Precise predictions at the LHC with the GENEVA Monte Carlo

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Particle Physics Seminar University of Vienna, 23rd November

Overview of the talk

- Motivation
- The Geneva method GENEVA http://geneva.physics.lbl.gov
- Colour singlet processes: Photon-pair production (in general diboson processes)
- Zero-jettiness resummation for top-quark pair production at the LHC
- Conclusions & Outlook

Work in collaboration with: S. Alioli, A. Gavardi, S. Kallweit, M. Lim, R. Nagar, D. Napoletano, L. Rottoli

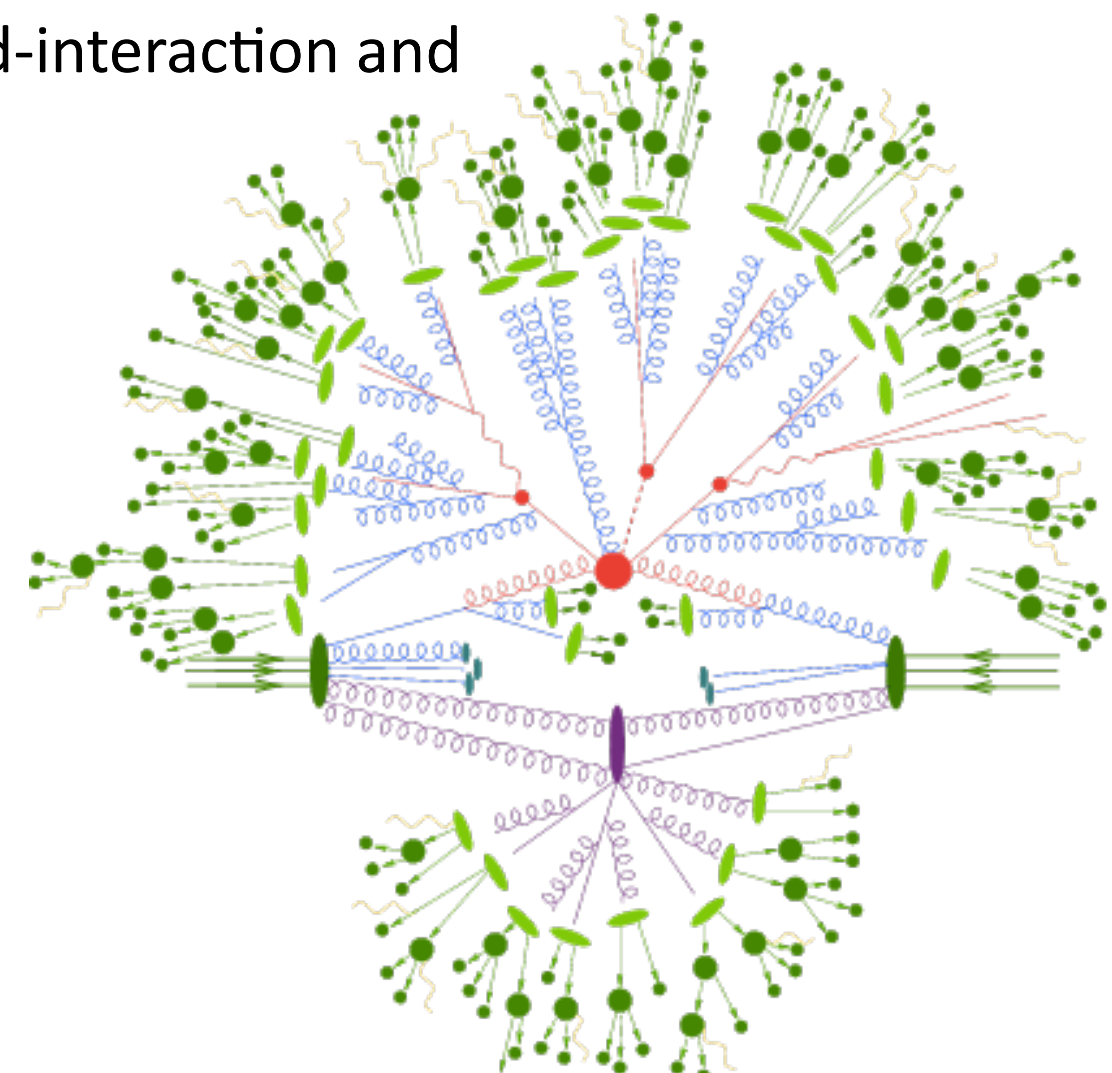
Motivation

- MC event generators are essential tools for particle physics phenomenology

- They provide realistic simulations: first principles QFT calculations are combined with parton showers and hadronization modelling

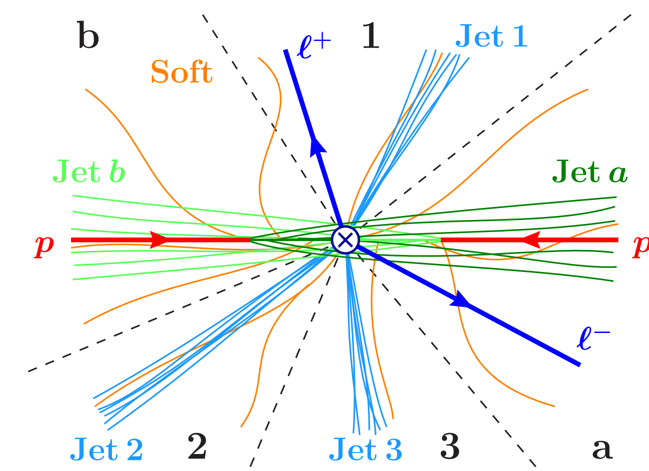
- They start from a perturbative description of the hard-interaction and predict the evolution of the event down to very small (nonperturbative) scales $\mathcal{O}(1)$ GeV

- State-of-the-art is the inclusion of partonic NNLO corrections. Several methods are available for colour-singlet processes (UNNLOPS, MiNNLOPS, GENEVA)



N-Jettiness and Factorization

- N-jettiness resolution variables: given an M-particle phase space point with $M \geq N$



$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$

- The limit $\mathcal{T}_N \to 0$ describes a N-jet event where the unresolved emissions can be either soft or collinear to the final state jets or initial state beams

- Color singlet final state, relevant variable is 0-jettiness aka “beam thrust”

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}| \, e^{-|\eta_k - Y|}$$

- Cross section factorizes in the limit $\mathcal{T}_0 \to 0$ [Stewart, Tackmann,Waalewijn `09,`10], three different scales arise

$$\mu_H = Q, \quad \mu_B = \sqrt{Q\mathcal{T}_0}, \quad \mu_S = \mathcal{T}_0$$

$$\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \overset{\text{NNLO}}{\boxed{H_{ij}^{\gamma\gamma}(Q^2, t, \mu_H)}} U_H(\mu_H, \mu) \left\{ \left[\overset{\text{NNLO}}{\boxed{B_i(t_a, x_a, \mu_B)}} \otimes U_B(\mu_B, \mu) \right] \right.$$
$$\left. \times \left[\boxed{B_j(t_b, x_b, \mu_B)} \otimes U_B(\mu_B, \mu) \right] \right\} \otimes \left[\underset{\text{NNLO}}{\boxed{S(\mu_s)}} \otimes U_S(\mu_S, \mu) \right]$$

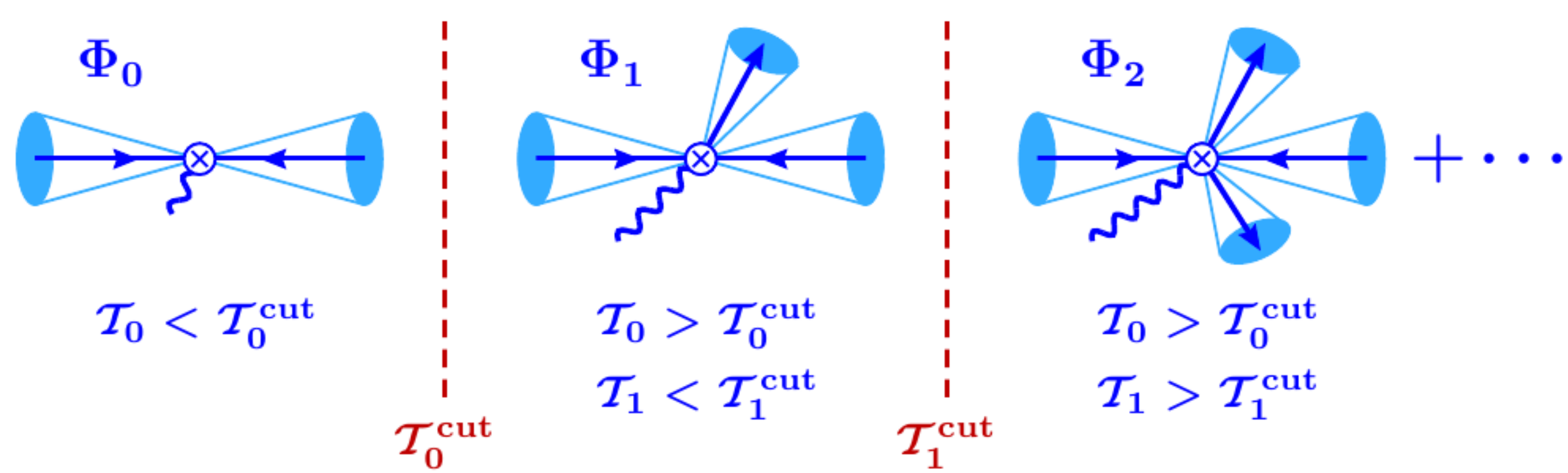
Monte Carlo implementation

- GENEVA [Alioli,Bauer,Berggren,Tackmann, Walsh `15], [Alioli,Bauer,Tackmann,Guns `16], [Alioli,Broggio,Lim, Kallweit,Rottoli `19],[Alioli,Broggio,Gavardi,Lim,Nagar,Napoletano,Kallweit,Rottoli `20-`21] combines 3 theoretical tools that are important for QCD predictions into a single framework
  - fully differential fixed-order calculations, up to NNLO via 0-jettiness or q_T subtraction
  - up to NNLL` resummation for 0-jettiness in SCET or N^{3}LL for q_T via RadISH for colour singlet processes
  - shower and hadronize events (PYTHIA8)
- IR-finite definition of events based on resolution parameters $\mathcal{T}_0^{\rm cut}$ (or $p_T^{\rm cut}$) and $\mathcal{T}_1^{\rm cut}$

$$\Phi_0 \text{ events:} \qquad \frac{d\sigma_0^{\rm MC}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}) ,$$

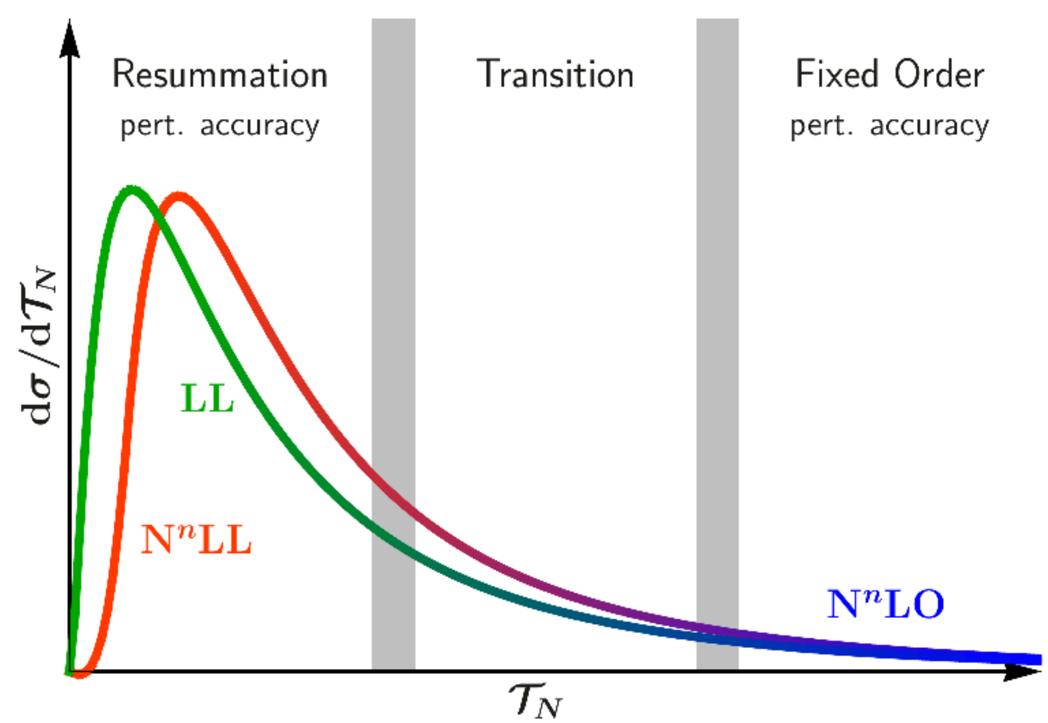
$$\Phi_1 \text{ events:} \qquad \frac{d\sigma_1^{\rm MC}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}; \mathcal{T}_1^{\rm cut}) ,$$

$$\Phi_2 \text{ events:} \qquad \frac{d\sigma_{\geq 2}^{\rm MC}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}, \mathcal{T}_1 > \mathcal{T}_1^{\rm cut})$$



N-Jettiness and Resummation

- At NNLO one needs a 0-jet and a 1-jet resolution parameters
- Emissions below $\mathcal{T}_N^{\mathrm{cut}}$ are unresolved (integrated over) and the kinematic considered is the one of the event before extra emissions
- Emissions above $\mathcal{T}_N^{\mathrm{cut}}$ are kept and the full kinematics is considered
- When we take $\mathcal{T}_N^{\mathrm{cut}} \to 0$, large logarithms of $\mathcal{T}_N^{\mathrm{cut}}$, $\mathcal{T}_N$ appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum



Monte Carlo implementation

0-jet events

$$\frac{\mathrm{d}\sigma_0^{\mathrm{MC}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\,\theta_{\mathrm{iso}}^{\mathrm{PS}}(\Phi_0) + \frac{\mathrm{d}\sigma_0^{\mathrm{nons}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_0^{\mathrm{nons}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \left\{\frac{\mathrm{d}\sigma_0^{\mathrm{NNLO}_0}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{\mathrm{NNLO}_0}\right\}\theta_{\mathrm{iso}}^{\mathrm{PS}}(\Phi_0)$$

At $\mathcal{O}(\alpha_s^2)$ assumed exact cancellation between NNLO and resummed expanded singular contributions

≥ 1-jet events (Split between 1 and ≥ 2 events via $\mathcal{T}_1$ resolution variable)

$$\frac{\mathrm{d}\sigma_{\geq 1}^{\mathrm{MC}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0}\mathcal{P}(\Phi_1)\theta\left(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}\right)\,\theta_{\mathrm{iso}}^{\mathrm{PS}}(\Phi_1)\theta_{\mathrm{iso}}^{\mathrm{proj}}(\tilde{\Phi}_0) + \frac{\mathrm{d}\sigma_{\geq 1}^{\mathrm{nons}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq 1}^{\mathrm{nons}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \boxed{\underbrace{\frac{\mathrm{d}\sigma_{\geq 1}^{\mathrm{NLO}_1}}{\mathrm{d}\Phi_1}}_{\text{Diphoton+jet at NLO. Divergent for } \mathcal{T}_0 \to 0}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\,\theta_{\mathrm{iso}}^{\mathrm{PS}}(\Phi_1) - \underbrace{\left[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0}\mathcal{P}(\Phi_1)\right]_{\mathrm{NLO}_1}}_{\text{Resummed Expanded Divergent for } \mathcal{T}_0 \to 0}}\,\theta_{\mathrm{iso}}^{\mathrm{PS}}(\Phi_1)\,\theta_{\mathrm{iso}}^{\mathrm{proj}}(\tilde{\Phi}_0)\,\theta\left(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}\right)$$

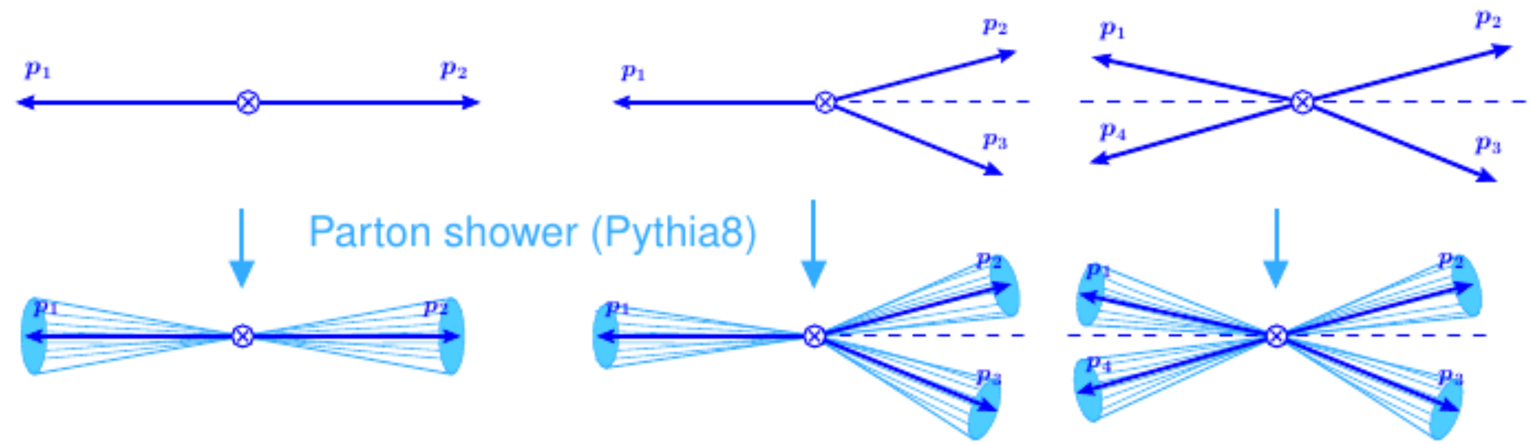
$P(\Phi_1)$ splitting function

$$\int \frac{d\Phi_1}{d\Phi_0 dT_0} P(\Phi_1) = 1$$

The sum is a non singular contribution

Matching to a parton shower

- Parton shower makes the calculation differential in higher multiplicities by filling the 0- and 1-jet exclusive bins with radiation and by adding more emissions to the inclusive 2-jet bin



