ASPECTS OF THRESHOLD RESUMMATION AT NEXT-TO-LEADING POWER

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OUTLINE

- **Particle scattering near threshold**
- **Factorization and resummation at NLP**
- **New partonic channels in DIS and DY**

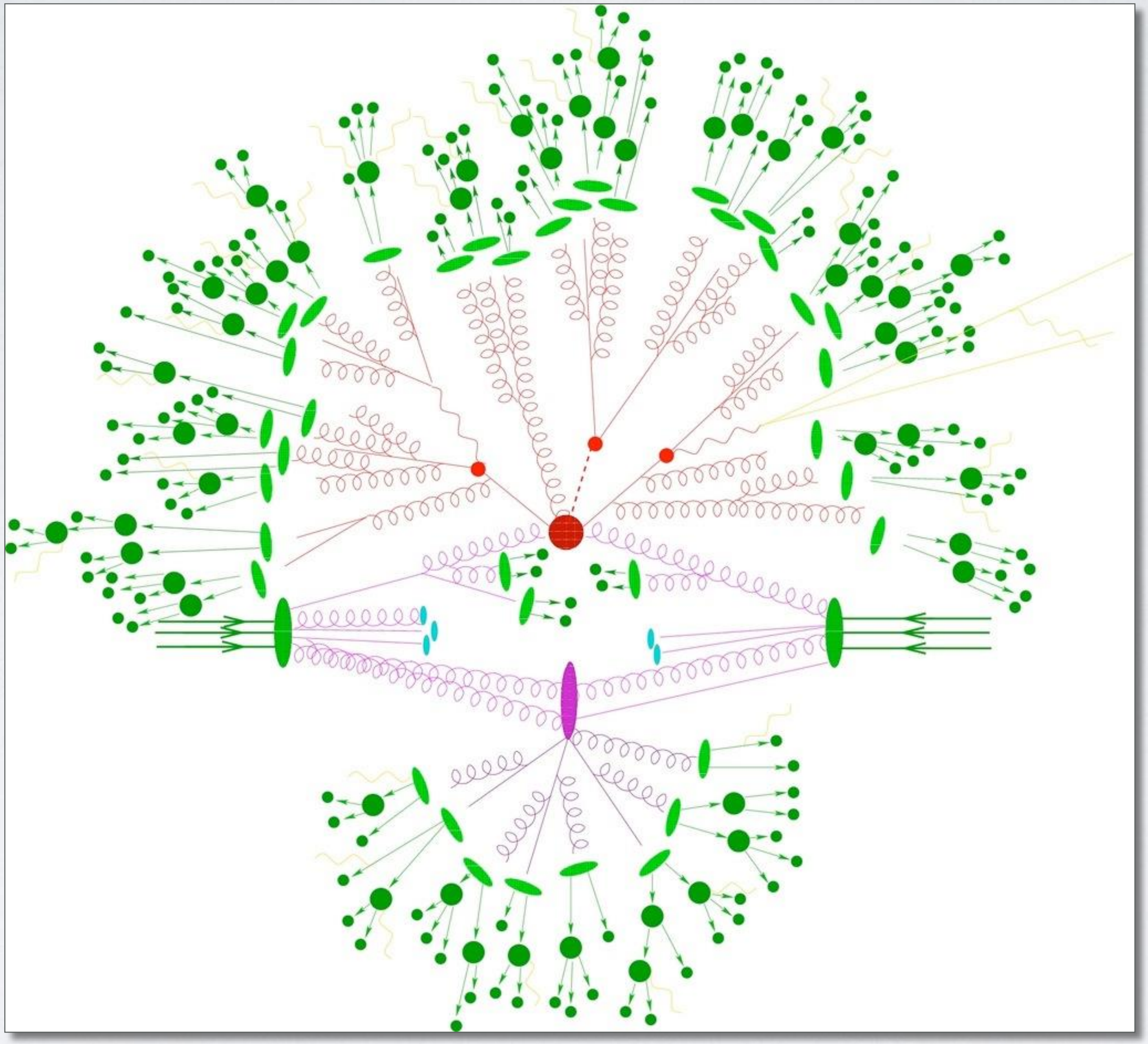
JHEP 10 (2020), 196, [arXiv:2008.04943],

with M. Beneke, M. Garny, S. Jaskiewicz, R. Szafron and J. Wang.

[arXiv: 2109.09752],

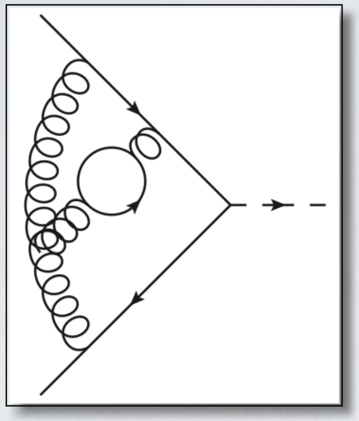
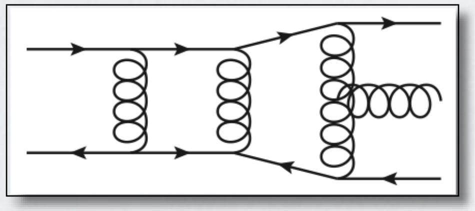
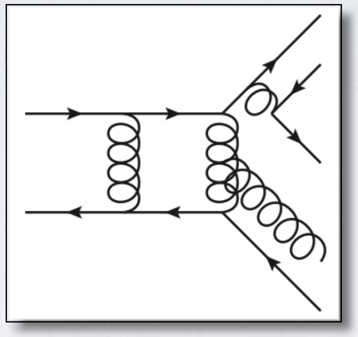
with M. van Beekveld and C. D. White.

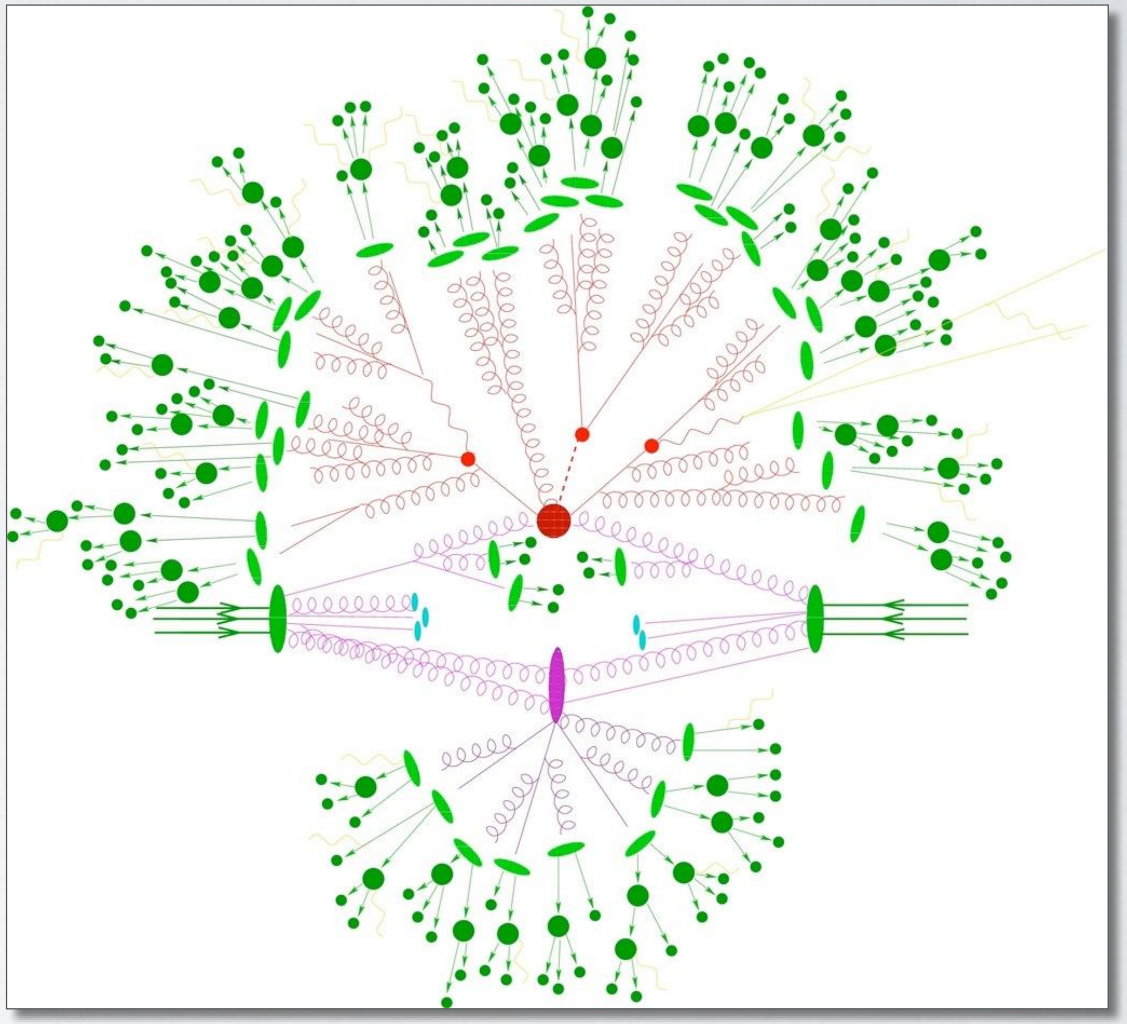
PARTICLE SCATTERING NEAR THRESHOLD



PRECISION FOR COLLIDER PHYSICS

- Hard scattering processes are calculated in perturbation theory.



- Going beyond NNLO and N3LO is difficult, yet necessary to match the precision of current and forthcoming experiments!
- Loop and phase space integrals:
  - Analytic vs numerical evaluation
  - Space of functions
  - Infrared divergences
  - Large logarithms

This talk

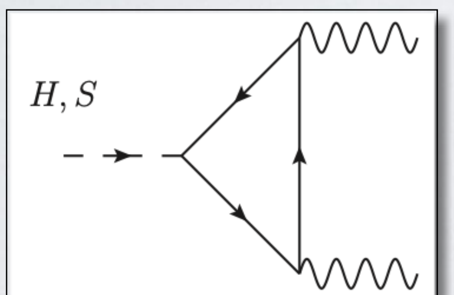
PRECISION FOR COLLIDER PHYSICS

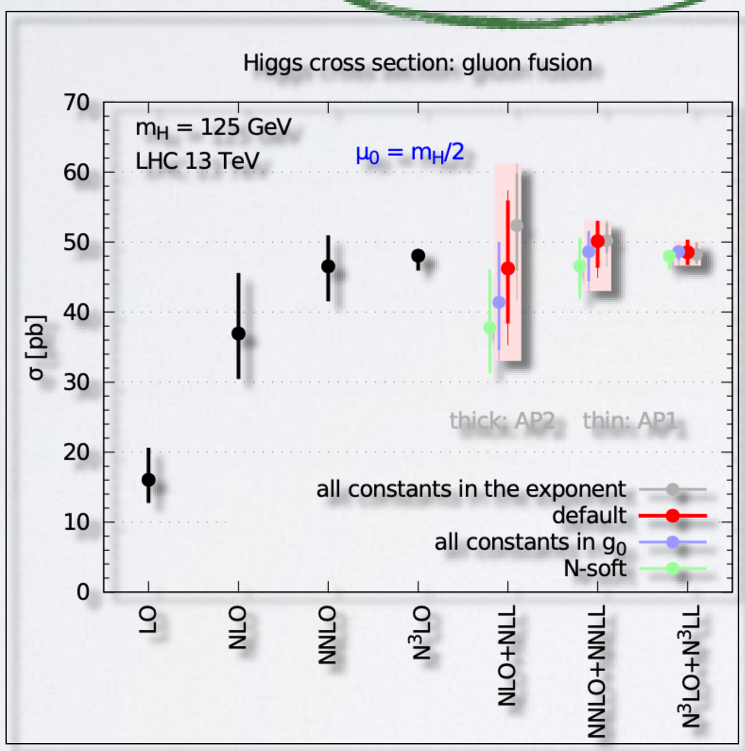
- The presence of largely different scales gives rise to large logarithms:

$$d\sigma \sim 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots$$

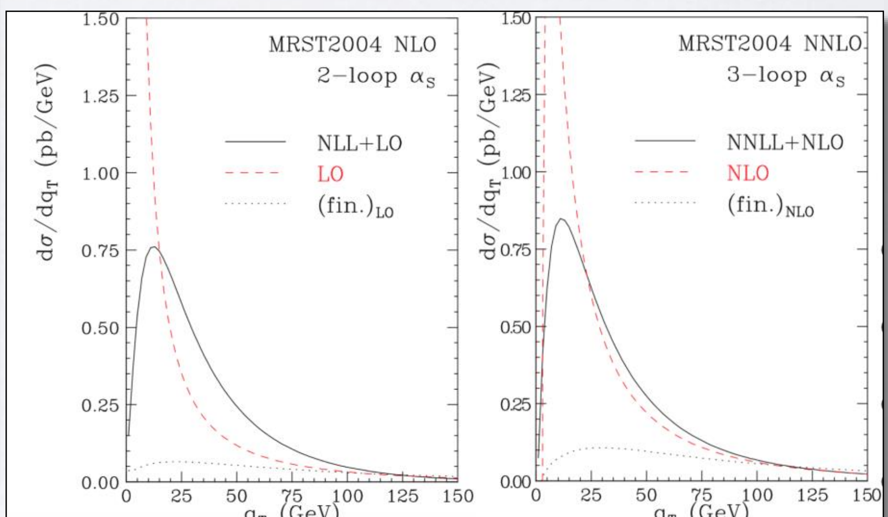
$$\sim \log^2(1-z)$$

$$\sim \log^2 \frac{m_H^2}{m_b^2}$$

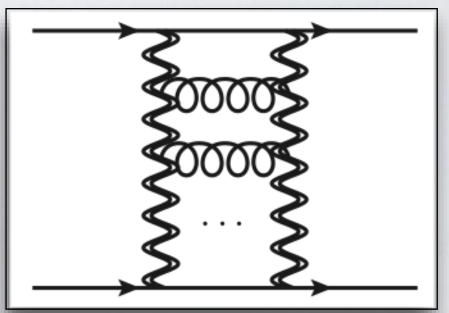




$$\sim \log \frac{m_H^2}{p_T^2}$$



$$\sim \log \frac{s}{-t}$$



- Large logarithms spoil the convergence of the perturbative series:

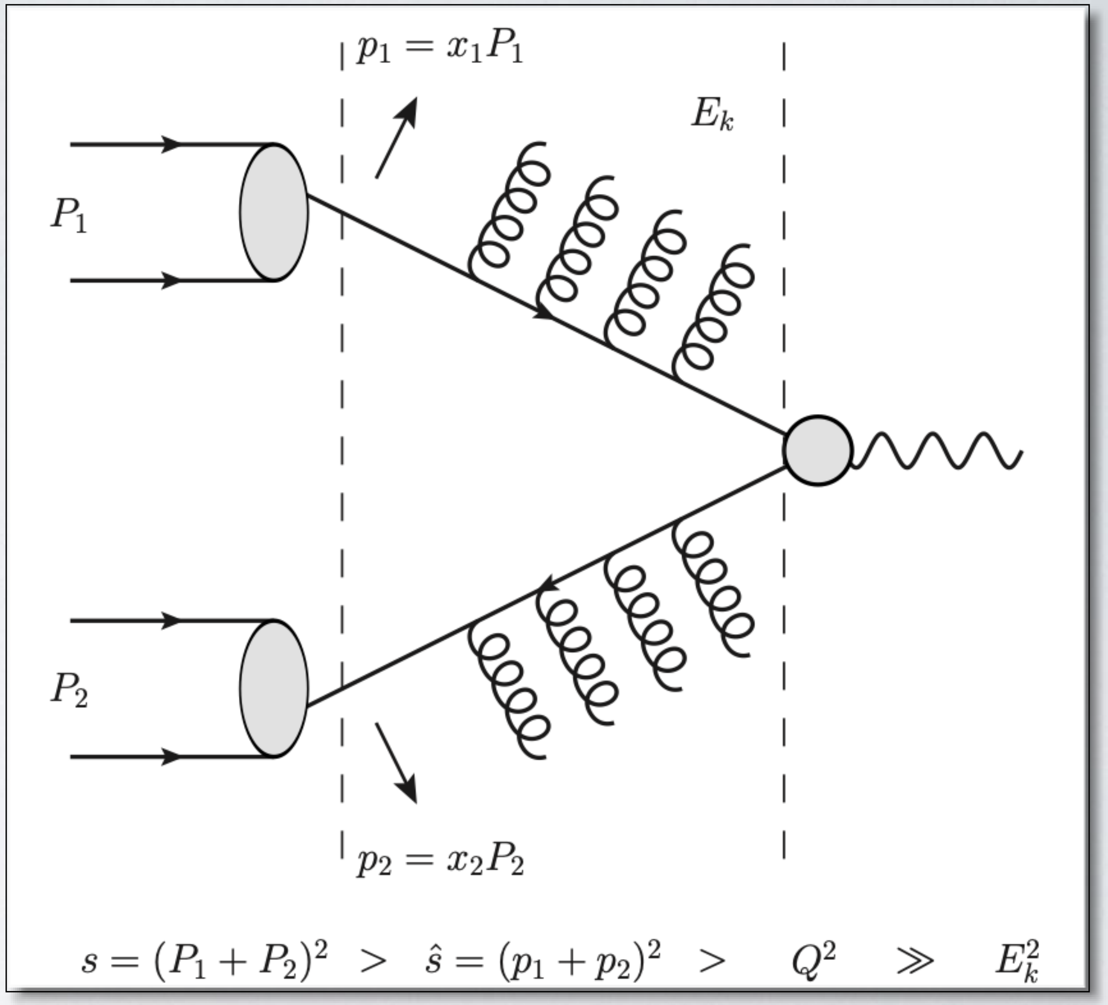
$\rightarrow$ need resummation.

