

Strongly and weakly interacting dark matter candidates

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Overview



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Strongly interacting massive particles (SIMPs)

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Low-energy effective description of dark Sp(4) theories (arXiv:2202.05191)

Project and Groups



Freeze out and thermal relic dark matter

WIMP paradigm



WIMP dark matter

- $\bullet\,$ DM in thermal equilibrium with SM particle via annihilations DM DM \rightarrow SM SM
- \bullet decoupled from thermal equilibrium, when universe cooled down \rightarrow freeze out
- DM relic abundance $\Omega_{dm} \sim 0.26$
- cold dark matter (non-relativistic) with mass $m_{\text{WIMP}} \sim O(\text{GeV} \text{TeV})$
- $\bullet\,$ searched for ~ 35 yr, WIMPs have not been observed

- The WIMP paradigm is being challenged (e.g. neutrino floor in direct detection experiments)
- Are there different signatures from another thermal DM?
- Is there an alternative DM candidate we can search for?

IF YES:

- What would happen if dark matter was secluded from the SM sector?
- What would its mass be?
- What interactions would appear?
- How would it interact with SM?

Production of SIMPs in the early stages of the universe



Strongly interacting massive particles (SIMPs) as DM

- $3 \rightarrow 2$ annihilation implies heating of DM sector
- DM must be coupled to SM \Rightarrow opens door to phenomenology, $m_{\text{SIMP}} \sim O(MeV)$
- DM abundance regulated by number changing process in DM sector only



Production of SIMPs in the early stages of the universe

SIMP paradigm



Strongly interacting massive particles (SIMPs) as DM

- $3 \rightarrow 2$ annihilation implies heating of DM sector
- DM must be coupled to SM \Rightarrow opens door to phenomenology, $m_{\text{SIMP}} \sim O(MeV)$
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- build a model for SIMPs
- use QCD-like physics
- chiral symmetry breaking \Rightarrow Goldstone bosons = DM composite states
- what interactions and masses would appear?
- want to introduce small mass splitting in SIMP masses
- what interactions and masses would appear in the non-degenerate case?
- relevant for future DM physics

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UV Lagrangian and spontaneous symmetry breaking UV Lagrangian

- We consider $N_f = 2$ fermions in pseudo-real representation with gauge group $\operatorname{Sp}(4)_c$
- All fundamental representations of Sp(2N) are pseudo-real
- UV Lagrangian (massless fermions):

$$\mathcal{L}_{\rm UV} = \sum_{q=u,d} \bar{q} i \gamma^{\mu} \partial_{\mu} q \quad \rightarrow \quad \mathcal{L}_{\rm UV} = \Psi^{\dagger} i \bar{\sigma}^{\mu} \partial_{\mu} \Psi \qquad \text{with } \Psi = \begin{pmatrix} u_L \\ d_L \\ -S \sigma^2 \bar{u}_R^T \\ -S \sigma^2 \bar{d}_P^T \end{pmatrix} \tag{1}$$

- $S \dots$ antisymmetric matrix in color space σ^2 ... Pauli matrix in flavor space
- Global flavor symmetry is enlarged to SU(4)
- 2 flavor QCD: fermions in complex representation with global flavor symmetry $SU(2)_L \times SU(2)_R \subset SU(4)$
- Global flavor symmetry spontaneously broken

$$\mathrm{SU}\left(4\right) \stackrel{SSB}{\longrightarrow} \mathrm{Sp}\left(4\right)$$

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Breaking patterns - explicit symmetry breaking

symmetry explicitly broken by mass term

$$\mathcal{L}_{\rm UV} = \bar{q}i\gamma^{\mu}\partial_{\mu}q + \bar{q}Mq \quad \rightarrow \quad \mathcal{L} = \Psi^{\dagger}i\bar{\sigma}^{\mu}D_{\mu}\Psi - \frac{1}{2}\Psi^{T}\sigma^{2}SM\Psi + \text{h.c.}$$
(2)

• degenerate mass matrix: M = m E

$$E = \begin{pmatrix} 0 & \mathbb{I}_{2 \times 2} \\ -\mathbb{I}_{2 \times 2} & 0 \end{pmatrix}.$$
 (3)

 $SU(4) \xrightarrow{explicit} Sp(4)$

• non-degenerate mass Matrix

$$m_u \neq m_d \Rightarrow M = M_{\text{deg}} + \Delta M = \begin{pmatrix} 0 & 0 & m_u & 0\\ 0 & 0 & 0 & m_d\\ -m_u & 0 & 0 & 0\\ 0 & -m_d & 0 & 0 \end{pmatrix}$$
(4)

