Jefferson Lab

Universität Wien
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New approach to (semi) inclusive DIS with QED & QCD factorization

Wally Melnitchouk
— Jefferson Lab —

collaborators: Tianbo Liu (Shandong U)
Jianwei Qiu (JLab)
Nobuo Sato (JLab)

arXiv:2008.02895, PRD (2021)
arXiv:2108.13371, JHEP (2021)

Outline

- Lepton-hadron scattering and hadron structure
  - → main source of information about quark/gluon (parton) structure of hadrons, via QCD factorization theorems
  - → leptons radiate photons, so QED radiation effects may complicate "Born" level interpretations
  - → new factorized approach treats QED+QCD radiation on equal footing

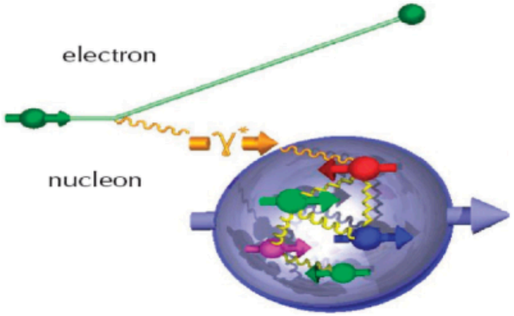
- Inclusive DIS
  - → radiative contributions *vs.* "radiative corrections"

- Semi-inclusive DIS
  - → no unique photon-hadron frame for simple TMD interpretation
    - → mixing of spin asymmetries

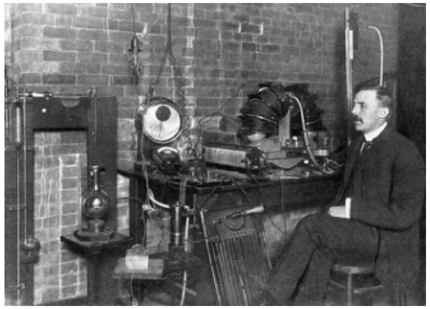
- Outlook

Lepton-hadron scattering

The new generation of “Rutherford” experiments for probing hadron structure:

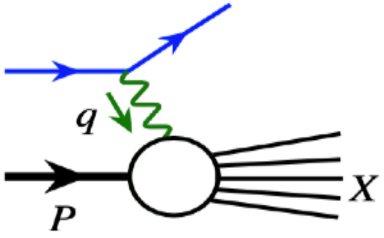


- ✧ **A controlled “probe” – virtual photon**



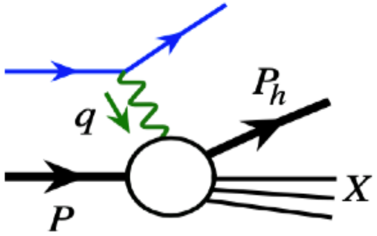
- ✧ **Inclusive events**: e+p/A → e’+X

  Detect only the scattered lepton in the detector



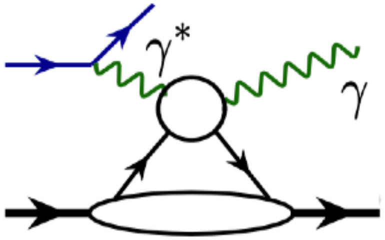
- ✧ **Semi-Inclusive events**: e+p/A → e’+h(p,K,p,jet)+X

  Detect the scattered lepton in coincidence with identified hadrons/jets



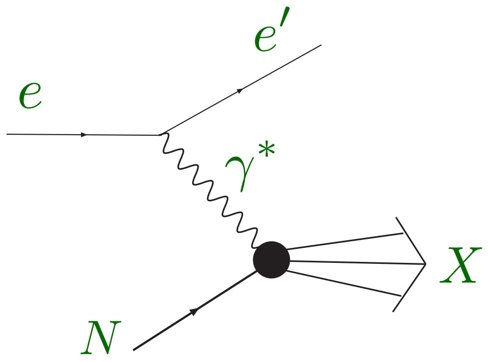
- ✧ **Exclusive events:** e+p/A → e’+ p’/A’+ h(p,K,p,jet)

  Detect every thing including scattered proton/nucleus (or its fragments)



Inclusive deep-inelastic scattering

- Most information on PDFs obtained from lepton-hadron deep-inelastic scattering (DIS)



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\,\frac{F_1}{M}+\frac{F_2}{\nu}\right)$$

$$x_B = \frac{Q^2}{2M\nu}$$

Bjorken-x

$$Q^2 = \vec{q}^{\,2} - \nu^2 \qquad \nu = E - E'$$

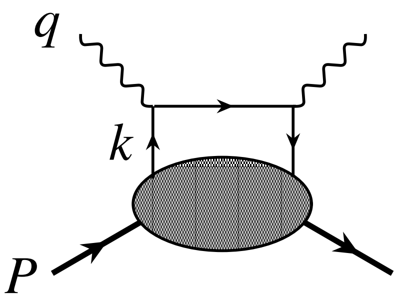
$$y = \frac{\nu}{E}$$

inelasticity

⟶ structure function given as convolution of hard Wilson coefficient with PDF

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \int_{x_B}^{1} \frac{dx}{x}\, C_q\!\left(\frac{x_B}{x}, \alpha_s\right) q(x, Q^2)$$

$$\longrightarrow x_B \sum_q e_q^2\, q(x_B, Q^2)$$ at LO $$C_q \longrightarrow \delta\!\left(1 - \frac{x_B}{x}\right)$$



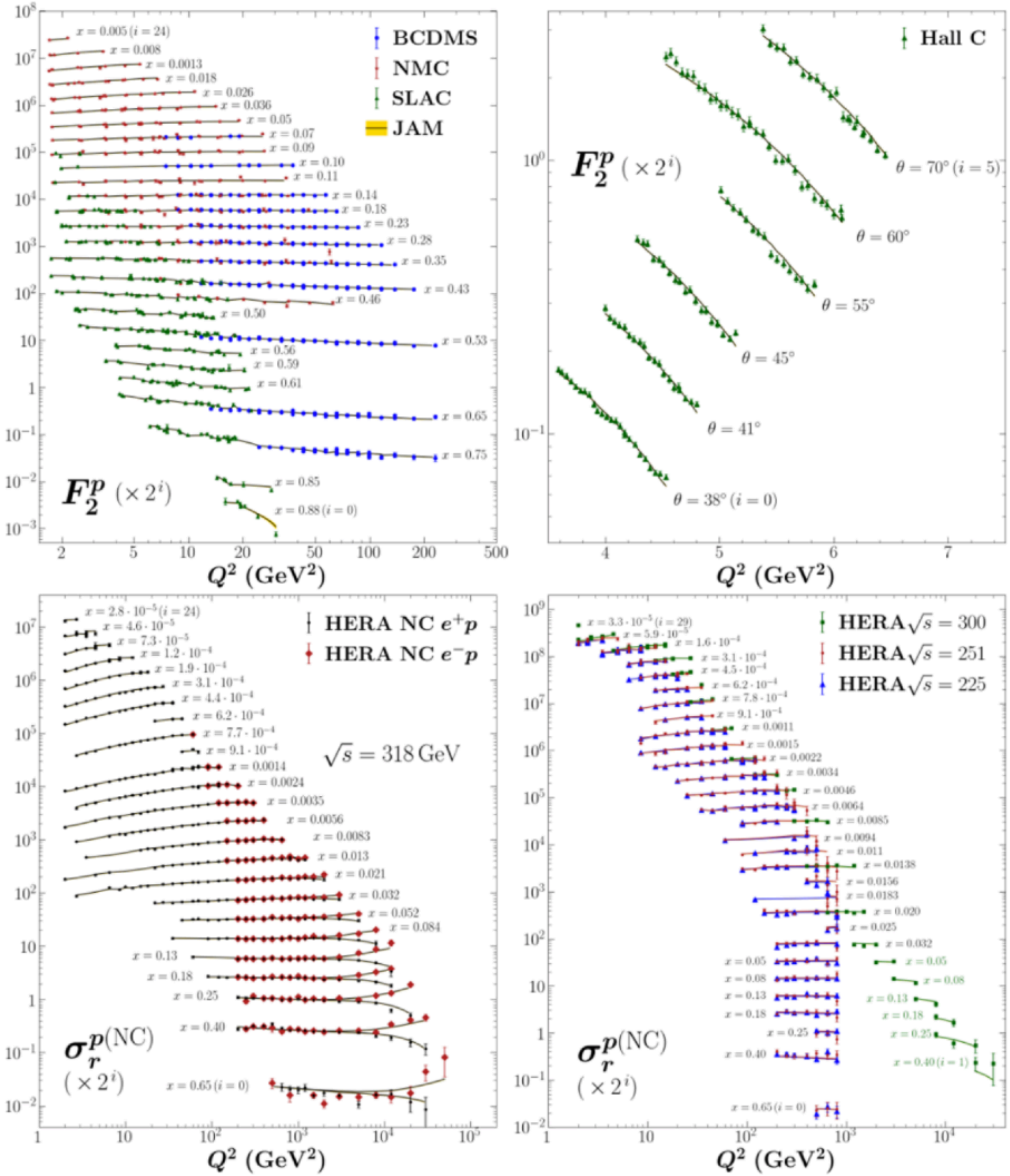
$$\boxed{q(x) = \int_{-\infty}^{\infty} d\xi^- e^{-ixP^+\xi^-} \langle\, P \,|\, \overline{\psi}(\xi^-)\, \gamma^+ \mathcal{W}(\xi^-, 0)\, \psi(0) \,|\, P \,\rangle}$$