PARTICLE SCATTERING NEAR THRESHOLD

- Consider the DY invariant mass distribution:

$$\frac{d\sigma}{dQ^2} = \tau\,\tilde{\sigma}_0(Q^2)\int_\tau^1 \frac{dz}{z}\,\mathcal{L}_{ab}\left(\frac{\tau}{z}\right)\Delta_{ab}(z),$$

$$\mathcal{L}_{ab}(y) = \int_y^1 \frac{dx}{x}\,f_{a/A}(x)\,f_{b/B}\left(\frac{y}{x}\right).$$



- Near partonic threshold:

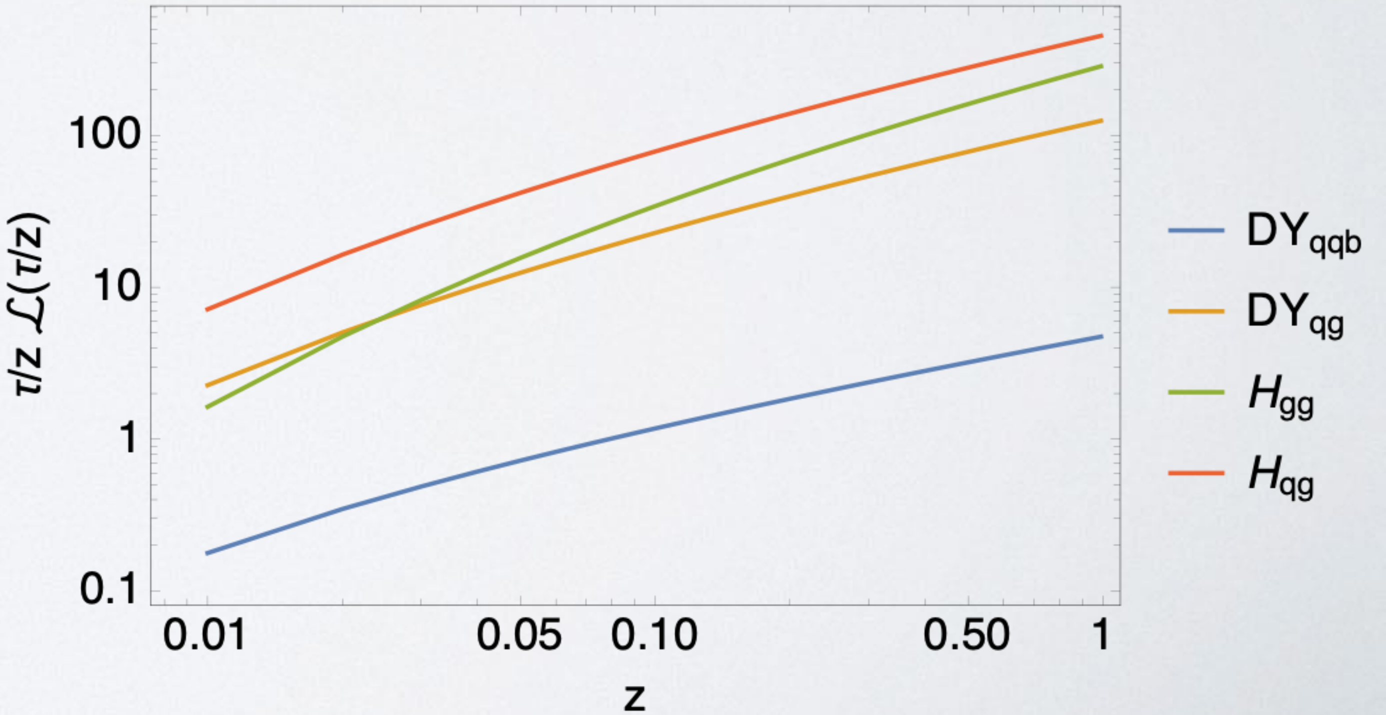
$$\tau = \frac{Q^2}{s}, \quad z = \frac{Q^2}{\hat{s}}, \quad (z \geq \tau), \quad z \to 1,$$

the partonic cross section has the singular expansion

$$\Delta_{ab}(z) = \sum_{n=0}^{\infty}\alpha_s^n\Bigg[c_n\delta(1-z) + \sum_{m=0}^{2n-1}\left(c_{nm}\left[\frac{\ln^m(1-z)}{1-z}\right]_+ + d_{nm}\ln^m(1-z)\right) + \ldots\Bigg].$$

LP terms

NLP terms



PARTICLE SCATTERING NEAR THRESHOLD: LP

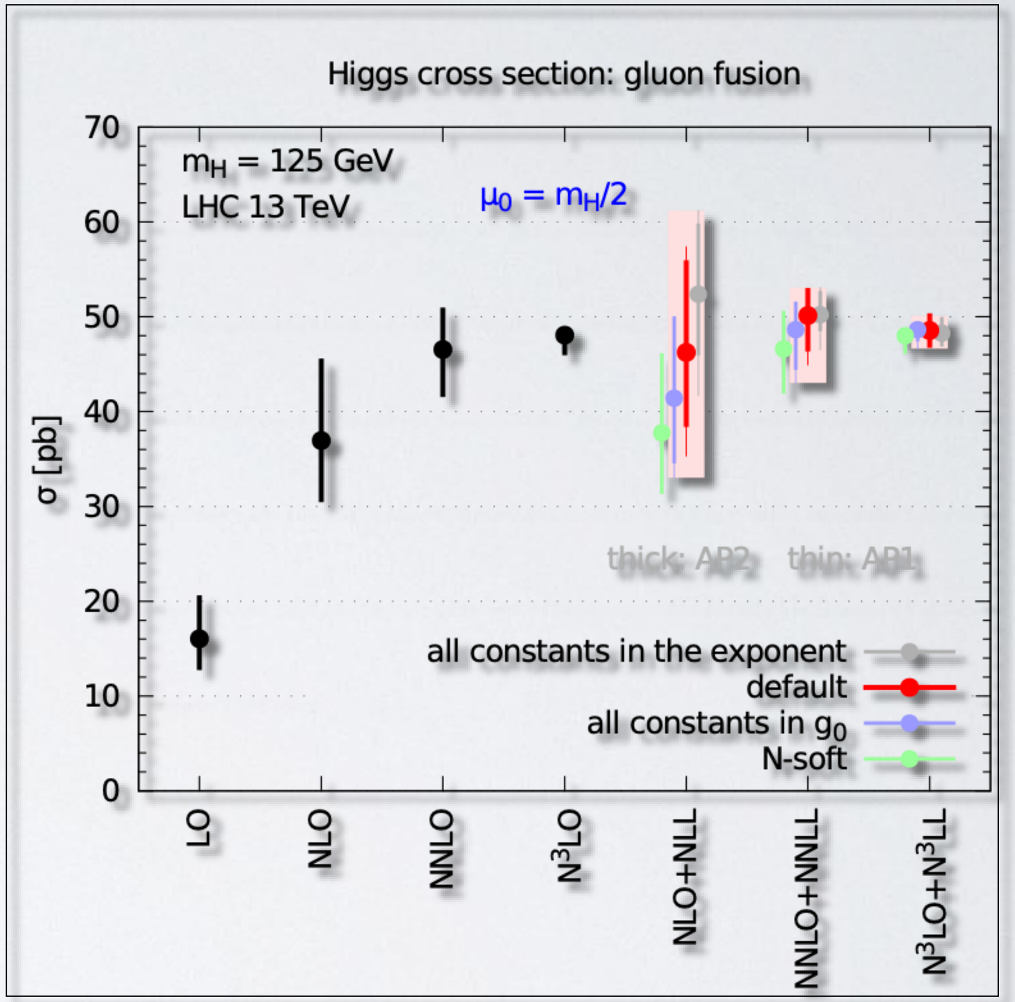
$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(-1)} \left.\frac{\log^m(1-z)}{1-z}\right|_+ + \dots$$

- Large threshold logarithms spoil the reliability of the perturbative expansion and needs to be resummed

- Resummation of LP logarithms is well established: it relies on factorization and exponentiation properties of soft radiation.

- The resummation of threshold logarithms leads to a more reliable perturbative expansion.

- More relevant for the production of heavy final states ($HH,\ t\bar{t},\ t\bar{t}W,\ t\bar{t}H,\ \dots$)



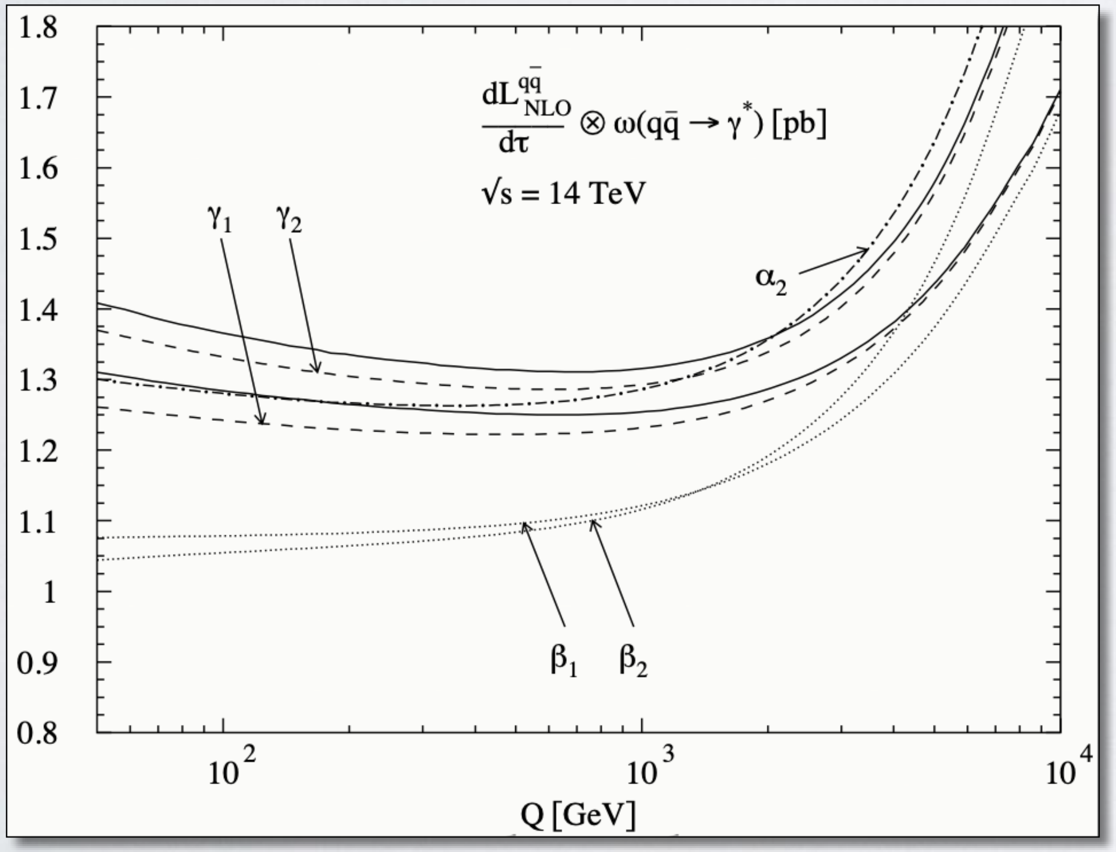
Bonvini, Marzani, Muselli, Rottoli 2016

PARTICLE SCATTERING NEAR THRESHOLD: NLP

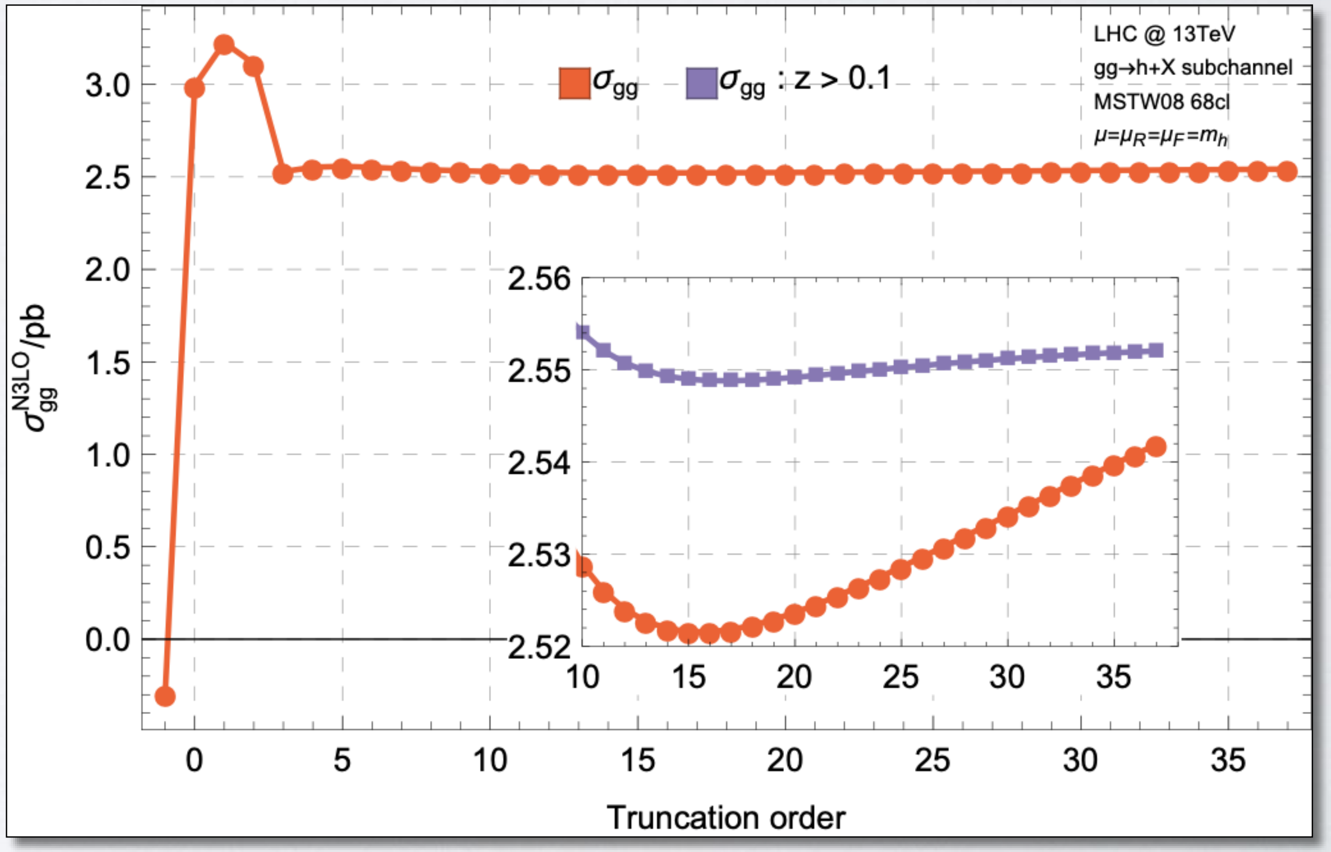
- What about NLP and higher power terms?

$$\Delta_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \ldots \right].$$

- Can be relevant for precision physics!
- Interesting problem: probes all-order structures beyond the semi-classical approximation.



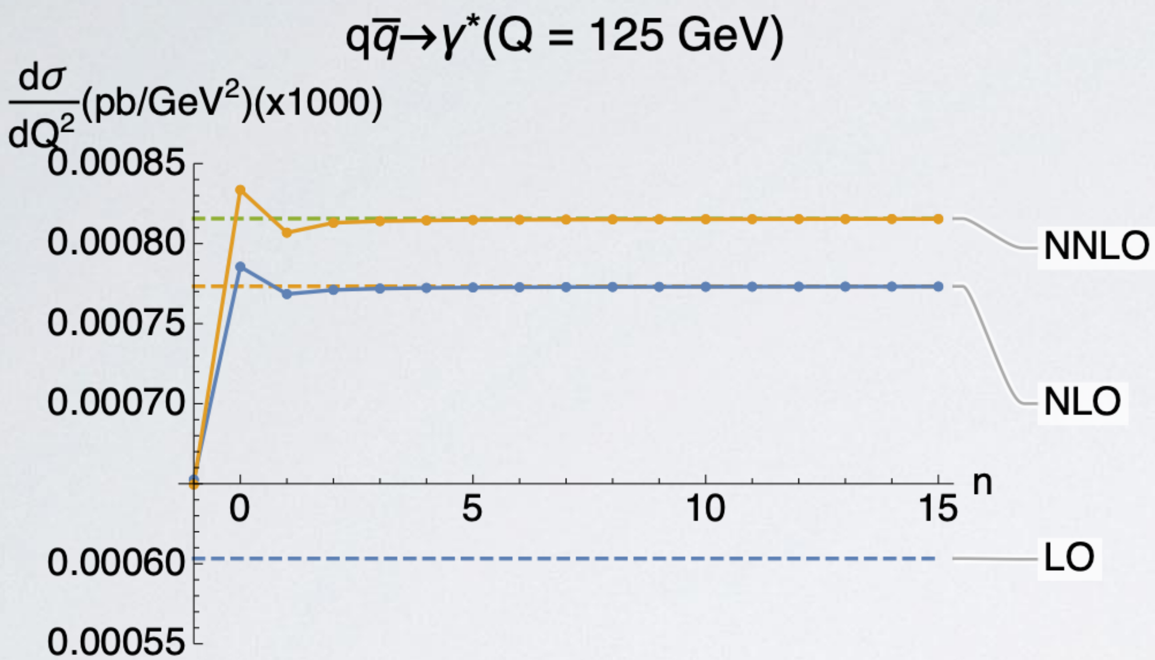
Kramer, Laenen, Spira, 1998

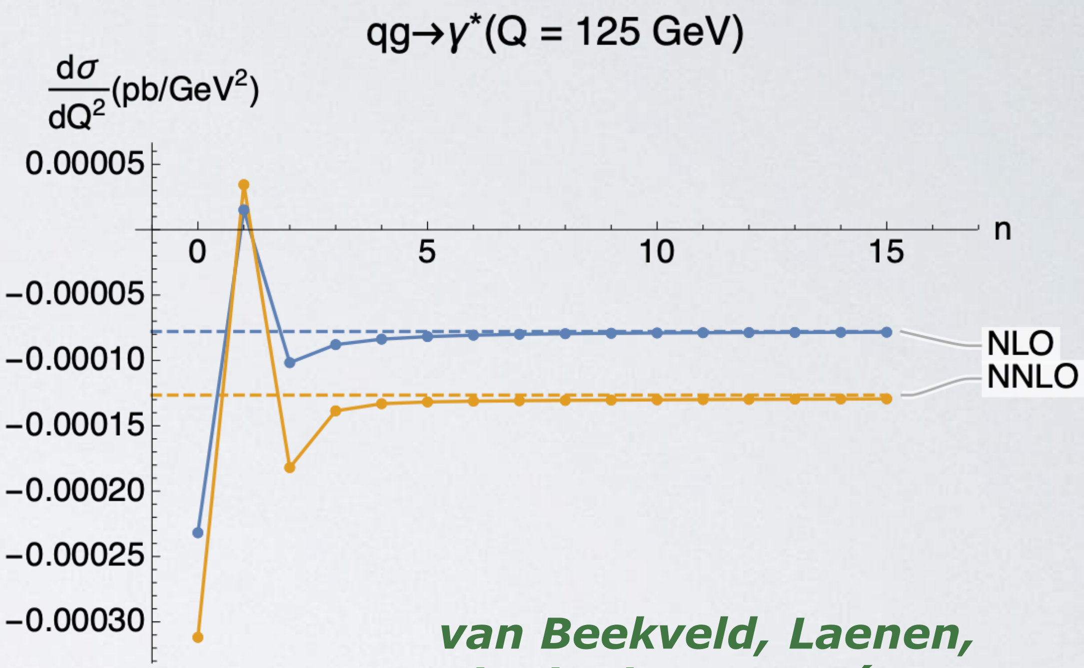


Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 2015

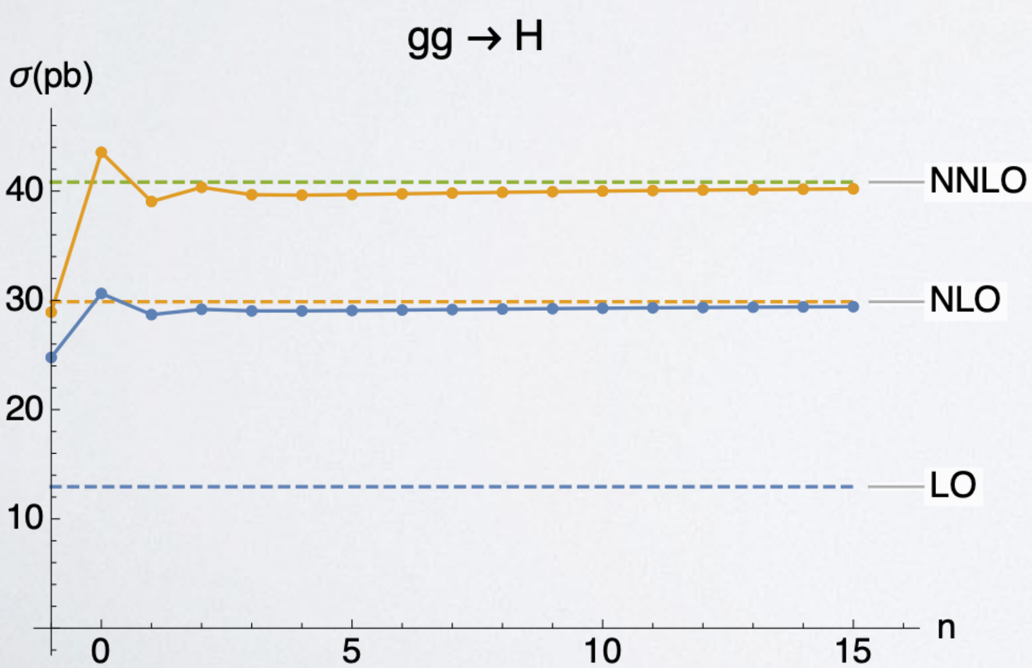
PARTICLE SCATTERING NEAR THRESHOLD: NLP

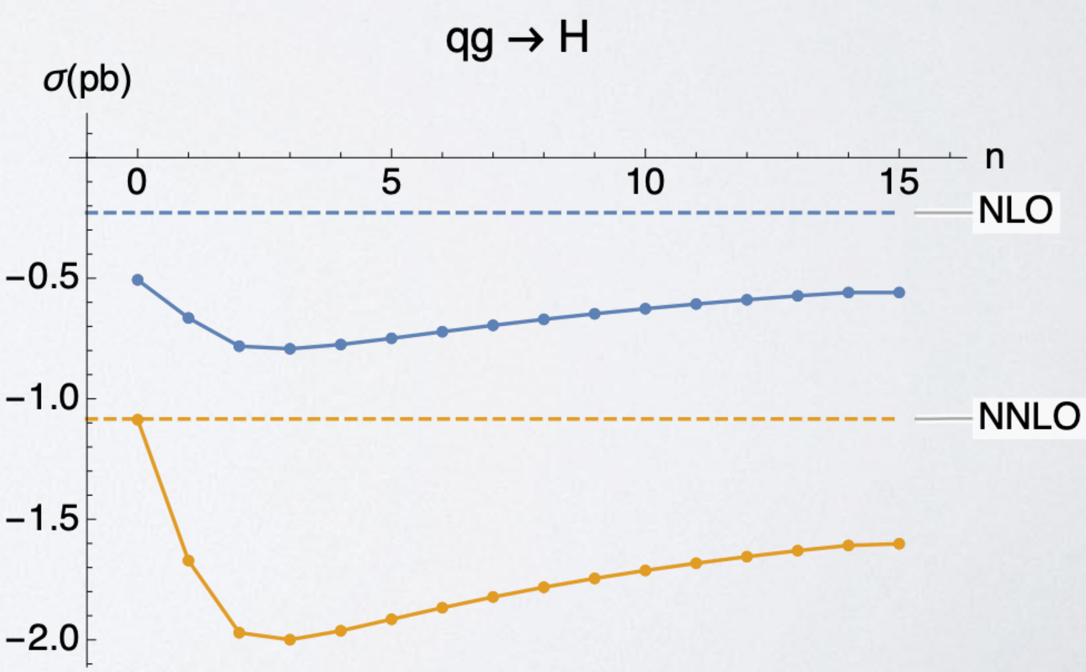
- At NLP more production channels contribute. Convergence is faster for leading channels.





van Beekveld, Laenen, Sinninghe Damsté, LV, 2021.





