TMD factorization beyond the leading power

based on [2109.09771]

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Transverse momentum dependent (TMD) factorization theorems and distributions is vastly expending area of physics.

Theory

It was originated in early 80's, but got a boost in early 2010's (proof of LP factorization theorem) [Collins,2011; SCET 2011]. Nowadays theory is as good as collinear factorization

- Full N^3LO evolution
- Many coefficient functions at NLO/NNLO/N^3LO
- Continue to expand: new processes (jets, heavy quarks,...), lattice, ...

Phenomenology

Phenomenology of TMD is in the process of development

- Many facilities have dedicated TMD program: **COMPASS, JLab, RHIC** (HERMES)
- Can be also observable at **LHC, BaBar, BELLE**.
- Significant part of physics programme for future **EIC**.
- First global extractions [Scimemi,AV,1912.06532]=SV19, [Bacchetta,et al,1912.07550]=Pavia19

Transverse momentum dependent (TMD) factorization theorems and distributions is vastly expending area of physics.

Today's talk

TMD operator expansion

- Novel approach to TMD factorization theorem
- Direct generalization of operator product expansion
- Has common features with high-energy expansion and SCET
- Elegant and "simple" internal structure
- I will demonstrate next-to-leading power (NLP) expression at NLO(!)
- Still in development (based [AV,Moos,Scimemi,2109.09771] [Rodini,AV,2022]...)

Outline

- Introduction to TMD factorization
- Introduction to TMD operator expansion
- Review of results and outlook

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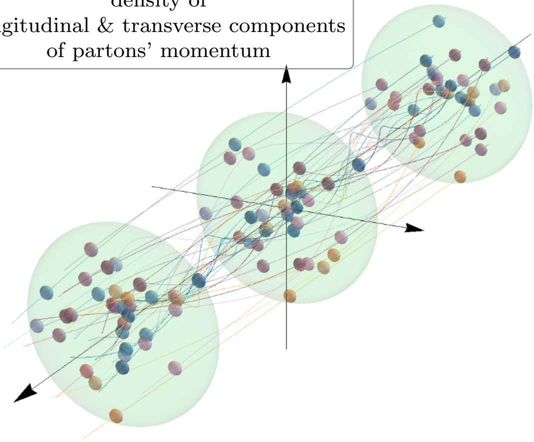
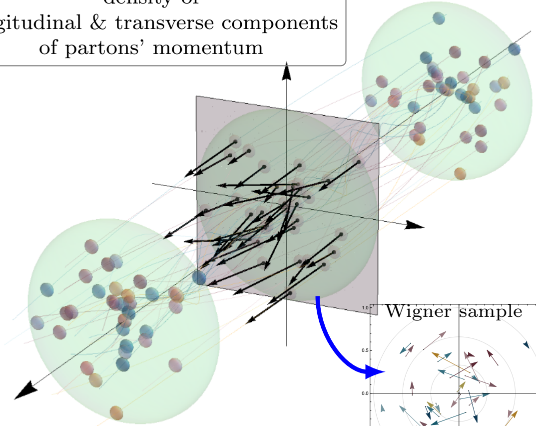
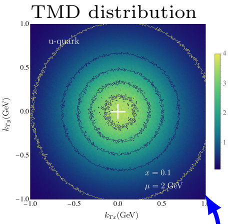
TMD distributions measure density of longitudinal & transverse components of partons' momentum

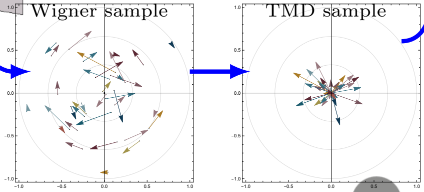


TMD distributions measure density of longitudinal & transverse components of partons' momentum



[Bury,Prokudin,AV, 2103.03270]





Selected features of TMD distributions (1)

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Presence of additional vector (k_T) reveals many structures

- 8 TMDPDFs already at LP
- + 2 TMDFFs
- Numerous spin-dependent effects are described by LP TMD factorization (while they are of NLP/NNLP in collinear factorization)

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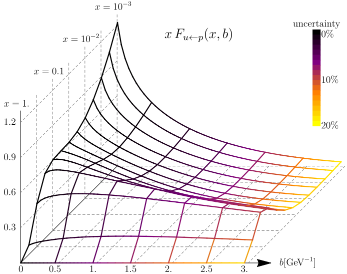
Selected features of TMD distributions (2)

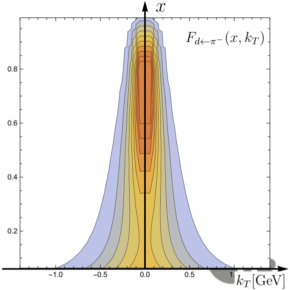
$$F(x,b)=\int \frac{d^2k_T}{(2\pi)^2}e^{-i(kb)_T}F(x,k_T)$$

- TMD factorization is *naturally* formulated in the position space
- Simple evolution equation

TMDs in b-space



TMDs in k_T-space

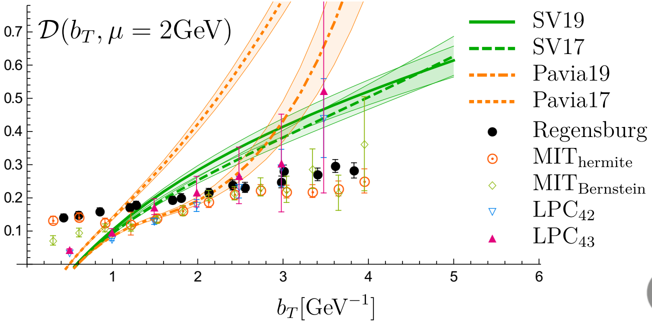


Selected features of TMD distributions (3)

TMD distributions has two scales and obey a pair of evolution equations

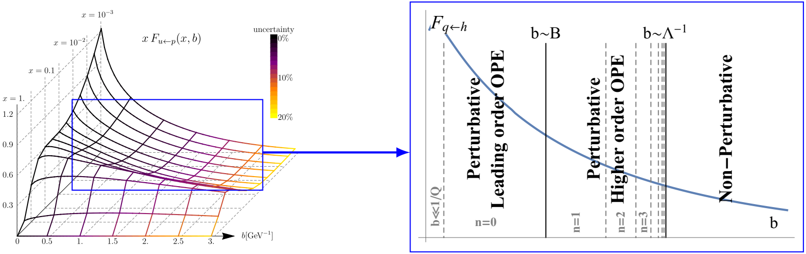
$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \gamma_F(\mu,\zeta)F(x,b;\mu,\zeta), \qquad \zeta \frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(b,\mu)F(x,b;\mu,\zeta).$$

$\mathcal{D} = -2K$ is the Collins-Soper kernel. **Nonperturbative!**
Measures the properties of QCD vacuum [AV,PRL,2020]



Selected features of TMD distributions (4)

TMD factorization $\longleftrightarrow$ resummation



$$F(x,b) = \left[1 + \alpha_s \left(p(x)\ln(b^2\mu^2) + ...\right) + \alpha_s^2 ...\right] \otimes q(x) + b^2 ... + ...$$

Lead.power OPE
up N^{3}LO

Higher power OPE
[Moos,AV,2008.01744]

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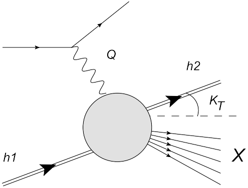
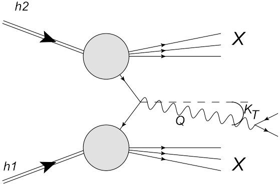
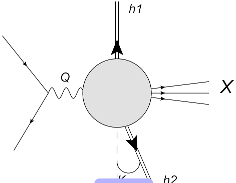
The main processes are