$$x = \frac{k^+}{P^+}$$

Inclusive deep-inelastic scattering

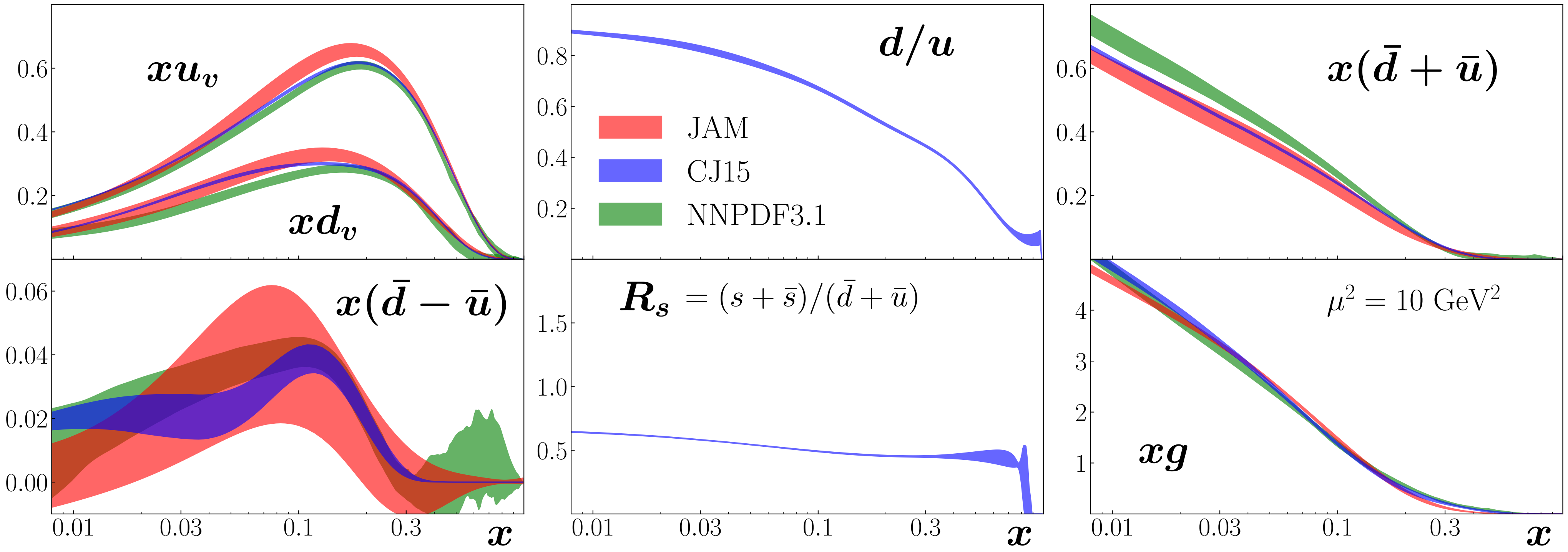
→ good description of data and extraction of PDFs over several orders of x & Q^2 !

→ one of great success stories of QCD & QCD factorization!



Inclusive deep-inelastic scattering

→ global QCD analysis of DIS and other high-energy scattering data has provided detailed information on momentum (& spin) distributions of partons in the nucleon



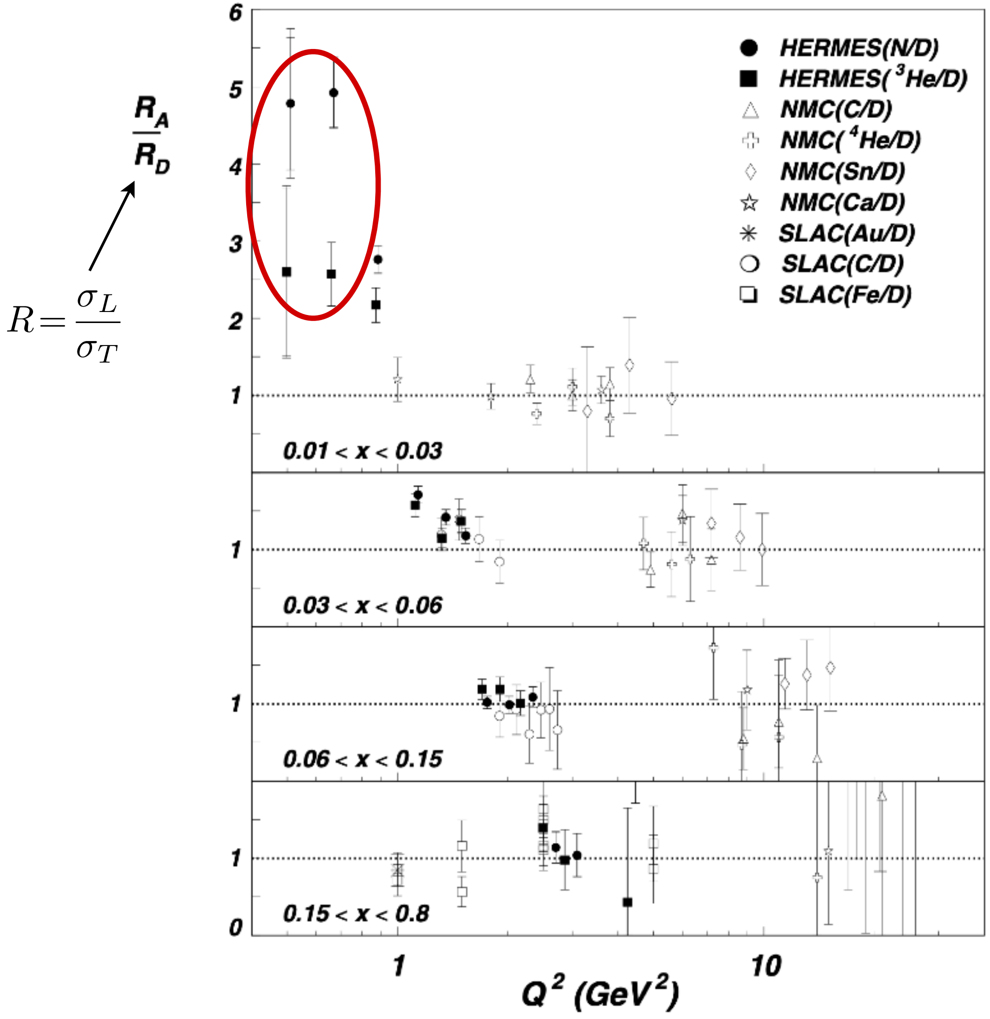
Moffat, WM, Rogers, Sato (2021)

***JAM** (Jefferson Lab Angular Momentum) Collaboration*

Inclusive deep-inelastic scattering

→ QED radiation effects important!