FACTORIZATION AND RESUMMATION AT NLP

$$\frac{d\sigma}{d\xi} \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(\xi)}{\xi}\right]_+ + d_{nm} \ln^m(\xi) \right) + \ldots \right].$$

- Understanding the factorization and resummation of large logarithms at next-to-leading power (NLP) has been subject of intense work in the past few years!

- Drell-Yan, Higgs and DIS near threshold

  Del Duca, 1990; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016;
  Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019;
  van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021;
  Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;
  Beneke, Broggio, Jaskiewicz, LV, 2019;
  Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019, 2020.

- Operators and Anomalous dimensions

  Larkoski, Neill, Stewart 2014;
  Moult, Stewart, Vita 2017; Feige, Kolodrubetz, Moult, Stewart 2017;
  Beneke, Garny, Szafron, Wang, 2017, 2018, 2019.

- Thrust

  Moult, Stewart, Vita, Zhu 2018, 2019.

- pT and Rapidity logarithms

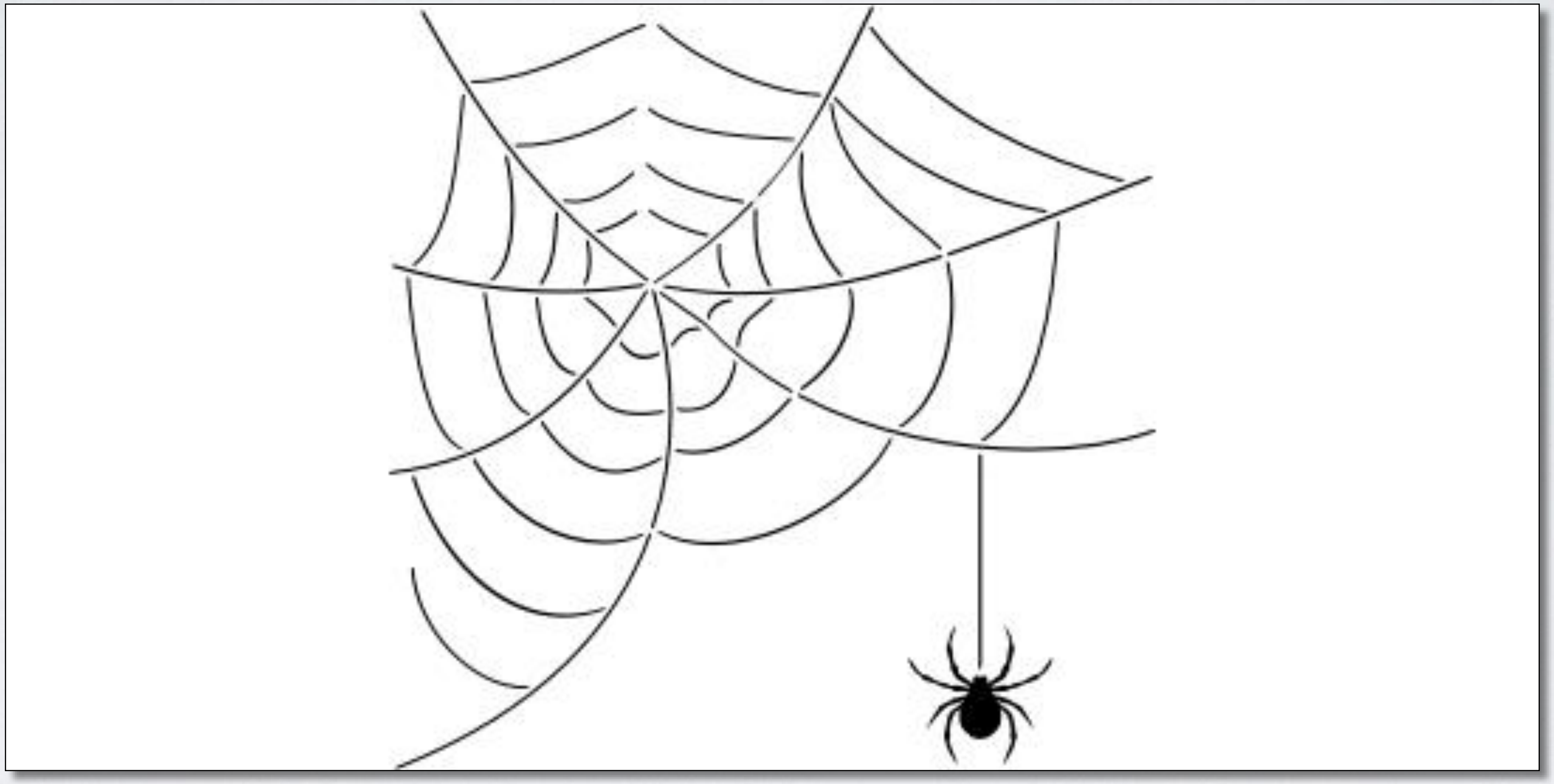
  Ebert, Moult, Stewart, Tackmann, Vita, 2018,
  Moult, Vita Yan 2019;
  Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020.

- Mass effects

  Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang, Fleming, 2020;
  Liu, Mecaj, Neubert, Wang, 2020;
  Anastasiou, Penin, 2020.

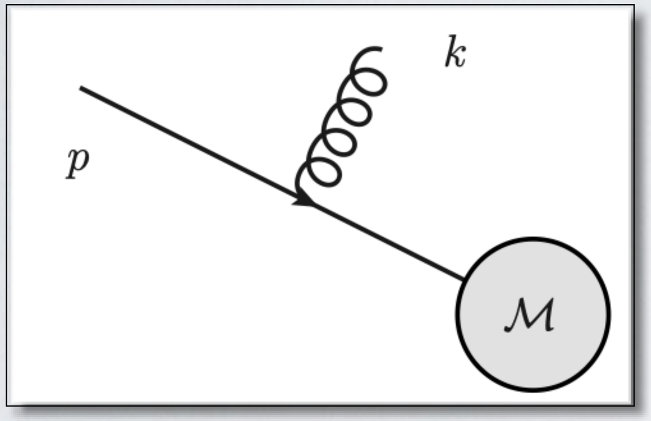
And many more!
[O(50 publications) and counting]

SCATTERING NEAR THRESHOLD: LP VS NLP



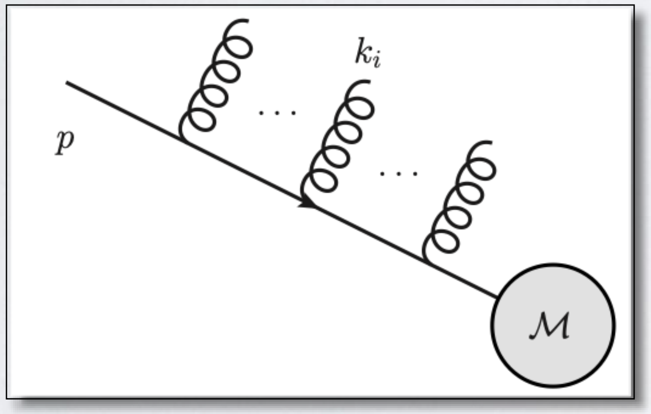
FACTORISATION OF SOFT GLUONS AT LP

- Emission of soft gluons from an energetic parton (quark):



$$= \mathcal{M}\,\frac{\not{p}-\not{k}}{2p\cdot k}\,\gamma^\mu\,T^A\,u(p) \;\sim\; \mathcal{M}\,\frac{p^\mu}{p\cdot k}\,T^A\,u(p).$$

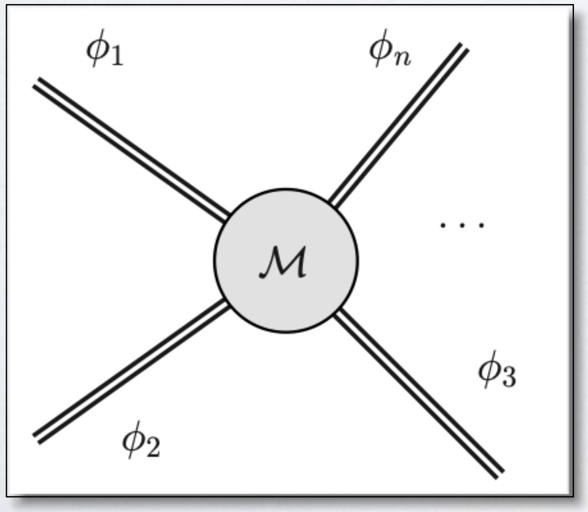
- Emission of multiple soft gluons factorises:



$$\sim \mathcal{M}\,\mathcal{S}\,u(p), \qquad \mathcal{S} = \langle 0|\Phi_\beta(-\infty,0)|0\rangle,$$

$$\Phi_\beta(\lambda_1,\lambda_2) = \mathcal{P}\exp\left\{i\,g_s\int_{\lambda_1}^{\lambda_2} d\lambda\;\beta\cdot A(\lambda\beta)\right\}.$$

- In general



$$\sim \mathcal{M}\,\mathcal{S}\,u(p_1)\bar{v}(p_2)\ldots\bar{u}(p_n),$$

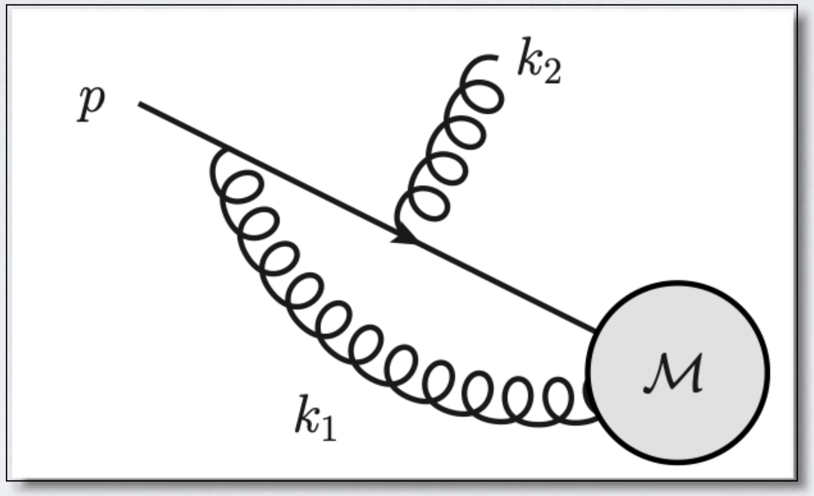
$$\mathcal{S} = \langle 0|\Phi_1\ldots\Phi_n|0\rangle \sim e^{\mathcal{W}_E}.$$

Collins, Soper,Sterman, 1989;

Gardi, Laenen, Stavenga, White, 2010;
Gardi, Smillie, White, 2013

EXPANSION BY REGIONS

- Beyond leading power one has non-trivial effects due to virtual gluons:



- The loop momentum runs over all scales, cannot be treated as soft:

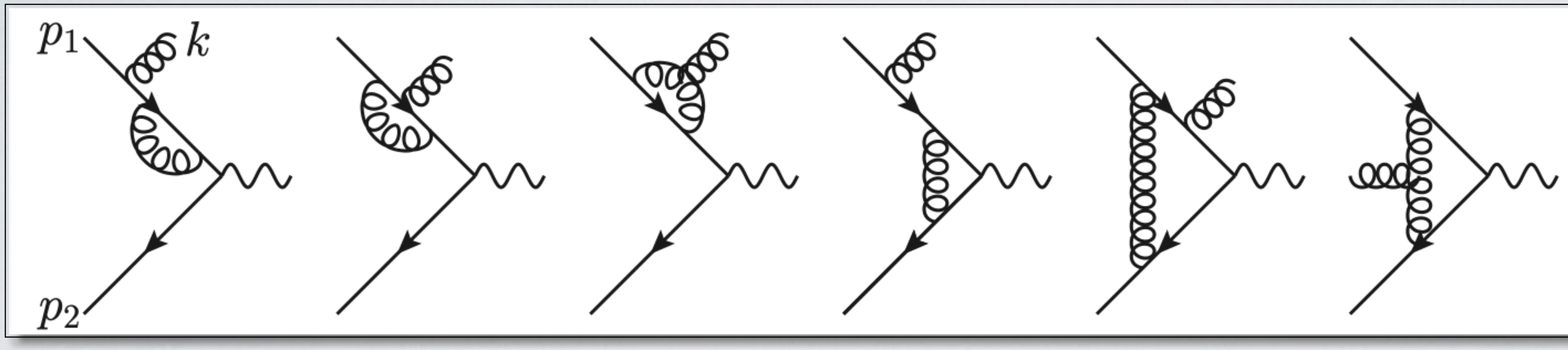
$$k = n_+ \cdot k \frac{n_-}{2} + n_- \cdot k \frac{n_+}{2} + k_\perp,$$

$$n_\pm^2 = 0, \qquad n_- \cdot n_+ = 2, \quad n_- \sim \frac{p}{\hat{s}},$$

$$\begin{aligned} \text{hard:} \qquad & k \sim \sqrt{\hat{s}}(1,1,1) \\ \text{collinear:} \qquad & k \sim \sqrt{\hat{s}}(1,\lambda,\lambda^2) \\ \text{anti-collinear:} \qquad & k \sim \sqrt{\hat{s}}(\lambda^2,\lambda,1) \\ \text{soft:} \qquad & k \sim \sqrt{\hat{s}}(\lambda^2,\lambda^2,\lambda^2), \qquad \lambda \ll 1. \end{aligned}$$

Bonocore, Laenen, Magnea, LV, White, 2014

EXPANSION BY REGIONS: LP



$$\hat{s} = (p_1 + p_2)^2,$$
$$t = (p_1 - k)^2,$$
$$u = (p_2 - k)^2.$$

- Virtual gluons gives non-analytical contributions ∝ to the scales of the problem: LP

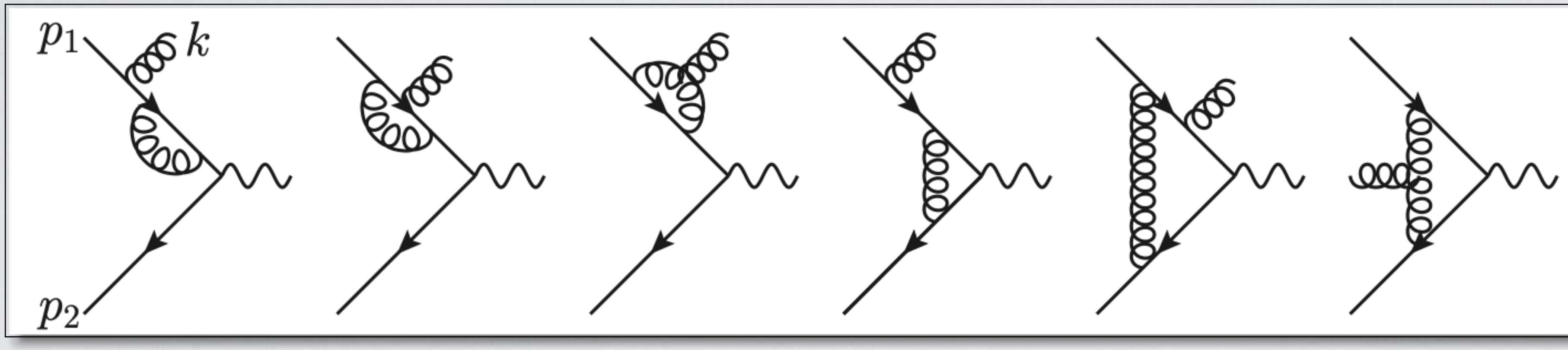
$$|\mathcal{M}|^2 \propto \overbrace{\frac{\hat{s}^2}{tu}}^{\text{LP}} \left\{ C_F^2 \overbrace{\left(\frac{\mu^2}{-s}\right)^{\epsilon}}^{\text{hard}} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \ldots \right) + C_A C_F \overbrace{\left(\frac{\hat{s}\,\mu^2}{t\,u}\right)^{\epsilon}}^{\text{soft}} \left(-\frac{1}{\epsilon^2} + \ldots \right) \right\} + \ldots$$

Factorisation

$$S\left[\frac{\hat{s}\,\mu^2}{t\,u}, \epsilon\right] \times H\left[\frac{\mu^2}{-\hat{s}}, \epsilon\right]$$

- Factorisation: physics at different scales is uncorrelated.

EXPANSION BY REGIONS: NLP



$$\hat{s} = (p_1 + p_2)^2, \quad t = (p_1 - k)^2, \quad u = (p_2 - k)^2.$$

- Virtual gluons gives non-analytical contributions $\propto$ to the scales of the problem: NLP

$$|\mathcal{M}|^2 \propto C_F^2 \Bigg\{ \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{hard}}{\left(\frac{\mu^2}{-\hat{s}}\right)^{\epsilon}} \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon} + \ldots \right) + \Bigg[\overset{\text{NLP}}{\frac{s}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \Bigg] \left(-\frac{2}{\epsilon} + \ldots \right) \Bigg\}$$

$$+ C_A C_F \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{soft}}{\left(\frac{\hat{s}\,\mu^2}{t\,u}\right)^{\epsilon}} \left(-\frac{1}{\epsilon^2} + \ldots \right) + \Bigg[\overset{\text{NLP}}{\frac{\hat{s}}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \Bigg] \left(-\frac{5}{2} + \ldots \right) \Bigg\} + \ldots$$

$\downarrow$

Factorization?

