RESUMMATION OF SUPER-LEADING LOGARITHMS

(SOLVING A 16-YEAR OLD QCD PROBLEM)

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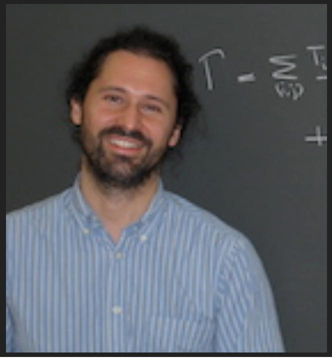
JOHANNES GUTENBERG UNIVERSITY MAINZ

THEORETICAL PHYSICS SEMINAR — ESI, VIENNA, 3 MAY 2022

T. BECHER, MN, D.Y. SHAO, PHYS. REV. LETT. 127 (2021) 212002



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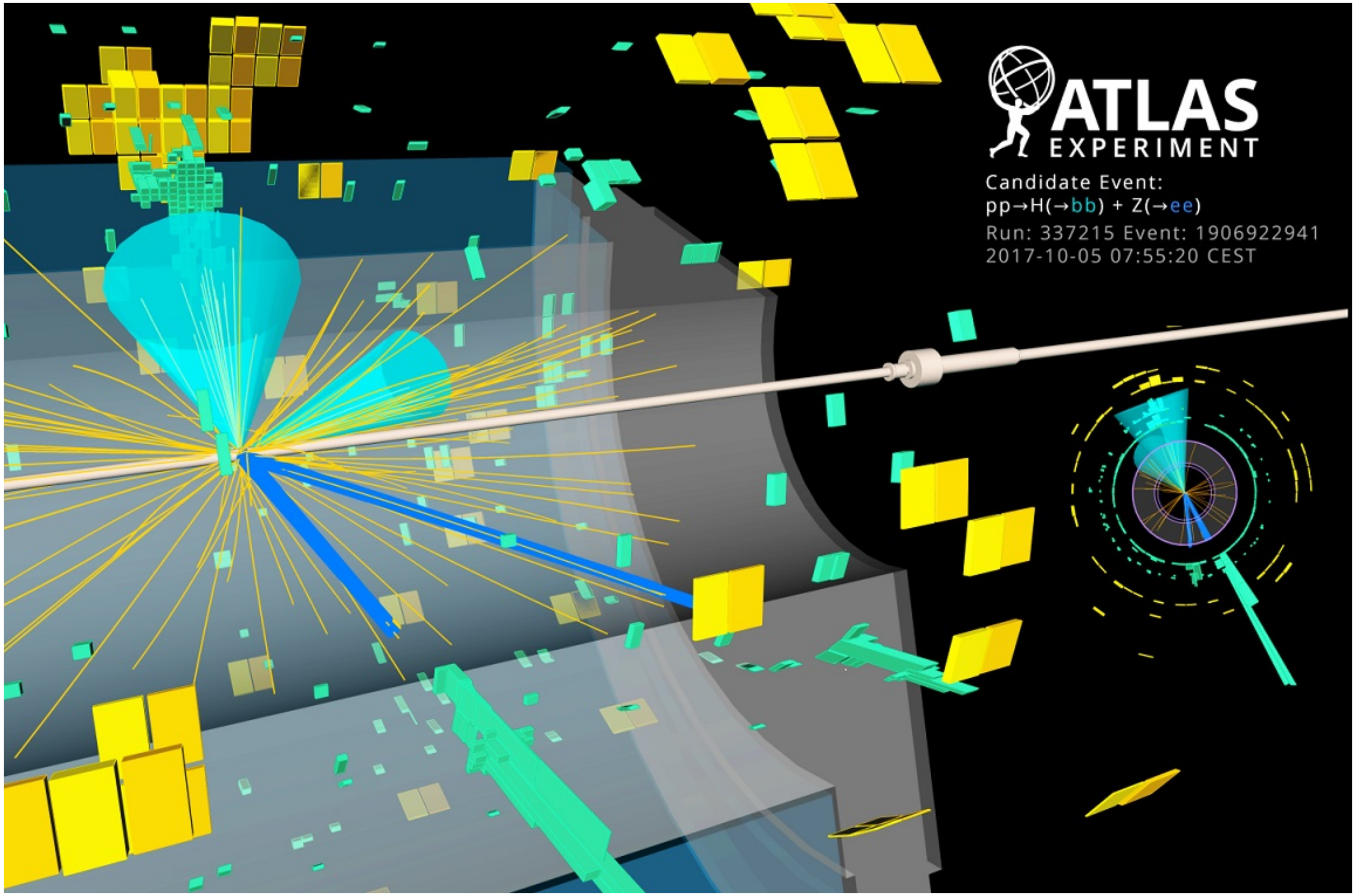
LARGE LOGARITHMS IN JET PROCESSES



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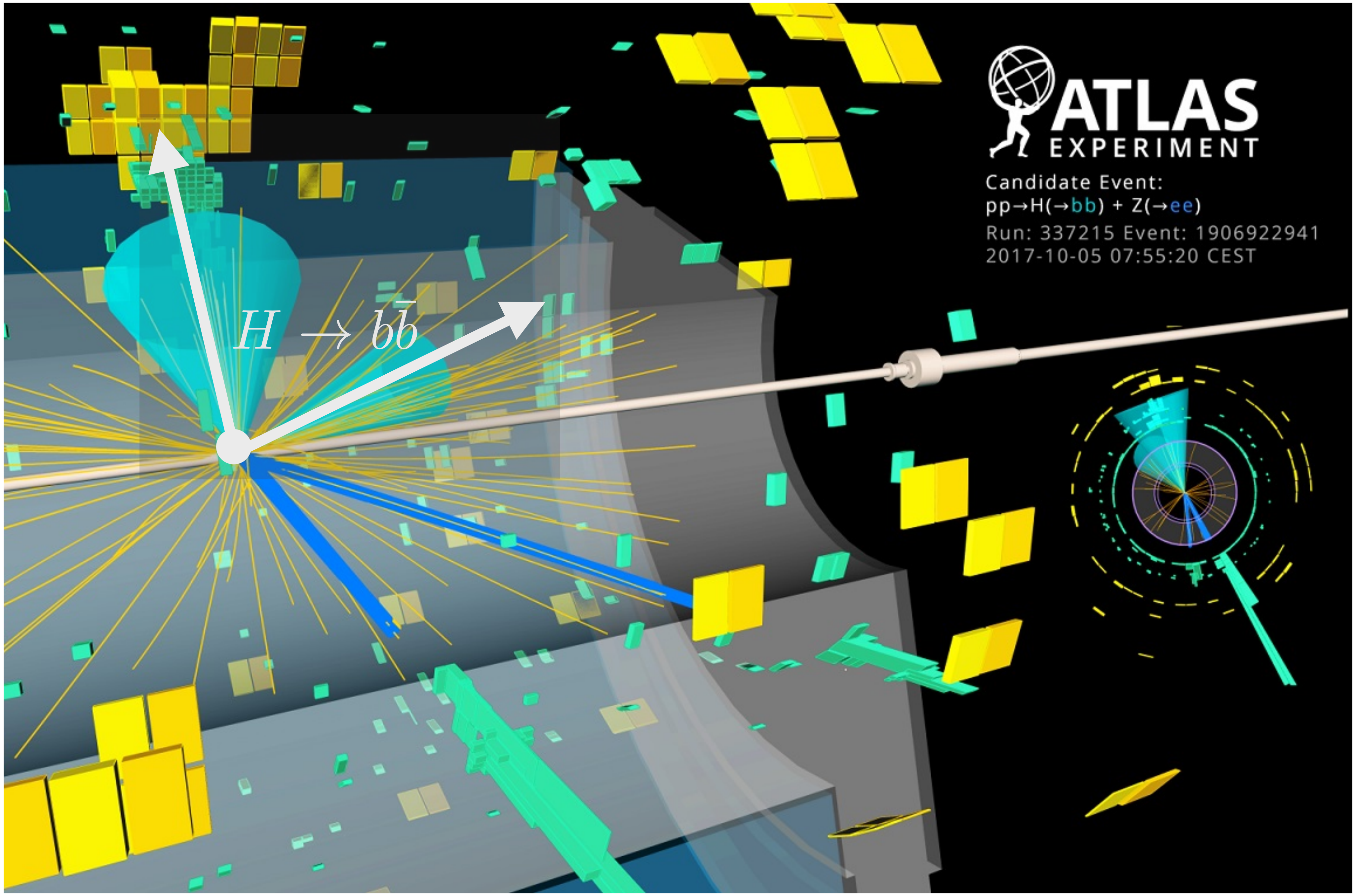

