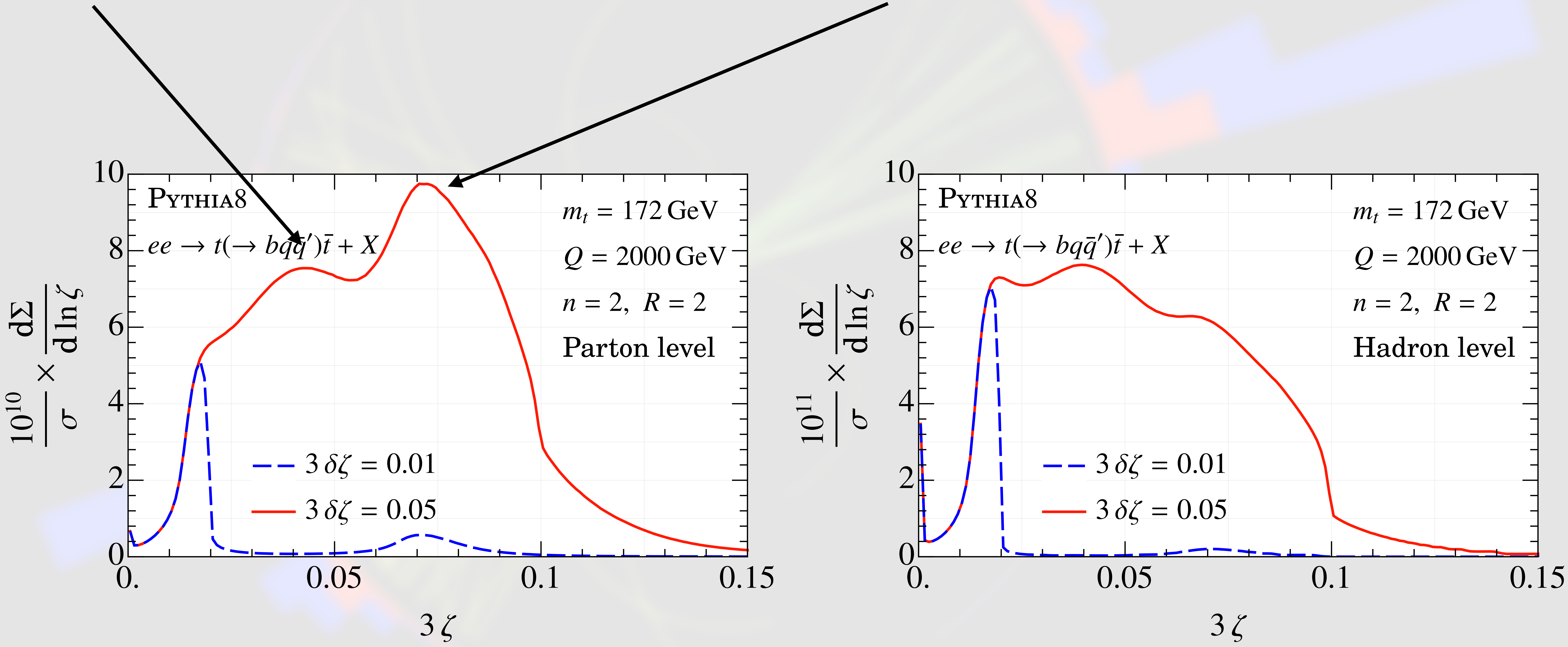


Comparison with Vincia



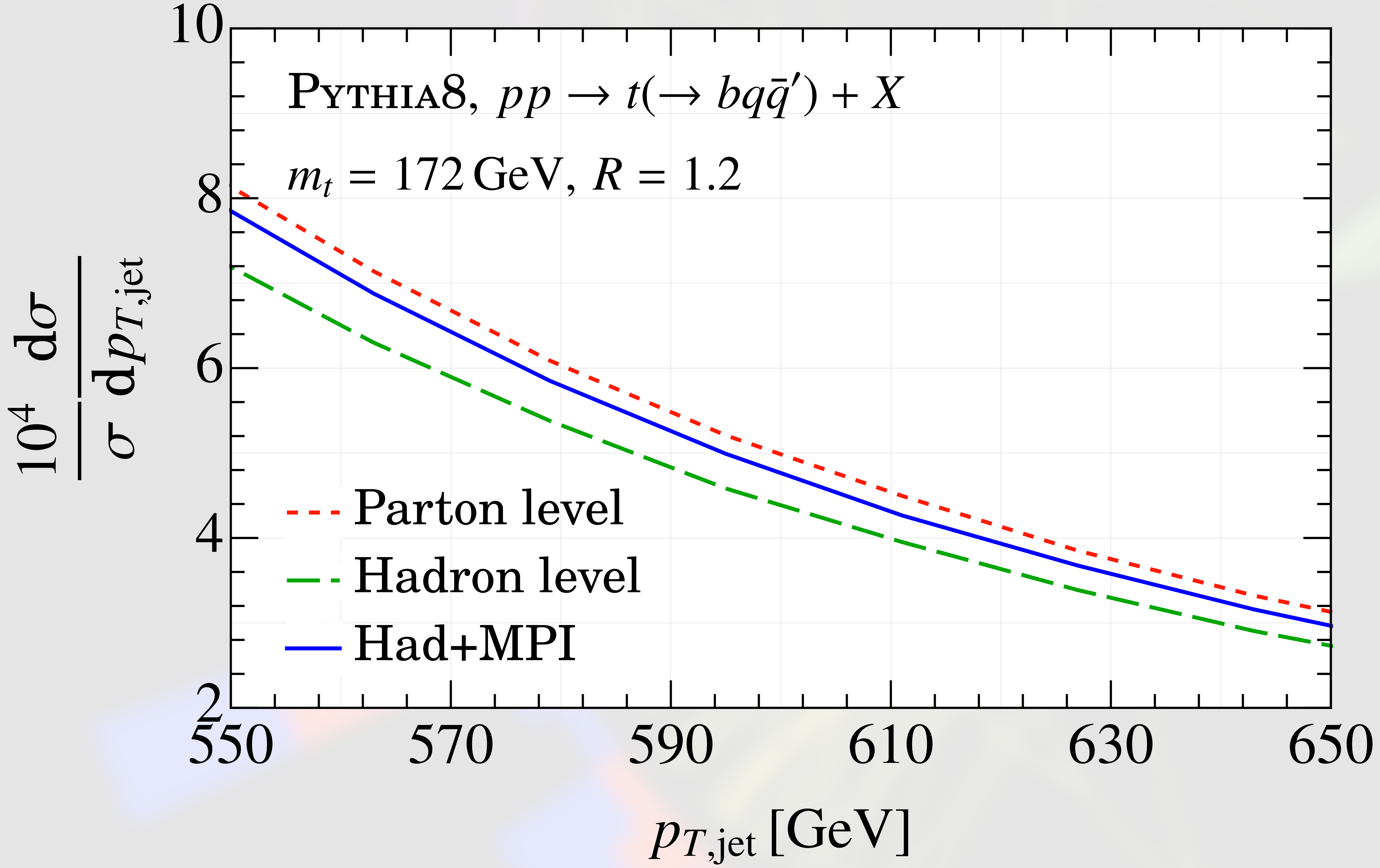
Relaxing asymmetry cut at parton and hadron level