 $SU(4) \xrightarrow{explicit} Sp(2)_u \times Sp(2)_d$

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UV-theory to effective theory - existence of pseudo-Goldstone bosons

- Whenever a continuous global symmetry is broken, the Goldstone theorem guarantees the existence of low-energy Goldstone bosons
- SU(4) \xrightarrow{SSB} Sp(4) leads to existence of massless Goldstone bosons
- Number of Goldstone bosons is determined by the dimension of the coset space SU(4)/Sp(4)
- Counting:

 $\left. \begin{array}{l} SU(4) \text{ has 15 generators} \\ Sp(4) \text{ has 10 generators} \end{array} \right\} \Rightarrow 5 \text{ massles Goldstone bosons } \pi \in SU(4)/Sp(4) \end{array} \right\}$

- Goldstone bosons determined by broken generators T^A , $A = 1, \ldots, 5$
- Additional symmetry breaking due to mass term (explicit breaking)
- symmetry is not exact \Rightarrow Goldstone gain non-zero mass (pseudo-Goldstone bosons)

$$m_{\pi} \neq 0$$

UV-Lagrangian to effective Lagrangian for fermions (dark quarks) with degenerate masses

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UV-theory to effective theory - kinetic term

• UV-Lagrangian

$$\mathcal{L}_{\rm UV} = \Psi^{\dagger} \mathrm{i} \gamma^{\mu} \partial_{\mu} \Psi \qquad \text{with } \Psi = \begin{pmatrix} \psi_L \\ -S \sigma^2 \bar{\psi}_R^T \end{pmatrix}$$
(5)

- inputs to build chiral Lagragian:
 - parameterization of the chiral field (degenerate vacuua)

$$\Sigma = V \Sigma_0 V^T$$
, where $V = \exp\left(i\frac{\pi_n(x)T^n}{f_\pi}\right)$ (6)

- respecting symmetries:
 - -) Lorentz invariance (only even number of derivatives possible)
 - -) chiral symmetry invariance (only combination of Σ and Σ^{\dagger})
 - -) P,C invariance
- ▶ low energy \Rightarrow smallest possible number of derivatives
- effective Lagrangian:

$$L_2 = \frac{f_\pi^2}{4} \operatorname{Tr} \left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] \tag{7}$$

From UV Lagrangian to effective Lagrangian - degenerate mass term

• UV-Lagrangian

$$\mathcal{L}_{\rm UV} = \bar{q}i\gamma^{\mu}\partial_{\mu}q + \bar{q}Mq \quad \rightarrow \quad \mathcal{L}_{\rm UV} = \Psi^{\dagger}i\bar{\sigma}^{\mu}D_{\mu}\Psi - \frac{1}{2}\Psi^{T}\sigma^{2}SM\Psi + \text{h.c.}$$
(8)

• How to obtain mass term in the effective Lagrangian?

- take symmetry transformation, $\Psi \to U\Psi$, $U \in SU(4)$
- treat M as a spurion field
- chiral invariance $\Rightarrow M \rightarrow U^* M U^{\dagger}$

• total effective Lagrangian:

$$L_2 = \frac{f_\pi^2}{4} \operatorname{Tr} \left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] + 2\mu^3 \left(\operatorname{Tr} \left[M \Sigma \right] + \operatorname{Tr} \left[\Sigma^\dagger M^\dagger \right] \right)$$
(10)

From UV Lagrangian to effective Lagrangian - Goldstone mass

• effective Lagrangian in terms of Goldstone fields π (degenerate fermion masses)

$$\mathcal{L}_{\pi} = \operatorname{Tr} \partial_{\mu} \pi \partial^{\mu} \pi - m_{\pi}^{2} \operatorname{Tr} \pi^{2} + \frac{m_{\pi}^{2}}{3f_{\pi}^{2}} \operatorname{Tr} \pi^{4} - \frac{2}{3f_{\pi}^{2}} \operatorname{Tr} \left(\pi^{2} \partial^{\mu} \pi \partial_{\mu} \pi - \pi \partial^{\mu} \pi \pi \partial_{\mu} \pi \right)$$
(11)