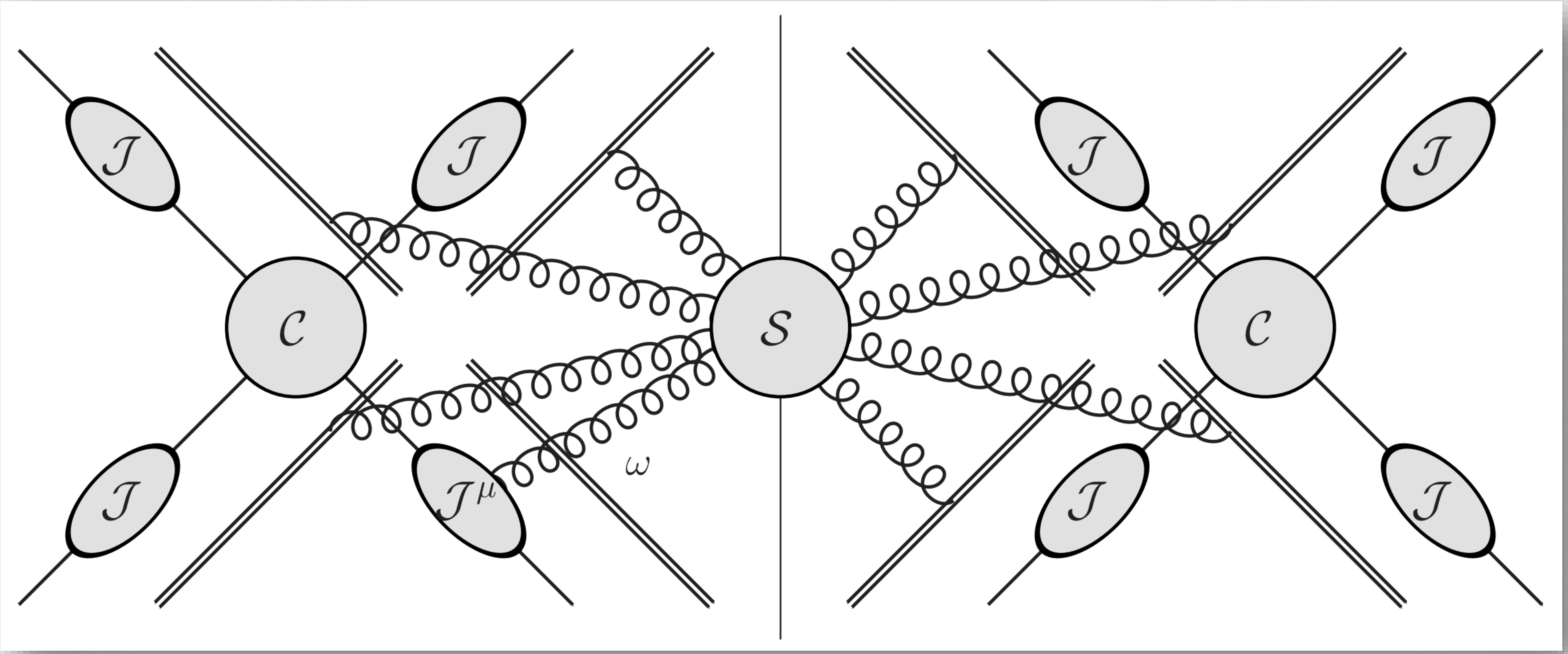
$\downarrow$

Bonocore, Laenen, Magnea, LV, White, 2014

$$S\left[\frac{\hat{s}\,\mu^2}{t\,u}, \epsilon\right] \times J\left[\frac{\mu^2}{-t}, \epsilon\right] \times \bar{J}\left[\frac{\mu^2}{-u}, \epsilon\right] \times H\left[\frac{\mu^2}{-\hat{s}}, \epsilon\right]$$

- Need an effective approach to take into account hard, collinear and soft modes.
- Two approaches: ~ Diagrammatic; ~ Soft Collinear Effective Field Theory.

FACTORIZATION AND RESUMMATION: DIAGRAMMATIC VS SCET

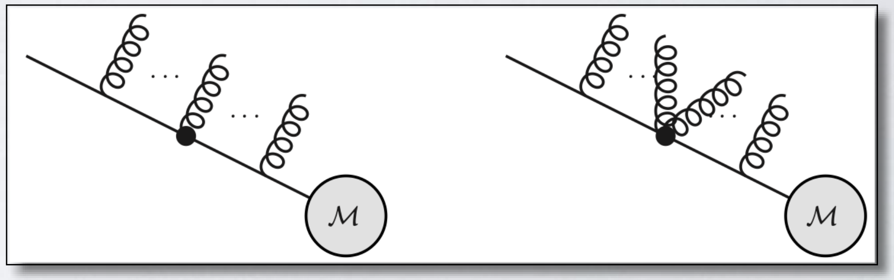


DIAGRAMMATIC FACTORIZATION AT NLP: DRELL YAN

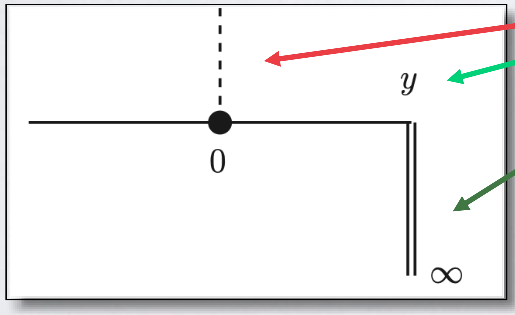
- Describe momentum regions in terms of universal functions in QCD:
- For instance, for Drell-Yan we have
  - "Derivative" of the non-radiative amplitude,
  - "Generalized" soft function,
  - "Radiative" jet function,

Low 1958, Burnett,Kroll 1968

$$\frac{k^\nu}{p_l\cdot k}\left[p_{l,\nu}\frac{\partial}{\partial p_l^\mu}-p_{l,\mu}\frac{\partial}{\partial p_l^\nu}\right]\mathcal{A}(p_j)$$



$$J_{\mu,a}\,(p,n,k)\,u(p)\;=\;\int d^dy\;\mathrm{e}^{-\mathrm{i}(p-k)\cdot y}\;\langle 0\,|\,\Phi_n(\infty,y)\,\psi(y)\,j_{\mu,a}(0)\,|\,p\rangle\;.$$



Del Duca 1990,

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016

$$F_p(-\infty,0)=\mathcal{P}\exp\left[g\int\frac{d^dk}{(2\pi)^d}A_\mu(k)\left(-\frac{p^\mu}{p\cdot k}+\frac{k^\mu}{2p\cdot k}-k^2\frac{p^\mu}{2(p\cdot k)^2}\right.\right.$$
$$\left.\left.-\frac{\mathrm{i}k_\nu\Sigma^{\nu\mu}}{p\cdot k}\right)+\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}A_\mu(k)A_\nu(l)\left(\frac{\eta^{\mu\nu}}{2p\cdot(k+l)}+\ldots\right)\right].$$

Laenen, Magnea, Stavenga, White, 2009, 2010

Bonocore, Laenen, Magnea, LV, White,2016

- One has

$$\mathcal{A}_{\mu,a}(p_j,k)\;=\;\sum_{i=1}^{2}\left(\frac{1}{2}\,\widetilde{\mathcal{S}}_{\mu,a}(p_j,k)+g\,\mathbf{T}_{i,a}\,G^{\nu}_{i,\mu}\,\frac{\partial}{\partial p_i^\nu}+J_{\mu,a}\,(p_i,n_i,k)\right)\mathcal{A}(p_j)-\mathcal{A}^{\widetilde{\mathcal{J}}}_{\mu,a}(p_j,k)\,,$$

for $n_1 = p_2$, $n_2 = p_1$.

(Removes soft-collinear overlap in the radiative jet)

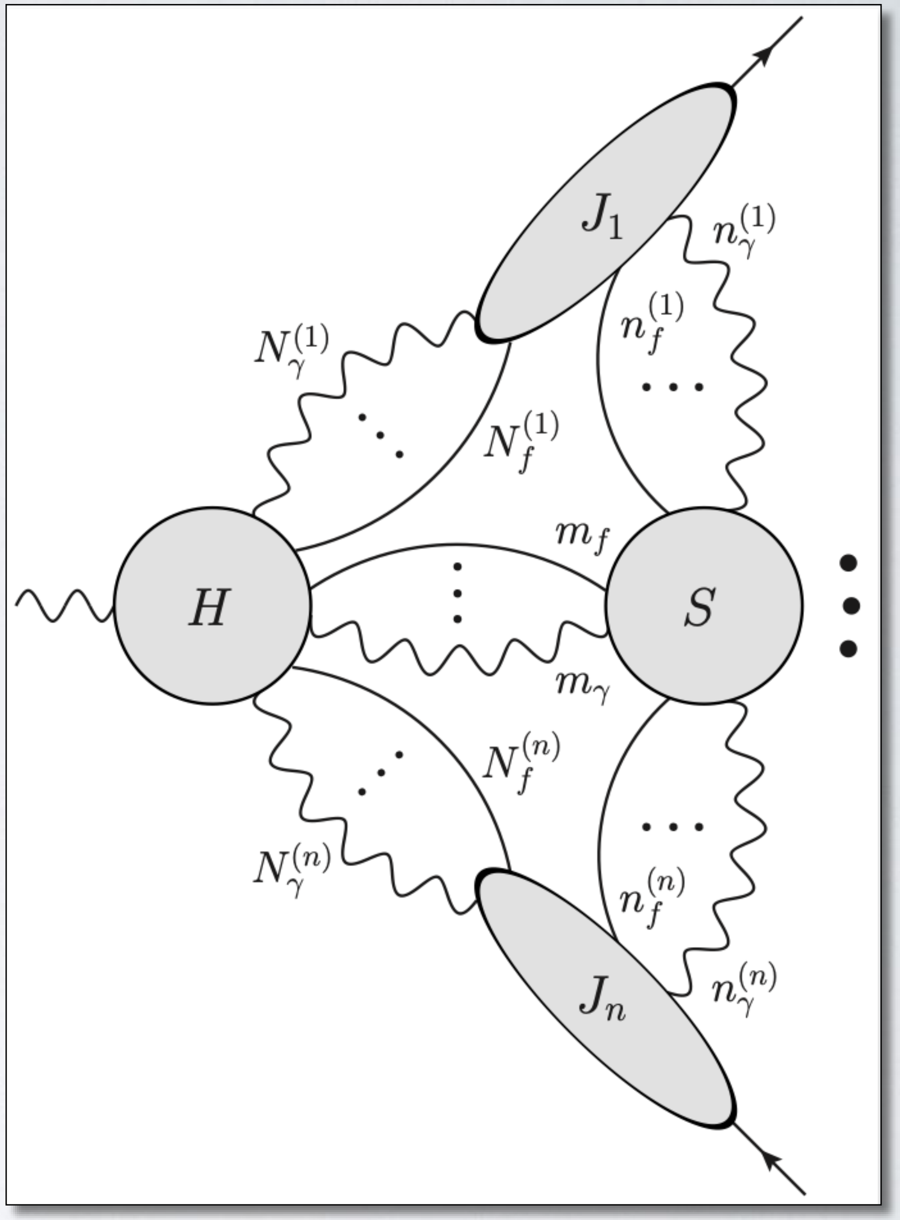
DIAGRAMMATIC FACTORIZATION AT NLP

- Exponentiation (resummation): investigate the combinatorial structure of all-order classes of diagrams, constructed with the functions defined before, contributing to a given logarithmic accuracy at NLP (more later).

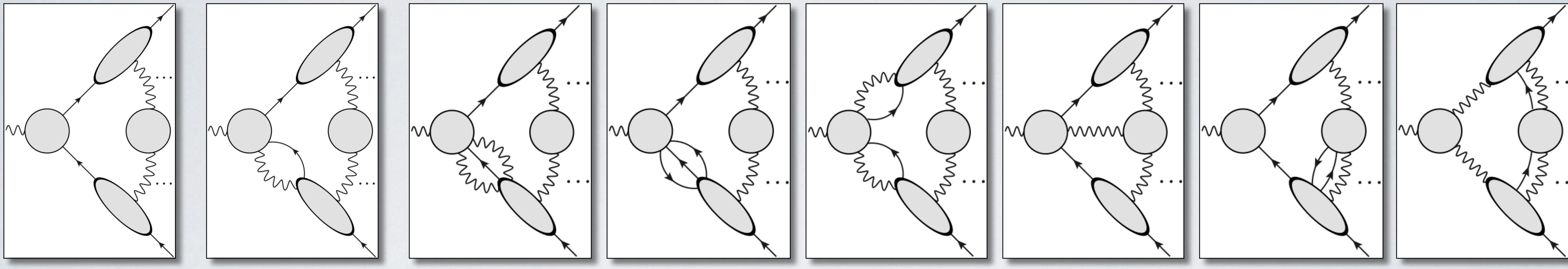
    Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019; van Beekveld, LV, White 2021

- The example discussed so far accurately describes DY up to NNLO. In general, one needs to take into account
    - processes with more than two external directions;
    - factorization beyond one loop;
    - Multiple soft gluon emission.

- Task: obtain a classification of the jet-like structures, consisting of virtual radiation collinear to any of the *n* external hard particles, contributing at subleading power in a parametrically small scale, corresponding to a fermion mass or a soft external momentum.



DIAGRAMMATIC FACTORISATION AT NLP



$\gamma = 0$ $\gamma = 1$ $\gamma = 2$

- Work in progress: in QED one obtains the all-orders factorization formula

$$\mathcal{M}^{\mathrm{LP}} = \left(\prod_{i=1}^{n} J_{(f)}(\hat{p}_i) \right) \otimes H(\hat{p}_1, \ldots, \hat{p}_n) \, S(n_i \cdot n_j) \,,$$

Laenen, Sinninghe Damsté, LV, Waalewijn, Zoppi, 2020

$$\mathcal{M}^{\mathrm{NLP}}_{\mathrm{coll}} = \sum_{i=1}^{n} \left(\prod_{j \neq i} J^{j}_{(f)} \right) \left[J^{i}_{(f\gamma)} \otimes H^{i}_{(f\gamma)} + J^{i}_{(f\partial\gamma)} \otimes H^{i}_{(f\partial\gamma)} \right] S \; + \sum_{i=1}^{n} \left(\prod_{j \neq i} J^{j}_{(f)} \right) J^{i}_{(f\gamma\gamma)} \otimes H^{i}_{(f\gamma\gamma)} \, S$$
$$+ \sum_{i=1}^{n} \left(\prod_{j \neq i} J^{j}_{(f)} \right) J^{i}_{(fff)} \otimes H^{i}_{(fff)} \, S \; + \sum_{1 \leq i \leq j \leq n} \left(\prod_{k \neq i,j} J^{k}_{(f)} \right) J^{i}_{(f\gamma)} J^{j}_{(f\gamma)} \otimes H^{ij}_{(f\gamma)(f\gamma)} \, S \,.$$

- The main message: in general, more types of jet functions are needed, involving two or more particles along a given collinear direction.
- Possible drawbacks: soft-collinear overlap, power expansion of the factorized functions.

FACTORIZATION AND RESUMMATION IN SCET AT LP

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\rm SCET} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002.

$$\mathcal{O}_n = \int dt_1 \ldots dt_n \, \mathcal{C}(t_1, \ldots, t_n) \, \phi_1(t_1 n_{1+}) \ldots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \ldots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

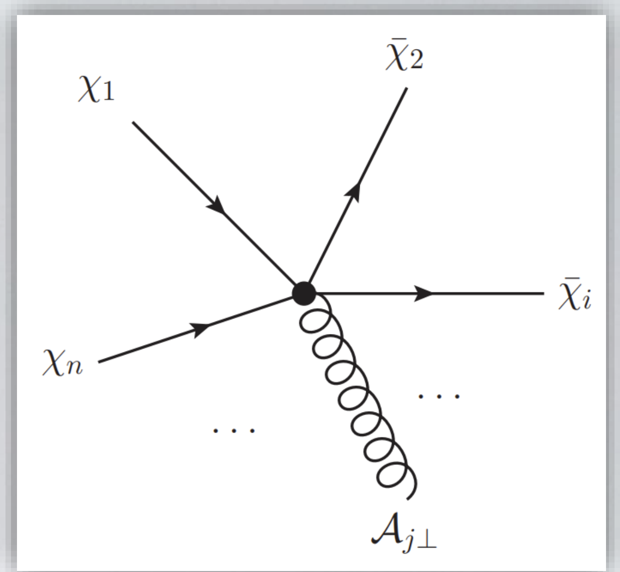
Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

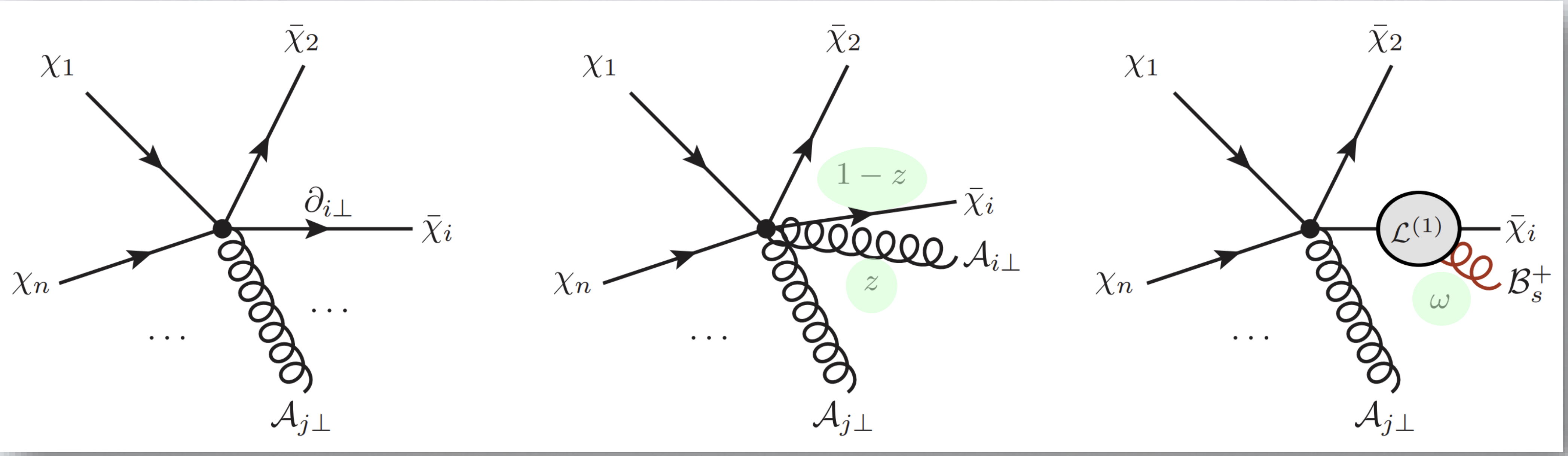
FACTORIZATION IN SCET: LP VS NLP

- Leading power (LP):
  - N-jet operators;
  - Soft-collinear decoupling.



- Next-to-leading power (NLP):
  - Kinematic suppression;
  - Multi-particle emission along the same collinear direction;
  - No soft-collinear decoupling.

