Electroweak Effects in Boosted Top Jet Production

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Seminar Particle Physics WS21

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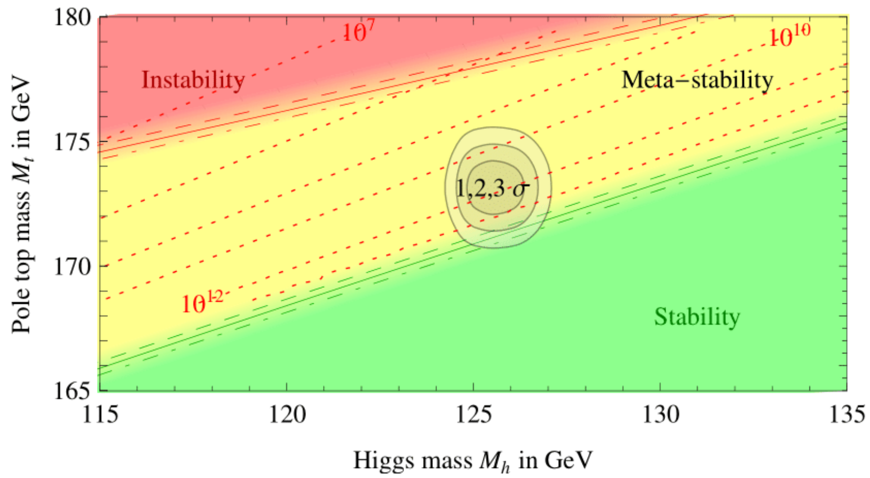
The top quark

- Top quark heaviest known elementary particle to date
- Unstable particle, with width $\Gamma_t \gg \Lambda_{\mathrm{QCD}}$
- Reduced non-perturbative impact („bare" quark)
- Couples to all sectors of the SM
- Top parameters important:
  - Virtual top effects
  - EW vacuum stability
  - Consistency test of SM
- Important process: top pair-production
- Precision studies at future linear colliders
  - Threshold scans
  - Direct reconstruction (from top decay products)

$$m_t^{\text{pole}} = 172.5 \pm 0.7\,\text{GeV} \qquad \text{(cross section)}$$
$$m_t^{\text{MC}} = 172.76 \pm 0.30\,\text{GeV} \qquad \text{(direct reconstruction)}$$

[PDG, 20]



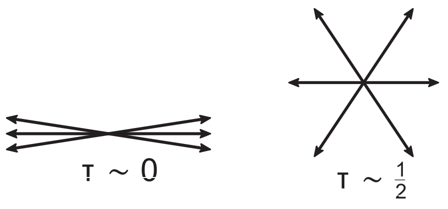
[Degrassi et al, 13]

$$\Delta m_t = 50\,\text{MeV} \qquad \text{(threshold scan)}$$
$$\Delta m_t = 100\,\text{MeV} \qquad \text{(direct reconstruction)}$$

Projected uncertainties for ILC500 [Zarnecki et al, 11]

N-jettiness

- Event shapes
  - Functions of (hadronic) kinematics
  - Characterize the shape of the event



e.g. thrust (massless)

- This work: *N-jettiness* τ_N [Stewart et al, 10]

Jet directions (pre-determined)

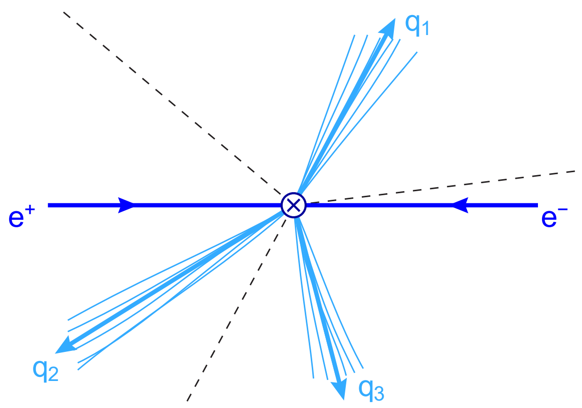
$$\tau_N \equiv \sum_k \min_i \frac{2q_i \cdot p_k}{Q_i}$$

Particle momenta

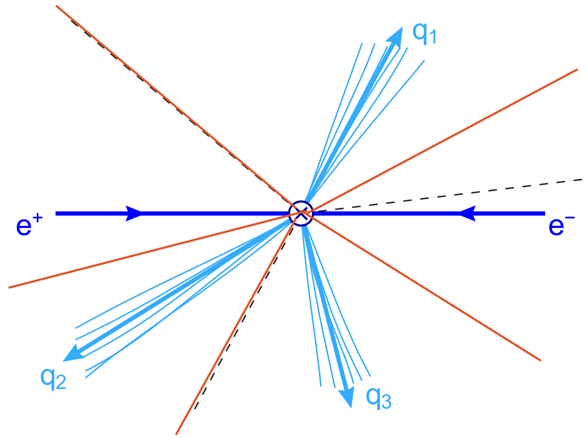
  - Measure for how N-jet-like an event is
  - τ_N small → N well-separated jets
  - Assigns each momentum to a jet sector
  - Related up to power-corrections $O(\tau_N)$ to other event shapes

- Top mass extractions from top mass-sensitive event shapes
  -> peak position for 2-jettiness

- <u>Important:</u> treating EW (QED) effects necessitates including the beam (jets)



Jet sectors without beam (boundaries dashed)



Jet sectors including beam (boundaries red)

2-jettiness for $t\bar{t}$ production

- Leading-order: Breit-Wigner function

$$\frac{1}{\sigma_0}\left(\frac{d\sigma}{d\tau_2}\right)^{(0)} = f_{\mathrm{BW}}\left(\tau_2 - \tau_{\min}, \frac{2\Gamma_t}{Q\varrho_t}\right)$$

$Q = \sqrt{\hat{s}}$ ... CM energy

$\varrho_t = Q/m_t$ ... boost factor

$\beta_t = \sqrt{1 - 4/\varrho_t}$ ... quark velocity

$\tau_{\min} = 1 - \beta_t$ ... stable threshold

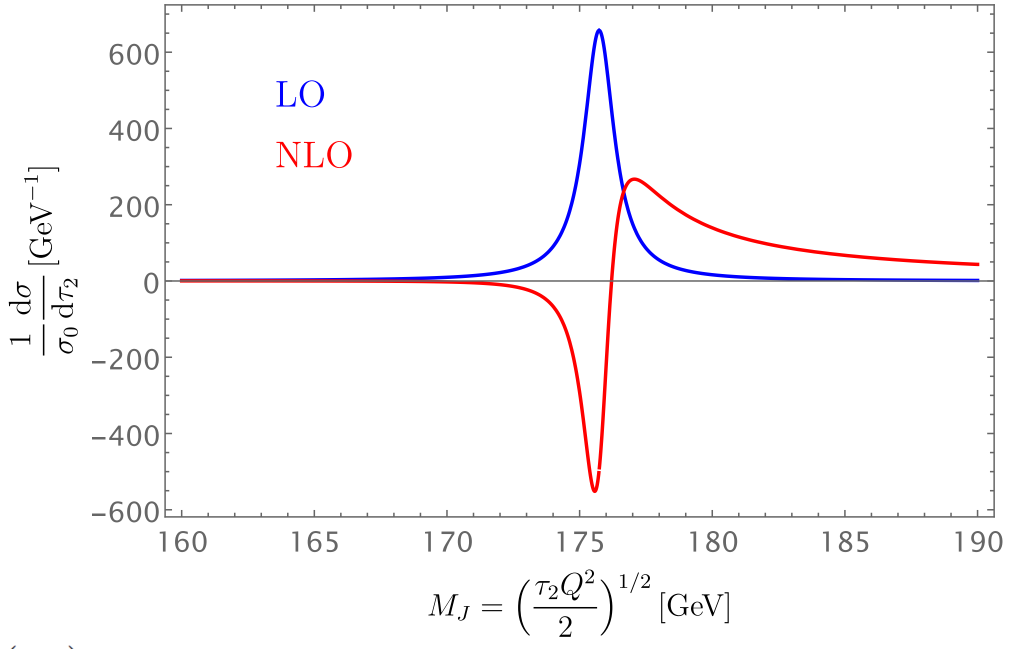
scale	magnitude [GeV]
Q	1000
m_t	173
Γ_t	1.4
Λ_{QCD}	< 1

