The GeoSMEFT & some applications

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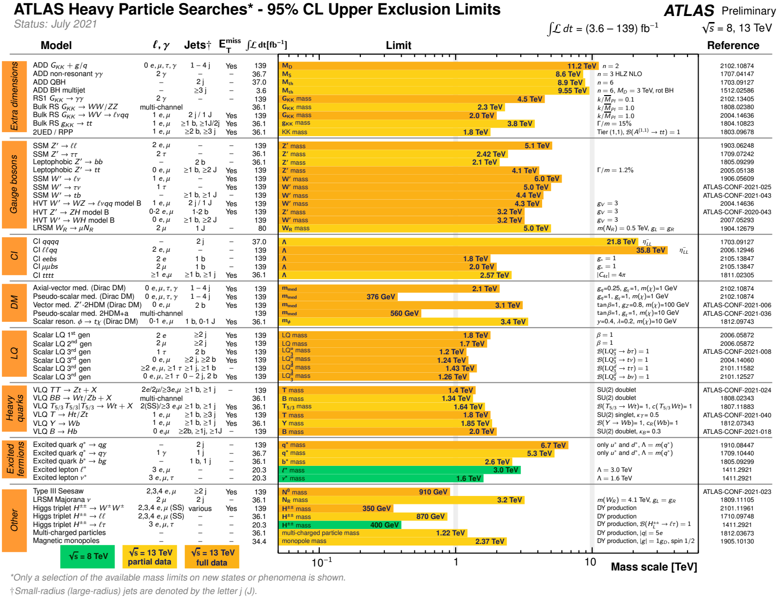
Based on: TC, A. Helset, A. Martin, M. Trott, arXiv:2102.02819

TC, A. Martin, M. Trott, arXiv:2107.07470

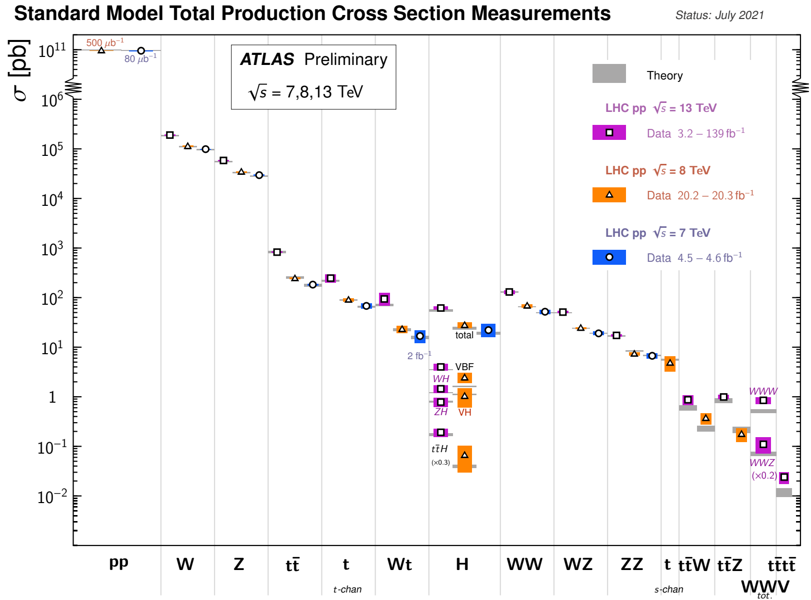
TC, arXiv:2106.10284

Outline

ATLAS Exotics Summary

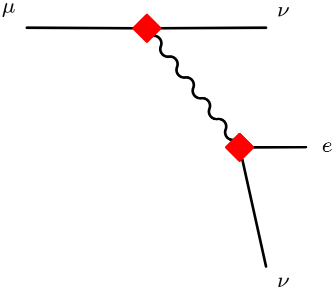


ATLAS SM Summary



The Fermi-theory example

In the SM



$$\mathcal{M} \sim \frac{g_W^2}{2} \frac{(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e)}{k^2 - M_W^2}$$

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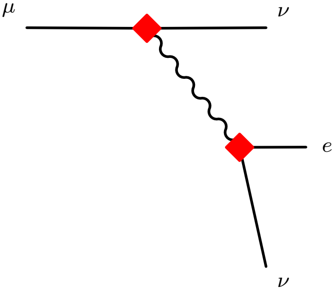
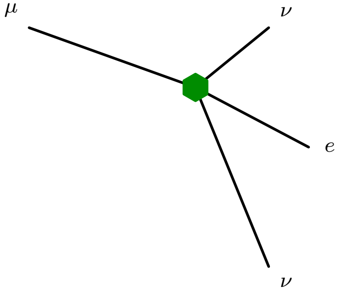


In the Fermi theory



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$$\frac{1}{M_W^2} (\bar{\psi}\gamma^\mu P_L \psi)^2$$

$$\mathcal{M} \sim -\frac{g_{\mathrm{W}}^2}{2M_W^2} (\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) + \cdots$$

The Fermi-theory example

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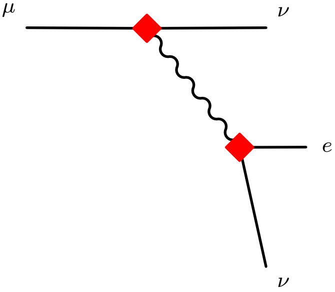

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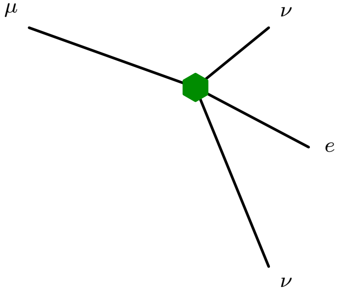

$\frac{1}{M_W^2}(\bar{\psi}\gamma^\mu P_L \psi)^2$

$\frac{1}{M_W^4}\partial^2(\bar{\psi}\gamma^\mu P_L \psi)^2$

$$\mathcal{M} \sim -\frac{g_{\rm W}^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) - \frac{g_{\rm W}^2 k^2}{2M_W^4}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) + \cdots$$

EFTs

Λ_{NP}

The major underlying assumption of any EFT

$\Lambda_{\text{NP}} \gg E$ of the scale of experiments/measurements

E–scale of experiments

EFTs

Λ_{NP}

Top down EFTs:

- example: SM $\rightarrow$ Fermi theory
- Start w/ ultraviolet (UV)–model in mind (Seesaw, SUSY, etc)
- "integrate out" heavy degrees of freedom at UV scale
- model dependent relations between ops, $Q_i^{d>4}$
- Q_i are only in terms of IR degrees of freedom

E–scale of experiments

EFTs

Λ_{NP}

Bottom up EFTs:

- example: Fermi theory
- Start w/ infrared (IR)–model in mind (QED, SM)
- using symmetries of model put together ops, $Q^{d>4}$
- truncate EFT at some $\mathcal{O}(1/\Lambda)$
- constrain Q in experiment & infer properties of NP at Λ
- Qs are unrelated $\to$ model independent
- Qs are only in terms of IR degrees of freedom

E–scale of experiments

SMEFT

In studying NP at $\Lambda_{\text{NP}} \gg v$, we employ the Standard Model EFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots \qquad \mathcal{L}_d = \sum_i c_i Q_i$$

The SMEFT is formed of $\mathcal{L}_{\text{SM}}$ and Q of $d > 4$ respecting SM symmetries & c_i embedding UV physics

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a Taylor series in $\frac{v}{\Lambda}$, $\frac{p}{\Lambda} \ll 1$

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The SMEFT is:
a Taylor series in $\frac{v}{\Lambda}, \frac{p}{\Lambda} \ll 1$

The leading operator:

$$\mathcal{L}_5 = c_{\alpha\beta}(\bar{L}^c_\alpha \tilde{H})(\tilde{H}^\dagger L_\beta) \sim v^2 \bar{\nu}_\alpha \nu_\beta$$

$$\Rightarrow m_\nu \sim v^2/\Lambda$$

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The SMEFT is
a Taylor series in $\frac{v}{\Lambda}$,

$$c_i \Leftrightarrow \frac{c_i}{\Lambda^2}$$

$$c_i^{(n)} \Leftrightarrow \frac{c_i^{(n)}}{\Lambda^{n-4}}$$

T is formed of $\mathcal{L}_{\rm SM}$ and
specting SM symmetries
bedding UV physics

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$$\Rightarrow m_\nu \sim v^2/\Lambda$$

The SMEFT at dimension-six

D6 operators from SM field content ⇒ SMEFT @ D6

Type I: X^3		Type II, III: H^6, H^4D^2		Type V: Ψ^2H^3 + h.c.	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	Q_{eH}	$(H^\dagger H)(\bar{L}eH)$
$Q_{\tilde{G}}$	$f^{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{uH}	$(H^\dagger H)(\bar{Q}u\tilde{H})$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	Q_{HD}	$(H^\dagger D^\mu H)^*(H^\dagger D^\mu H)$	Q_{dH}	$(H^\dagger H)(\bar{Q}dH)$
$Q_{\tilde{W}}$	$\epsilon^{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