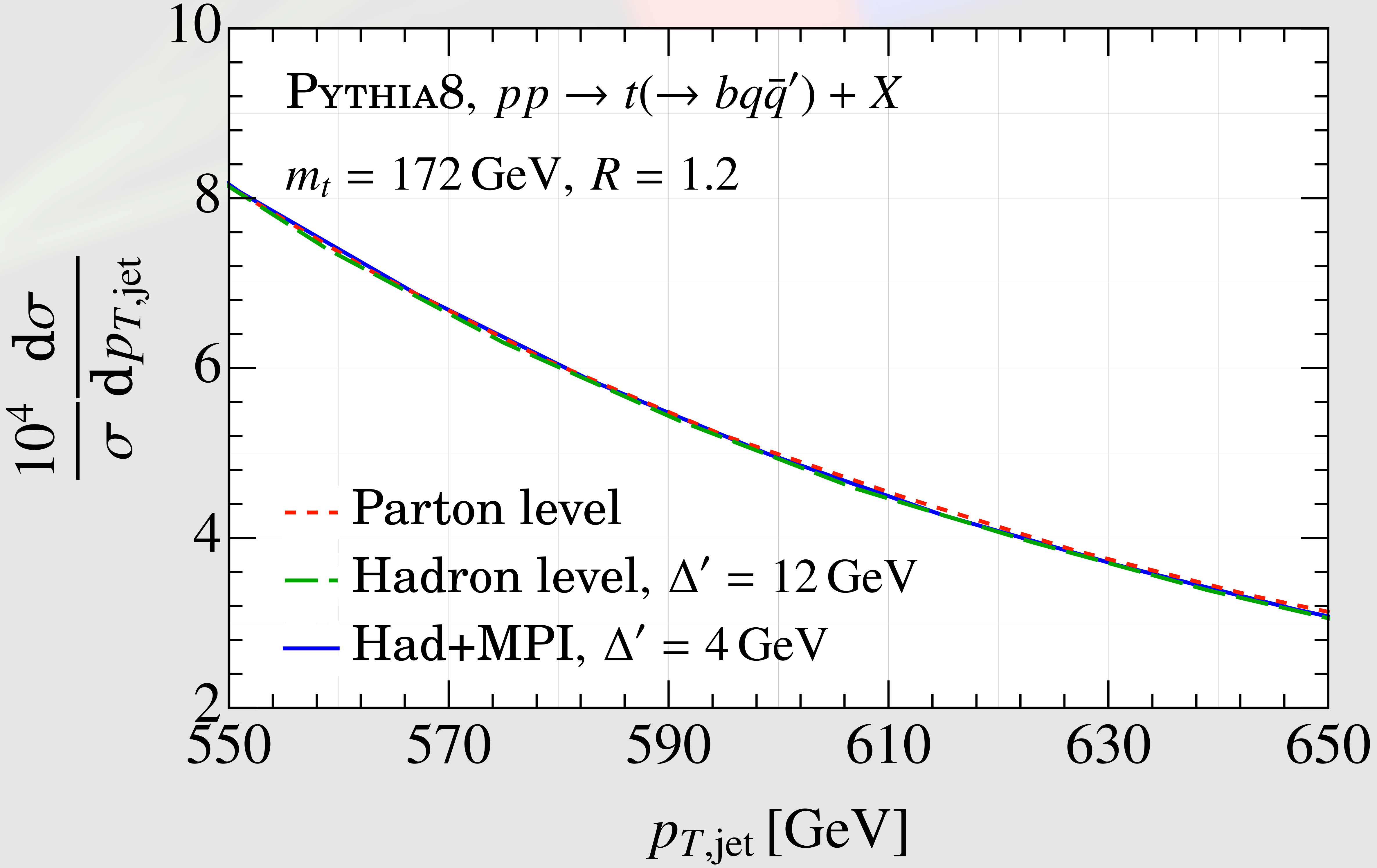
This peak ($W \to q\bar{q}'$?) persists at hadron level, while this one ($t \to bq\bar{q}'$) requires asymmetry cut.