Goldstone matrix

$$\pi \equiv \sum_{n=1}^{5} \pi_n T^n = \frac{1}{2\sqrt{2}} \begin{pmatrix} \pi_3 & \pi_1 - i\pi_2 & 0 & \pi_5 - i\pi_4 \\ \pi_1 + i\pi_2 & -\pi_3 & -\pi_5 + i\pi_4 & 0 \\ 0 & -\pi_5 - i\pi_4 & \pi_3 & \pi_1 + i\pi_2 \\ \pi_5 + i\pi_4 & 0 & \pi_1 - i\pi_2 & -\pi_3 \end{pmatrix}$$
(12)

• masses of Goldstone fields

$$m_{\pi} = \frac{8\mu^3 m}{f_{\pi}^2}$$
(13)

- kinetic term implies 4-point interaction $\Rightarrow 2$ to 2 scattering for free
- Semi-annihilation and 5-point interaction are absent, because ${\rm Tr}\,[\pi^n]=0$ for n odd
- How to include 3 to 2 interaction?

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From UV Lagrangian to effective Lagrangian - Wess-Zumino-Witten term

• effective Lagrangian:

$$L_2 = \frac{f_\pi^2}{4} \operatorname{Tr} \left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] + 2\mu^3 \left(\operatorname{Tr} \left[M \Sigma \right] + \operatorname{Tr} \left[\Sigma^\dagger M^\dagger \right] \right)$$
(14)

- 3 to 2 process \Rightarrow need 5 point interaction term
- 5-point interaction out of Goldstones \Rightarrow Wess-Zumino-Witten interaction

$$L_{WZW} = \frac{-iN_c}{240\pi^2} \int \text{Tr}\left[(\Sigma^{\dagger} d\Sigma)^5 \right]$$
(15)

• WZW in terms of Goldstone fields (degenerate masses $N_f = 2$)

$$L_{\rm WZW} = \frac{8N_c}{15\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} \operatorname{Tr} \left[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi\right] + \mathcal{O}\left(\pi^6\right)$$
(16)

• WZW term allows only interactions with Goldstones of different generation

$$\epsilon^{\mu\nu\rho\sigma}\pi_1\partial_\mu\pi_2\partial_\nu\pi_3\partial_\rho\pi_4\partial_\sigma\pi_5\tag{17}$$

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UV-Lagrangian to effective Lagrangian for fermions (dark quarks) with non-degenerate masses

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Reminder - UV-theory for non-degenerate quark masses

symmetry explicitly broken by mass term

$$\mathcal{L} = \Psi^{\dagger} \mathbf{i} \bar{\sigma}^{\mu} D_{\mu} \Psi - \frac{1}{2} \Psi^{T} \sigma^{2} S M \Psi + \text{h.c.}$$
(18)

• degenerate mass matrix: M = m E

$$E = \begin{pmatrix} 0 & \mathbb{I}_{2 \times 2} \\ -\mathbb{I}_{2 \times 2} & 0 \end{pmatrix}.$$
 (19)

 $SU(4) \xrightarrow{explicit} Sp(4)$

• non-degenerate mass Matrix

$$m_u \neq m_d \Rightarrow M = M_{\text{deg}} + \Delta M = \begin{pmatrix} 0 & 0 & m_u & 0\\ 0 & 0 & 0 & m_d\\ -m_u & 0 & 0 & 0\\ 0 & -m_d & 0 & 0 \end{pmatrix}$$
(20)

 $SU(4) \xrightarrow{explicit} Sp(2)_u \times Sp(2)_d$

UV-theory for non-degenerate quark masses

• splitting of decay constants and vacuum condensates

$$f_{\pi} \xrightarrow{m_u \neq m_d} \begin{cases} f_{\pi} & \text{for } \pi_{1,2,4,5} \\ f_{\pi_3} & \text{for } \pi_3 \end{cases}, \quad \mu^3 \xrightarrow{m_u \neq m_d} \begin{cases} \mu_u^3 = \frac{1}{2} \langle u^T \sigma_2 S E_2 u \rangle \\ \mu_d^3 = \frac{1}{2} \langle d^T \sigma_2 S E_2 d \rangle \end{cases}, \quad (21)$$

chiral Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm WZW} = \frac{4\mu_u^6}{(\mu_u^3 + \mu_d^3)^2} \left(\frac{f_\pi^2}{4} \operatorname{Tr} \left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] - \frac{1}{2} \mu_u^3 \left(\operatorname{Tr} \left[M \Sigma \right] + \text{h.c.} \right) \right) + \frac{\mu_d^6 \sqrt{\mu_u^6 + \mu_d^6} \left(\mu_u^3 + \mu_d^3 \right)^5}{32\sqrt{2} \mu_u^{24}} \mathcal{L}_{\rm WZW}^{\rm deg.}$$
(22)