Type IV: $X^2\Phi^2$		Type VI: Ψ^2HX		Type VII: Ψ^2H^2D	
Q_{HG}	$(H^\dagger H)G_{\mu\nu}^AG^{A\mu\nu}$	Q_{eW}	$(\bar{L}\sigma^{\mu\nu}e)\tau^IHW_{\mu\nu}^I$	$Q_{HL}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$
$Q_{H\tilde{G}}$	$(H^\dagger H)\tilde{G}_{\mu\nu}^AG^{A\mu\nu}$	Q_{eW}	$(\bar{L}\sigma^{\mu\nu}e)\tau^IHB_{\mu\nu}$	$Q_{HL}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{L}\tau^I\gamma^\mu L)$
Q_{HW}	$(H^\dagger H)W_{\mu\nu}^IW^{I\mu\nu}$	Q_{uG}	$(\bar{Q}\sigma^{\mu\nu}T^Au)\tilde{H}G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$
$Q_{H\tilde{W}}$	$(H^\dagger H)\tilde{W}_{\mu\nu}^IW^{I\mu\nu}$	Q_{uW}	$(\bar{Q}\sigma^{\mu\nu}u)\tau^I\tilde{H}W_{\mu\nu}^I$	$Q_{HQ}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
Q_{HB}	$(H^\dagger H)B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{Q}\sigma^{\mu\nu}u)\tilde{H}B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}\tau^I\gamma^\mu q)$
$Q_{H\tilde{B}}$	$(H^\dagger H)\tilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{Q}\sigma^{\mu\nu}T^Ad)HG_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$
Q_{HWB}	$(H^\dagger\tau^IH)W_{\mu\nu}^IB^{\mu\nu}$	Q_{dW}	$(\bar{Q}\sigma^{\mu\nu}d)\tau^IHW_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$Q_{H\tilde{W}B}$	$(H^\dagger\tau^IH)\tilde{W}_{\mu\nu}^IB^{\mu\nu}$	Q_{dB}	$(\bar{Q}\sigma^{\mu\nu}d)\tilde{H}B_{\mu\nu}$	Q_{Hud}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu d)$

Type VIII: $5\times(\bar{L}L)(\bar{L}L)+7\times(\bar{R}R)(\bar{R}R)+8\times(\bar{L}L)(\bar{R}R)$
$+(\bar{L}R)(\bar{R}L)+4[(\bar{L}R)(\bar{L}R)+\text{h.c.}]=25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi)$

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D6 operators from SM field content ⇒ SMEFT @ D6

Type I: X^3		Type II, III: H^6, H^4D^2		Type V: Ψ^2H^3 + h.c.	
Q_G	f				$(H^\dagger H)(\bar{L}eH)$
$Q_{\tilde{G}}$	f				$(H^\dagger H)(\bar{Q}u\tilde{H})$
Q_W	ϵ				$(H^\dagger H)(\bar{Q}dH)$
$Q_{\tilde{W}}$	ϵ				

Typ				VII: Ψ^2H^2D	
Q_{HG}					$H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$
$Q_{H\tilde{G}}$					$i\overleftrightarrow{D}^I_\mu H)(\bar{L}\tau^I\gamma^\mu L)$
Q_{HW}					$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$
$Q_{H\tilde{W}}$					$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
Q_{HB}					$i\overleftrightarrow{D}^I_\mu H)(\bar{q}\tau^I\gamma^\mu q)$
$Q_{H\tilde{B}}$		Q_{dG}		Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$
Q_{HWB}	$(H^\dagger\tau^I H)W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{Q}\sigma^{\mu\nu}d)\tau^I HW^I_{\mu\nu}$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$Q_{H\tilde{W}B}$	$(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{Q}\sigma^{\mu\nu}d)\tilde{H}B_{\mu\nu}$	Q_{Hud}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu d)$

1. Take Field Content:
   X – Field Strengths
   Ψ – Fermions
   H – Higgs doublets
   D – Covariant Derivatives
2. Form D6 operators:
   X^3, H^6, H^4D^2, Ψ^2H^3, H^2X^2, Ψ^2HX, Ψ^2H^2D, Ψ^4
3. Remove, e.g., $XD\Psi^2$ and $(DX)^2$ via field redef./EOM

$$\text{Type VIII: } 5\times(\bar{L}L)(\bar{L}L) + 7\times(\bar{R}R)(\bar{R}R) + 8\times(\bar{L}L)(\bar{R}R)$$
$$+(\bar{L}R)(\bar{R}L) + 4[(\bar{L}R)(\bar{L}R) + \text{h.c.}] = 25(\bar{\Psi}\Psi)(\bar{\Psi}\Psi)$$

SMEFT: Effective Vertices

T3: $Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$

T3: $Q_{HD} = (H^\dagger D^\mu H)^*(H^\dagger D^\mu H)$

T4: $Q_{HV} = (H^\dagger H)V^{\mu\nu}V_{\mu\nu}$

T4: $Q_{HWB} = (H^\dagger \tau^I H)W^I_{\mu\nu}B^{\mu\nu}$

T5: $Q_{\psi H} = (H^\dagger H)(\bar{\Psi} H \psi)$

T7: $Q^{(3)}_{HL} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{L}\gamma^\mu L)$

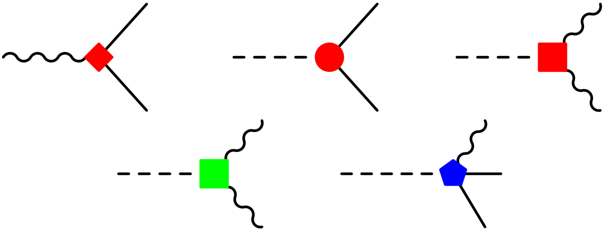
T7: $Q^{(1,3)}_{H\Psi} = (H^\dagger \overleftrightarrow{D}_\mu H)(\bar{\Psi}\gamma^\mu \Psi)$

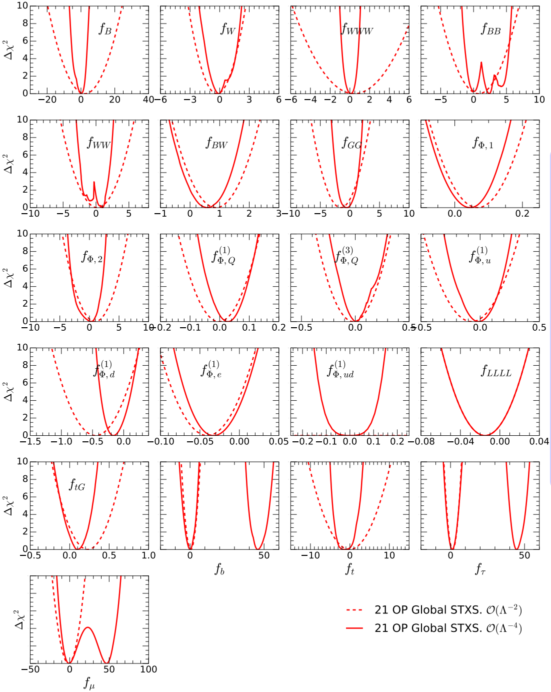
T7: $Q_{H\psi} = (H^\dagger \overleftrightarrow{D}_\mu H)(\bar{\psi}\gamma^\mu \psi)$

T8: $Q_{LL} = (\bar{L}\gamma^\mu L)(\bar{L}\gamma^\mu L)$

SM–like

Non-SM-like kinematic structure





Almeida, Alves, Éboli, Gonzalez-Garcia
arXiv:2108.04828

Uses:

- EWPD
- EW diboson production
- Higgs data

dashed - $\mathcal{O}\left(\frac{1}{\Lambda^2}\right)$

solid - $\sigma\left(\mathcal{O}\frac{1}{\Lambda^4}\right) \times \mathrm{BR}\left(\mathcal{O}\frac{1}{\Lambda^4}\right)$

Ambiguities of $(\mathrm{D6})^2$

Imagine we match a UV model ⇒ this IR Lagrangian: (top-down)

$$\mathcal{L}_{\mathrm{IR}} = \mathcal{L}_{\mathrm{SM}} + \left(\frac{c_{eH}^{(6)}}{\Lambda^2} (H^\dagger H) \bar{L} e H + \frac{c_{eH}^{(8)}}{\Lambda^4} (H^\dagger H)^2 \bar{L} e H + h.c. \right)$$

Equivalence theorem ⇒ can xform fields (consistently) & the S-matrix remains invariant:

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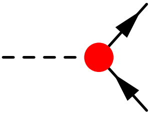
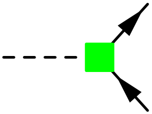
Equivalence theorem ⇒ can xform fields (consistently) & the S-matrix remains invariant:

$$L \to L + \alpha(H^\dagger H)L$$

$$\begin{aligned}\mathcal{L}_{\rm IR} \to \mathcal{L}_{\rm SM} & \\ & + \left(\frac{c_{eH}^{(6)}}{\Lambda^2} - \alpha Y \right)(H^\dagger H)\bar{L}eH + \left(\frac{c_{eH}^{(8)}}{\Lambda^4} + \alpha\frac{c_{eH}^{(6)}}{\Lambda^2} \right)(H^\dagger H)^2\bar{L}eH + \alpha\frac{c_{eH}^{(8)}}{\Lambda^4}(H^\dagger H)^3\bar{L}eH + h.c. \\ & + \frac{i\alpha}{2}\bar{L}\gamma^\mu L\partial_\mu(H^\dagger H) + \frac{i\alpha^2}{2}\bar{L}\gamma^\mu L\partial_\mu(H^\dagger H) + i\alpha(H^\dagger H)\bar{L}\not{D}L + \frac{i\alpha^2}{2}(H^\dagger H)^2\bar{L}\not{D}L + h.c.\end{aligned}$$





Ambiguities of $(\mathrm{D6})^2$ II

In the original IR model as well as for keeping full α dependence:

$$|\mathcal{M}_{h\to\bar{e}e}|^2 = |\mathcal{M}_{h\to\bar{e}e}|^2_\alpha = \frac{1}{v^2}\left[2\bar{m}_e - \sqrt{2}v^3\left(\frac{c^{(6)}_{eH}}{\Lambda^2} + \frac{c^{(8)}_{eH}}{\Lambda^4}v^2\right)\right]^2 (p_1\cdot p_2 - \bar{m}_e^2)$$

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Taking $\alpha \to \frac{\alpha}{\Lambda^2}$, truncating at $\frac{1}{\Lambda^4}$, we compare:

$$\begin{aligned}
|\mathcal{M}_{h\to\bar{e}e}|^2 &= \left[\frac{4\bar{m}_e^2}{v^2} - 4\sqrt{2}\bar{m}_e v\frac{c_{eH}^{(6)}}{\Lambda^2} + 4\sqrt{2}\bar{m}_e v^3\frac{c_{eH}^{(8)}}{\Lambda^4} - 2v^4\frac{[c_{eH}^{(6)}]^2}{\Lambda^4}\right](p_1\cdot p_2 - \bar{m}_e^2) \\
|\mathcal{M}_{h\to\bar{e}e}|^2_\alpha &= \left[\frac{4\bar{m}_e^2}{v^2} - 4\sqrt{2}\bar{m}_e v\frac{c_{eH}^{(6)}}{\Lambda^2} + 4\sqrt{2}\bar{m}_e v^3\frac{c_{eH}^{(8)}}{\Lambda^4} - 2v^4\frac{[c_{eH}^{(6)}]^2}{\Lambda^4}\right](p_1\cdot p_2 - \bar{m}_e^2) \\
&\quad +8\frac{\bar{m}_e^2\alpha}{\Lambda^2}\left(\delta^{(6)}_{\bar{L}\not{D}L} - \delta_{\bar{L}eH}\right)(p_1\cdot p_2 - \bar{m}_e^2) \\
&\quad -2\sqrt{2}\frac{c_{eH}^{(6)}\alpha\bar{m}_e v^3}{\Lambda^4}\left(2\,\delta^{(8)}_{\bar{L}eH} - 3\delta^{(6)}_{\bar{L}\not{D}L} + \delta^{(6)}_{Y}\right)(p_1\cdot p_2 - \bar{m}_e^2) \\
&\quad -4\frac{\bar{m}_e^2 v^2\alpha^2}{\Lambda^4}\left[2\delta^{(6)}_{\bar{L}\not{D}L}\delta^{(6)}_{Y} - 3\left(\delta^{(6)}_{\bar{L}eH}\right)^2 + \delta^{(8)}_{\bar{L}\not{D}L}\right](p_1\cdot p_2 - \bar{m}_e^2)
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Taking $\alpha \to$

Consistent calcs in the SMEFT:

1. Equiv. theorem $\Rightarrow$ S-matrix invariance
2. Fundamental to reduction of basis in EFTs
3. Unique solution only consistent order by order in $\frac{1}{\Lambda}$

$$|\mathcal{M}_{h\to\bar{e}e}|^2 \quad \ldots \quad {}_1\cdot p_2 - \bar{m}_e^2)$$

$$\begin{aligned}
|\mathcal{M}_{h\to\bar{e}e}|^2_\alpha = & \left[\frac{4\bar{m}_e^2}{v^2} - 4\sqrt{2}\bar{m}_e v\frac{c^{(6)}_{eH}}{\Lambda^2} + 4\sqrt{2}\bar{m}_e v^3\frac{c^{(8)}_{eH}}{\Lambda^4} - 2v^4\frac{[c^{(6)}_{eH}]^2}{\Lambda^4}\right](p_1\cdot p_2 - \bar{m}_e^2) \\
& +8\frac{\bar{m}_e^2\alpha}{\Lambda^2}\left(\delta^{(6)}_{\bar{L}\not{D}L} - \delta_{\bar{L}eH}\right)(p_1\cdot p_2 - \bar{m}_e^2) \\
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\end{aligned}$$

the geoSMEFT (op forms $\mathcal{W}^{A,\mu\nu}\mathcal{W}^{B}_{\mu\nu}$)

(the very simplified version, see Helset, Martin, Trott arXiv:2001.01453)

1. Take SM field content, X, Ψ, DH, D
2. Form Lorentz invariant, but gauge variant combos of 2 or 3 fields
3. stick towers of $H^\dagger$, H, and/or τ^A $\rightarrow$ gauge invariant
   use Hilbert Series to confirm all operators included/no redundancy
   (see e.g. Lehman & Martin 2015, Henning et al. 2015)

The operators are:

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@ D4: $-\frac{1}{4}W^{A,\mu\nu}W^{A}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

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@ D6 $\frac{1}{\Lambda^2}$: $c_{HB}|H|^2B^{\mu\nu}B_{\mu\nu}$ $\quad$ $c_{HW}|H|^2W^{A,\mu\nu}W^{A}_{\mu\nu}$ $\quad$ $c_{HWB}(H^\dagger\sigma^a H)W^{A}_{\mu\nu}B^{\mu\nu}$

the geoSMEFT (op forms $\mathcal{W}^{A,\mu\nu}\mathcal{W}^{B}_{\mu\nu}$)

(the very simplified version, see Helset, Martin, Trott arXiv:2001.01453)

1. Take SM field content, X, Ψ, DH, D
2. Form Lorentz invariant, but gauge variant combos of 2 or 3 fields
3. stick towers of $H^\dagger$, H, and/or τ^A $\rightarrow$ gauge invariant
   use Hilbert Series to confirm all operators included/no redundancy
   (see e.g. Lehman & Martin 2015, Henning et al. 2015)

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$c^{(8)}_{HW,2}(H^\dagger\sigma^a H)(H^\dagger\sigma^b H)W^{\mu\nu}_{a}W_{b,\mu\nu}$

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$c^{(8)}_{HW,2}(H^\dagger\sigma^a H)(H^\dagger\sigma^b H)W^{\mu\nu}_a W_{b,\mu\nu}$

@ D10 $\frac{1}{\Lambda^6}$: $c^{(8)}_{HB}|H|^6B^{\mu\nu}B_{\mu\nu}$ $\quad c^{(8)}_{HW}|H|^6W^{A,\mu\nu}W^A_{\mu\nu}$ $\quad c^{(8)}_{HWB}|H|^4(H^\dagger\sigma^a H)W^A_{\mu\nu}B^{\mu\nu}$

$c^{(8)}_{HW,2}|H|^2(H^\dagger\sigma^a H)(H^\dagger\sigma^b H)W^{\mu\nu}_a W_{b,\mu\nu}$

$\cdots$ *ad infintum* $\cdots$

Operator forms $\mathcal{W}^{A,\mu\nu}\mathcal{W}^{B}_{\mu\nu}$ Part II

Defining:

$$\mathcal{W}^A_\mu = \{W^1_\mu, W^2_\mu, W^3_\mu, B_\mu\}$$

We can simplify the all orders expression:

$$\begin{aligned}\mathcal{L}_{\mathcal{W}^A\mathcal{W}^B} &= -\frac{1}{4}W^{A,\mu\nu}W^A_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + c_{HB}|H|^2B^{\mu\nu}B_{\mu\nu} + c_{HW}|H|^2W^{A,\mu\nu}W^A_{\mu\nu} + \cdots \\ &\equiv -\frac{1}{4}g_{AB}\mathcal{W}^{A,\mu\nu}\mathcal{W}^B_{\mu\nu}\end{aligned}$$