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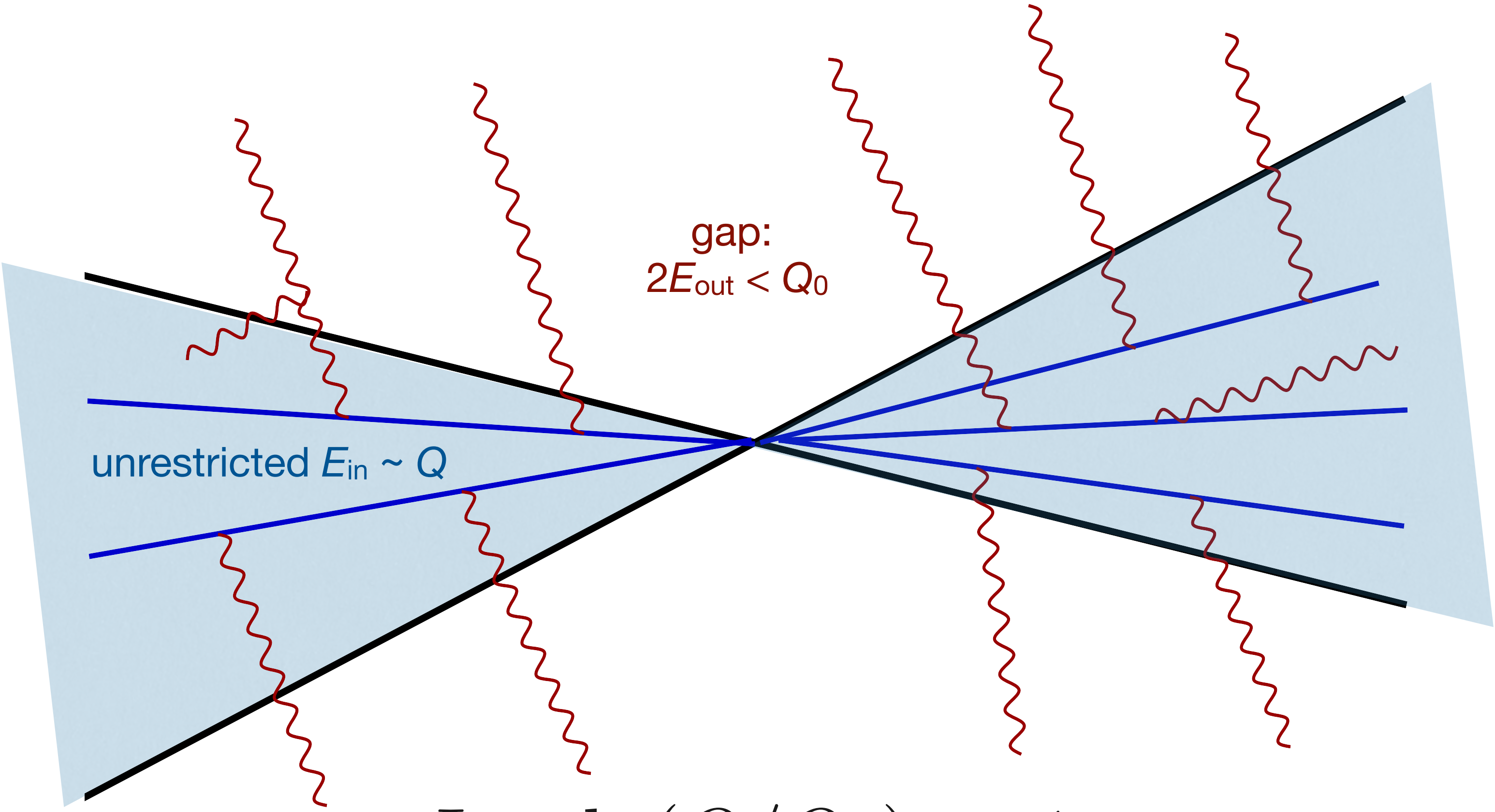
LARGE LOGARITHMS IN JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion:

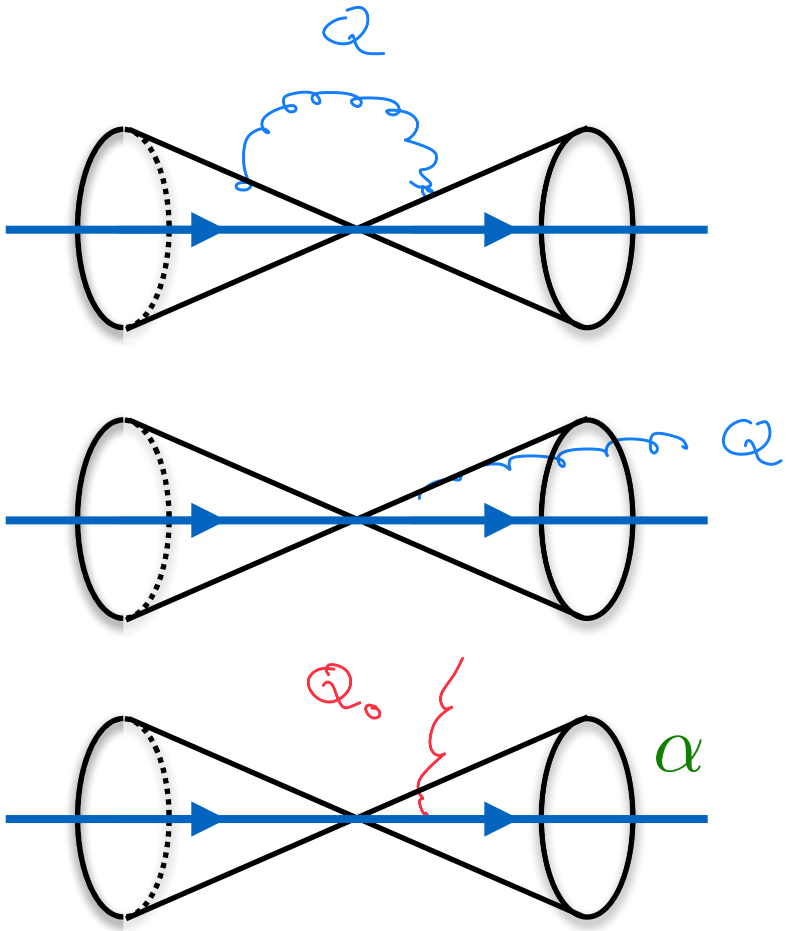
$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 \right\}$$

$\uparrow$

state-of-the-art: 2-loop order

LARGE LOGARITHMS IN JET PROCESSES

Incomplete cancellation of soft and collinear IR divergences:



$$r = \tan^2(\alpha/2)$$

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left[-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} + \dots \right] \qquad (D = 4 - 2\epsilon)$$

soft & collinear

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left[+\frac{4}{\epsilon^2} + \frac{6}{\epsilon} - \frac{4\ln(r)}{\epsilon} + \dots \right]$$

wide-angle soft

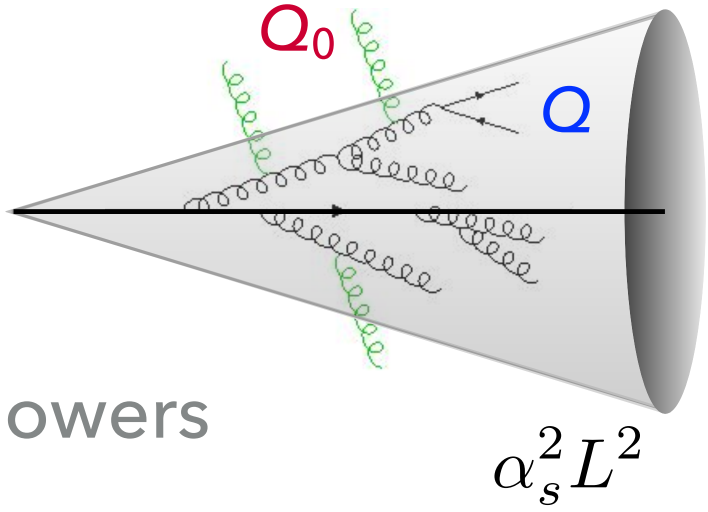
$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q_0^2}\right)^{\epsilon} \left[\frac{4\ln(r)}{\epsilon} + \dots \right]$$

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \, 4\ln(r) \ln \frac{Q^2}{Q_0^2} + \dots$$