- Not allowed to affect the accuracy of the cross sections reached at partonic level

- $\mathcal{T}_i^{\rm cut}$ constraints must be respected by the shower

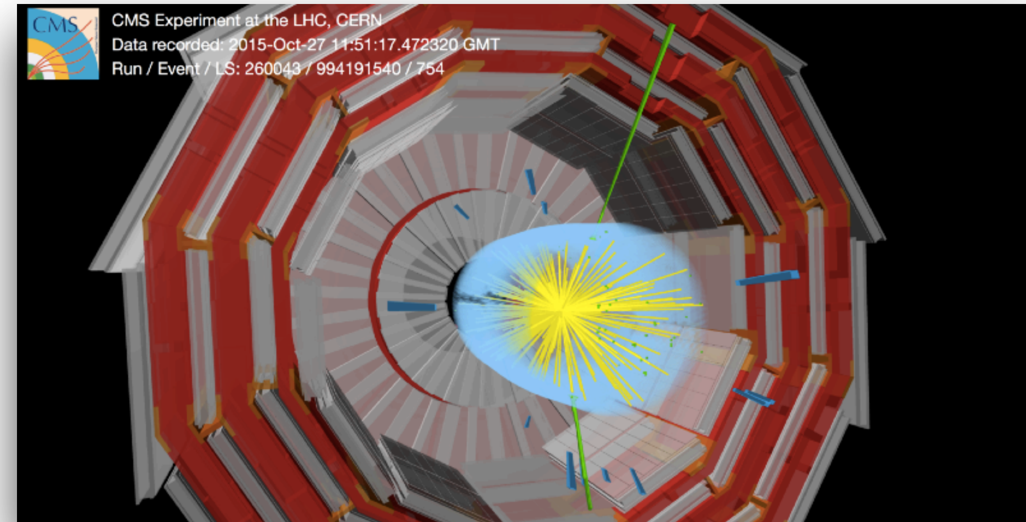
- Φ_0 events have $\mathcal{T}_0 = 0$. The shower should restore the emissions which were integrated, but should respect the constraint $\mathcal{T}_0(\Phi_N) < \mathcal{T}_0^{\rm cut}$. The shape is completely given by PYTHIA

- Φ_1 events, the first shower emission should satisfy $\mathcal{T}_1(\Phi_2) < \mathcal{T}_1^{\rm cut}$ and $\mathcal{T}_0(\Phi_2) = \mathcal{T}_0(\Phi_1)$ (map) $\longrightarrow$ First emission is done in GENEVA after that $\mathcal{T}_1(\Phi_N) < \mathcal{T}_1^{\rm cut}$

- Φ_2 events (>95% of total cross section) with nonzero values of $\mathcal{T}_0$ and $\mathcal{T}_1$: PYTHIA first emission affects the $\mathcal{T}_0$ distribution only beyond NNLL' on average

Photon pair production process

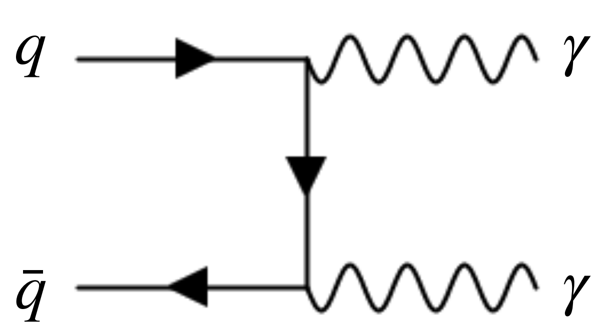
- Production of a pair of "isolated" photons is one of the most interesting processes at the LHC



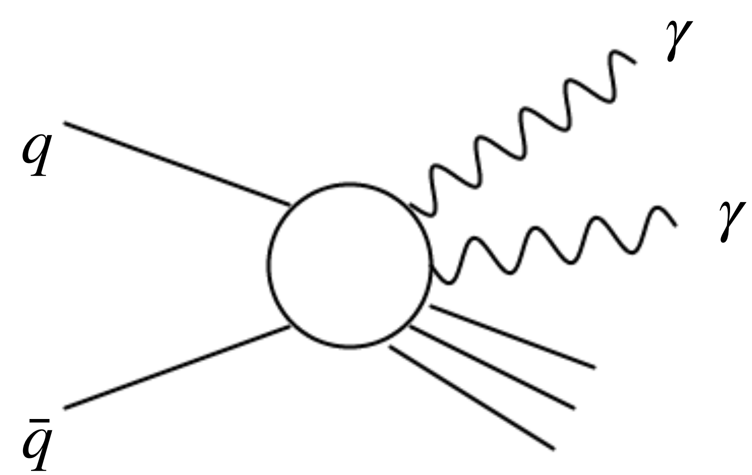
- Boosted by the discovery of the Higgs boson via its decay mode into two photons
- Experimentally clean final state and high production rate
- Search for new heavy resonances in the diphoton invariant mass spectrum

$$pp \to \gamma\gamma + X$$

LO



$+\ldots$

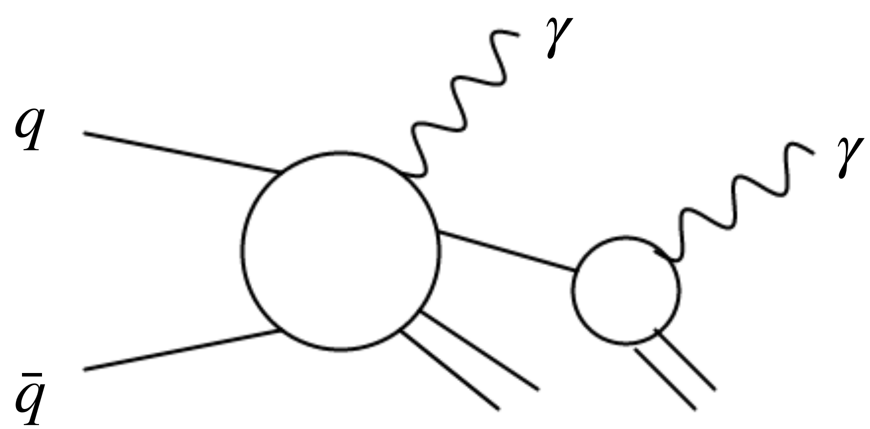


Direct Component

- LO contribution is already divergent due to collinear QED singularities, kinematical cuts are required ($p_T^{\gamma_h} > p_T^{\gamma_h\,\mathrm{cut}}$ and $p_T^{\gamma_s} > p_T^{\gamma_s\,\mathrm{cut}}$)

Photon Isolation

- Second production mechanism: fragmentation process of a quark or a gluon into a photon. Very different signature compared to direct photon production

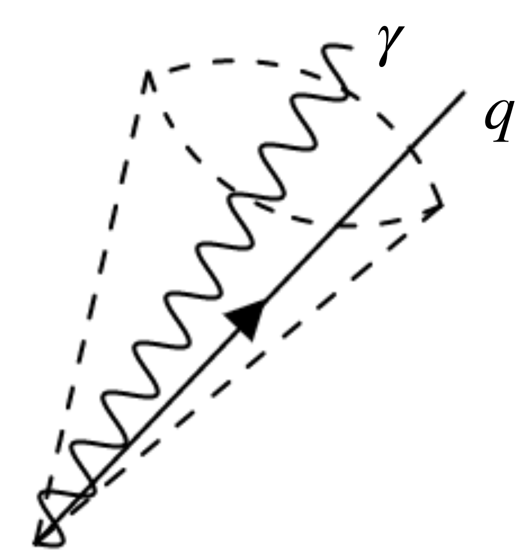


Fragmentation contribution
[Binoth, Guillet, Pilon, Werlen `02]

- Separate direct photons from the rest of the hadrons in the event via Isolation procedures:

  - Fixed-Cone isolation: construct a cone with fixed radius R_{iso} around the photon direction. One then restricts the amount of hadronic energy inside the cone. A photon is considered isolated when $E_T^{\text{had}}(R_{\text{iso}})$ is smaller than a fixed numerical value E_T^{thres}. Sensitive to fragmentation contributions

$$R_{\text{iso}}^2 = (y - y_\gamma)^2 + (\phi - \phi_\gamma)^2$$



Photon Isolation criteria

- Smooth-Cone isolation [Frixione `98]: initial cone with fixed radius R_{iso} + a series of smaller sub-cones with radius $r \leq R_{\text{iso}}$ are considered

$$E_T^{\text{had}}(r) \leq E_T^{\max}\,\chi(r; R_{\text{iso}})\,, \quad \text{for all sub-cones with } r \leq R_{\text{iso}}$$

isolation function smooth function monotonically decreases and vanishes when the sub-cone radius vanishes

$$\chi(r; R_{\text{iso}}) = \left(\frac{1-\cos r}{1-\cos R_{\text{iso}}}\right)^n$$

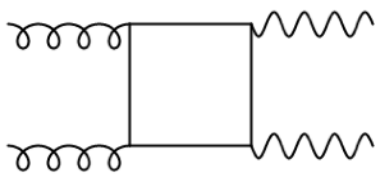
$$\chi(r; R_{\text{iso}}) \to 0, \quad r \to 0$$

- Smooth-cone: removes the fragmentation component and quark-photon collinear QED divergences (direct well defined by itself). But ALL experimental analyses use a fixed-cone isolation algorithm!

- Hybrid isolation: theoretical calculation is initially carried out using the smooth-cone isolation with a small radius parameter R_{iso}. Second step: the fixed-cone isolation with R $\gg$ R_{iso} is applied to the events which passed the smooth-cone criterion.

Available theoretical calculations

- DIPHOX Full NLO for direct and fragmentation contribution + Box contribution [Binoth, Guillet, Pilon, Werlen `02]



- 2γNNLO NNLO with q_T subtraction method [Catani, Cieri, de Florian, Ferrera, Grazzini `12]
  MATRIX NNLO with q_T subtraction method [Grazzini, Kallweit, Wiesemann `17]

- MCFM NNLO with N-jettiness subtraction [Campbell, Ellis, Li, Williams `16]

- NNLOJET NNLO via Antenna subtraction [Gehrmann, Glover, Huss, Whitehead `20]

- Resummation of the small transverse momentum of the photon pair: NNLL RESBOS, 2γRes, reSolve N^3LL CuTe-MCFM, MATRIX+RadISH

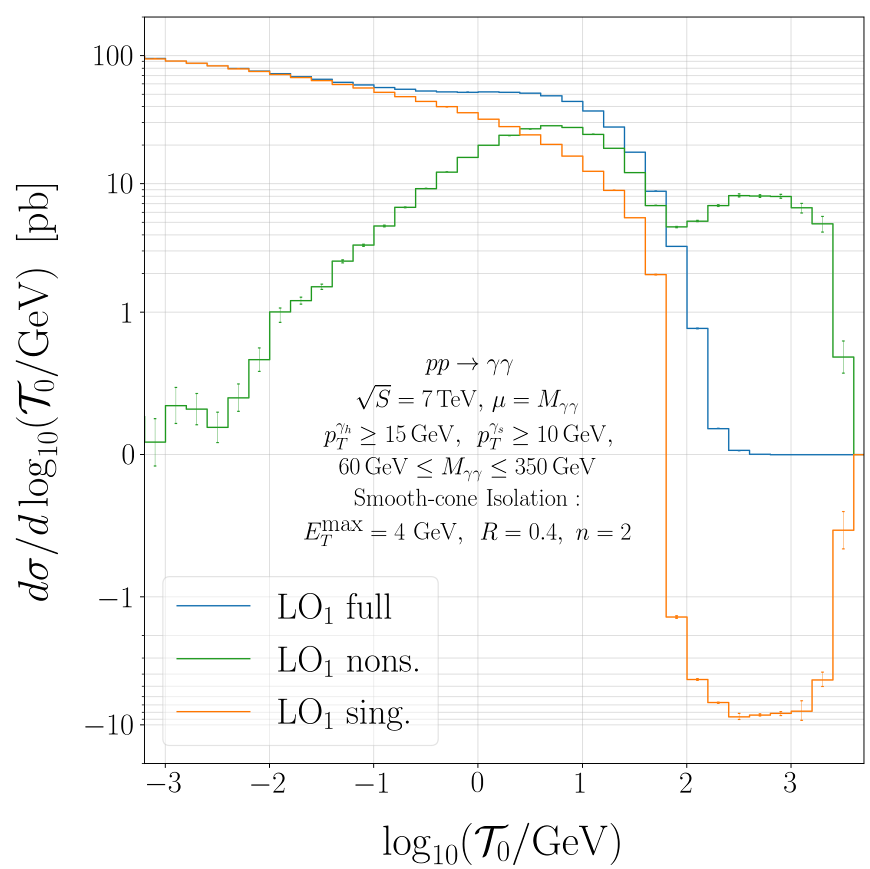
- EW Corrections [A. Bierweiler, T. Kasprzik and J. H. Kuehn `13], [M. Chiesa, N. Greiner, M. Schoenherr and F. Tramontano `17]

- Event generation at NLO matched to PS: SHERPA [Hoeche, Schumann, Siegert `09], HERWIG [Corcella et al. `01], POWHEG [L. D'Errico, P. Richardson `11]

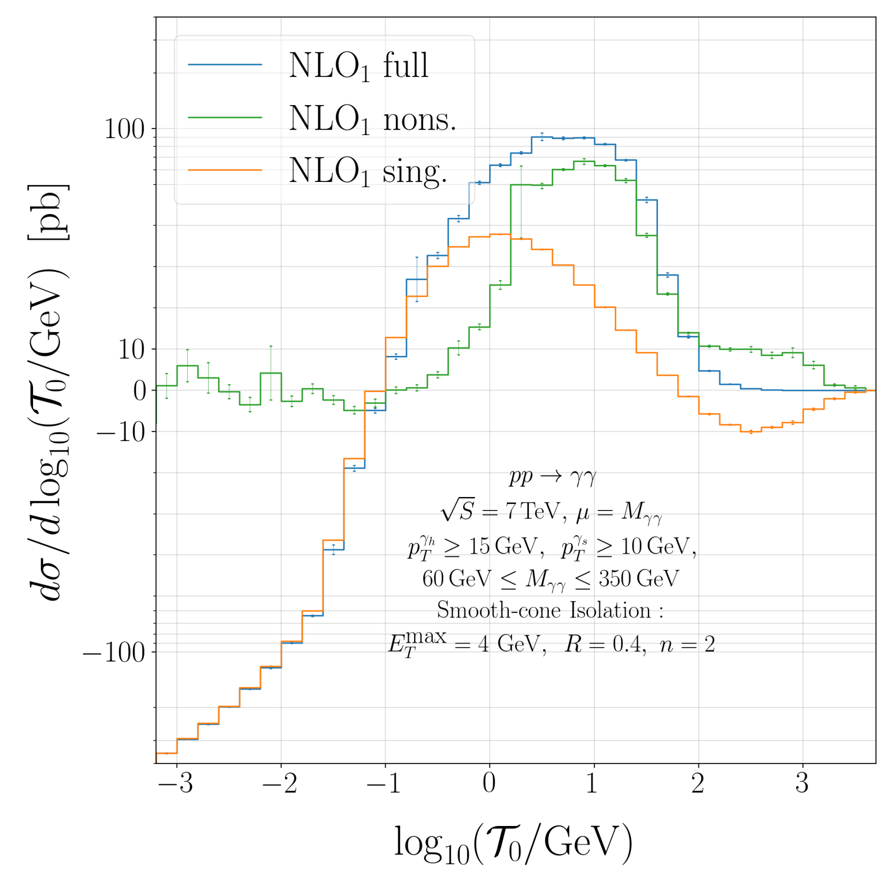
- GENEVA event generation at NNLO+NNLL` accuracy with N-jettiness subtraction matched to PS [S.Alioli, AB, A.Gavardi, S.Kallweit, M.Lim, R.Nagar, D.Napoletano, L.Rottoli `20] JHEP 04 (2021) 041