Operator forms $\mathcal{W}^{A,\mu\nu}\mathcal{W}^{B}_{\mu\nu}$ Part II

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$$\begin{aligned}g_{AB} \quad = \quad & \left[1 - 4\sum_{n=0}^{\infty}\left(c_{HW}^{(6+2n)}(1-\delta_{A4}) + c_{HB}^{(6+2n)}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\ & - \sum_{n=0}^{\infty}c_{HW,2}^{(8+2n)}\left(\frac{\phi^2}{2}\right)^n(\phi_I\Gamma^I_{A,J}\phi^J)(\phi_L\Gamma^L_{B,K}\phi^K)(1-\delta_{A4})(1-\delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty}c_{HWB}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^n\right]\left[(\phi_I\Gamma^I_{A,J}\phi^J)(1-\delta_{A4})\delta_{B4} + (A\leftrightarrow B)\right]\end{aligned}$$

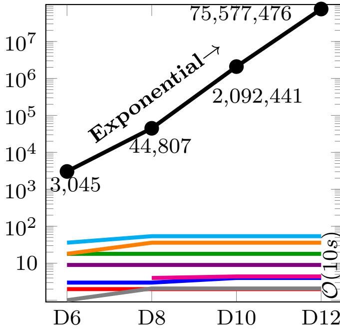
three–point functions from geoSMEFT

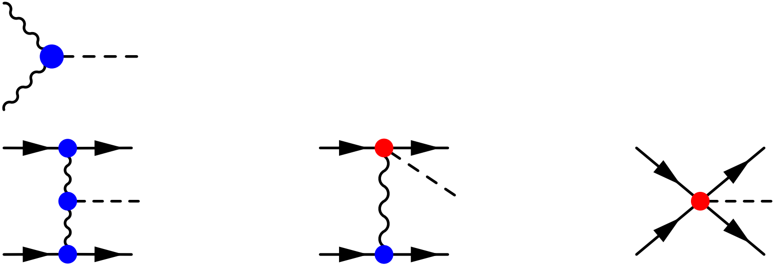
operator form	shifts:
$h_{IJ}(D\phi)^I(D\phi)^J$	SM 3–point functions + Masses
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	SM triple gauge couplings + $h(\partial V)^2$ + mixing angles
$Y^\psi\bar{\Psi}_L\psi_R + h.c.$	SM Yukawas+ψ masses
$L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$	SM gauge-fermion couplings
$d^\psi_A W^{A,\mu\nu}(\bar{\psi}\sigma_{\mu\nu}\psi)$	Dipoles
$f_{ABC}W^{A,\mu\nu}W^B_{\nu\rho}W^{C,\rho}_\mu$	new TGCs $(\partial V)^3$
$\kappa^A_{IJ}(D_\mu\phi)^I(D_\nu\phi)^J W^A_{\mu\nu}$	new TGCs $(\partial h)^2(\partial V)$, removed from D6 in Warsaw

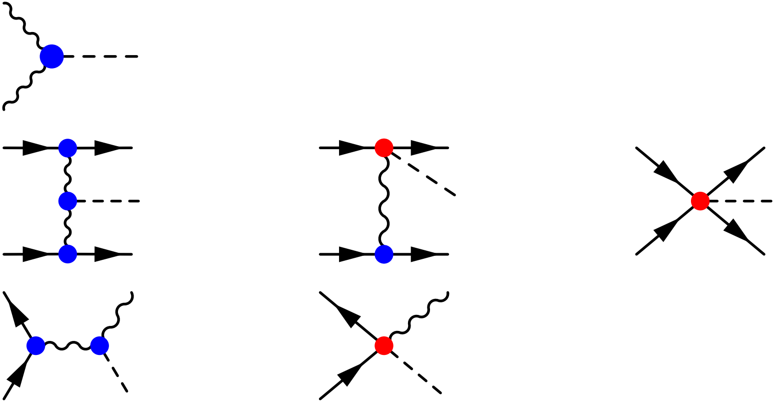
Saturation of number of operators

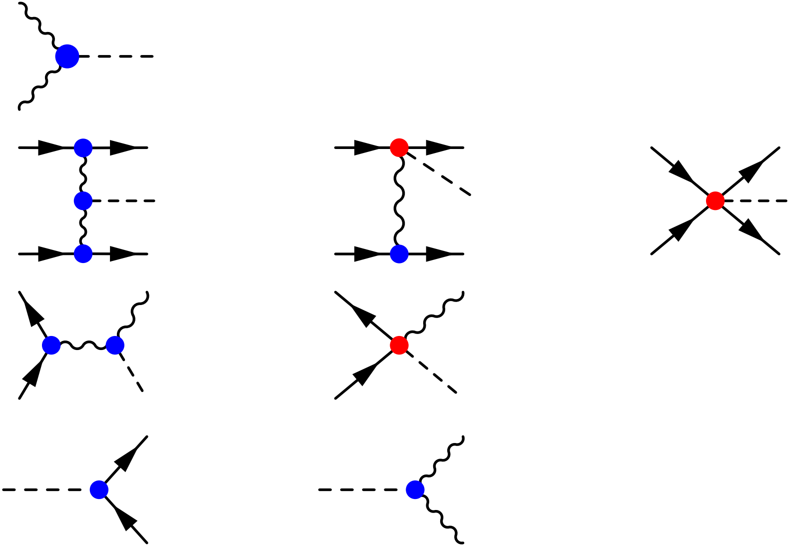
(This information is contained in the Hilbert Series)
(see e.g. Lehman & Martin 2015, Henning et al. 2015)

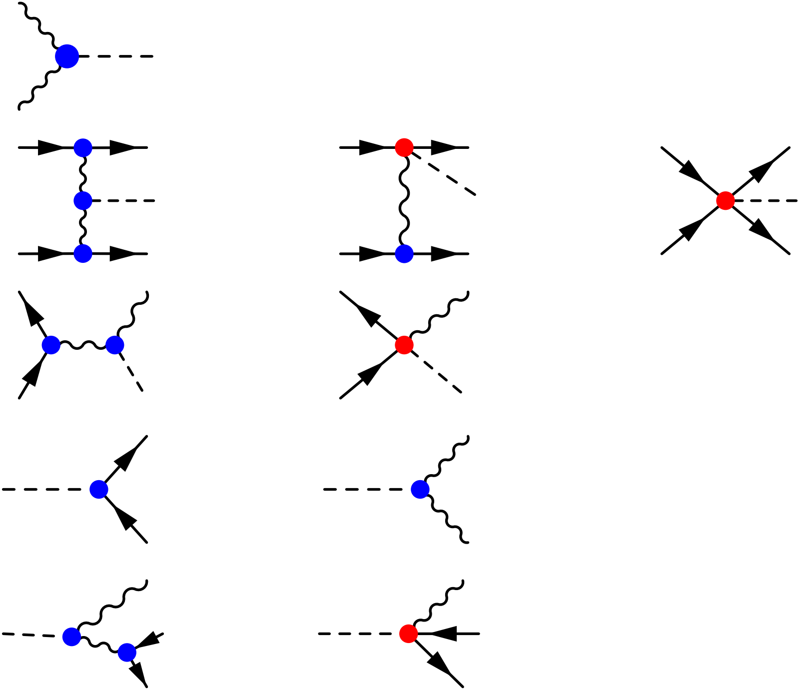
Operator form:	Mass Dimension 6	8	10
$h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	3	4	4
$k_{IJA}(D^\mu\phi)^I(D^\nu\phi)^J W^A_{\mu\nu}$	0	3	4
$f_{ABC}W^A_{\mu\nu}W^{B,\nu\rho}W^{C,\mu}_\rho$	1	2	2
$Y^\psi_{pr}\bar{\Psi}_L\psi_R + h.c.$	$2N_f^2$	$2N_f^2$	$2N_f^2$
$d^{\psi,pr}_A\bar{\Psi}_L\sigma_{\mu\nu}\psi_R W^{\mu\nu}_A + h.c.$	$4N_f^2$	$6N_f^2$	$6N_f^2$
$L^{\psi_R}_{pr,J,A}(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2
$L^{\Psi_L}_{pr,J,A}(D^\mu\phi)^J(\bar{\Psi}_{p,L}\gamma_\mu\sigma_A\Psi_{r,L})$	$2N_f^2$	$4N_f^2$	$4N_f^2$

