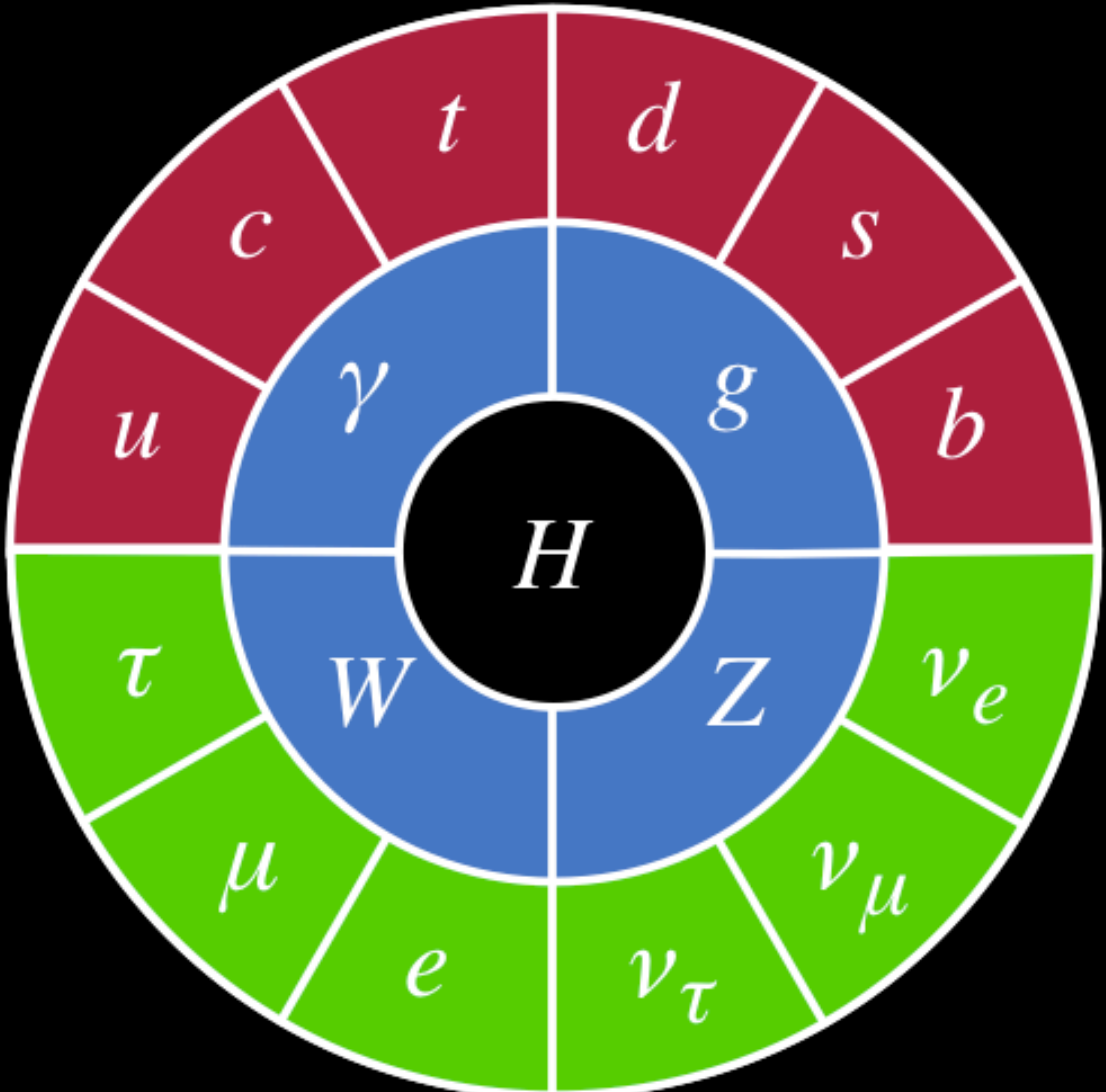
Quantum Computing for Colliders

Christian Bauer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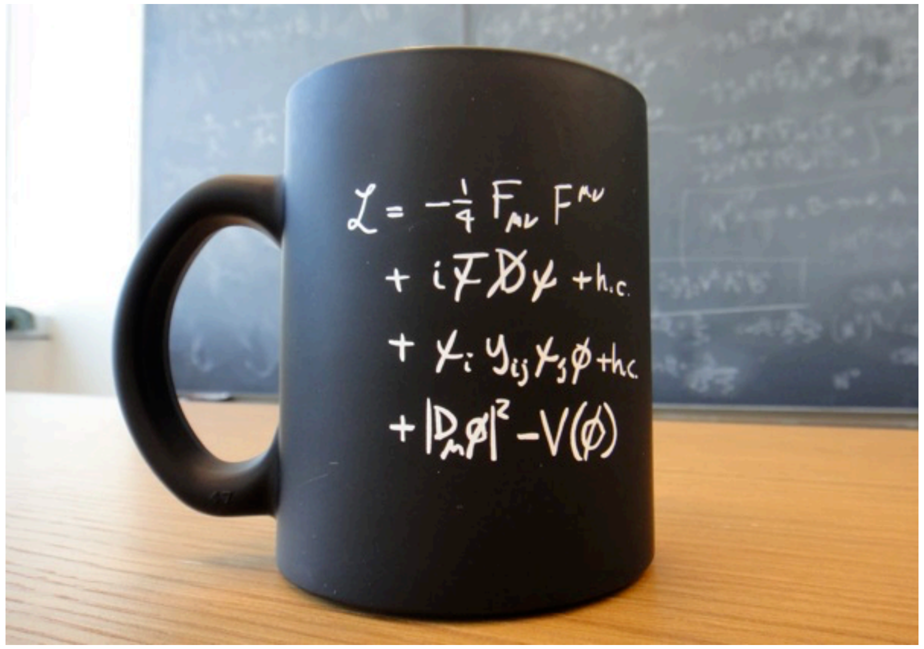
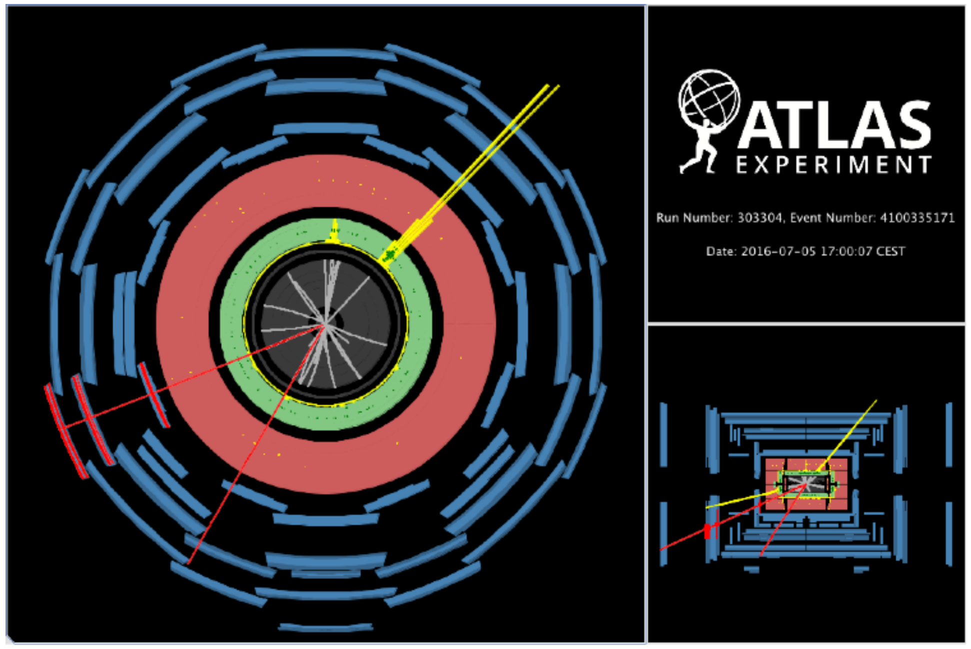
Christian Bauer
Quantum Computing for Colliders

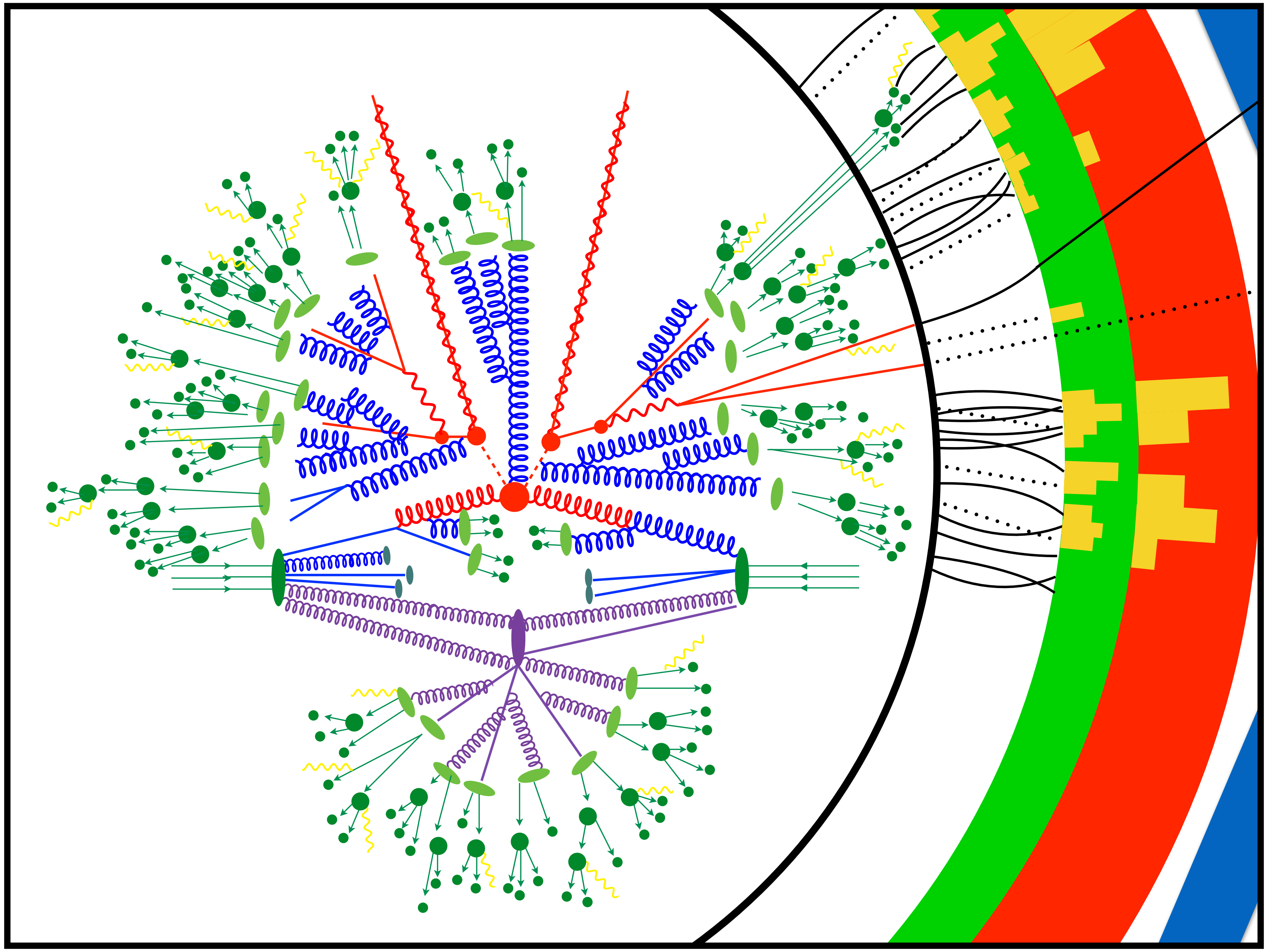
To truly understand if Standard Model describes data observed at LHC, need to connect theory and data