Beneke, Garny, Szafron, Wang, 2017,2018



DRELL-YAN AT LP IN SCET

- Drell-Yan momentum modes:

$$p_c = (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1,\lambda^2,\lambda), \qquad p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2_{\mathrm{QCD}}/Q, \Lambda_{\mathrm{QCD}}),$$

$$p_{\bar{c}} = (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2,1,\lambda), \qquad p_{\bar{c}-\mathrm{PDF}} \sim (\Lambda^2_{\mathrm{QCD}}/Q, Q, \Lambda_{\mathrm{QCD}}),$$

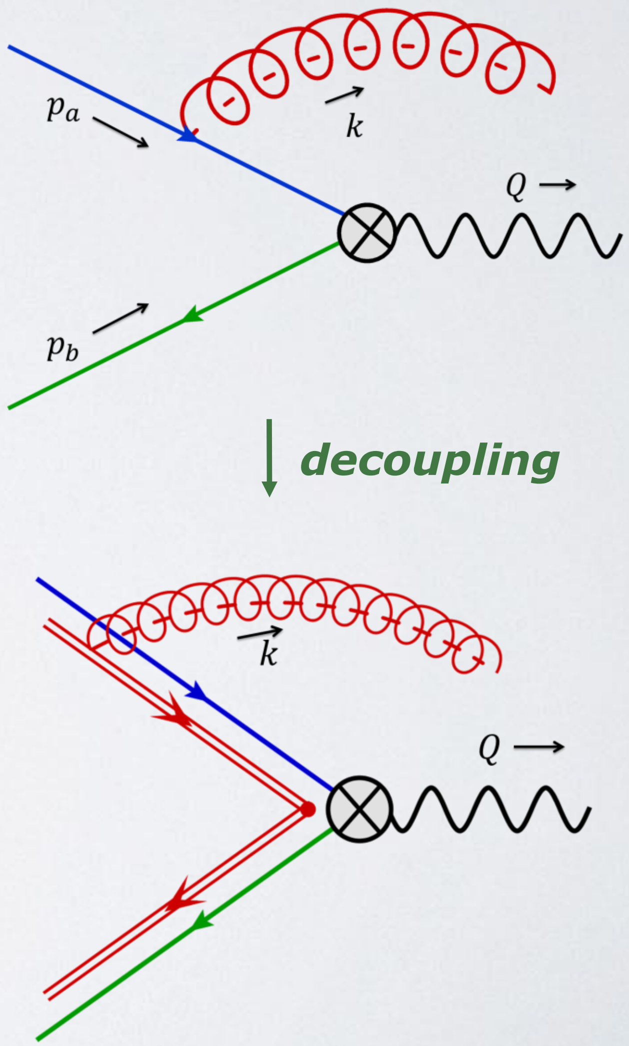
$$p_s = (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2,\lambda^2,\lambda^2), \qquad Q^2\lambda^2 = Q(1-z) \gg \Lambda_{\mathrm{QCD}}.$$

- At LP the QCD DY current matches to

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \tilde{C}^{A0}(t,\bar{t})\, J^{A0,A0}_\mu(t,\bar{t}),$$

where

$$J^{A0A0}_\rho = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\,\gamma_{\perp\rho}\,\chi_c(tn_+)$$
$$= \bar{\chi}^{(0)}_{\bar{c}}(\bar{t}n_-) Y^\dagger_-(0)\,\gamma_{\perp\rho} Y_+(0)\,\chi^{(0)}_c(tn_+),$$



- After decoupling, soft interaction factorise into soft Wilson lines.
- Initial and final state factorise into soft and PDF-collinear sector:

$$|X\rangle = |X^{\mathrm{PDF}}_{\bar{c}}\rangle |X^{\mathrm{PDF}}_c\rangle |X_{\mathrm{s}}\rangle.$$

DRELL-YAN AT LP IN SCET

- The invariant mass distribution at LP reads

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm EM}^2}{3N_cQ^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\,\hat{\sigma}_{q\bar{q}}^{\rm LP}(z),$$

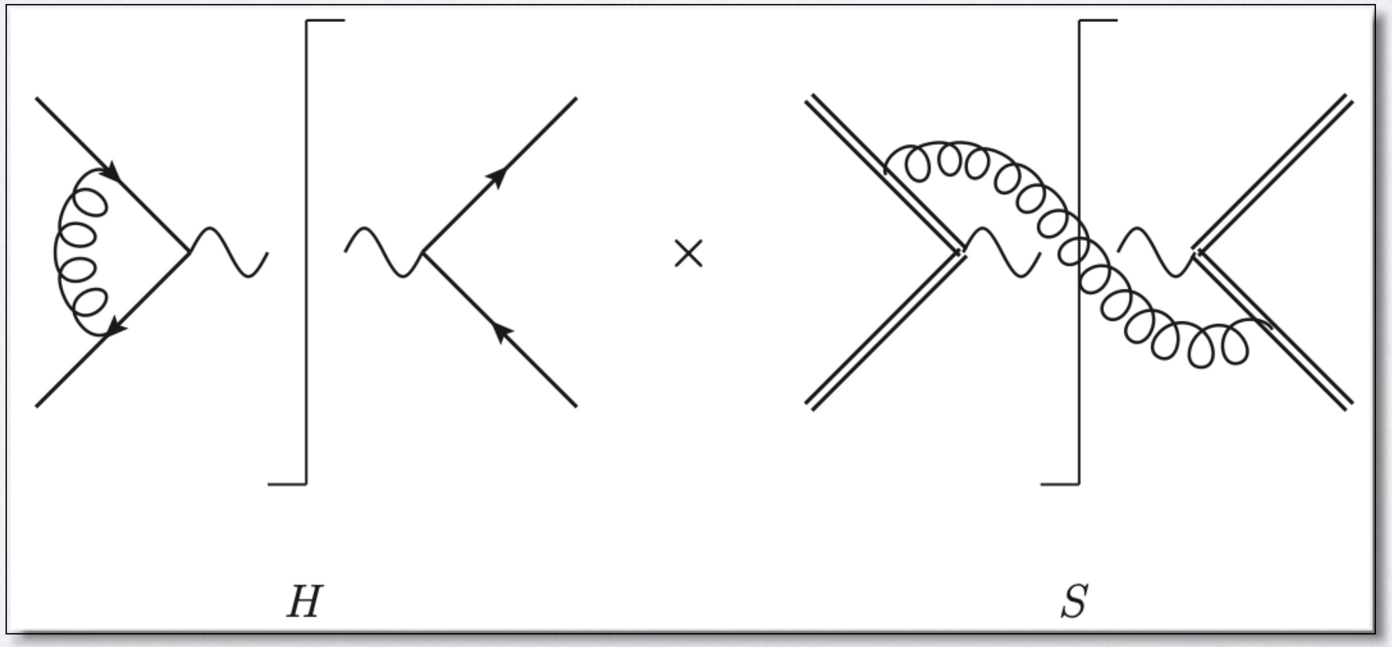
Sterman, 1987, Catani, Trentadue, 1989, Korchemsky, Marchesini 1993, Becher,Neubert, Xu 2007

where

$$\hat{\sigma}_{q\bar{q}}^{\rm LP}(z) = |C(Q^2)|^2\, Q\, S_{\rm DY}(Q(1-z)),$$

- The soft function is a vacuum matrix element of Wilson lines:

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}{\rm Tr}\langle 0|\bar{\mathbf{T}}\big[(Y_+^\dagger Y_-)(x^0)\big]\,\mathbf{T}\big[(Y_-^\dagger Y_+)(x^0)\big]|0\rangle.$$



DRELL-YAN AT NLP IN SCET

- Schematic factorization formula at NLP: we expect

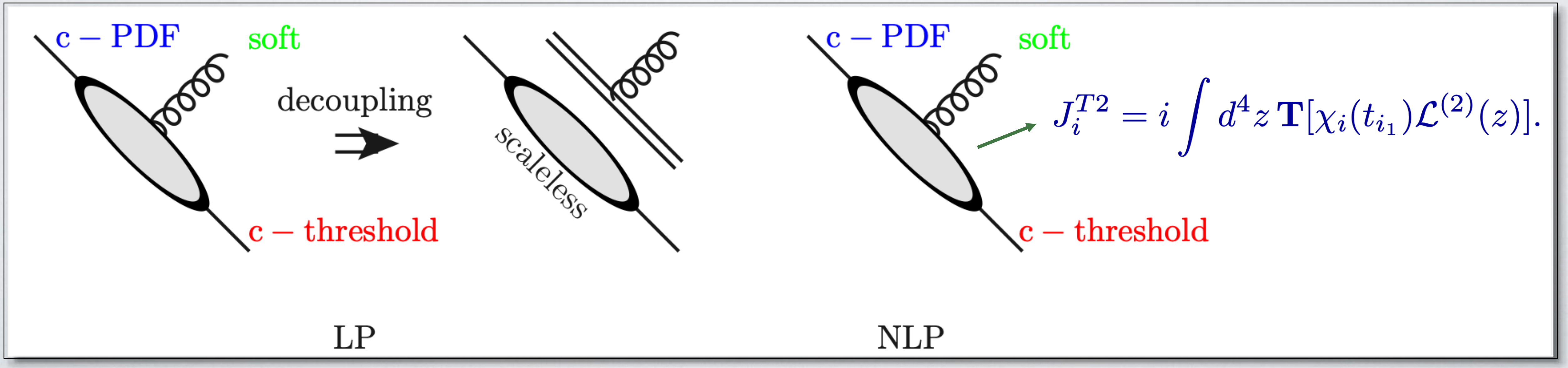
$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{EM}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{q\bar{q}}^{\mathrm{NLP}}(z),$$

where

$$\hat{\sigma}_{NLP} = \sum_{\mathrm{terms}} [C \otimes J \otimes \bar{J}] \otimes S,$$

  - C is the hard Wilson matching coefficient,
  - S is a *generalized* soft function,
  - J is a new collinear function.

- The collinear function is trivial at LP, because all threshold collinear modes are scaleless.
- The collinear scale is induced by the injection of a soft momentum.



DRELL-YAN AT NLP IN SCET

- This is provided at subleading power by soft-collinear interactions. For instance:

$$J_c^{T2}(t) = i \int d^4z \, \mathbf{T}\left[\chi_c(tn_+)\, \mathcal{L}_{2\xi}^{(2)}(z)\right] ,$$

where

$$\mathcal{L}_{2\xi}^{(2)}(z) = \frac{1}{2}\,\bar{\chi}_c(z)\, z_\perp^\mu \, z_\perp^\nu \left[i\partial_\nu \, in_-\partial \, \mathcal{B}_\mu^+(z_-)\right] \frac{\not{n}_+}{2}\, \chi_c(z)\,, \qquad \mathcal{B}_\pm^\mu = Y_\pm^\dagger \left[i\, D_s^\mu \, Y_\pm\right] ,$$

- The matrix element reads

$$\langle X|\bar{\psi}\gamma^\rho\psi(0)|A(p_A)B(p_B)\rangle = \int dt\, d\bar{t}\, C^{A0,A0}(t,\bar{t})\, \langle X_{\bar{c}}^{\rm PDF}|\bar{\chi}_{\bar{c},\alpha a}(\bar{t}n_-)|B(p_B)\rangle \gamma^\rho_{\perp,\alpha\gamma}$$
$$\times\, i\int d^4z\, \langle X_c^{\rm PDF}|\frac{1}{2} z_\perp^\nu z_\perp^\mu (in_-\partial_z)^2\, \mathbf{T}\left[\chi_{c,\gamma f}\,(tn_+)\, \bar{\chi}_c\,(z)\; \mathbf{T}^A \frac{\not{n}_+}{2}\chi_c\,(z)\right]|A(p_A)\rangle$$
$$\times\, \langle X_s|\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af} \frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}_{\perp\nu}^{+A}\,(z_-)\right)|0\rangle\,.$$

Non-scaleless collinear matrix element

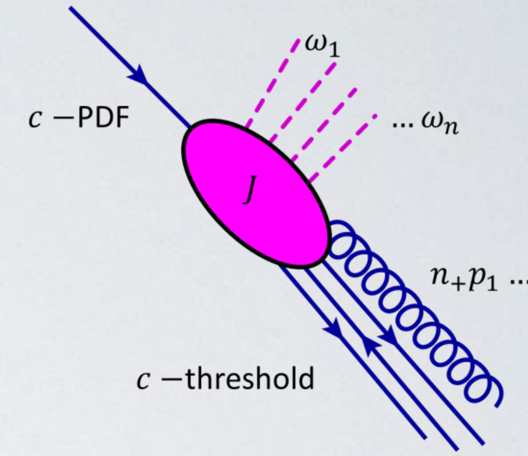
(Recall)

$$J_{\mu,a}\,(p,n,k)\, u(p) \; = \; \int d^dy\; \mathrm{e}^{-\mathrm{i}(p-k)\cdot y} \left\langle 0\right| \Phi_n(\infty,y)\, \psi(y) \;\; \underbrace{j_{\mu,a}}_{-g\overline{\psi}(x)\,\gamma^\mu\,\mathbf{T}_a\,\psi(x)+\ldots} \left|p\right\rangle .$$

DRELL-YAN AT NLP IN SCET

- This is easily generalized at any subleading power: there can be many Lagrangian insertions, each with its own ω_i conjugate to the large component of the collinear momentum.

$$i^m \int \{d^4 z_j\}\ \mathbf{T}\Big[\{\psi_c(t_k n_+)\} \times \Big\{\mathcal{L}^{(l)}(z_j)\Big\}\Big]$$
$$= 2\pi \sum_i \int du \int \{dz_{j-}\}\ \tilde{J}_i\left(\{t_k\}, u; \{z_{j-}\}\right)\ \chi_c^{\mathrm{PDF}}(un_+)\ \mathfrak{s}_i(\{z_{j-}\}),$$

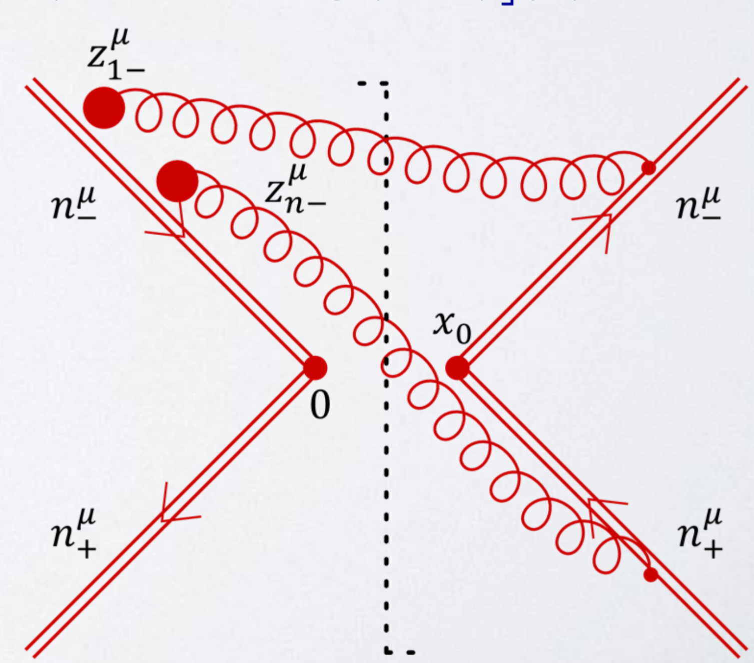


- After taking the matrix element squared, this gives a generalized soft functions:

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(z_{-j})}{4\pi} e^{-i\omega_j z_{-j}} \right)$$
$$\times\ \mathrm{Tr}\langle 0|\bar{\mathbf{T}}\big[(Y_+^\dagger Y_-)(x^0)\big]\mathbf{T}\big[(Y_-^\dagger Y_+)(x^0) \times \mathcal{L}_s^n(z_{1-}) \times \ldots \times \mathcal{L}_s^n(z_{n-})\big]|0\rangle.$$

which are equivalent to the generalized Wilson lines built in terms of NLP webs in the diagrammatic approach.

Beneke, Broggio, Jaskiewicz, LV, 2019



DRELL-YAN AT NLP IN SCET

- Up to NLP one has:

Beneke, Broggio, Jaskiewicz, LV, 2019

$$\Delta^{dyn}_{\rm NLP}(z) = -\frac{2}{(1-\epsilon)}\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_\perp^\rho\right]_{\beta\gamma}$$
$$\times \int d(n_+p)\; C^{A0,A0}(n_+p, x_b n_-p_B)\, C^{*A0A0}(x_a\, n_+p_A,\, x_b n_-p_B)$$
$$\times \sum_{i=1}^{5}\int \{d\omega_j\}\; J_i(n_+p, x_a\, n_+p_A; \{\omega_j\})\; S_i(\Omega; \{\omega_j\}) + \text{h.c.}\,.$$

- Compare fixed order expansion against results in literature.
  - At NLO this gives the contribution:

$$\Delta^{dyn\,(1)}_{\rm NLP}(z) = 4\, Q\, H^{(0)}(Q^2) \int d\omega\, J^{(0)}_{1,2}(x_a\, n_+p_A; \omega)\; S^{(1)}_1(\Omega; \omega)$$

DRELL-YAN AT NLP IN SCET

- At NNLO:

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-hard}}(z) = 2\,Q \int d\omega\, S^{(1)}_1\,(\Omega;\,\omega\,)\,\Bigg(\Big(H^{(1)}(Q^2)\,J^{(0)}_{1,2}\,(x_a n_+ p_A;\omega) - C^{*A0\,(0)}\,(\,x_a n_+ p_A,\,x_b n_- p_B)\,J^{(0)}_{1,1}\,(x_a n_+ p_A;\omega) \times \frac{\partial}{\partial x_a(n_+ p_A)} C^{A0\,(1)}(x_a n_+ p_A, x_b n_- p_B)\Bigg) + \mathrm{h.c.},$$

Beneke, Broggio, Jaskiewicz, LV, 2019

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-coll}}(z) = 4\,Q\,H^{(0)}(Q^2)\,\int d\omega\,J^{(1)}_{1,2}\,(x_a\,n_+ p_A;\omega)\;S^{(1)}_1(\Omega;\omega)\,,$$

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-soft}}(z) = -\frac{4}{(1-\epsilon)}\,Q\,\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\,\gamma^{\rho}_{\perp}\right]_{\beta\gamma}\,H^{(0)}(Q^2) \times \sum_{i=1}^{5} \int \{d\omega_j\}\;J^{(0)}_i\,(x_a\,n_+ p_A;\{\omega_j\})\;S^{(2)}_i(\Omega;\{\omega_j\})\,.$$

Broggio, Jaskiewicz, LV, 2021

DRELL-YAN AT NLP IN SCET

- We reproduce the 1real 1-virtual and 2-real contribution to DY.
- The ingredients to reproduce 1r1v correction in DY are the 1-loop collinear and soft functions:

$$J^{K\,(1)}_{1,2;\gamma\beta,fq}\,(n_+p_A;\,\omega) = \frac{\alpha_s\, e^{\epsilon\,\gamma_E}}{4\pi}\,\frac{1}{(n_+p_A)}\delta_{\gamma\beta}\mathbf{T}^K_{fq}\left(\frac{\omega\, n_+p_A}{\mu^2}\right)^{-\epsilon}\frac{\Gamma[1+\epsilon]\Gamma[1-\epsilon]^2}{(\epsilon-1)(\epsilon+1)\Gamma[2-2\epsilon]}$$
$$\times\left(C_F\left(-\frac{4}{\epsilon}+3+8\epsilon+\epsilon^2\right)-C_A\left(-5+8\epsilon+\epsilon^2\right)\right)\,,$$

$$S_1\,(\Omega,\omega) = \frac{\alpha_s\, C_F}{2\pi}\,\frac{\mu^{2\epsilon}\, e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\,\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^{\epsilon}}\theta(\omega)\theta(\Omega-\omega)\,+\,\mathcal{O}(\alpha_s^2)\,.$$

- However: the convolution is regularized by dimensional regularization. For resummation, we treat the two object independently, and expand in ϵ prior to performing the convolution.

$$\int d\omega\;\underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}}\;\underbrace{\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^{\epsilon}}}_{\text{soft piece}}\,.$$

- Technical problem that needs to be solved to achieve resummation beyond LL.

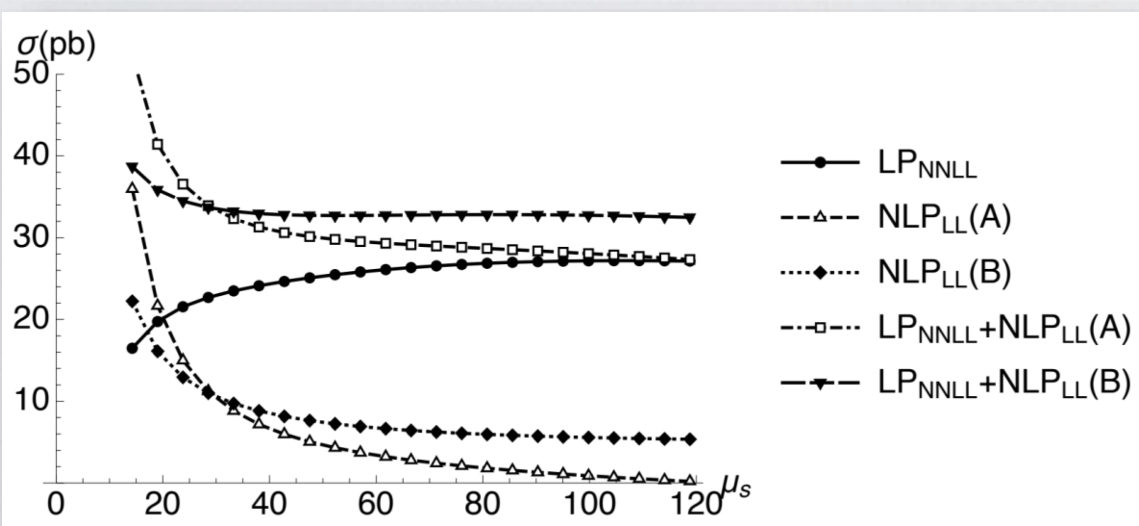
Studies in:* *Moult, Stewart, Vita, Zhu, 2019;
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020;
Liu, Mecaj, Neubert, Wang, Fleming, 2019, 2020;

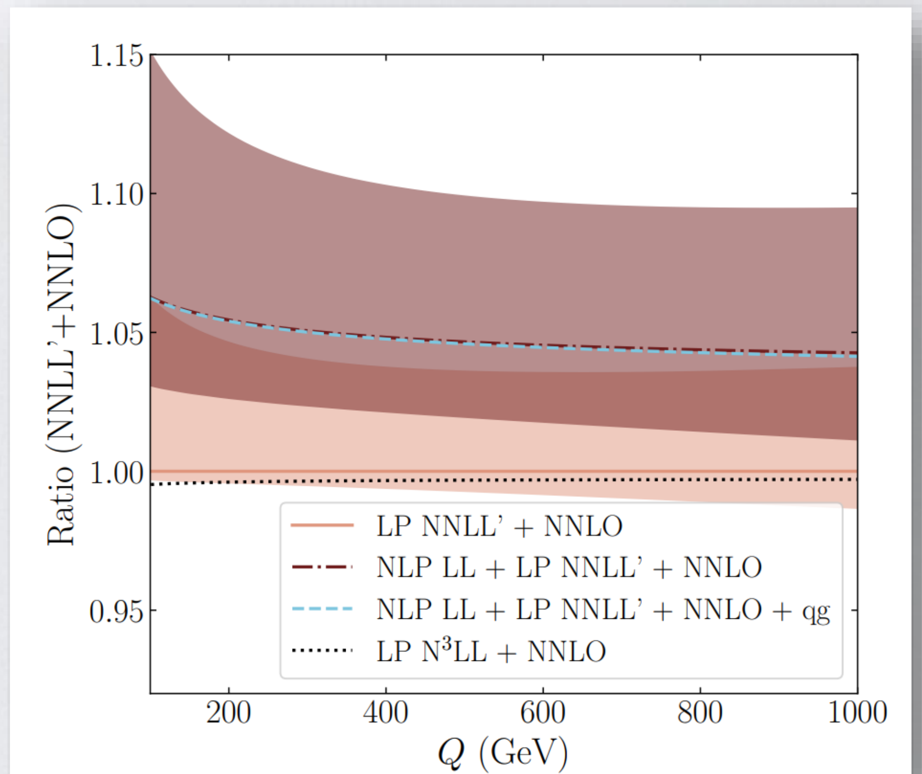
LL RESUMMATION AT NLP

- For leading channels like $q\bar{q}$ in Drell-Yan or gg in Higgs production, it turns out that the collinear function contributes only starting at NLL accuracy.
- This means that at LL accuracy only the hard and soft functions contribute. The divergent contribution problem can be easily overcome, and LLs can be resummed.