→ *e.g.* "HERMES effect"

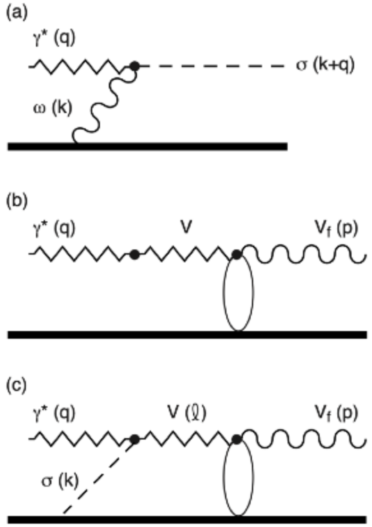


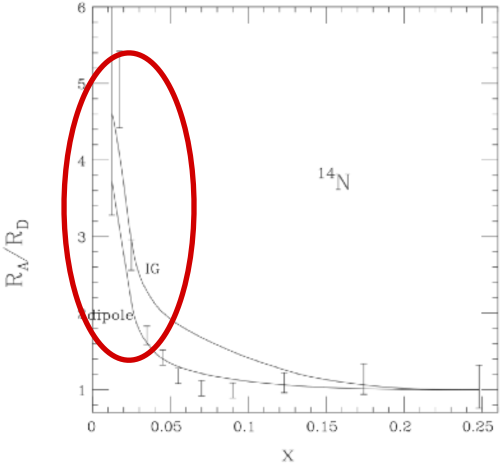
$R = \frac{\sigma_L}{\sigma_T}$

Phys. Lett. B ***475***, *386 (2000)*

lots of theoretical interpretations

e.g., coherent scattering from σ, ω mesons



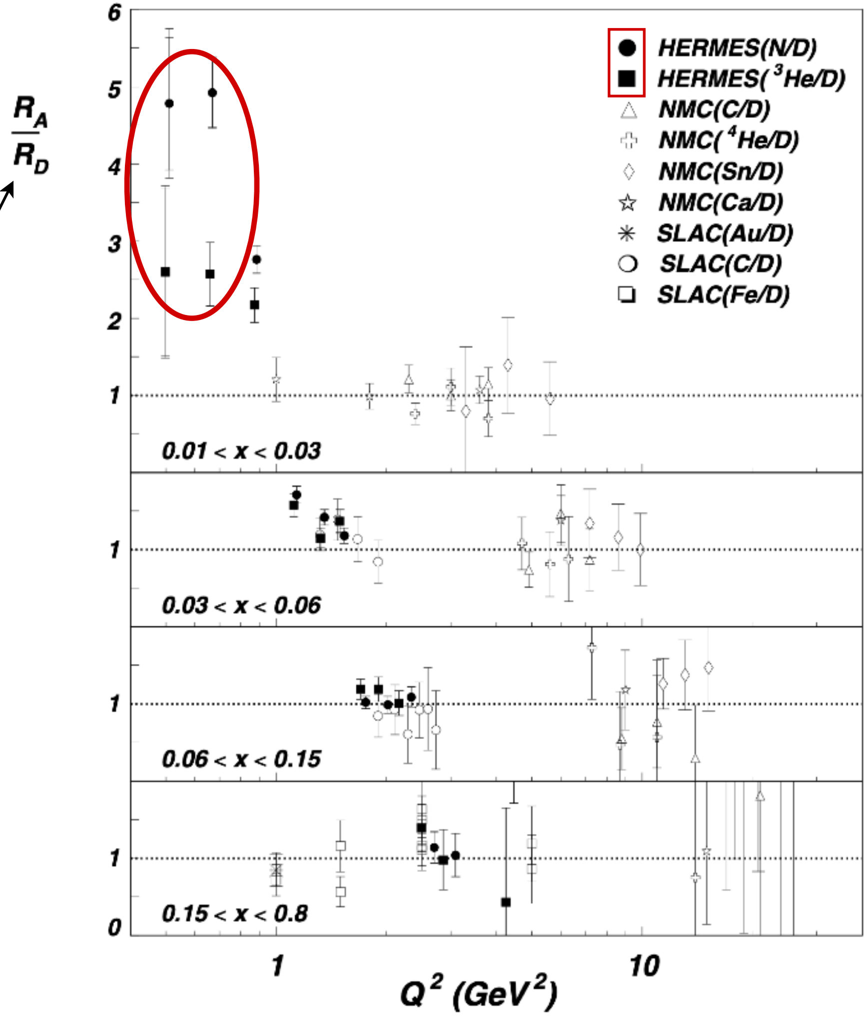


Miller, Brodsky, Karliner
Phys. Lett. B ***481***, *245 (2000)*

Inclusive deep-inelastic scattering

→ QED radiation effects important!

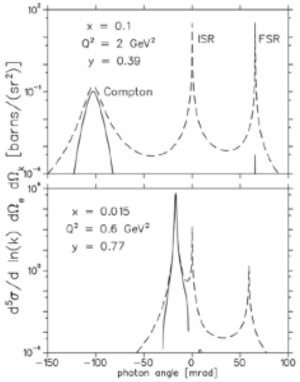
→ *e.g.* "HERMES effect"

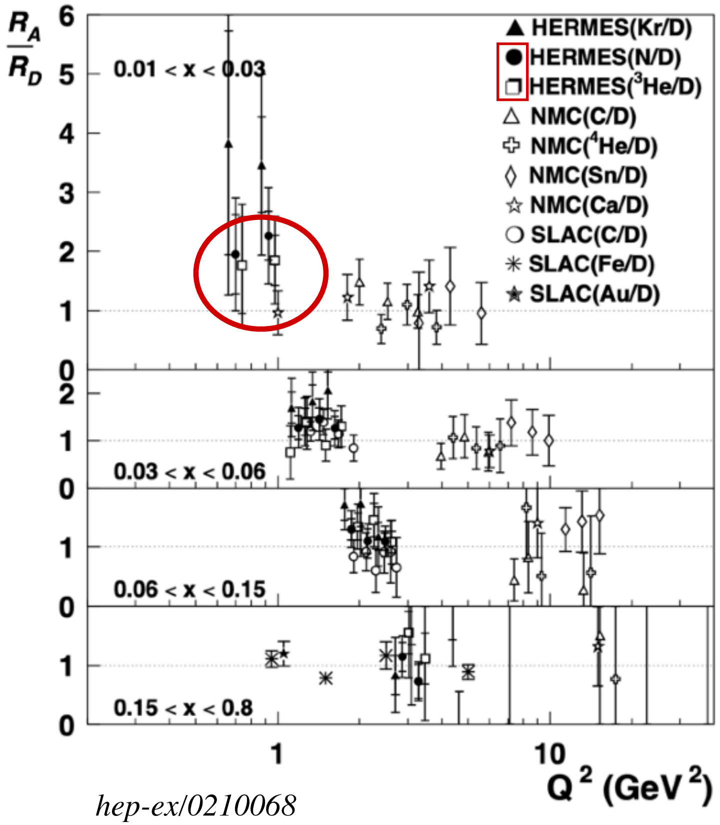


$$R = \frac{\sigma_L}{\sigma_T}$$

Phys. Lett. B ***475***, *386 (2000)*

A-dependent tracking inefficiency of spectrometer, not recognized in original analysis, had significant effect on radiative corrections!



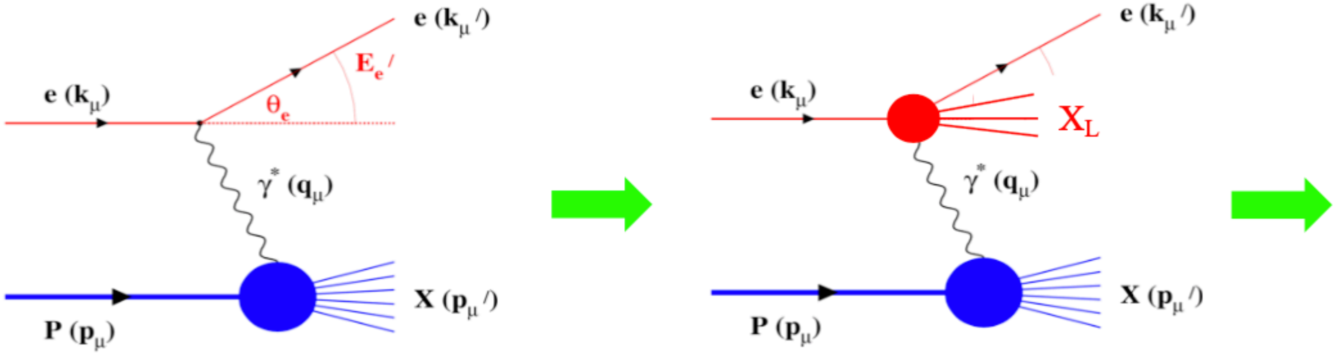


hep-ex/0210068

→ effect disappears!