- Of interest: boosted jet regime: $E_{jet}^2 \gg p_{jet}^2 \sim m_t^2$, i.e. small τ_2
- Radiative corrections (QCD NLO and beyond) very sizable
- Complex beyond LO because of multiple scales
- Due to scale separation, e.g. in the peak region

$$Q^2 \gg m_t^2 \gg Q^2\tau_2 - m_t^2 \gtrsim m_t\Gamma_t \gg \varrho_t^{-1} m_t\Gamma_t \gtrsim \Lambda_{\mathrm{QCD}}$$

  - large logs: $\alpha_s^n \ln^k \tau_2$
  - Sudakov logs: $\alpha_s^n \ln^k \varrho_t$

$\Longrightarrow$ Resummation necessary



- Factorization properties of N-jettiness: $\hat{\tau}_2 = \hat{\tau}_c + \hat{\tau}_s + \mathcal{O}(\tau_2)$

Indicates power-suppressed/non-singular terms

SCET

- Factorization of observable into soft and collinear contributions
  -> use *Soft-Collinear Effective Theory* (**SCET**) [Stewart et al, 01]

- Power-counting parameters for boosted regime:
  $\lambda = \sqrt{\tau_2} \ll 1$ and $\varrho_t^{-1} = m_t/Q \ll 1$

- SCET reproduces (singular) leading-power contributions in τ_2 and the asymptotic limit in ϱ_t^{-1}

- Lagrangians decouple:

$$\mathcal{L} = \mathcal{L}_{\text{SCET}} + \mathcal{O}(\lambda) = \mathcal{L}_s(A_s, \xi_s) + \sum_n \mathcal{L}_c(A_n, \xi_n) + \mathcal{O}(\lambda)$$

- Hard modes are integrated out of Lagrangian and encoded in Wilson coefficients, e.g.: current:

$$O^\mu = \overline{\psi}\gamma^\mu\psi = C(Q,\mu)\ \bar{\xi}_n W_n Y_n^\dagger \gamma^\mu Y_n W_{\bar{n}}^\dagger \xi_n = C(Q,\mu)\ \mathcal{O}^\mu$$

- Interactions and operators in SCET dictated by <u>individual</u> soft and collinear gauge invariance

$$p^\mu = \frac{\bar{n}^\mu}{2} p^+ + \frac{n^\mu}{2} p^- + p_\perp^\mu$$

$$p^2 = p^+ p^- - \vec{p}_\perp^{\,2}$$

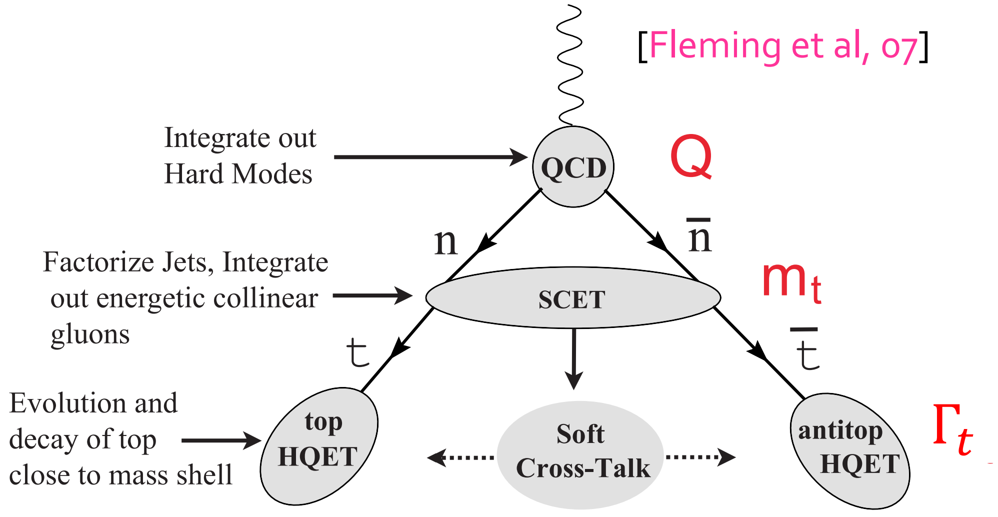
mode	scaling
hard	$Q(1,1,1)$
n-collinear	$Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$
(ultra-)soft	$Q(\lambda^2, \lambda^2, \lambda^2)$
mass-mode n-collinear	$Q(\varrho_t^{-2}, 1, \varrho_t^{-1})$
mass-mode $\bar{n}$-collinear	$Q(1, \varrho_t^{-2}, \varrho_t^{-1})$
mass-mode soft	$Q(\varrho_t^{-1}, \varrho_t^{-1}, \varrho_t^{-1})$

$$W_n(x) = \mathrm{P}\exp\left[ig_s \int_{-\infty}^{0} \mathrm{d}s\ \bar{n}\cdot A_n(x+\bar{n}s)\right]$$

$$Y_n(x) = \mathrm{P}\exp\left[ig_s \int_{0}^{\infty} \mathrm{d}s\ \bar{n}\cdot A_s(x+\bar{n}s)\right]$$

(boosted) HQET

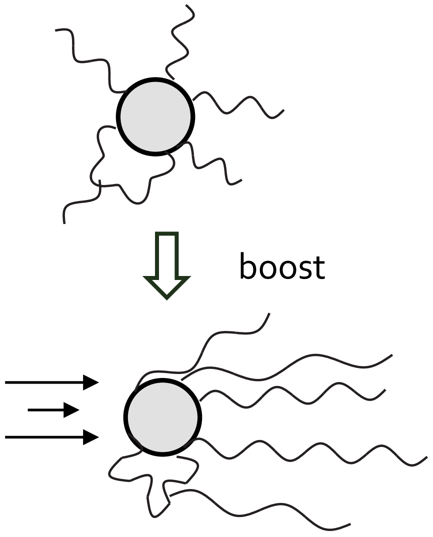
[Fleming et al, 07]



- For unstable quarks: additional large logs $\alpha_s^n \ln^k \Gamma_t/m_t$ in the tail and peak regions (where $m_t^2 \gg Q^2\tau_2 - m_t^2 \sim m_t\Gamma_t$) that cannot be resummed with SCET
- Origin = *ultracollinear* radiation: radiation that is ultrasoft (+ isotropic) in heavy quark rest frame, but boosted by ϱ_t along jet direction
- *Heavy Quark Effective Theory* **(HQET):** [Georgi et al, 90]
  integrate out modes with off-shellness m_t^2, only ultrasoft gluons remain

$$\psi \to h_v : \quad p^\mu = m_t v^\mu + k^\mu, \quad v = (1,0,0,0), \quad k \sim \Lambda_{\mathrm{QCD}}$$

- **bHQET:** [Fleming et al, 07]
  radiation is boosted (here IR scale for ultracollinear radiation is Γ_t instead of Λ_{QCD}):



$$\xi_n \to h_v : \quad p^\mu = m_t v^\mu + k^\mu, \quad v = (\varrho_t^{-1}, \varrho_t, 0_\perp), \quad k \sim \Gamma_t(\varrho_t^{-1}, \varrho_t, 1)$$

$v^2 = 1$ $\qquad$ $k^2 \sim \Gamma_t^2 \ll m_t^2$

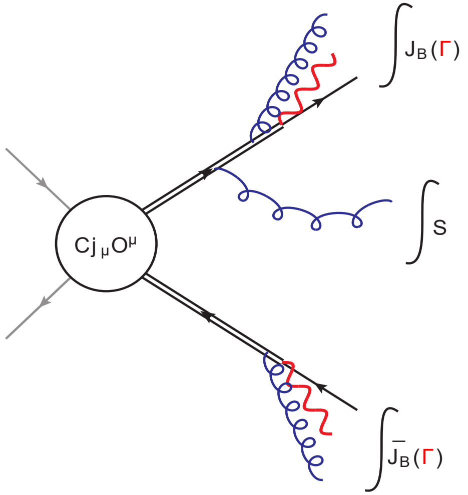
- Matching SCET jet fields onto bHQET jet fields:

$$W_n^\dagger \xi_n = C_M(m_t, \mu)\, W_n^\dagger h_v$$

Factorization for 2-jettiness in pure QCD

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_2} = H_Q(Q,\mu) \times H_M(\varrho_t, m_t, \mu)$$

$$\times\, J_{B,t}^{(\Gamma_t)}(Q^2\tau_2, m_t, \mu) \otimes \bar{J}_{B,\bar{t}}^{(\Gamma_t)}(Q^2\tau_2, m_t, \mu) \otimes S(Q\tau_2,\mu) + \mathcal{O}\left(\lambda, \varrho_t^{-1}, \frac{\Gamma_t}{m_t}\right)$$



[Fleming et al, 07]