Analysis of $p_{T,\text{jet}}$ shifts

Here we show $p_{T,\text{jet}}$ shifts relative to parton level:





Obtaining the top mass from multiple $p_{T,\text{jet}}$ bins

1. Parameterize the all orders peak position:

$$\zeta_{\text{peak}}^{(pp)} = 3(1+\mathcal{O}(\alpha_s))\frac{m_t^2}{f(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}})^2} \equiv 3(1+\mathcal{O}(\alpha_s))\frac{m_t^2}{\left(p_{T,\text{jet}} + \Delta(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}})\right)^2}$$

2. Work with

$$\rho^2(\zeta_{\text{peak}}^{(pp)\text{v}}, p_{T,\text{jet}}^{\text{v}}) = \left(\zeta_{\text{peak}}^{(pp)\text{ref}} - \zeta_{\text{peak}}^{(pp)\text{v}}\right)\left(\frac{3(1+\mathcal{O}(\alpha_s))}{(p_{T,\text{jet}}^{\text{v}})^2} - \frac{3(1+\mathcal{O}(\alpha_s))}{(p_{T,\text{jet}}^{\text{ref}})^2}\right)^{-1},$$

3. Define

$$\Delta^{\text{ref}} \equiv \Delta(p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}), \qquad \Delta^{\text{v}}(p_{T,\text{jet}}^{\text{v}} - p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}) \equiv \Delta(p_{T,\text{jet}}^{\text{v}}, m_t, \alpha_s, \Lambda_{\text{QCD}}) - \Delta^{\text{ref}}$$