LARGE LOGARITHMS IN JET PROCESSES

Non-global logarithms at lepton colliders

- high-energetic radiation restricted to certain regions (inside jets)
- soft radiation from secondary emissions inside jets leads to intricate pattern of large logarithms that do not exponentiate

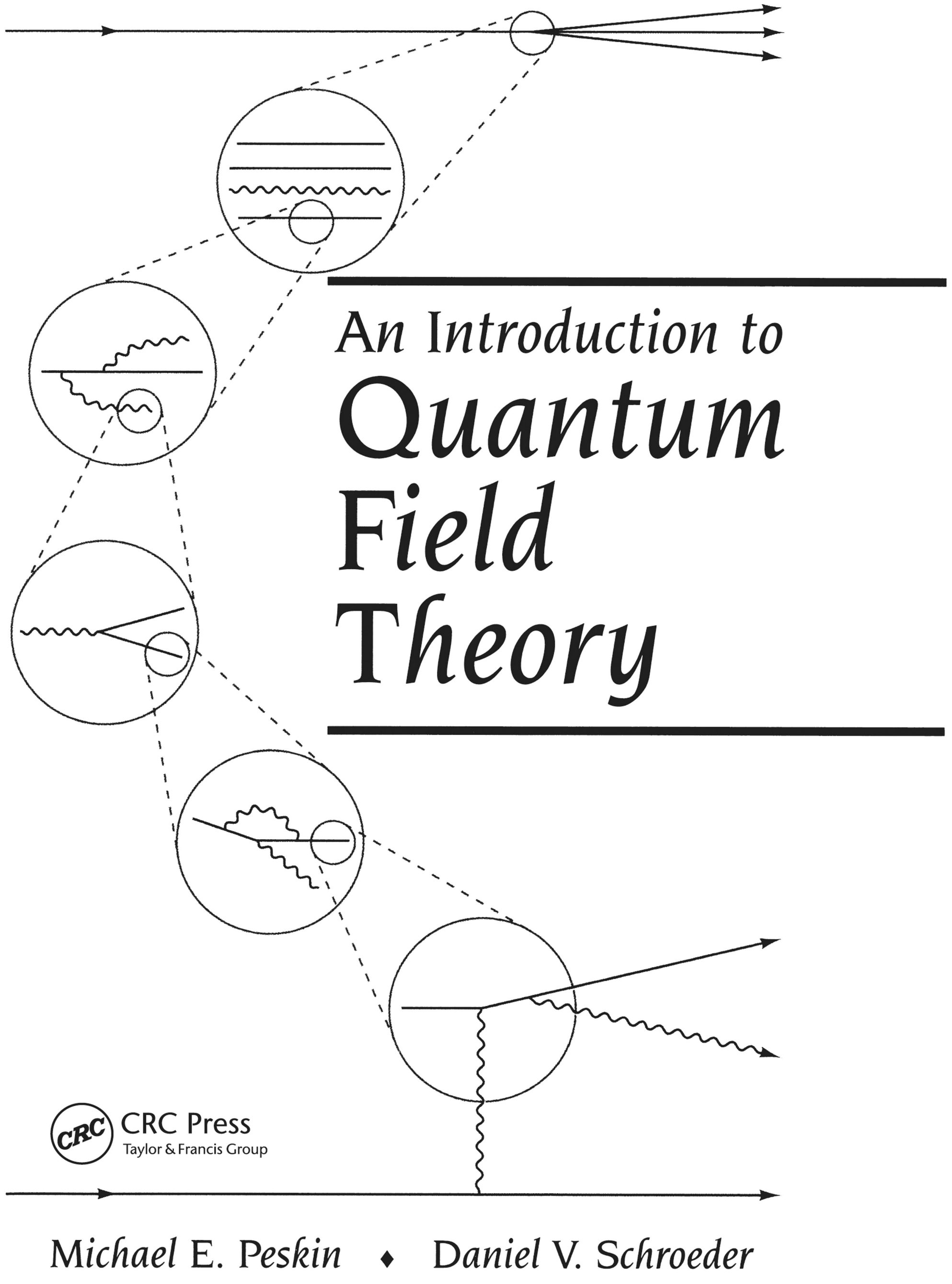


- "non-global" logarithms not contained in parton showers
- single-logarithmic effects $\sim (\alpha_s L)^n$ at lepton colliders
- resummation in large-N_c limit using BMS integral equation

J. Banfi, G. Marchesini, G. Smye: JHEP 08 (2002) 006

At hadron colliders, non-global logarithms take on more intricate form, and no generalization of BMS equation exists!

FACTORIZATION THEOREM FOR JETS AT LEPTON COLLIDERS



An Introduction to
Quantum
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CRC Press
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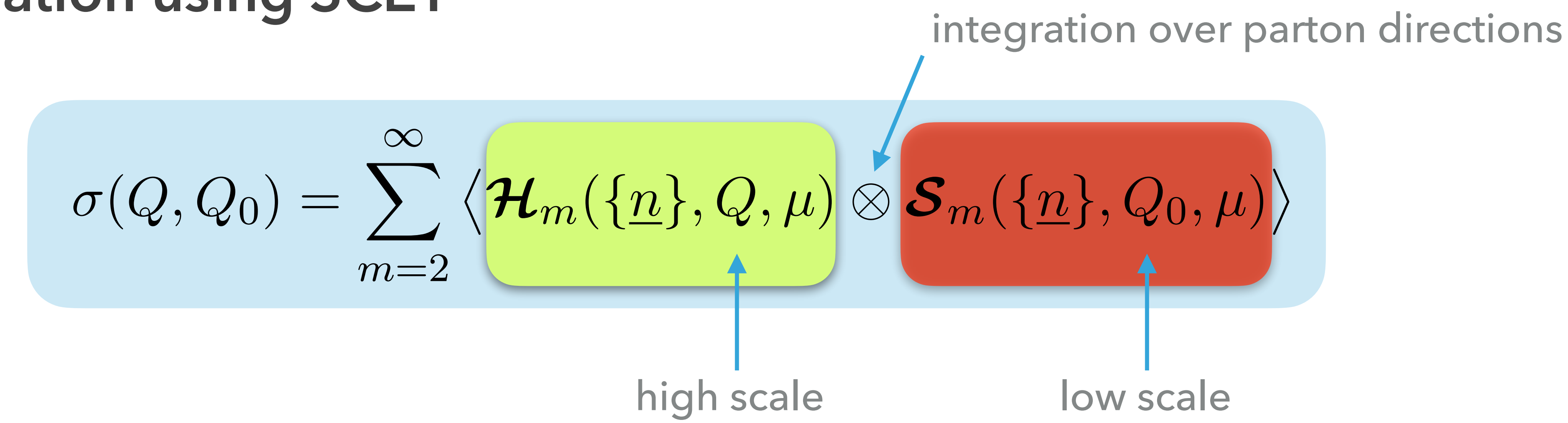
Michael E. Peskin ◆ Daniel V. Schroeder

FACTORIZATION THEOREM FOR JETS AT LEPTON COLLIDERS

Scale separation using SCET



integration over parton directions

$$\sigma(Q,Q_0) = \sum_{m=2}^{\infty} \Big\langle \boxed{\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu)} \otimes \boxed{\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu)} \Big\rangle$$

high scale

low scale

T. Becher, M. Neubert, L. Rothen, D. Y. Shao (2015, 2016)

Rigorous operator definitions:

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) = \frac{1}{2Q^2}\sum_{\text{spins}}\prod_{i=1}^{m}\int \frac{dE_i\,E_i^{d-3}}{(2\pi)^{d-2}}\,|\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|\;(2\pi)^d\,\delta\Big(Q-\sum_{i=1}^{m}E_i\Big)\,\delta^{(d-1)}(\vec{p}_{\text{tot}})\,\Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) = \sum\!\!\!\!\!\!\!\int_{X_s} \langle 0|\,\boldsymbol{S}_1^\dagger(n_1)\,\dots\,\boldsymbol{S}_m^\dagger(n_m)\,|X_s\rangle\langle X_s|\,\boldsymbol{S}_1(n_1)\,\dots\,\boldsymbol{S}_m(n_m)\,|0\rangle\,\theta(Q_0-E_{\text{out}})$$