For this, need to be able to go from Lagrangian to fully exclusive events

One of the holy grails of HEP is the full simulation of scattering processes at colliders



One of the holy grails of HEP is the full simulation of scattering processes at colliders

Dream would be to literally compute the full S-matrix

Perform measurement of final state at time T

Create initial state with 2 protons at time -T

$$\left| \langle X(T) \,|\, U(T,-T) \,|\, pp(-T) \rangle \right|^2$$

Perform time evolution with full SM Hamiltonian from initial time -T to final time T

One of the holy grails of HEP is the full simulation of scattering processes at colliders

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Create initial state with 2 protons at time -T

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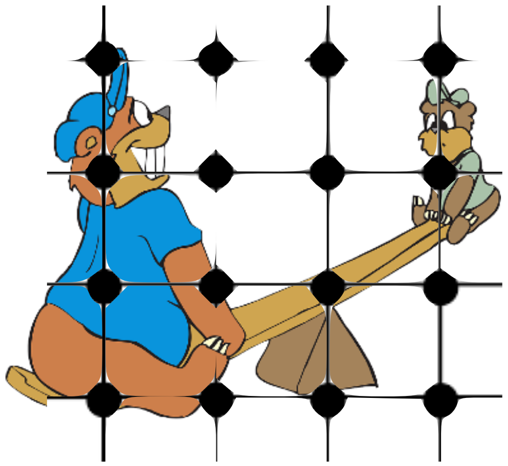
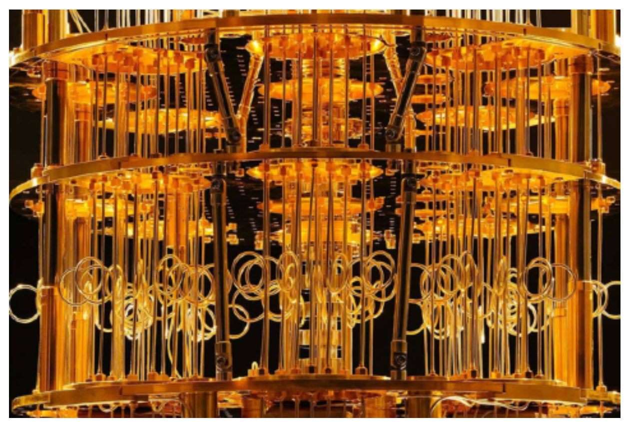
Perform time evolution with full SM Hamiltonian from initial time -T to final time T

1. This clearly requires Quantum Physics (Quantum Field Theory)
2. This is something that is not even remotely feasible using classical computers
3. Would revolutionize how we can compare experimental collider measurements with theoretical predictions

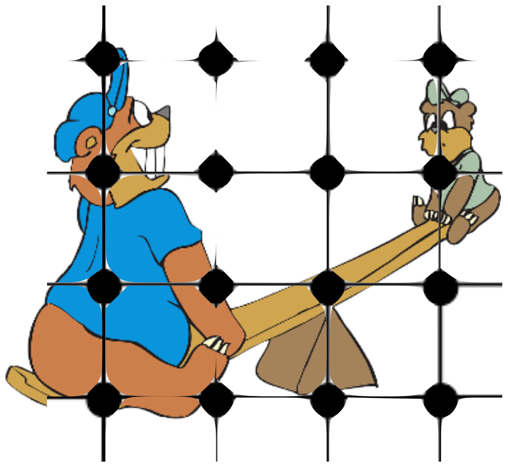


General principles



Soft functions on a quantum computer

Dealing with errors in quantum computers



General principles

One can turn the QFT calculation into a QM calculation by discretization / digitization

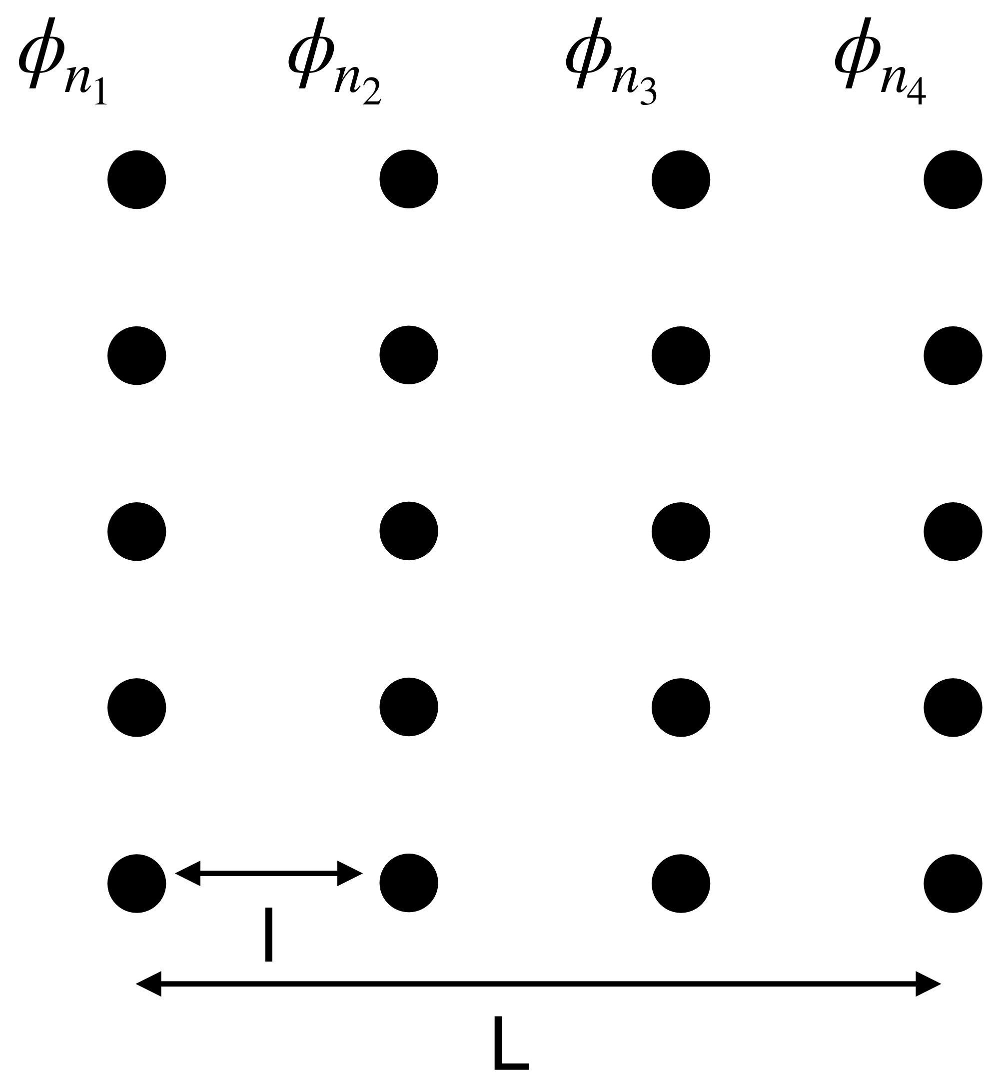
$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

All elements in this expression in terms of fields $\phi(x)$
Both position x and field $\phi(x)$ are continuous

Discretizing position x and digitizing field value $\phi(x)$ turn continuous (QFT) problem into discrete (QM) problem

Basic idea is to map the infinite Hilbert space of QFT on a finite dimensional HS making this a QM problem

Instead of having a continuous field ϕ at each position x, we put a digitized field ϕ_n at discrete points x_k arranged on a lattice



$$L = N\, l$$

Hilbert space has dimension

$$\left(n_\phi\right)^{N^d}$$

n_ϕ : # of digitized field values
n_L : # of lattice points per dim
d : # of dimensions

Problem reduced to matrix multiplication

Basic idea is to map the infinite Hilbert space of QFT on a finite dimensional HS making this a QM problem

$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

3 basic steps:

1. Create an initial state vector at time (-T) of two proton wave packets
2. Evolve this state forward in time from to time T using the Hamiltonian of the full interacting field theory
3. Perform a measurement of the state