4. Solve for ρ

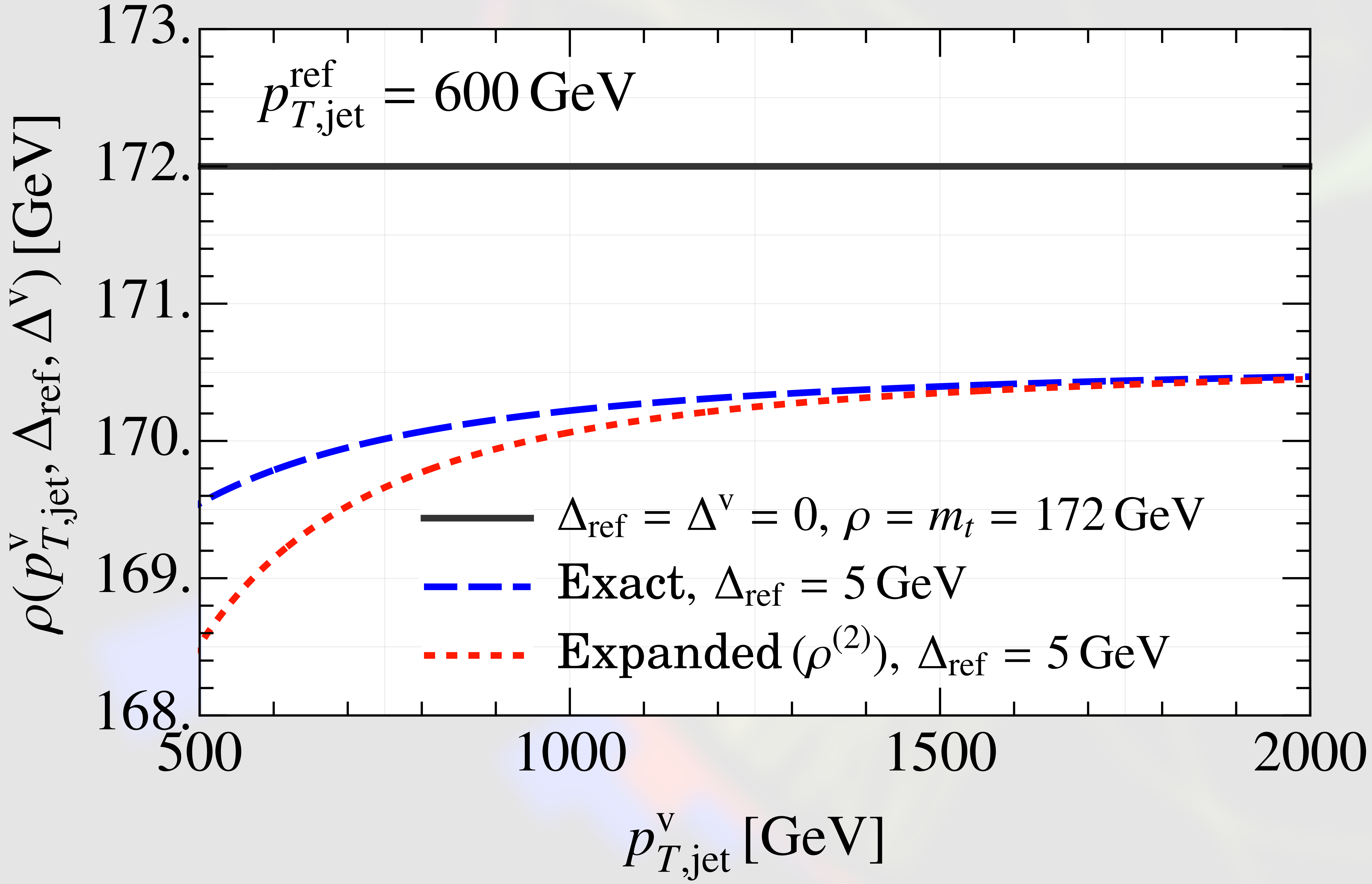
$$\rho(p_{T,\text{jet}}^{\text{v}}, \Delta^{\text{ref}}, \Delta^{\text{v}}) = \sqrt{F_{\text{pert}}}\frac{p_{T,\text{jet}}^{\text{ref}}}{p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}}}\left(1 - \frac{2p_{T,\text{jet}}^{\text{ref}}\Delta^{\text{ref}} + (\Delta^{\text{ref}})^2}{2(p_{T,\text{jet}}^{\text{v}})^2} + \frac{\left(p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}}\right)^2(\Delta^{\text{ref}} + \Delta^{\text{v}})}{8(p_{T,\text{jet}}^{\text{v}})^3} + \ldots\right)$$