SIDIS



Drell-Yan



SIA

q is momentum of initiating EW-boson

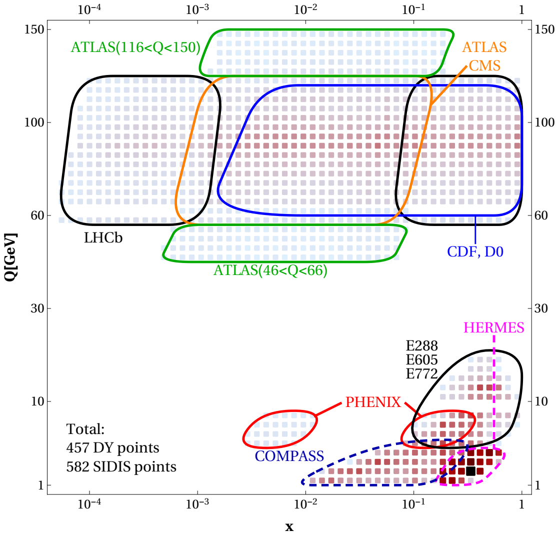
$q^2 = \pm Q^2$

q_T^μ transverse component

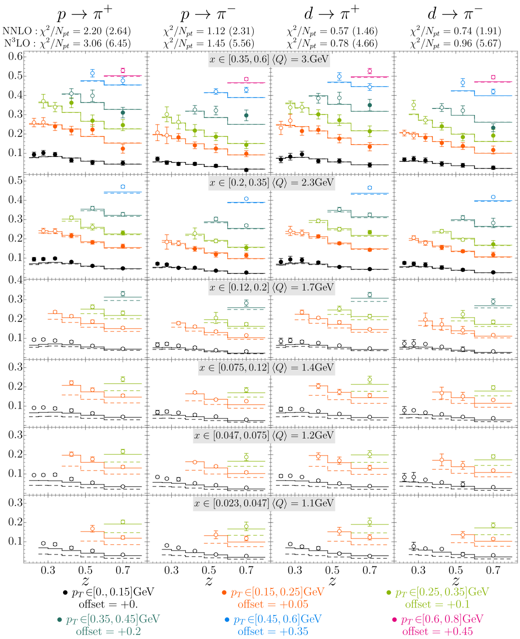
$$\begin{cases} Q^2 \gg \Lambda^2_{QCD} \\ Q^2 \gg q_T^2 \end{cases}$$

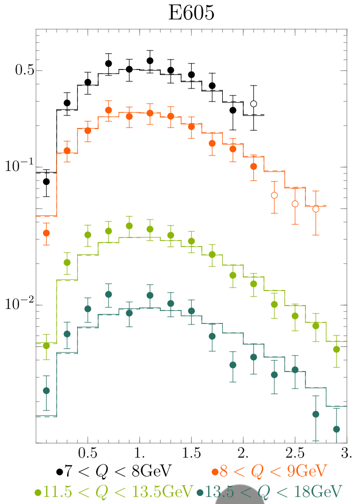
$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2) + ...$$

[Scimemi,AV,1912.06532] = **SV19**



- N^3LO perturbative input
- 1039 data points (DY+SIDIS) in fit
  - $2 < Q < 150$GeV
  - $10^{-4} < x < 1$
- ~ 1500 extra points described
- `artemide`
- Further expansions πDY, SSA





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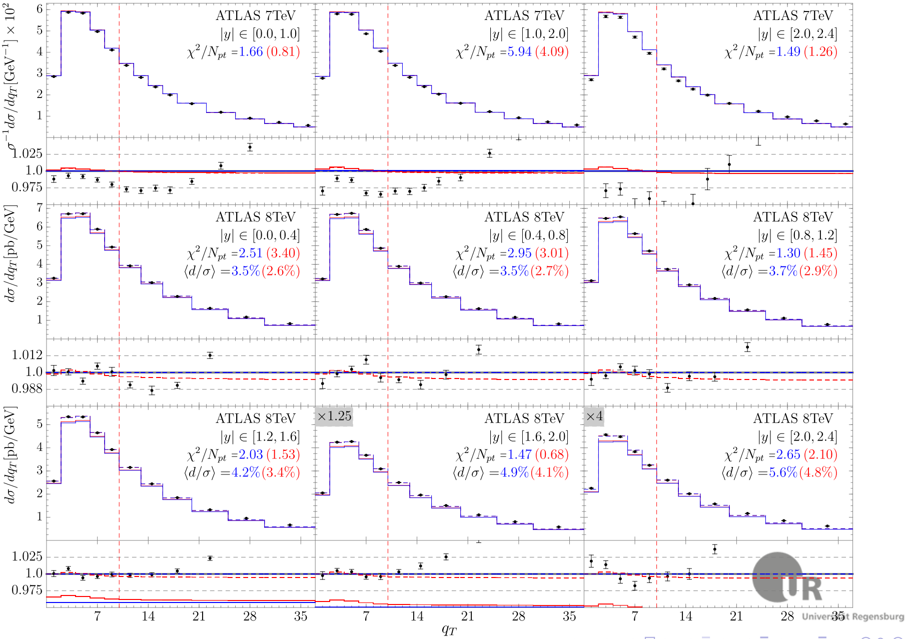
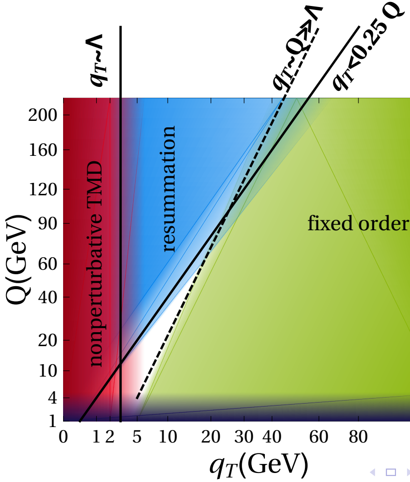


ATLAS 7TeV
$|y| \in [0.0, 1.0]$
χ^2/N_{pt} =1.66 (0.81)
ATLAS 7TeV
$|y| \in [1.0, 2.0]$
χ^2/N_{pt} =5.94 (4.09)
ATLAS 7TeV
$|y| \in [2.0, 2.4]$
χ^2/N_{pt} =1.49 (1.26)
ATLAS 8TeV
$|y| \in [0.0, 0.4]$
χ^2/N_{pt} =2.51 (3.40)
$\langle d/\sigma \rangle$ =3.5%(2.6%)
ATLAS 8TeV
$|y| \in [0.4, 0.8]$
χ^2/N_{pt} =2.95 (3.01)
$\langle d/\sigma \rangle$ =3.5%(2.7%)
ATLAS 8TeV
$|y| \in [0.8, 1.2]$
χ^2/N_{pt} =1.30 (1.45)
$\langle d/\sigma \rangle$ =3.7%(2.9%)
ATLAS 8TeV
$|y| \in [1.2, 1.6]$
χ^2/N_{pt} =2.03 (1.53)
$\langle d/\sigma \rangle$ =4.2%(3.4%)
×1.25
ATLAS 8TeV
$|y| \in [1.6, 2.0]$
χ^2/N_{pt} =1.47 (0.68)
$\langle d/\sigma \rangle$ =4.9%(4.1%)
×4
ATLAS 8TeV
$|y| \in [2.0, 2.4]$
χ^2/N_{pt} =2.65 (2.10)
$\langle d/\sigma \rangle$ =5.6%(4.8%)
$\sigma^{-1} d\sigma/dq_T [\text{GeV}^{-1}] \times 10^2$
$d\sigma/dq_T$[pb/GeV]
$d\sigma/dq_T$[pb/GeV]
q_T

Factorization regions

$q_T \lesssim 0.25Q$ TMD factorization $= \begin{cases} q_T \lesssim \Lambda & \text{nonpertrubative regime} \\ q_T \gg \Lambda & \text{"resummation" regime} \end{cases}$

$q_T \sim Q \gg \Lambda$ fixed order computation



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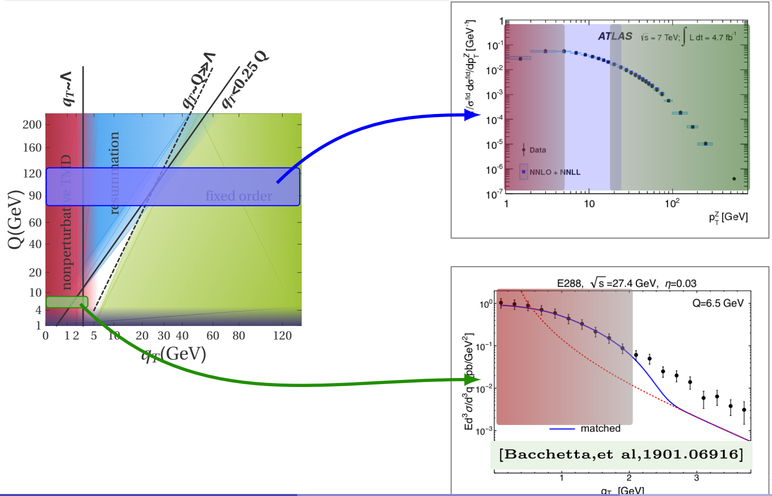
Factorization regions

$$q_T \lesssim 0.25Q \quad \text{TMD factorization} \quad = \begin{cases} q_T \lesssim \Lambda & \text{nonpertrubative regime} \\ q_T \gg \Lambda & \text{"resummation" regime} \end{cases}$$

$$q_T \sim Q \gg \Lambda \quad \text{fixed order computation}$$



[Bacchetta,et al,1901.06916]

Transverse momentum dependent factorization

$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2)$$

LP term is studied VERY WELL!