Let's try to estimate the resources we need to simulate physics at the LHC

Energy rage that can be described by lattice is given by

$$\frac{1}{Nl} \lesssim E \lesssim \frac{1}{l}$$

To simulate full energy range of LHC need

$$100\,\mathrm{MeV} \lesssim E \lesssim 7\,\mathrm{TeV}$$

This needs $\mathcal{O}(70{,}000^3) \sim 10^{14}$ lattice sites

Assume I need at least 5 bit digitization $\Rightarrow n_\phi = 2^5 = 32$

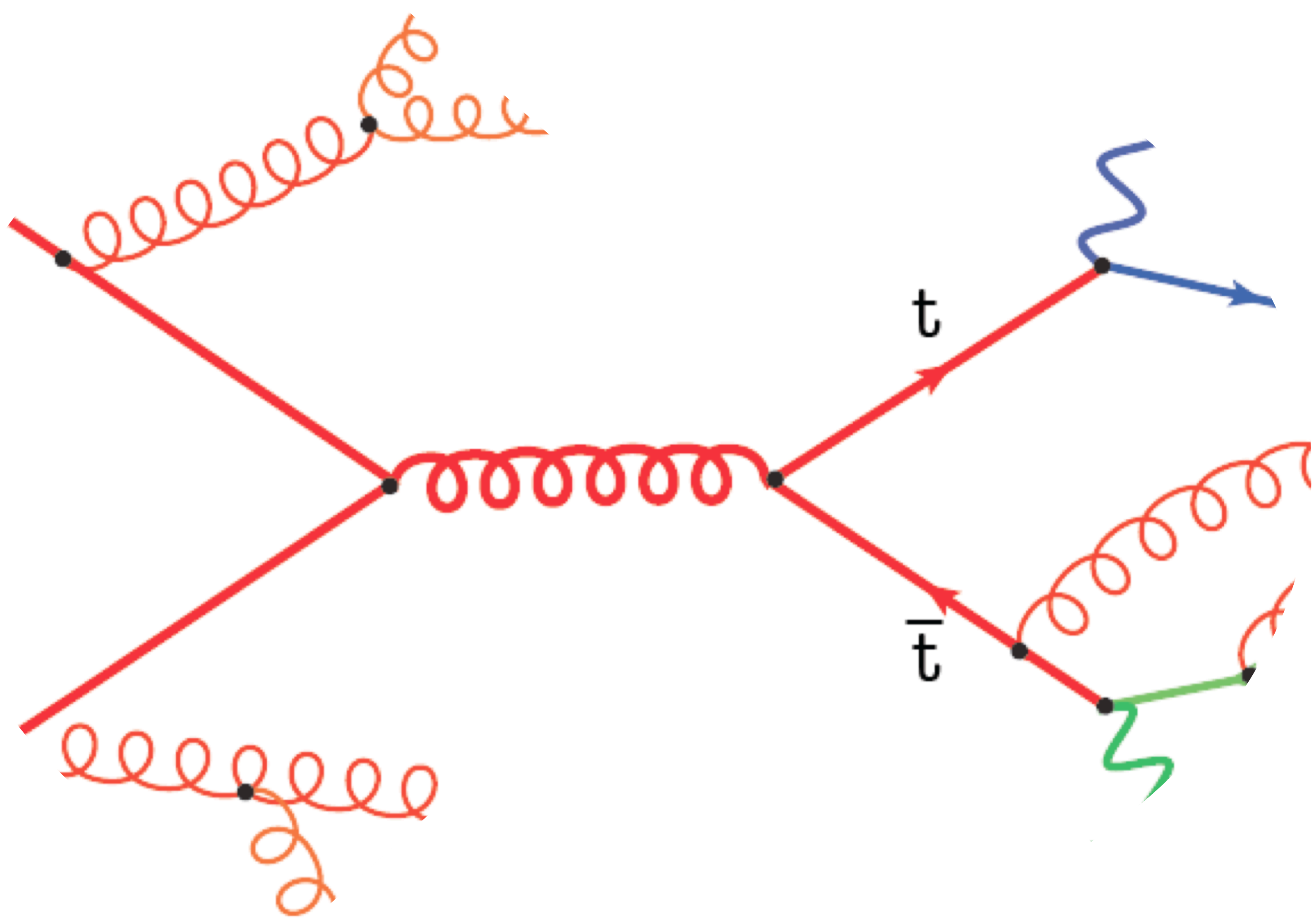
Dimension of Hilbert space is

$$32^{10^{14}} \sim \infty$$

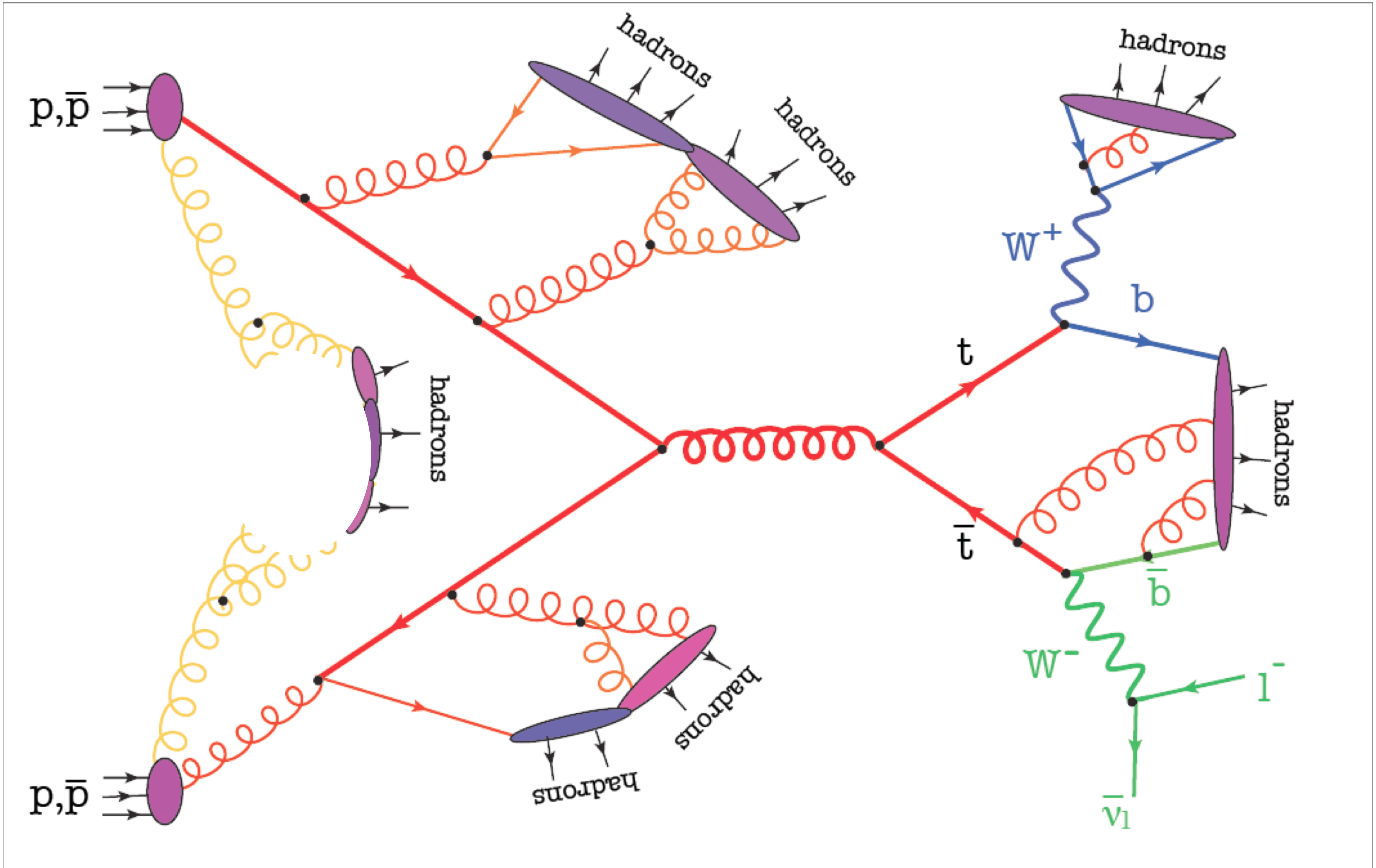
Clearly completely impossible to perform such a calculation

Typical event at LHC involves very different energy scales:
High energy / short distance: Perturbation Theory

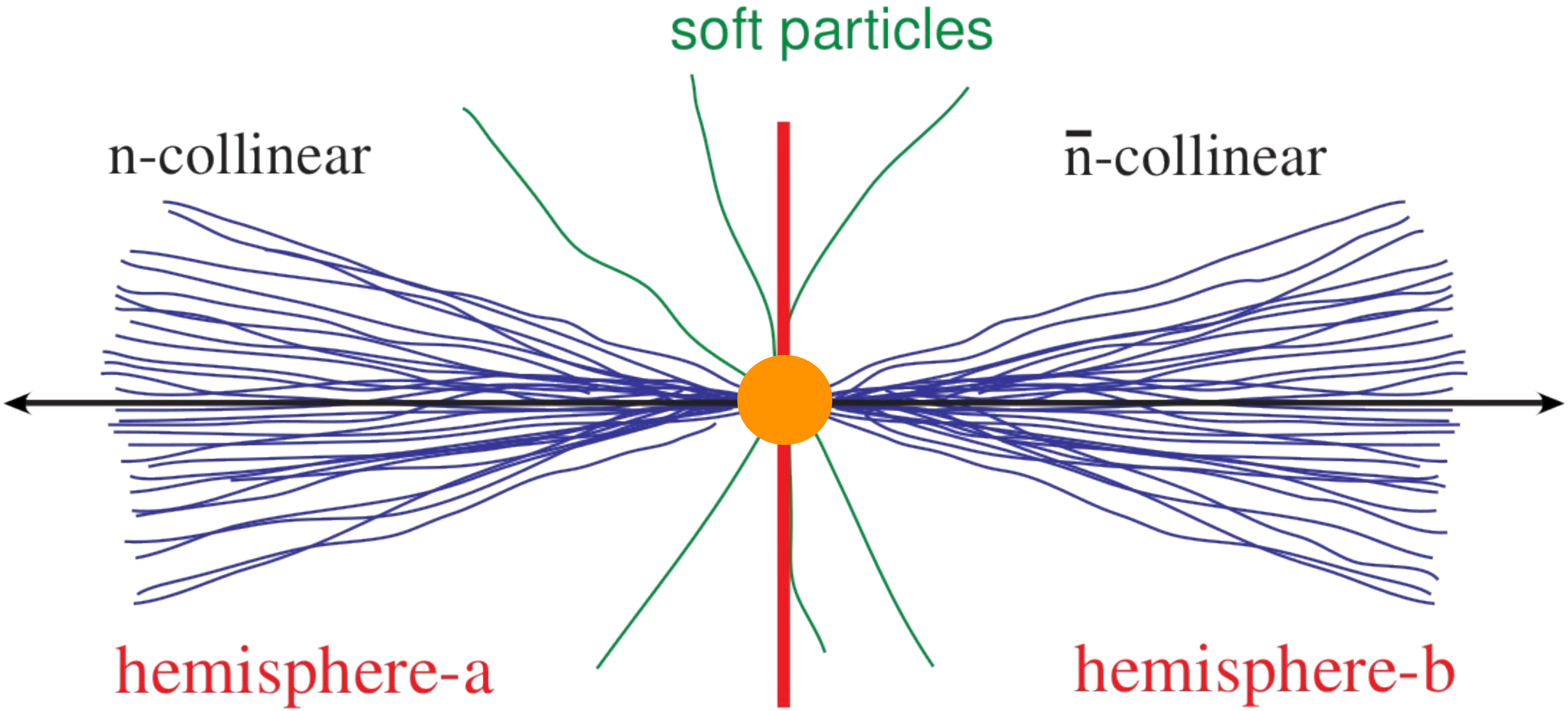
Typical event at LHC involves very different energy scales: Medium energy / medium distance: Parton shower



Typical event at LHC involves very different energy scales: Low energy / long distance: soft radiation / hadronization



Can separate physics into three main categories: Hard, Collinear, Soft



Hard: Q
Collinear: m_J
Soft: m_J^2/Q

$$m_J^2/Q \ll m_J \ll Q$$

Soft-Collinear Effective Theory
SCET

CWB, Fleming, Luke ('00)
CWB, Fleming, Pirjol, Stewart ('00)