QED radiation in DIS

- In presence of QED radiation (from initial or final leptons), momentum transfer not unique



$$q_\mu \rightarrow \hat{q}_\mu$$

$$Q^2 = -q^2 \rightarrow \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \rightarrow \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot\hat{q}}$$

$$E'\frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s}\frac{1}{Q^4}L^{\mu\nu}(k,k;q)W_{\mu\nu}(q,P) \quad\Longrightarrow\quad E'\frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s}\int d^4\hat{q}\left(\frac{1}{\hat{q}^2}\right)^2 \widetilde{L}^{\mu\nu}(k,k;\hat{q})W_{\mu\nu}(\hat{q},P)$$

$$\widetilde{L}^{\mu\nu}(k,k;\hat{q}) = \sum_{X_L}\int\prod_{i\in X_L}\frac{d^3k_i}{(2\pi)^3 2E_i}\,\delta^{(4)}\Big(k-k'-\hat{q}-\sum_{i\in X_L}k_i\Big)\,\langle k|j^\mu(0)|k'X_L\rangle\langle k'X_L|j^\nu(0)|k\rangle$$

$$= -\widetilde{g}^{\mu\nu}(\hat{q})\,L_1 + \frac{\widetilde{k}^\mu\widetilde{k}^\nu}{k\cdot k'}L_2 + \frac{\widetilde{k'}^\mu\widetilde{k'}^\nu}{k\cdot k'}L_3 + \frac{\widetilde{k}^\mu\widetilde{k'}^\nu + \widetilde{k'}^\mu\widetilde{k}^\nu}{2k\cdot k'}L_4$$

$$\rightarrow 2\big(k^\mu k'^\nu + k'^\mu k^\nu - k\cdot k' g^{\mu\nu}\big)\,\delta^{(4)}(k-k'-\hat{q})$$

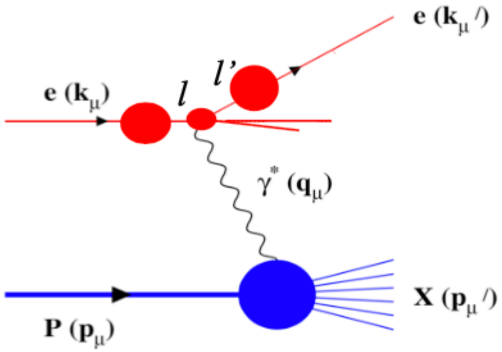
$$\widetilde{g}^{\mu\nu}(\hat{q}) = g^{\mu\nu} - \frac{\hat{q}^\mu\hat{q}^\nu}{\hat{q}^2}$$

$$\widetilde{k}^\mu = \widetilde{g}^{\mu\nu}(\hat{q})k_\nu$$

→ **4 Lepton structure functions! Small α leads to $\hat{q}_T^2 \ll \widehat{Q}^2$ in lepton back-to-back frame!**

QED radiation in DIS

- Apply collinear factorization (with one-photon exchange approximation)



$\boxed{\ell = \xi\, k}$ $\boxed{\ell' = k'/\zeta}$

$$E_{k'}\frac{d^3\sigma_{kP\to k'X}}{d^3k'} \approx \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\, D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^1 d\xi\, f_{e/e}(\xi,\mu^2)\left[E_{l'}\frac{d^3\hat{\sigma}_{lP\to l'X}}{d^3l'}\right]_{l=\xi k,\, l'=k'/\zeta}$$

$$\approx \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\, D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^1 d\xi\, f_{e/e}(\xi,\mu^2)$$

$$\times\frac{\alpha^2}{\hat{y}\,\widehat{Q}^4}\left[\hat{x}_B\hat{y}^2\, F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right) F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- **QED radiation prevents a well-defined "photon-hadron" frame**
- **Radiation is IR sensitive as** $m_e/Q \to 0$**, into LDFs & LFFs**
- **Hadron is probed by** $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$

$x_B \to \hat{x}_B \in [x_B, 1]$ $\widehat{Q}^2_{\min} = Q^2\,\dfrac{(1-y)}{(1-x_B\,y)}$ $\widehat{Q}^2_{\max} = Q^2\,\dfrac{1}{(1-y+x_B\,y)}$

QED radiation in DIS

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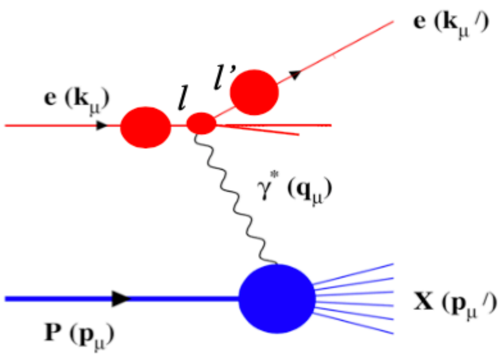

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$$\approx \int_{\zeta_{\rm min}}^{1}\frac{d\zeta}{\zeta^2}\,D_{e/e}(\zeta,\mu^2)\int_{\xi_{\rm min}}^{1} d\xi\, f_{e/e}(\xi,\mu^2)$$

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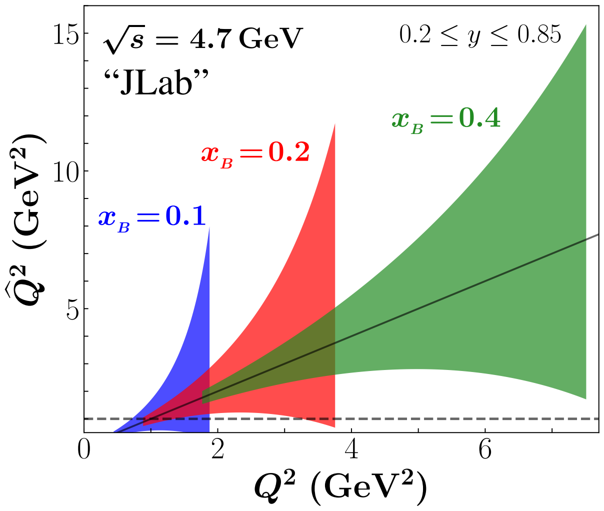

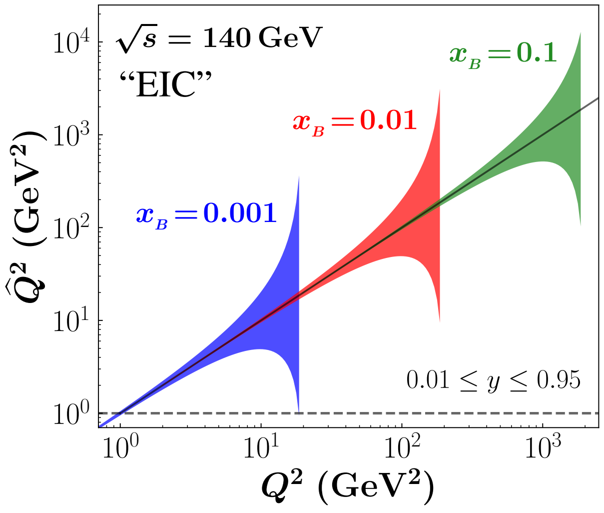

QED radiation in DIS

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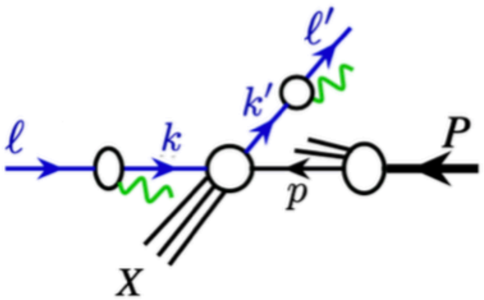


LFF

LDF

$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\rm min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\rm min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$

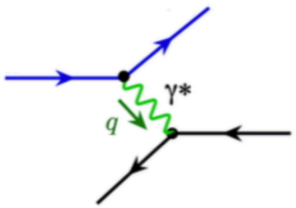
$$\times\int_{x_{\rm min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2)\;+\;\cdots$$