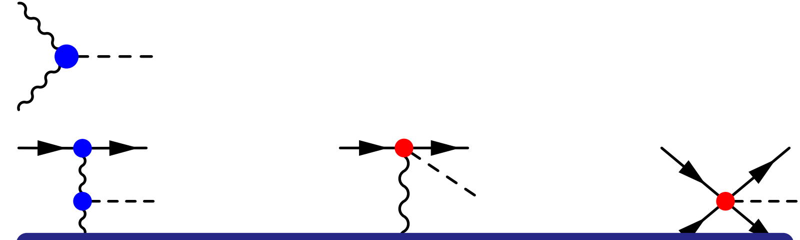










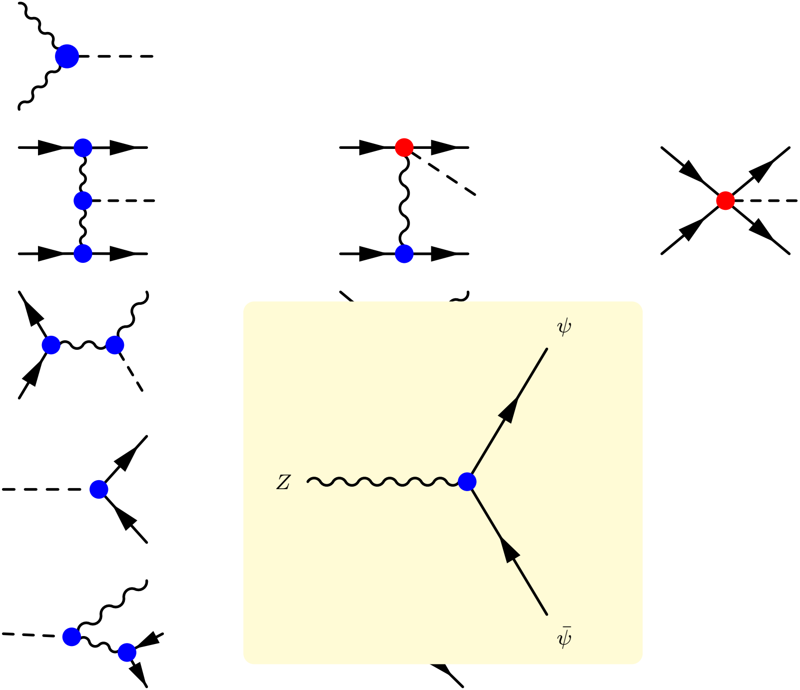


To-do list/(soon-to-be) low hanging fruit:

1. H production up to and including 4-point functions (soon)
2. H decay up to and including 4-point functions (soon)
3. Package for global fitters/experimentalists to obtain results above

4. dibosons
5. tops

6. differential/kinematics to try and isolate operator contributions (See e.g. TC, T. Rasmussen arXiv:2110.03694)
7. Simulations for (HL-)LHC and future colliders



ψ
Z
$\bar{\psi}$

geoSMEFT summary

1. all 3-point functions defined for both $\frac{v^2}{\Lambda^2}$ and $\frac{p^2}{\Lambda^2}$ expansions
   for 3pt only: $\{m_i^2\}$
2. largest Higgs production xs (hgg) can be defined in geoSMEFT
   ✓ + D6-loop (TC, A Martin, M Trott, arXiv:2107.07470)
3. largest Higgs decay (hbb) + most accurately measured ($h\gamma\gamma$)
   for $h\gamma\gamma$:✓ + D6-loop (TC, A Martin, M Trott, arXiv:2107.07470)
4. many other all-orders results (e.g. $hZ\gamma$)
   C. Hays, A. Helset, A. Martin, M Trott, arXiv:2007.00565
5. Z-pole predictions can be derived to all orders
6. currently expanding to include 4-point functions
   only all orders in $\frac{v^2}{\Lambda^2}$ expansion
   for 4pt have infinitely many $\frac{p^2}{\Lambda^2}$ operators: $\{s^n, t^m, m_i^2\}$

geoSMEFT on the Z-pole

operator form	shifts:
$h_{IJ}(D\phi)^I(D\phi)^J$	SM 3–point functions + Masses
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	SM triple gauge couplings + $h(\partial V)^2$ + mixing angles
$L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$	SM gauge-fermion couplings

$$\psi = \{Q, L, u_R, d_R, e_R\}$$

$$g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2\, s^2_{\theta_Z}\, Q_\psi - \sigma_3) + \bar{v}_T\, \langle L^\psi_{3,4}\rangle + \sigma_3 \bar{v}_T\, \langle L^\psi_{3,3}\rangle\right]$$

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$$\bar{m}^2_Z = \frac{\bar{g}_Z}{4}^2 \langle h_{33}\rangle \bar{v}^2_T \qquad s^2_{\theta_Z} = f(\langle g_{AB}\rangle, g_1, g_2)$$

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$$\bar m_Z^2 = \frac{\bar g_Z}{4}^2 \langle h_{33}\rangle \bar v_T^2 \qquad s^2_{\theta_Z} = f(\langle g_{AB}\rangle, g_1, g_2)$$

$$\Gamma_{Z\to\bar\psi\psi} = \frac{N_c^\psi}{24\pi}\bar m_Z |g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar m_\psi^2}{\bar m_Z^2}\right)^{3/2}$$

Z-pole pheno, arXiv:2102.02819

1. **Calculate the following:**
   $\Gamma_{e,\mu}$, Γ_τ, Γ_ν, Γ_c, Γ_b, Γ_Z,
   R_l, R_c, R_b, $A^l_{\rm FB}$, $A^c_{\rm FB}$, $A^b_{\rm FB}$, $\sigma^0_{\rm Had}$
2. **Supplement with SM predictions up to two-loops**
   Awramik et al. hep-ph/0608099, Dubovyk et al. arXiv:1906.08815
   Freitas arXiv: 1401.2447, Awramik et al. hep-ph/0311148
3. **Interpolate SM pred. to** $\{M_W, M_Z, G_F\}$ **input scheme from** $\{\alpha, M_Z, G_F\}$
   $\rightarrow$ allows for comparison of scheme dependence

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   Need alternative approach to studying impact of D8
   Brivio, Trott, arXiv:1703.10924

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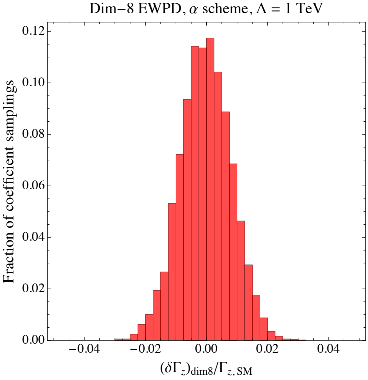
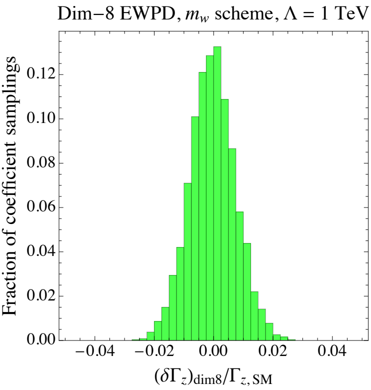
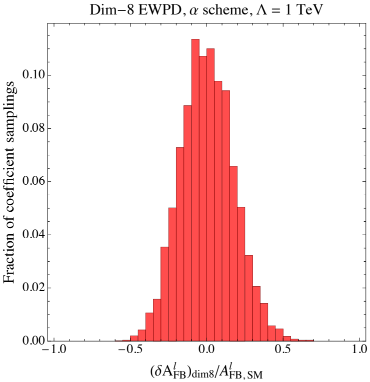
   Brivio, Trott, arXiv:1703.10924
5. **use $\delta\mathcal{O}_8/\mathcal{O}_{\rm SM}$ as a measure of impact of D8 operators on $\mathcal{O}$**
6. **take $\Lambda = 1$ TeV**
7. **randomly sample values of c_i**
   assume WCs are 0 ± 1 ("tree"–)
   and $0 \pm .01$ ("loop"–induced operator assumption)

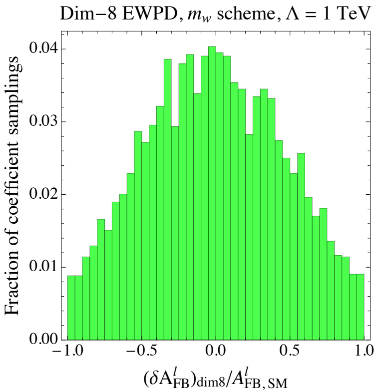
Z-pole pheno, arXiv:2102.02819





Z-pole pheno, arXiv:2102.02819





Z-pole pheno, arXiv:2102.02819

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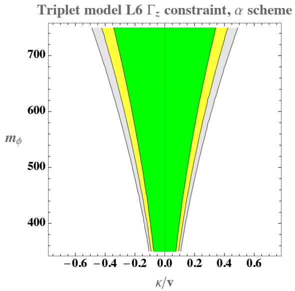
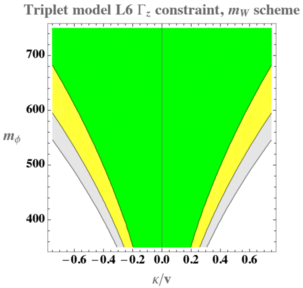
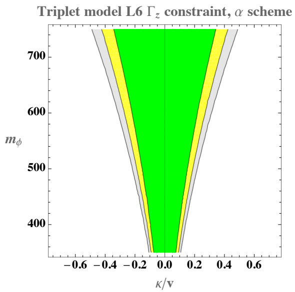
   $\rightarrow$ allows for comparison of scheme dependence
4. ***Z*–pole data alone is not able to constrain all contributing WCs**
   Need alternative approach to studying impact of D8
   Brivio, Trott, arXiv:1703.10924
5. **Take triplet scalar model w $Y_\Phi = 0$ and match to SMEFT to D8**
6. **Perform χ^2 analysis to *Z*–pole data**
   gives constraints in the M_Φ–κ plane ($2\kappa H^\dagger \tau^a H \Phi^a$, $\eta |H|^2 \Phi^2$ w/ $\eta = .1$)