- Hard and mass-mode functions:

$$H_Q(Q,\mu) = |C_Q(Q,\mu)|^2, \qquad H_M(m_t,\mu) = |C_M(\varrho_t, m_t, \mu)|^2$$

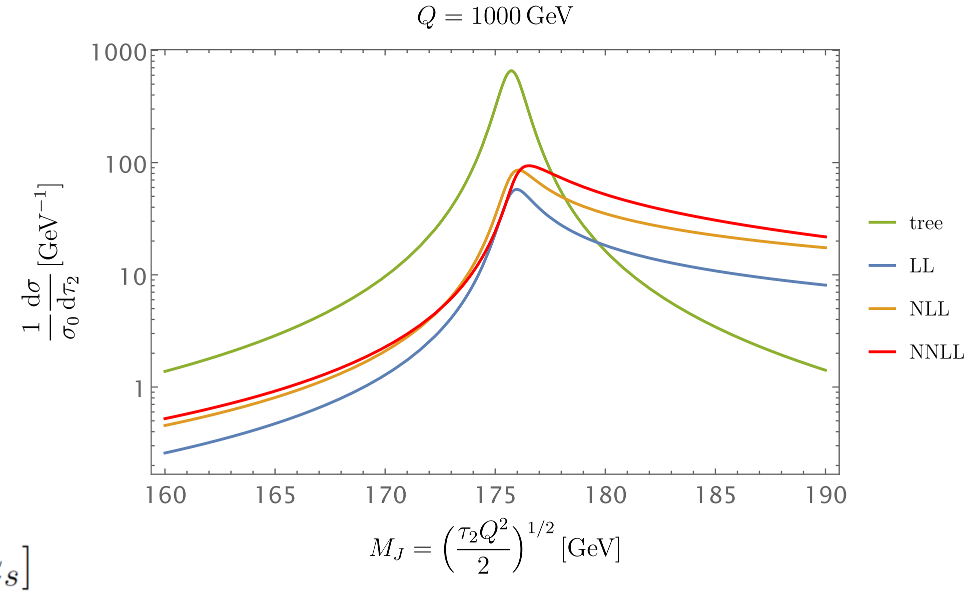
- bHQET jet function

$$J_{B,t}^{(\Gamma_t)}(p^2, m_t, \mu) = \mathrm{Im}\left[\frac{\mathrm{i}}{4\pi N_c m_t}\int \mathrm{d}^4x\, \mathrm{e}^{-\mathrm{i}px}\langle 0|T[W_n^\dagger h_v](0)[\bar{h}_v W_n](x)\,\not{\bar{n}}|0\rangle\right]$$

- Soft function

$$S_\tau(Q\tau_2,\mu) = \frac{1}{N_c}\langle 0|[\bar{Y}_{\bar{n}}^\dagger Y_n](0)\,\delta(Q\tau_2 - Q\hat{\tau}_s)\,[Y_n^\dagger \bar{Y}_{\bar{n}}](0)|0\rangle$$

- SCET provides the possibility to resum large logarithmic corrections by renormalization group evolution



$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}F(t,\mu) = \gamma_F(t,\mu)\,F(t,\mu) \qquad \gamma_F(t,\mu) = \Gamma_{\mathrm{cusp}}[\alpha_s]\,\ln(\mathrm{i}\mu\mathrm{e}^{\gamma_E}t) + \gamma[\alpha_s]$$

QCD state-of-the-art (peak region)

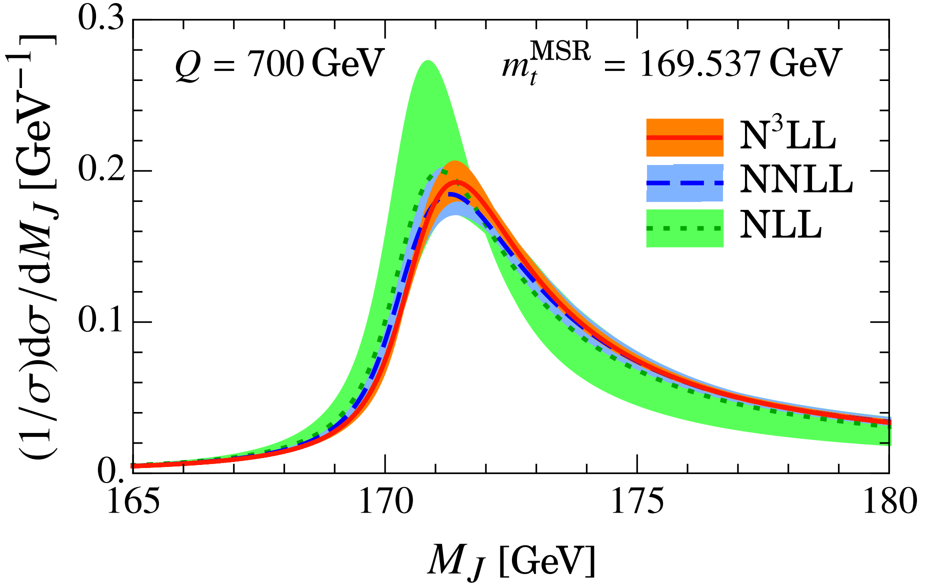
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_2} = H_Q(Q,\mu)\times H_M(\varrho_t, m_t, \mu)$$

$$\times J_{B,t}^{(\Gamma_t)}(Q^2\tau_2, m_t, \mu)\otimes \bar{J}_{B,\bar{t}}^{(\Gamma_t)}(Q^2\tau_2, m_t, \mu)\otimes S(Q\tau_2,\mu) + \mathcal{O}\left(\lambda, \varrho_t^{-1}, \frac{\Gamma_t}{m_t}\right)$$

	$F(\mu_F)$	$\Gamma_{\rm cusp}, \beta$	γ
LL	0	1	0
NLL	0	2	1
NLL'	1	2	1
N^{2}LL	1	3	2
N^{3}LL	2	4	3

Loop-orders of resummation ingredients

- State-of-the-art QCD:
  - NLL QCD + LO EW (width Γ_t in peak region) [Fleming et al, 07, 08]
  - 2-loop bHQET jet function [Stewart et al, 08]
  - 2-loop SCET-bHQET current matching [Pathak et al, 15]
  - N3LL-study (peak region) [Hoang et al, 20]
  - 2-loop SCET massive jet function [Lepenik et al, 18]
- EW only LO so far -> this work's aim: subleading EW effects
- QCD field-theoretic subtleties (not covered now):
  - Non-perturbative corrections
  - Renormalons
  - Top mass schemes (MSR mass)
  - Profiles (τ_2-dependent canonical scales μ_F)
  - Secondary mass effects



[Hoang et al, 20]

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Electroweak sector

- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group:

$$\begin{aligned} \mathrm{i}D_\mu &= \mathrm{i}\partial_\mu + g_3\, T^A G^A_\mu + g_2\, t^a W^a_\mu + g_1\, Y B_\mu \\ &= \mathrm{i}\partial_\mu + g_s T^A G^A_\mu + g_W\, t^\pm W^\pm_\mu + g_Z\, t^Z Z_\mu - e\, Q A_\mu \end{aligned} \qquad \alpha_i \equiv \frac{g_i^2}{4\pi}$$

- Matter content:

$$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, \quad d_R, \qquad \ell = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R$$

- Fundamental Higgs with Yukawa couplings (top only):

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi)^2$$

$\mu^2 = 0$: symmetric/unbroken phase
$\mu^2 \neq 0$: broken phase

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h + \mathrm{i}\chi), \end{pmatrix}, \qquad \mathcal{L}_{\text{Yuk}} = -y_t\, \bar{q} \tilde{\Phi} u_R + \text{h.c}$$

EW couplings and Sudakov logs

- Couplings (at $\mu \sim m_t$):

$$\alpha_s \sim \frac{1}{10}, \qquad \alpha_W \sim \alpha_Z \sim \frac{1}{30}, \qquad \alpha_{\rm em} \sim \frac{1}{125}, \qquad \alpha_t \sim \frac{1}{15}$$

- Evolution coupled beyond LL ($i, j = 1,2,3,t$):

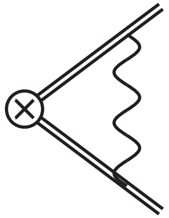
$$\beta_i[\{\alpha_j\}] = 2\alpha_i \Big(\prod_j \sum_{l_j=0}^{\infty} \Big) \beta_i^{(l_1,l_2,l_3,l_t)} \left(\frac{\alpha_1}{4\pi}\right)^{l_1} \left(\frac{\alpha_2}{4\pi}\right)^{l_2} \left(\frac{\alpha_3}{4\pi}\right)^{l_3} \left(\frac{\alpha_t}{4\pi}\right)^{l_{y_t}}$$

$$\cancel{\ln \frac{\mu}{\mu_0} = \int_{\mu_0}^{\mu} \frac{\mathrm{d}\alpha_s}{\beta[\alpha_s]}}$$