SCET: **Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;**
Diagrammatic: ***Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019***

Phenomenological analysis in: **Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019;**
van Beekveld, Laenen, Sinninghe Damsté, LV, 2021.



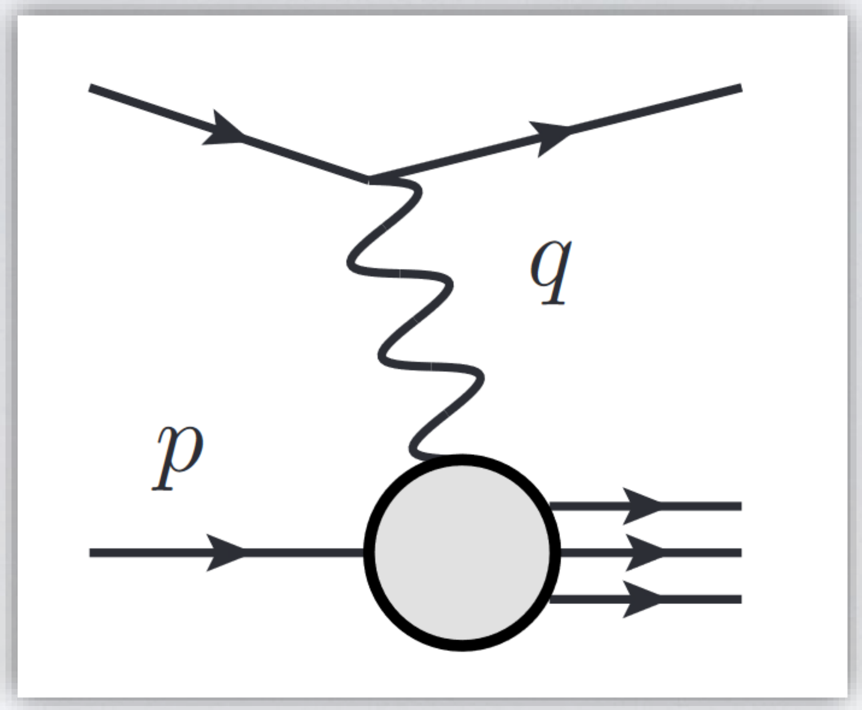


ENDPOINT DIVERGENCES AND RESUMMATION

Addressing the problem of endpoint divergences in the leading channels is not an easy task: one has to deal with several operators/contributions.

- Things are different for subleading channels (e.g. *qg* in Drell-Yan): they start at NLP, and LLs are generated by all momentum regions. Better place to start investigating endpoint divergences.

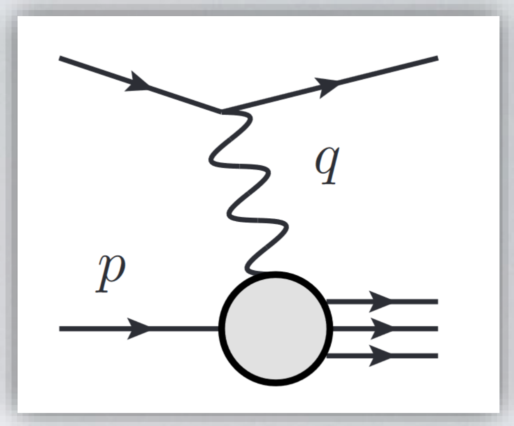
THRESHOLD RESUMMATION AT NLP IN OFF-DIAGONAL CHANNELS: DEEP INELASTIC SCATTERING



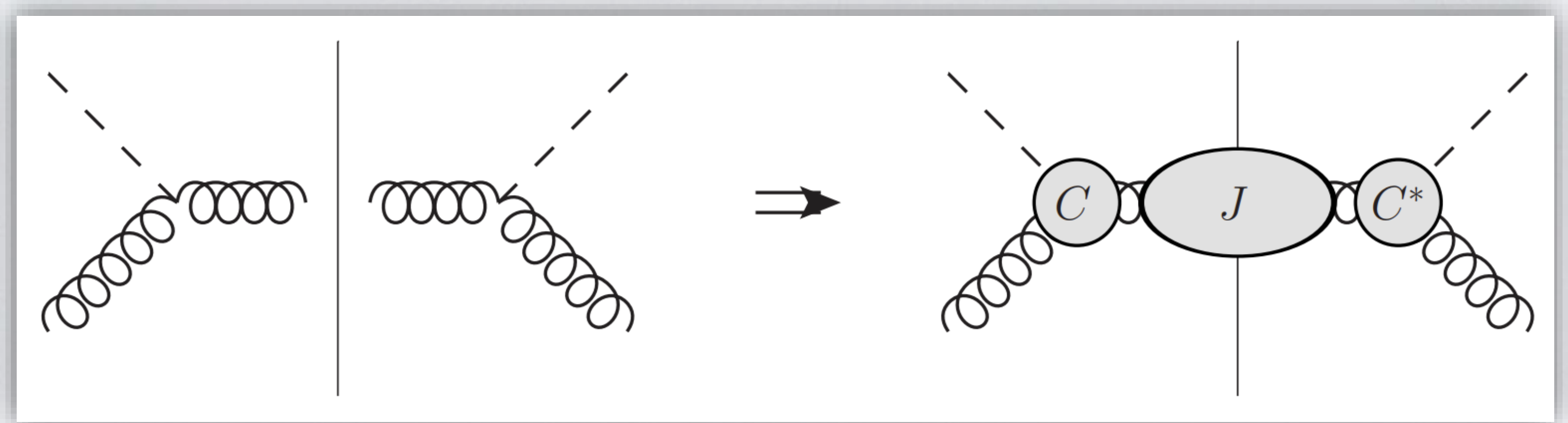
DEEP INELASTIC SCATTERING

- Deep inelastic scattering (DIS) near threshold develops a hierarchy of scales:

$$Q^2 \gg P_X^2 \sim Q^2(1-x), \qquad \text{with} \qquad x \equiv \frac{Q^2}{2p\cdot q} \to 1.$$



- Factorization and resummation well understood at LP:



Sterman 1987; Catani, Trentadue 1989; Korchemsky, Marchesini, 1993; Moch, Vermaseren, Vogt 2005; Becher, Neubert, Pecjak, 2007

$$W_\phi = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\,\langle N(P)|\big[G^A_{\mu\nu}G^{\mu\nu A}\big](x)\big[G^B_{\rho\sigma}G^{\rho\sigma B}\big](0)|N(P)\rangle$$

$$= |C(Q^2,\mu)|^2\int_x^1 \frac{d\xi}{\xi}\, J\Big(Q^2\frac{1-\xi}{\xi},\mu\Big)\,\frac{x}{\xi} f_g\Big(\frac{x}{\xi},\mu\Big).$$

Short-distance coefficient and jet function are single scale object – resummation obtained by solving the corresponding RGE.

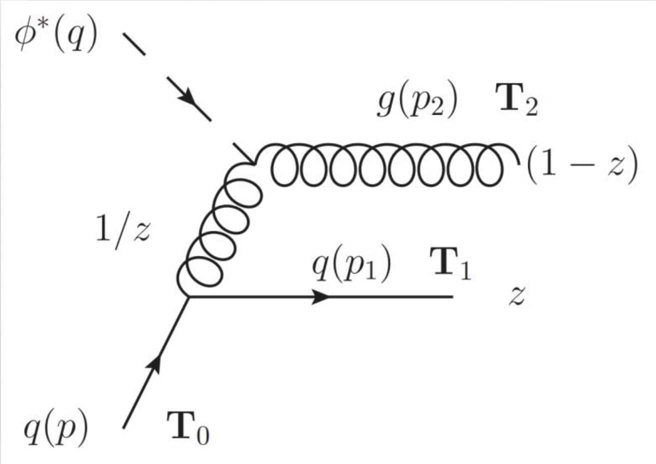
DIS: OFF-DIAGONAL CHANNEL

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

- The off-diagonal channel $q(p)+\phi^*(q)\to X(p_X)$ contributes to DIS at NLP. Consider the partonic structure function

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz\,\left(\frac{\mu^2}{s_{qg}z\bar{z}}\right)^{\epsilon}\mathcal{P}_{qg}(s_{qg},z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}, \qquad \mathcal{P}_{qg}(s_{qg},z)\equiv \frac{e^{\gamma_E\epsilon}Q^2}{16\pi^2\Gamma(1-\epsilon)}\frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2}.$$

with momentum fraction $z\equiv \dfrac{n_-p_1}{n_-p_1+n_-p_2}$, and $\bar{z}=1-z$.



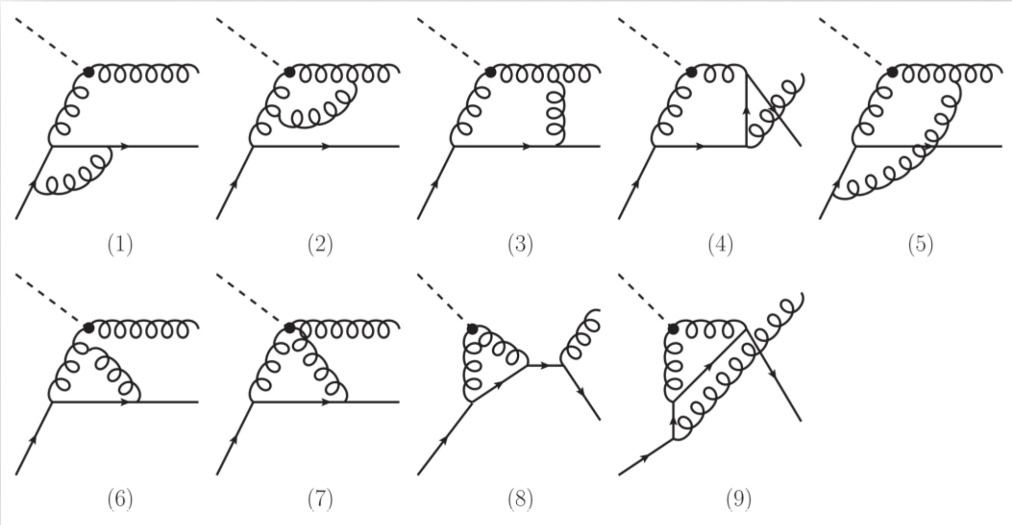
- At LO one has

$$\mathcal{P}_{qg}(s_{qg})|_{\rm tree} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z}, \quad\Rightarrow\quad W_{\phi,q}\Big|^{\rm NLP}_{\mathcal{O}(\alpha_s),\,\text{leading pole}} = -\frac{1}{\epsilon}\frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{Q^2(1-x)}\right)^{\epsilon}.$$

The single pole originate from $z\to 0$, due to the $1/z$ of the momentum distribution function.

- At NLO:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\rm loop} = \mathcal{P}_{qg}(s_{qg},z)|_{\rm tree}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}$$
$$\cdot\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}+\mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left.+\,\mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}-\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}+\left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right)+\mathcal{O}(\epsilon^{-1}).$$



ON THE ENDPOINT DIVERGENCES

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

- The **T1.T2** term contains a single pole, but: promoted to leading pole after integration!
- Compare exact integration:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3},$$

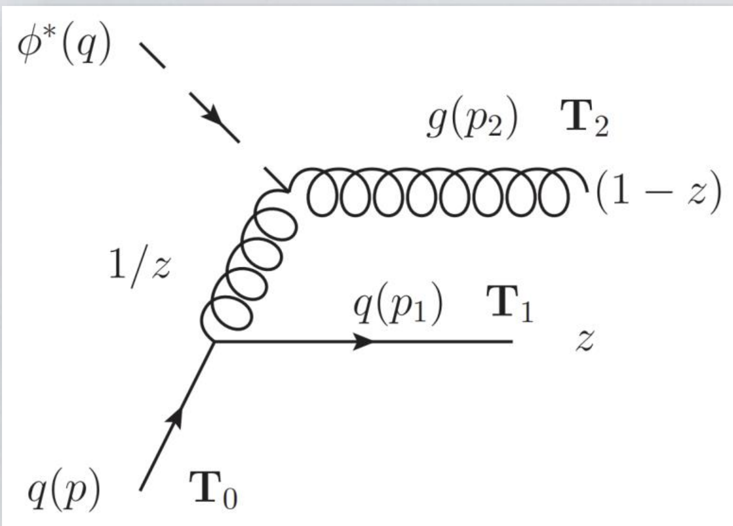
vs integration after expansion:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots\,.$$

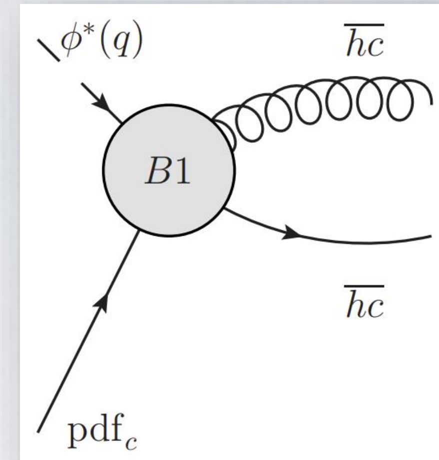
- Expansion in ε not possible before integration!
- The pole associated to **T1.T2** does not originate from the standard cups anomalous dimension.

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

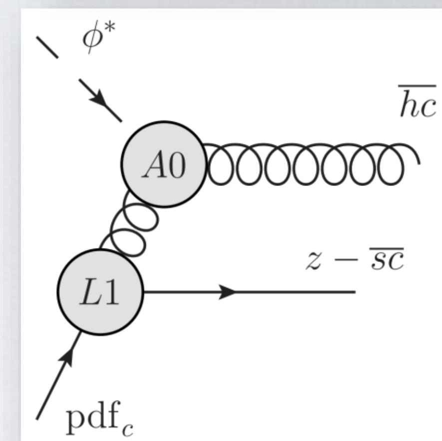
- What happens for $z \to 0$?



For $z \sim 1$ intermediate propagator is hard



For $z \ll 1$ intermediate propagator cannot be integrated out



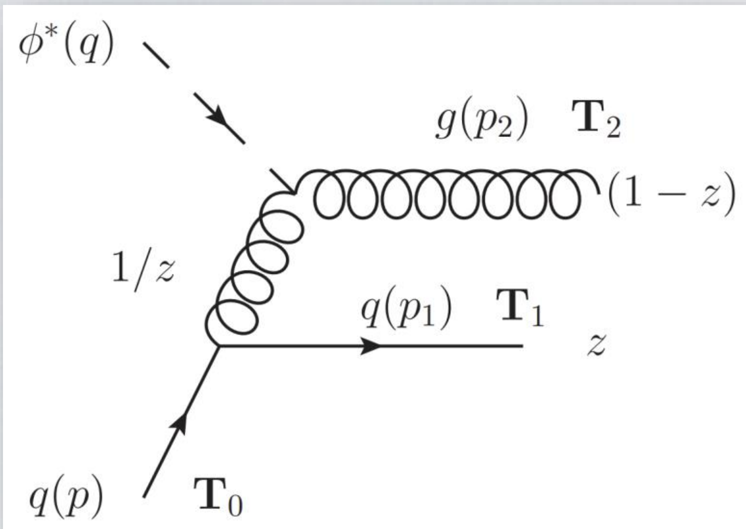
- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: need "z-SCET".
- z-modes are non-physical! Not related to external scales of the problem.

Name	$(n_+l\,,\, l_\perp\,,\, n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

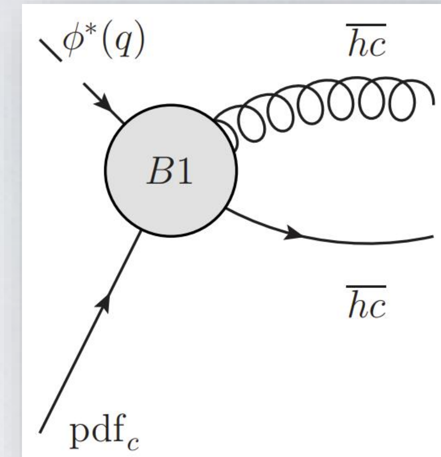
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

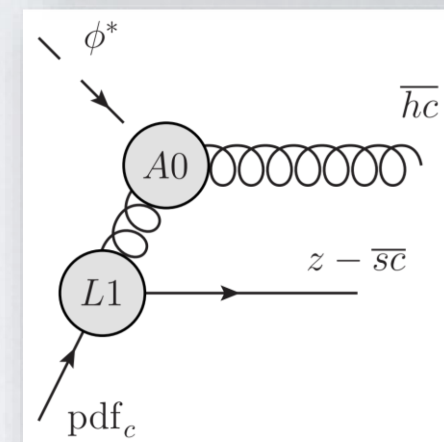
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



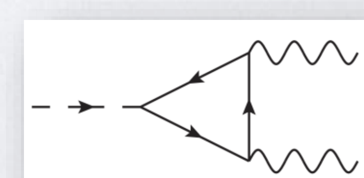
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: need "z-SCET".
- z-modes are non-physical! Not related to external scales of the problem.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}}\, J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2)\int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2) D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}}\, J^{B1}_{z-\overline{sc}}.$$

- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



DIS FACTORIZATION

- Re-factorization is nontrivial: needs to be embedded in a complete EFT description of DIS:
- Physical modes:

"z-SCET" is here

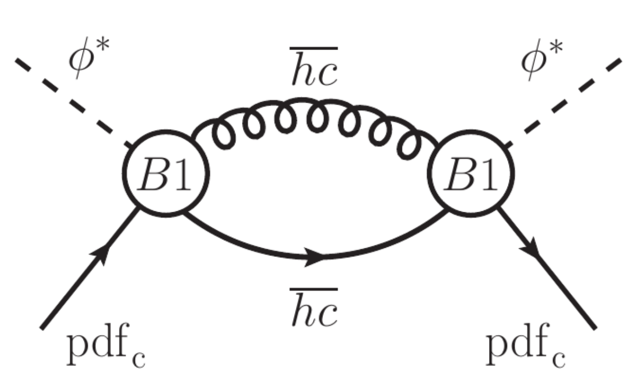
Perturbative modes

Hard:	$p^2 = Q^2,$	
Hard-collinear:	$p^2 = Q^2\lambda^2 = Q^2/N,$	
Collinear:	$p^2 = \Lambda^2,$	
Soft-collinear:	$p^2 = \Lambda^2\lambda^2 = \Lambda^2/N.$	

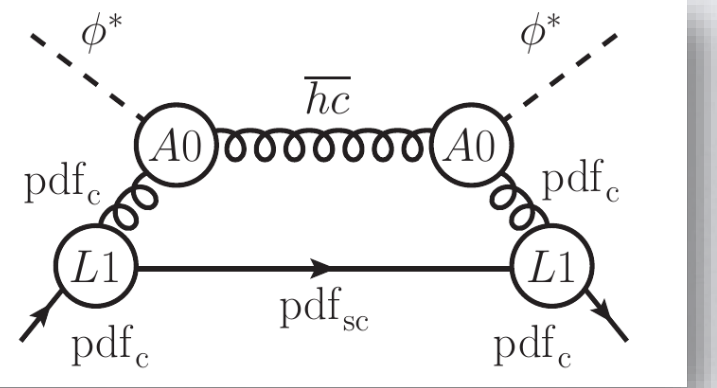
Non-perturbative modes

$$g(N) \equiv \int_0^1 dx\, x^{N-1}\, g(x),$$

$$x \to 1 \;\Leftrightarrow\; N \to \infty.$$



Time-ordered product contribution



B-type current contribution

- Both terms contain endpoint divergences in the convolution integral.
- We could reshuffle factorization theorem;
  → however, use d-dimensional consistency conditions to start with. ⟶ ***Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020***

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Hadronic structure function is finite:

$$W = \sum_i W_{\phi,i}\, f_i = \sum_i \tilde{C}_{\phi,k} \tilde{f}_k\,, \qquad \text{with} \qquad \tilde{f}_k = Z_{ki} f_i, \quad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki}.$$

- Focus on the bare functions: at LP a single channel contribute:

$$\sum_i (W_{\phi,i} f_i)^{LP} = W_{\phi,g}^{LP} f_g^{LP}.$$

- Work in d-dimensions: ε regularizes endpoint divergences in convolution integrals. The general expansion of the cross section reads

$$W_{\phi,g}\, f_g = f_g(\Lambda) \times \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n}} \sum_{k=0}^{n} \sum_{j=0}^{n} b_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^{\epsilon} + \mathcal{O}\left(\frac{1}{N}\right).$$

- In this equation:

Each hard loop gives $\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}$, each hard-collinear loop gives $\left(\frac{\mu^2}{Q^2} N\right)^{\epsilon}$,

Each collinear loop gives $\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}$, each soft-collinear loop gives $\left(\frac{\mu^2}{\Lambda^2} N\right)^{\epsilon}$.