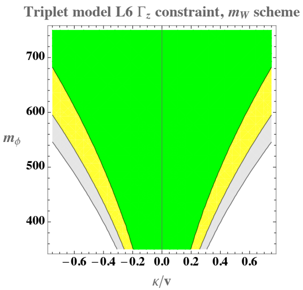
Z-pole pheno, arXiv:2102.02819

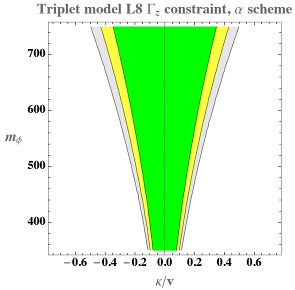


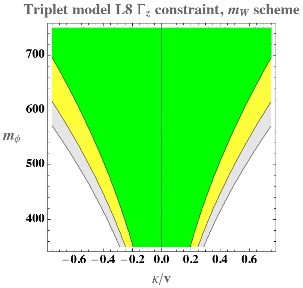


Z-pole pheno, arXiv:2102.02819

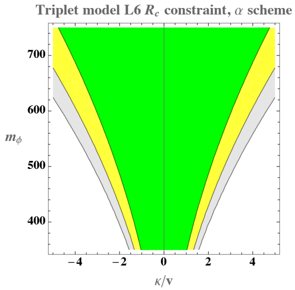


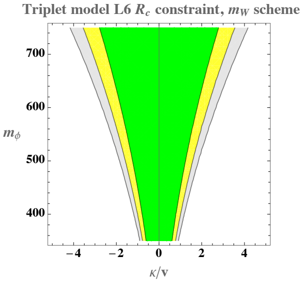






Z-pole pheno, arXiv:2102.02819





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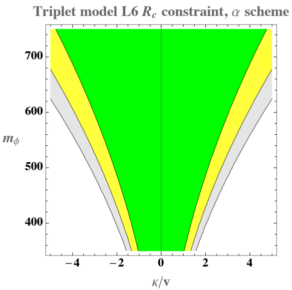

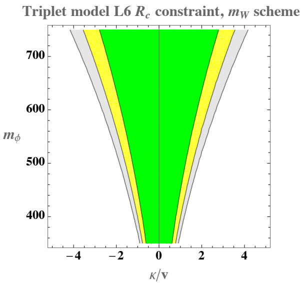

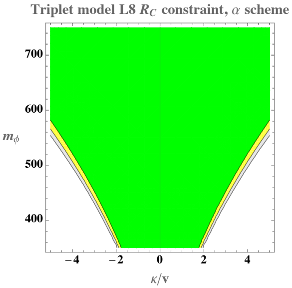

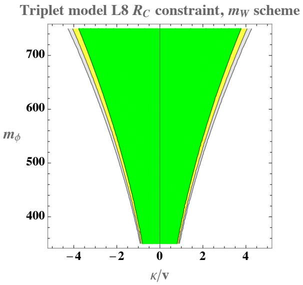

Z-pole quick conclusions

The full set of Z-pole observables have been calculated to $\frac{1}{\Lambda^4}$ in arXiv:2102.02819

- not enough data to constrain all the WCs with Z-pole data alone
- explored random sampling of WCs and impact of D8
- explored matching to D8 and impact of D8
- indicates naive D8 impact of $\sim$ % in most cases, but EWPD is per mil
- in phenomenological ex. w matching, slight broadening of 99% region large broadening of 68 and 95% regions

Ward identities & the (geo)SMEFT

TC, A. Helset, M. Trott arXiv:1909.08470

1. Let the generator of 1-particles irreducible diagrams be Γ
2. Using the background field method $\Rightarrow$ Γ is invariant under gauge transforms:

$$\frac{\delta\Gamma}{\delta\alpha^B} = 0 = \left(\partial^\mu \delta^A_B - \tilde{\epsilon}^A_{BC} W^{C,\mu}\right) \frac{\delta\Gamma}{\delta W^\mu_A} - \frac{\tilde{\gamma}^I_{B,J}}{2}\phi^J \frac{\delta\Gamma}{\delta\phi^I} + \text{(fermions)}$$

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3. Apply *all-orders xforms* to the mass eigenstates, $\langle\sqrt{g}^{AB}\rangle U_{BC}$, $\langle\sqrt{h}^{IJ}\rangle V_{JK}$:

$$\frac{\delta\Gamma}{\delta\beta^B} = 0 = \partial^\mu \frac{\delta\Gamma}{\delta A^{X,\mu}} - \frac{\delta\Gamma}{\delta A^{C,\mu}} \bar{\epsilon}^C_{XY} A^{Y,\mu} - \frac{\delta\Gamma}{\delta\Phi^K} V^{KJ} \langle\sqrt{h}_{JI}\rangle \bar{\gamma}^I_{XL} \langle\sqrt{h}^{LM}\rangle V_{MN} \Phi^N$$

β, $\bar{\epsilon}$, $\bar{\gamma}$ are rotated $\{\alpha, \tilde{\epsilon}, \tilde{\gamma}\}$

Ward identities & the (geo)SMEFT

TC, A. Helset, M. Trott arXiv:1909.08470

1. Let the generator of 1-particles irreducible diagrams be Γ

2. Using the background field method $\Rightarrow$ Γ is invariant under gauge transforms:

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β, $\bar{\epsilon}$, $\bar{\gamma}$ are rotated $\{\alpha, \tilde{\epsilon}, \tilde{\gamma}\}$

4. Obtain Ward IDs by taking variations w/r to fields
then setting fields to their vevs ($A^\mu \to 0,\ \Phi^{I\neq 4} \to 0,\ \Phi^4 = v$)

Ward identities & the (geo)SMEFT

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Ward identities & the (geo)SMEFT

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Ward identities & the (geo)SMEFT

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3. Take a variation w/r to Φ^3 and choose $X = 3$:

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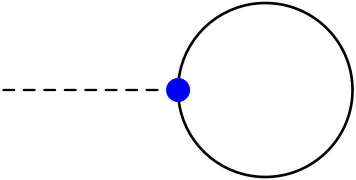
The tadpole in the geoSMEFT

TC, arXiv:2106.10284



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operator form	shifts	contributes?
$h_{IJ}(D\phi)^I(D\phi)^J$	SM 3pt + M	
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	SM TGCs + $h(\partial V)^2$ + θ	
$Y^\psi\bar{\Psi}_L\psi_R + h.c.$	SM Yukawas+M_ψ	
$L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$	SM-like $V\psi\psi$ couplings	
$d^\psi_A W^{A,\mu\nu}(\bar{\psi}\sigma_{\mu\nu}\psi)$	Dipoles	
$f_{ABC}W^{A,\mu\nu}W^B_{\nu\rho}W^{C,\rho}_\mu$	new TGCs $(\partial V)^3$	
$\kappa^A_{IJ}(D_\mu\phi)^I(D_\nu\phi)^J W^A_{\mu\nu}$	new TGCs $(\partial h)^2(\partial V)$	

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$L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$	SM-like $V\psi\psi$ couplings	no, $(H^\dagger \overleftrightarrow{D} H)$
$d^\psi_A W^{A,\mu\nu}(\bar{\psi}\sigma_{\mu\nu}\psi)$	Dipoles	no, too many fields
$f_{ABC}W^{A,\mu\nu}W^B_{\nu\rho}W^{C,\rho}_\mu$	new TGCs $(\partial V)^3$	no, too many fields
$\kappa^A_{IJ}(D_\mu\phi)^I(D_\nu\phi)^J W^A_{\mu\nu}$	new TGCs $(\partial h)^2(\partial V)$	no, momentum dependent

The simple example

All "Class 5" operators are summed in the field-space connection Y^ψ:

$$\mathcal{L}_{\rm cl5} = \mathcal{Y}^\psi(\phi)\bar{\Psi}\psi$$

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$$\begin{aligned}\{h, \bar{\psi}, \psi\} &= -i\left\langle \frac{\delta \mathcal{Y}^{\psi}}{\delta h} \right\rangle \\ &= i\frac{\langle\sqrt{h}^{44}\rangle}{v}\bar{M}_{\psi} - i\frac{\langle\sqrt{h}^{44}\rangle}{\sqrt{2}}\sum_{n=0}^{\infty} c_{\psi H}^{(6+2n)}\frac{v^{2n+2}}{2^{n+1}}(2n+2)\end{aligned}$$