This was a very brief review of
LP TMD factorization

Now let's turn to power corrections

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Transverse momentum dependent factorization

$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} \Big\{ |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2) \qquad \longleftarrow \text{LP}$$

$$+ \frac{q_T}{Q} [C_2(Q) \otimes F_3(x, b; Q, Q^2) F_4(x, b; Q, Q^2)](x_1, x_2) \qquad \longleftarrow \text{NLP}$$

$$+ \frac{q_T^2}{Q^2} [C_3(Q) \otimes F_5(x, b; Q, Q^2) F_6(x, b; Q, Q^2)](x_1, x_2) \qquad \longleftarrow \text{NNLP}$$

$$+ \ldots$$

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▶ Sub-leading power observables

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{h}\cdot k_T}{M_h}\left(xeH_1^{\perp} + \frac{M_h}{M}f_1\frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{h}\cdot p_T}{M}\left(xg^{\perp}D_1 + \frac{M_h}{M}h_1^{\perp}\frac{\tilde{E}}{z}\right)\right]$$

twist-3 pdf — unpolarized PDF — twist-3 t-odd PDF — Boer-Mulders

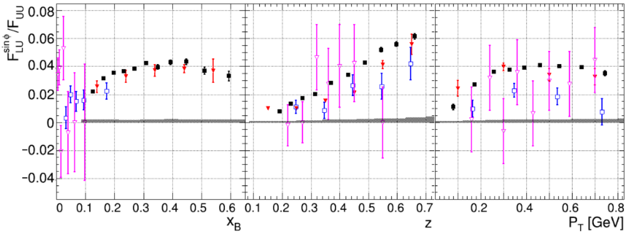
Collins FF — twist-3 FF — unpolarized FF — twist-3 FF

by Timothy B. Hayward at QCD-N

To describe it, one needs TMD factorization at NLP.

- ▶ **JLab**
- ▶ LHC

[CLAS, 2101.03544]



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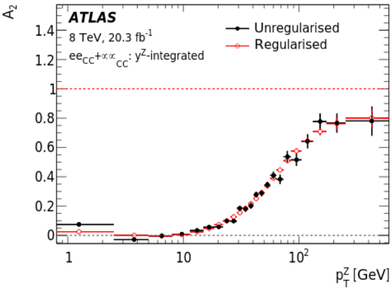
Motivation

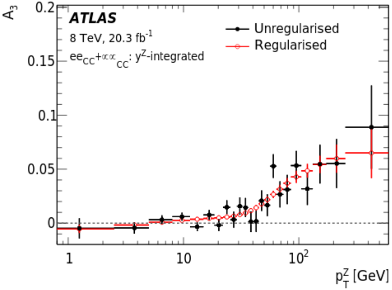
▶ **Sub-leading power observables**

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}\Big\{(1+\cos^2\theta)+\frac{1}{2}A_0(1-3\cos^2\theta)+A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

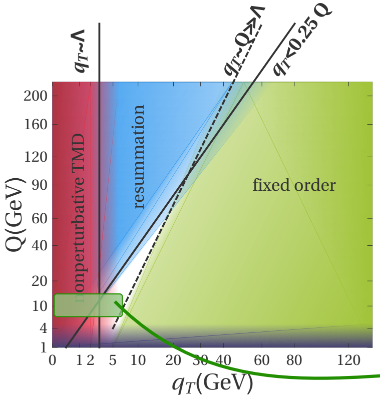
To describe it, one needs TMD factorization at NNLP.

- ▶ JLab
- ▶ **LHC**



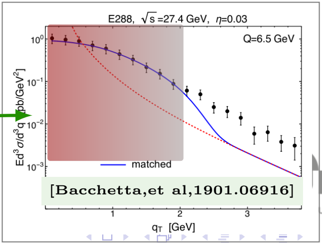


- Sub-leading power observables
- **Increase of applicability domain**



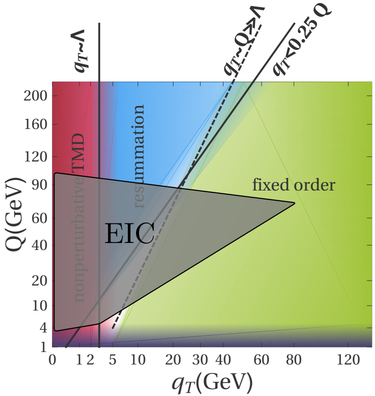
LP TMD factorization has limited region of application.

For SIDIS it cuts **the most part** of the data



Motivation

- Sub-leading power observables
- **Increase of applicability domain**



Phase space of EIC is centered directly in the transition region

COMPASS, JLab have large contribution of power corrections

Motivation

- Sub-leading power observables
- Increase of applicability domain
- **Restoration of broken properties**

LP TMD factorization breaks EM-gauge invariance

$$W^{\mu\nu} = \int dy e^{iqy} \langle J^\mu(y) J^\nu(0) \rangle$$

$$q_\mu W^{\mu\nu} = 0$$

$$W^{\mu\nu}_{\text{LP}} = g_T^{\mu\nu} |C_V|^2 \mathcal{F}(F_1 F_2)$$

$$q_\mu W^{\mu\nu}_{\text{LP}} \sim q_T^\nu$$

- The violation is of the NLP
- Similar problem with frame-dependence (GTMD case)
- The problem is not unique, e.g. collinear factorization for DVCS

Sources of power corrections

*(exact)=known at all powers

$$\frac{d\sigma}{dP.S.} = \sigma_{PS} L_{\mu\nu} W^{\mu\nu}$$

Phase space PC (exact)

e.g. SIDIS $\sigma_{PS} = \dfrac{\pi}{\sqrt{1+\gamma^2 \frac{\mathbf{p}_{h\perp}^2}{z^2 Q^2}}}$

Hadronic tensor (e.g. DY)

$$W^{\mu\nu} = \int \frac{d^4 y e^{i(yq)}}{(2\pi)^4} \langle p_1 p_2 | J^\mu(y) | X\rangle \langle X | J^\nu | p_1 p_2\rangle$$

Factorized in powers of

$$\frac{q_T}{q^+}, \quad \frac{q_T}{q^-}$$

Leptonic tensor (exact)

e.g. un.DY with fid.cuts

$$L^{\mu\nu} \sim (l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}(ll'))\mathcal{P}$$

- l, l' with transverse parts
- $\mathcal{P}$ fiducial part

Power corrections due to frame choice (exact)

$$p_1^+ \gg p_1^-, \qquad p_2^- \gg p_2^+$$

e.g. SIDIS $q_T^2 = \dfrac{p_\perp^2}{z^2} \dfrac{1+\gamma^2}{1-\gamma^2 \frac{p_\perp^2}{z^2 Q^2}}$

Sources of power corrections

*(exact)=known at all powers

$$\frac{d\sigma}{dP.S.} = \sigma_{PS} L_{\mu\nu} W^{\mu\nu}$$

Phase space PC (exact)

e.g. SIDIS $\sigma_{PS} = \frac{\pi}{\sqrt{1 + \ldots}}$

Hadronic tensor (e.g. DY)

$$W^{\mu\nu} = \int \frac{d^4 y e^{i(yq)}}{(2\pi)^4} \langle p_1 p_2 | J^\mu(y) | X \rangle \langle X | J^\nu | p_1 p_2 \rangle$$

QCD Factorization (this talk)

Factorized in powers of $\frac{q_T}{q^+}, \frac{q_T}{q^-}$

Leptonic tensor (exa...)

e.g. un.DY with fid.cuts

$$L^{\mu\nu} \sim (l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}(ll'))\mathcal{P}$$

- l, l' with transverse parts
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Power corrections due to frame choice (exact)

$$p_1^+ \gg p_1^-, \qquad p_2^- \gg p_2^+$$

e.g. SIDIS $q_T^2 = \frac{p_\perp^2}{z^2} \frac{1+\gamma^2}{1 - \gamma^2 \frac{p_\perp^2}{z^2 Q^2}}$

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There are already computations of TMD factorization at NLP/NNLP

- Small-x-like
  - Balitksy [1712.09389],[2012.01588],...
  - Nefedov, Saleev, [1810.04061],[1906.08681]
- SCET
  - Ebert, et al [2112.07680] *tree order*
  - Inglis-Whalen, et al [2105.09277]
  - Beneke, et al, [1712.04416],[1808.04742],... *not TMD, but closely related*
- Boer, Mulders, Pijlman [hep-ph/0303034]
- ...