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Invoking pole cancellations one has the obvious condition:

$$\sum_{k=0}^{n}\sum_{j=0}^{n} b_{kj}^{(n)} = 0\,,$$

  however, also all poles of the type

$$(\ln N)^r \left(\ln\frac{\Lambda}{Q}\right)^s \times \frac{1}{\epsilon^{2n-r-s}}\,.$$

  need to cancel. In the end one has the conditions

$$\sum_{k=0}^{n}\sum_{j=0}^{n} j^r\, k^s\, b_{kj}^{(n)} = 0 \qquad \text{for } s+r<2n,\ r,s\geq 0\,.$$

- At order n this gives $n^2 + 2n$ equations, for $(n+1)^2$ coefficients $b_{kj}^{(n)}$: the system can be solved in terms of n unknown coefficient, one per order n.
- Let us consider the n-loop hard coefficients $b_{n0}^{(n)}$: assuming exponentiation of the one-loop hard coefficient $b_{10}^{(1)}$ one has

$$W_{\phi,g}^{LP,LL}\Big|_{\text{hard loops}} = \exp\left[-\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\right], \qquad \Rightarrow \qquad b_{n0}^{(n)} = (-4C_A)^n\,.$$

- This provides the single condition required at LP to fix all of the $b_{kj}^{(n)}$:

$$(W_{\phi,g}\, f_g)^{LP,LL} = \exp\left[\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left\{\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}\right\}(N^{\epsilon}-1)\right] f_g(\Lambda)\,.$$

RESUMMATION AT LP

- The exponentiation of the one loop correction is actually well known. In SCET hard modes are given as a short-distance coefficient of the LP operator

$$J^{A0} = 2g_{\mu\nu}\, n_-\partial\mathcal{A}^{\mu A}_{\perp hc}(sn_-)n_+\partial\mathcal{A}^{\nu A}_{\perp c}(tn_+)\,,$$

See for instance Becher, Neubert, Pecjak, 2007 for Photon DIS

whose anomalous dimension reads

$$\Gamma(\alpha_s,\mu) \equiv -\frac{dZ}{d\ln\mu}Z^{-1} = \Gamma_{\rm cusp}(\alpha_s)\ln\frac{Q^2}{\mu^2} + \gamma(\alpha_s)\,, \qquad \text{with} \qquad \frac{d\alpha_s}{d\ln\mu} \equiv -2\epsilon\alpha_s + \beta(\alpha_s)\,.$$

- The bare coefficient function is given by

$$C^{A0}_{\rm bare} = Z(\mu)C^{A0}(\mu)\,, \quad \text{with} \quad \ln Z(\mu) = \int_0^{\alpha_s(\mu)}\frac{d\alpha}{\alpha}\frac{1}{2\epsilon-\beta(\alpha)/\alpha}\left(\Gamma(\alpha,\mu) - \int_0^{\alpha}\frac{d\alpha'}{\alpha'}\frac{2\Gamma_{\rm cusp}(\alpha')}{2\epsilon-\beta(\alpha')/\alpha'}\right)\,.$$

- At LL accuracy

$$\Gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s C_A}{\pi}\,, \qquad \gamma(\alpha_s) = 0\,, \qquad \beta(\alpha_s) = 0\,,$$

- Which gives

$$W^{LP,LL}_{\phi,g}\Big|_{\rm hard\ loops} = |C^{A0}_{\rm bare}|^2_{LL} = \exp\left(2\ln Z^{LL}(Q)\right) = \exp\left[-\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\right]\,.$$

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Consider now the expansion to NLP:

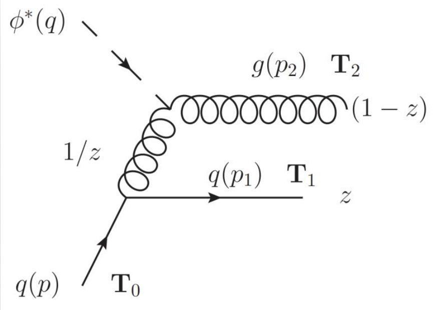
$$\sum_i (W_{\phi,i} f_i)^{NLP} = W_{\phi,q}^{NLP} f_q^{LP} + W_{\phi,\bar{q}}^{NLP} f_{\bar{q}}^{LP} + W_{\phi,g}^{NLP} f_g^{LP} + W_{\phi,g}^{LP} f_g^{NLP} .$$

- The general expansion of the cross section reads.

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^{\epsilon} + f_{\bar{q}}(\Lambda),\ f_g(\Lambda)\ \text{terms}\,.$$

- Invoking pole cancellations:

$$\sum_{k=0}^{n} \sum_{j=0}^{n} j^r k^s c_{kj}^{(n)} = 0 \qquad \text{for } s + r < 2n - 1,\ r, s \geq 0\,,$$



  allows $(n+1)^2$ coefficients $c_{kj}^{(n)}$ to be determined from $2n^2$-n equations.

- Three unknowns: these can be reduced using information from the region expansion:
  - $c_{n0}^{(n)} = 0$ (final state cannot be purely hard)
  - $c_{00}^{(n)} = 0$ (without any hard or anti-hardcollinear loops there must be at least one soft-collinear loop).
  - Non-trivial condition is given by $c_{n1}^{(n)}$: this is the n-loop hard region!

D-DIMENSIONAL CONSISTENCY CONDITIONS

Similar conjecture "soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019.

- Assume exponentiation of 1-loop result:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\boxed{\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

- Restricting to the hard region and substituting color operators one has

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A-C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

- With $f_i(\mu) = U_{ij}(\mu) f_j(\Lambda)$ one has

$$\sum_i (W_{\phi,i} f_i)^{NLP}\Big|_{\propto f_q(\Lambda)} = \left(W_{\phi,q}^{NLP} U_{qq}^{LP} + W_{\phi,g}^{LP} U_{gq}^{NLP}\right) f_q(\Lambda)\,.$$

- Inserting the result above in the end one has

$$W_{\phi,q}^{NLP,LP} = -\frac{1}{2N}\frac{C_F}{C_F-C_A}\frac{\epsilon N^{\epsilon}}{N^{\epsilon}-1}\left(\exp\left[\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^{\epsilon}-1)\right] - \exp\left[\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^{\epsilon}-1)\right]\right),$$

$$U_{gq}^{NLP,LP} = -\frac{1}{2N}\frac{C_F}{C_F-C_A}\frac{\epsilon N^{\epsilon}}{N^{\epsilon}-1}\left(\exp\left[-\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}(N^{\epsilon}-1)\right] - \exp\left[-\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}(N^{\epsilon}-1)\right]\right).$$

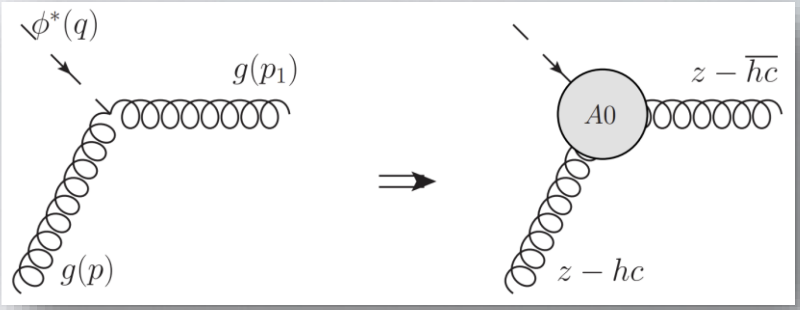
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

RESUMMATION FROM RE-FACTORIZATION: A GLIMPSE

- Is it possible to achieve this in SCET? Another look at re-factorization:

$$C^{B1}(Q,z)J^{B1}(z) \stackrel{z\to 0}{\longrightarrow} C^{A0}(Q^2)\int d^4x\,\mathbf{T}\Big[J^{A0},\mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = C^{A0}(Q^2)D^{B1}(zQ^2,\mu^2)J^{B1}_{z-\overline{sc}}.$$

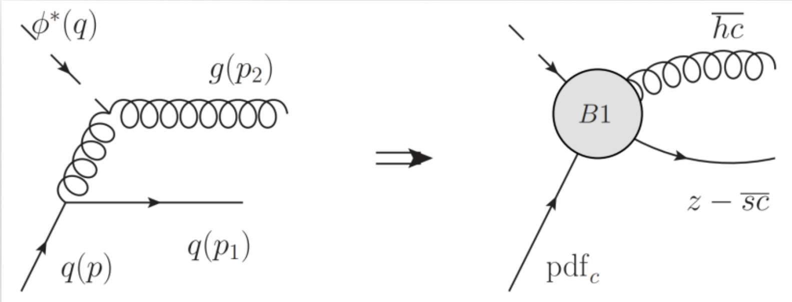
- Integrate out hard modes (solve RGEs in d-dimensions)



$$\frac{d}{d\ln\mu}C^{A0}(Q^2,\mu^2) = \frac{\alpha_s C_A}{\pi}\ln\frac{Q^2}{\mu^2}\,C^{A0}(Q^2,\mu^2)\,.$$

$$\Rightarrow \qquad \left[C^{A0}\left(Q^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right].$$

- Integrate out z-hardcollinear modes



$$\frac{d}{d\ln\mu}D^{B1}\left(zQ^2,\mu^2\right) = \frac{\alpha_s}{\pi}\left(C_F-C_A\right)\ln\frac{zQ^2}{\mu^2}\,D^{B1}\left(zQ^2,\mu^2\right).$$

$$\Rightarrow \qquad \left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right].$$

- This reproduces

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}+\left(C_A-C_F\right)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

OFF-DIAGONAL DIS: FINITE STRUCTURE FUNCTION

- Integrating P_{qg} we get W:

$$W_{\phi,q}\Big|_{q\phi^*\to qg} = \int_0^1 dz\left(\frac{\mu^2}{s_{qg}z\bar{z}}\right)^{\epsilon}\mathcal{P}_{qg}(s_{qg},z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}},$$

- Furthermore, recall

$$W = \sum_i W_{\phi,i}f_i = \sum_k \tilde{C}_{\phi,k}\tilde{f}_k, \quad \Rightarrow \quad W^{NLP}_{\phi,q} = \tilde{C}^{NLP}_{\phi,q}Z^{LP}_{qq} + \tilde{C}^{LP}_{\phi,g}Z^{NLP}_{gq}.$$

- We obtain a solution for $\tilde{C}$:

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

$$\tilde{C}^{NLP,LL}_{\phi,q}\Big|_{\epsilon\to 0} = \frac{1}{2N\ln N}\frac{C_F}{C_F-C_A}\Bigg(\mathcal{B}_0(a)\exp\left[C_A\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln^2 N+\ln N\ln\frac{\mu^2}{Q^2}\right)\right]$$
$$-\exp\left[\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2 N+\ln N\ln\frac{\mu^2}{Q^2}\right)\right]\Bigg), \qquad \text{with} \qquad a = \frac{\alpha_s}{\pi}(C_F-C_A)\ln^2 N,$$

and

$$P_{ij} = -\gamma_{ij} = \frac{dZ_{ik}}{d\ln\mu}(Z^{-1})_{kj}, \qquad \gamma^{NLP,LL}_{gq}(N) = -\frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathcal{B}_0(a), \qquad \mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n,$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, …

- Reproduces earlier conjecture by *Vogt, 2010*.

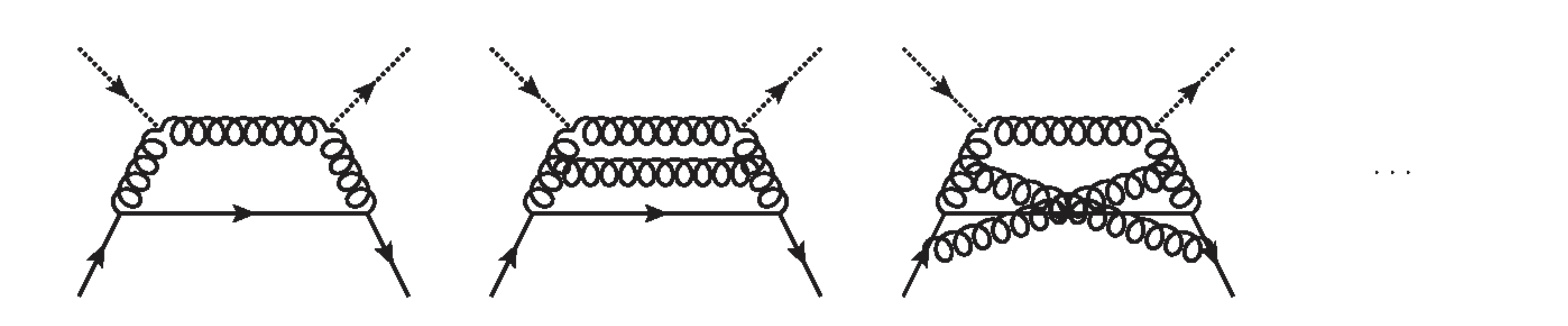
OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

- Resummation has been obtained assuming the exponentiation of the 1-real 1 virtual-hard contribution to the partonic structure function. The exponentiation can be phrased in the context of refactorization in SCET.

- Resummation of the whole structure function can be achieved starting from the exponentation of a single region, because of consistency conditions for pole cancellation.

- Exponentation of the hard region fixes the tower of coefficients $c_{n1}^{(n)}$.

- Determining another tower of coefficients should allow one to achieve resummation as well. The tower of all-real soft emissions $c_{nn}^{(n)}$ is particularly suitable for a diagrammatic approach.



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OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

- The tower of soft real emissions can be determined based on the following considerations:
- In a physical polarization gauge in which

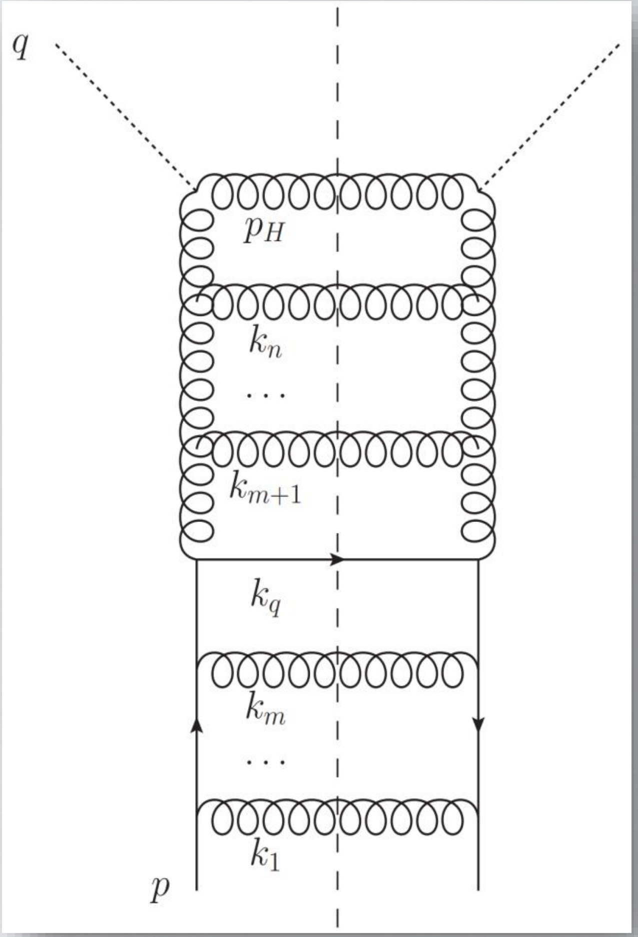
$$\sum_{\text{pols.}} \epsilon^{\dagger}_{\mu}(k)\epsilon_{\nu}(k) = -\eta_{\mu\nu} + \frac{k_\mu c_\nu + k_\nu c_\mu}{c\cdot k}\,, \qquad c = q + xp.$$

Gribov, Lipatov, 1972; Dokshitzer, Diakonov, Troian, 1980; Dokshitzer, Khoze, Mueller, Troian, 1991.

only ladder diagrams contribute to the LLs.

- The power suppression is given by the soft quark polarization sum; gluon emissions are eikonal (LP):

$$\overline{|\mathcal{M}_{qh\to qg_1\ldots g_n}|^2} = \frac{|\lambda|^2 C_F^{m+1} C_A^{n-m} g_s^{2(n+1)}}{8\mu^{(d-4)(n+1)}} \left(\prod_{i=1}^{n} \frac{2q\cdot p\, p\cdot k_i}{q'\cdot k_i}\right) \mathrm{Tr}[\not{p}\gamma^\beta \not{k}_q \gamma^\alpha] \left(-\eta_{\alpha\beta} + \frac{q'_\alpha p_{H,\beta} + q'_\beta p_{H,\alpha}}{q'\cdot p_H}\right)$$
$$\times \frac{1}{(p\cdot k_1)^2 [p\cdot(k_1+k_2)]^2 \ldots [p\cdot(k_1+\ldots+k_m+k_q)]^2 \ldots [p\cdot(k_1+\ldots+k_n+k_q)]^2}.$$



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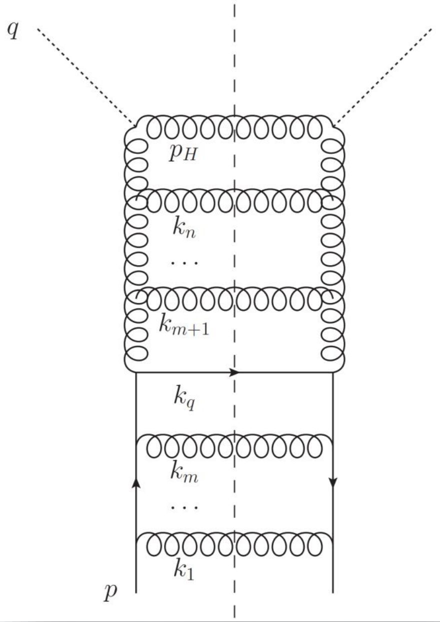
- Phase space can be also approximated to LP, and factorizes in Laplace space:

$$\int \mathrm{d}\Phi^{(n+2)} = \frac{2\pi}{(4\pi)^{\frac{(n+1)d}{2}}} (Q^2)^{n+(n+1)\frac{(d-4)}{2}} \frac{x^{-(n+1)\frac{(d-4)}{2}-n}(1-x)^{-(n+1)\frac{d-2}{2}}}{\Gamma\left(\frac{d-2}{2}\right)^{n+1}} \int_{-i\infty}^{i\infty} \frac{\mathrm{d}T}{2\pi i}\, e^{T(1-x)}$$
$$\times \int \mathrm{d}\alpha_q\, \mathrm{d}\beta_q\, (\alpha_q\,\beta_q)^{\frac{d-4}{2}}\, e^{-T(\alpha_q+\beta_q)} \left[\prod_{i=1}^{n} \int \mathrm{d}\bar{\alpha}_i\, \mathrm{d}\bar{\beta}_i\, (\bar{\alpha}_i\,\bar{\beta}_i)^{\frac{d-4}{2}}\, e^{-T(\bar{\alpha}_i+\bar{\beta}_i)}\right], \quad \bar{\alpha}_i \simeq \frac{q'\cdot k}{p\cdot q}, \quad \bar{\beta}_i \simeq \frac{p\cdot k}{p\cdot q}.$$

OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY



- In Mellin space one obtains

$$W_{\phi,q}^{(n+1)} = -\left(\sum_{m=0}^{n} C_F^{m+1} C_A^{n-m}\right) \frac{2}{\epsilon}\frac{N^\epsilon}{N}\left(\frac{4N^\epsilon}{\epsilon^2}\right)^n \frac{1}{(n+1)!},$$

which sums to

$$W_{\phi,q}\Big|_{\mathrm{LL}} = \sum_{n=1}^{\infty} a_s^n W_{\phi,q}^{(n)} = -\frac{2a_s C_F}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_F - C_A}\left(\frac{4a_s N^\epsilon}{\epsilon^2}\right)^{-1}\left\{\exp\left[\frac{4a_s C_F N^\epsilon}{\epsilon^2}\right] - \exp\left[\frac{4a_s C_A N^\epsilon}{\epsilon^2}\right]\right\}.$$

- The full result is found requiring that virtual corrections modify the real emission contributions at each order, removing singularities which are simultaneously soft and collinear: this leads to the parametrization

$$W_{\phi,q}\Big|_{\mathrm{LL}} = -\frac{2a_s C_F}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_F - C_A}\left(\frac{4a_s (N^\epsilon + \lambda_1)}{\epsilon^2}\right)^{-1} \exp\left[\frac{4a_s(\lambda_2 C_F + \lambda_3 C_A)}{\epsilon^2}\right]$$
$$\times\left\{\exp\left[\frac{4a_s C_F (N^\epsilon + \lambda_4)}{\epsilon^2}\right] - \exp\left[\frac{4a_s C_A (N^\epsilon + \lambda_5)}{\epsilon^2}\right]\right\}.$$

which can be easily fixed, given that the system is overconstrained. We find

$$W_{\phi,q}\Big|_{\mathrm{LL}} = -\frac{2a_s C_F}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_F - C_A}\left(\frac{4a_s (N^\epsilon - 1)}{\epsilon^2}\right)^{-1}\left\{\exp\left[\frac{4a_s C_F (N^\epsilon - 1)}{\epsilon^2}\right] - \exp\left[\frac{4a_s C_A (N^\epsilon - 1)}{\epsilon^2}\right]\right\},$$

which reproduces the previous result.