FACTORIZATION THEOREM FOR JETS AT LEPTON COLLIDERS

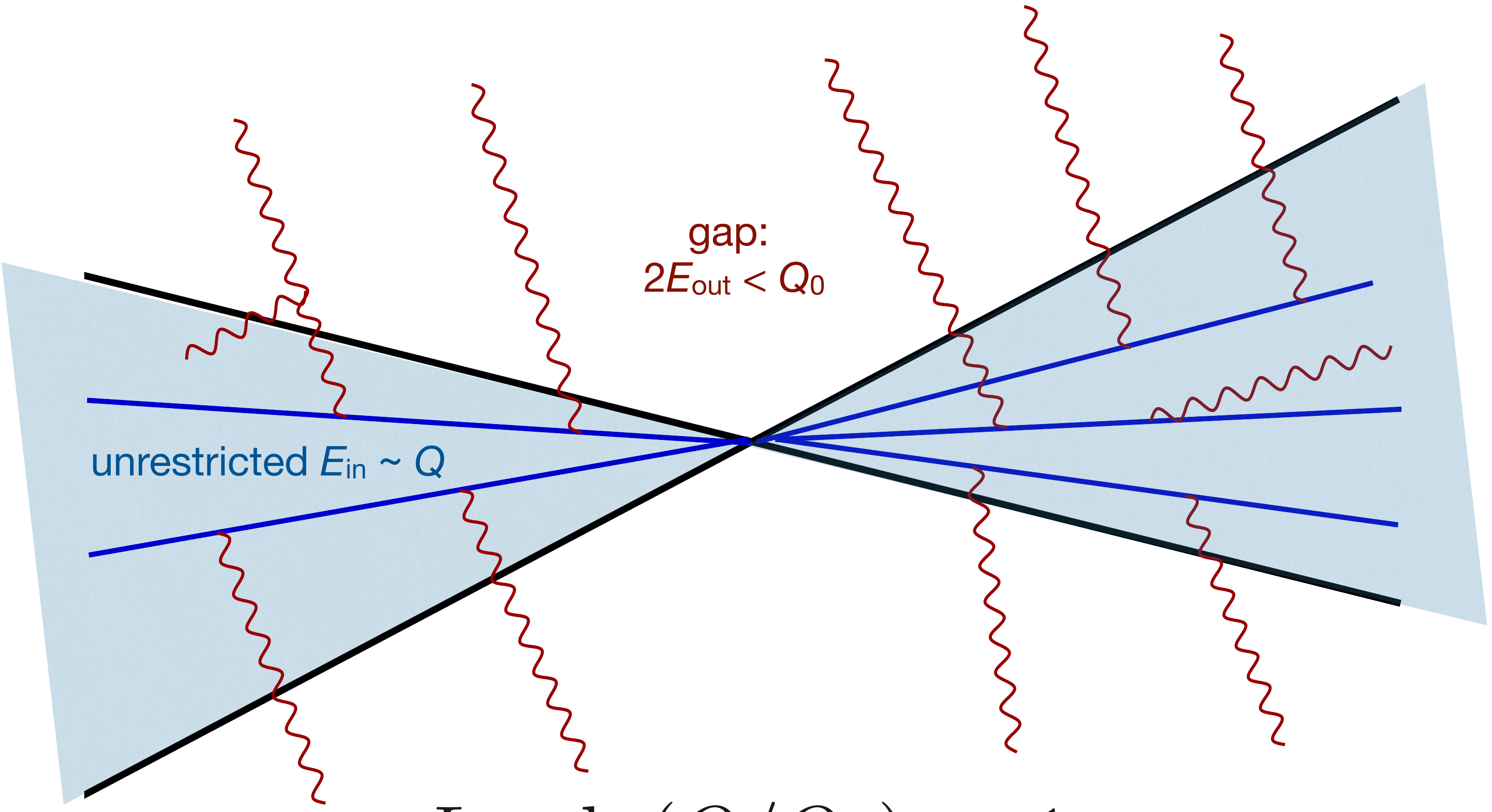
Scale separation using SCET

$$\sigma(Q,Q_0)=\sum_{m=2}^{\infty}\big\langle \underbrace{\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu)}_{\text{high scale}} \overset{\text{integration over parton directions}}{\otimes} \underbrace{\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu)}_{\text{low scale}}\big\rangle$$

T. Becher, M. Neubert, L. Rothen, D. Y. Shao (2015, 2016)

- solving **RG equations** for hard functions allows one to resum all large logarithms (including the NGLs) to all orders of perturbation theory
- not limited to leading logarithms or large-N_c limit!

LARGE LOGARITHMS IN JET PROCESSES AT HADRON COLLIDERS



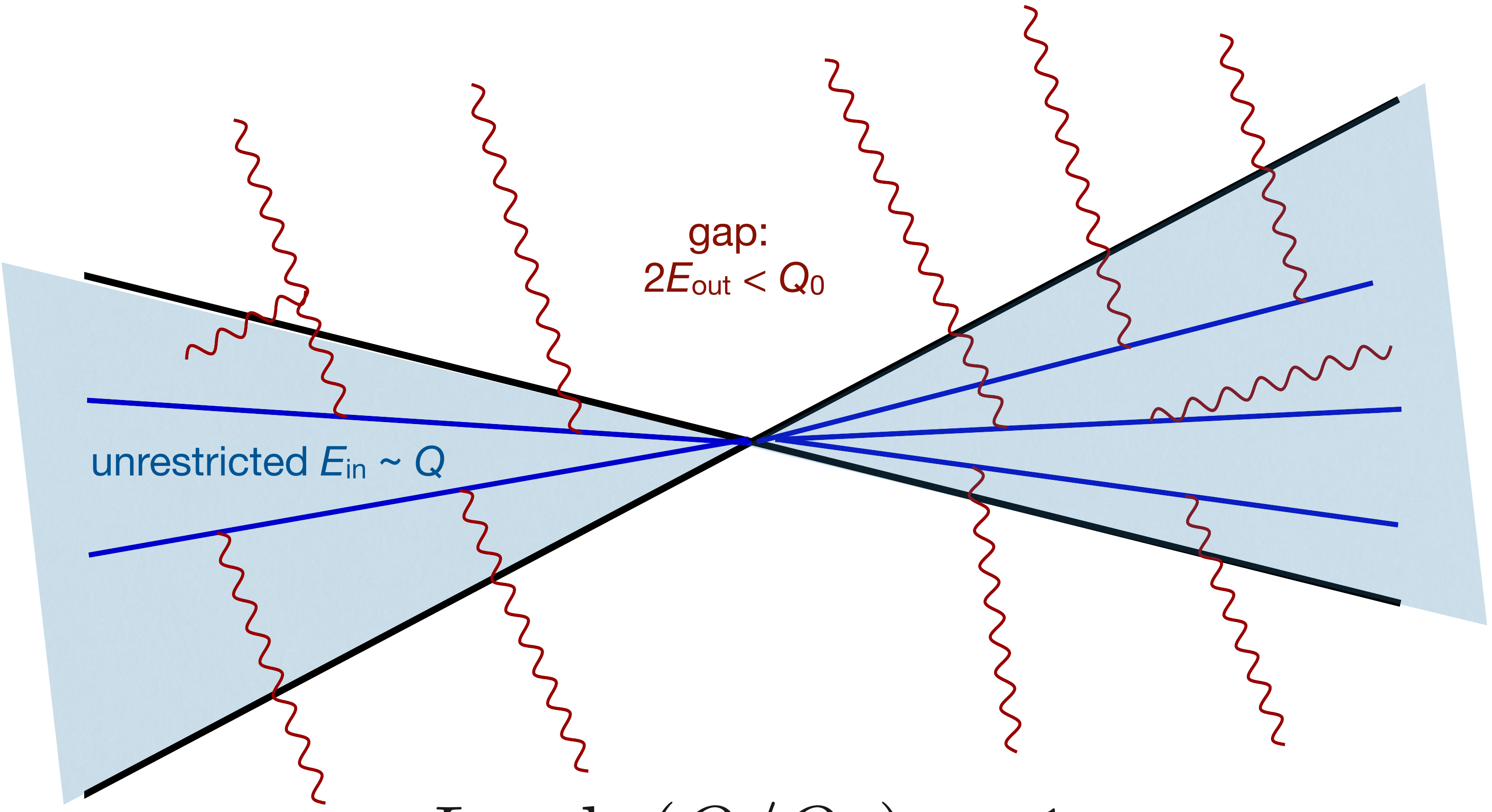
$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion includes “super-leading” logarithms:

$$\sigma \sim \sigma_{\rm Born} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 \, L^5 + \alpha_s^5 \, L^7 + \dots}_{\text{formally larger than } O(1)}\}$$