TMD operator expansion

- Based on the experience of higher-twist, and higher power computations in collinear factorization
  - Systematicness of OPE
  - Operator level
  - Position space [a lot of simplification for beyond leading twist]
- Has common parts with small-x and SCET computations
- **Generalization of ordinary background method**

Background field method for parton physics
(in a nutshell)

$$\langle h|T\, J^{\mu}(z)J^{\nu}(0)|h\rangle = \int [D\bar{q}DqDA]e^{iS_{\rm QCD}}\Psi^*[\bar{q},q,A]J^{\mu}(z)J^{\nu}(0)\Psi[\bar{q},q,A]$$

Cannot be integrated since Ψ is unknown

Background field method for parton physics
(in a nutshell)

$$\langle h|T\,J^\mu(z)J^\nu(0)|h\rangle = \int [D\bar{q}DqDA]e^{iS_{\text{QCD}}}\Psi^*[\bar{q},q,A]J^\mu(z)J^\nu(0)\Psi[\bar{q},q,A]$$

Parton model

Ψ contains only collinear particles

$$\Psi[\bar{q},q,A] \to \Psi[\bar{q}_{\bar{n}},q_{\bar{n}},A_{\bar{n}}]$$

$$\{\partial_+,\partial_-,\partial_T\}q_{\bar{n}} \lesssim \{1,\lambda^2,\lambda\}q_{\bar{n}}$$

Integral can be partially computed

Background field method for parton physics (in a nutshell)

$$\langle h|T\, J^\mu(z) J^\nu(0)|h\rangle = \int [D\bar q D q D A] e^{iS_{\rm QCD}} \Psi^*[\bar q, q, A] J^\mu(z) J^\nu(0) \Psi[\bar q, q, A]$$

Parton model

Ψ contains only collinear particles

$\Psi[\bar q, q, A] \to \Psi[\bar q_{\bar n}, q_{\bar n}, A_{\bar n}]$

$\{\partial_+, \partial_-, \partial_T\} q_{\bar n} \lesssim \{1, \lambda^2, \lambda\} q_{\bar n}$

Background technique

$$q = q_{\bar n} + \psi$$
$$A = A_{\bar n} + B$$

- $q_{\bar n}, A_{\bar n}$: background (external field)
- ψ, B: dynamical (to be integrated)

Integral can be partially computed

$$\langle h|T\, J^\mu(z) J^\nu(0)|h\rangle = \int [D\bar q_{\bar n} D q_{\bar n} D A_{\bar n}] e^{iS_{\rm QCD}} \Psi^*[\bar q, q, A] \mathcal{J}^{\mu\nu}_{\rm eff}[\bar q_{\bar n}, q_{\bar n}, A_{\bar n}](z) \Psi[\bar q, q, A]$$

$$\mathcal{J}^{\mu\nu}_{\rm eff} = \int [D\bar\psi D\psi D B] e^{iS_{\rm QCD} + iS_{\rm back}[\bar q, q, A]} J^\mu[q+\psi](z) J^\nu[q+\psi](0)$$

Generating function for operator product expansion

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Background QCD with 2-component background

$$q \to q_n + q_{\bar{n}} + \psi \qquad A^\mu \to A^\mu_n + A^\mu_{\bar{n}} + B^\mu$$

- **Technical note:** S_{QCD} for 2-component background has 1PI vertices!

collinear-fields
(associated with hadron 1)

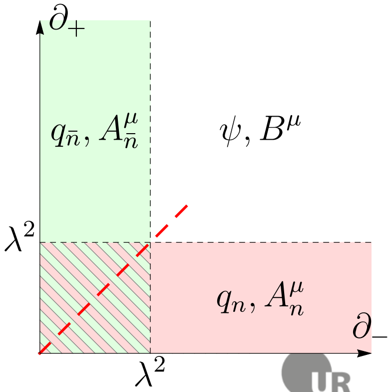
$$\{\partial_+, \partial_-, \partial_T\}\, q_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\, q_{\bar{n}},$$

$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\, A^\mu_{\bar{n}},$$

anti-collinear-fields
(associated with hadron 2)

$$\{\partial_+, \partial_-, \partial_T\}\, q_n \lesssim Q\{\lambda^2, 1, \lambda\}\, q_n,$$

$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_n \lesssim Q\{\lambda^2, 1, \lambda\}\, A^\mu_n.$$



TMD operator expansion
is conceptually similar to ordinary OPE
The only difference is counting rule for y

$$W^{\mu\nu}_{\text{DY}} = \int \frac{d^4y}{(2\pi)^4} e^{-i(yq)} \sum_X \langle p_1, p_2 | J^{\mu\dagger}(y) | X \rangle \langle X | J^{\nu}(0) | p_1, p_2 \rangle,$$

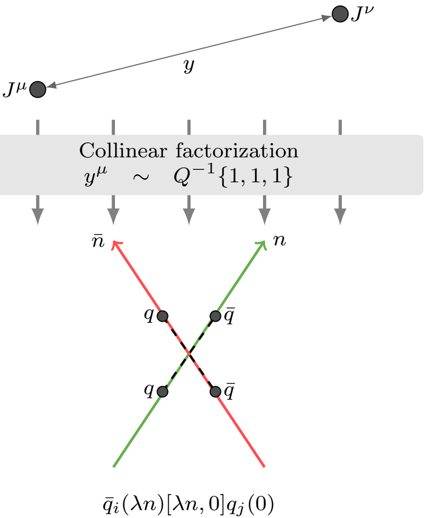
$$W^{\mu\nu}_{\text{SIDIS}} = \int \frac{d^4y}{(2\pi)^4} e^{i(yq)} \sum_X \langle p_1 | J^{\mu\dagger}(y) | p_2, X \rangle \langle p_2, X | J^{\nu}(0) | p_1 \rangle,$$

$$W^{\mu\nu}_{\text{SIA}} = \int \frac{d^4y}{(2\pi)^4} e^{i(yq)} \sum_X \langle 0 | J^{\mu\dagger}(y) | p_1, p_2, X \rangle \langle p_1, p_2, X | J^{\nu}(0) | 0 \rangle.$$

$$(q \cdot y) \sim 1 \qquad \Rightarrow \qquad \{y^+, y^-, y_T\} \sim \{\frac{1}{q^-}, \frac{1}{q^+}, \frac{1}{q_T}\} \sim \frac{1}{Q}\{1, 1, \lambda^{-1}\}$$

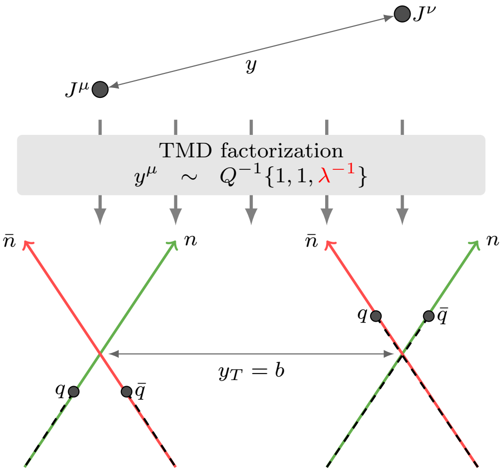
To be accounted in operator expansion

$$z_T^\mu \partial_\mu q \sim \text{NLP}, \qquad y_T^\mu \partial_\mu q \sim \text{LP}$$



TMD operator expansion
has different geometry

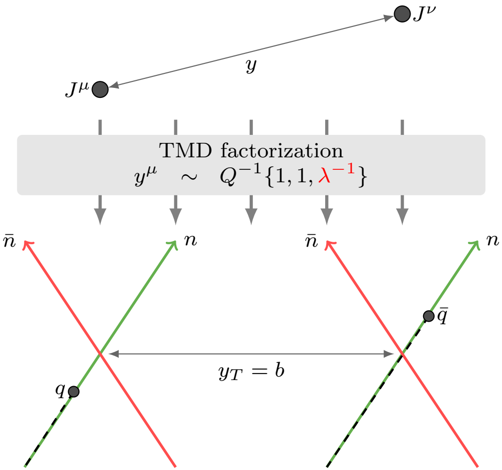
Two
light-cone operators
$\Downarrow$
Two
parton distribution function
PDFs & FFs



TMD operator expansion
has different geometry

Four
semi-compact
light-cone operators
⇓
Two
TMD distributions
TMDPDFs & TMDFFs

TMD operator expansion has different geometry



$$\bar q_i(\lambda n + b)[\lambda n + b, \pm\infty n + b] \quad [\pm\infty n, 0] q_j(0)$$

Four
semi-compact
light-cone operators
$\Downarrow$
Two
TMD distributions
TMDPDFs & TMDFFs

TMD-twist

Each light-cone operator must be twist-decomposed

- Geometrical twist = dimension - spin (projected to light-cone)
- Half-integer spin operators
  - $(\bar q\gamma^+\gamma^-)_i$ = twist-1 $(\frac{3}{2}-\frac{1}{2})$
  - $(\bar q\gamma^-\gamma^+)_i$ = twist-indefinite $\Rightarrow$ EOM $\Rightarrow$ $\underbrace{\left(\bar q\gamma^+\frac{\overleftarrow{\not\partial}_T}{\overleftarrow{\partial}_+}\right)_i}_{\text{tot.der. twist-1}} + \underbrace{\int(\bar q\gamma^\mu F_{\mu+}\gamma^+)_i}_{\text{twist-2}}$