Singular and Nonsingular contributions

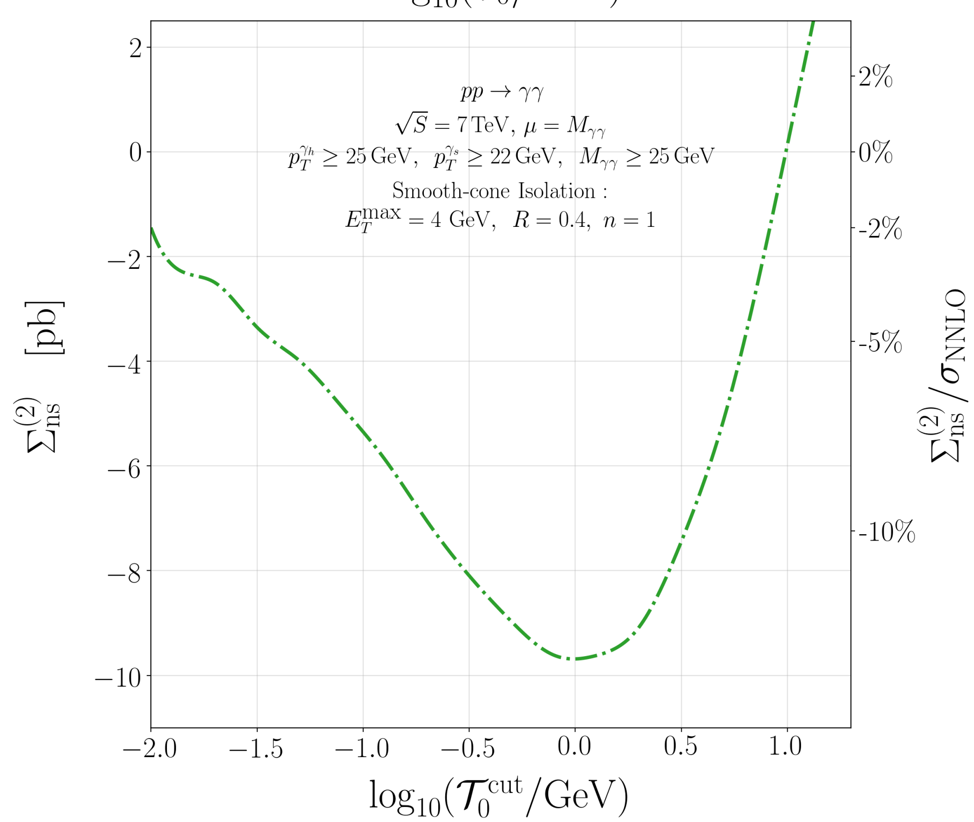
LO_1 vs Resummed expanded



NLO_1 vs Resummed expanded

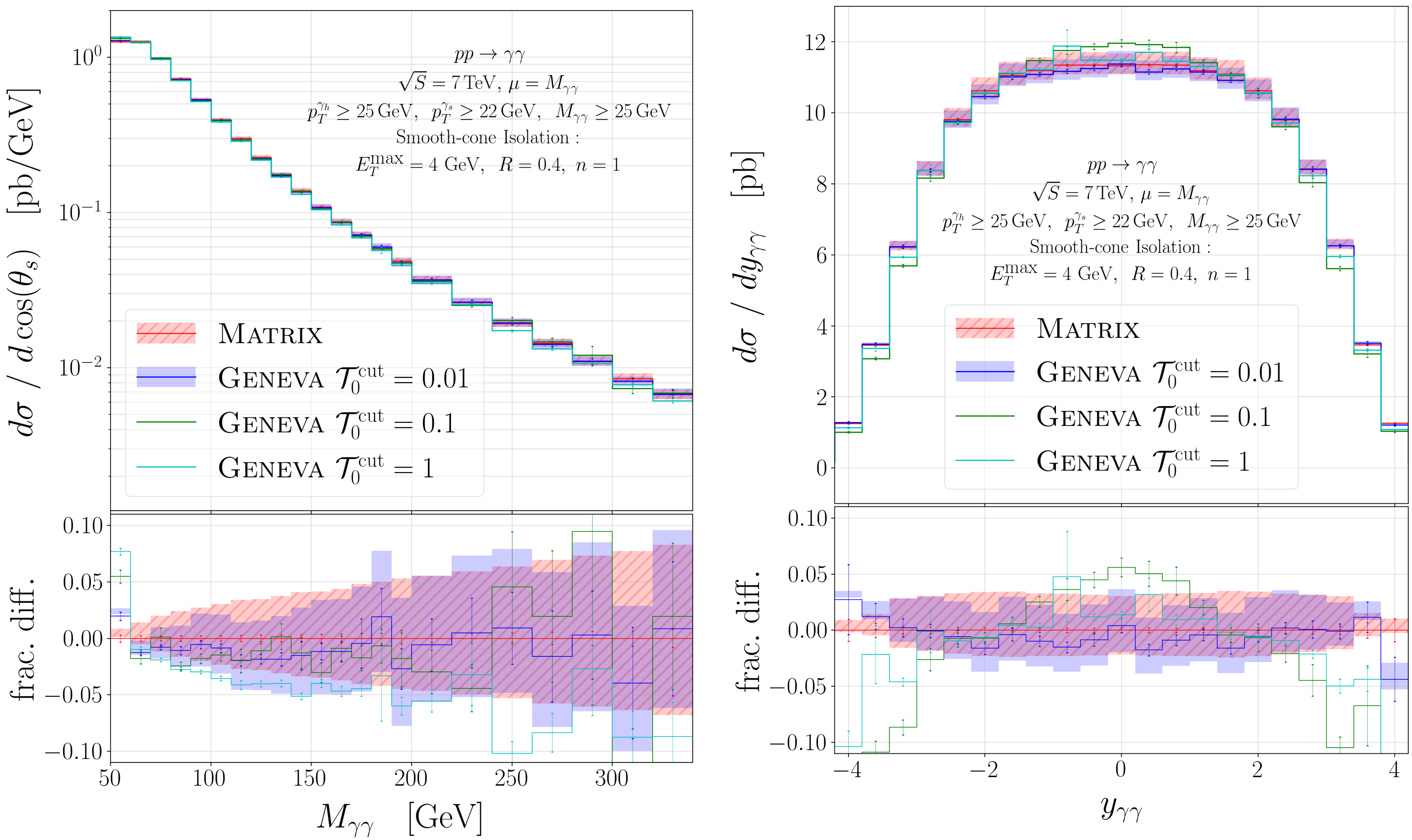


Size of power corrections is very challenging



Size of the missing non singular contributions below the cut as a function of $\mathcal{T}_0^{\mathrm{cut}}$

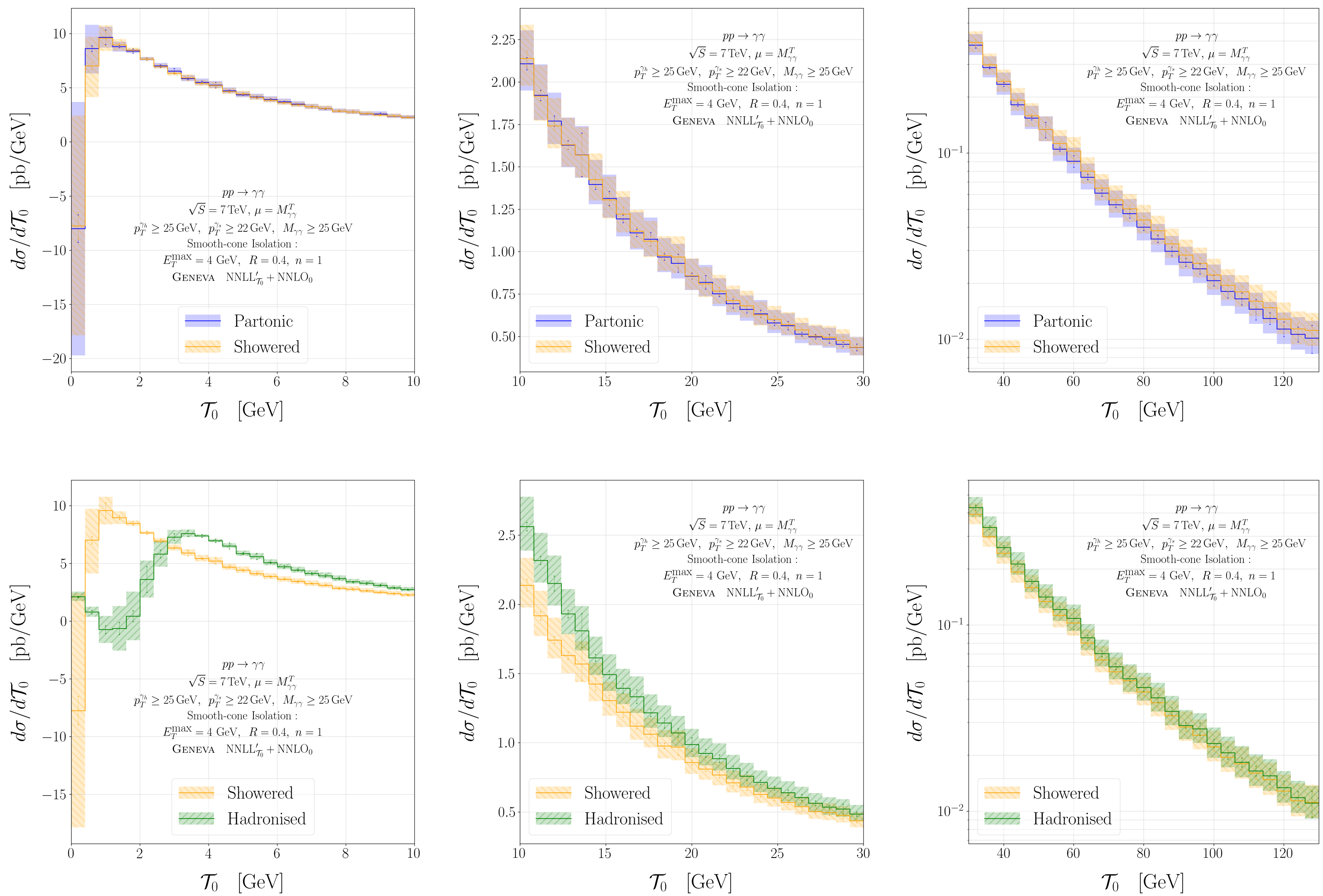
NNLO validation against MATRIX



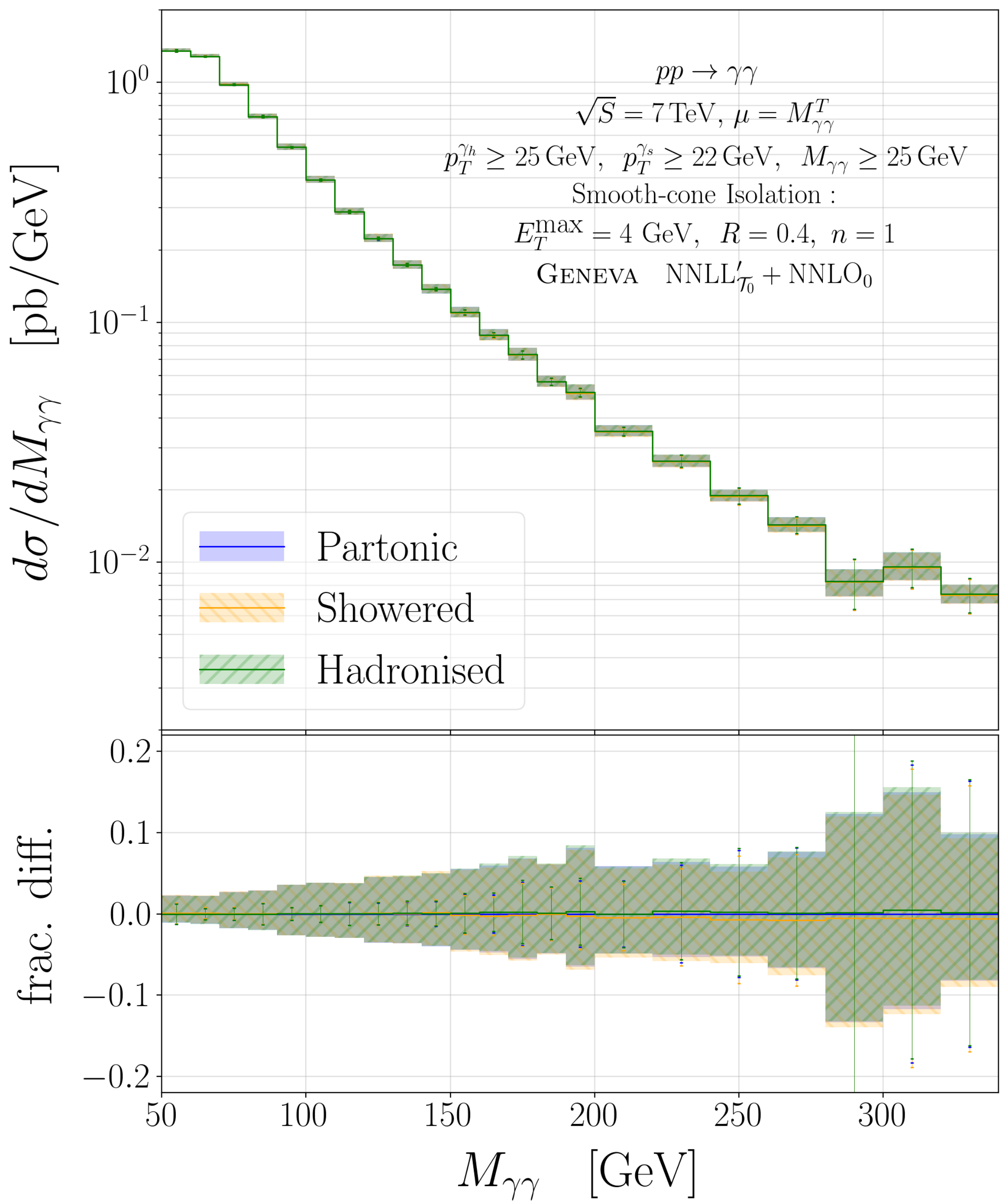
$q\bar{q}$ channel only, good agreement with independent NNLO computation with MATRIX

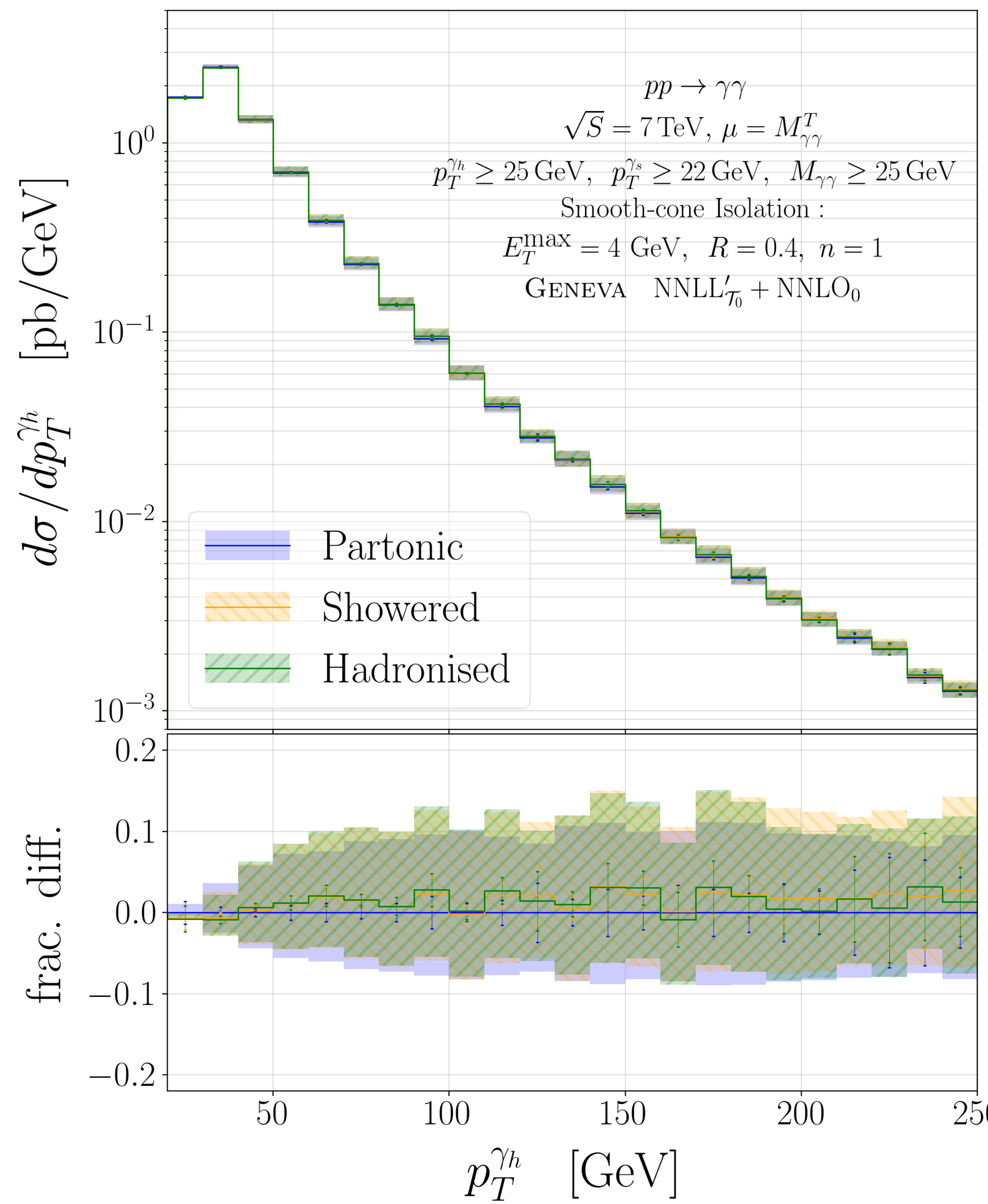
Adding the Shower (PYTHIA8)

- Parton-level result is NNLO+NNLL` accurate
- Parton shower should not affect the accuracy of the cross section reached at partonic level
- Constraints on event definition must be respected
- Accuracy is numerically well-preserved after the shower



Adding the Shower (PYTHIA8)