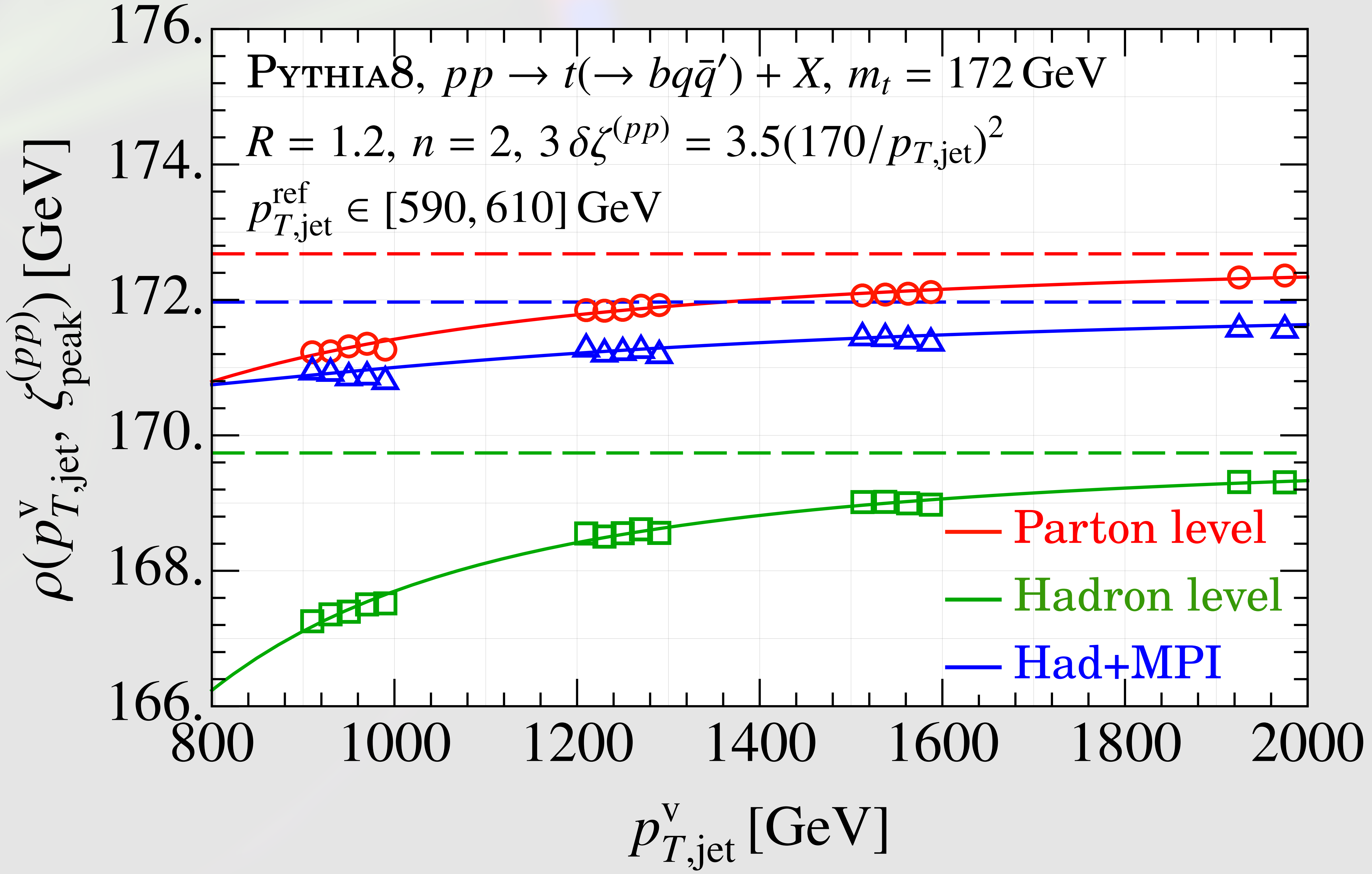
5. The asymptotic value for $p_{T,\text{jet}}^{\text{v}}$ depends only on m_t and Δ^{ref}.

Obtaining the top mass from multiple $p_{T,\mathrm{jet}}$ bins

Fit function:

$$\rho = \rho_{\mathrm{asy}} + c_2 (p^{\mathrm{v}}_{T,\mathrm{jet}})^{-2} + c_3 (p^{\mathrm{v}}_{T,\mathrm{jet}})^{-3}$$