PDF

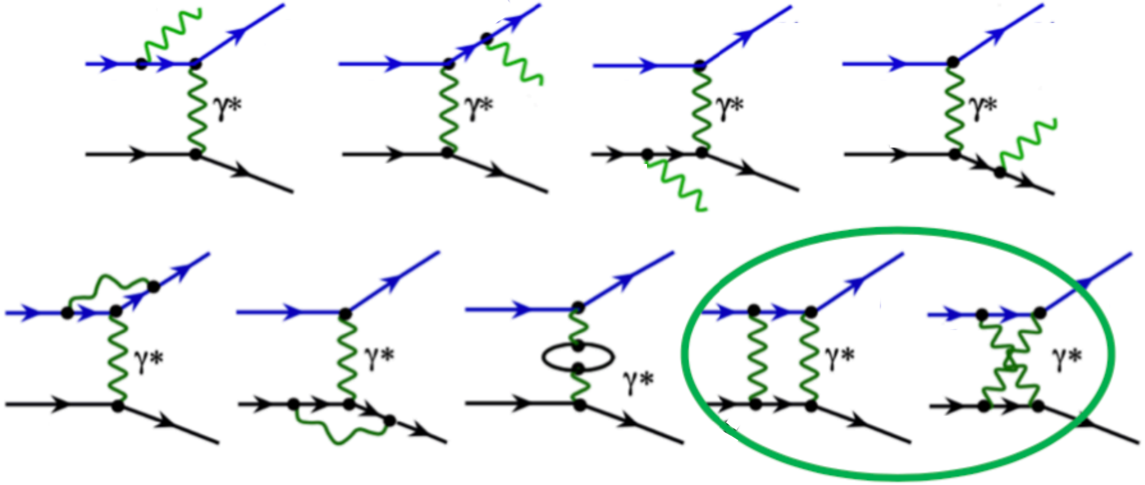
short-distance hard part

Calculated hard parts in power of $\alpha^m\alpha_s^n$ **:**

LO



NLO:

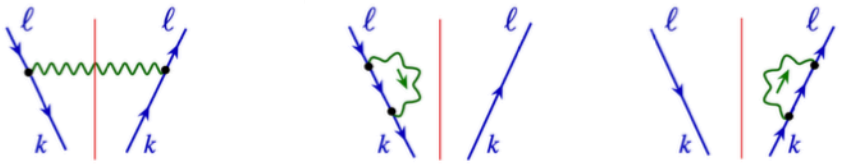


Beyond one-photon exchange

→ lepton distribution function (LDF) & lepton fragmentation function (LFF) do not decouple from parton distribution!

QED radiation in DIS

☐ **Lepton distribution function** (LDF)



$$f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2\,m_e^2}\right]_+$$

☐ **Lepton evolution**

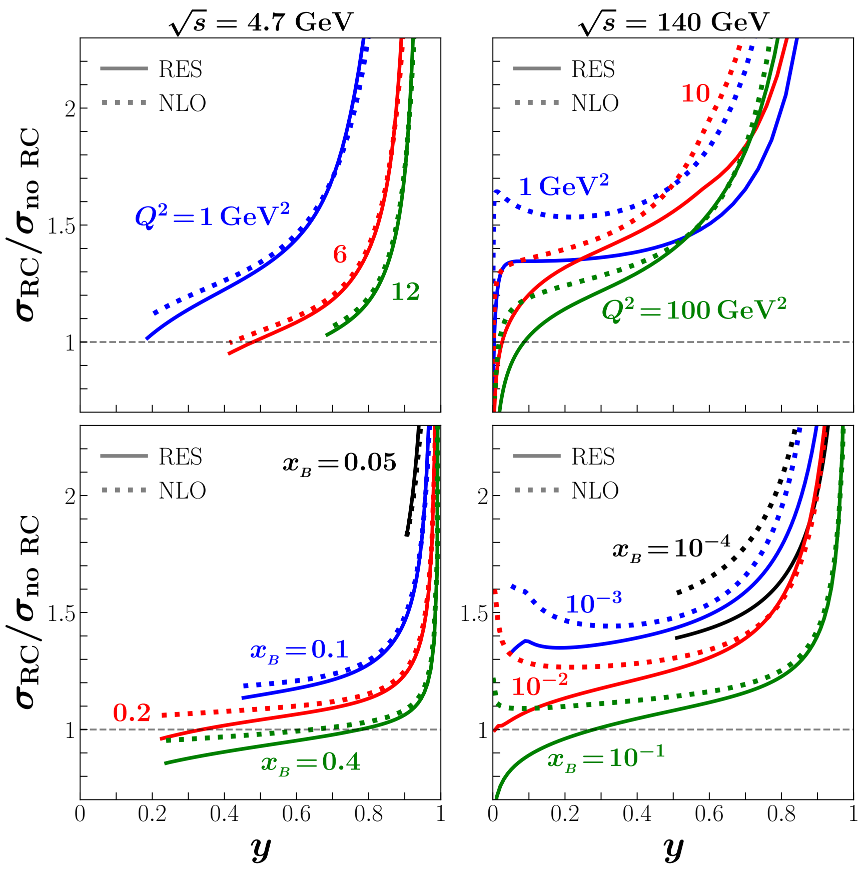
$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}f_{e/e}(\xi,\mu^2) = \int_\xi^1\frac{\mathrm{d}\xi'}{\xi'}P_{ee}\left(\frac{\xi}{\xi'},\alpha\right)f_{e/e}(\xi',\mu^2)$$

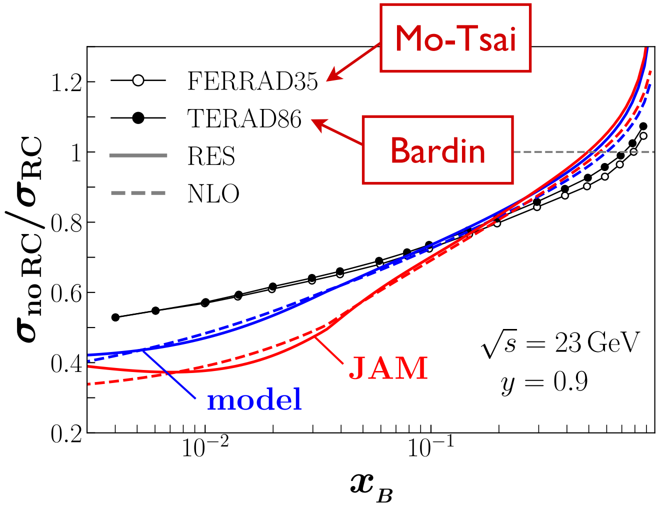
☐ **Lepton fragmentation function** (LFF)

$$D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2\,m_e^2}\right]_+$$

→ compare with existing calculations

→ RC depends on input PDF!





Mo-Tsai

Bardin

QED radiation in DIS

QED radiative corrections vs. radiative contributions

QED radiative corrections:

$$\sigma_{\rm obs}(x_B,Q^2) \stackrel{?}{=} R_{\rm QED}(x_B,Q^2;x_{B,\rm true},Q^2_{\rm true}) \times \sigma_{\rm Born}(x_{B,\rm true},Q^2_{\rm true}) + \sigma_X(x_B,Q^2)$$

- The correction factors R_{QED} and σ_x should not depend on the hadron structure that we wish to extract, and they can be systematically calculated in QED to high precision;
- The effective scale Q^2_{true} for the Born cross section σ_{Born} should be large enough to keep the "true" scattering within the DIS regime.
- Extraction of $\sigma_{\rm Born}$ is an inverse problem

QED radiative contributions:

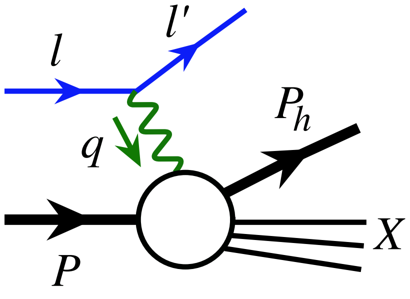
$$\sigma_{\rm obs}(x_B,Q^2) = \sigma_{\rm lep}^{\rm univ}(\mu^2;m_e^2) \otimes \sigma_{\rm had}^{\rm univ}(\mu^2;\Lambda^2_{\rm QCD}) \otimes \widehat{\sigma}_{\text{IR-safe}}(\hat{x}_B,\widehat{Q}^2,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2},\frac{m_e^2}{Q^2}\right)$$