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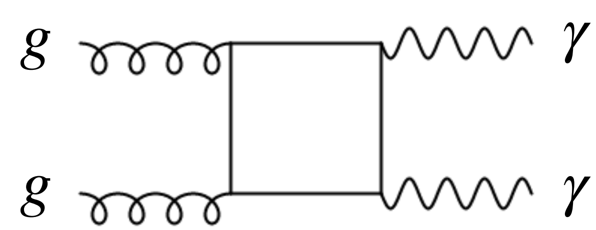
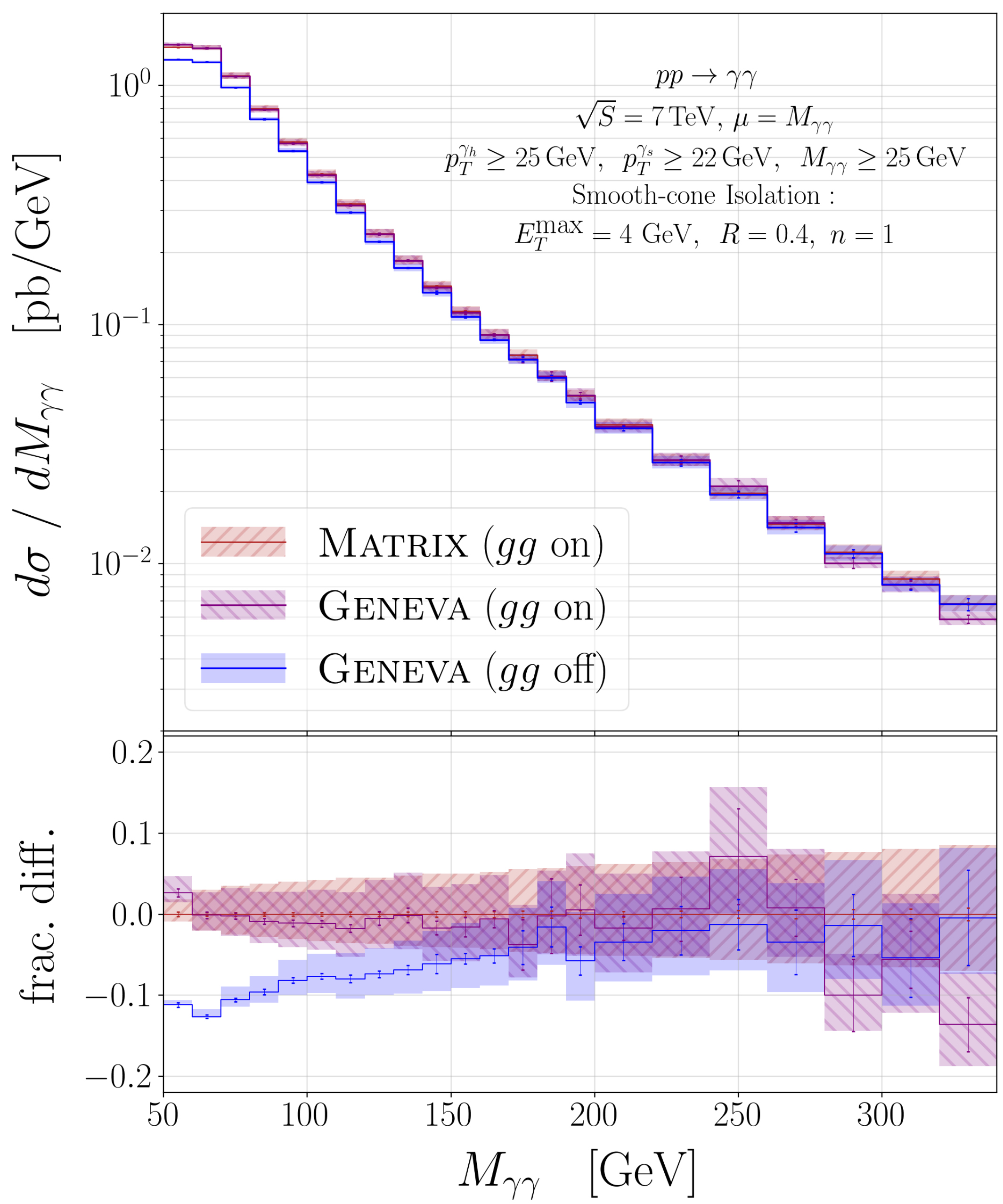
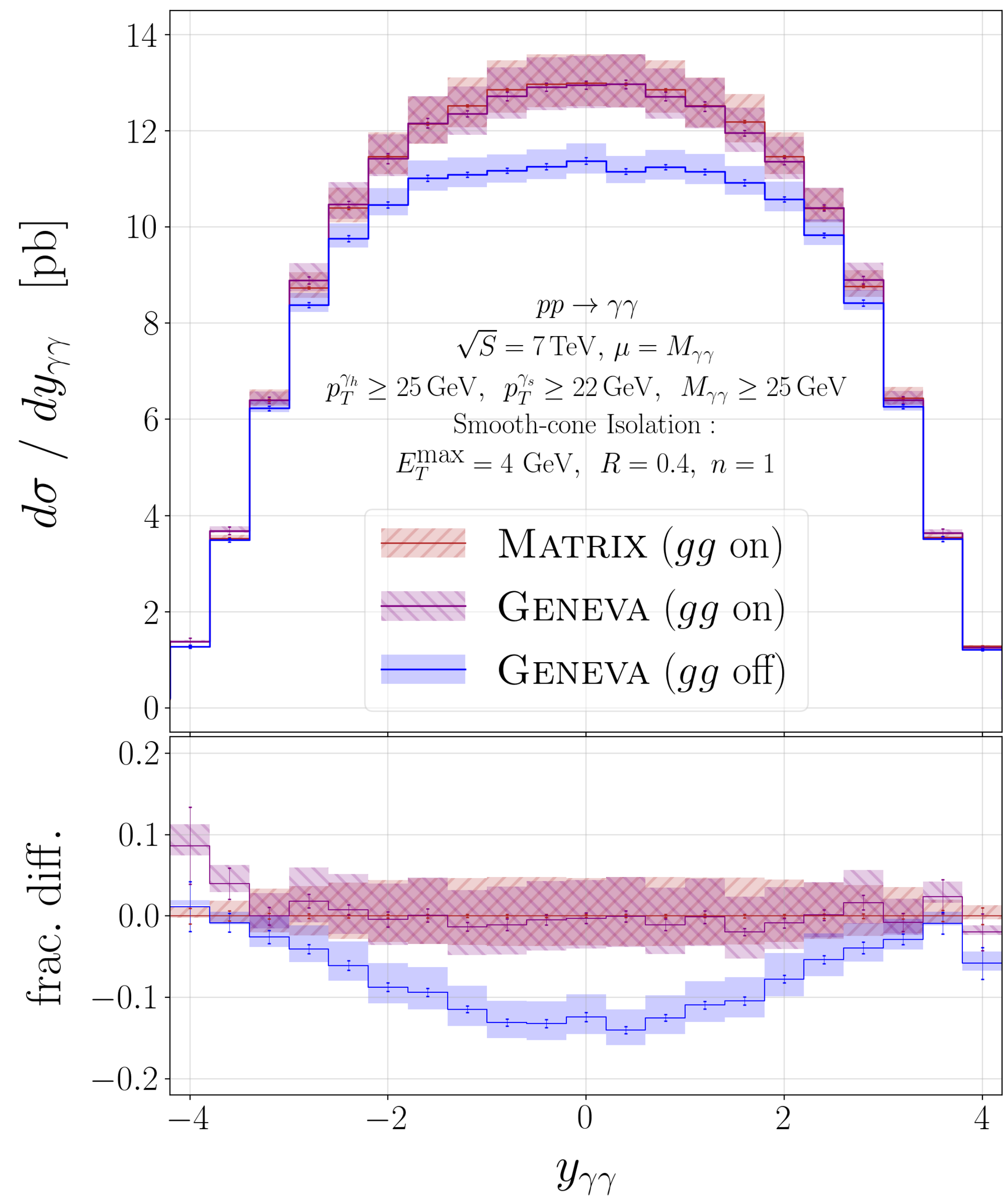


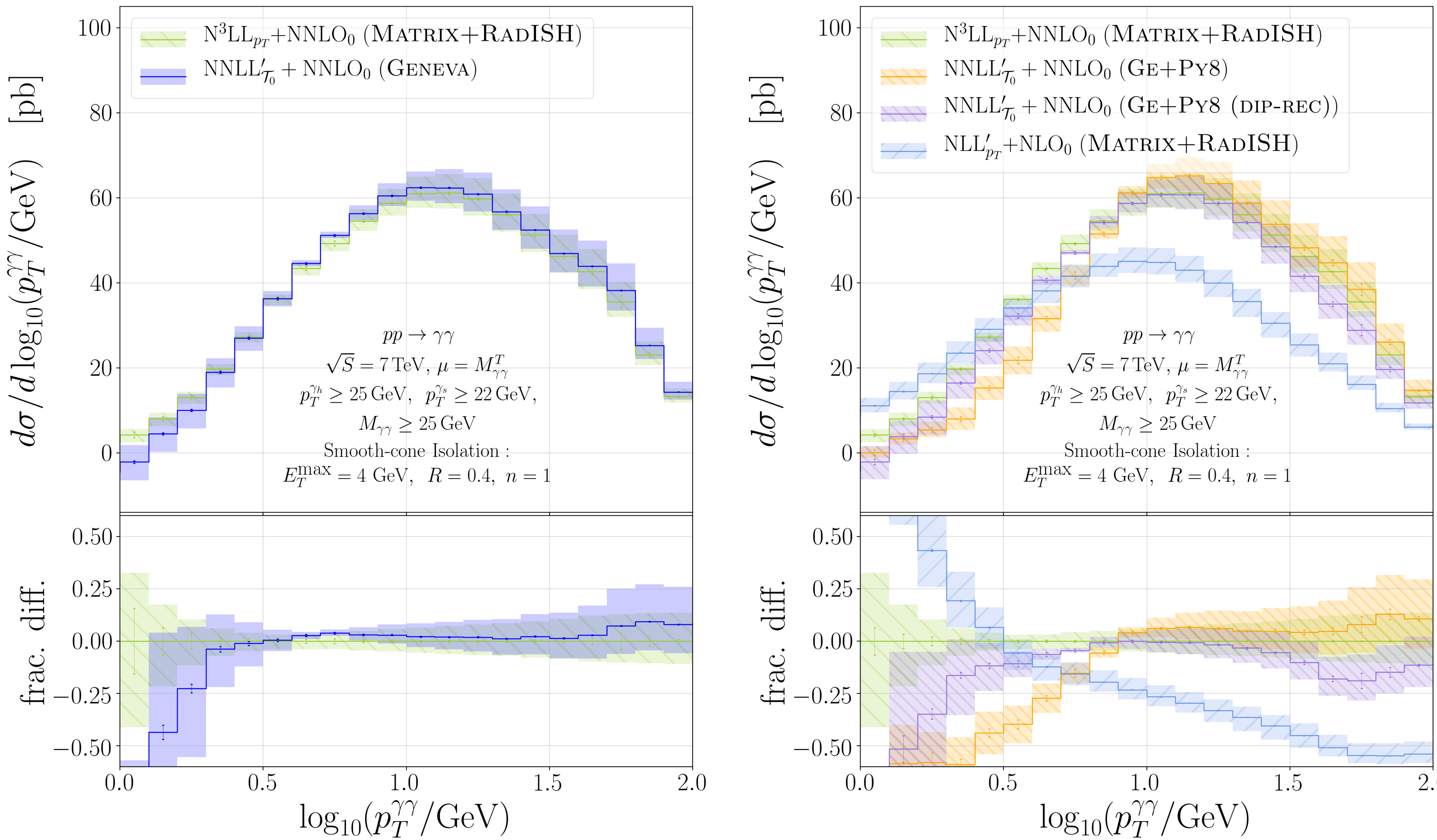
included gg box contribution starts at NNLO, large effect





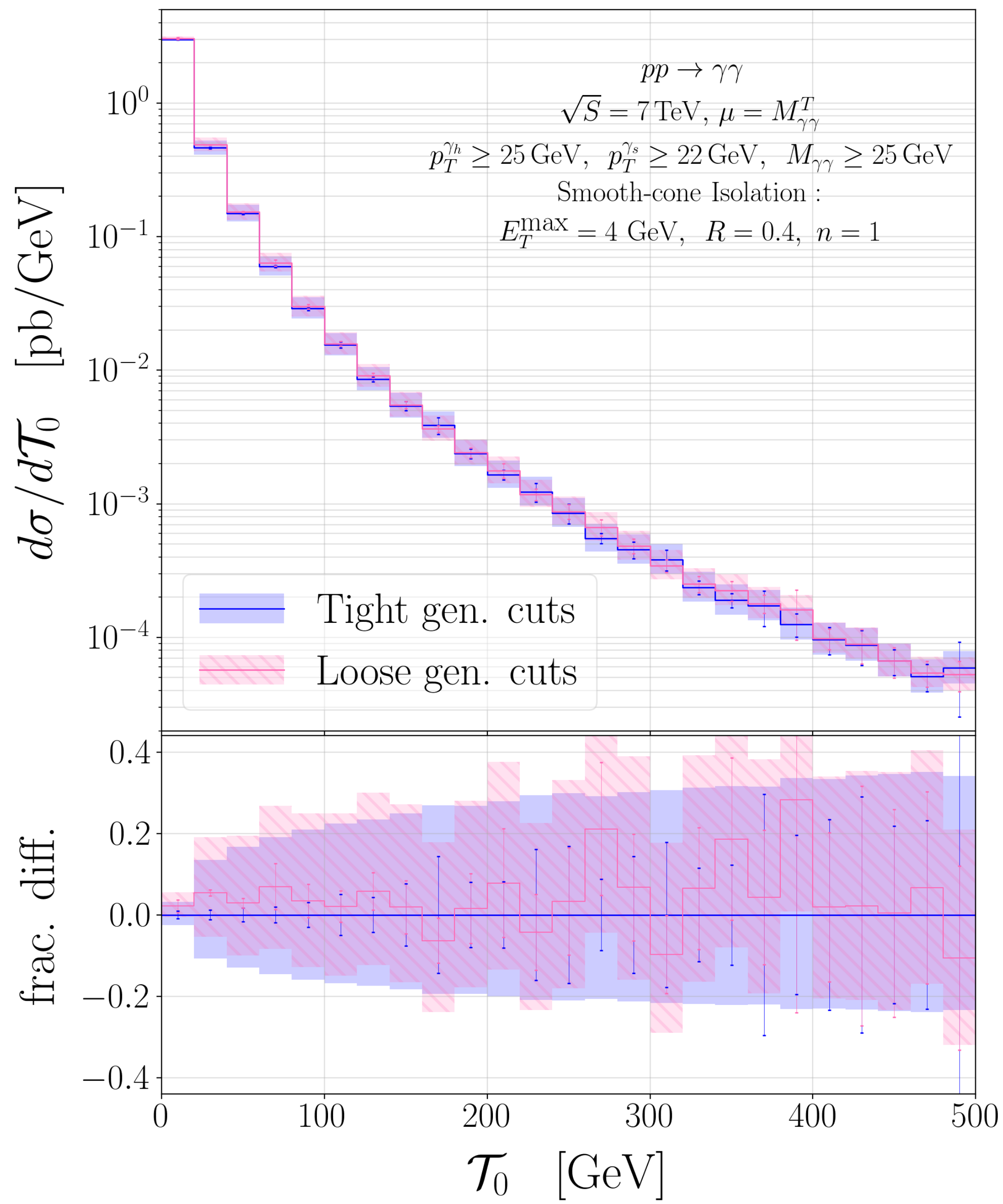
GENEVA vs q_T resummation

- Inclusive quantities are not modified, changes are expected in exclusive observables
- Shower recoil schemes large impact in predictions of colour singlet p_T

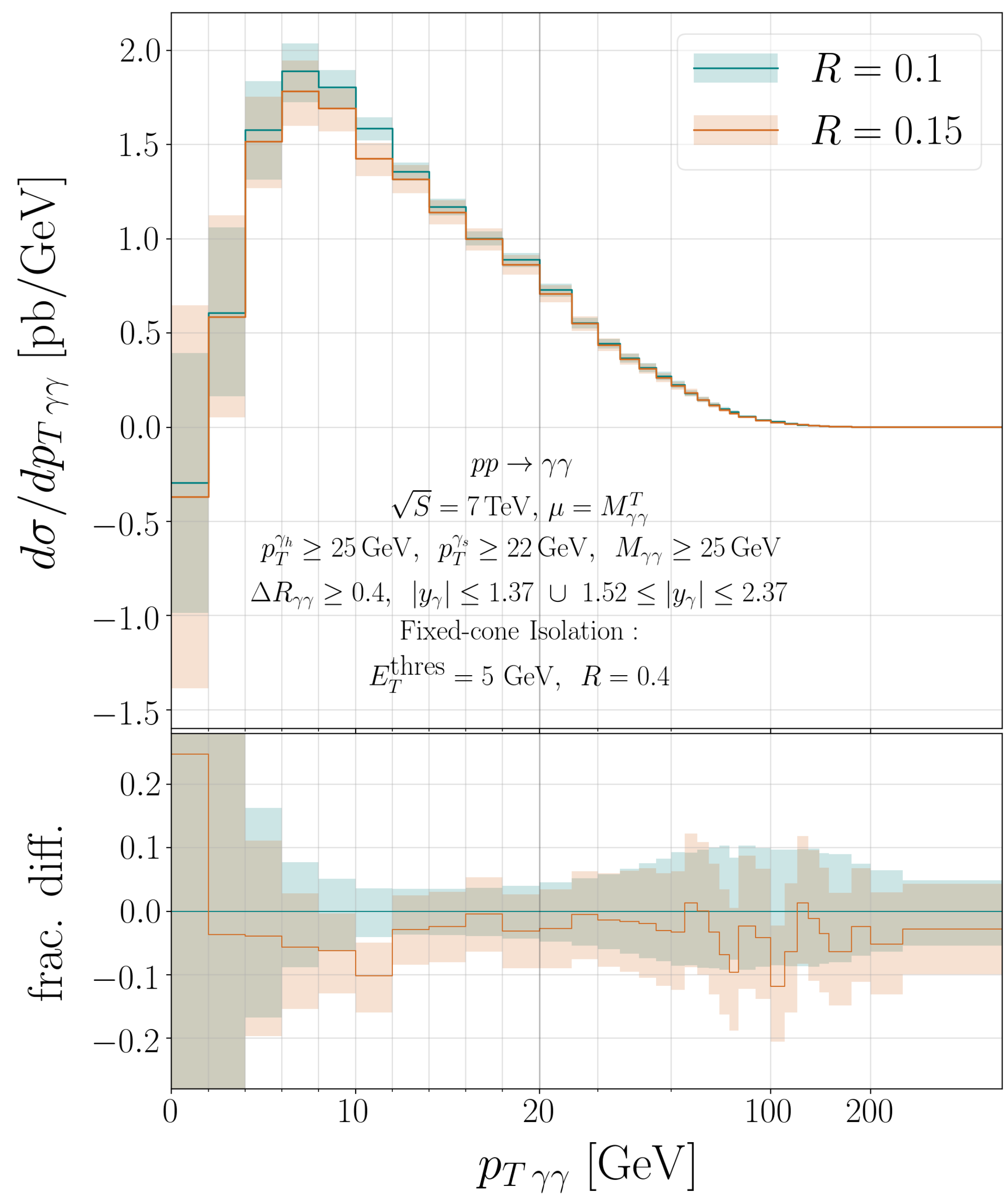


Event Generation and Analysis Cuts

- Study dependence on generation cuts: compare tight generation cuts with loose generation and tight analysis cuts
- Parton level results are not dependent so much on the exact choice
- Shower can reshuffle momenta, larger effects