Formal understanding of QCD	Proofs of factorization	Jet substructure	Event generation
Fixed order calculations	Jet quenching in heavy Ion collisions	Flavor physics	Parton distribution functions
Resummed calculations	Non-global logarithms	Quarkonia physics	Parton showers

Soft-Collinear Effective Theory
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Effective theories allow to separate short and long distance physics from one another

Goal is to separate ingredients that are calculable in perturbation theory from those that really benefit from non-perturbative techniques

Effective Field Theories (SCET)

$$d\sigma = H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

Most interesting object in above equation is the soft function S, which as discussed lives at the lowest energies

For 1TeV jets with 100GeV mass, find

$$\Lambda_S = (100\,\mathrm{GeV})^2/(1000\,\mathrm{GeV}) = 10\,\mathrm{GeV}$$

Let's try to estimate the resources we need to simulate physics at the LHC

Energy rage that can be described by lattice is given by

$$\frac{1}{Nl} \lesssim E \lesssim \frac{1}{l}$$

As I will argue later, can use effective field theories to limit required range to

$$100\,\mathrm{MeV} \lesssim E \lesssim 10\,\mathrm{GeV}$$

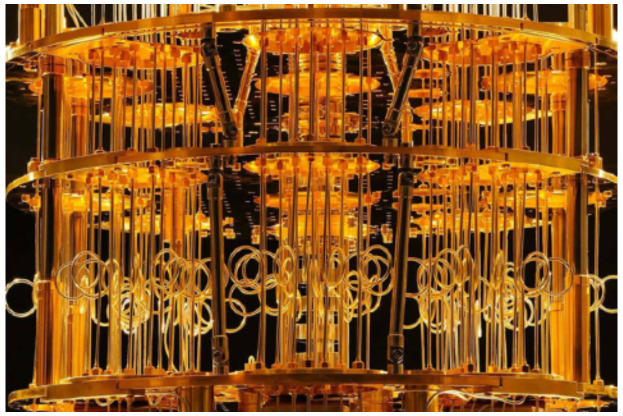
This needs $\mathcal{O}(100^3) \sim 10^6$ lattice sites

Dimension of Hilbert space is

$$32^{10^6} \sim \infty$$

While $32^{10^6} \ll 32^{10^{14}}$,

still completely impossible to perform such a calculation



Soft functions on a quantum computer

Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1]* Keith S. M. Lee,[2] John Preskill[3]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.

Science 336 (2012) 1130

The resources on a quantum computer are much smaller, but still very large

From the discussion before, size of Hilbert space to simulate full LHC given by

$$\dim(H) \sim 32^{10^{14}}$$

This Hilbert space can be encoded in

$$n_Q = \ln_2\left[\dim(H)\right] \sim 5 \times 10^{14}$$

While this is much, much smaller, still inconceivable to have a system of this size in any of our lifetimes

Crucial thing to realize is that we don't need quantum computer for most of this physics

First, for most observables not interested in the most general high energy process (typically care about events with relatively small number of jets)

Second, perturbation theory works very well for high energy processes with limited number of final state particles

Should use Quantum Computers only for those calculations that are not possible using known techniques

Combine quantum computing with EFTs

Soft function is the expectation value of a "Wilson line" operator between initial and final state

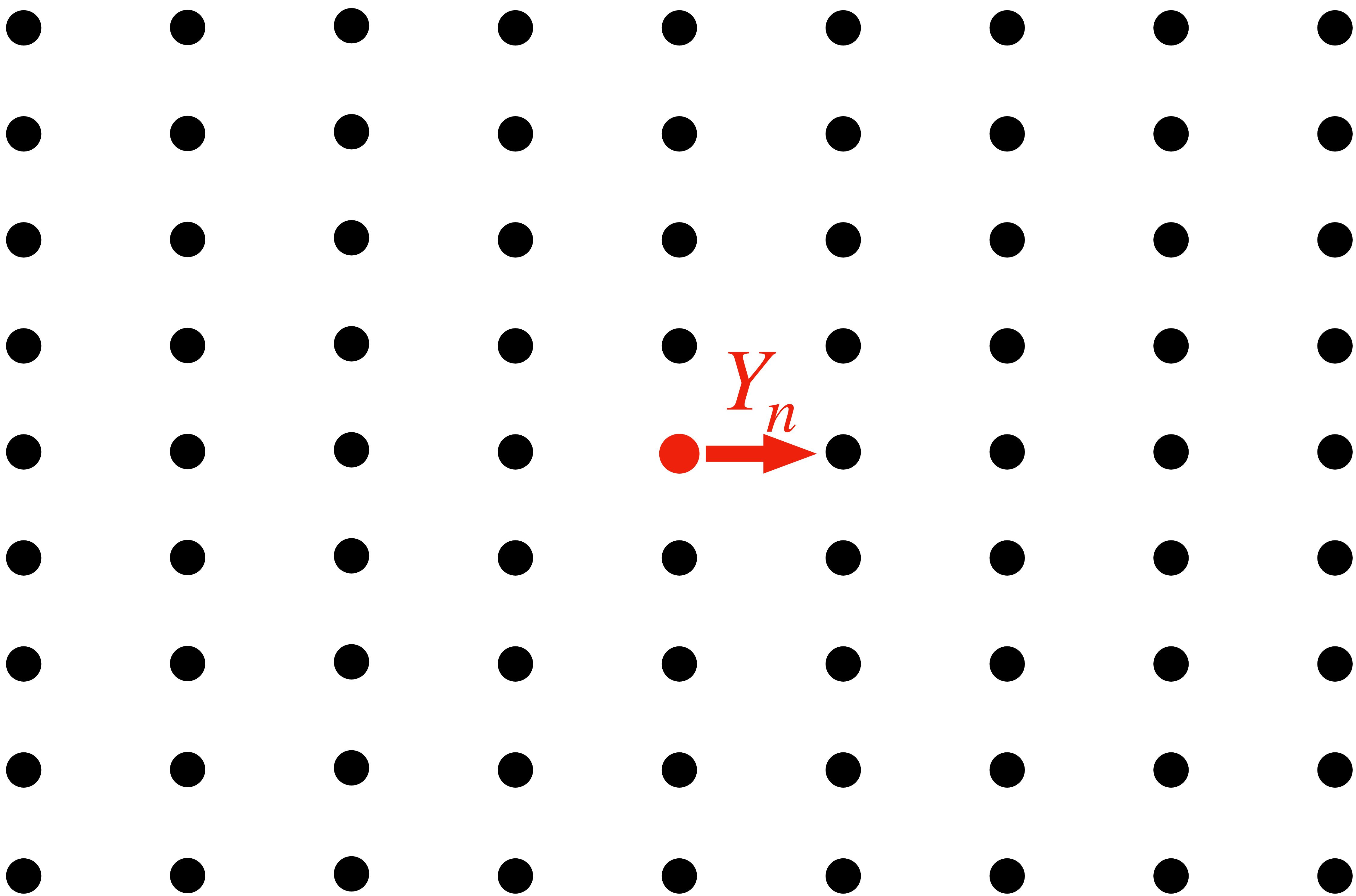
Soft function can be written as

$$S = \Big| \langle X | \, T[Y_n Y_{\bar{n}}^\dagger] \, | \Omega \rangle \Big|^2$$

$$Y = \mathrm{P}\exp\left[ig \int_0^\infty \mathrm{d}s \, \phi(ns) \right] \qquad ns = (s, 0, 0, s)$$

How does this look like on a lattice?

A Wilson line is a relatively simple object on a lattice



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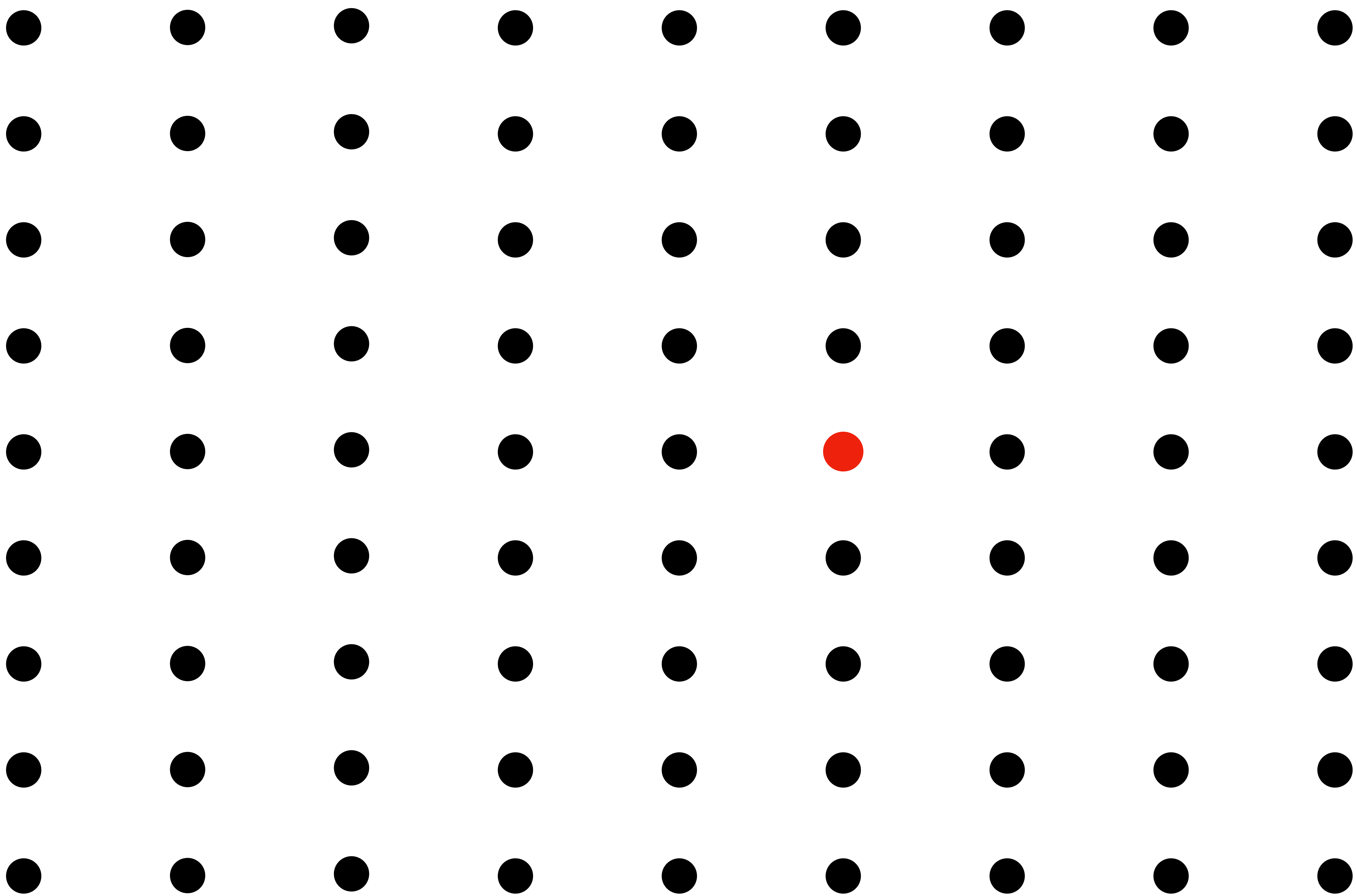