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OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

- The same procedures can be easily adapted to the subleading qg channel in Drell-Yan (and Higgs production).
- Consistency conditions can be studied to determine the smallest set of parameters necessary to determine the whole partonic cross section;
- The set of parameters can be determined
  - by assuming exponentiation of a given region, justified within a refactorization approach.
  - by direct calculation of the ladder diagrams contributing to the real emission.
- Either way, one in the end reproduces an earlier conjecture in Lo Presti, Almasy, Vogt 2014:

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang, 2020 (unpublished)

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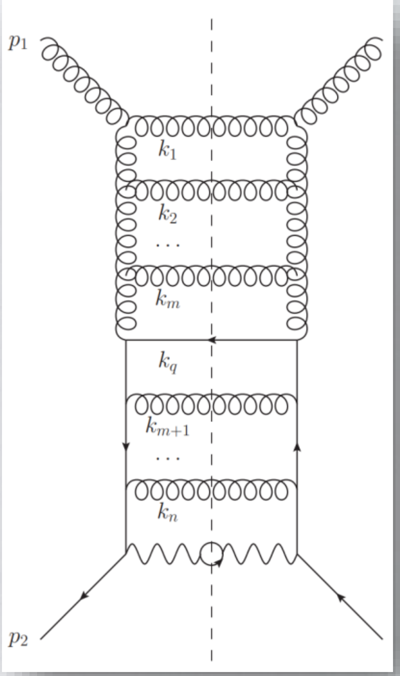


$$W_{\mathrm{DY},g\bar{q}}\Big|_{\mathrm{LL}} = -\frac{T_R}{2(C_F-C_A)}\frac{1}{N}\frac{\epsilon(N^{\epsilon-1})}{N^\epsilon-1}\exp\left[\frac{4a_sC_F(N^\epsilon-1)}{\epsilon^2}\right] \times\left\{\exp\left[\frac{4a_sC_FN^\epsilon(N^\epsilon-1)}{\epsilon^2}\right]-\exp\left[\frac{4a_sC_AN^\epsilon(N^\epsilon-1)}{\epsilon^2}\right]\right\},$$

$$\widetilde{C}_{\mathrm{DY},g\bar{q}}\Big|_{\mathrm{LL}} = \frac{T_R}{C_A-C_F}\frac{1}{2N\ln N}\left[e^{8C_Fa_s\ln^2N}\mathcal{B}_0[4a_s(C_A-C_F)\ln^2N]-e^{(2C_F+6C_A)a_s\ln^2N}\right].$$

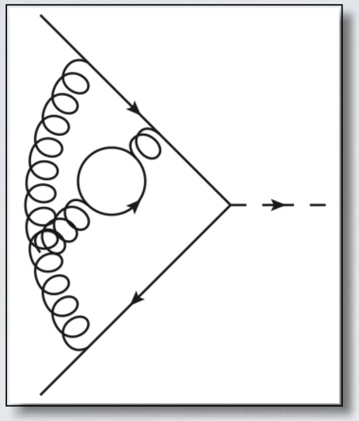
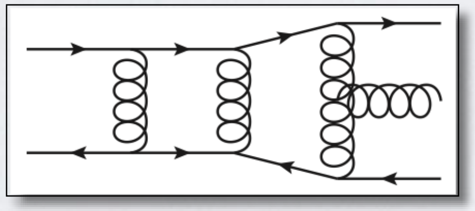
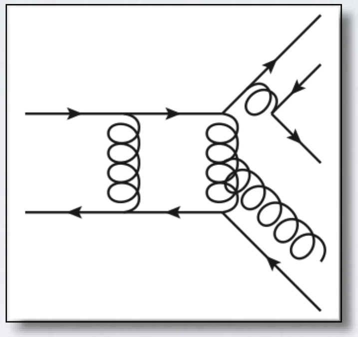
CONCLUSION

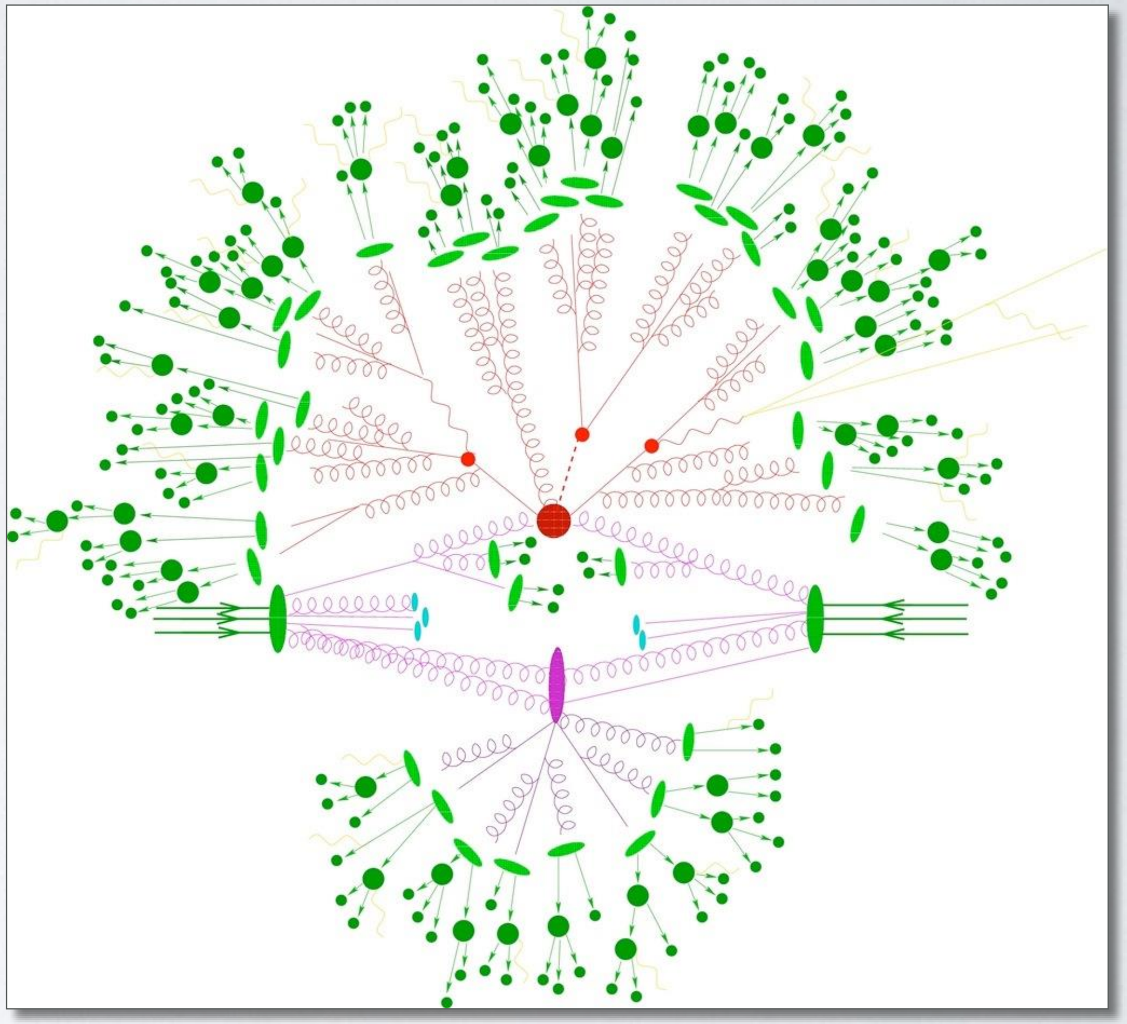
- The general structure of factorization theorems describing soft radiation beyond LP is now understood.
- However, it involves divergences in convolution integrals: new “internal” modes appear due to endpoint divergences. Rigorous factorization and resummation still possible, but highly non-trivial.
- Relationships between bare and renormalized objects need to be better understood.
- We provided an explicit example (off-diagonal DIS qg channel) where these problems can be studied, obtaining new insight.
- To do:
  $\rightarrow$ derive a refactorization framework for resummation near threshold in SCET;
  $\rightarrow$ extend the diagrammatic approach beyond LLs.
- More in general:
  $\rightarrow$ phenomenological analysis of relevant processes for the LHC;
  $\rightarrow$ extend resummation at NLP to other kinematic limits (small pT, small β, etc).

EXTRA

PRECISION FOR COLLIDER PHYSICS

- Hard scattering processes are calculated in perturbation theory.



- Going beyond NNLO and N3LO is difficult, yet necessary to match the precision of current and forthcoming experiments!
- Loop and phase space integrals:
  - Analytic vs numerical evaluation
  - Space of functions
  - Infrared divergences
  - Large logarithms

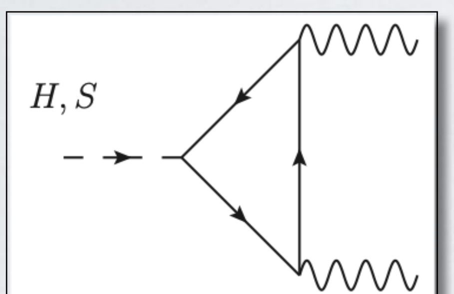
$\Big\}$ ***High-energy limit***

PRECISION FOR COLLIDER PHYSICS

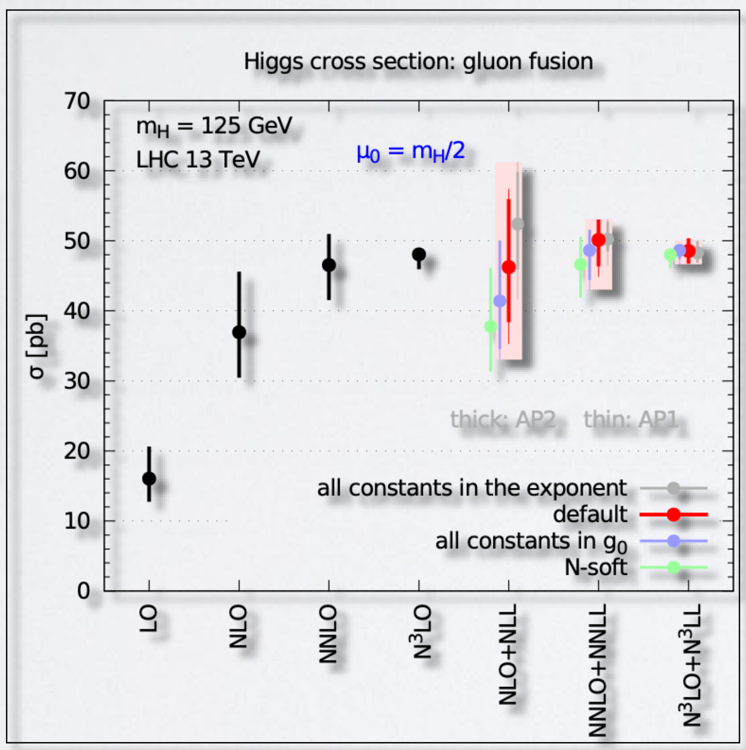
- The presence of largely different scales gives rise to large logarithms:

$$d\sigma \sim 1 + \alpha_s(L+1) + \alpha_s^2(L^2+L+1) + \ldots$$

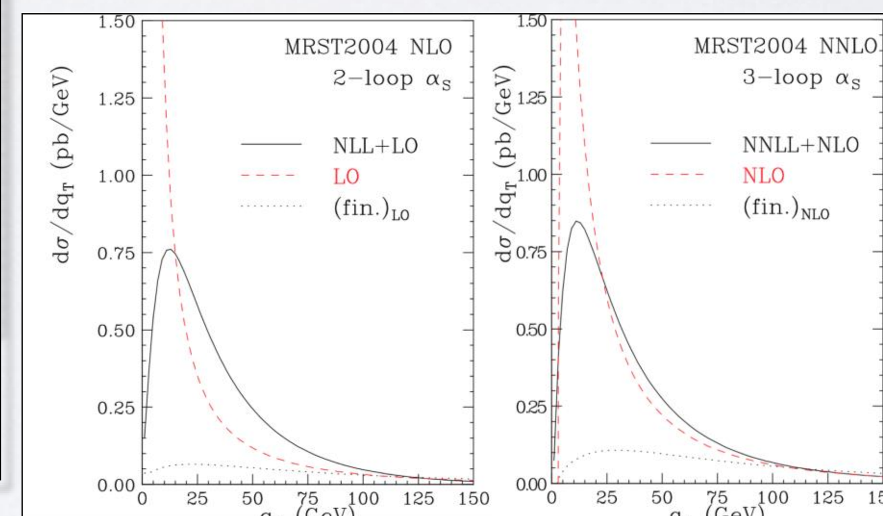
$$\sim \log^2 \frac{m_H^2}{m_b^2}$$



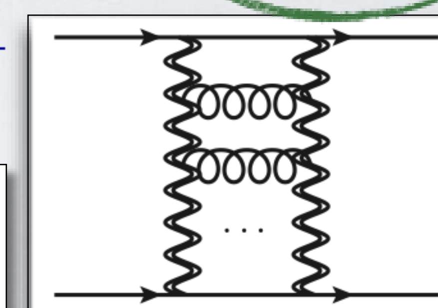
$$\sim \log^2(1-z)$$



$$\sim \log \frac{m_H^2}{p_T^2}$$



$$\sim \log \frac{s}{-t}$$



- Studies of scattering amplitudes to high-perturbative order, bootstrap approach for infrared divergences, phenomenology.

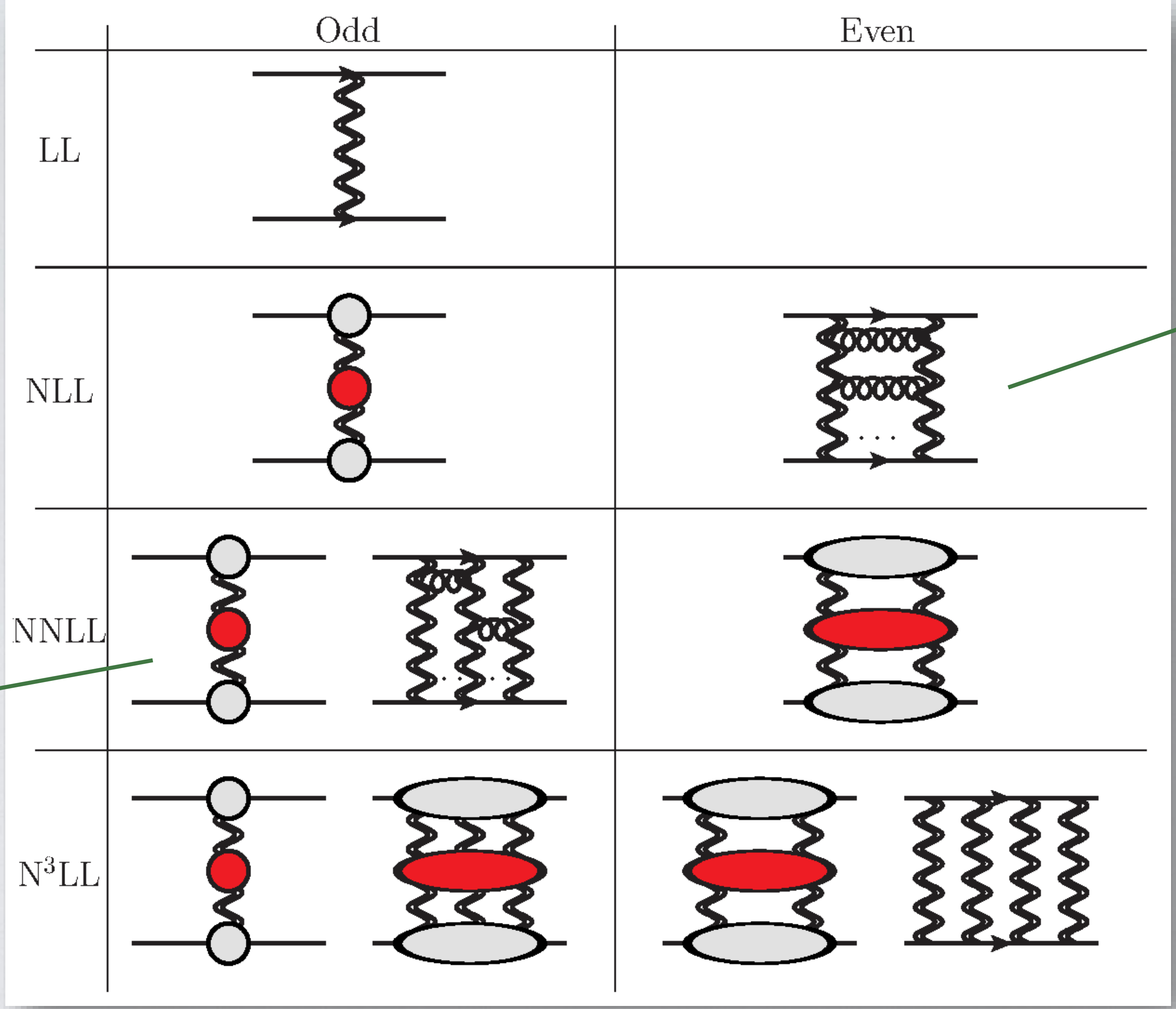
HIGH-ENERGY LIMIT

Developed a framework for the calculation of amplitudes in the high-energy limit, by solving iteratively rapidity evolution equation (Baliktsky-JIMWLK).

Analysed to 2 loops in Del Duca, Falcioni, Magnea, Vernazza 2014;

Calculated to 3 loops in Caron-Huot, Gardi, LV, 2017;

Calculated to 4 loops in Falcioni, Gardi, Milloy, LV, 2020; Falcioni, Gardi, Maher, Milloy, LV, 2021.



IR rivergences calculated to all orders in Caron-Huot, Gardi, Reichel, LV, 2017;

Finite terms calculated to 13 loops in Caron-Huot, Gardi, Reichel, LV, 2020.