Partonic

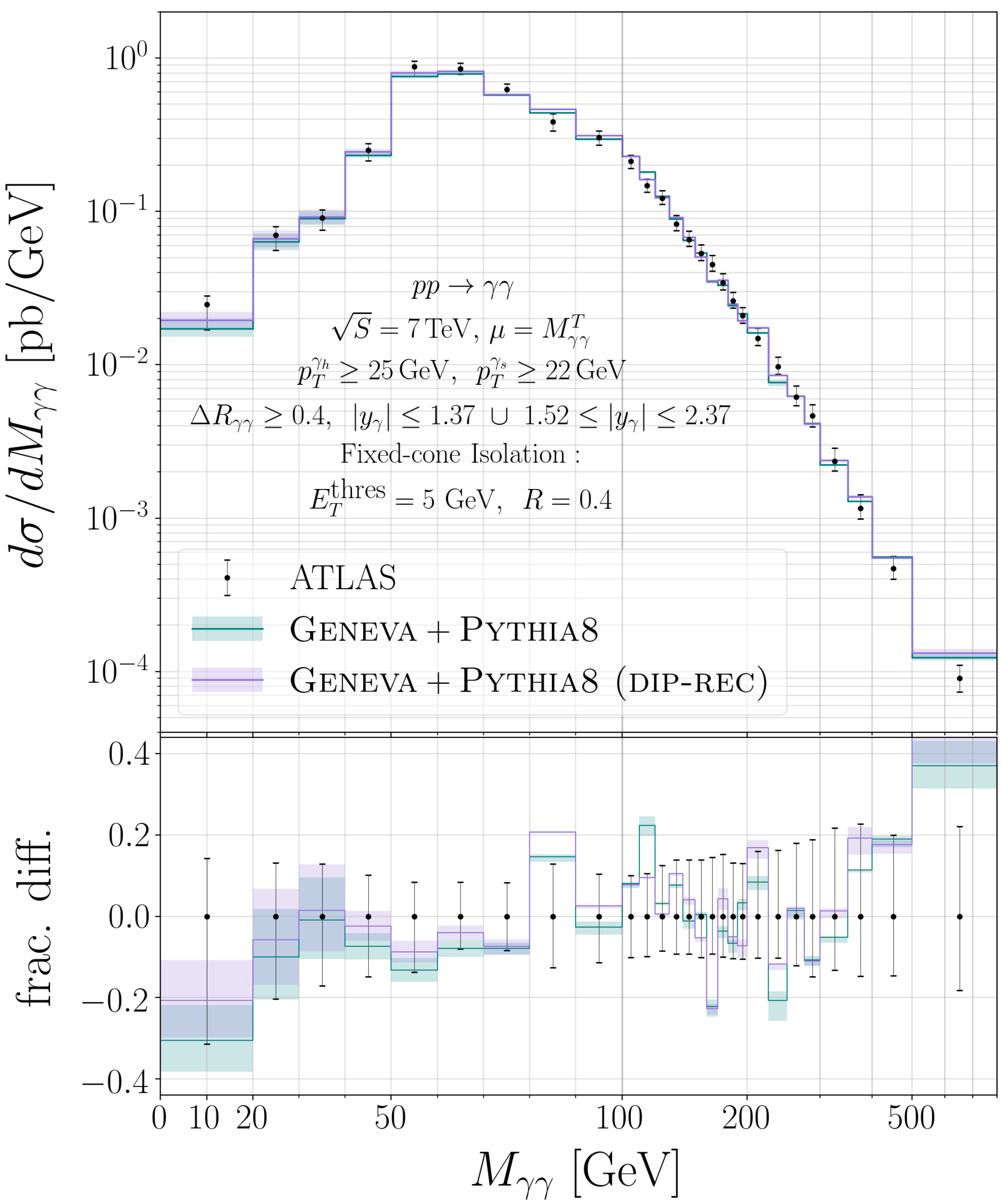


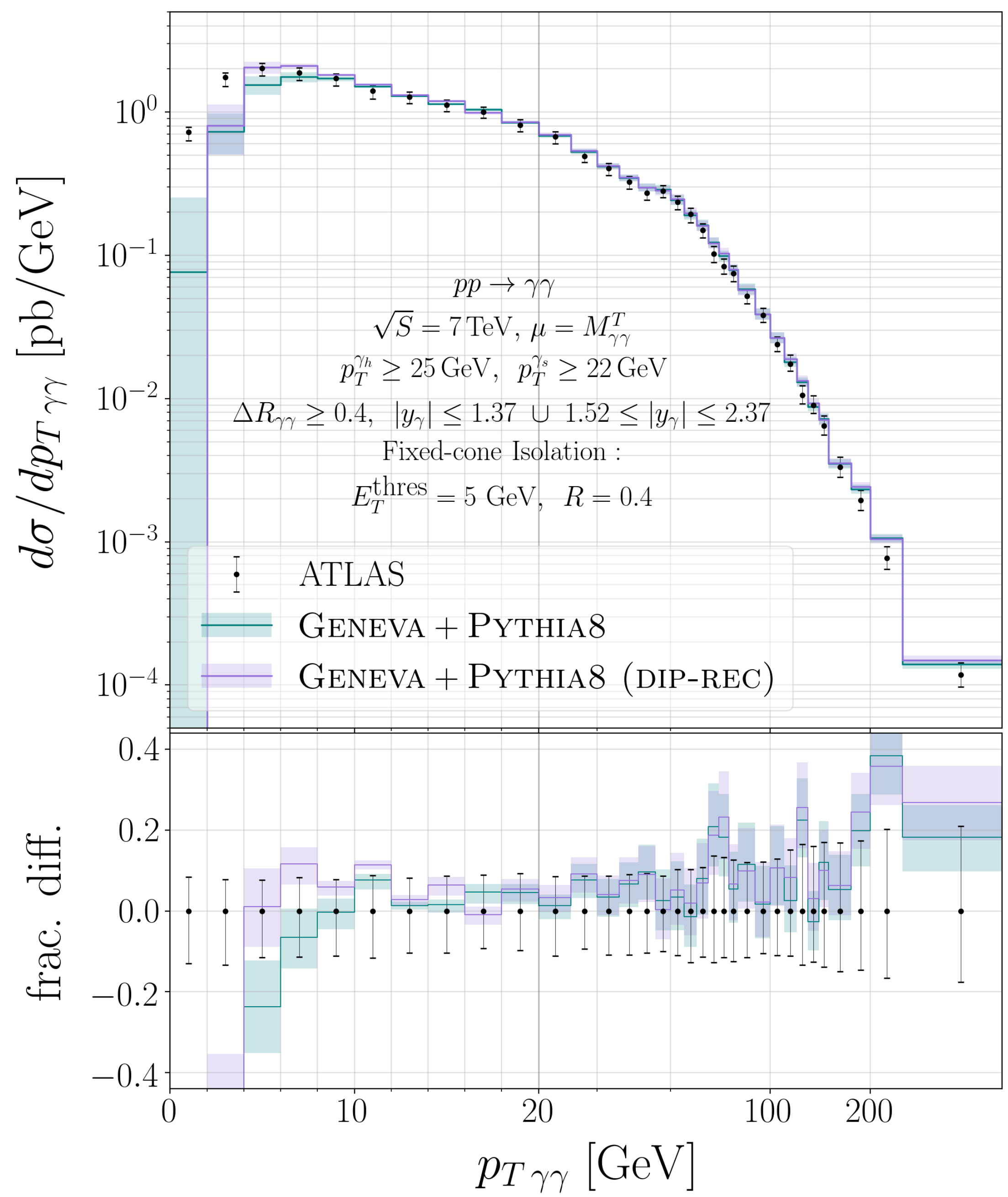
Showered

Comparison to ATLAS data LHC 7 TeV

Hybrid isolation procedure.
Process-defining cuts at generation level

$$p_T^{\gamma_h} \geq 18 \text{ GeV}, \quad p_T^{\gamma_s} \geq 15 \text{ GeV}, \quad M_{\gamma\gamma} \geq 1 \text{ GeV}$$
$$E_T^{\max} = 4 \text{ GeV}, \quad R_{\text{iso}} = 0.1, \quad \text{and} \quad n = 1\,.$$



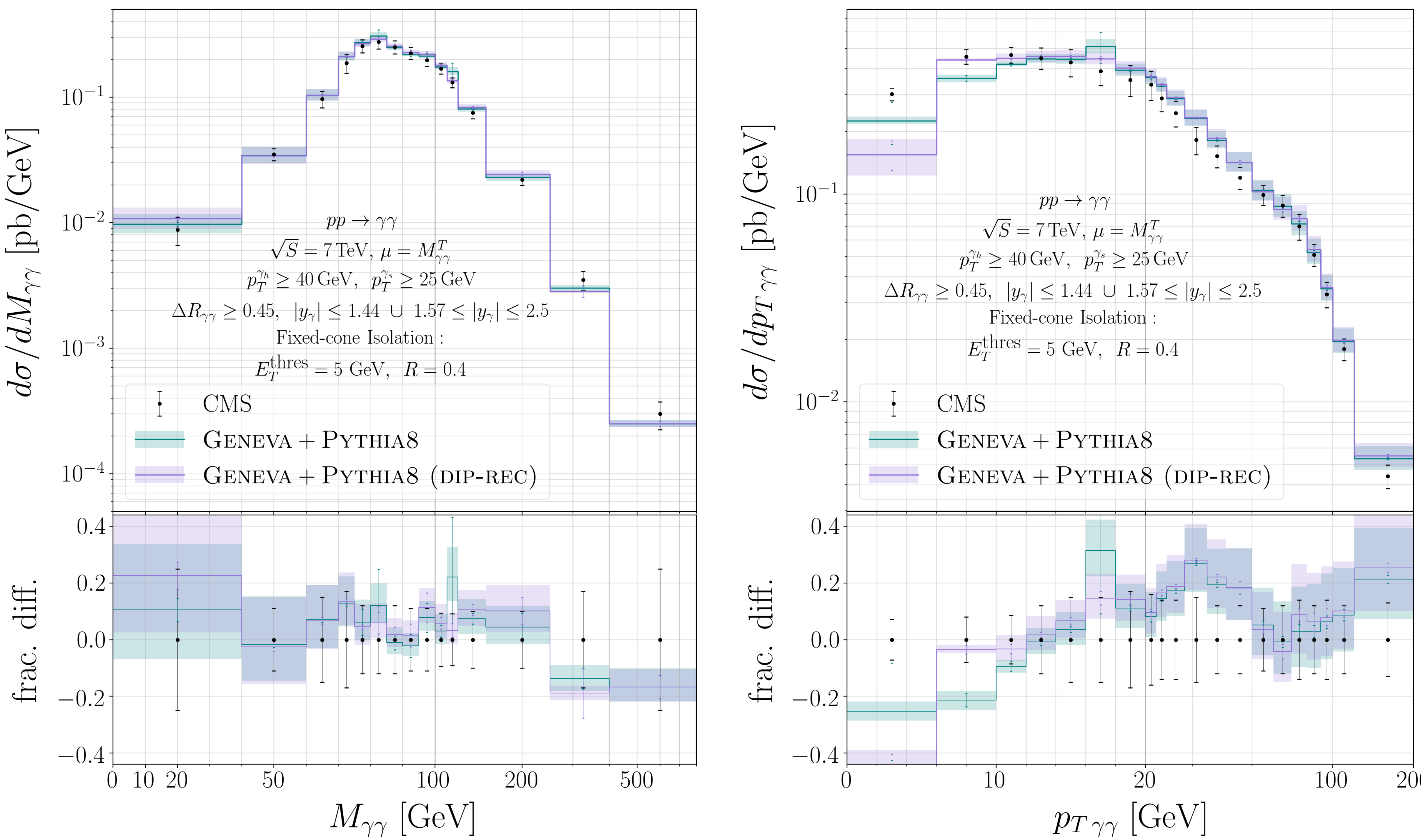


2-loop top massive effects not yet included in qqbar channel. EW effects also important at high $M_{\gamma\gamma}$

ATLAS [arXiv:1211.1913]

Comparison to CMS data LHC 7 TeV

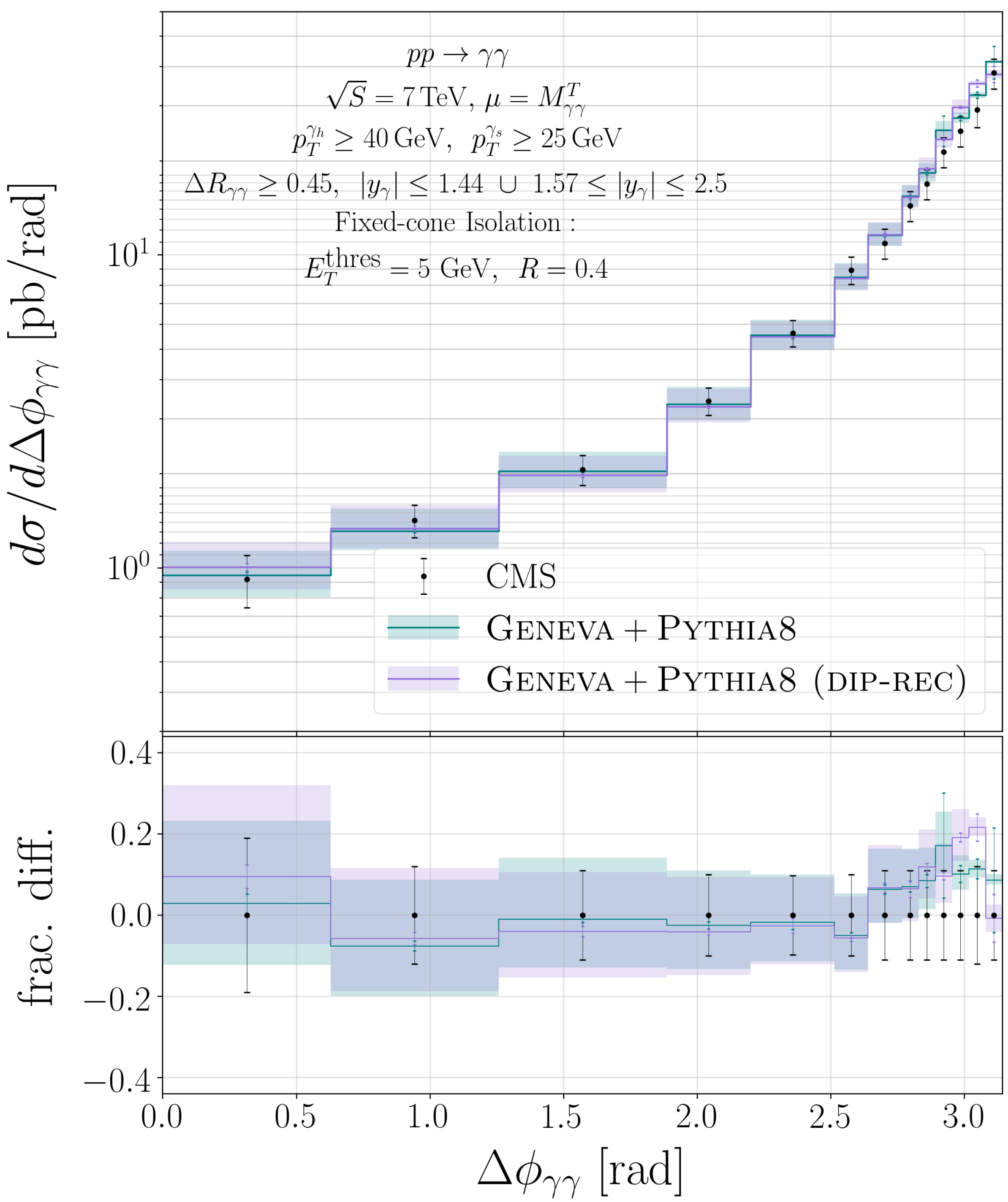
Hybrid isolation procedure (smooth-cone at generation with $R_{\text{iso}} = 0.1$)

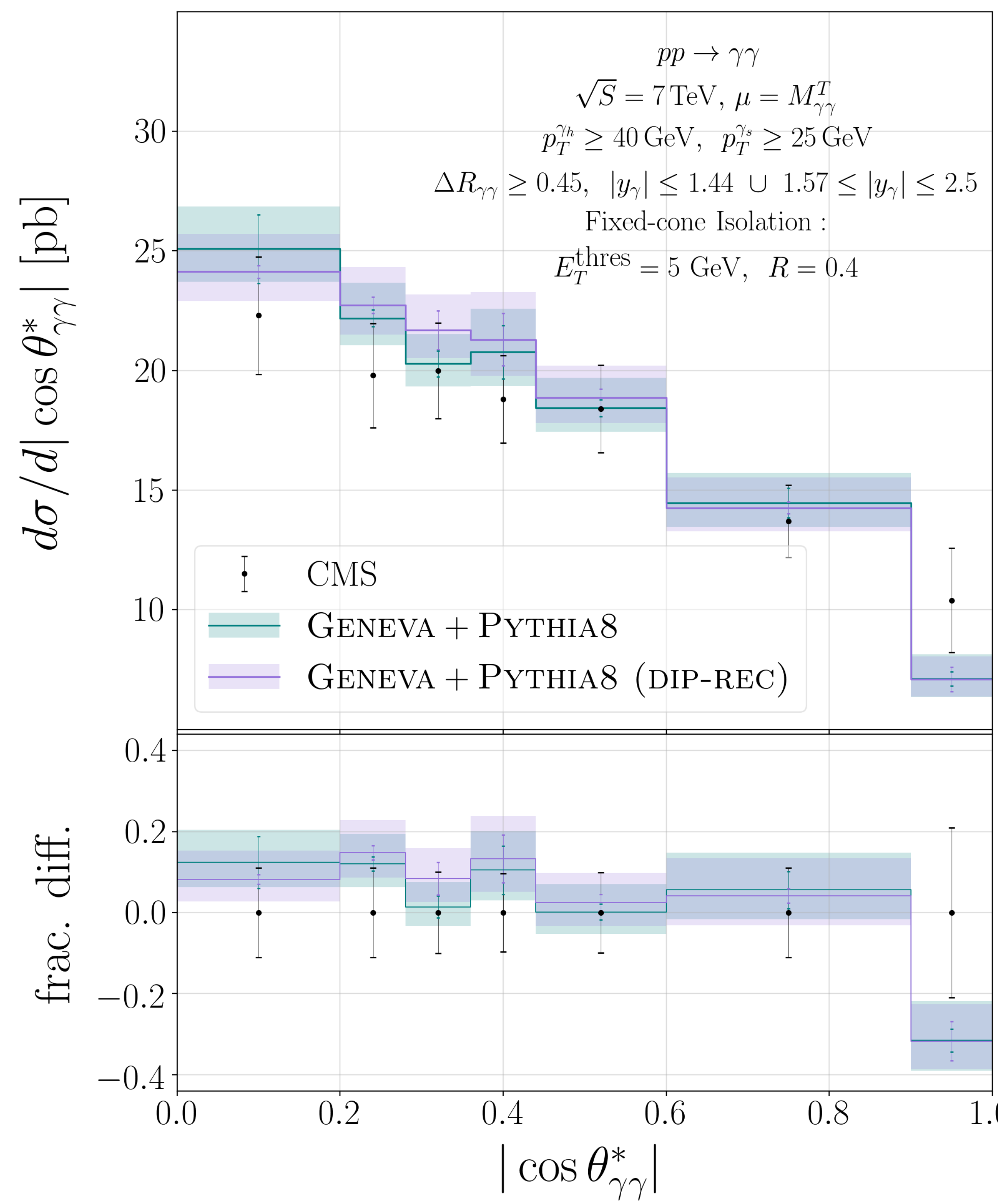


CMS [arXiv:1405.7225]