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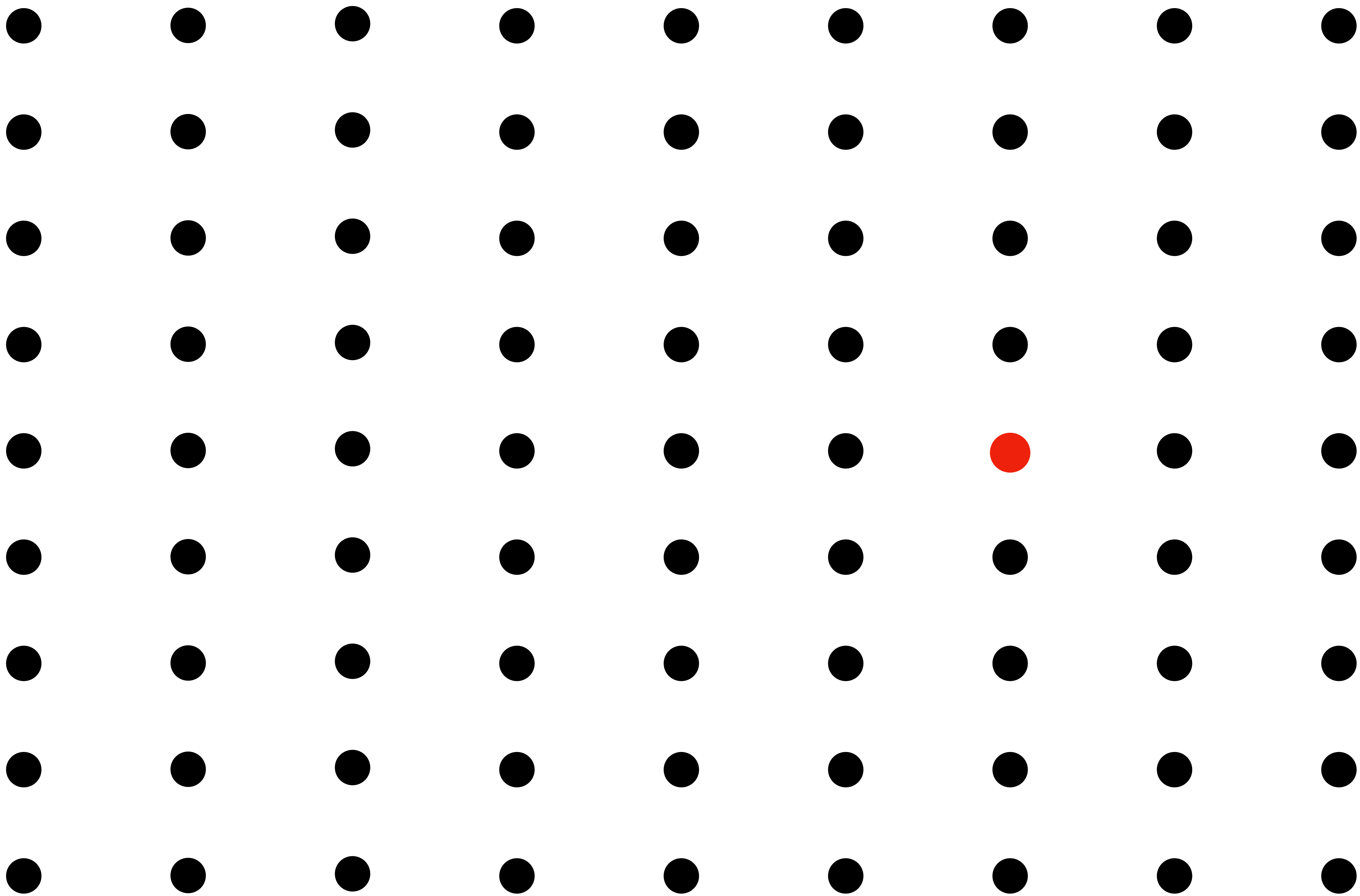

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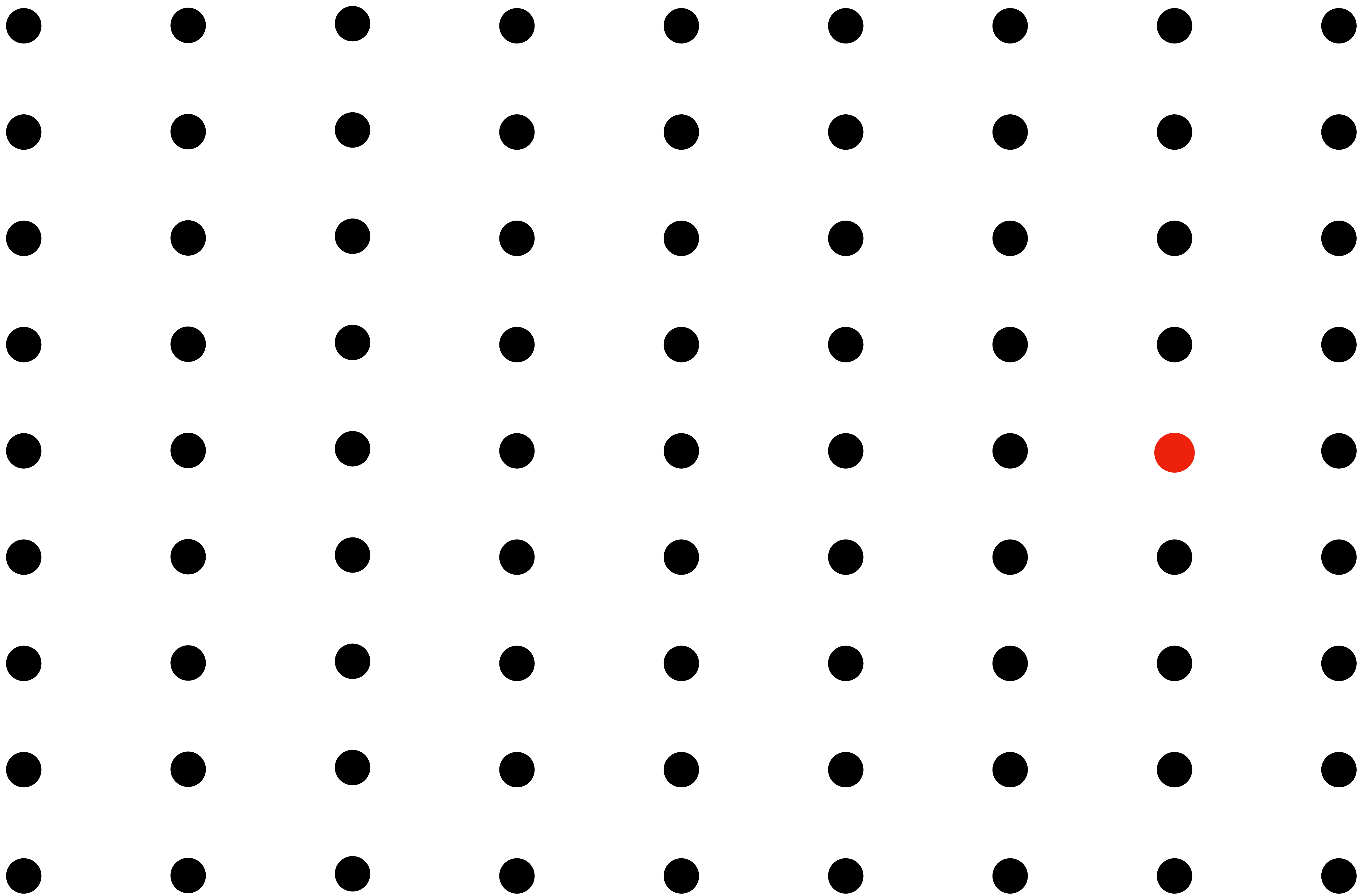

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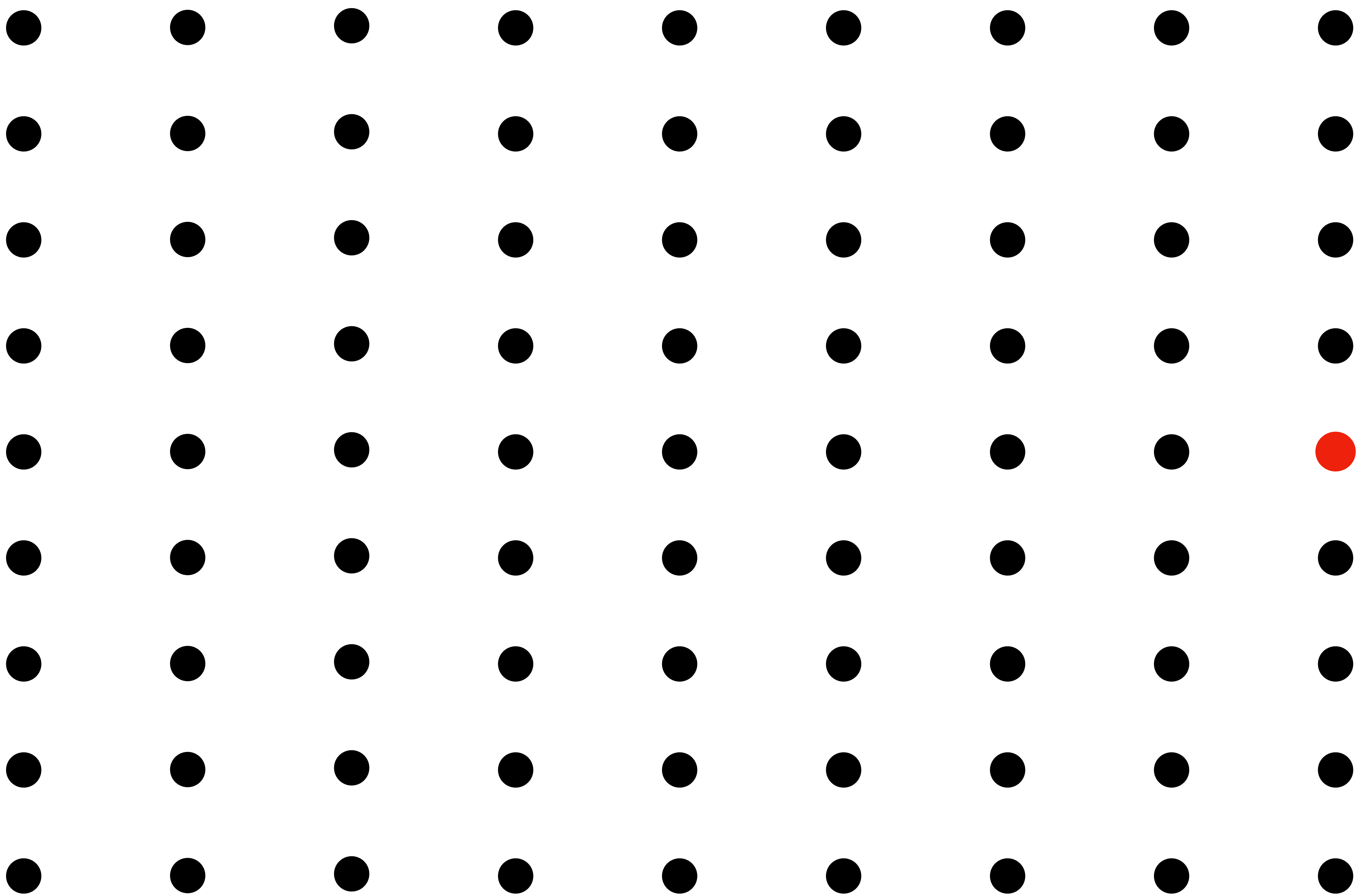