J. R. Forshaw, A. Kyrieleis, M. H. Seymour: JHEP 08 (2006) 031

LARGE LOGARITHMS IN JET PROCESSES AT HADRON COLLIDERS

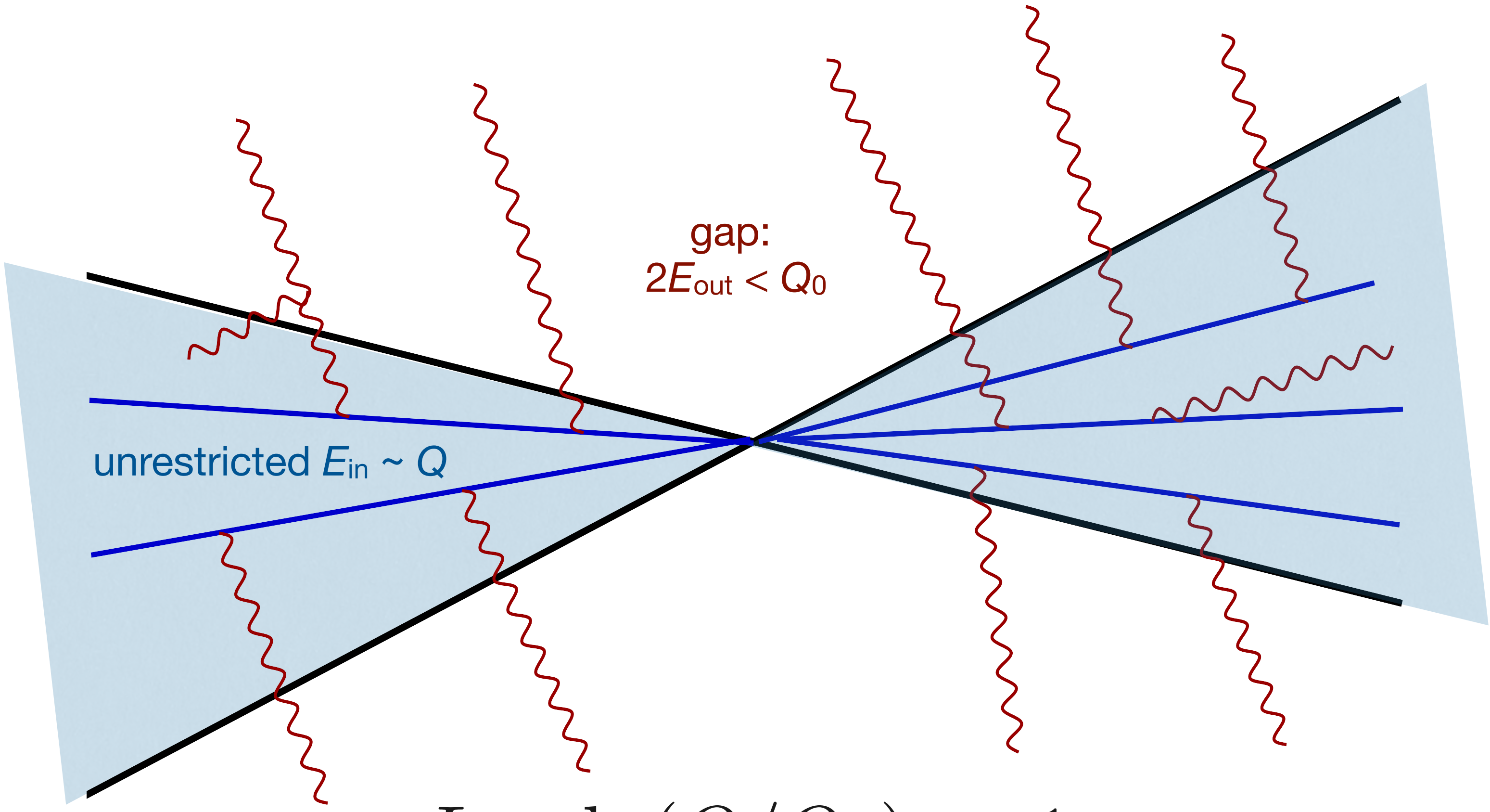


$$L = \ln(Q/Q_0) \gg 1$$

Really, double logarithmic series starting at 3-loop order:

$$\sigma \sim \sigma_{\text{Born}} \times \Big\{1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \underset{(\Im\text{m}\, L)^2}{\pi^2}) \underbrace{\big[\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots\big]}_{\text{formally larger than } O(1)}\Big\}$$

LARGE LOGARITHMS IN JET PROCESSES AT HADRON COLLIDERS



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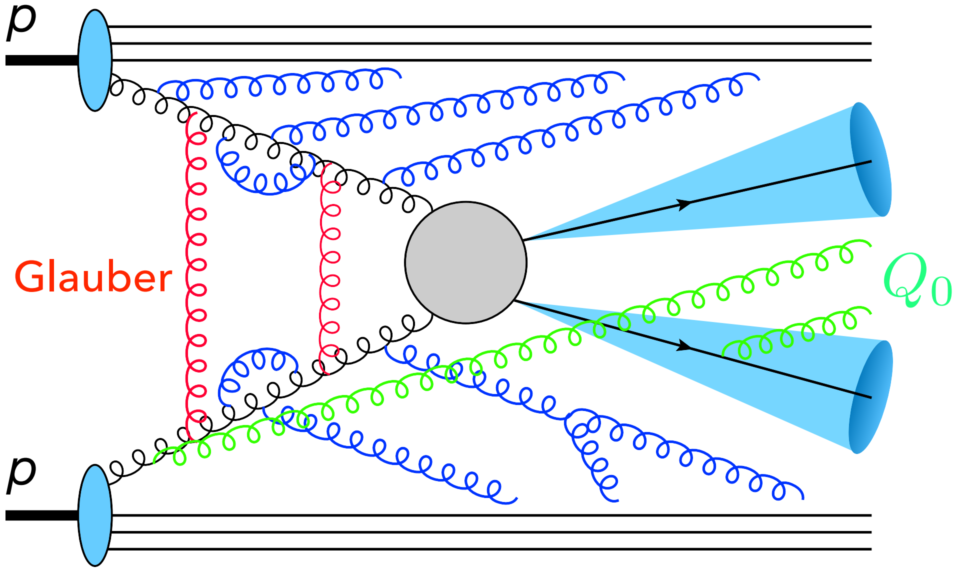
COULOMB PHASES BREAK COLOR COHERENCE

Super-leading logarithms



- breakdown of color coherence due to a subtle quantum effect: soft gluon exchange between initial-state partons
- soft anomalous dimension:

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

T. Becher, M. Neubert (2009)

where $s_{ij} > 0$ if particles i and j are both in initial or final state

- imaginary part (only at hadron colliders):

$$\boxed{\mathrm{Im}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = -2\pi\,\gamma_{\rm cusp}(\alpha_s)\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2} + (\dots)\,\underset{\substack{\uparrow \\ \text{irrelevant}}}{\mathbf{1}}$$

THEORY OF NON-GLOBAL LHC OBSERVABLES

Novel factorization theorem from SCET

$$\sigma_{2\to M}(Q,Q_0)=\sum_{a,b=q,\bar{q},g}\int dx_1 dx_2 \sum_{m=2+M}^{\infty}\big\langle \underbrace{\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)}_{\text{high scale}} \otimes \underbrace{\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu)}_{\text{low scale}}\big\rangle$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

Differences with NGLs at e^+e^- colliders:

- low-energy matrix elements now contain soft Wilson lines plus collinear fields for incoming partons
- low-energy EFT contains Glauber gluons mediating non-trivial interactions between soft and collinear partons I. Rothstein, I. Stewart (2016)
  $\Rightarrow$ breakdown of “naive” factorization

THEORY OF NON-GLOBAL LHC OBSERVABLES

Novel factorization theorem from SCET

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1\, dx_2 \sum_{m=2+M}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \big\rangle$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

high scale

low scale

Renormalization-group equation:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\},Q,\mu) = -\sum_{m\le l} \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)\, \boldsymbol{\Gamma}_{ml}^{H}(\{\underline{n}\},Q,\mu)$$

operator in color space and in the infinite space of parton multiplicities

All-order summation of large logarithmic corrections, including the super-leading logarithms!