Comparison to CMS data LHC 7 TeV

Hybrid isolation procedure (initial smooth-cone $R_{\text{iso}} = 0.1$)





Cosine of the photon angle in the Collins-Soper frame

Zero-jettiness resummation for top-quark pair production at the LHC

Based on arXiv:2111.03632, S. Alioli, AB, M.A. Lim

0-jettiness resummation for $t\bar{t}$ production

- Top-quark properties are very interesting, interplay with the Higgs sector
- It is desirable to have a NNLO+PS calculation. Extrapolation from fiducial to inclusive phase space is done using NLO event generators, see for example [Behring, Czakon, Mitov, Papanastasiou, Poncelet `19])
- Recently, NNLO+PS for $t\bar{t}$ production available via MINNLOPS formalism [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20]
- Including higher-order resummation can improve the description of observables (this is the case of the GENEVA generator)

0-jettiness resummation for $t\bar{t}$ production

- To reach NNLO+PS accuracy in GENEVA
  - NLO calculations for $t\bar{t}$ and $t\bar{t}$+jet
  - Resummed calculation at NNLL\` in the resolution variable $\mathcal{T}_0$ (or q_T)
  - q_T resummation via SCET (NNLL in 1307.2464) or direct QCD [1408.4564], [1806.01601] NNLL' ingredients (soft functions) in [1901.04005], [Angeles-Martinez, Czakon, Sapeta 1809.01459] but they are not publicly available
  - 0-jettines resummation is used for colour-singlet in GENEVA, must be extended for $t\bar{t}$ production
  - Definition of 0-jettiness has to be adapted with top-quarks in the final state, we choose to treat them like EW particles and exclude them from the sum over radiation
  - We first need to develop the resummation framework

Factorization

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \hat{s}$ are all hard scales. In case of boosted regime $M_{t\bar{t}} \gg m_t$ situation similar to [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

Hard functions (color matrices)

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b \boxed{B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)}\, \mathrm{Tr}\left[\boxed{\mathbf{H}_{ij}(\Phi_0, \mu)}\boxed{\mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)}\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of functions

$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Hard functions

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO it was first computed in [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{\Gamma}_H(M,\beta_t,\theta,\mu)\,\mathbf{H}(M,\beta_t,\theta,\mu) + \mathbf{H}(M,\beta_t,\theta,\mu)\,\mathbf{\Gamma}_H^\dagger(M,\beta_t,\theta,\mu)$$

Solution:

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^\dagger(M,\beta_t,\theta,\mu_h,\mu)$$

$$\boxed{\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)}$$

We have split the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\mathbf{\Gamma}_H(M,\beta_t,\theta,\mu) = \Gamma_{\text{cusp}}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha_s)$$ [Ferroglia, Neubert, Pecjak, Yang,`09]

$$\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha}{\beta(\alpha)}\boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)$$

We evaluate the matrix exponential **u** as a series expansion in α_s [1003.5827], [Buchalla,Buras,Lautenbacher `96]

Soft functions

We computed the soft functions matrices at NLO which were unknown for this observable

$$\mathbf{S}^{(1)}_{\text{bare},\,ij}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = \sum_{\alpha,\,\beta} \boldsymbol{w}_{ij}^{\alpha\beta}\, \hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu)$$

$$\hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = -\frac{2(\mu^2 e^{\gamma_E})^\epsilon}{\pi^{1-\epsilon}} \int \mathrm{d}^d k \frac{v_\alpha \cdot v_\beta}{v_\alpha \cdot k\, v_\beta \cdot k}\, \delta(k^2)\Theta(k^0)$$
$$\times \left[\delta(k_a^+ - k\cdot n_a)\Theta(k\cdot n_b - k\cdot n_a)\,\delta(k_b^+) + \delta(k_b^+ - k\cdot n_b)\Theta(k\cdot n_a - k\cdot n_b)\,\delta(k_a^+)\right]$$

One can average over the two hemisphere momenta, the soft function satisfies the RG equation in Laplace space

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) = \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s\dagger}\right] \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) + \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s}\right]$$

Solution in momentum space, where we used the consistency relation among anomalous dimensions $\boldsymbol{\gamma}^s = \boldsymbol{\gamma}^h + \gamma^B\, \mathbf{1}$

$$\mathbf{S}_B(l^+, \beta_t, \theta, \mu) = \exp\left[4S(\mu_s, \mu) + 2a_{\gamma^B}(\mu_s, \mu)\right]$$
$$\times\; \mathbf{u}^\dagger(\beta_t, \theta, \mu, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s}, \beta_t, \theta, \mu_s)\, \mathbf{u}(\beta_t, \theta, \mu, \mu_s)\, \frac{1}{l^+}\left(\frac{l^+}{\mu_s}\right)^{2\eta_s} \frac{e^{-2\gamma_E \eta_s}}{\Gamma(2\eta_s)}$$

$$\eta_s \equiv -2a_\Gamma(\mu_s, \mu)$$

Beam functions

The beam functions are given by convolutions of perturbative kernels with the standard PDFs $f_i(x,\mu)$

$$B_i(t,z,\mu) = \sum_j \int_z^1 \frac{d\xi}{\xi} I_{ij}(t,z/\xi,\mu) f_j(\xi,\mu)$$

I_{ij} kernels are known up to N^{3}LO, process independent

RG equation in Laplace space is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \Big[-2\,\Gamma_{\rm cusp}(\alpha_s)\,L_c + \gamma_i^B(\alpha_s)\Big]\tilde{B}_i(L_c,z,\mu)$$

with solution in momentum space

$$\boxed{B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right]\tilde{B}(\partial_{\eta_B},z,\mu_B)\frac{1}{t}\left(\frac{t}{\mu_B^2}\right)^{\eta_B}\frac{e^{-\gamma_E\eta_B}}{\Gamma(\eta_B)}}$$

where $\eta_B \equiv 2a_\Gamma(\mu_B,\mu)$ and the collinear log is given by $L_c = \ln\left(M\kappa/\mu^2\right)$

Resummed result for the cross section

We can combine the solutions for the hard, soft and beam functions to obtain

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s)$$
$$\times \operatorname{Tr}\left\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathbf{H}(M, \beta_t, \theta, \mu_h)\, \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \right\}$$
$$\times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})}$$

where

$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) =$$
$$\exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \right]$$

and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

Resummed result for the cross section

- We have

  - hard functions at NLO

  - soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we only miss $\delta(\mathcal{T}_0)$ terms at NNLO

  - beam functions at NNLO (both for $q\bar{q}$ and gg channels)

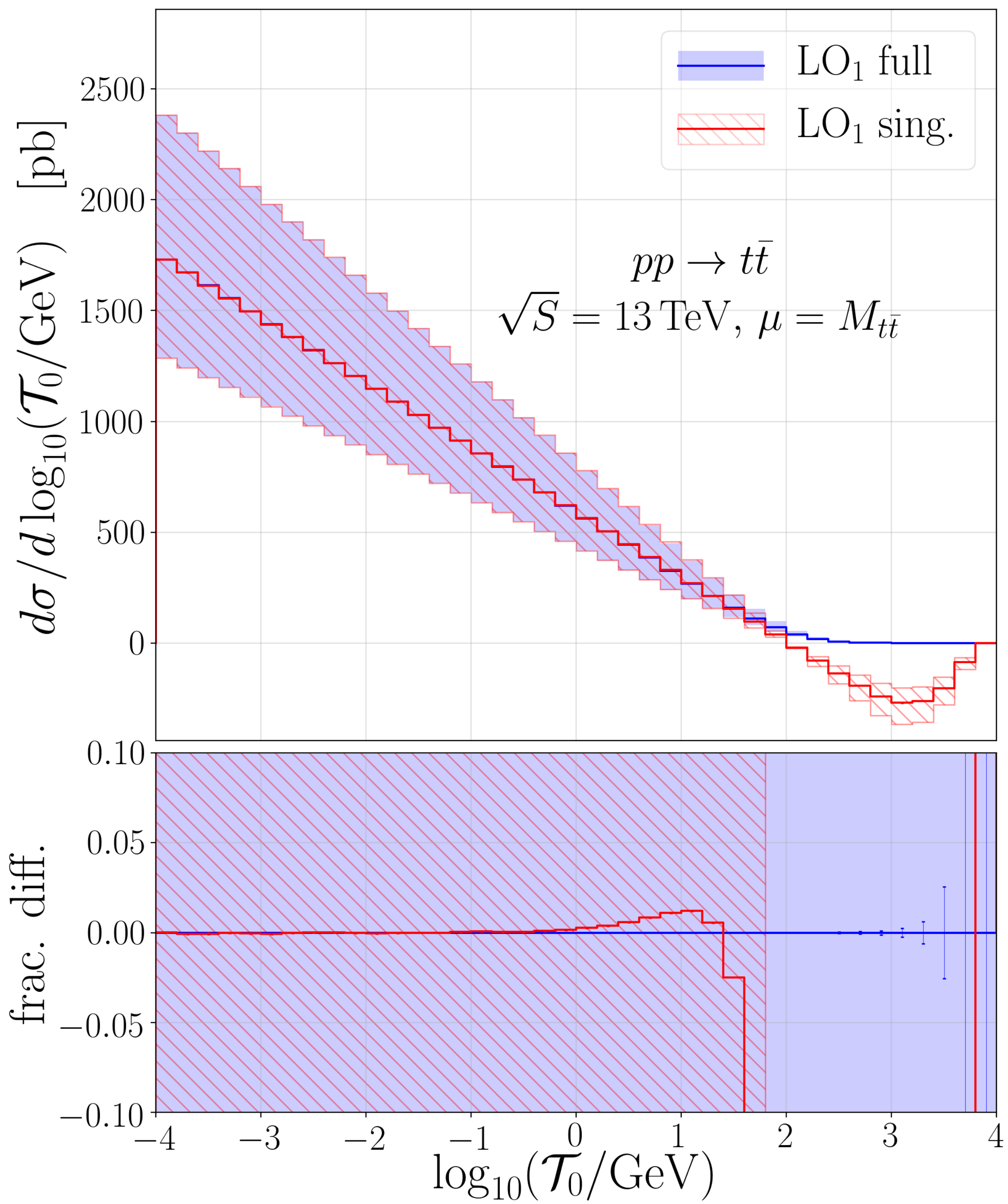
  - two-loop anomalous dimensions

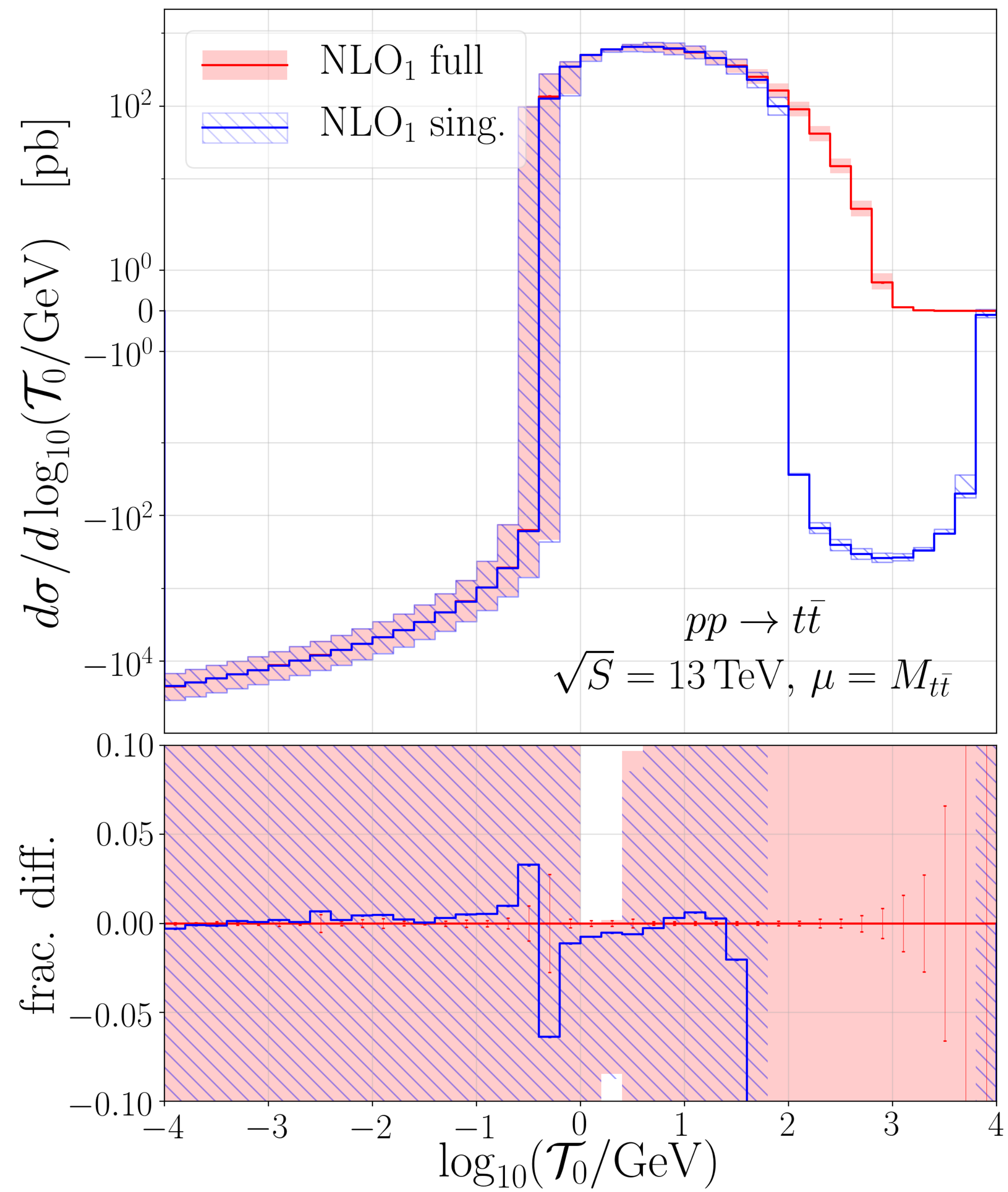
- We can resum to NNLL. We are missing $\delta(\mathcal{T}_0)$ terms (NNLO hard functions and NNLO soft). If we include everything we know we obtain a $\mathrm{NNLL}'_{\mathrm{a}}$ result

- We construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for $\mathcal{T}_0 > 0$)

Resummed results

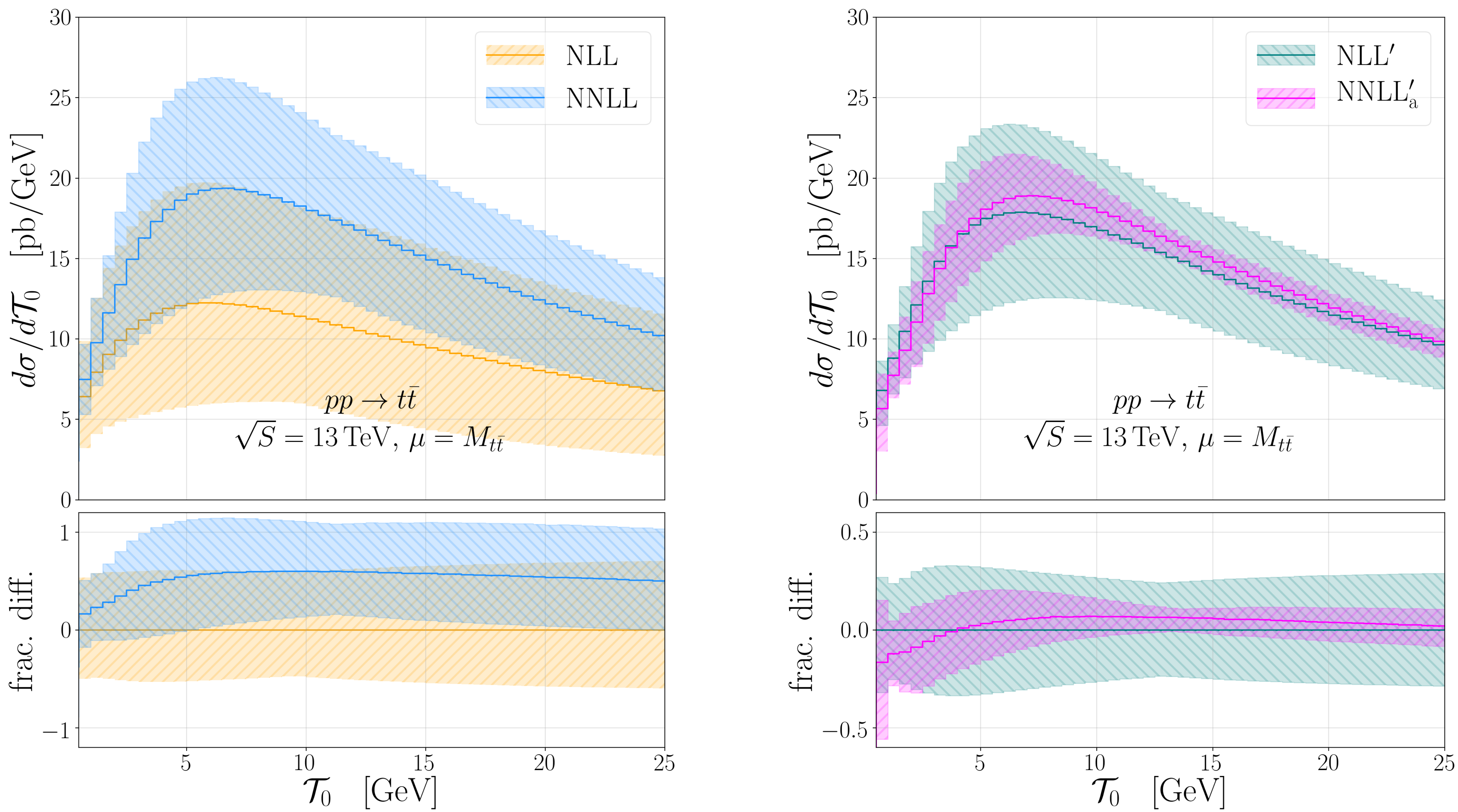
Fixed-order comparisons, approximate NLO and approximate NNLO vs LO_1 and NLO_1