Goldstone masses

$$m_{\pi}^{2} \equiv m_{\pi_{1,2,4,5}}^{2} = \frac{2\mu_{u}^{6}(m_{u} + m_{d})}{f_{\pi}^{2}(\mu_{u}^{3} + \mu_{d}^{3})}, \quad m_{\pi_{3}}^{2} = \frac{2\mu_{u}^{6}(m_{u}\mu_{u}^{3} + m_{d}\mu_{d}^{3})}{f_{\pi}^{2}(\mu_{u}^{6} + \mu_{d}^{6})}.$$
 (23)

• results supported by lattice simulations

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Lattice simulations



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Multiplet structure



- under "naive" parity $P: \psi(\mathbf{x}, t) \to \gamma_0 \psi(-\mathbf{x}, t)$ 5 Goldstones = $\begin{cases} 3 \text{ Pseudoscalars } \pi^A, \pi^B, \pi^C \\ 2 \text{ Scalars, } \pi^D, \pi^E \end{cases}$
- under parity $D: \psi(\mathbf{x}, t) \to \pm i\gamma_0 \psi(-x, t)$ 5 Goldstones are Pseudoscalars

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DM couple to SM

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- introduce a new Abelian dark gauge group U(1)' with (massive) gauge field V^{μ}
- V^{μ} kinetically mix with SM hypercharge

$$\mathcal{L}_{\rm int} = \frac{\varepsilon}{2\cos\theta_W} B_{\mu\nu} V^{\mu\nu},\tag{24}$$

where $B_{\mu\nu}$ and $V^{\mu\nu}$ are the $U(1)_D$ and $U(1)_Y$ field strengths • covariant derivative:

$$\partial_{\mu}\Psi^{ia} \to D_{\mu}\Psi^{ia} = \partial_{\mu}\Psi^{ia} + ie_D V_{\mu} \mathcal{Q}_{ij}\Psi^{ja}.$$
 (25)

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DM coupled to SM

- Gauging the theory under U(1)' may provide a source of explicit global symmetry breaking
- study how the gauge interaction term in the Lagrangian,

$$\mathcal{L} \supset -e_D V_\mu \left(\left(\Psi^i \right)_a^\dagger \overline{\sigma}^\mu \mathcal{Q}_{ij} \Psi^{ja} \right)$$
(26)

transforms under the remaining flavor symmetry Sp(4) as

$$\Psi^{ia} \to V_{ij}\Psi^{ja} = \left(1 + i\theta^N T^N_{ij} + \dots\right)\Psi^{ja},\tag{27}$$

• variation in the Lagrangian density

$$\delta \mathcal{L} = i e_D V_\mu \left((\Psi)^{\dagger} \overline{\sigma}_{\mu} \theta^N \left[\mathcal{Q}, T^N \right] \Psi \right), \qquad (28)$$

• remaining flavour symmetry is spanned by those generators T^N of Sp(4) that commute with Q.

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Charge Assignment \mathcal{Q}	Breaking Pattern	Multiplet Structure
(+a, -a, -a, +a)	$Sp(4) \rightarrow SU(2) \times U(1)$	$\begin{pmatrix} \pi^C \\ \pi^D \\ \pi^E \end{pmatrix}, \begin{pmatrix} \pi^A \\ \pi^B \end{pmatrix}$
(+a,+a,-a,-a)	$Sp(4) \rightarrow SU(2) \times U(1)$	$egin{pmatrix} \pi^C \ \pi^A \ \pi^B \end{pmatrix}, \ egin{pmatrix} \pi^D \ \pi^E \end{pmatrix}$
$(+a,+b,-a,-b)$, $a\neq b$	$Sp(4) \rightarrow U(1) \times U(1)$	$(\pi^C), \ \begin{pmatrix} \pi^A \\ \pi^B \end{pmatrix}, \ \begin{pmatrix} \pi^D \\ \pi^E \end{pmatrix}$
$\left(\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$Sp(4) \rightarrow SU(2) \times U(1)$	$\begin{pmatrix} \pi^{C} \\ \pi^{A,B} \\ \pi^{E,D} \end{pmatrix}, \begin{pmatrix} \pi^{D,E} \\ \pi^{B,A} \end{pmatrix}$
other off-diagonal assignments	$Sp(4) \rightarrow U(1) \times U(1)$	$(\pi^C), \begin{pmatrix} \pi^A\\ \pi^E \end{pmatrix}, \begin{pmatrix} \pi^B\\ \pi^D \end{pmatrix}$ or sim