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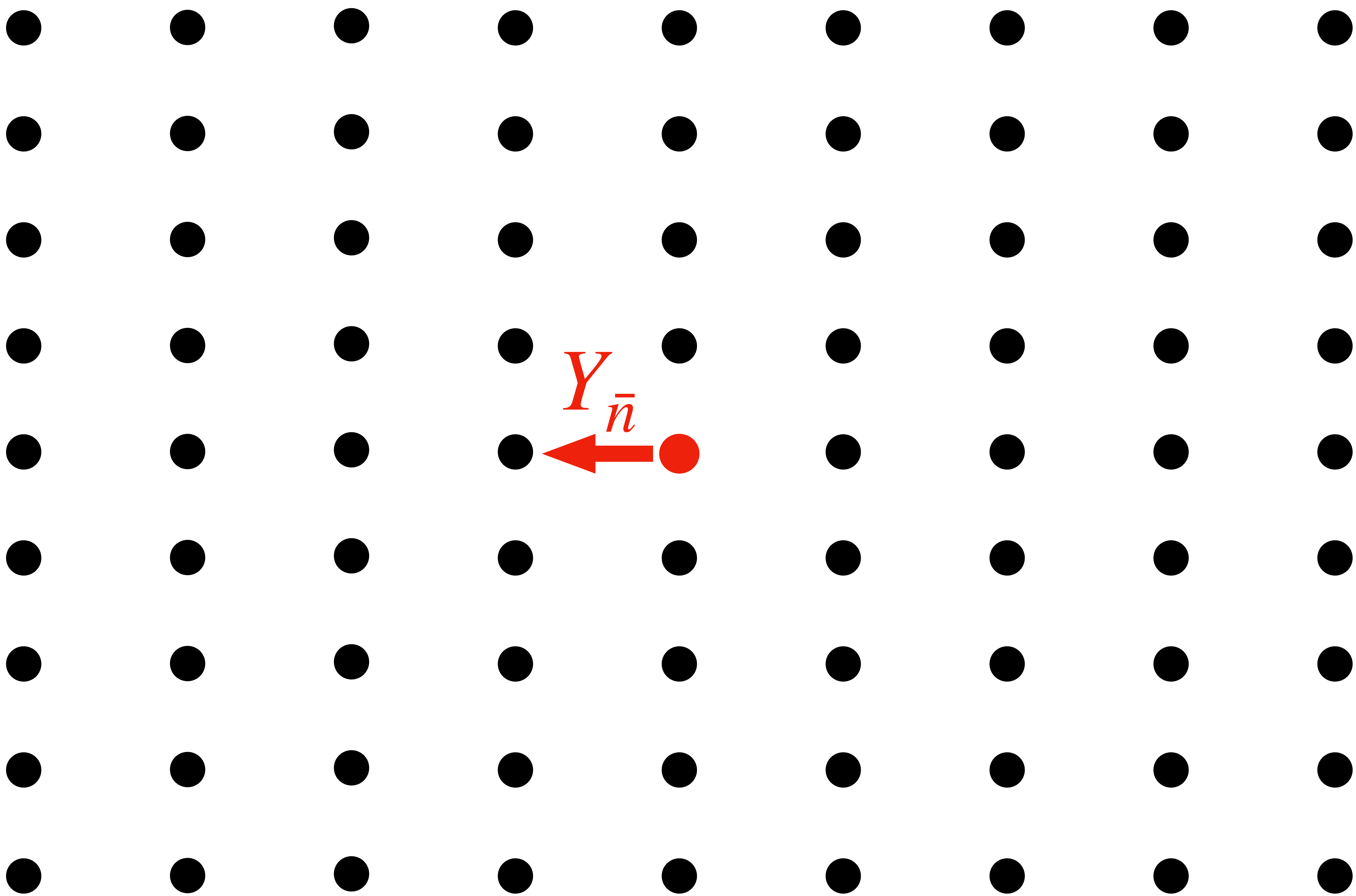

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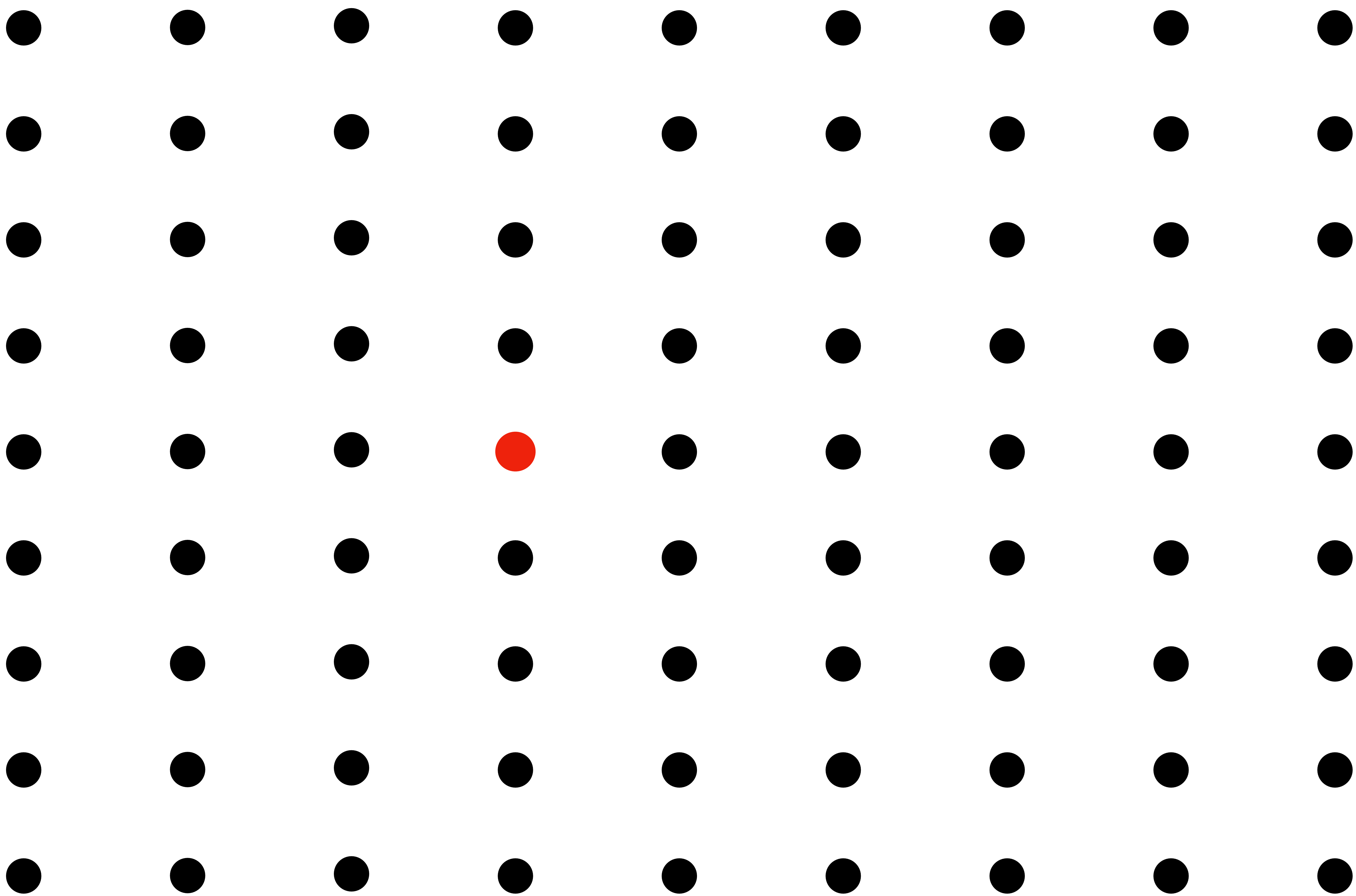