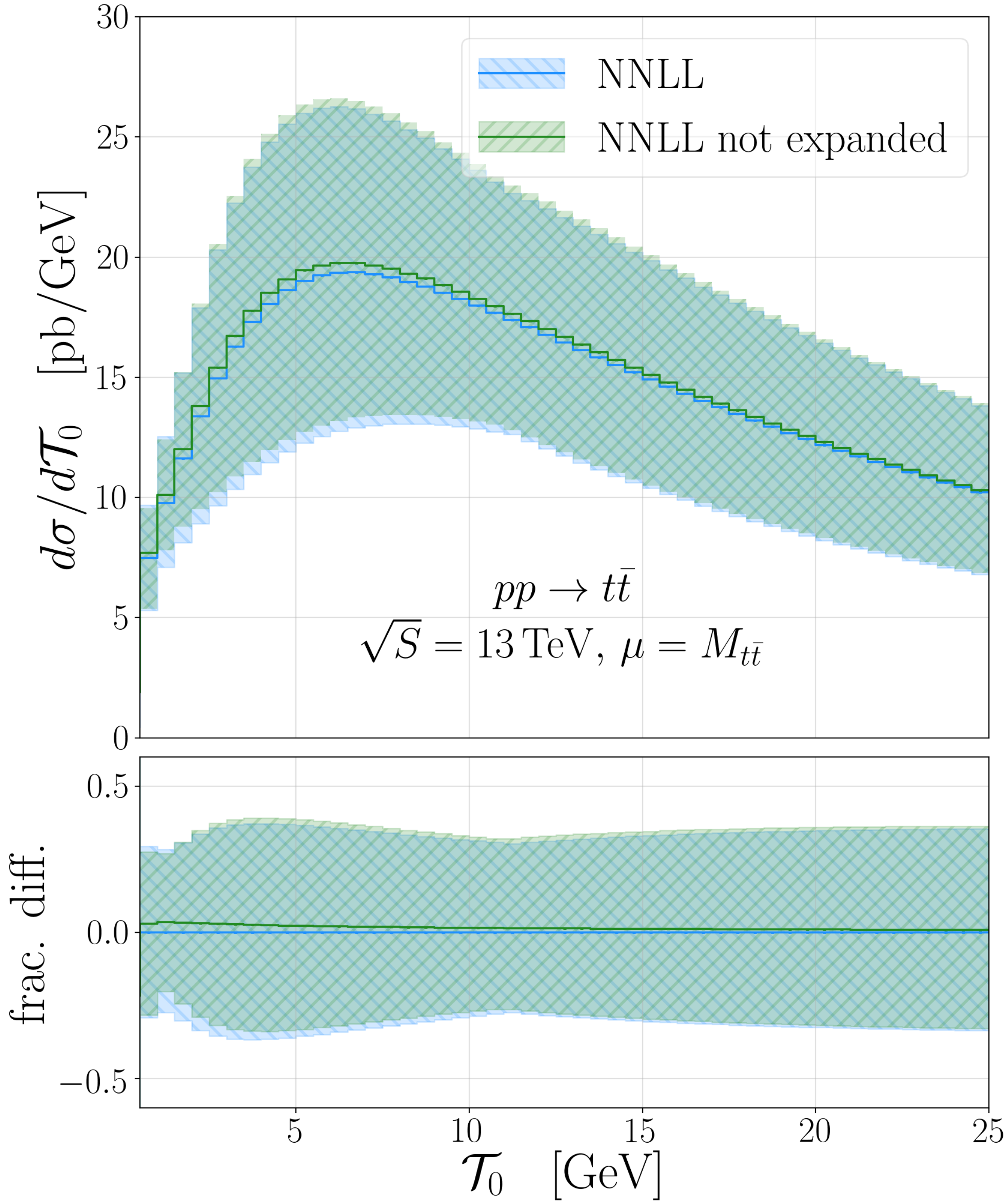
Resummed results

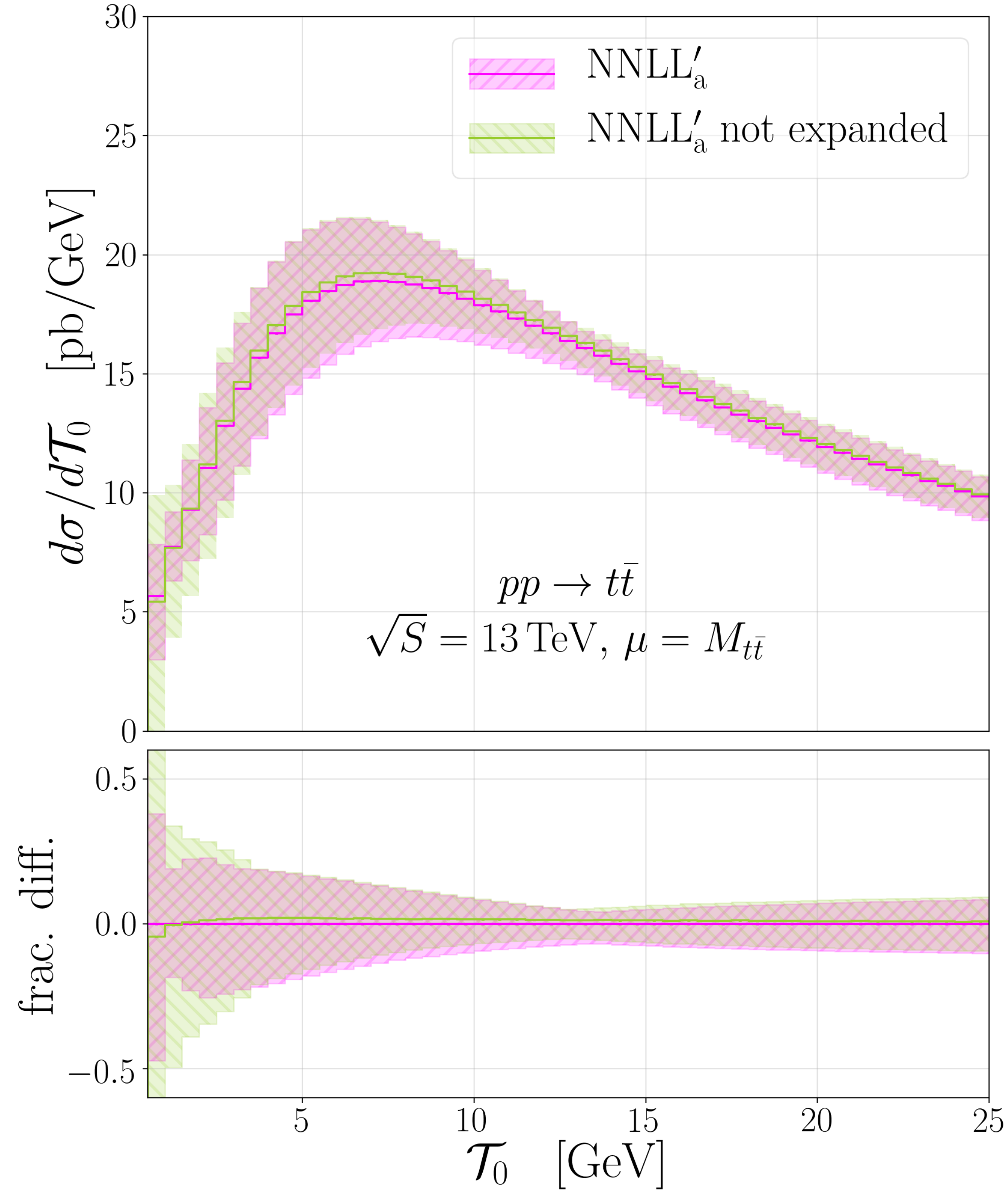
NNLL$'_a$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



Resummed results

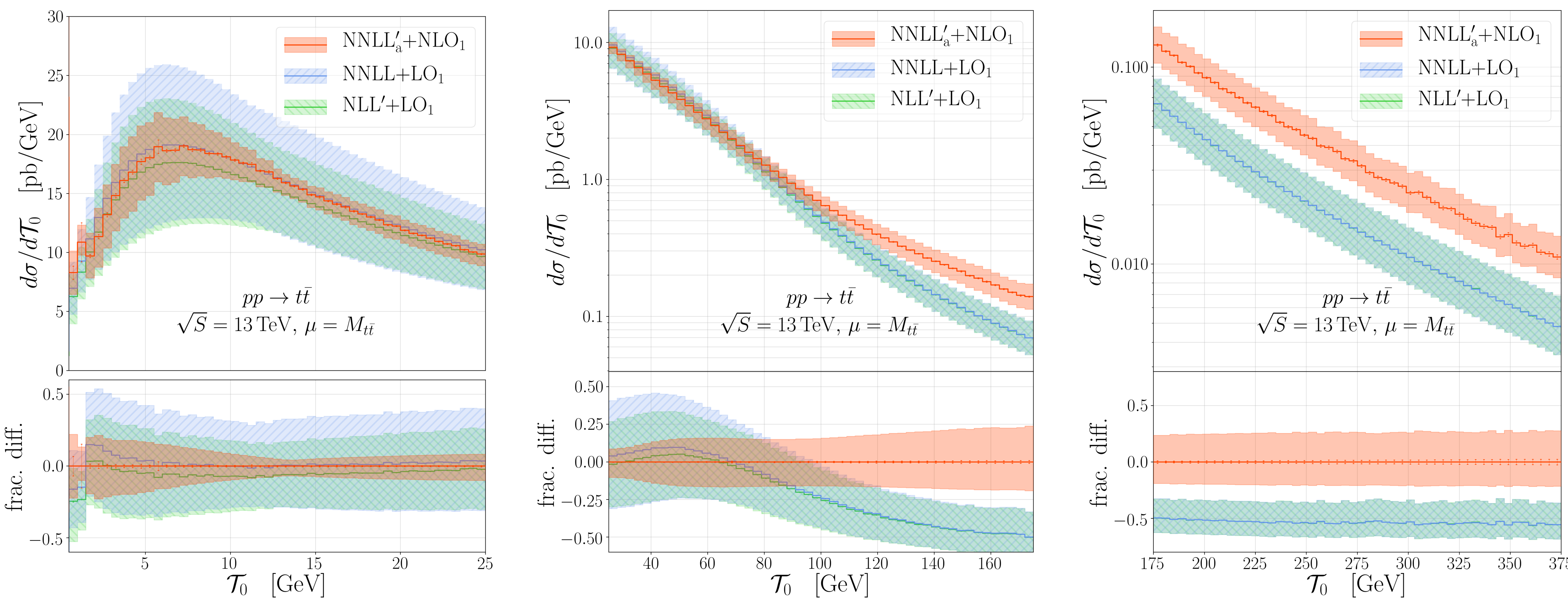
The evolution matrix **u** is evaluated in α_s expansion, we can choose to expand or not expand U, the difference is quite small





Matched results to fixed-order

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$



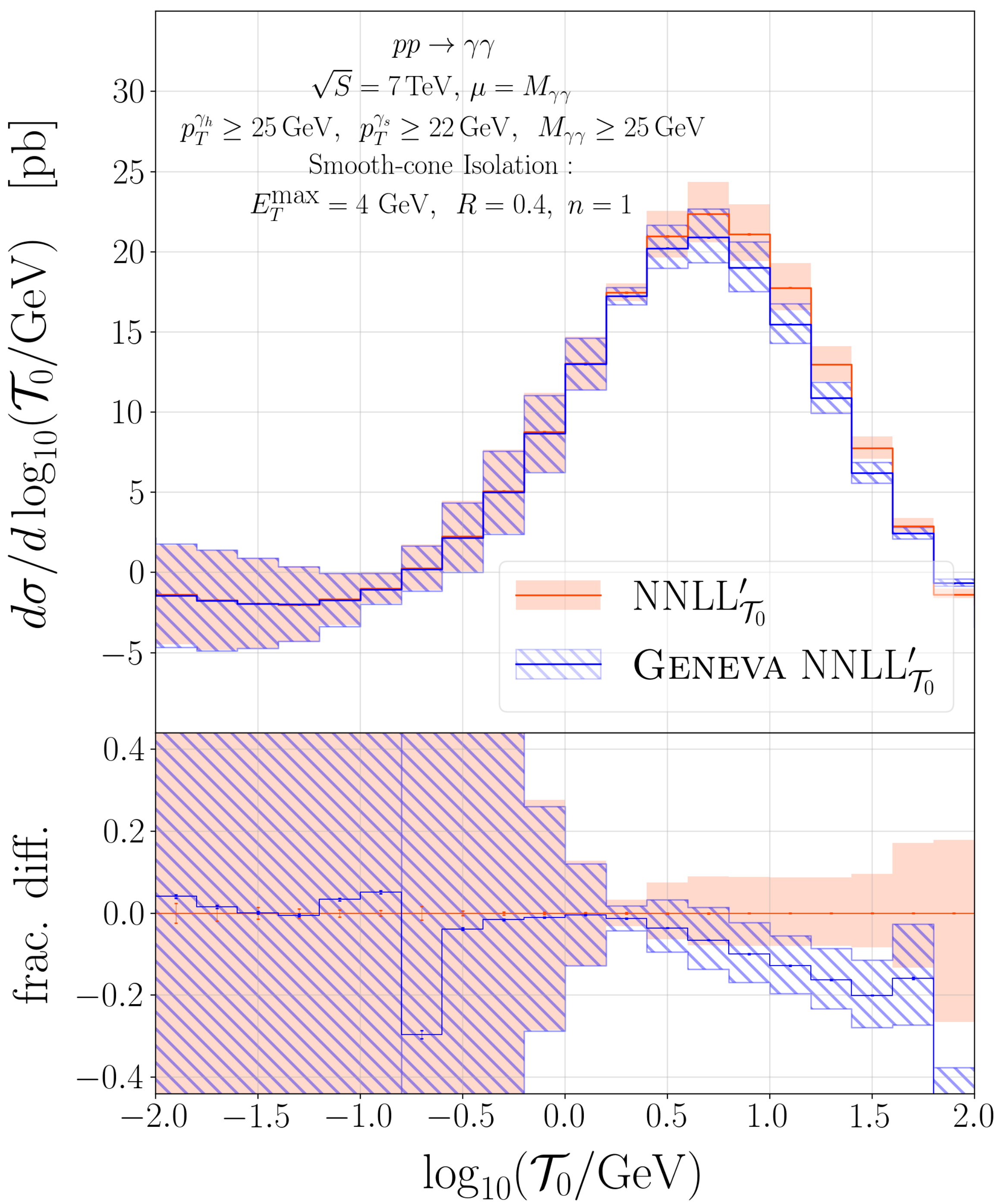
Outlook

Thank you!

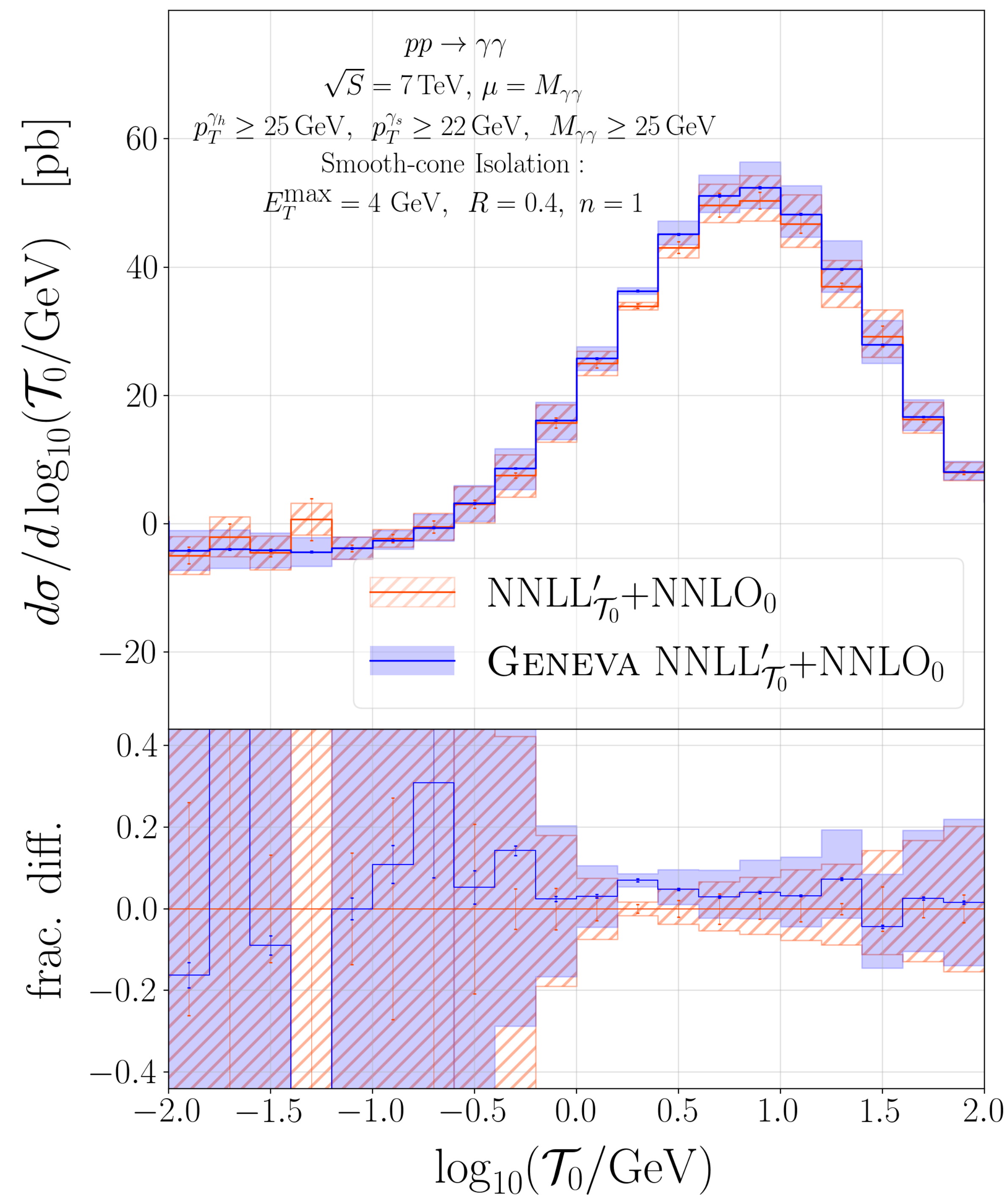
Backup slides

Monte Carlo implementation

Geneva is equivalent to standard resummation only in the $\mathcal{T}_0 \to 0$ limit, away from this limit same result only if one cuts on quantities preserved by $\Phi_1 \to \Phi_0$