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The contribution to the tadpole is then simply:

$$T_H^{\psi} = -\frac{N_c \bar{M}_{\psi}}{4\pi^2}\left\langle \frac{\delta \mathcal{Y}^{\psi}}{\delta h} \right\rangle \bar{M}_{\psi}^2 A_0(\bar{M}_{\psi})$$

$$T_H^W = \frac{\bar{M}_W^2}{16\pi^2}\left[(\sqrt{g}^{11})^2\left\langle\frac{\delta g_{11}}{\delta\hat{h}}\right\rangle - \frac{2}{v}\sqrt{h}^{44} - \left\langle\frac{\delta h_{11}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{11})^2\right]\left[2\bar{M}_W^2 - 3A_0(\bar{M}_W) - \xi_W A_0(\sqrt{\xi_W}\bar{M}_W)\right]$$

$$T_H^Z = \frac{\bar{M}_Z^2}{32\pi^2}\left[\Sigma_{ZZ} - \frac{2}{v}\sqrt{h}^{44} - \left\langle\frac{\delta h_{33}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{33})^2\right]\left[2\bar{M}_Z^2 - 3A_0(\bar{M}_Z) - \xi_W A_0(\sqrt{\xi_W}\bar{M}_Z)\right]$$

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$$T_H^{u^\pm} = \frac{\bar{M}_W^2}{8\pi^2}\left[\left\langle\frac{\delta g_{11}}{\delta\hat{h}}\right\rangle(\sqrt{g}^{11})^2 - \frac{2}{v}\sqrt{h}^{44} - \left\langle\frac{\delta h_{11}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{11})^2\right]\xi_W A_0(\sqrt{\xi_W}\bar{M}_W)$$

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$$T_H^{\Phi^\pm} = \frac{\bar{M}_W^2}{16\pi^2}\left[\frac{2}{v}\sqrt{h}^{44} + \left\langle\frac{\delta h_{11}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{11})^2 + \left\langle\frac{\delta g^{11}}{\delta\hat{h}}\right\rangle(\sqrt{g}^{11})^2\right]\xi_W A_0(\sqrt{\xi_W}\bar{M}_W)$$
$$+\frac{v}{32\pi^2}(\sqrt{h}^{11})^2\sqrt{h}^{44}\left(4\lambda - \sum_{n=3}^{\infty}\frac{1}{2^{n-3}}\binom{n}{1,1,n-2}v^{2n-4}c_H^{(2n)}\right)A_0(\sqrt{\xi_W}\bar{M}_W)$$

$$T_H^{\Phi^0} = \frac{\bar{M}_Z^2}{32\pi^2}\left[\frac{2}{v}\sqrt{h}^{44} + \left\langle\frac{\delta h_{33}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{33})^2 - \Sigma_{ZZ}\right]\xi_W A_0(\sqrt{\xi_W}\bar{M}_Z)$$
$$+\frac{v}{64\pi^2}(\sqrt{h}^{33})^2\sqrt{h}^{44}\left(4\lambda - \sum_{n=3}^{\infty}\frac{1}{2^{n-3}}\binom{n}{1,1,n-2}v^{2n-4}c_H^{(2n)}\right)A_0(\sqrt{\xi}\bar{M}_Z)$$

The one-loop tadpole in the geoSMEFT

$$
\begin{aligned}
T_H^{V,u,\Phi} \quad = \quad & \frac{\bar{M}_W^2}{16\pi^2}\left[(\sqrt{g}^{11})^2\left\langle\frac{\delta g_{11}}{\delta\hat{h}}\right\rangle - \frac{2}{v}\sqrt{h}^{44} - \left\langle\frac{\delta h_{11}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{11})^2\right]\left[2\bar{M}_W^2 - 3A_0(\bar{M}_W)\right] \\
& + \frac{\bar{M}_Z^2}{32\pi^2}\left[\Sigma_{ZZ} - \frac{2}{v}\sqrt{h}^{44} - \left\langle\frac{\delta h_{33}}{\delta\hat{h}}\right\rangle(\sqrt{h}^{33})^2\right]\left[2\bar{M}_Z^2 - 3A_0(\bar{M}_Z)\right] \\
& + (\lambda, c_H \text{ dependence})
\end{aligned}
$$

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$$
\begin{aligned}
T_H \quad = \quad & -\sum_{\psi} \frac{N_c \bar{M}_\psi}{4\pi^2} \left\langle \frac{\delta \mathcal{Y}^\psi}{\delta \hat{h}} \right\rangle A_0(\bar{M}_\psi) \\
& + \frac{\bar{M}_W^2}{16\pi^2} \left[(\sqrt{g}^{11})^2 \left\langle \frac{\delta g_{11}}{\delta \hat{h}} \right\rangle - \frac{2}{v} \sqrt{h}^{44} - \left\langle \frac{\delta h_{11}}{\delta \hat{h}} \right\rangle (\sqrt{h}^{11})^2 \right] \left[2\bar{M}_W^2 - 3A_0(\bar{M}_W) \right] \\
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& + \frac{v}{32\pi^2} (\sqrt{h}^{11})^2 \sqrt{h}^{44} \left(4\lambda - \sum_{n=3}^{\infty} \frac{1}{2^{n-3}} \binom{n}{1,1,n-2} v^{2n-4} c_H^{(2n)} \right) A_0(\sqrt{\xi_W} \bar{M}_W) \\
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& + \frac{1}{32\pi^2} (\sqrt{h}^{44})^2 \left[\bar{M}_H^2 \left\langle \frac{\delta h_{44}}{\delta \hat{h}} \right\rangle + v \sqrt{h}^{44} \left(6\lambda - \sum_{n=3}^{\infty} \frac{1}{2^{n-1}} \binom{2n}{1,2,2n-3} v^{2n-4} c_H^{(2n)} \right) \right]
\end{aligned}
$$

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Summary:

1. Allows for the all orders calculation of the one-loop tadpole
2. Far more compact expressions $\Rightarrow$ easier to see cancelation between V, u^V, ϕ^V terms
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Who cares?

1. In an FJ tadpole scheme the tadpole is removed by the counter term
   $\Rightarrow$ it just turns up in the "tree level" results
2. The tadpole is *gauge dependent*
   $\Rightarrow$ this gauge dependence is necessary to obtain gauge independent M_V and M_H
3. The vev and G_F are in 1:1 correspondence in the SM (at one loop)
   $\Rightarrow$ they are not in the SMEFT:

$$v^2 = \frac{1}{\sqrt{2}G_F} + \frac{\delta G_F}{G_F}$$

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Next step: 2 point functions @ one-loop

Conclusions

1. EFTs allow us to study the low energy impact of heavy decoupled new physics
2. The SMEFT is an excellent tool for precision BSM physics @ the LHC
3. The geoSMEFT allows us to make straightforward calculations beyond LO in $\frac{1}{\Lambda^2}$
   $\rightarrow$ it is only (currently) defined for 3pt functions
   $\rightarrow$ from 4 pt functions on there are infinitely many operators in the ∂ expansion
   $\rightarrow$ the most important Higgs observables already calculated, need 4pt to go further
4. The full set of Z-pole observables have been calculated to $\frac{1}{\Lambda^4}$ in arXiv:2102.02819
   $\rightarrow$ not enough data to constrain all the WCs with Z-pole data alone
   $\rightarrow$ explored random sampling of WCs and impact of D8
   $\rightarrow$ explored matching to D8 and impact of D8
   $\rightarrow$ indicates naive D8 impact of $\sim$ % in most cases, but EWPD is per mil
   $\rightarrow$ in phenomenological ex. w matching, slight broadening of 99% region
   large broadening of 68% and 95% regions
5. calculated the Tadpole @ one-loop
   $\rightarrow$ only diagram currently possible to calculate at one-loop
   $\rightarrow$ geoSMEFT "cleans up" calculations & clarifies cancellations
   $\rightarrow$ with 4pt functions defined (currently underway), can move on the 2-pt loops
   $\rightarrow$ enough of the D8 basis is included in 4-pt functions to begin observables

The geoSMEFT, or SMEFT to all orders $\frac{v}{\Lambda}$

(the very simplified version, see Helset, Martin, Trott arXiv:2001.01453)

1. Take SM field content, X, Ψ, DH, D
2. Form Lorentz invariant, but gauge variant combos of 2 or 3 fields
3. stick towers of $H^\dagger$, H, and/or τ^A $\rightarrow$ gauge invariant
   use Hilbert Series to confirm all operators included/no redundancy
   (see e.g. Lehman & Martin 2015, Henning et al. 2015)

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@ D6 $\frac{1}{\Lambda^2}$: $c_{H\Box}(H^\dagger H)\Box(H^\dagger H)$ & $c_{HD}(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$

@ D8 $\frac{1}{\Lambda^4}$: $c^{(8)}_{HD}(H^\dagger H)^2(D_\mu H)^\dagger(D^\mu H)$ & $c^{(8)}_{HD,2}(H^\dagger H)(H^\dagger \sigma_a H)(D_\mu H)^\dagger \sigma^a (D^\mu H)$

@ D10 $\frac{1}{\Lambda^6}$: $c^{(10)}_{HD}(H^\dagger H)^3(D_\mu H)^\dagger(D^\mu H)$ & $c^{(10)}_{HD,2}(H^\dagger H)^2(H^\dagger \sigma_a H)(D_\mu H)^\dagger \sigma^a (D^\mu H)$

@ D12 $\frac{1}{\Lambda^8}$: $c^{(12)}_{HD}(H^\dagger H)^4(D_\mu H)^\dagger(D^\mu H)$ & $c^{(12)}_{HD,2}(H^\dagger H)^3(H^\dagger \sigma_a H)(D_\mu H)^\dagger \sigma^a (D^\mu H)$

@ D14 $\frac{1}{\Lambda^{10}}$: $c^{(14)}_{HD}(H^\dagger H)^5(D_\mu H)^\dagger(D^\mu H)$ & $c^{(14)}_{HD,2}(H^\dagger H)^4(H^\dagger \sigma_a H)(D_\mu H)^\dagger \sigma^a (D^\mu H)$

and so on...