Case study: QGP in Pb-Pb

$$\langle N(\eta_1,\phi_1)N(\eta_2,\phi_2)\rangle = N_1 N_2 P(\eta_1,\eta_2)$$

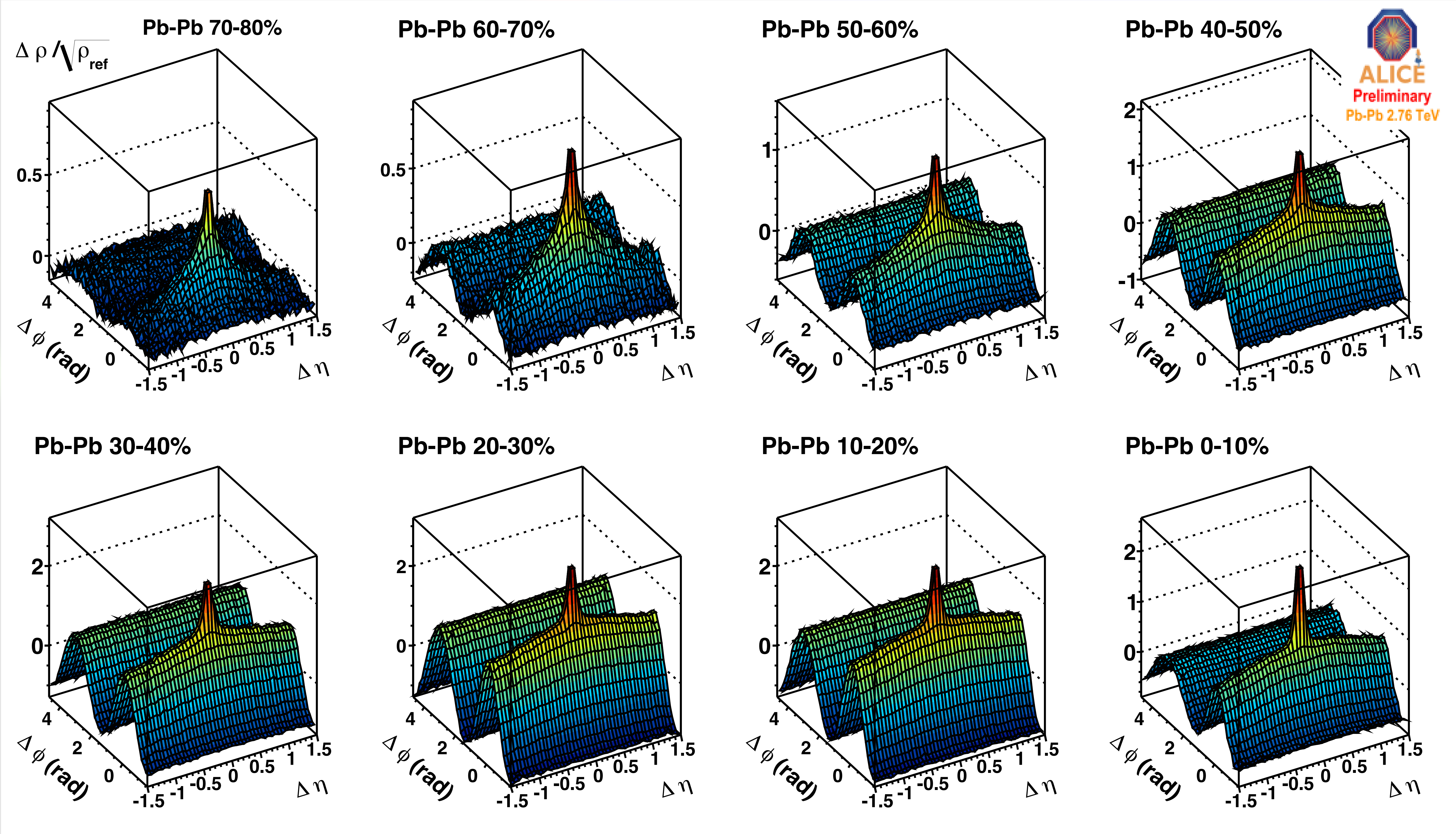
$$\sim \sum_X N_X(\eta_1,\phi_1)N_X(\eta_2,\phi_2)\langle \text{Pb-Pb}|X\rangle\langle X|\text{Pb-Pb}\rangle$$