• if π^C is singlet, it decays away, because it is not protected by flavor symmetry

U(1)' interactions with mesons

• interaction with gauge field V_{μ} with dark sector in the effective theory through covariant derivative

$$D_{\mu}\Sigma = \partial_{\mu}\Sigma + ie_D V_{\mu} (\mathcal{Q}\Sigma + \Sigma \mathcal{Q})$$
⁽²⁹⁾

- e_D is the U(1)' gauge coupling
- interaction depends on charge assignment Q
- for a charge assignment (+1, -1, -1, +1)

$$\mathcal{L}_{V-\pi} = -2ie_D V^{\mu} \left(\pi^A \partial_{\mu} \pi^B - \pi^B \partial_{\mu} \pi^A \right) + e_D^2 V_{\mu} V^{\mu} \pi^A \pi^B \tag{30}$$

- for (+1, +1, -1, -1) V_{μ} couples only to π^D, π^E
- one pair of Golstones carry U(1)' charge

- decays of Goldstone bosons occur through processes mediated by pairs of dark photons
- $\pi V V$ vertex is sourced by the gauged WZW action

$$\mathcal{L}_{\text{int}} \supset \frac{40iCe_D^2}{f_{\pi}^2} \varepsilon_{\mu\nu\alpha\beta} V^{\mu} \partial^{\nu} V^{\alpha} \operatorname{Tr} \left(\mathcal{Q}^2 \partial^{\beta} \pi \right)$$
(31)

• since all generators of SU(4)/Sp(4) are traceless \Rightarrow anomaly cancellation $\mathcal{L}_{int} = 0 \text{ for } \mathcal{Q}^2 \propto \mathbb{I}$

Conclusions

- SIMPs are composite states created through chiral symmetry breaking
- dynamics of SIMPs are described by effective (chiral) Lagrangian
- 2 to 2 scattering for free, 3 to 2 interaction given by WZW term
- SU(4) \xrightarrow{SSB} Sp(4)
- degenerate case: $SU(4) \xrightarrow{explicit} Sp(4)$
- non-degenerate case: SU(4) $\stackrel{explicit}{\rightarrow} Sp(2)_u \times Sp(2)_d$
- non-degenerate case: π^C lightest DM state
- multiplet structure given by symmetry $Sp(2)_u \times Sp(2)_d$
- DM couple to SM through dark U(1)' gauge field (kinetic mixing)
- breaking patterns depend on charge assignments
- can achieve stability of SIMPs

Dark radiation - indirect dark matter searches Sensitivity of direct detection experiments to neutrino dark radiation from dark matter decay and a modified neutrino-floor (arXiv:2008.13557)

Probing sub-eV Dark Matter decays with PTOLEMY (arXiv:2012.09704)

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Energy budget of the universe



Dark matter

- many efforts to find DM in the laboratory to understand its particle nature
- direct detection experiments designed to search for nuclear recoils induced by DM
- DM has not been directly observed in laboratories \Rightarrow look for other possible signals

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Energy budget of the universe



Dark radiation

- new form of relativistic particles produced by unstable dark matter (few % of total DM abundance) \rightarrow dark radiation (DR)
- \bullet number budget unknown \rightarrow large DR flux possible
- DR may leave a detectable signal in direct detection experiments

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Energy budget of the universe



Dark radiation x%

Dark radiation

- new form of relativistic particles produced by unstable dark matter (few % of total DM abundance) \rightarrow dark radiation (DR)
- \bullet number budget unknown \rightarrow large DR flux possible
- DR may leave a detectable signal in direct detection experiments

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"What is the maximum sensitivity to DR that we can reach in direct detection experiments?"