- Infrared sensitive QED contributions – divergent as $m_e/Q \to 0$, are absorbed to universal LDFs and LFFs
- Infrared safe QED contributions – finite as $m_e/Q \to 0$, are calculated order-by-order in power of α
- Power suppressed contributions as $m_e/Q \to 0$, are neglected

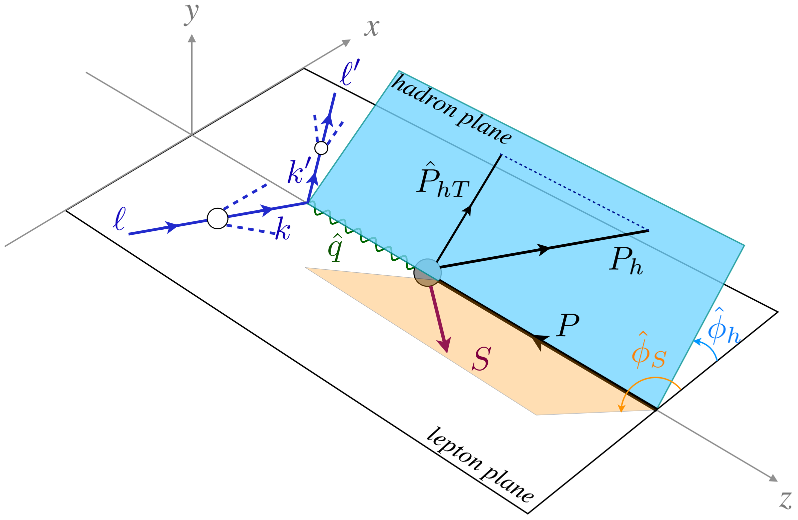
Predictive power: Universality of LDFs and LFFs, their evolution, calculable hard parts
Neglect power corrections

Semi-inclusive deep-inelastic scattering

- Differential cross section involves 18 structure functions



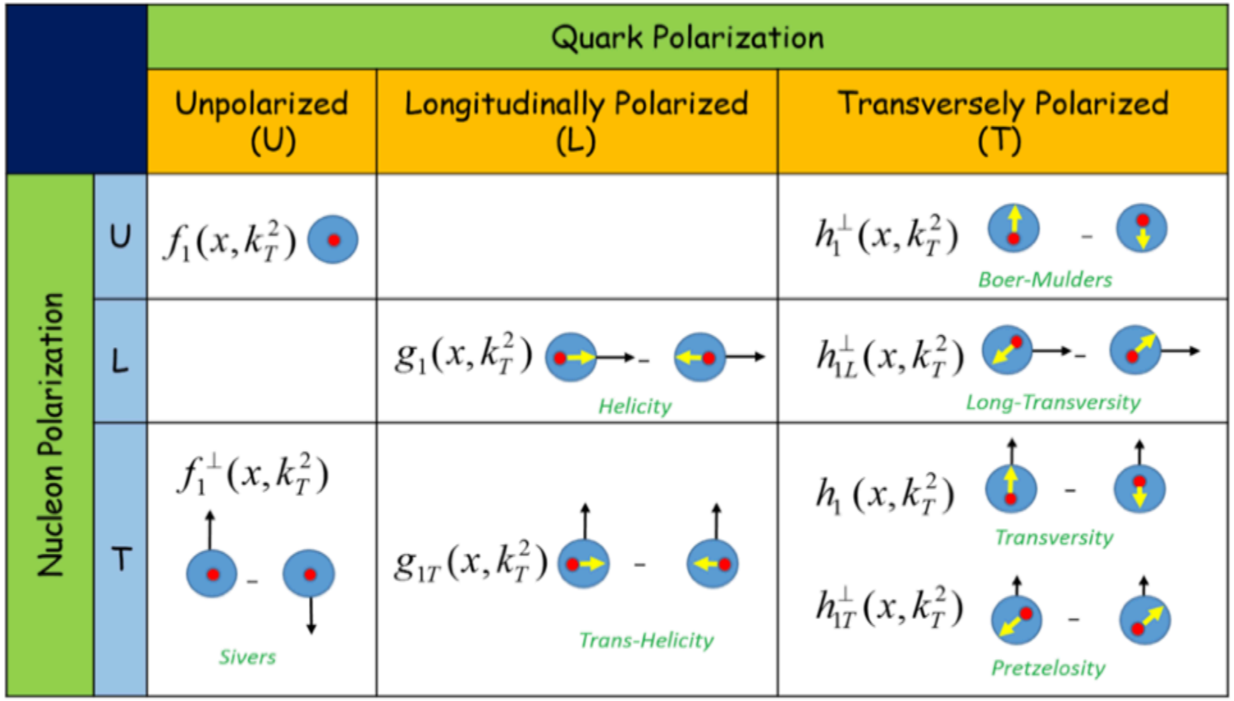
$$
\begin{aligned}
\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x_B y\, Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\bigg\{ & F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel \bigg[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \bigg] \\
& + S_\parallel\,\lambda_e \bigg[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h} \bigg] \\
& + |\boldsymbol{S}_\perp| \bigg[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \Big) \\
& \quad + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \bigg] \\
& + |\boldsymbol{S}_\perp|\lambda_e \bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \bigg] \bigg\}
\end{aligned}
$$



Semi-inclusive deep-inelastic scattering

- Goal to extract from SIDIS structure functions the transverse momentum dependent (TMD) distribution and fragmentation functions

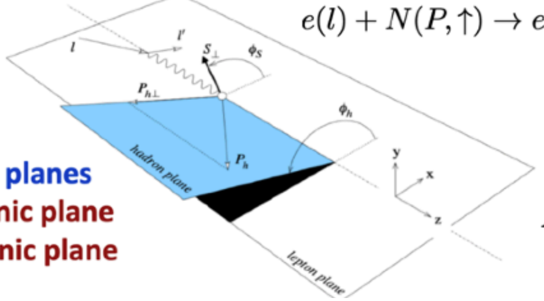
Quark TMDs with polarization



		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ Boer-Mulders
	L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^{\perp}(x,k_T^2)$ Long-Transversity
	T	$f_1^{\perp}(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Trans-Helicity	$h_1(x,k_T^2)$ Transversity $h_{1T}^{\perp}(x,k_T^2)$ Pretzelosity

Polarized SIDIS:

$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$



Two planes
Leptonic plane
Hadronic plane

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

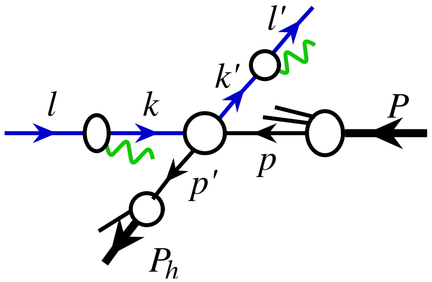
$$A_{UT}^{Collins} \propto \left\langle \sin(\phi_h + \phi_S) \right\rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \left\langle \sin(\phi_h - \phi_S) \right\rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \left\langle \sin(3\phi_h - \phi_S) \right\rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

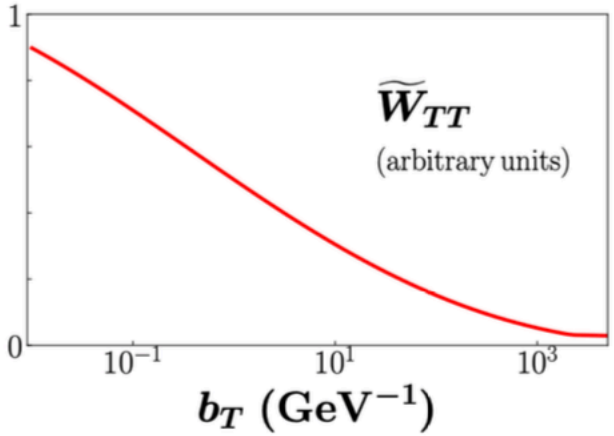
Semi-inclusive deep-inelastic scattering

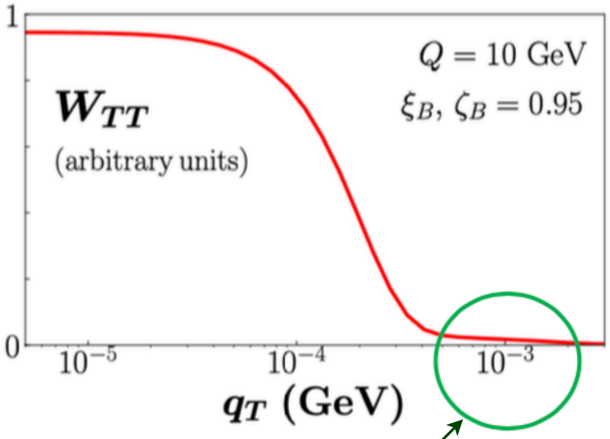
- Do we need to consider lepton TMDs?