Twist of the TMD operator is enumerated by twists of each light-cone components (N,M) =TMD-twist

e.g. usual TMD operator

$$\underbrace{\overbrace{\bar q(\lambda n+b)[\lambda n+b,\pm\infty n+b]}^{\text{twist-1}}\;\gamma^+\;\overbrace{[\pm\infty n,0]q(0)}^{\text{twist-1}}}_{\text{TMD-twist=(1,1)}}$$

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TMD operators of different TMD-twists

(1,1)

$O_{11}(z,b) = \bar{\xi}(zn+b)[...]\Gamma[...]\xi(0)$

$\Gamma = \{\gamma^+, \gamma^+\gamma^5, \sigma^{\alpha+}\}$
$\Rightarrow$ well known 8 TMD distributions

(1,2) & (2,1)

$$O_{21}(z_{1,2},b) = \bar{\xi}(z_1 n+b)[..]F_{\mu+}(z_2+b)[...]\Gamma[...]\xi(0)$$
$$O_{12}(z_{1,2},b) = \bar{\xi}(z_1 n+b)[...]\Gamma[...]F_{\mu+}(z_2)[..]\xi(0)$$

- $\Gamma = \{\gamma^+, \gamma^+\gamma^5, \sigma^{\alpha+}\}$
- 16 (?) TMD distributions
- Related by charge-conjugation $\Leftrightarrow$ complex/real

(1,3) & (3,1) & (2,2)

$$O_{31;1}(z_{1,2,3},b) = \bar{\xi}..F_{\mu+}..F_{\nu+}[...]\Gamma[...]\xi(0)$$
$$O_{22}(z_{1,2,3},b) = \bar{\xi}..F_{\mu+}[...]\Gamma[...]F_{\nu+}..\xi(0)$$
$$O_{31;2}(z_{1,2,3},b) = \bar{\xi}..(\bar{\xi}..\Gamma_2..\xi)[...]\Gamma[...]\xi(0)$$
$$O_{31;3}(z_{1,2},b) = \bar{\xi}..F_{-+}[...]\Gamma[...]\xi(0)$$
$$...$$

- Quasi-partonic and non-quasi-partonic

Operators with different TMD-twists do not mix
renormalization/evolution is independent
independent TMD distributions

Evolution of TMD distribution with TMD-twist=(N,M)

$$\Phi_{NM}(x_1,...,x_n,b) = \int dz_1...dz_n e^{-ip_+(x_1z_1+...+x_nz_n)}\langle p|\overline{U}_N(\{z_1,...\},b)U_M(\{...,z_k\},0_T)|p\rangle$$

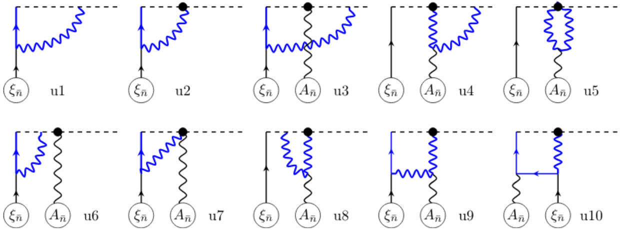
- Each light-cone operator U renormalizes independently (because there is a finite y_T in-between)

$$\mu\frac{d}{d\mu}U_N(\{z_1,...\},b) = \gamma_N \otimes U_N(\{z_1,...\},b)$$

- Light-cone operators with different N do not mix (Lorentz invariance!)
- Evolution of TMD distribution

$$\mu\frac{d}{d\mu}\Phi_{NM}(x_1,...,x_n,b) = (\overline{\gamma}_N + \gamma_M) \otimes \Phi_{NM}(x_1,...,x_n,b)$$

Evolution of a twist-2 semi-compact operator at LO



UV anomalous dimension of a **semi-compact** operator has two parts

Compart part	**Non-Compart part**
$\sigma \quad < \quad z_{\max}$	$z_{\max} \quad < \quad \sigma$
Reproduces elementry evolution kernel	Collinearly divergent (UV/collinear overlap)
Bukhvostov-Frolov-Lipatov-Kuraev for QP operator	Needs a regulator to compute (canceled by UV part of rap.divergence)
Braun-Manashov-Rohrwild for non-QP operators	

$$\gamma_2(\mu,\zeta) = a_s\Bigg\{2\mathbb{H}_1 + \Bigg[C_F\left(\frac{3}{2}+\ln\left(\frac{\mu^2}{\zeta}\right)\right) +2\left(C_F-\frac{C_A}{2}\right)\ln\left(\frac{q^+}{\hat{p}^+_\xi}\right)+C_A\ln\left(\frac{q^+}{\hat{p}^+_A}\right)\Bigg]\Bigg\}+O(a_s^2).$$

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Rapidity divergences
appears due to overlap of the fields in the soft region

collinear-fields & anti-collinear
are the same at

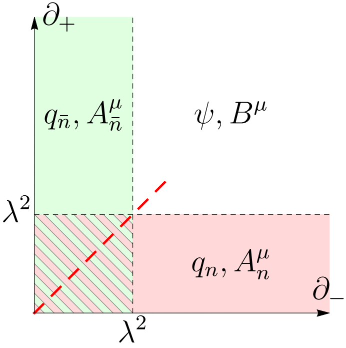
$$\{\partial_+, \partial_-, \partial_T\}\, q \lesssim Q\{\lambda^2, \lambda^2, \lambda\}\, q,$$

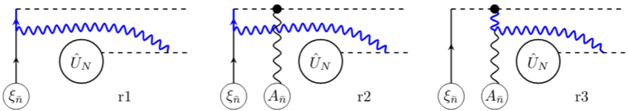
$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu \lesssim Q\{\lambda^2, \lambda^2, \lambda\}\, A^\mu,$$

- (or) Introduce separating-scale
- (or) Subtract by soft-factor
- (or) ...

⇒ multiplicative renormalization
[AV,1707.07606] ⇒ evolution equation with ζ

$$\zeta \frac{d}{d\zeta}\Phi_{NM}(\{z_1, ...\}, b) = -\mathcal{D}(b)\Phi_{NM}(\{z_1, ...\}, b)$$





$$\widetilde{R}\left(b^2, \frac{\delta^+}{\nu^+}\right) = 1 - 4a_s C_F \Gamma(-\epsilon)\left(-\frac{b^2\mu^2}{4e^{-\gamma_E}}\right)^{\epsilon} \ln\left(\frac{\delta^+}{\nu^+}\right) + O(a_s^2).$$

Rapidity divergence arise from the interaction with the far end of neighbour Wilson line

General facts

- Independent on the “type” of another operator
- Multiplicatively renormalizable (*)
- Same for all operators (up to color-representation) at LP, NLP, NNLP(!)

  * End-point divergences and derivatives of R cancel!

Evolution for quasi-partonic TMD operators (distributions)

$$\mu^2 \frac{d\Phi_{NM}}{d\mu^2}(\mu,\zeta) = (\overline{\gamma}_N(\mu,\zeta) + \gamma_M(\mu,\zeta)) \otimes \Phi_{NM}(\mu,\zeta)$$
$$\zeta \frac{d\Phi_{NM}}{d\zeta}(\mu,\zeta) = -\mathcal{D}(b,\mu) \otimes \Phi_{NM}(\mu,\zeta)$$

- γ_1 known up to NNLO
- γ_2 known up to LO
- γ_3 could be reconstructed at LO (if ever needed)
- $\mathcal{D} = -K/2$ is CS-kernel (non-perturbative)
  - Same for all QP operators!

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Computing TMD factorization

Keldysh thechnique to deal with causality structure → $J^{(+)\mu}(y)J^{(-)\nu}(0)$

Details & examples in [2109.09711]

Computing TMD factorization

Details & examples in [2109.09711]

$$J^{(+)\mu}(y)J^{(-)\nu}(0)$$

(power) Expand in background fields
sort operators by TMD-twist

$$\bar{q}_{\bar{n}}(y^- n + y_T)\gamma_T^\mu q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \bar{\psi}_{\bar{n}}(y)\gamma_T^\mu q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \dots$$
$$+n^\mu \bar{q}_{\bar{n}}(y^- n + y_T)\gamma^- q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \dots$$
$$+y^+ \bar{q}_{\bar{n}}(y^- n + y_T)\overleftarrow{\partial}_- \gamma^- q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \dots$$

Computing TMD factorization

Details & examples in [2109.09711]

$$J^{(+)\mu}(y)J^{(-)\nu}(0)$$

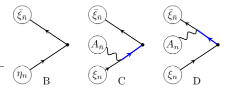
(power) Expand in background fields
sort operators by TMD-twist

$$\bar{q}_{\bar{n}}(y^- n + y_T)\gamma_T^\mu q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \bar{\psi}_{\bar{n}}(y)\gamma_T^\mu q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \ldots$$
$$+n^\mu \bar{q}_{\bar{n}}(y^- n + y_T)\gamma^- q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \ldots$$
$$+y^+ \bar{q}_{\bar{n}}(y^- n + y_T)\overleftarrow{\partial}_-\gamma^- q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \ldots$$