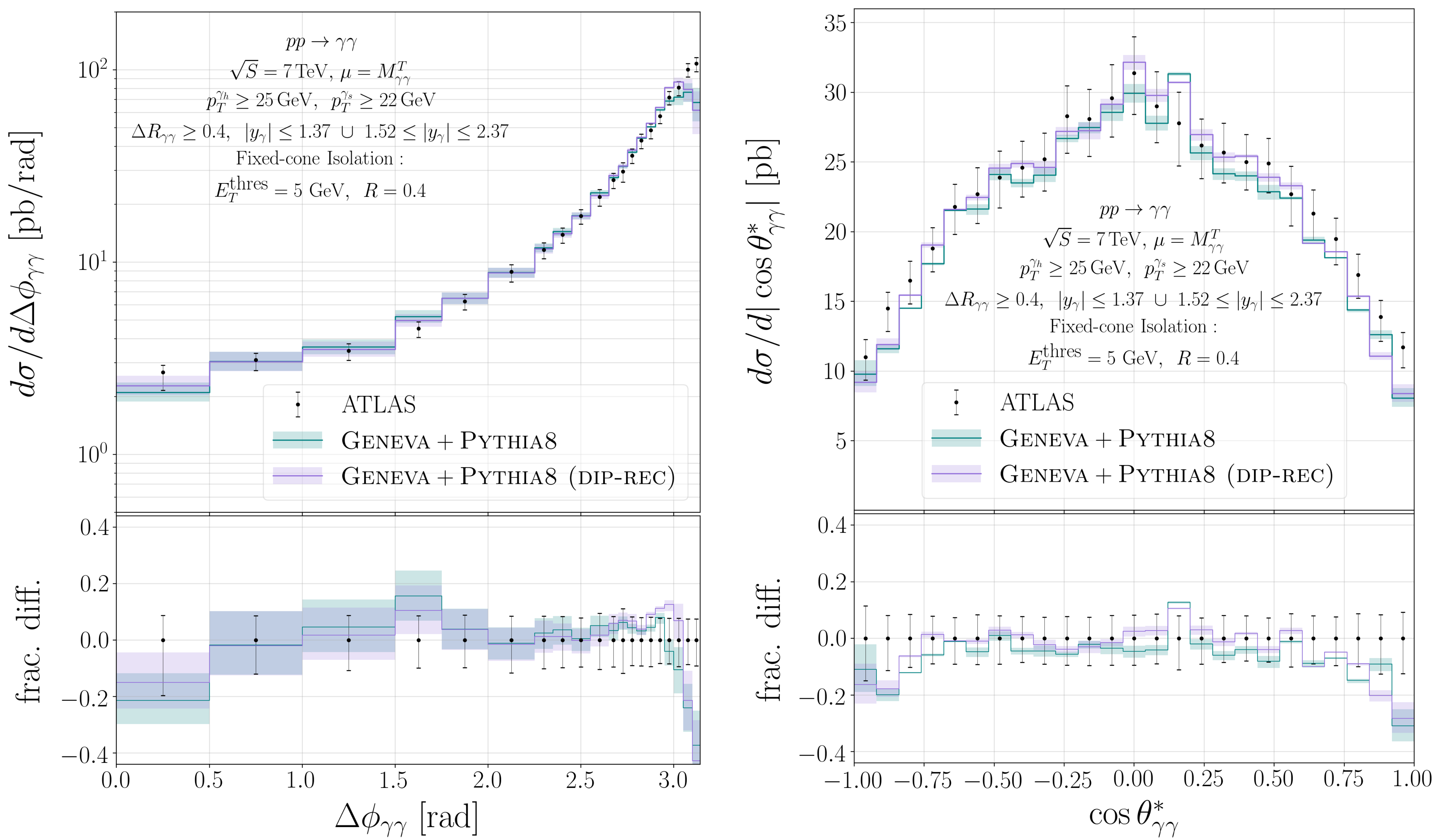
Resummed computation



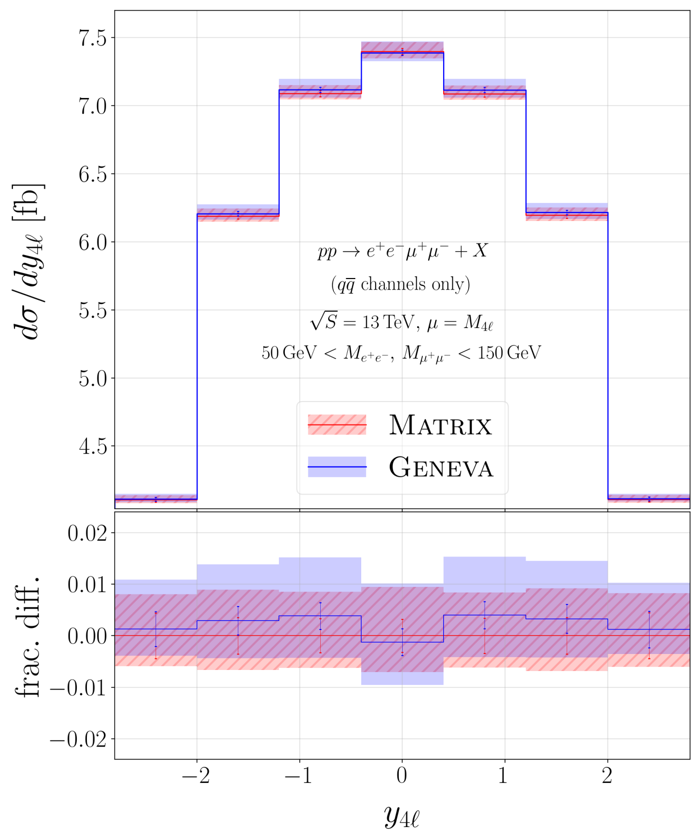
Matched computation

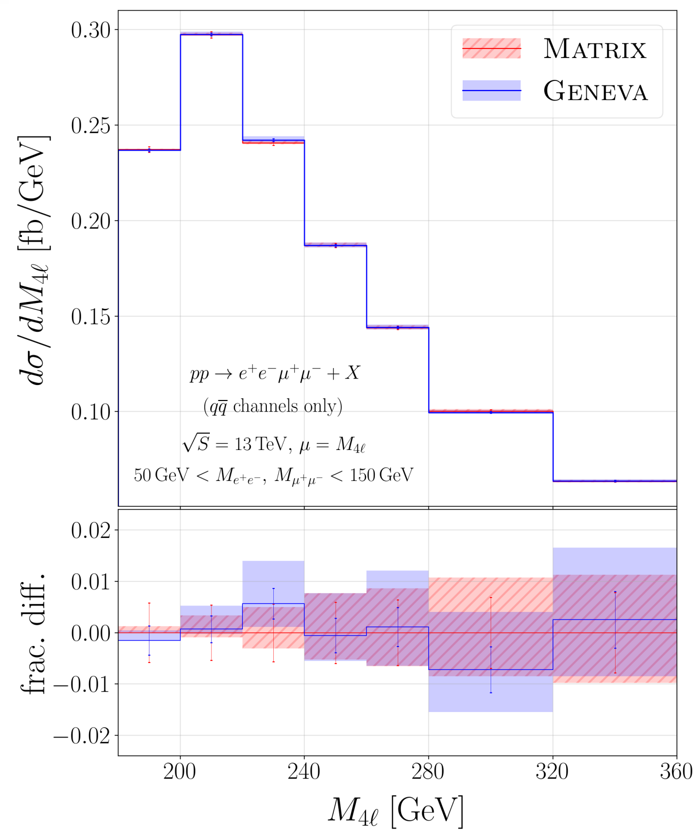
Comparison to ATLAS data LHC 7 TeV

Hybrid isolation procedure (initial smooth-cone $R_{\text{iso}} = 0.1$)



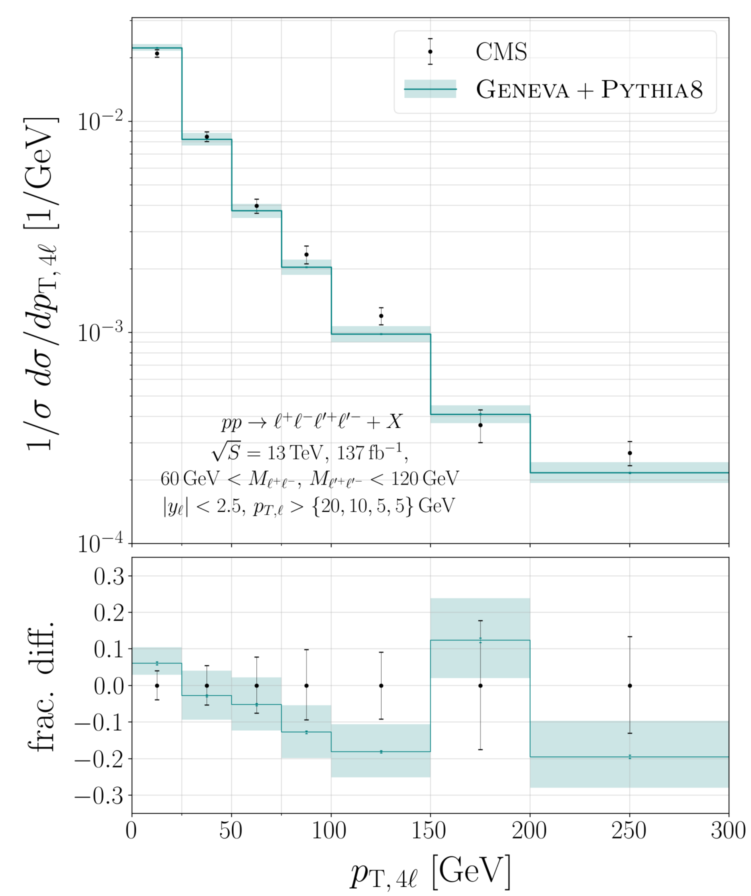
Diboson production $ZZ \to l^+l^-l'^+l'^-$

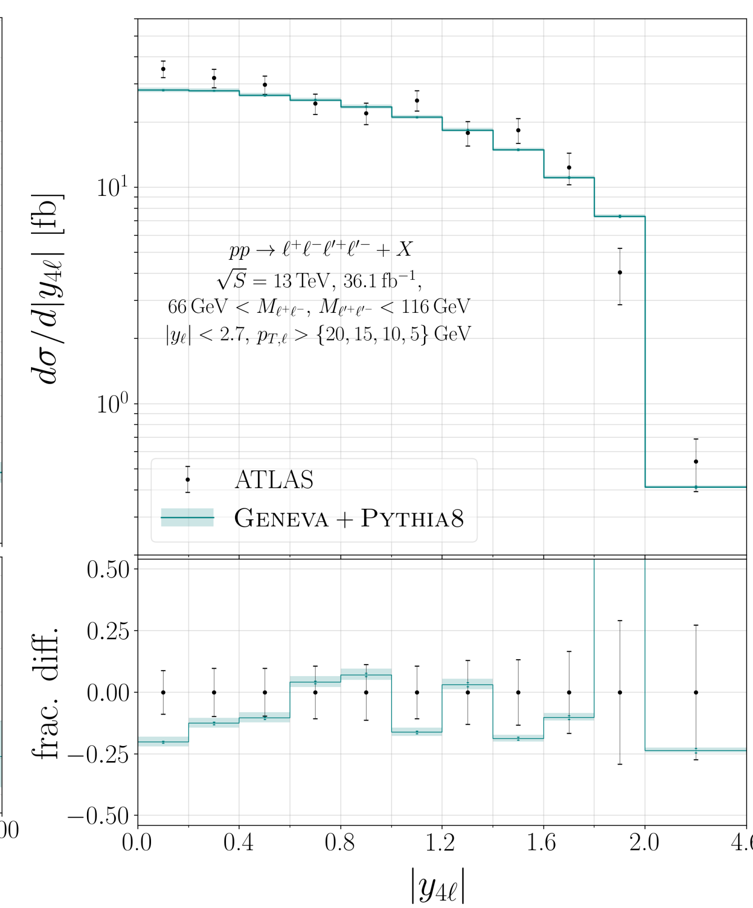




NNLO validation against MATRIX, complex kinematics dependence

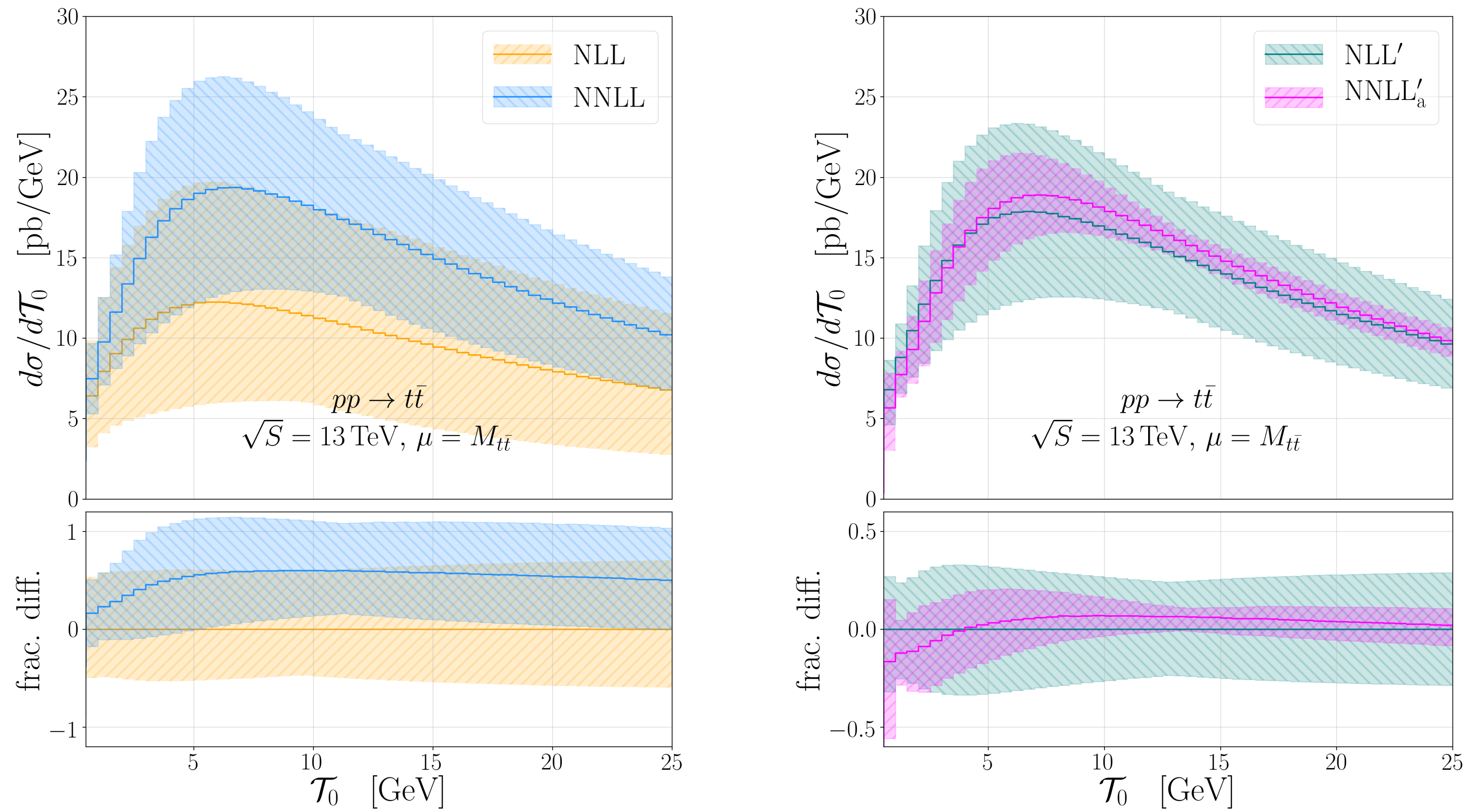
After the inclusion of the gg-channel at LO we compared to ATLAS and CMS





Resummed results

NNLL′ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



$$
\mu_H = \mu_{\rm NS}\,,\qquad
\mu_S(\mathcal{T}_0) = \mu_{\rm NS}\ f_{\rm run}(\mathcal{T}_0/M)\,,\qquad
\mu_B(\mathcal{T}_0) = \mu_{\rm NS}\ \sqrt{f_{\rm run}(\mathcal{T}_0/M)}
$$

$$
f_{\rm run}(y) = \begin{cases} y_0\big[1+(y/y_0)^2/4\big] & y \le 2y_0\,,\\ y & 2y_0 \le y \le y_1\,,\\ y + \frac{(2-y_2-y_3)(y-y_1)^2}{2(y_2-y_1)(y_3-y_1)} & y_1 \le y \le y_2\,,\\ 1 - \frac{(2-y_1-y_2)(y-y_3)^2}{2(y_3-y_1)(y_3-y_2)} & y_2 \le y \le y_3\,,\\ 1 & y_3 \le y\,. \end{cases}
$$

$$
y_0 = 1.0\,\mathrm{GeV}/M\,,\quad \{y_1, y_2, y_3\} = \{0.1, 0.175, 0.25\}
$$