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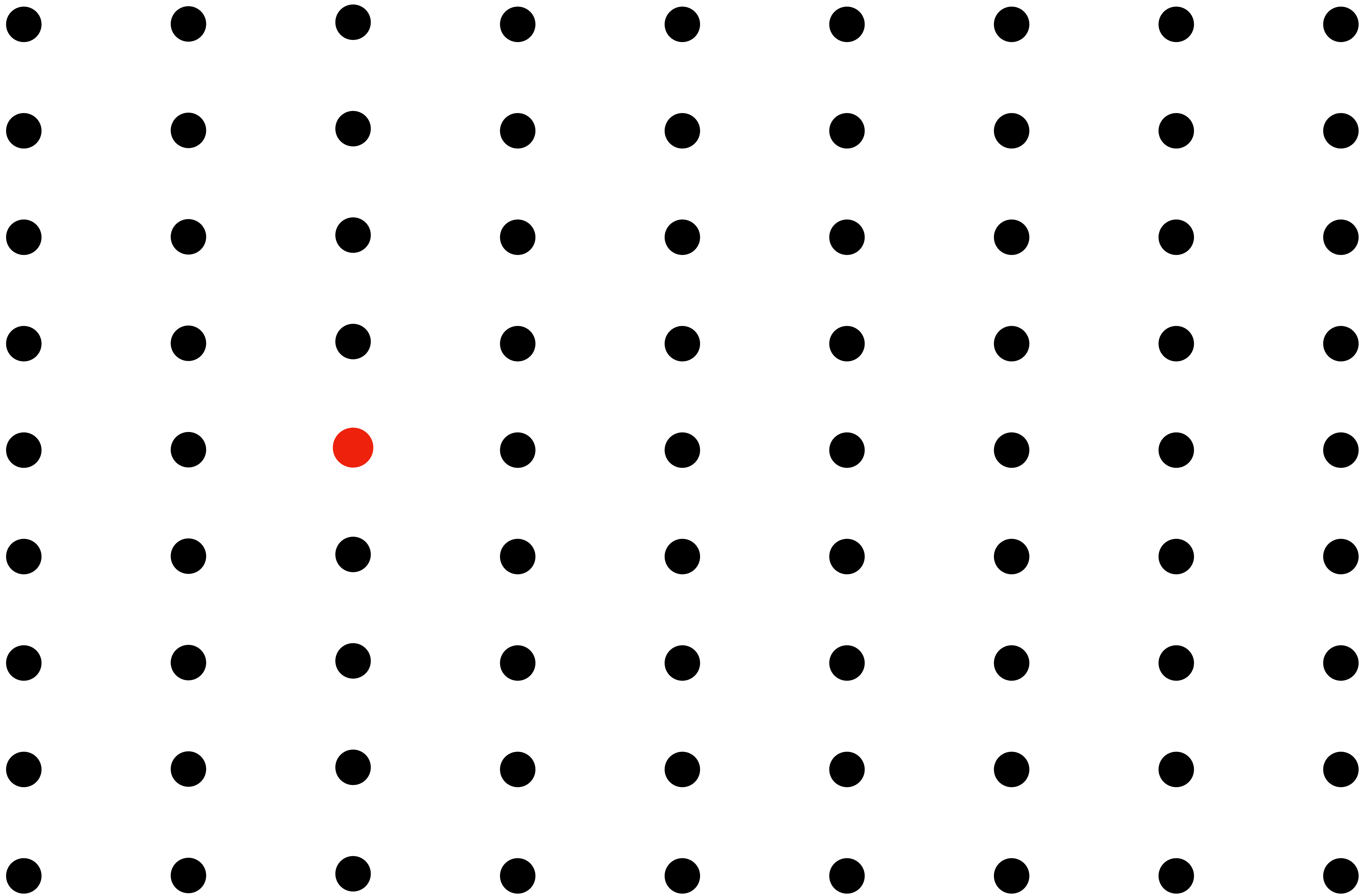

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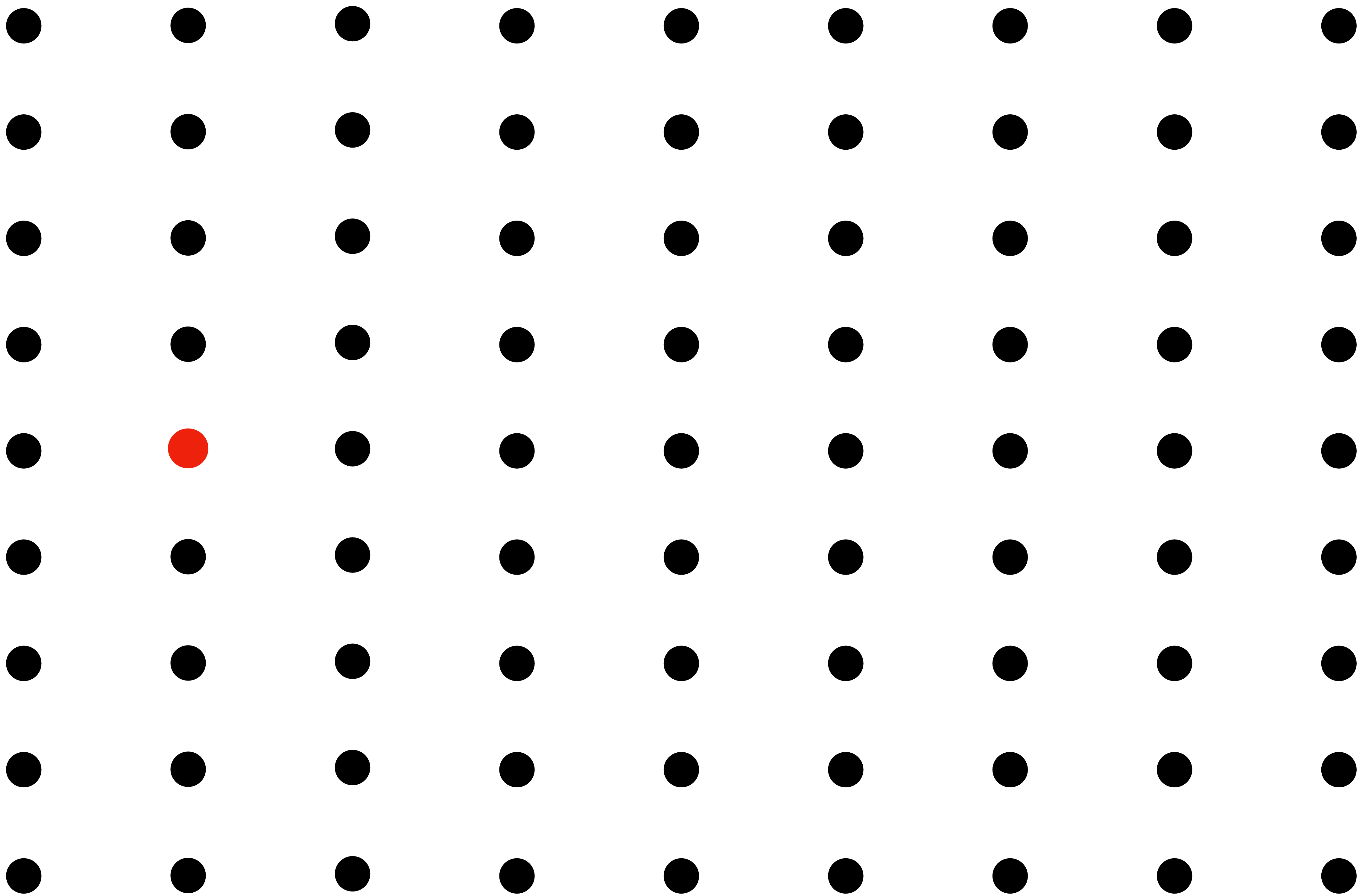

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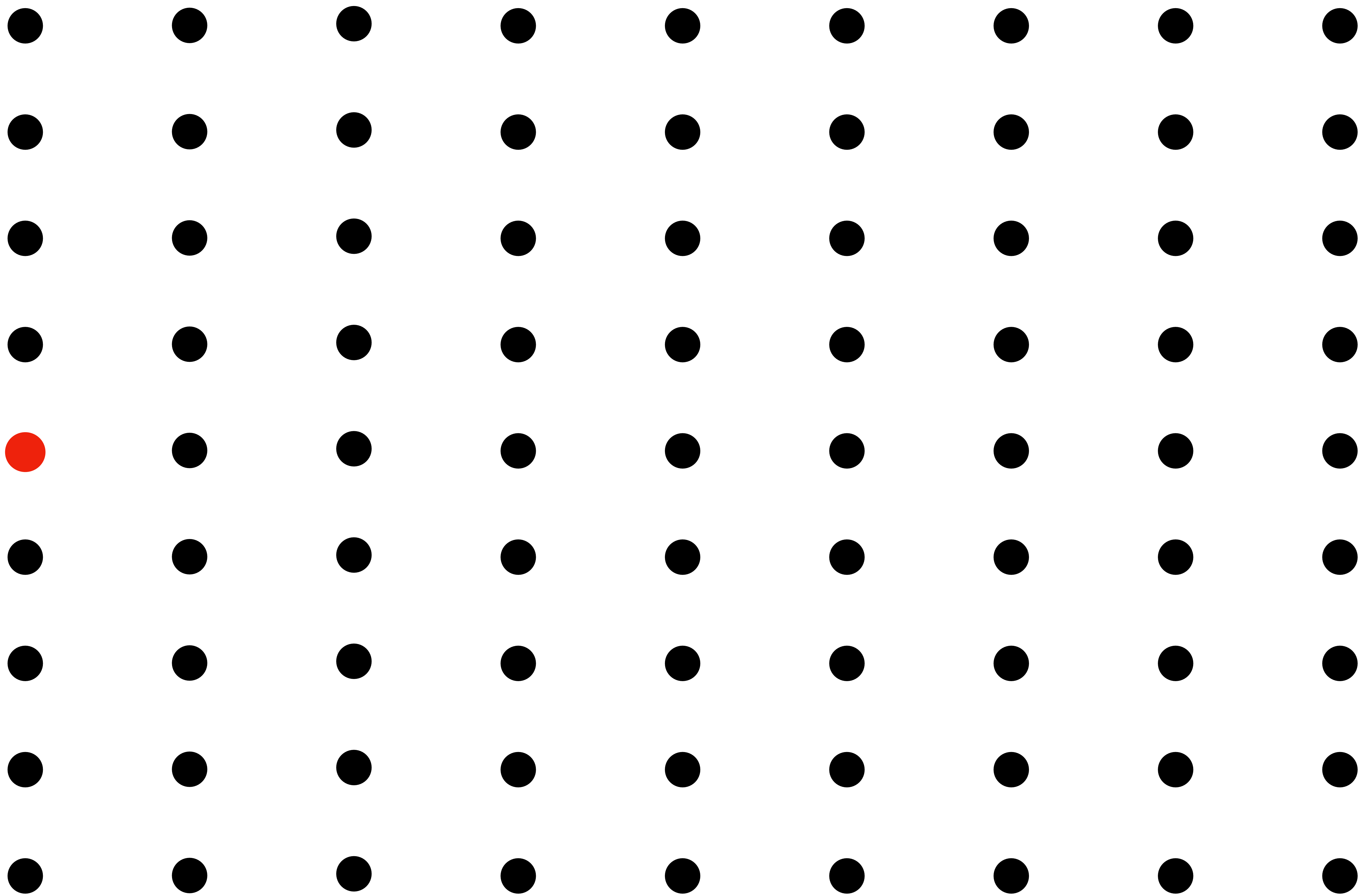