(loop) Integrate over fast components
with 2-bcg.QCD action

at least NLO is needed
to confirm factorization
(WL direction,
pole-cancelation)

$$\mathcal{J}^{\mu\nu}_{\rm NLP} = -\frac{n^\mu\gamma^\rho_{T,ij}\gamma^\nu_{T,kl} + n^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\Big(\frac{\partial_\rho}{\partial_+}\mathcal{O}^{li}_{11,\bar n}\overline{\mathcal{O}}^{jk}_{11,n} + \frac{\partial_\rho}{\partial_+}\overline{\mathcal{O}}^{jk}_{11,\bar n}\mathcal{O}^{li}_{11,n}\Big)$$
$$-\frac{\bar n^\mu\gamma^\rho_{T,ij}\gamma^\nu_{T,kl} + \bar n^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar n}\frac{\partial_\rho}{\partial_-}\overline{\mathcal{O}}^{jk}_{11,n} + \overline{\mathcal{O}}^{jk}_{11,\bar n}\frac{\partial_\rho}{\partial_-}\mathcal{O}^{li}_{11,n}\Big)$$
$$+ig\frac{\delta_{ij}\gamma^\nu_{T,kl}}{N_c}\Big\{\mathbb{O}^{li}_{21,\bar n}\left(\frac{\bar n^\mu}{\overrightarrow{\partial_-}} - \frac{n^\mu}{\overleftarrow{\partial_+}}\right)\overline{\mathcal{O}}^{jk}_{11,n} - \overline{\mathbb{O}}^{jk}_{21,\bar n}\left(\frac{\bar n^\mu}{\overrightarrow{\partial_-}} - \frac{n^\mu}{\overleftarrow{\partial_+}}\right)\mathcal{O}^{li}_{11,n}$$



Coincides with [Boer,Mulders,Pijlman,03]

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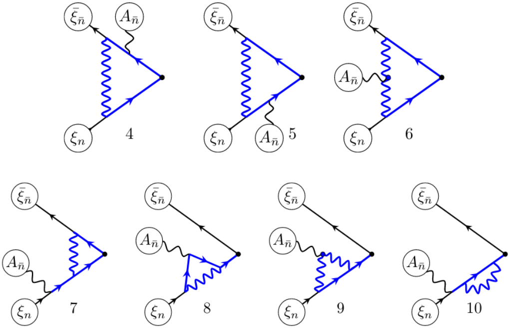
NLO computation



Main check of factorization: pole cancellation

$$\overbrace{F_1[\otimes Z_1 R]}^{\text{finite}} \otimes \underbrace{[Z_1^{-1}R^{-1}] \otimes H \otimes [Z_2^{-1}R^{-1}]}_{\text{finite}} \otimes \overbrace{[Z_2 R] \otimes F_2}^{\text{finite}}$$

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NLO computation

Extra facts

- At LP and NLP one Sudakov form factor is needed (exchange diagrams are NNLP)
- Computation for Sudakov is done for LP and NLP both at NLO
  - Position space
  - LP is well known (up to N^3LO) and coincides
  - Twist-(1,1) part of NLP is the same as LP
    - Required by EM gauge invariance **Non-trivial check**
  - Twist-(1,2) part is totally new
- The UV and rapidity divergences of NLP operators computed independently
  - (position space) BFLK part coincide with [Braun,Manashov,09]
  - (momentum space) "Coincides" with [Beneke, et al, 17] (up to missed channels)
- Checks
  - Pole parts of hard coefficient and operators cancel **very non-trivial check**
  - Some diagrams are computed in momentum space **check**

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Final expression: **TMD factorization at NLP**

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) \tag{6.17}$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right)$$

$$\times \left(\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x,\tilde{x},b) \right)$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)$$

$$\times \left(C_1^* C_2(\tilde{x}_{2,3}) \delta\left(\tilde{x}_1 - \frac{q^-}{p_2^-}\right) \mathcal{J}^{\mu\nu}_{1112}(x,\tilde{x},b) + C_2^*(\tilde{x}_{1,2}) C_1 \delta\left(\tilde{x}_3 + \frac{q^-}{p_2^-}\right) \mathcal{J}^{\mu\nu}_{1121}(x,\tilde{x},b) \right)$$

$$+ \dots \Bigg\}$$

Final expression: **TMD factorization at NLP**

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \left\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) \right. \quad (6.17)$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \ldots (x,\tilde{x},b)\Big) \ldots (x,\tilde{x},b)\Big)$$

$$\mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) = \frac{\gamma^{\mu}_{T,ij}\gamma^{\nu}_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big)$$
$$+ i\frac{n^{\mu}\gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + n^{\nu}\gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^+ N_c}\Big(\partial_\rho\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \partial_\rho\overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big)$$
$$+ i\frac{\bar{n}^{\mu}\gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + \bar{n}^{\nu}\gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^- N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\partial_\rho\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\partial_\rho\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big),$$

- Operators of $(1,1)\times(1,1)$ (ordinary TMDs)

$$\mathcal{O}^{ij}_{11}(x,b) = p_+ \int \frac{d\lambda}{2\pi} e^{-ix\lambda p_+} \bar{q}_j[\lambda n + b, \pm\infty n + b][\pm\infty n, 0] q_i$$

- Contains LP and NLP (total derivatives)
- Restores EM gauge invariance up to λ^3

$$q_\mu J^{\mu\nu}_{1111} \quad \sim \quad (p_1^- q_T \; + \; p_2^+ q_T) J_{1111}$$

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Final expression: **TMD factorization at NLP**

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x, \tilde{x}, b) \quad (6.17)$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right)$$

$$\times \left(\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}^{\mu\nu}_{1211}(x, \tilde{x}, b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x, \tilde{x}, b) \right)$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)$$

$$\ldots (x, \tilde{x}, b) \Big)$$

$$\mathcal{J}^{\mu\nu}_{1211}(x, \tilde{x}, b) =$$

$$\frac{ig}{x_2} \left(\frac{\bar{n}^\nu}{q^-} - \frac{n^\nu}{q^+} \right) \frac{\gamma^\mu_{T,ij} \delta_{kl}}{N_c} \left(\mathcal{O}^{jk}_{12,\bar{n}}(x, b) \overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x}, b) - \overline{\mathcal{O}}^{jk}_{12,\bar{n}}(x, b) \mathcal{O}^{li}_{11,n}(\tilde{x}, b) \right)$$

- Operators of $(1,2) \times (1,1)$

$$\mathcal{O}^{ij}_{12}(x_{1,2,3}, b) = p_+^2 \int \frac{dz_{1,2,3}}{2\pi} e^{-ix^i z_i p_+} \bar{q}_j [z_1 n + b, \pm\infty n + b][\pm\infty n, z_2 n] \gamma^\mu F_{\mu+}[z_2 n, z_3 n] q_i$$

- EM gauge invarint only up to NNLP

$$q_\mu J^{\mu\nu}_{1211} \quad \sim \quad (p_1^- \; + \; p_2^+) J_{1211}$$

Final expression: **TMD factorization at NLP**

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x, \tilde{x}, b) \qquad (6.17)$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right)$$

$$\times \left(\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}^{\mu\nu}_{1211}(x, \tilde{x}, b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x, \tilde{x}, b)\right)$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)$$

$$\ldots (x, \tilde{x}, b)\Big)$$

$$C_1 = 1 + a_s C_F \left(-\mathbf{L}_Q^2 + 3\mathbf{L}_Q - 8 + \frac{\pi^2}{6}\right) + O(a_s),$$

$$C_2(x_{1,2}) = 1 + a_s \Big[C_F \left(-\mathbf{L}_Q^2 + \mathbf{L}_Q - 3 + \frac{\pi^2}{6}\right) + C_A \frac{x_1 + x_2}{x_1} \ln\left(\frac{x_1 + x_2}{x_2}\right)$$

$$+ \left(C_F - \frac{C_A}{2}\right) \frac{x_1 + x_2}{x_2} \ln\left(\frac{x_1 + x_2}{x_1}\right) \left(2\mathbf{L}_Q - \ln\left(\frac{x_1 + x_2}{x_1}\right) - 4\right) \Big]$$

- Coefficient functions up to NLO
- C_1 is know up to N^{3}LO
- C_1 is same for LP, NLP, ... parts of operator $J^{\mu\nu}_{1111}$

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Conclusion

What I have not told:

- Process dependence and Wilson lines
- Cancellation between end-point divergences and derivatives of soft-factor
- Systematization of NLP TMD distributions, and expression for cross-section in these terms
- Matching to collinear factorization
- Application for different objects (lattice)

Roadmap for power corrections in TMD

- NLP/NLO (done) [2109.09771]
- NNLP (done)/NLO (in progress)
- Summation of descendants of LP $\Rightarrow$ restoration of EM gauge invariance (in progress)
- Phenomenology ...