Estimate of lepton transverse momentum generated by QED shower:

Resummation to lepton TMD





$$L_{\rho\sigma}(\xi_B, \zeta_B, Q^2, \hat{\boldsymbol{q}}_T^2) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}} \underline{\widetilde{W}_{\rho\sigma}(\xi_B, \zeta_B, Q^2, b)} + Y_{\rho\sigma}(\xi_B, \zeta_B, Q^2, \hat{\boldsymbol{q}}_T^2),$$

$$\widetilde{W}_{TT}(\xi_B, \zeta_B, Q^2, b) = 2\int_{\xi_B}^{1} \frac{d\xi}{\xi} \int_{\zeta_B}^{1} \frac{d\zeta}{\zeta^2} \underline{f(\xi)D(\zeta)} C_f(\lambda) C_D(\eta)$$
$$\times \exp\left\{ -\int_{\mu_b^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[A(\alpha(\mu')) \ln\frac{\mu_Q^2}{\mu'^2} + B(\alpha(\mu')) \right] \right\}$$

QED broadening for lepton much less than typical parton k_T !

⟶ use "hybrid" framework, with TMD factorization for hadrons, collinear factorization for leptons

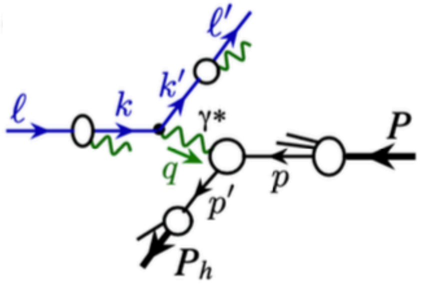
Semi-inclusive deep-inelastic scattering

❑ QED factorization of collision induced radiation – collinear:

$$E_{\ell'} E_{P_h} \frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}^3\ell'\,\mathrm{d}^3 P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^{1} \mathrm{d}\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \left[E_{k'} E_{P_h} \frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3 k'\,\mathrm{d}^3 P_h}\right]_{k=\xi\ell,\, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- **Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs**
- **IR safe contributions are calculated order-by-order in powers of α**
- **Neglect m_e/Q power suppressed contributions**

❑ "One photon"-approximation:



$$k = \xi\,\ell$$
$$k' = \ell'/\zeta$$

$$\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}x_B \mathrm{d}y\,\mathrm{d}\psi\,\mathrm{d}z_h\,\mathrm{d}\phi_h \mathrm{d}P_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} \frac{\mathrm{d}\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\, D_{e/j}(\zeta)$$
$$\times \frac{\hat{x}_B}{x_B\xi\zeta} \underbrace{\left[\frac{\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})}\left(1+\frac{\hat{\gamma}^2}{2\hat{x}_B}\right)\sum_n \hat{w}_n F_n^h(\hat{x}_B, \widehat{Q}^2, \hat{z}_h, \widehat{P}_{hT}^2)\right]}_{\text{Evaluated in a "virtual photon-hadron" frame}}$$

⟶ **Apply a (ξ,ζ)-dependent Lorentz transformation:**

$$\{\hat{q}, P, \widehat{P}_h\} \xrightarrow[(\xi,\zeta)]{} \{q, P, P_h\}$$

In a frame to compare with exp. measurements

Semi-inclusive deep-inelastic scattering

- ☐ **Case study F_{UU}:**

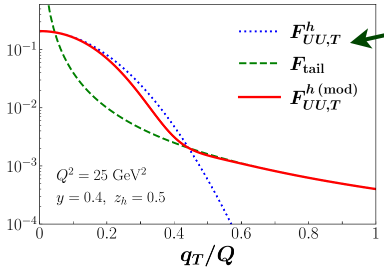
$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\rm min}}^1 d\zeta \int_{\xi_{\rm min(\zeta)}}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \underbrace{\left[\frac{\hat{x}_B}{x_B\, \xi\zeta}\right] \left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B \hat{y}\, \widehat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \widehat{Q}^2, \hat{z}, \widehat{P}_{hT})\right]}_{\boxed{\text{Evaluated in a “virtual photon-hadron” frame}}}$$

Unpolarized structure function:

$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

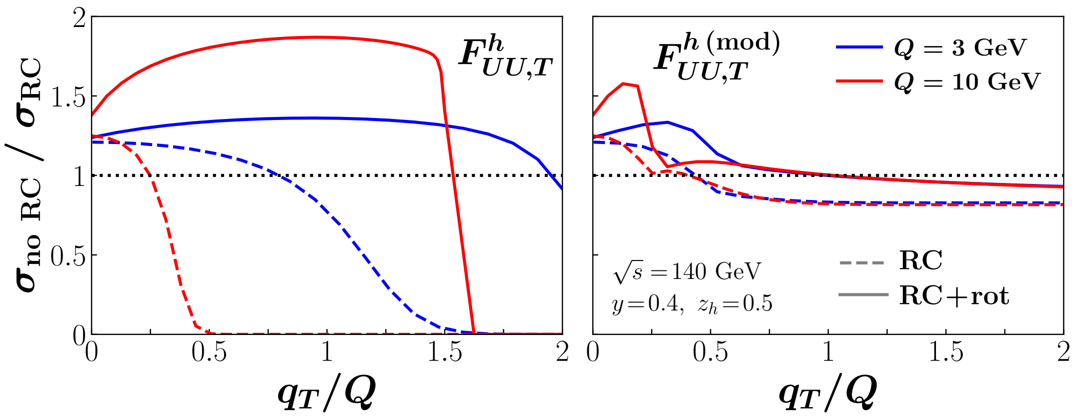
(ξ, ζ) **- Dependent Lorentz transformation**

Effectively, a rotation in hadron-rest frame



models for input SIDIS structure functions

$$F^h_{UU,T} \;\to\; F^{h\,({\rm mod})}_{UU,T} \;=\; F^h_{UU,T}\, R \;+\; (1-R) F_{\rm tail},$$



→ QED radiative effect depends strongly on hadronic input!

Semi-inclusive deep-inelastic scattering

❑ **Case study – single transverse spin asymmetry:**

$$
\begin{aligned}
\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2}{x_B y\, Q^2}\, \frac{y^2}{2\,(1-\varepsilon)} \Big\{ & F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel \Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \Big] \\
& + S_\parallel\, \lambda_e \Big[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \Big] \\
& + |\boldsymbol{S}_\perp| \Big[\sin(\phi_h - \phi_S) \Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \Big) \\
& \quad + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Big] \\
& + |\boldsymbol{S}_\perp| \lambda_e \Big[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Big] \Big\}
\end{aligned}
$$

Sivers function

pretzelocity
(set to zero)

Collins function

Semi-inclusive deep-inelastic scattering

☐ **Case study – single transverse spin asymmetry:**

e.g., for Sivers function, $\sin(\phi_h - \phi_S)$ modulation would be obtained from

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \overset{\text{no QED}}{=} \int \mathrm{d}\phi_h\,\mathrm{d}\phi_S\,\sin(\phi_h-\phi_S)\Big[\sin(\phi_h-\phi_S)F_{UT,T}^{\sin(\phi_h-\phi_S)} + \sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]$$