- Numerical solution with Mathematica sufficiently fast and precise for our purpose
- Radiative corrections involving massive gauge bosons get Sudakov log enhancement ($L \equiv \ln \frac{Q^2}{M_W^2} \sim 5$ at Q = 1 TeV)

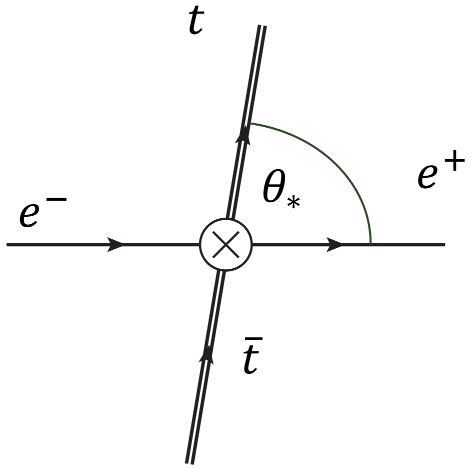
$$\mathrm{LL}: \ \alpha_W L^2 \sim \frac{5}{6} \qquad \mathrm{NLL}: \ \alpha_W L \sim \frac{1}{6} \qquad \delta\mathcal{M} \sim \alpha_W \left(c_2 L^2 + c_1 L + c_0\right)$$

- Note: enhancement large, but still perturbative, i.e. $(\alpha_W L^2)^n > (\alpha_W L^2)^{n+1}$
- Also: group theoretic/loop factors in c_i affect (relative) numerics, e.g.:



$$\sim \frac{\alpha_2}{2}\Big(- L^2 + 3L + \frac{\pi^2}{3} - \frac{7}{2}\Big)$$

SCET$_{\mathrm{EW}}$ and SCET$_\gamma$ [Chiu et al, 07, 08, 09]



- Since $v \ll Q$ (and thus $M_X \ll Q$), one should get the correct asymtptotic limit by a SCET-approach
- Note: we assume $|\hat{s}| \sim |\hat{t}| \sim |\hat{u}| \gg v^2$, i.e. $z = \cos\theta_* \sim 0$ (*central* scattering) for power-counting purposes
- Theory above EW scale M = $\mathbf{SCET_{EW}}$, with Wilson line

$$W_n(x) = \mathrm{P}\exp\left[\mathrm{i}\int_{-\infty}^{0} \mathrm{d}s\ \bar{n}\cdot\Big(g_s T^A G^A_\mu + g_W\, t^\pm W^\pm_\mu + g_Z\, t^Z Z_\mu - e\, Q A_\mu\Big)(x+\bar{n}s)\right]$$

- Theory below EW scale M = $\mathbf{SCET_\gamma}$, with Wilson line

$$w_n(x) = \mathrm{P}\exp\left[\mathrm{i}\int_{-\infty}^{0} \mathrm{d}s\ \bar{n}\cdot\Big(g_s T^A G^A_\mu - e\, Q A_\mu\Big)(x+\bar{n}s)\right]$$

- Operators now 4-fermion operators (with 4 lightcone directions n_i, $n_i^2 = 0$)

$$\mathcal{O}_I^{\mathrm{SCET_{EW}}} = \left(\bar{\ell}_2 W_2 Y_2^\dagger\, \Gamma_I \gamma^\mu Y_1 W_1^\dagger \ell_1\right)\ \left(\bar{q}_3 W_3 Y_3^\dagger\, \Gamma_I \gamma^\mu Y_4 W_4^\dagger q_4\right),$$

$$\mathcal{O}_i^{\mathrm{SCET_\gamma}} = \left([\bar{\ell}_2 w_2 y_2^\dagger]_{i_2} \gamma^\mu [y_1 w_1^\dagger \ell_1]_{i_1}\right)\ \left([\bar{q}_3 w_3 Y_3^\dagger]_{i_3} \gamma^\mu [y_4 w_4^\dagger q_4]_{i_4}\right)$$

lepton current quark current

Isospin structures

- 2 natural bases to express radiative corrections
- 6 „physical" combinations for $\Gamma_I \otimes \Gamma_I$ and isospin channels *i* out of 16 in total (= those with net $I_3 = 0$)
- The two bases can be related by an orthogonal transformation B_{iI}
- Channel space:
  + physically intuitive, suitable for QED + QCD and *Z*-contributions
  - no clear view on EW symmetry(-breaking) patterns
- Operator space:
  + convenient for symmetric SU(2) effects and *W*-contributions
  - not well-suited for effects at EW scale and below
- Scattering amplitude is some vector in either space, and (factorizable) radiative corrections *A* act as matrices:

$$\mathcal{M}_i = \sum_j A_{ij} \mathcal{M}_j^{\text{tree}} = \sum_I A_{iI} \mathcal{M}_I^{\text{tree}}$$

$$O_I = (\bar{\ell}\,\Gamma_I \ell)(\bar{q}\,\Gamma_I q) \equiv \Gamma_I \otimes \Gamma_I$$

$$\langle O_I \rangle_i = \langle i_3, i_4 | O_I | i_1, i_2 \rangle \equiv \mathcal{M}_{iI}$$

$$i \sim \begin{pmatrix} (\bar{\nu}\nu)(\bar{u}u) \\ (\bar{\nu}\nu)(\bar{d}d) \\ (\bar{e}e)(\bar{u}u) \\ (\bar{e}e)(\bar{d}d) \\ (\bar{e}\nu)(\bar{u}d) \\ (\bar{\nu}e)(\bar{d}u) \end{pmatrix}$$

channel space

$$\Gamma_I \otimes \Gamma_I \sim \begin{pmatrix} Y \otimes Y \\ t^a \otimes t^a \\ t^3 \otimes t^3 - \frac{1}{6} t^a \otimes t^a \\ t^3 \otimes Y \\ Y \otimes t^3 \\ t^+ \otimes t^- - t^- \otimes t^+ \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ b \end{matrix}}\right\} \text{symmetric} \\ \left.\vphantom{\begin{matrix} a \\ b \\ c \\ d \end{matrix}}\right\} \text{broken} \end{matrix}$$

operator space

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Master formula

$$\frac{d\sigma_{e^+e^-t\bar{t}}(P_-,P_+)}{d\tau_2\, d\Phi_2} = \sum_{\kappa,\rho} \frac{K_\Phi^{\kappa\rho}}{\mathcal{F}} \mathrm{Tr}\left[\rho_{e^+e^-t\bar{t}}\, \mathcal{M}_M H_H \mathcal{M}_M^\dagger\right]^{\kappa\rho} * J_{B,t}^{(\Gamma_t)} \otimes \bar{J}_{B,t}^{(\Gamma_t)} \otimes B_{e^-}^{\kappa|P_-} \otimes B_{e^+}^{\kappa|P_+} \otimes S_{e^+e^-t\bar{t}}$$

Hard phase space

Beam polarizations, $P_i \in [-1,1]$

Hard kinematics + flux

- Lepton chirality $\kappa = \pm 1$, quark chirality $\rho = \pm 1$
- Switching off QED below *M* (-> later work) simplifies formula to

$$\frac{d\sigma_{e^+e^-t\bar{t}}(P_-,P_+)}{d\tau_2\, dz} = \sum_{\kappa,\rho} \frac{K_\Phi^{\kappa\rho}\, \phi^\kappa(P_-,P_+)}{\mathcal{F}} \mathrm{Tr}\left[\rho_{e^+e^-t\bar{t}}\, \mathcal{M}_M H_Q \mathcal{M}_M^\dagger\right]^{\kappa\rho} * J_{B,t}^{(\Gamma_t)} \otimes \bar{J}_{B,t}^{(\Gamma_t)} \otimes S_{e^+e^-t\bar{t}}$$

CM frame now: $z = \cos\theta_*$

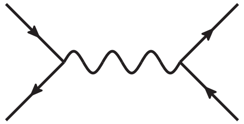
- Beam polarization function: $$\phi^\kappa(P_-,P_+) = \frac{1+\kappa P_-}{2}\frac{1-\kappa P_+}{2}$$
- Isospin density matrix (in channel space): $$\rho_{e^+e^-t\bar{t}} = \mathrm{diag}(0,0,1,0,0,0)$$

$$i \sim \begin{pmatrix} (\bar{\nu}\nu)(\bar{u}u) \\ (\bar{\nu}\nu)(\bar{d}d) \\ \boxed{(\bar{e}e)(\bar{u}u)} \\ (\bar{e}e)(\bar{d}d) \\ (\bar{e}\nu)(\bar{u}d) \\ (\bar{\nu}e)(\bar{d}u) \end{pmatrix}$$