- irreducible backgrounds from STANDARD NEUTRINO SOURCES (solar, atmospheric, supernova ν) in direct detection experiments
- backgrounds may mimic **D**R
- find region in parameter space of DR where DR and backgrounds are distinguishable

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DARK RADIATION

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2-body decay

$$X \to \nu\nu \tag{32}$$

parameter space of DR: τ_X, m_X, f_X ($f_X = 0.1$) benefit of scenario: interactions of neutrinos are known

Dark Radiation as SM neutrinos



Dark Radiation from decaying Dark Matter

DARK RADIATION vs. STANDARD NEUTRINO SOURCES

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Backgrounds vs. DR



- DR-induced recoil events may be misinterpreted as standard neutrino sources
- experiments lose sensitivity to DR at certain combinations of progenitor lifetime τ_X and mass m_X

How we can properly discriminate DR from standard neutrino sources? use hypothesis testing based on likelihood statistics

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Sensitivity

Discovery potential - DR discovery with 3σ significance

- find DR signals in the presence of standard neutrino sources (sol,atm,supernova)
- establish the minimum lifetime corresponding to the smallest DR flux, where DR and backgrounds are still distinguishable
- test DR+standard neutrino background hypothesis against standard neutrino only-background hypothesis
- generate mock data by Monte Carlo simulations, which represent recoil energies of DR and background rates
- general tool: likelihood function



• discovery region is superseded by SK constraints (grey) \Rightarrow direct detection experiments with $\varepsilon \lesssim O(1)$ ton yr cannot measure this form of DR

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Discovery potential - DR discovery with 3σ significance

- find DR signals in the presence of standard neutrino sources (sol,atm,supernova)
- establish the minimum lifetime corresponding to the smallest DR flux, where DR and backgrounds are still distinguishable
- test DR+standard neutrino background hypothesis against standard neutrino only-background hypothesis
- generate mock data by Monte Carlo simulations, which represent recoil energies of DR and background rates
- general tool: likelihood function



 discovery region for new dark states (semi sterile neutrinos) \Rightarrow larger discovery region by new interaction

DR in PTOLEMY

Detectability of DR in the future neutrino experiment PTOLEMY (on going)

- proposal to measure $C\nu B$ (relic neutrinos)
- $\bullet\,$ if ${\rm C}\nu{\rm B}$ measured, accurate measurement of m_{ν}
- detection via β -decaying nuclei (neutrino capture):

$$(A, Z) + \nu_e \to (A, Z + 1) + e^-$$
 (33)

• target material: 100 g tritium



DR in PTOLEMY

DR in PTOLEMY

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- another possibility: search for DR signals in PTOLEMY
- massive neutrinos: $X \to \nu \bar{\nu}$
- backgrounds: $C\nu B + \beta$ -decay from tritium



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Discovery potential

• background-only $H_0: C\nu B + \beta$ against signal+background $H_1: DR + C\nu B + \beta$



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- $\bullet~{\rm DR}$ in form of SM neutrinos almost completely excluded \Rightarrow increase exposure
- DR in form of baryonic neutrinos still detectable
- search for DR in PTOLEMY

Thank you for your attention!

Backup slides

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- Global flavor symmetry spontaneously broken, because vacuum condensate is not invariant under SU(4)
- Under an infinitesimal transformation under the group SU(4), the fermion field transforms as

$$\Psi \to \left(1 + i \sum_{a=1}^{15} \theta^a T^a\right) \Psi,\tag{34}$$

where T^{a} are generators of SU(4)

• chiral condensate becomes

$$\langle qq \rangle \rightarrow \langle qq \rangle - \frac{i}{2} \theta^a \Psi^T \sigma_2 S \left(ET^a + (T^a)^T E \right) \Psi + \text{ h.c.}$$
 (35)

• vacuum condensate is invariant if $ET^a + (T^a)^T E = 0 \Rightarrow$ invariant under Sp(4)

Multiplet structure

$$\pi_{2} = \frac{i}{\sqrt{2}} \left(\bar{d}\gamma_{5}u - \bar{u}\gamma_{5}d \right) \qquad \qquad \pi^{B} = -\bar{d}\gamma_{5}u \qquad \qquad 0^{-} 0^{-}$$

$$egin{array}{lll} \pi_3 &=& rac{\sqrt{2}}{\sqrt{2}} \left(ar{u} \gamma_5 u - d \gamma_5 d
ight) & \pi^{\mathbb{C}} &=& rac{1}{\sqrt{2}} \left(ar{u} \gamma_5 u - d \gamma_5 d
ight) & 0^- & 0^- \ \pi^D &=& ar{d} \gamma_5 S C ar{u}^T & 0^+ & 0^- \end{array}$$

$$\pi_5 = \frac{\sqrt{1}}{\sqrt{2}} \left(\bar{d}\gamma_5 SC \bar{u}^T + d^T SC \gamma_5 u \right) \quad \pi^E = d^T SC \gamma_5 u \qquad \qquad 0^+ \quad 0^-$$