THEORY OF NON-GLOBAL LHC OBSERVABLES

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- low-energy matrix element:

$$\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_{\mathbf{s}})$$

- hard-scattering functions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \mathbf{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

THEORY OF NON-GLOBAL LHC OBSERVABLES

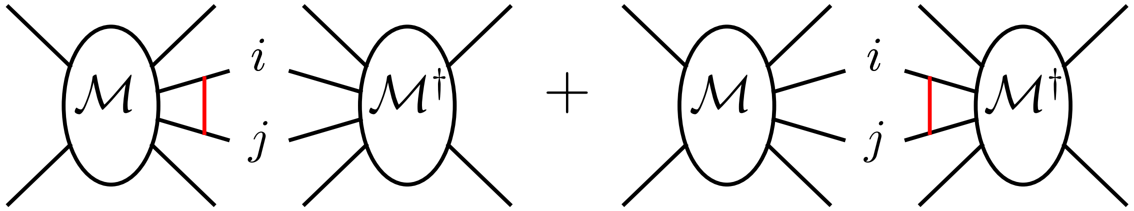
Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- anomalous-dimension matrix:

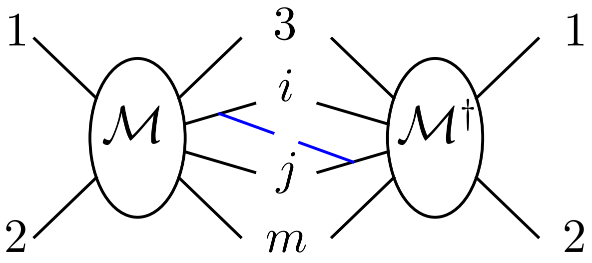
$$\boldsymbol{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_4 & \boldsymbol{R}_4 & 0 & 0 & \cdots \\ 0 & \boldsymbol{V}_5 & \boldsymbol{R}_5 & 0 & \cdots \\ 0 & 0 & \boldsymbol{V}_6 & \boldsymbol{R}_6 & \cdots \\ 0 & 0 & 0 & \boldsymbol{V}_7 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- action on hard functions:

$$\boldsymbol{\mathcal{H}}_m \boldsymbol{V}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \boldsymbol{R}_m = \sum_{(ij)}$$



THEORY OF NON-GLOBAL LHC OBSERVABLES

Detailed structure of the anomalous-dimension coefficients

- virtual and real contributions contain collinear singularities, which must be regularized and subtracted:

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

- with:

$$\boldsymbol{V}^G = -8i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

Coloumb phase

$$\left.\begin{aligned} \boldsymbol{V}_i^c &= 4C_i\,\boldsymbol{1} \\ \boldsymbol{R}_i^c &= -4\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R}\,\delta(n_k - n_i) \end{aligned}\right\}$$

soft & collinear terms

$T_{L,i}$

THEORY OF NON-GLOBAL LHC OBSERVABLES

Comments on notation

- color generators $\boldsymbol{T}_{L,i}$ act on the amplitude (multiply hard functions from the left)
- color generators $\boldsymbol{T}_{R,i}$ act on the complex conjugate amplitude (multiply hard functions from the right)
- real-emission terms take an amplitude with *m* partons and turn it into an amplitude with (*m*+1) partons:

$$\boldsymbol{\mathcal{H}}_m\,\boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}=\boldsymbol{T}_i^a\,\boldsymbol{\mathcal{H}}_m\,\boldsymbol{T}_j^{\tilde{a}}$$

where $a, \tilde{a}$ are color indices of the emitted gluon (symbol ∘ indicates the additional color space of the new parton)

THEORY OF NON-GLOBAL LHC OBSERVABLES

Detailed structure of the anomalous-dimension coefficients

- virtual and real contributions contain collinear singularities, which must be regularized and subtracted:

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\[1em] \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

- with:

$$\overline{\boldsymbol{V}}_m = 2\sum_{(ij)} \left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right) \int\frac{d\Omega(n_k)}{4\pi}\,\overline{W}_{ij}^k$$

$$\overline{\boldsymbol{R}}_m = -4\sum_{(ij)} \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}\,\overline{W}_{ij}^{m+1}\,\Theta_{\text{hard}}(n_{m+1})$$

THEORY OF NON-GLOBAL LHC OBSERVABLES

Detailed structure of the anomalous-dimension coefficients

- virtual and real contributions contain collinear singularities, which must be regularized and subtracted:

$$\left.\begin{array}{l} \boldsymbol{V}_m = \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \displaystyle\sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\[1.5em] \boldsymbol{R}_m = \overline{\boldsymbol{R}}_m + \displaystyle\sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{array}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

- with:

subtracted dipole:

$$\overline{\boldsymbol{V}}_m = 2\sum_{(ij)} \left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right) \int \frac{d\Omega(n_k)}{4\pi}\, \overline{W}_{ij}^k$$

$$\overline{\boldsymbol{R}}_m = -4\sum_{(ij)} \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}\, \overline{W}_{ij}^{m+1}\, \Theta_{\text{hard}}(n_{m+1})$$

$$W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

$$\begin{aligned} W_{ij}^k\, f(n_k) &= \overline{W}_{ij}^k\, f(n_k) \\ &\quad + \frac{1}{n_i\cdot n_k} f(n_i) + \frac{1}{n_j\cdot n_k} f(n_j) \end{aligned}$$

THEORY OF NON-GLOBAL LHC OBSERVABLES

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ with the highest number of insertions of Γ_c

- three properties simplify the calculation:

  - color coherence in absence of Glauber phases (sum of soft emissions off collinear partons has same effect as soft emission of parent parton):

    $$\boldsymbol{\mathcal{H}}_m\,\boldsymbol{\Gamma}^c\,\overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m\,\overline{\boldsymbol{\Gamma}}\,\boldsymbol{\Gamma}^c$$

  - collinear safety (singularities from real and virtual emission cancel):

    $$\langle\boldsymbol{\mathcal{H}}_m\,\boldsymbol{\Gamma}^c\otimes\mathbf{1}\rangle = 0$$

  - cyclicity of the trace:

    $$\big\langle\boldsymbol{\mathcal{H}}_m\,\boldsymbol{V}^G\otimes\mathbf{1}\big\rangle = 0$$

THEORY OF NON-GLOBAL LHC OBSERVABLES

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ with the highest number of insertions of Γ_c

- under the color trace, insertions of Γ_c are non-zero only if they come in conjunction with (at least) two Glauber phases and one $\overline{\Gamma}$
- relevant color traces:

$$C_{rn} = \big\langle \boldsymbol{\mathcal{H}}_{2\,\to\,M}\,(\boldsymbol{\Gamma}^c)^r\,\boldsymbol{V}^G\,(\boldsymbol{\Gamma}^c)^{n-r}\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}\otimes\mathbf{1}\big\rangle$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Simplifications for (anti-)quark-initiated processes

- in the fundamental representation, symmetrized products of color generators can be reduced ($\sigma_i = \pm 1$ for (anti-)quarks):

$$\{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\} = \frac{1}{N_c}\,\delta_{ab} + \sigma_i\, d_{abc}\, \boldsymbol{T}_i^c$$

- simple results in terms of three non-trivial color structures:

$$C_{rn} = -2^{8-r}\pi^2\,(4N_c)^n \left\{ \sum_{j=3}^{M+2} J_j \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \left[(\boldsymbol{T}_1 - \boldsymbol{T}_2)\cdot \boldsymbol{T}_j - 2^{r-1} N_c\,(\sigma_1 - \sigma_2)\, d_{abc}\, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \right] \right\rangle \right.$$

$$\left. - \, 2\,(1-\delta_{r0})\, J_2 \, \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \left[C_F\, \boldsymbol{1} + (2^r - 1)\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \right] \right\rangle \right\}$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

RESUMMATION OF SUPER-LEADING LOGARITHMS

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$$\left. - 2\,(1-\delta_{r0})\, J_2 \;\left\langle \boldsymbol{\mathcal{H}}_{2\to M} \left[C_F\, \boldsymbol{1} + (2^r - 1)\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \right] \right\rangle \right\}$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

- kinematic information contained in $(M+1)$ angular integrals:

$$J_j = \int \frac{d\Omega(n_k)}{4\pi} \left(W_{1j}^k - W_{2j}^k \right) \Theta_{\text{veto}}(n_k)\,; \quad \text{with} \quad W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k\, n_j \cdot n_k}$$

- reproduces all that is known about SLLs (and much more…)

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result (valid for arbitrary representations)

$$C_{rn} = -256\pi^2 \left(4N_c\right)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{S}_i \right\rangle \right]$$