Hard matching

- Results chiral; for simplicity focus on exclusively left-chiral case
- Hard SM tree-level (expand in $M/Q \ll 1$):

$$C_I^{\text{tree}} \sim (\star\ \star\ 0\ 0\ 0\ 0)^T$$



$$\Gamma_I \otimes \Gamma_I \sim \begin{pmatrix} Y \otimes Y \\ t^a \otimes t^a \\ t^3 \otimes t^3 - \frac{1}{6} t^a \otimes t^a \\ t^3 \otimes Y \\ Y \otimes t^3 \\ t^+ \otimes t^- - t^- \otimes t^+ \end{pmatrix}$$

- SCET_{EW} -operators equipped with Wilson coefficients C_I

$$\mathcal{O}_I^{\text{SCET}_{\text{EW}}} \sim \sum_{ij} \{\tilde{\chi}_n\}_i^{\text{EW}}\, S_{ij}^{\text{EW}} B_{jI}, \qquad \mathcal{L}_{\text{eff}} \sim \sum_I \mathcal{O}_I^{\text{SCET}_{\text{EW}}} C_I$$

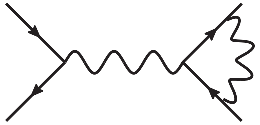
$$\{\chi_n\}_i = \chi_{i_1} \overline{\chi}_{i_2} \chi_{i_3} \overline{\chi}_{i_4}$$

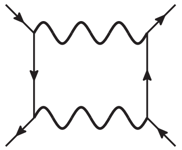
$$S_{ij} = (Y_1)_{j_1 i_1} (\overline{Y}_2)_{i_2 j_2} (Y_3)_{i_3 j_3} (\overline{Y}_4)_{j_4 i_4}$$

- Matching at high scale $\mu_Q \sim Q$ and evolution from $\mu_Q \sim Q$ to $\mu_M \sim M$

[Chiu et al, 08]

$$\langle q\bar{q}|\ell\bar{\ell}\rangle_i = \sum_I \langle q\bar{q}|\mathcal{O}_I^{\text{SCET}_{\text{EW}}}(\mu)|\ell\bar{\ell}\rangle_i\, C_I(Q,\mu)$$





...

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_I = \sum_J \gamma_{Q,IJ}^{\text{EW}} C_J \quad \to \quad C_I(Q,\mu_M) = \sum_J U_{Q,IJ}^{\text{EW}}(\mu_M \leftarrow \mu_Q)\, C_J(Q,\mu_Q)$$

- C and evolution γ_Q^{EW} are free of symmetry-breaking (SB) effects
- <u>Open question</u>: consistent (coherent) inclusion of tree power-corrections

$$(\gamma_Q^{\text{EW}})_{IJ} \sim \begin{pmatrix} \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & 0 & 0 & 0 & 0 \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \end{pmatrix}$$

singlet part gauge-independent

Non-singlet part gauge-dependent

Mass-mode matching

- Matching at EW scale $\mu_M \sim M$ onto QED + QCD (SCET_γ) [Chiu et al, 09]

$$\mathcal{O}_I^{\mathrm{SCET_{EW}}}(\mu) = \sum_J \mathcal{O}_J^{\mathrm{SCET}_\gamma}(\mu)\,\mathcal{M}_{M,JI}(M,\mu) \qquad \mathcal{O}_i^{\mathrm{SCET}_\gamma} \sim \{\chi_n\}_i^\gamma\, S_i^\gamma$$

$$\mathcal{M}_{M,IJ} \sim \begin{pmatrix} \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \end{pmatrix}$$

- Contains SB effects (most easily seen in operator space)
- Anomalous dimensions free of SB effects above and below M

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\mathcal{M}_M(M,\mu) = \mathcal{M}_M(M,\mu)\,\gamma_Q^{\mathrm{EW}} - \gamma_Q^\gamma\,\mathcal{M}_M(M,\mu)$$

$$\gamma^{(\nu)}_{M,IJ} \sim \begin{pmatrix} \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \end{pmatrix}$$

- Rapidity log remains in matching: $\ln \mathcal{M}_M(M,\mu) \sim \alpha \ln M/Q \times \ln M/\mu_M$
- „Soft-collinear" factorization (= regulator-dependent split) of matching at $\mu_M \sim M$

$$\mathcal{M}_M(M,Q/M,\mu) = \mathcal{M}_c(M,Q/\nu,\mu)\,\mathcal{M}_s(M,M/\nu,\mu)$$

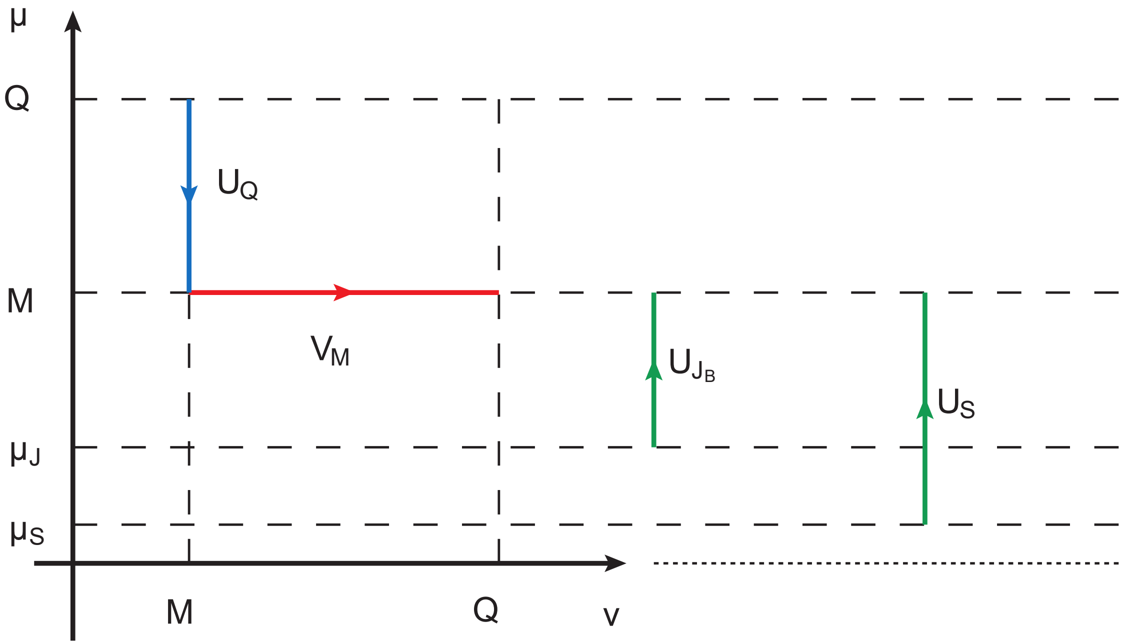
- Using symmetric rapidity regulator allows resummation by RG method [Rothstein et al, 11]

Rapidity evolution gauge-independent

$$\nu\frac{\mathrm{d}}{\mathrm{d}\nu}\mathcal{M}_c = \mathcal{M}_c\,\gamma_M^{(\nu)} \qquad \nu\frac{\mathrm{d}}{\mathrm{d}\nu}\mathcal{M}_s = -\gamma_M^{(\nu)}\,\mathcal{M}_s \longrightarrow \mathcal{M}_c(M,Q/\nu_Q,\mu)\,V_M(M,\nu_Q \leftarrow \nu_M)\,\mathcal{M}_s(M,M/\nu_M,\mu)$$