Operator forms $(D^\mu\phi)^I(D^\mu\phi)^J$

Simplify things a bit:

$$\phi^I = \{\phi_1, \phi_2, \phi_3, \phi_4\} \quad \Leftrightarrow \quad H = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 - i\phi_3 \end{pmatrix}$$

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Now we can define:

$$\begin{aligned} \mathcal{L}_{(D\phi)(D\phi)} &= (D_\mu H)^\dagger (D^\mu H) + c_{H\Box} Q_{H\Box} + c_{HD} Q_{HD} + \cdots \\ &\equiv h_{IJ} (D^\mu\phi)^I (D_\mu\phi)_J \end{aligned}$$

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$$\begin{aligned} h_{IJ} &= \left[1 + \phi^2 \tfrac{c_{H\Box}}{\Lambda^2} + \textstyle\sum_{n=0}^\infty \left(\tfrac{\phi^2}{2}\right)^{n+2} \left(\tfrac{c_{HD}^{(8+2n)} - c_{HD,2}^{(8+2n)}}{\Lambda^{2+2n}}\right)\right]\delta_{IJ} \\ &\quad + \tfrac{\Gamma^I_{A,J}\phi_K \Gamma^K_{A,L}\phi^L}{2}\left(\tfrac{c_{HD}^{(6)}}{2\Lambda^2} + \textstyle\sum_{n=0}^\infty \left(\tfrac{\phi^2}{2}\right)^{n+1} \tfrac{c_{H,D2}^{(8+2n)}}{\Lambda^{2+2n}}\right) \end{aligned}$$

Truncation error

An amplitude squared in the SMEFT is defined perturbatively as:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\rm SM}|^2 + \frac{2}{\Lambda^2}{\rm Re}\left[\mathcal{M}^*_{\rm SM}\mathcal{M}_6\right] + \frac{1}{\Lambda^4}\left(|\mathcal{M}_6|^2 + 2{\rm Re}\left[\mathcal{M}^*_{\rm SM}\left(\mathcal{M}_{6^2} + \mathcal{M}_8\right)\right]\right) + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$

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LO SMEFT contribution

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Frequently used to "approximate" truncation error sometimes is **bigger** than LO contribution

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LO SMEFT contribution

Full NLO result possible with geoSMEFT
⇒ More consistent definition of truncation error

Frequently used to “approximate” truncation error sometimes is **bigger** than LO contribution

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LO SI

A more consistent definition of truncation error

1. Calculate consistently to D8 (geoSMEFT)
2. Many new open parameters appear, $c_i^{(8)}$ (can't be constrained by current experiments)
3. Vary the new parameters in some way ⇒ infer truncation error (similar to above examples)

h geoSMEFT
of truncation

sometimes is **bigger** than LO contribution

Finite field renormalizations in the SMEFT

Recall:

$$c_{H\Box}(H^\dagger H)\Box(H^\dagger H) \to \frac{1}{4}(v^2+2vh+h^2)\Box(v^2+2vh+h^2) = -v^2 c_{H\Box}(\partial^\mu h)(\partial_\mu h) + \cdots$$

We need to redefine h to have a “canonical kinetic form”:

$$\begin{aligned}
h &\to \frac{h'}{\sqrt{1-2v^2 c_{H\Box}}} = \left(1 + v^2 c_{H\Box} + \frac{3v^2}{2} c_{H\Box}^2 + \cdots\right) h' \\
\frac{1}{2}(1-2v^2 c_{H\Box})(\partial_\mu h)(\partial^\mu h) &\to \frac{1}{2}(\partial_\mu h')(\partial^\mu h') \\
\frac{\sqrt{2}m_\psi}{v} h\bar{\psi}\psi &\to \frac{\sqrt{2}m_\psi}{v}\left(1 + v^2 c_{H\Box} + \frac{3v^2}{2} c_{H\Box}^2 + \cdots\right) h'\bar{\psi}\psi
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Rinse and repeat for Q_{HD}, $Q_{HD}^{(8)}$, $Q_{HD,2}^{(8)}$, Q_{HB}, Q_{HW}, Q_{HWB}, $Q_{HB}^{(8)}$, $Q_{HW}^{(8)}$, $Q_{HW,2}^{(8)}$, $Q_{HWB}^{(8)}$

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Finite field renormalizations in the SMEFT

Recall:

$$\begin{aligned}\mathcal{L}_{\rm cl3} &= h_{IJ}(D^\mu\phi)^I(D^\mu\phi)^J \\ \mathcal{L}_{\rm cl4} &= -\frac{1}{4}g_{AB}\mathcal{W}^{A,\mu\nu}\mathcal{W}^B_{\mu\nu}\end{aligned}$$

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Defining the expectation of the field-space connection:

$$\langle M\rangle \equiv M|_{\phi_4\to v,\phi_{i\neq 4}\to 0}$$

Then the canonically normalized fields are simply:

$$\begin{aligned}A^C_\mu \equiv \{W^+,W^-,Z,\gamma\} &= \sqrt{\langle g^{CB}\rangle}U_{BA}\mathcal{W}^A_\mu\\ \Phi^K \equiv \{\Phi^-,\Phi^+,\chi,h\} &= \sqrt{\langle h^{KJ}\rangle}V_{JI}\phi^I\end{aligned}$$

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Which gives:

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Finite field renormalizations in the SMEFT

Recall:

$$\mathcal{L} \quad = \quad h \quad (D^\mu\phi)^I(D^\mu\phi)^J$$

$$g^{AB}g_{BC} = \delta^A_C \quad \& \quad U^{AB}U_{BC} = \delta^A_C$$
$$h^{IJ}h_{JK} = \delta^I_K \quad \& \quad V^{IJ}V_{JK} = \delta^I_K$$

Defining the expectation of th

$$U_{BC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \bar{c}_W & \bar{s}_W \\ 0 & 0 & -\bar{s}_W & \bar{c}_W \end{pmatrix}$$

Then the canonically normalized fields are simply:

$$\begin{aligned} A^C_\mu \equiv \{W^+, W^-, Z, \gamma\} &= \sqrt{\langle g^{CB}\rangle}U_{BA}\mathcal{W}^A_\mu \\ \Phi^K \equiv \{\Phi^-, \Phi^+, \chi, h\} &= \sqrt{\langle h^{KJ}\rangle}V_{JI}\phi^I \end{aligned}$$

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Finite field renormalizations in the SMEFT

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$$\mathcal{L}_{\ldots} = h_{\ldots}(D^\mu\phi)^I(D^\mu\phi)^J$$

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Defining the expectation of th

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

TC, arXiv:2010.15852

"The Feynman Rules for the SMEFT in the background field gauge"

Only tool for FR to consistently derive these corrections at $\frac{1}{\Lambda^4}$

Partial D8 basis results, recently updated with Cl7 and Cl15 (leptons) to D8, TC, T Rasmussen, arXiv:2110.03694

Th

$$\Phi^K \equiv \{\Phi^-, \Phi^+, \chi, h\} \quad = \quad \sqrt{\langle h^{KJ}\rangle} V_{JI}\phi^I$$

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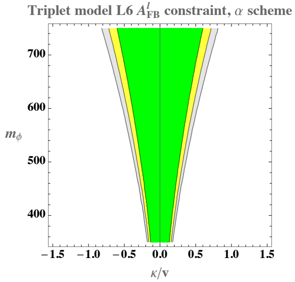
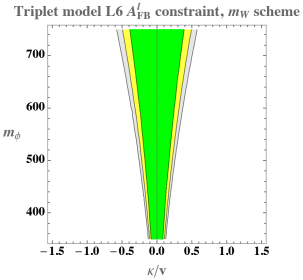
Z-pole pheno, arXiv:2102.02819





Z-pole pheno, arXiv:2102.02819