T. Becher, M. Neubert, D. Y. Shao: in preparation

- basis of 10 color structures:

$$\boldsymbol{O}_1^{(j)} = f_{abe} f_{cde}\, \boldsymbol{T}_2^a \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\}\, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_2^{(j)} = d_{ade} d_{bce}\, \boldsymbol{T}_2^a \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\}\, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_3^{(j)} = \boldsymbol{T}_2^a \{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\}\, \boldsymbol{T}_j^b - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_4^{(j)} = 2C_1\, \boldsymbol{T}_2 \cdot \boldsymbol{T}_j - 2C_2\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_j$$

$$\boldsymbol{S}_1 = f_{abe} f_{cde}\, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\}\, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^d\}$$

$$\boldsymbol{S}_2 = d_{ade} d_{bce}\, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\}\, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^d\}$$

$$\boldsymbol{S}_3 = d_{ade} d_{bce} \left[\boldsymbol{T}_2^a \left(\boldsymbol{T}_1^b \boldsymbol{T}_1^c \boldsymbol{T}_1^d \right)_+ + (1 \leftrightarrow 2) \right]$$

$$\boldsymbol{S}_4 = \{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\}\, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^b\}$$

$$\boldsymbol{S}_5 = \boldsymbol{T}_1 \cdot \boldsymbol{T}_2$$

$$\boldsymbol{S}_6 = \boldsymbol{1}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result (valid for arbitrary representations)

$$C_{rn} = -256\pi^2\,(4N_c)^{n-r}\left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)}\,\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\boldsymbol{O}_i^{(j)}\rangle - J_2\sum_{i=1}^{6} d_i^{(r)}\,\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\boldsymbol{S}_i\rangle\right]$$

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- coefficient functions:

$$c_1^{(r)} = 2^{r-1}\left[(3N_c+2)^r + (3N_c-2)^r\right]$$

$$c_2^{(r)} = 2^{r-2}N_c\left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} - \frac{(2N_c)^{r+1}}{N_c^2-4}\right]$$

$$c_3^{(r)} = 2^{r-1}\left[(3N_c+2)^r - (3N_c-2)^r\right]$$

$$c_4^{(r)} = 2^{r-1}\left[\frac{(3N_c+2)^r}{N_c+1} + \frac{(3N_c-2)^r}{N_c-1} - \frac{2N_c^{r+1}}{N_c^2-1}\right]$$

$$d_1^{(r)} = 2^{3r-1}\left[(N_c+1)^r + (N_c-1)^r\right] - 2^{r-1}\left[(3N_c+2)^r + (3N_c-2)^r\right]$$

$$d_2^{(r)} = 2^{3r-2}N_c\left[\frac{(N_c+1)^r}{N_c+2} + \frac{(N_c-1)^r}{N_c-2}\right] - 2^{r-2}N_c\left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2}\right]$$

$$d_3^{(r)} = 2^{r-1}N_c\left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} - \frac{(2N_c)^{r+1}}{N_c^2-4}\right]$$

$$d_4^{(r)} = 2^{3r-1}\left[(N_c+1)^r - (N_c-1)^r\right] - 2^{r-1}\left[(3N_c+2)^r - (3N_c-2)^r\right]$$

$$\begin{aligned} d_5^{(r)} &= 2^r\,(C_1+C_2)\left[\frac{N_c+2}{N_c+1}\,(3N_c+2)^r - \frac{N_c-2}{N_c-1}\,(3N_c-2)^r - \frac{2N_c^{r+1}}{N_c^2-1}\right] \\ &\quad - \frac{2^{r-1}N_c}{3}\left[(N_c+4)\,(3N_c+2)^r + (N_c-4)\,(3N_c-2)^r - (2N_c)^{r+1}\right] \end{aligned}$$

$$\begin{aligned} d_6^{(r)} &= 2^{3r+1}C_1C_2\left[(N_c+1)^{r-1} + (N_c-1)^{r-1}\right](1-\delta_{r0}) \\ &\quad - 2^{r+1}C_1C_2\left[\frac{(3N_c+2)^r}{N_c+1} + \frac{(3N_c-2)^r}{N_c-1} - \frac{2N_c^{r+1}}{N_c^2-1}\right] \end{aligned}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result (valid for arbitrary representations)

$$C_{rn} = -256\pi^2 \, (4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{S}_i \right\rangle \right]$$

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- series of SLLs, starting at 3-loop order:

$$\sigma_{\rm SLL} = \sigma_{\rm Born} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \, \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Summation of super-leading logarithms for $qq \to qq$ scattering:

$$\sigma_{\rm SLL}^{(S)} = -\sigma_{\rm Born}\,\underbrace{\boxed{\frac{16\alpha_s L}{27 N_c \pi}}}_{\text{1-loop factor}}\,\Delta Y \left(\frac{N_c \alpha_s}{\pi}\,\pi^2\right) w\;{}_2F_2\!\left(1,1;2,\frac{5}{2};-w\right)$$

$$w = \frac{N_c \alpha_s}{\pi}\,L^2$$

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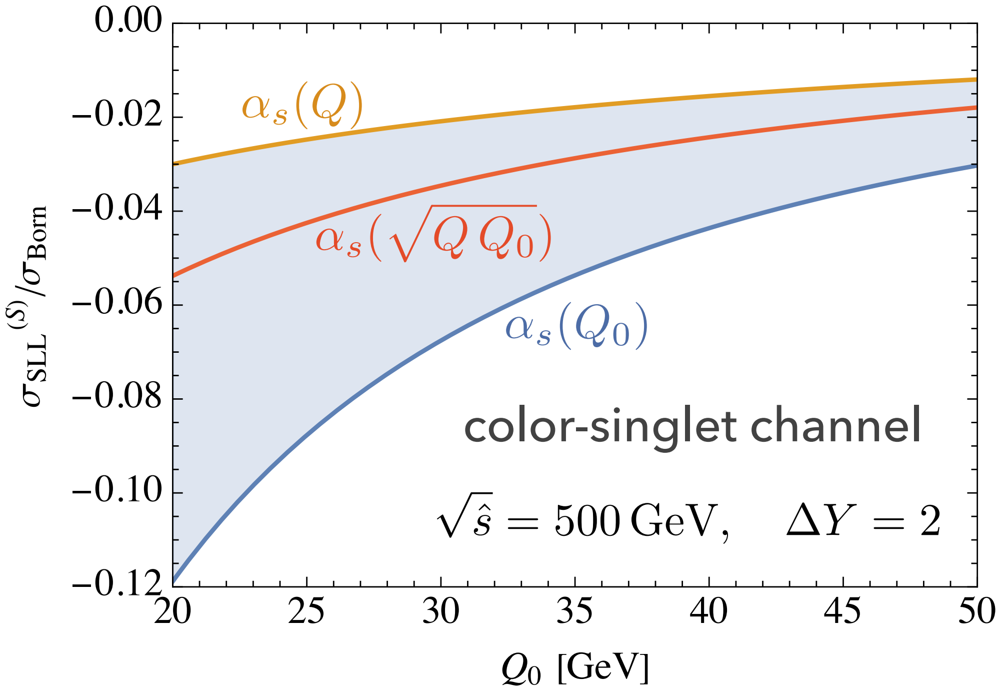

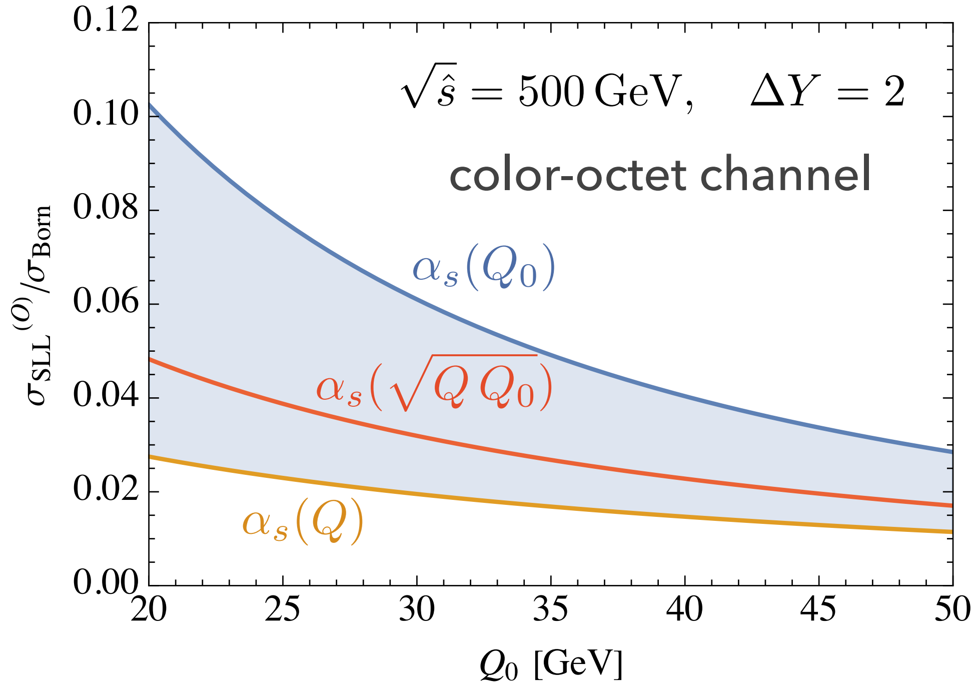

RESUMMATION OF SUPER-LEADING LOGARITHMS

Summation of super-leading logarithms for $qq \to qq$ scattering:

$$\sigma_{\text{SLL}}^{(S)} = -\sigma_{\text{Born}} \boxed{\frac{16\alpha_s L}{27 N_c \pi}} \Delta Y \left(\frac{N_c \alpha_s}{\pi} \pi^2 \right) w \, {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right)$$