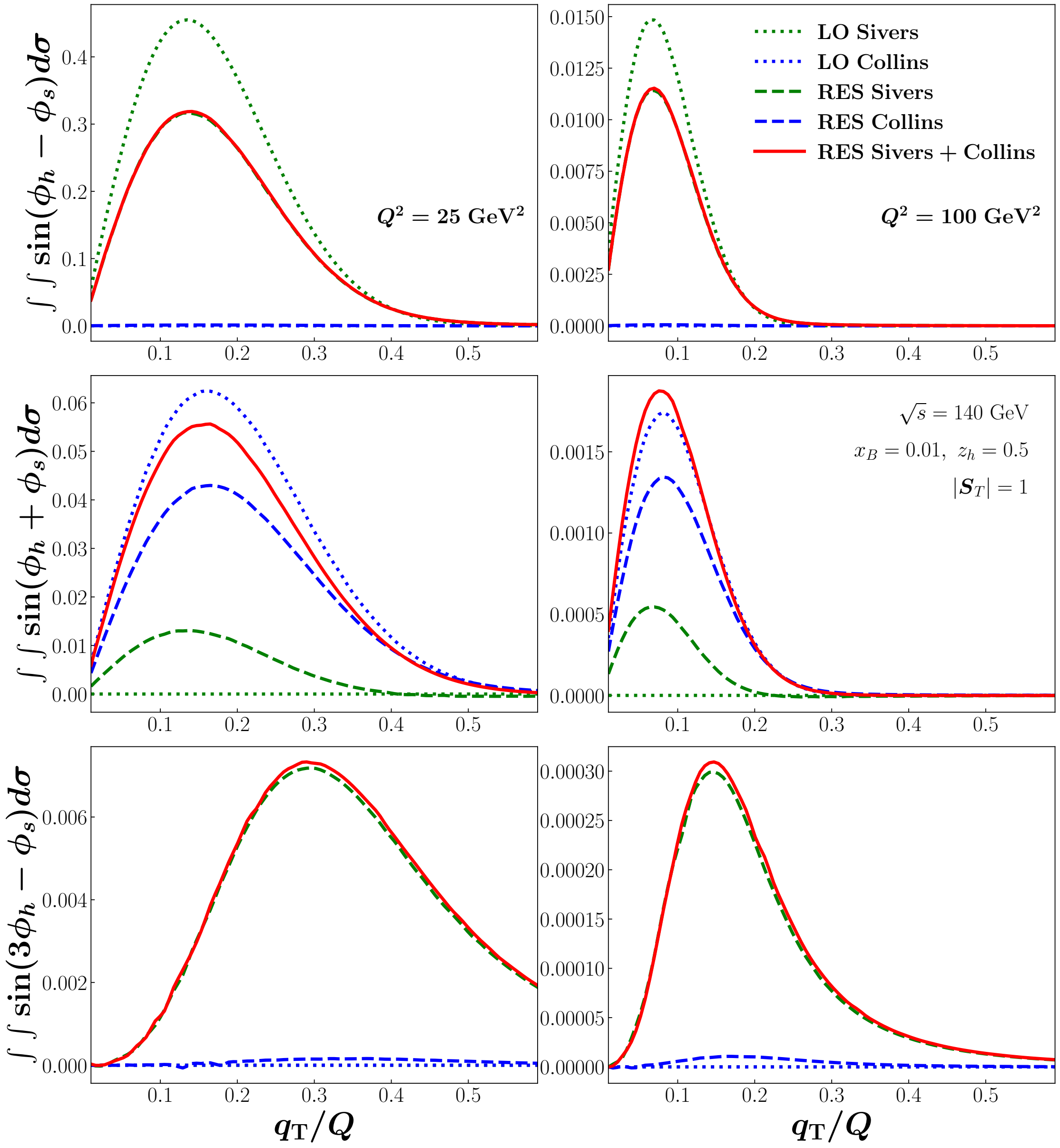
since $\int d\phi_h d\phi_S\ \sin(\phi_h-\phi_S)\ \sin(\phi_h+\phi_S) = 0$

⟶ with QED radiation, projecting phases are no longer orthonormal

$$\int \mathrm{d}\phi_h\,\mathrm{d}\phi_S\,\sin(\phi_h-\phi_S)\,\sin(\hat{\phi}_h+\hat{\phi}_S)\ \neq\ 0$$

⟶ “leakage” between various asymmetries

Semi-inclusive deep-inelastic scattering



→ "LO" = no QED

"RES" = QED resummed

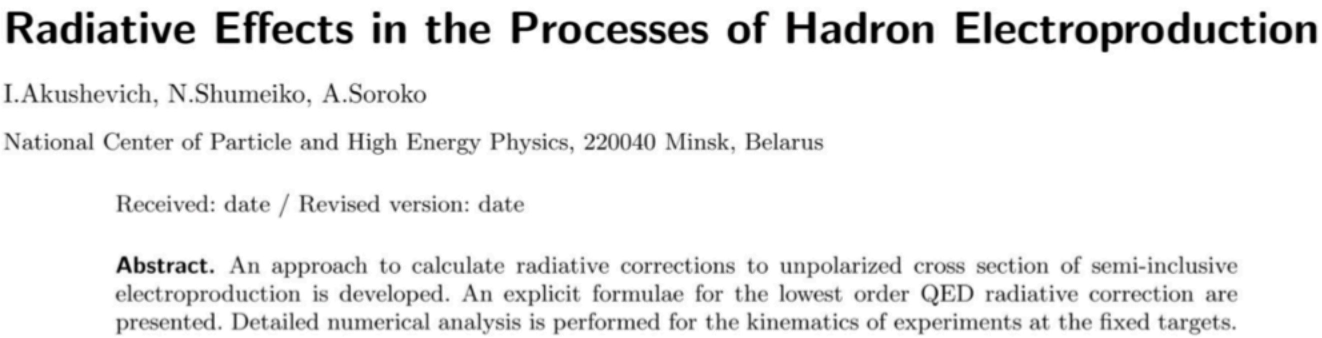
→ leakage from Sivers to Collins

→ cannot isolate QED-free signals

… QED corrections inherently model dependent!

Semi-inclusive deep-inelastic scattering

- **QED radiative corrections:**



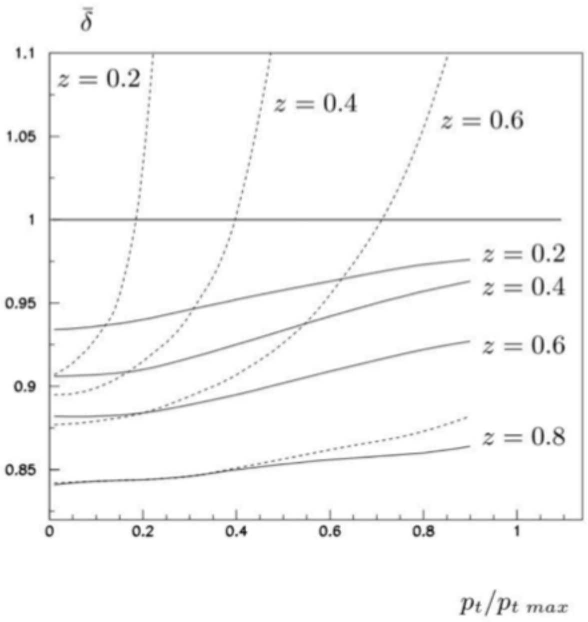
Radiative Effects in the Processes of Hadron Electroproduction

I.Akushevich, N.Shumeiko, A.Soroko

National Center of Particle and High Energy Physics, 220040 Minsk, Belarus

Received: date / Revised version: date

Abstract. An approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed. An explicit formulae for the lowest order QED radiative correction are presented. Detailed numerical analysis is performed for the kinematics of experiments at the fixed targets.



Similar trends. Eg. RCs depends on hadronic input

- **Our formalism is different. It does not depend on hadronic input, which is what we want to probe!**
- **Our formalism is organized in terms of IR safe quantities and universal functions – advantage of factorization**

Outlook

- Factorized approach treats QED & QCD radiation on equal footing

  ⟶ QED radiation integral part of production cross section, not universal "correction factor"

  ⟶ all QED & QCD hard parts IR safe; IR sensitivity included in universal lepton distribution and fragmentation functions

- More important for less inclusive processes, such as SIDIS

  ⟶ no well-defined photon-hadron frame in presence of radiation

  ⟶ radiative effects more pronounced at high momentum transfer — new paradigm for future data analyses at EIC energies

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Danke!