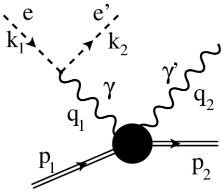
TMD operator expansion – an efficient approach to TMD factorization beyond LP

- Operator level / Position space / All processes
- Strict & intuitive rules for operator sorting (TMD-twist)

Thank you for attention!

The most efficient way to study power corrections: **OPE + background formalism**

- Many results (so far) unreachable by other methods
  - Twist-3, twist-4 evolution kernels [Braun,Manashov,08-09]
  - Coefficient function for various observables (e.g. quasi-PDFs at twist-3 [Braun,Ji,AV,20-21])
  - All-Power corrections (DVCS [Braun,Manashov,17-21], target-mass corrections to TMDs [Moos,AV,20])
- Clear and strict formulation ⇒ Simple computation
- Twist-decomposition

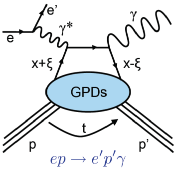


DVCS

$$J^\mu(z)J^\nu(0) \xrightarrow{\text{OPE}} \sum_{n=0}^{\infty} z^n [C_n^{\mu\nu} \otimes O_n](z^+)$$

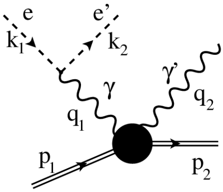
Leading power ⇒ GPDs

violates EM Ward identities and translation invariance



The most efficient way to study power corrections: **OPE + background formalism**

- Many results (so far) unreachable by other methods
  - Twist-3, twist-4 evolution kernels [Braun,Manashov,08-09]
  - Coefficient function for various observables (e.g. quasi-PDFs at twist-3 [Braun,Ji,AV,20-21])
  - All-Power corrections (DVCS [Braun,Manashov,17-21], target-mass corrections to TMDs [Moos,AV,20])
- Clear and strict formulation ⇒ Simple computation
- Twist-decomposition



DVCS

$$J^\mu(z)J^\nu(0) \xrightarrow{\text{OPE}} \sum_{n=0}^{\infty} z^n [C_n^{\mu\nu} \otimes O_n](z^+)$$

power	operators		
0	$\bar q[..]q$		
1	$\bar q[..]q$	$\bar q F_{\mu+}[..]q$	
2	$\bar q[..]q$	$\bar q F_{\mu+}[..]q$	$\bar q F_{\mu+}F_{\nu+}[..]q$ $\bar q[..]q\bar q[..]q$ $\bar q F_{\mu+}[..]\gamma^- q$
...	tw2	tw3	tw4

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The most efficient way to study power corrections: **OPE + background formalism**

- Many results (so far) unreachable by other methods
  - Twist-3, twist-4 evolution kernels [Braun,Manashov,08-09]
  - Coefficient function for various observables (e.g. quasi-PDFs at twist-3 [Braun,Ji,AV,20-21])
  - All-Power corrections (DVCS [Braun,Manashov,17-21], target-mass corrections to TMDs [Moos,AV,20])
- Clear and strict formulation ⇒ Simple computation
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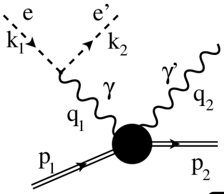

DVCS

TMD factorization has same structure

$$\xrightarrow{\text{OPE}} \sum^{\infty} z^n [C_n^{\mu\nu} \otimes O_n](z^+)$$

All properties restored

All properties restored

power	operators		
0	$\bar q[..]q$		...
1	$\bar q[..]q$	$\bar q F_{\mu+}[..]q$	
2	$\bar q[..]q$	$\bar q F_{\mu+}[..]q$	$\bar q F_{\mu+}F_{\nu+}[..]q$ $\bar q[..]q\bar q[..]q$ $\bar q F_{\mu+}[..]\gamma^- q$
...	tw2	tw3	tw4

Independent
Do not mix

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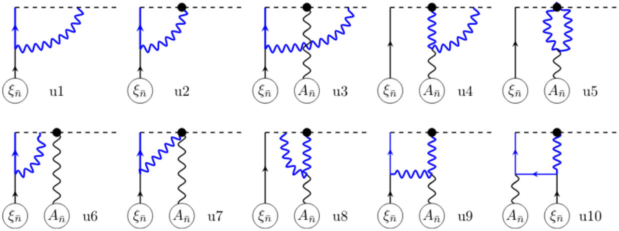


(LO) UV anomalous dimension of a **semi-compact** operator has two parts

Compart part

$\sigma < z_{\max}$

Reproduce elementry evolution kernel

Bukhvostov-Frolov-Lipatov-Kuraev for QP operator

Braun-Manshov-Rohrwild for non-QP operators

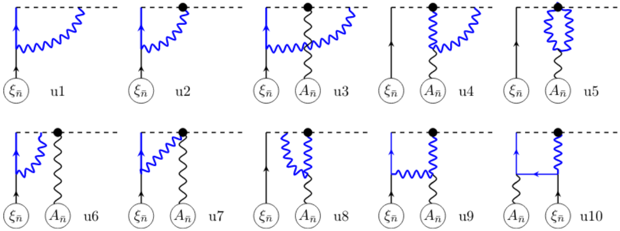
Non-Compart part

$z_{\max} < \sigma$

Collinearly divergent

(UV/collinear overlap)

Needs a regulator to compute (canceled by UV part of rap.divergence)



$$U^{\mu}_{2,\bar{n}}(\{z_1,z_2\},b) = g[Ln+b, z_1 n + b] F^{\mu+}_{\bar{n}}[z_1 n + b, z_2 n + b]\xi_{\bar{n}}(z_2 n + b),$$

$$\text{diag}_{u2} + ... + \text{diag}_{u10} = \frac{a_s}{\epsilon}\Bigg\{ \gamma_T^{\mu}\gamma_T^{\nu}\mathbb{H}_1 U^{\nu}_{2,\bar{n}} + \gamma_T^{\nu}\gamma_T^{\mu}\mathbb{H}_2 U^{\nu}_{2,\bar{n}}$$
$$+\left[C_F\left(2+2\ln\left(\frac{\delta^+}{q^+}\right)\right) + 2\left(C_F - \frac{C_A}{2}\right)\ln\left(\frac{q^+}{is\hat{p}^+_{\xi}}\right) + C_A \ln\left(\frac{q^+}{is\hat{p}^+_A}\right)\right]U^{\mu}_{2,\bar{n}}\Bigg\},$$

- Confirmed by direct computation
- Same structure for QP operators of higher twists
- Non-QP operator different... (in progress)

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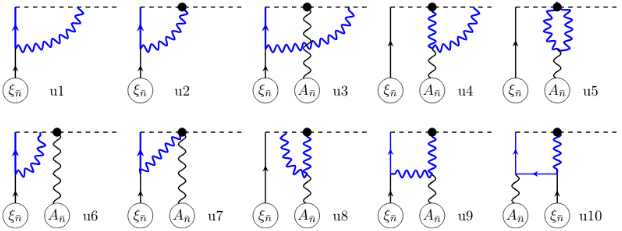

$$U^{\mu}_{2,\bar n}(\{z_1,z_2\},b) = g[Ln+b,z_1n+b]F^{\mu+}_{\bar n}[z_1n+b,z_2n+b]\xi_{\bar n}(z_2n+b),$$

$$\text{diag}_{u2}+...+\text{diag}_{u10} = \frac{a_s}{\epsilon}\Bigg\{\gamma_T^{\mu}\gamma_T^{\nu}\mathbb{H}_1U^{\nu}_{2,\bar n}+\gamma_T^{\nu}\gamma_T^{\mu}\mathbb{H}_2U^{\nu}_{2,\bar n}$$
$$+\Big[C_F\Big(2+2\ln\Big(\frac{\delta^+}{q^+}\Big)\Big)+2\Big(C_F-\frac{C_A}{2}\Big)\ln\Big(\frac{q^+}{is\hat p_\xi^+}\Big)+C_A\ln\Big(\frac{q^+}{is\hat p_A^+}\Big)\Big]U^{\mu}_{2,\bar n}\Bigg\},$$

$$2\mathbb{H}_1U(x_1,x_2) = C_A\delta_{x_1}U(0,x_2)+\int dv\Bigg\{C_A(\theta(v,x_1)-\theta(-v,-x_1))\frac{x_1}{x_1+v}\Big[\tag{C.6}$$
$$\frac{U(x_1,x_2)-U(x_1+v,x_2-v)}{v}+\Big(\frac{x_2}{x_1+v}-\frac{x_1}{x_1+x_2}\Big)\frac{U(x_1+v,x_2-v)}{x_1+x_2}\Big]$$
$$+C_A(\theta(v,x_2)-\theta(-v,-x_2))\frac{x_2}{x_2+v}\Big[$$
$$\frac{U(x_1,x_2)-U(x_1-v,x_2+v)}{v}-\frac{2x_1+x_2}{(x_1+x_2)^2}U(x_1-v,x_2+v)\Big]$$
$$+2\Big(C_F-\frac{C_A}{2}\Big)\frac{1}{x_1+v}\Big[\frac{-x_1^2}{(x_1+x_2)^2}(\theta(v,x_1)-\theta(-v,-x_1))$$
$$+\Big(\Big(\frac{x_1}{x_1+x_2}\Big)^2-\Big(\frac{v}{x_2-v}\Big)^2\Big)(\theta(v,-x_2)-\theta(-v,x_2))\Big]U(x_2-v,x_1+v)\Bigg\},$$