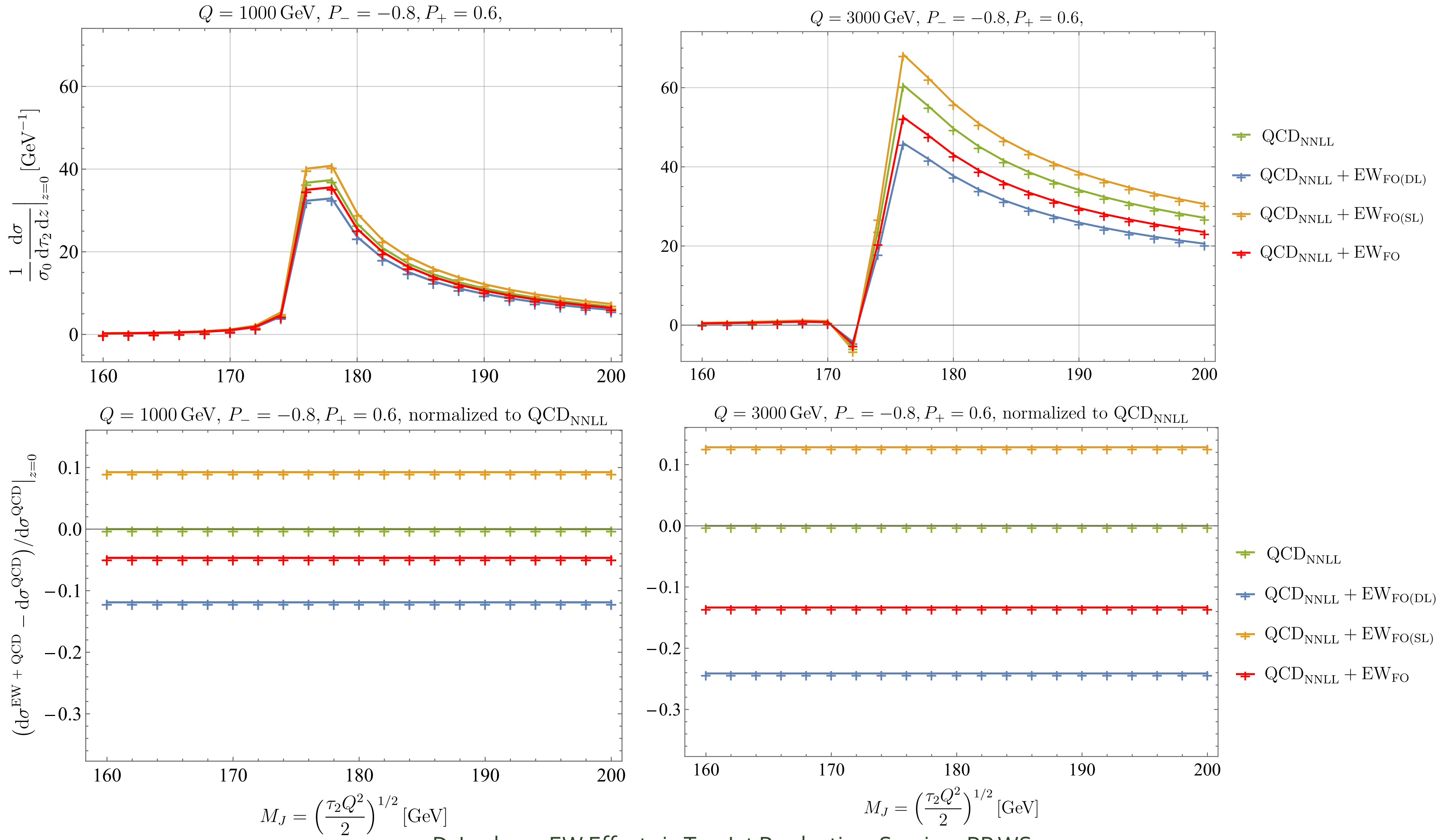
EW resummation: summary

- Resummation in virtuality sums EW Sudakov logs between Q and $M \rightarrow$ no SB effects
- Symmetric tree-level gives consistent resummed results
  -> open question: coherent inclusion of power-corrections at tree-level
- Matching:
  - high-scale Q: free of SB effects
  - EW scale M: SB effects (boson masses, isospin dependence)
- Rapidity resummation at EW scale M consistent at NLL
  -> open question: field-theoretic definition of „mass-mode collinear" contribution (Wilson line vs. Method-of-Regions)
- Resummation below M (QCD + QED) understood
  -> QED contributions to be included

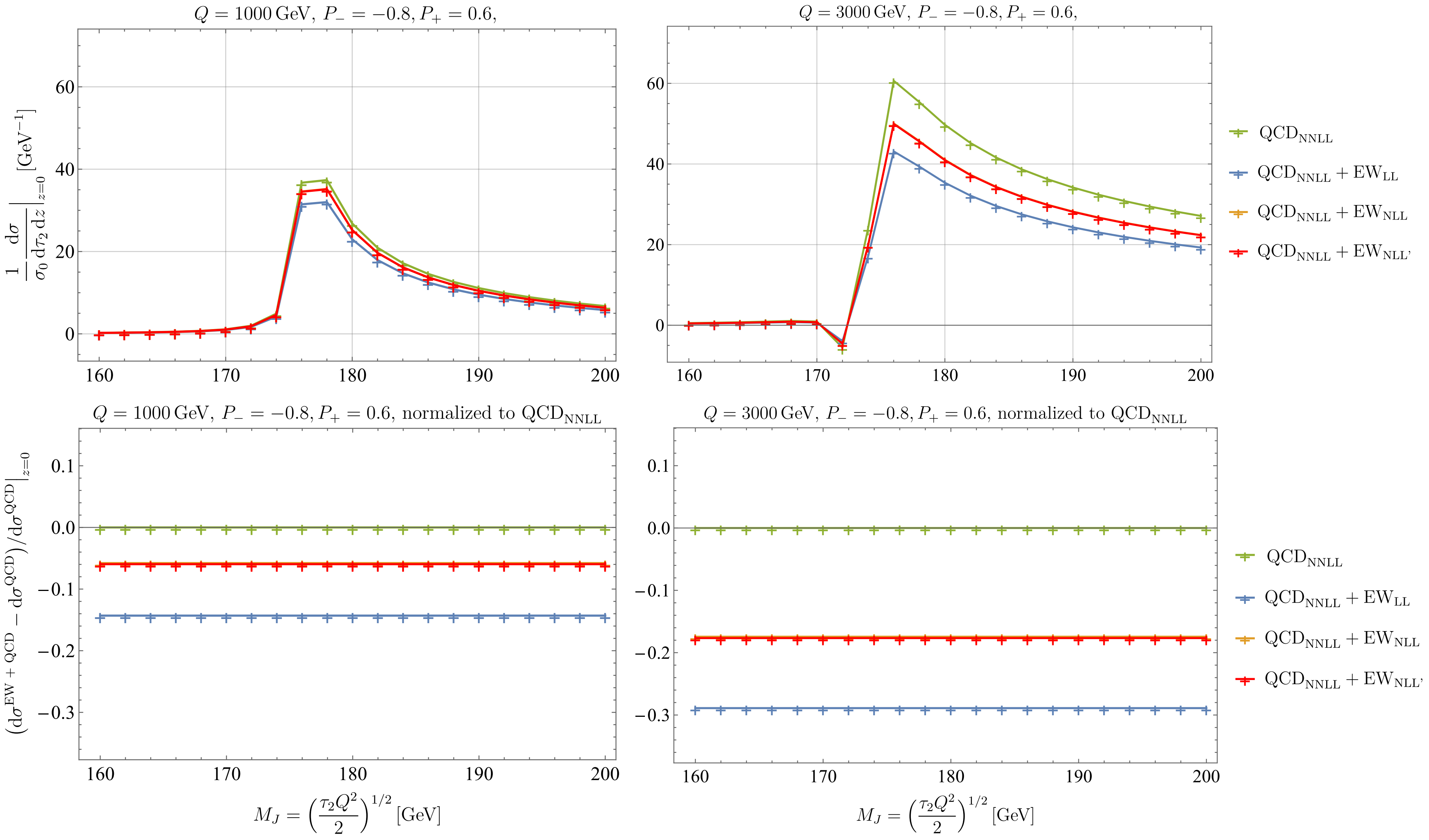


Resummation path: all quantities are evolved to $\mu_M \sim M$

Fixed-order EW contributions



Resummed EW contributions



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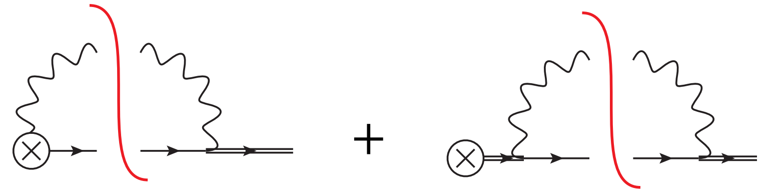
Imaginary parts from *bW*-cuts

$$\frac{\mathrm{d}\sigma_{e^+e^-t\bar{t}}(P_-,P_+)}{\mathrm{d}\tau_2\,\mathrm{d}z} = \sum_{\kappa,\rho} \frac{K_\Phi^{\kappa\rho}\,\phi^\kappa(P_-,P_+)}{\mathcal{F}}\,\mathrm{Tr}\big[\rho_{e^+e^-t\bar{t}}\,\mathcal{M}_c\,\mathcal{M}_s H_Q \mathcal{M}_s^\dagger \mathcal{M}_c^\dagger\big]^{\kappa\rho} * J_{B,t}^{(\Gamma_t)} \otimes \bar{J}_{B,t}^{(\Gamma_t)} \otimes S_{e^+e^-t\bar{t}}$$