$$= \sum_X \langle \text{Pb-Pb}|\widehat{N}(\eta_1,\phi_1)\widehat{N}(\eta_2,\phi_2)|X\rangle\langle X||\text{Pb-Pb}\rangle$$

$$= \langle \text{Pb-Pb}|\widehat{N}(\eta_1,\phi_1)\widehat{N}(\eta_2,\phi_2)|\text{Pb-Pb}\rangle$$

$$\frac{dN}{d^2p\,d^2k\,d\eta\,d\xi} = \langle \hat{\sigma}(k)\hat{\sigma}(p)\rangle_{P,T}$$

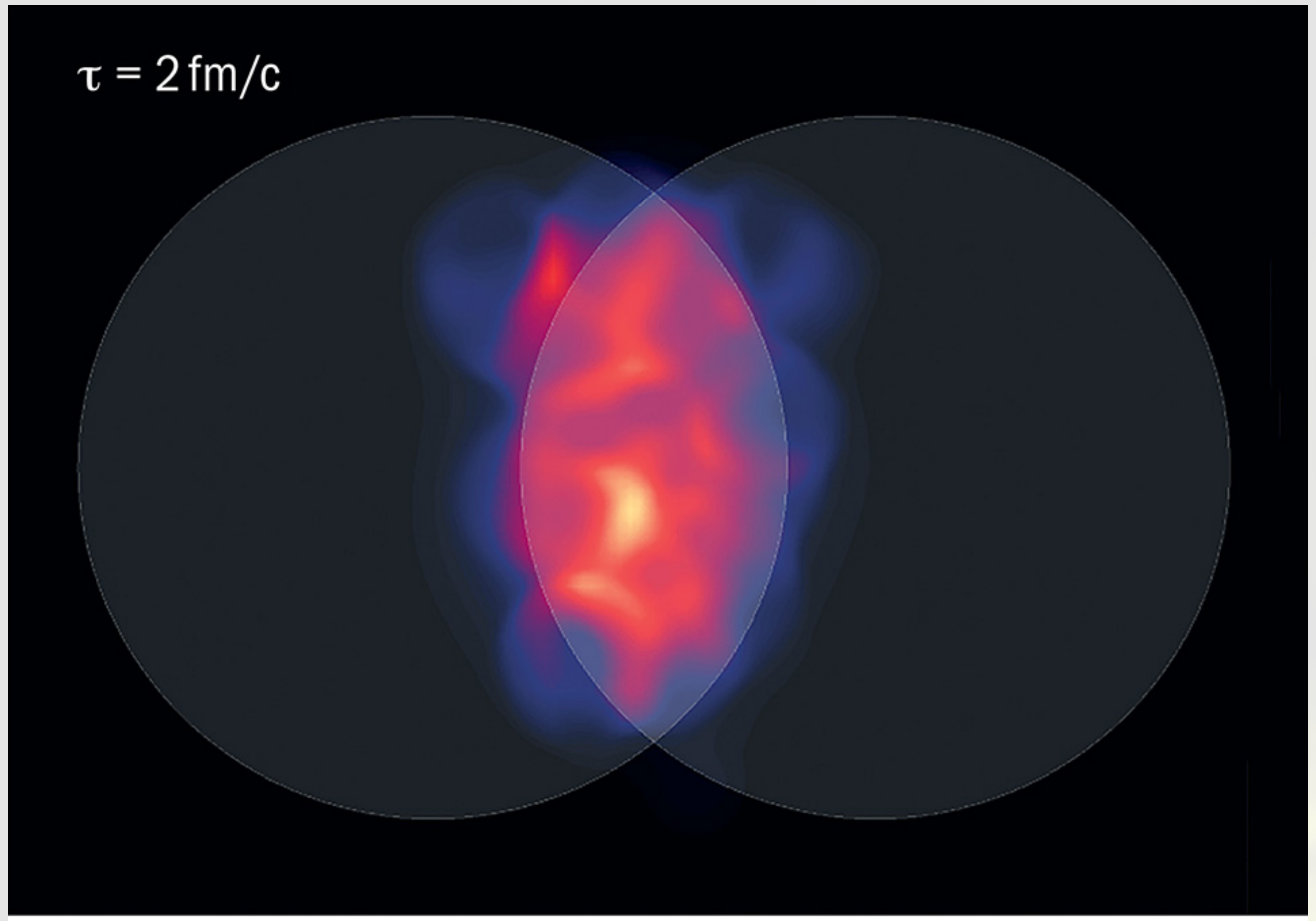
Kovner, Lubinsky 1211.1928

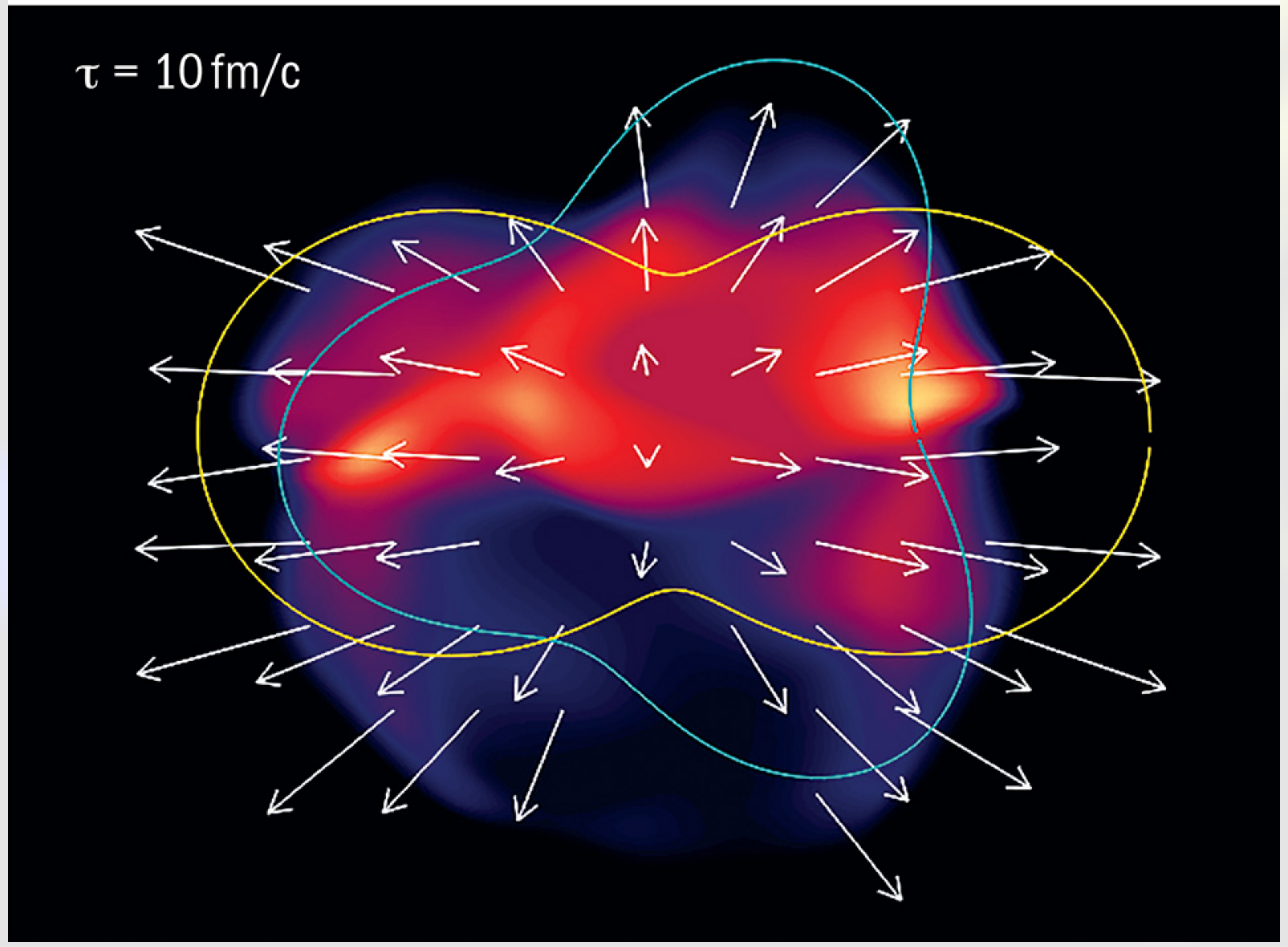


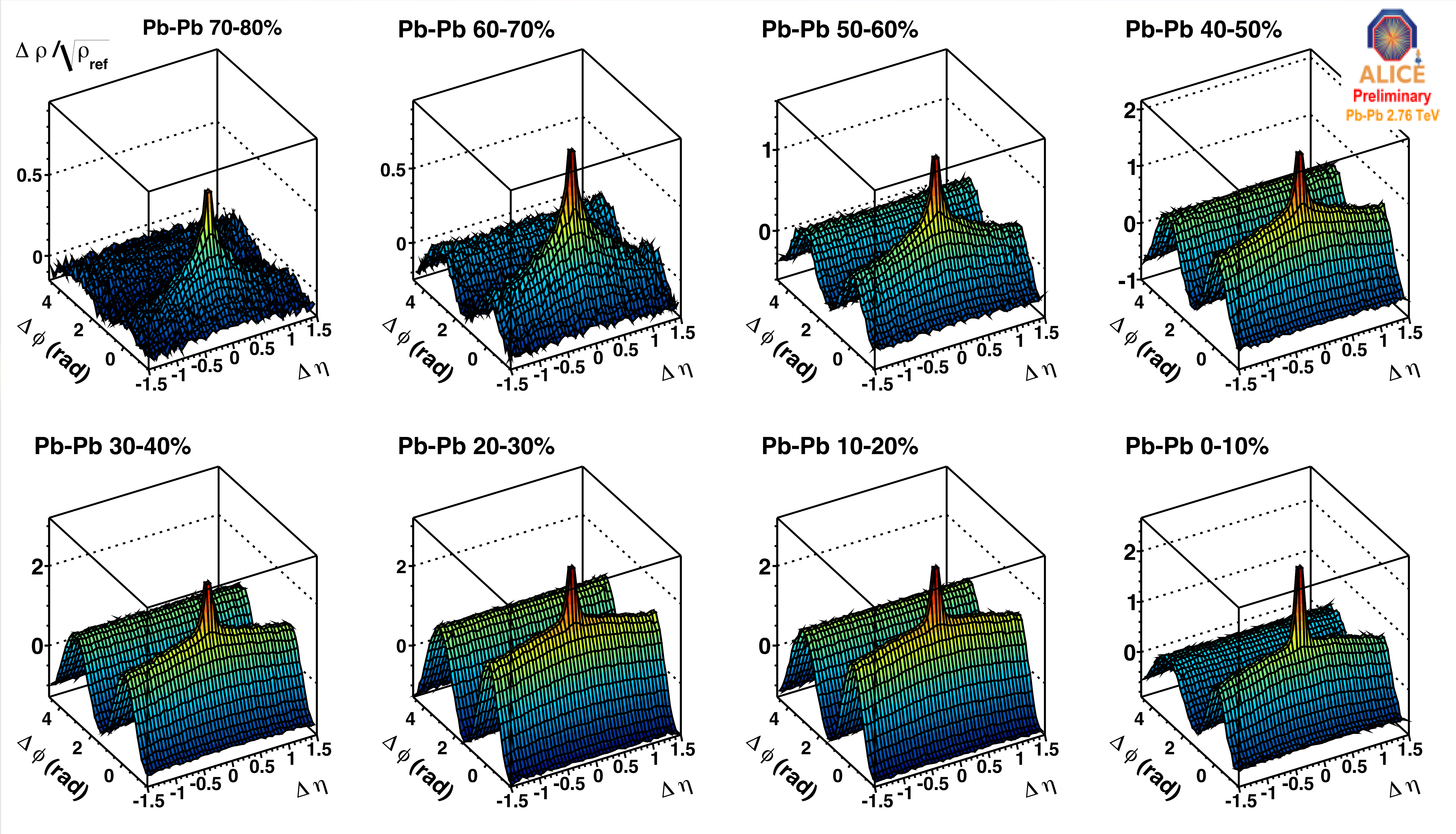
Timmins 1106.6057

Fully inclusive and hence nice properties

Case study: QGP in Pb-Pb







Timmins 1106.6057