• under "naive" parity
$$P: \psi(\mathbf{x}, t) \to \gamma_0 \psi(-\mathbf{x}, t)$$

5 Goldstones =
$$\begin{cases} 3 \text{ Pseudoscalars } \pi^A, \pi^B, \pi^C \\ 2 \text{ Scalars, } \pi^D, \pi^E \end{cases}$$

- under parity $D: \psi(\mathbf{x}, t) \to \pm i\gamma_0 \psi(-x, t)$
 - 5 Goldstones are Pseudoscalars

D

Including spin-1 states - ρ -mesons

- 5 axial-vector and 10 vector mesonic states under parity D: $\rho_{\mu} = \sum_{a=1}^{10} \rho_{\mu}^{a} T^{a}$.
- add kinetic and mass terms for the spin-1 states to chiral Lagrangian \mathcal{L}_{π}

$$\mathcal{L}_{\rho} = -\frac{1}{2} \operatorname{Tr} \left(\rho_{\mu\nu} \rho^{\mu\nu} \right) + \frac{1}{2} m_{\rho}^{2} \operatorname{Tr} \left(\rho_{\mu} \rho^{\mu} \right)$$
(36)

• interactions between Goldstone bosons and spin-1 states

$$D_{\mu}\Sigma = \partial_{\mu}\Sigma + ig_{\rho\pi\pi} \left[\rho_{\mu}\Sigma + \Sigma\rho_{\mu}^{T} \right], \qquad (37)$$

axial-vectors heavier than vectors

$$m_a^2 = m_\rho^2 / Z^2. (38)$$

• axial-vectors not important for DM physics

Neutrino fluxes



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Neutrino sources



• Solar neutrinos produced in the Sun by fusion processes

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Neutrino sources



atmospheric neutrinos

- Cosmic rays collide with nuclei in the atmosphere
- hadronic showers: pions, kaons, neutrons, protons
- muon decay: $\mu^- \to e^- + \bar{\nu}_{\mu} + \nu_e$, $\mu^+ \to e^+ + \nu_{\mu} + \bar{\nu}_e$
- electromagnetic showers: production of photons and electrons

Irreducible neutrino background

coherent scattering

$$\nu + N \rightarrow \nu + N$$

• spin-independent neutrino-nucleus cross section

$$\frac{d\sigma_{N\nu}(E_{\nu}, E_R)}{dE_R} = \frac{Q_W^2 G_F^2 m_N F(\vec{q}\,)^2}{4\pi} \left[1 - \frac{E_R m_N}{2E_{\nu}^2} \right],\tag{40}$$

Fermi constant $G_F = 1.1663787(6) \cdot 10^{-5} \text{ GeV}^{-2}$ weak charge $Q_W = (4 \sin^2 \theta_W - 1)Z + N$, Weinberg angle θ

• Helm form factor

$$F(\vec{q}) = \frac{3j_1(\vec{q}\,R)}{\vec{q}\,R} e^{-\vec{q}\,^2 s^2/2} \tag{41}$$

• Coherence lost, when momentum transfer too large

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(39)

WIMP event mimicked by neutrino backgrounds



- Large number of neutrino-induced nuclear recoils at $E_R \sim$ few keV
- Relevant neutrino backgrounds: solar, atmospheric and supernova neutrinos
- ⁸B- ν mimic WIMP with $\sigma_n = 4.9 \cdot 10^{-45} \text{ cm}^2$, $m_{\chi} = 6 \text{ GeV}$ atm. ν mimic WIMP with $\sigma_n = 2 \cdot 10^{-49} \text{ cm}^2$, $m_{\chi} = 100 \text{ GeV}$

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Dark radiation from an anomalous neutrino source

Two-body decay into neutrino pair (monochromatic injection)

$$X \to \nu\nu, \quad \frac{dN}{dE_{\nu}} = N_{\nu}\delta(E_{\nu} - E_{\rm in}), \quad N_{\nu} = 2$$
 (42)

$$\frac{d\Phi_{\nu,\text{gal.}}}{dE_{\nu}} = N_{\nu} \frac{\kappa e^{-\frac{t_0}{\tau_X}}}{m_X \tau_X} r_{\odot} \rho_{\odot} \langle J_{\text{dec}}(\theta) \rangle \delta(E_{\nu} - E_{\text{in}}), \quad E_{\text{in}} = \frac{m_X}{2}$$
(43)

$$\frac{d\Phi_{\nu,\text{e.g.}}}{dE_{\lambda}} = N_{\nu} \frac{\kappa \Omega_{\text{dm}} \rho_{\text{crit}}}{H_0 m_X} \frac{1}{E_{\nu}} \frac{\frac{1}{\tau_X} e^{-\frac{\tau(\alpha-1)}{\tau_X}}}{\sqrt{\alpha^3 \Omega_M + \Omega_\Lambda}} \theta(\alpha-1), \ \alpha = E_{\text{in}}/E_{\nu}$$
(44)