- There are imaginary parts in $\mathcal{M}_c$ due to *bW*-cuts
- Diagrammatically at $O(\alpha_W)$:

$\mathcal{M}_c$ =



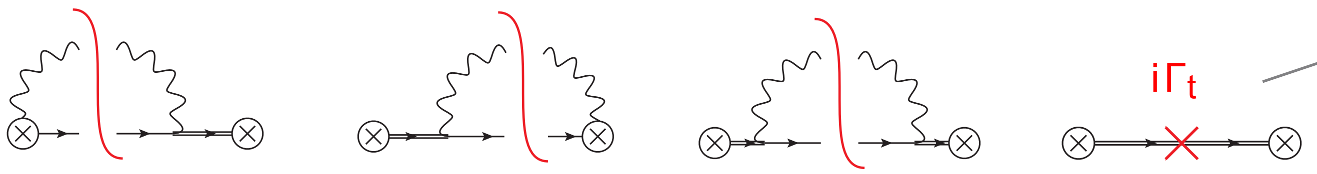
- These imaginary parts cancel in the combination $\mathcal{M}_c\mathcal{M}_c{}^\dagger$
- Resonant/non-resonant interference contributions are obtained by modifying jet function definition

$$\mathrm{Im}\left[\mathrm{Tr}\big[\widetilde{\mathcal{M}}_c^\dagger \rho_{e^+e^-t\bar{t}}\mathcal{M}_c \ldots\big] \times \frac{\mathrm{i}}{4\pi N_c m_t}\int \mathrm{d}^4x\; \mathrm{e}^{-\mathrm{i}px}\langle 0|T[W_n^\dagger h_v](0)[\bar{h}_v W_n](x)\,\not{\bar{n}}|0\rangle\right]$$

- Important: the *bW*-cuts in $\widetilde{\mathcal{M}}_c{}^\dagger$ have the same sign as in $\mathcal{M}_c$!!

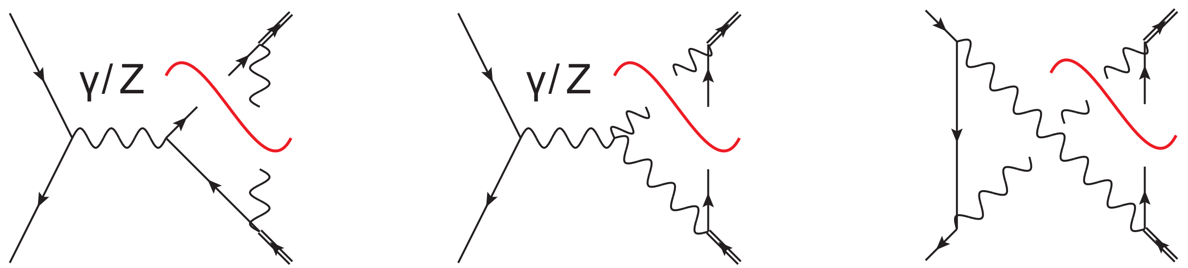
EFT vs. full theory cuts

- $\mathrm{Im}\Big[\,\mathrm{Tr}\big[\widetilde{\mathcal{M}}_c^{\dagger}\rho_{e^+e^-t\bar{t}}\mathcal{M}_c\ldots\big]\Big]$ diagrammatically at $O(\alpha_W)$



Width already part of bHQET Lagrangian and appearing in jet functions

- The corresponding full theory cuts are reproduced

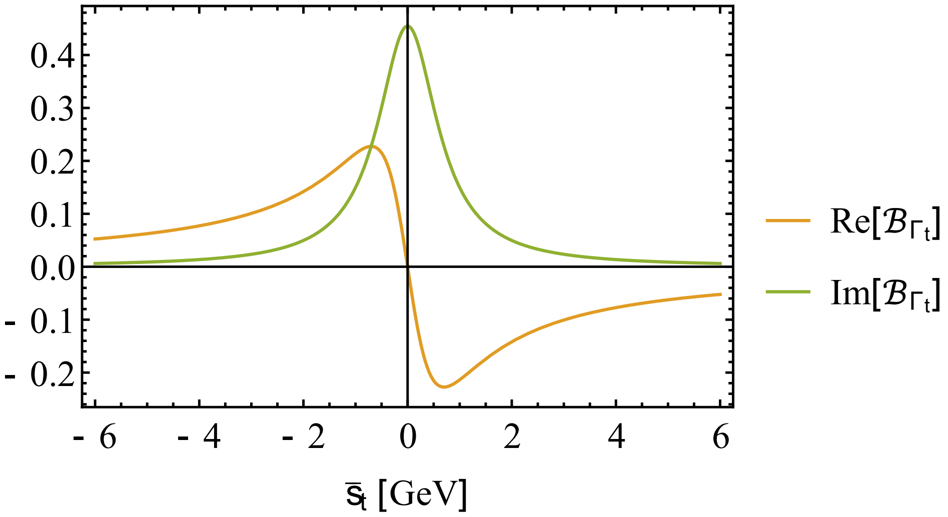


- <u>Important</u>: the mechanism needs
  - ➢ Wilson lines (i.e. factorization of $\mathcal{M}_c$ into top and anti-top contributions) -> cf. rapidity discussion
  - ➢ Factorization of ultracollinear (QCD) from mass-mode (EW) effects -> checked diagrammatically at $O(\alpha_s\alpha_W)$

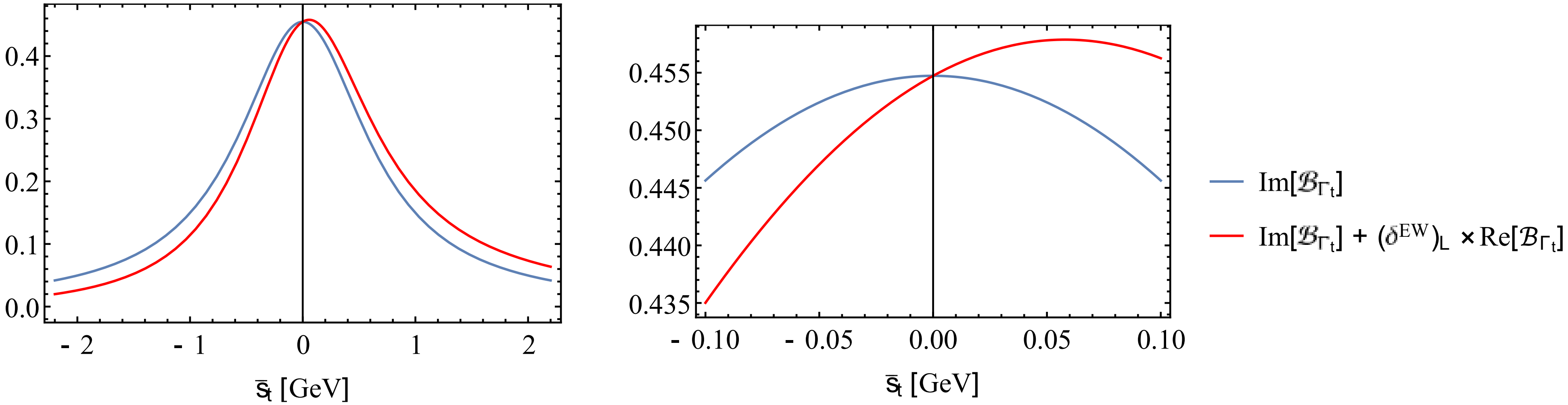
Numerical example

- Interference with tree-level bHQET jet function

$$\mathrm{i}\,\langle 0|\mathrm{T}\,h_t\bar{h}_t|0\rangle^{(\Gamma_t)\mathrm{tree}} \sim -\frac{1}{\pi}\frac{1}{\bar{s}_t+\mathrm{i}\Gamma_t/2} = \mathcal{B}_{\Gamma_t}(\hat{s}_t), \qquad \bar{s}_t = \hat{s}_t/2$$



$$\mathrm{Im}\big[(A+\mathrm{i}B)\mathcal{B}_{\Gamma_t}\big] =$$
$$= A\,\mathrm{Im}\big[\mathcal{B}_{\Gamma_t}\big] + B\,\mathrm{Re}\big[\mathcal{B}_{\Gamma_t}\big]$$