1-loop factor

$$w = \frac{N_c \alpha_s}{\pi} L^2$$

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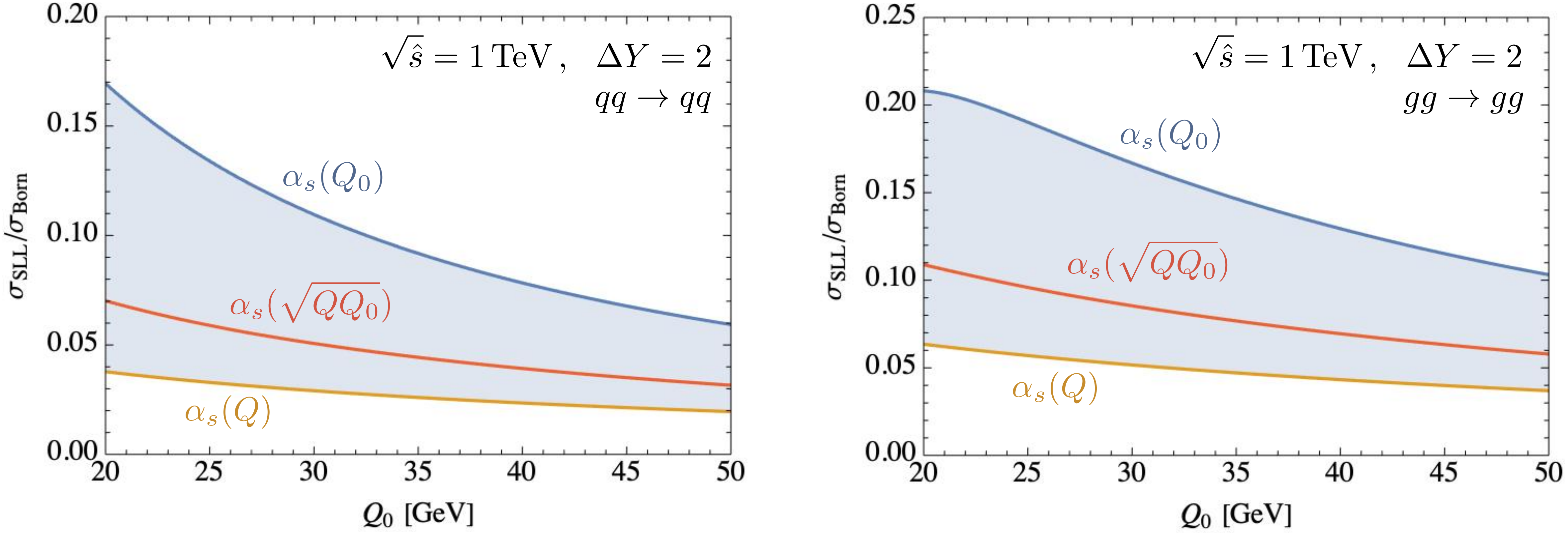
- asymptotic behavior for $L \to \infty$:

$$w \, {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right) \to \frac{3}{2} \left[\ln(4w) + \gamma_E - 2 \right]$$

- very different from standard Sudakov double logarithms $\sim e^{-cw}$
- **expect even larger effects for gluon-initiated processes!**

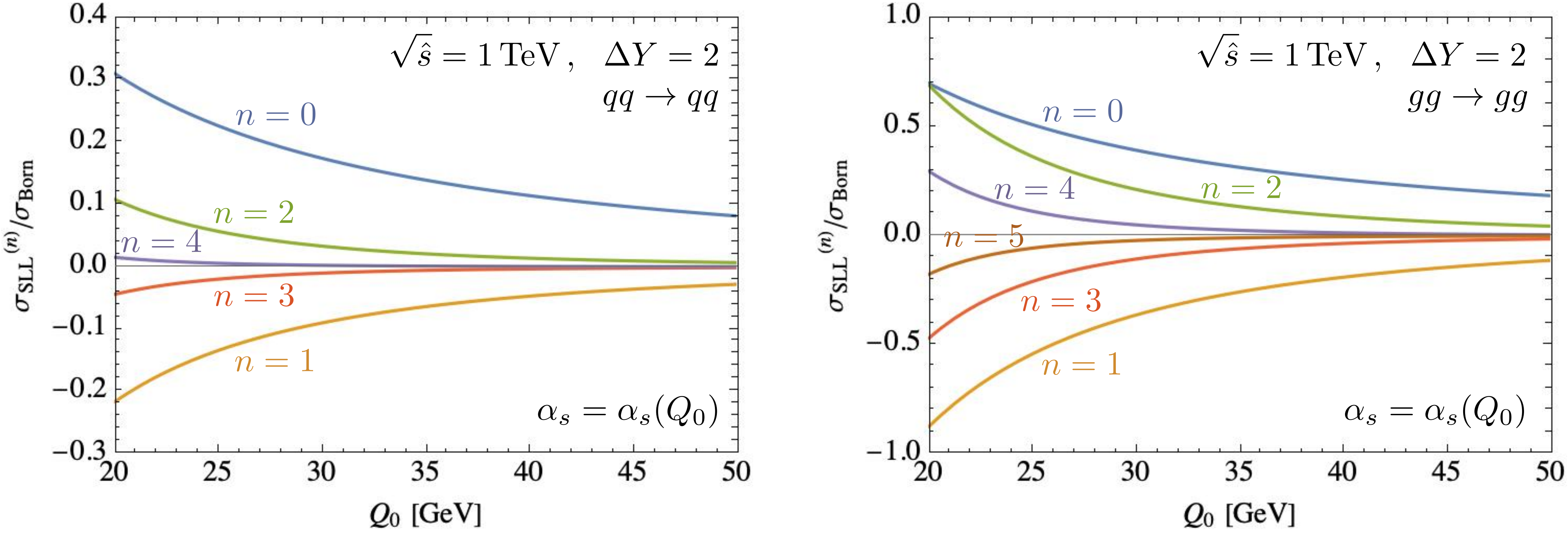
RESUMMATION OF SUPER-LEADING LOGARITHMS

Super-leading logs for $qq \to qq$ and $gg \to gg$ scattering at $\sqrt{\hat{s}} = 1$ TeV:



RESUMMATION OF SUPER-LEADING LOGARITHMS

Super-leading logs for $qq \to qq$ and $gg \to gg$ scattering at $\sqrt{\hat{s}} = 1$ TeV:



$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^{n+3} L^{2n+3}\, \frac{(-4)^n\, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r\,(r!)^2}\, C_{rn}$$

IMPORTANT REMARKS

- SCET-based approach solves 16-year old QCD problem, extending existing results to all orders of perturbation theory and to arbitrary $2 \to M$ hard-scattering processes
- master formula also applies to cases where $M = 1$ or even $M = 0$, which were not considered before (SLLs start at 4- and 5-loop order, respectively)
- relevant for both SM phenomenology (e.g. $pp \to h + \text{jet}$) and New-Physics searches (e.g. WIMP searches in $pp \to \text{jet} + \not{E}_T$)

CONCLUSIONS

Toward a complete theory of LHC jet processes

- powerful new factorization theorem derived using SCET
- in future, extension to massive final-state partons and calculations beyond leading logarithms (important for scale setting)
- detailed study of low-energy matrix elements using SCET with Glauber gluons will offer an *ab initio* understanding of violations of conventional factorization (perturbative part of “underlying event”)
- results very relevant for future improvements of parton showers
- new levels of precision in predictions for important LHC processes