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Likelihood statistics

• Likelihood function:

$$\mathcal{L}\left(\tau_{X},\vec{\phi_{\nu}}\right) = \frac{\left(\mu_{\mathrm{DR}}\left(\tau_{X}\right) + \sum_{j=1}^{n_{\nu}} \mu_{\nu}\left(\phi_{\nu}^{j}\right)\right)^{N_{\mathrm{obs}}} e^{-\left(\mu_{\mathrm{DR}}\left(\tau_{X}\right) + \sum_{j=1}^{n_{\nu}} \mu_{\nu}\left(\phi_{\nu}^{j}\right)\right)}}{N_{\mathrm{obs}}!} \times \prod_{i=1}^{\mu_{\mathrm{obs}}} \frac{f\left(E_{R_{i}},\tau_{X},\vec{\phi_{\nu}}\right)}{\mu_{\mathrm{DR}}\left(\tau_{X}\right) + \sum_{j=1}^{n_{\nu}} \mu_{\nu}\left(\phi_{\nu}^{j}\right)} \times \prod_{j=1}^{n_{\nu}} L_{j}\left(\phi_{\nu}^{j}\right)}$$
(45)

• measure of discrepancy is obtained by the test statistics (likelihood ratio)

$$q_0 = -2\ln\lambda(0) = \begin{cases} -2\ln\frac{L(\Gamma_X = 0, \vec{\phi}_\nu; m_X | E_R)}{L(\hat{\Gamma}_X, \vec{\phi}_\nu; m_X | E_R)} & \text{for } \hat{\Gamma}_X > 0\\ 0 & \text{for } \hat{\Gamma}_X \le 0, \end{cases}$$

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Improve sensitivity

- reducing uncertainty of neutrino fluxes
- measure direction of recoil energies when hitting solar neutrino background
- using different detector materials for additional information

other DR sources

- new particle with different strength
- DR from multi-body decays
- evaporating primordial back holes or emerging from supernova explosions

Neutrino flux - $\mathrm{C}\nu\mathrm{B}$



- C ν B neutrino flux at $E_{\nu} \sim 10^{-4} \text{ eV} \Rightarrow$ nuclear recoils of $E_R \lesssim 10^{-15} \text{ eV}$ in direct detection \rightarrow impossible to measure, large thresholds
- detection via β -decaying nuclei (neutrino capture): $(A, Z) + \nu_e \rightarrow (A, Z + 1) + e^-$
- C ν B rate: $\Gamma_{\rm CNB} \approx (4 \text{ or } 8) \text{ yr}^{-1} \left(\frac{M_{\rm T}}{100 \text{ g}} \right)$

Detection of $\mathrm{C}\nu\mathrm{B}$

• Consider unstable nucleus (A, Z), that decays through β -decay,

 $(A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e$

releasing energy Q_{β}

• There exists a threshold-less reaction of neutrino capture

 $(A,Z) + \nu_e \to (A,Z+1) + e^-$

- β -decays create a background for neutrino capture, but we can distinguish them using different kinematics
- PTOLEMY will measure neutrino mass m_{ν}
- $C\nu B$ rate:

$$\Gamma_{\rm CNB} \approx (4 \text{ or } 8) \text{ yr}^{-1} \left(\frac{M_{\rm T}}{100 \text{ g}} \right)$$





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Suppression factor (Pauli-blocking) and accumulating neutrinos

- galactic rate is affected by Pauli-blocking issues
- galactic rate suppressed by Pauli-blocking
- Non-escaping neutrinos: galactic neutrinos from DM decay are injected at non-relativistic speeds with $v_{\nu} \ll v_{\rm esc} \sim 10^{-3}c$

• happens when
$$\frac{m_{\rm DM} - 2m_{\nu}}{m_{\rm DM}} \lesssim 10^{-6}$$

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