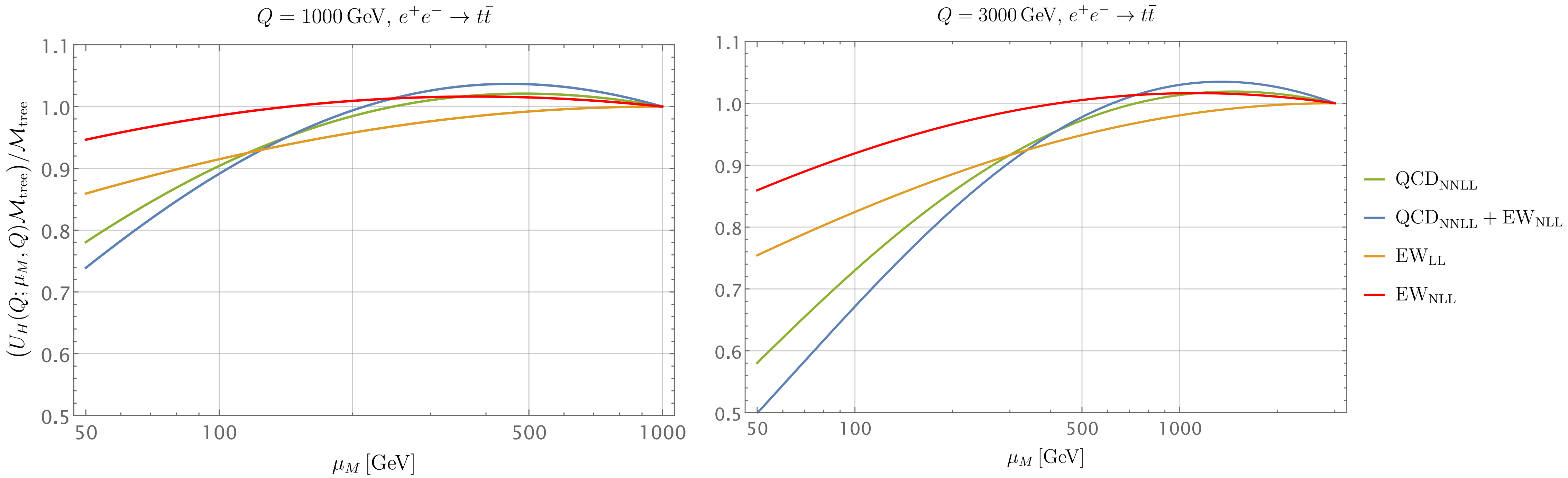
Summary and outlook

- Summary:
  - NLO EW effects incorporated into factorized calculation for top jet pair production (following [Chiu et al, 07, 08, 09])
  - Set up formalism for 2-jettiness cross section (NNLL QCD + NLL' EW) for linear collider environment (including beam polarization effects)
  - Reorganized language concerning isospin structures/operators for more transparent look on SB mechanism and (potential) limitations of the factorized approach
  - Resonant/non-resonant interference: proofs of factorization (diagrammatically at $O(\alpha_s \alpha_W)$) and tree-level study

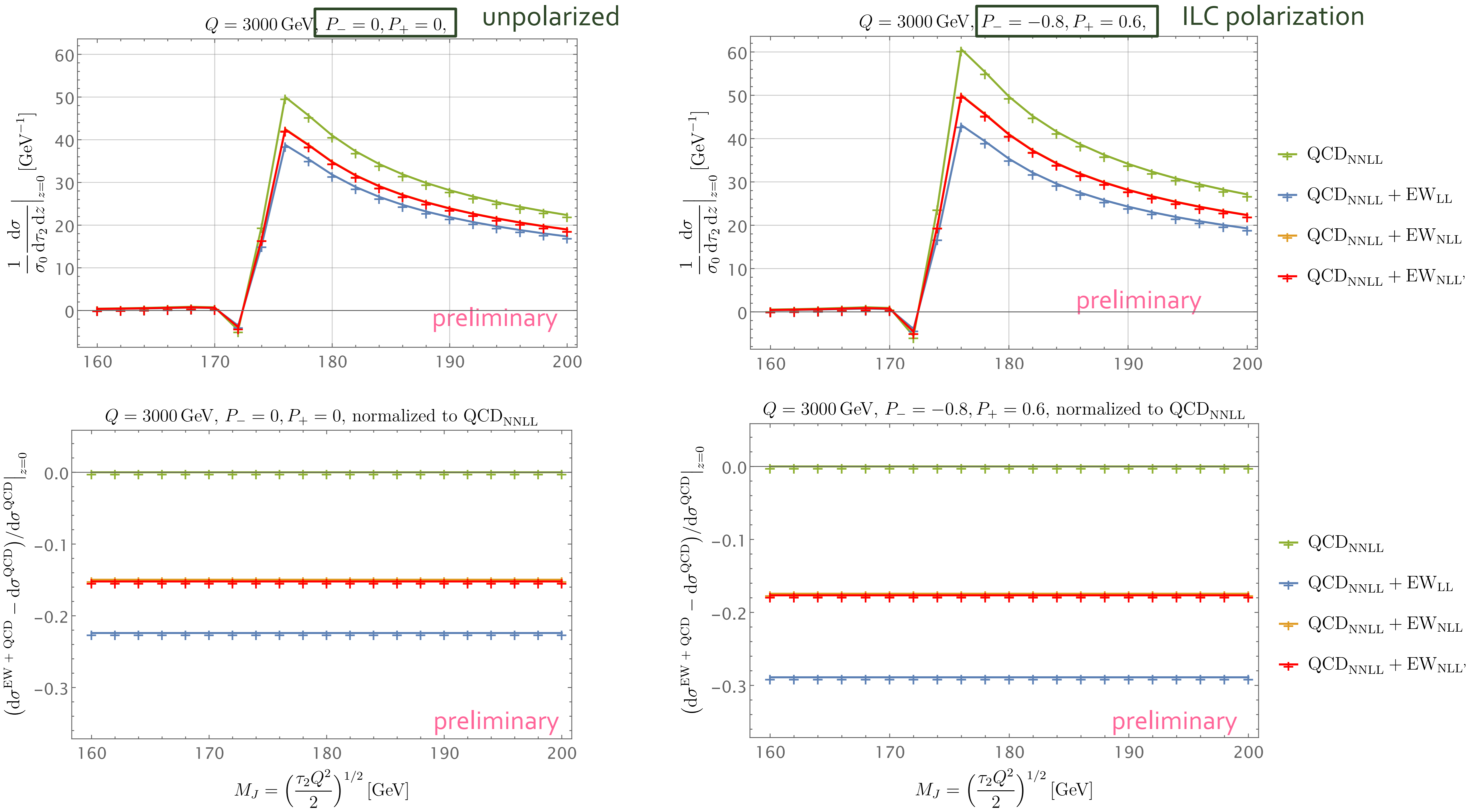
- Outlook:
  - Extend numerical studies of EW resummation effects for 2-jettiness including QCD
  - Coherent treatment of resonant/non-resonant interference and QCD effects
  - Improve understanding of rapidity resummation concept (limitations of EW Wilson line formalism)
  - QED below M
  - Extension to LHC observables

Backup

Sudakov evolution



Chirality of operators shown: left-left (for left-right and right-left: SU(2)-effects roughly half the size)


unpolarized
ILC polarization
$Q = 3000\,\mathrm{GeV}, \boxed{P_- = 0, P_+ = 0,}$
$Q = 3000\,\mathrm{GeV}, \boxed{P_- = -0.8, P_+ = 0.6,}$
$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_2\,dz}\Big|_{z=0}\,[\mathrm{GeV}^{-1}]$
$Q = 3000\,\mathrm{GeV},\ P_- = 0, P_+ = 0$, normalized to $\mathrm{QCD_{NNLL}}$
$Q = 3000\,\mathrm{GeV},\ P_- = -0.8, P_+ = 0.6$, normalized to $\mathrm{QCD_{NNLL}}$
$\left(d\sigma^{\mathrm{EW+QCD}} - d\sigma^{\mathrm{QCD}}\right)/d\sigma^{\mathrm{QCD}}\Big|_{z=0}$
$M_J = \left(\frac{\tau_2 Q^2}{2}\right)^{1/2}\,[\mathrm{GeV}]$
$\mathrm{QCD_{NNLL}}$
$\mathrm{QCD_{NNLL} + EW_{LL}}$
$\mathrm{QCD_{NNLL} + EW_{NLL}}$
$\mathrm{QCD_{NNLL} + EW_{NLL'}}$
preliminary

Forward scattering (45°)

Central scattering (90°)