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Process dependence

The background can be taken in any gauge (since it is gauge invariant)

- Light-cone gauge kills operators with $A_{+,\bar{n}}$ and $A_{-,n}$ (~ 1 in power counting).
- Convenient choice of gauges
  - Collinear field $A_+ = 0$
  - Anti-Collinear field $A_- = 0$
  - Dynamical field: **Feynman gauge**
- **However** one needs to specify boundary condition. The result depends on it.

$$A^{\mu}_{\bar{n}}(z) = -g\int_{-\infty}^{0} d\sigma F^{\mu+}_{\bar{n}}(z+n\sigma) \quad \textbf{vs.} \quad A^{\mu}_{\bar{n}}(z) = -g\int_{+\infty}^{0} d\sigma F^{\mu+}_{\bar{n}}(z+n\sigma)$$

$$\bar{q}[z, z-\infty n] \quad \textbf{vs.} \quad \bar{q}[z, z+\infty n]$$

etc.

To specify boundary and WL direction, we should go to NLO

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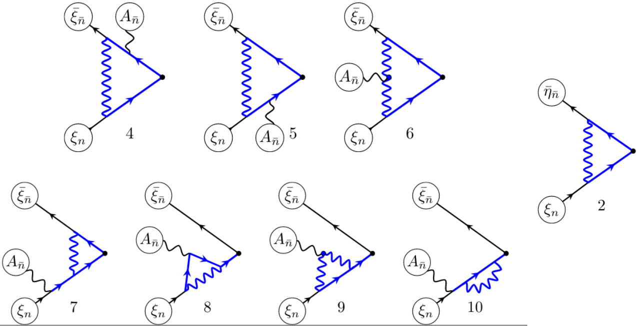
NLO expression in position space



$$\text{diag}_4 + ... + \text{diag}_{10} + \text{diag}_2^{\bar\xi A\xi-\text{part}} = g a_s \frac{\Gamma(-\epsilon)\Gamma(1-\epsilon)\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)}$$

$$\int \frac{dz^+dz^-}{4^\epsilon \pi}\frac{1}{[-2z^+z^-+i0]^{1-\epsilon}}\int_0^1 ds\Big\{(z^+\bar n^\mu - z^- n^\mu)C_F\frac{2-\epsilon}{\epsilon}\mathcal{K}(1,1)$$

$$-\left(C_F\frac{\epsilon(1+\epsilon)}{(1-\epsilon)^2}+C_A\frac{1-\epsilon-\epsilon^2}{(1-\epsilon)^2}\right)\Big[(\epsilon z^- n^\mu + (1-\epsilon)z^+\bar n^\mu)\mathcal{K}(s,1) - z^+\bar n^\mu\mathcal{K}(0,1)\Big]$$

$$+\left(C_F-\frac{C_A}{2}\right)\frac{2(1-\epsilon-\epsilon^2)}{\epsilon(1-\epsilon)^2}\Big[(\epsilon z^- n^\mu + (1-\epsilon)z^+\bar n^\mu)\mathcal{K}(1,s) - z^+\bar n^\mu\mathcal{K}(1,0)\Big]\Big\},$$

$$\mathcal{K}(s,t) = \bar\xi_{\bar n}(sz^-n)\not{A}_{\bar n,T}(tz^-n)\xi_n(z^+\bar n)$$

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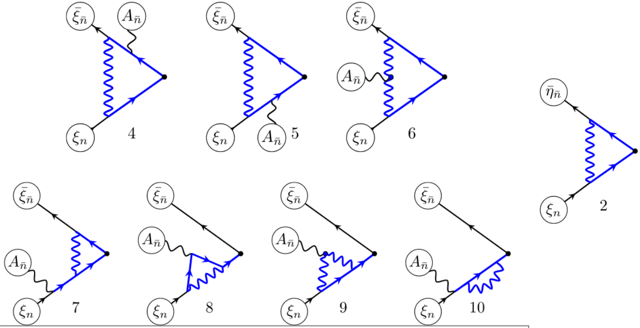
NLO expression in position space



Depends on boundary conditions

$$\text{diag}_4+\dots+\text{diag}_{10}+\text{diag}_2^{\bar\xi A\xi-\text{part}}=g a_s\frac{\Gamma(-\epsilon)\Gamma(1-\epsilon)\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)}$$
$$\int\frac{dz^+dz^-}{4^\epsilon\pi}\frac{1}{[-2z^+z^-+i0]^{1-\epsilon}}\int_0^1 ds\Big\{(z^+\bar n^\mu-z^-n^\mu)C_F\frac{2-\epsilon}{\epsilon}\mathcal{K}(1,1)$$
$$-\Big(C_F\frac{\epsilon(1+\epsilon)}{(1-\epsilon)^2}+C_A\frac{1-\epsilon-\epsilon^2}{(1-\epsilon)^2}\Big)\Big[(\epsilon z^-n^\mu+(1-\epsilon)z^+\bar n^\mu)\mathcal{K}(s,1)-z^+\bar n^\mu\mathcal{K}(0,1)\Big]$$
$$+\Big(C_F-\frac{C_A}{2}\Big)\frac{2(1-\epsilon-\epsilon^2)}{\epsilon(1-\epsilon)^2}\Big[(\epsilon z^-n^\mu+(1-\epsilon)z^+\bar n^\mu)\mathcal{K}(1,s)-z^+\bar n^\mu\mathcal{K}(1,0)\Big]\Big\},$$

$$\mathcal{K}(s,t)=\bar\xi_{\bar n}(sz^-n)\not{A}_{\bar n,T}(tz^-n)\xi_n(z^+\bar n)$$

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NLO expression in position space

$$I = \int_{-\infty}^{\infty} dz^+ dz^- \frac{f_{\bar{n}}(z^-) f_n(z^+)}{[-2z^+z^- + i0]^\alpha}$$

f's are TMDPDFs or TMDFFs →

	for DY	for SIDIS	for SIA	
$f_{\bar{n}}(z^-)$ is analytical in	lower	lower	upper	half-plane.
$f_n(z^+)$ is analytical in	lower	upper	upper	half-plane.

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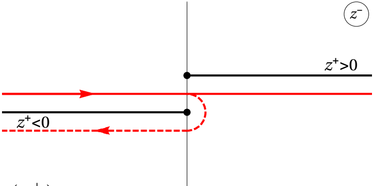

$$I = \int_{-\infty}^{0} dz^+ \frac{f_n(z^+)}{(-2z^+)^\alpha} \left(I_0 + I_1 + I_2 + I_\infty\right), \qquad I_C = \int_C \frac{f_{\bar{n}}(z^-)}{(z^-)^\alpha}$$

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NLO expression in position space

$$I=\int_{-\infty}^{\infty}dz^+dz^-\frac{f_{\bar n}(z^-)f_n(z^+)}{[-2z^+z^-+i0]^\alpha}$$

f's are TMDPDFs or TMDFFs →

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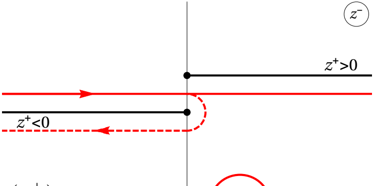

$$I=\int_{-\infty}^{0}dz^+\frac{f_n(z^+)}{(-2z^+)^\alpha}\left(I_0+I_1+I_2+I_\infty\right),\qquad I_C=\int_C\frac{f_{\bar n}(z^-)}{(z^-)^\alpha}$$

I_∞ → 0

for DY:	$\lim\limits_{z^-\to-\infty}A^\mu_{\bar n}(z)=0,$	$\lim\limits_{z^+\to-\infty}A^\mu_n(z)=0,$
for SIDIS:	$\lim\limits_{z^-\to+\infty}A^\mu_{\bar n}(z)=0,$	$\lim\limits_{z^+\to-\infty}A^\mu_n(z)=0,$
for SIA:	$\lim\limits_{z^-\to+\infty}A^\mu_{\bar n}(z)=0,$	$\lim\limits_{z^+\to+\infty}A^\mu_n(z)=0.$

Fields at ∞

(= interaction with transverse link)

→ Reproduce ordinary rules!