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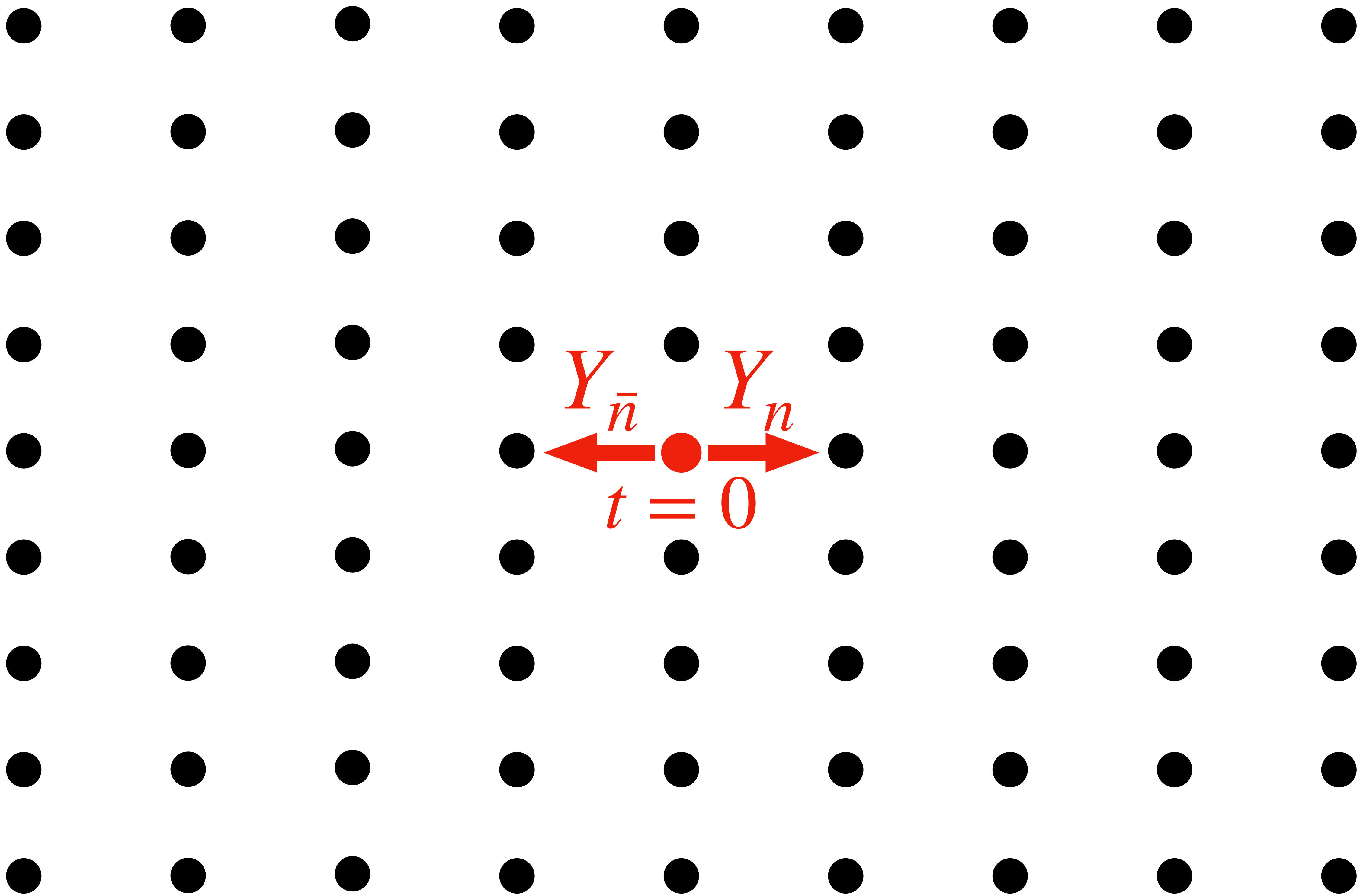

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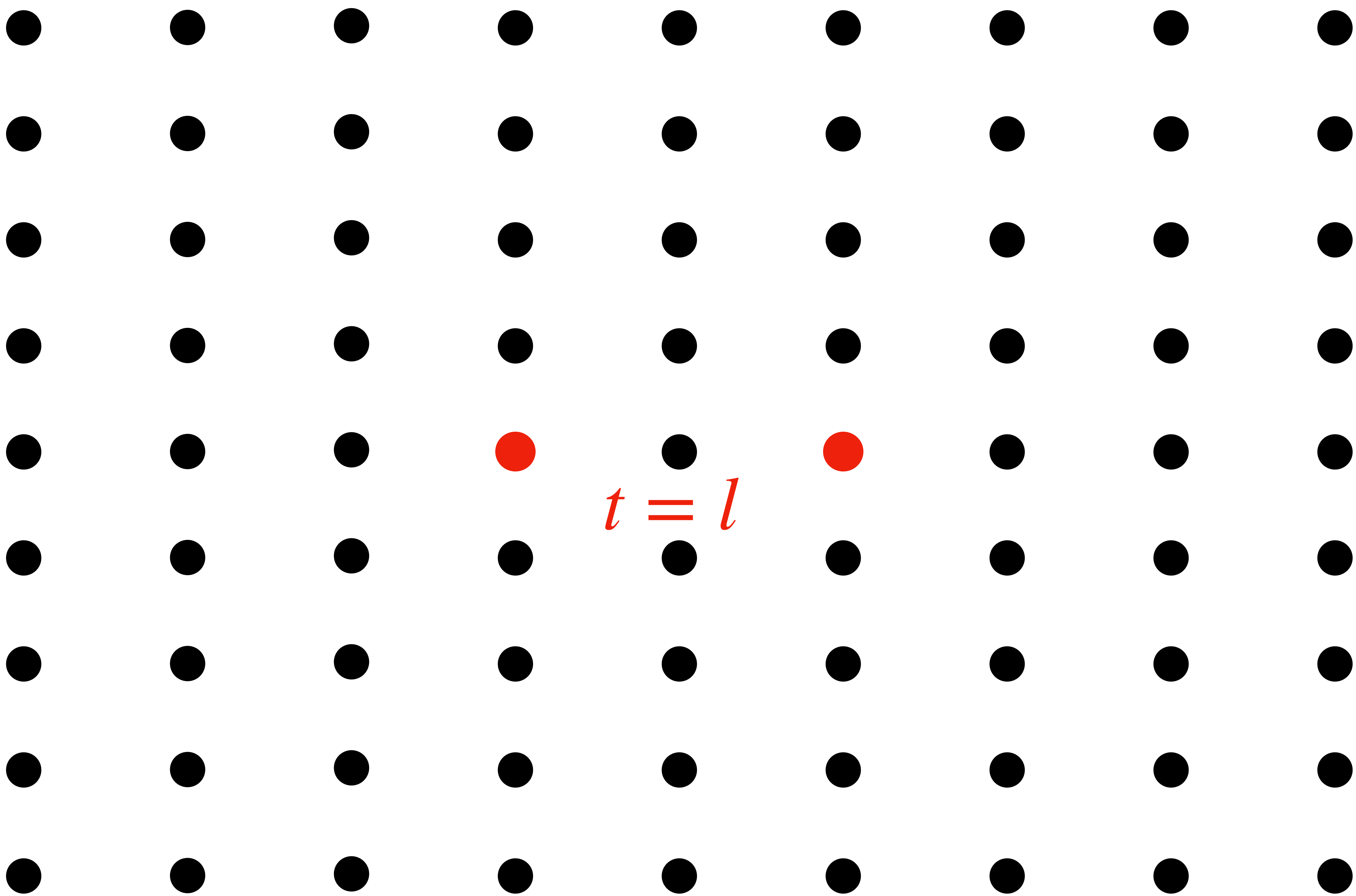

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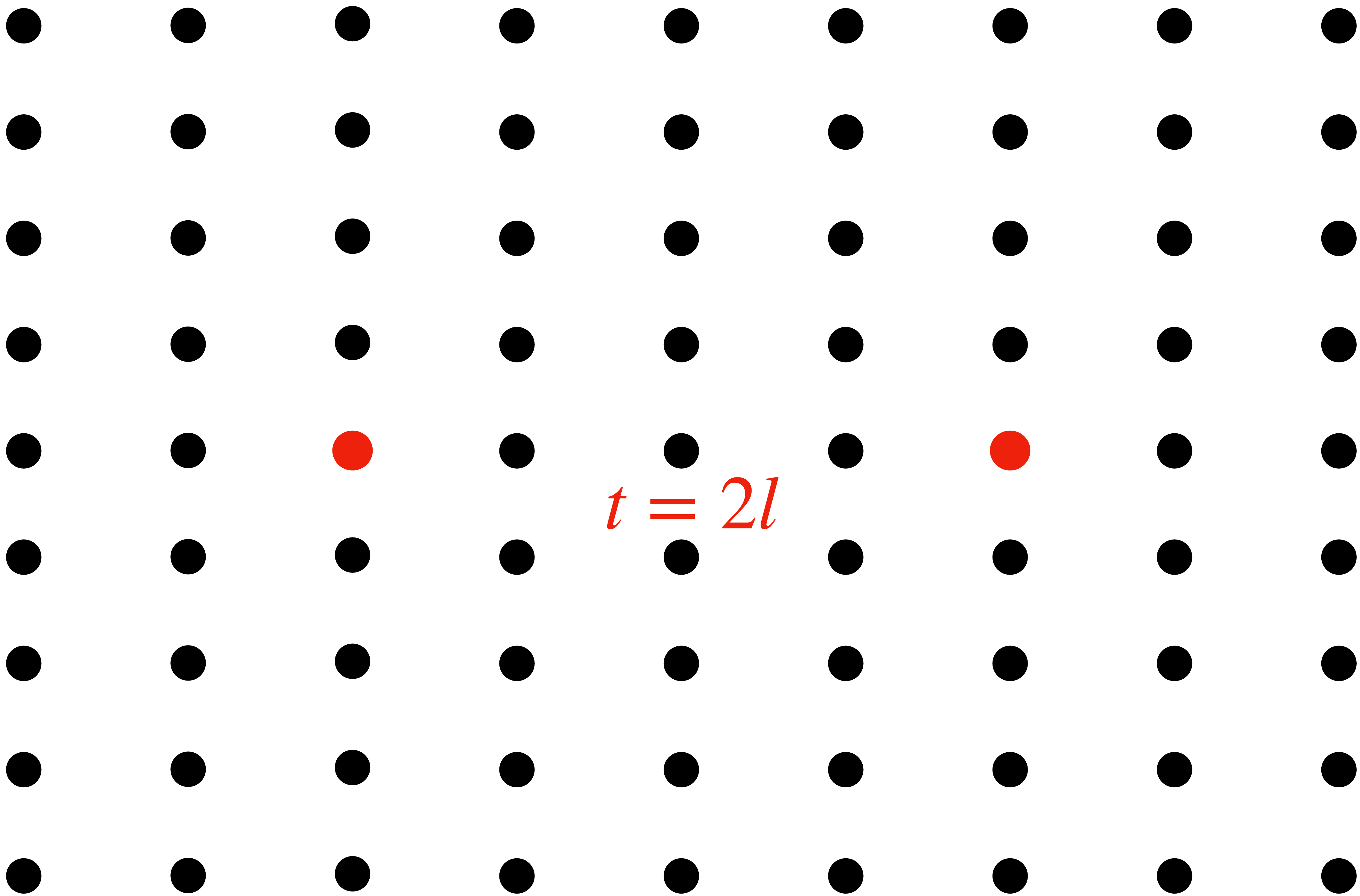

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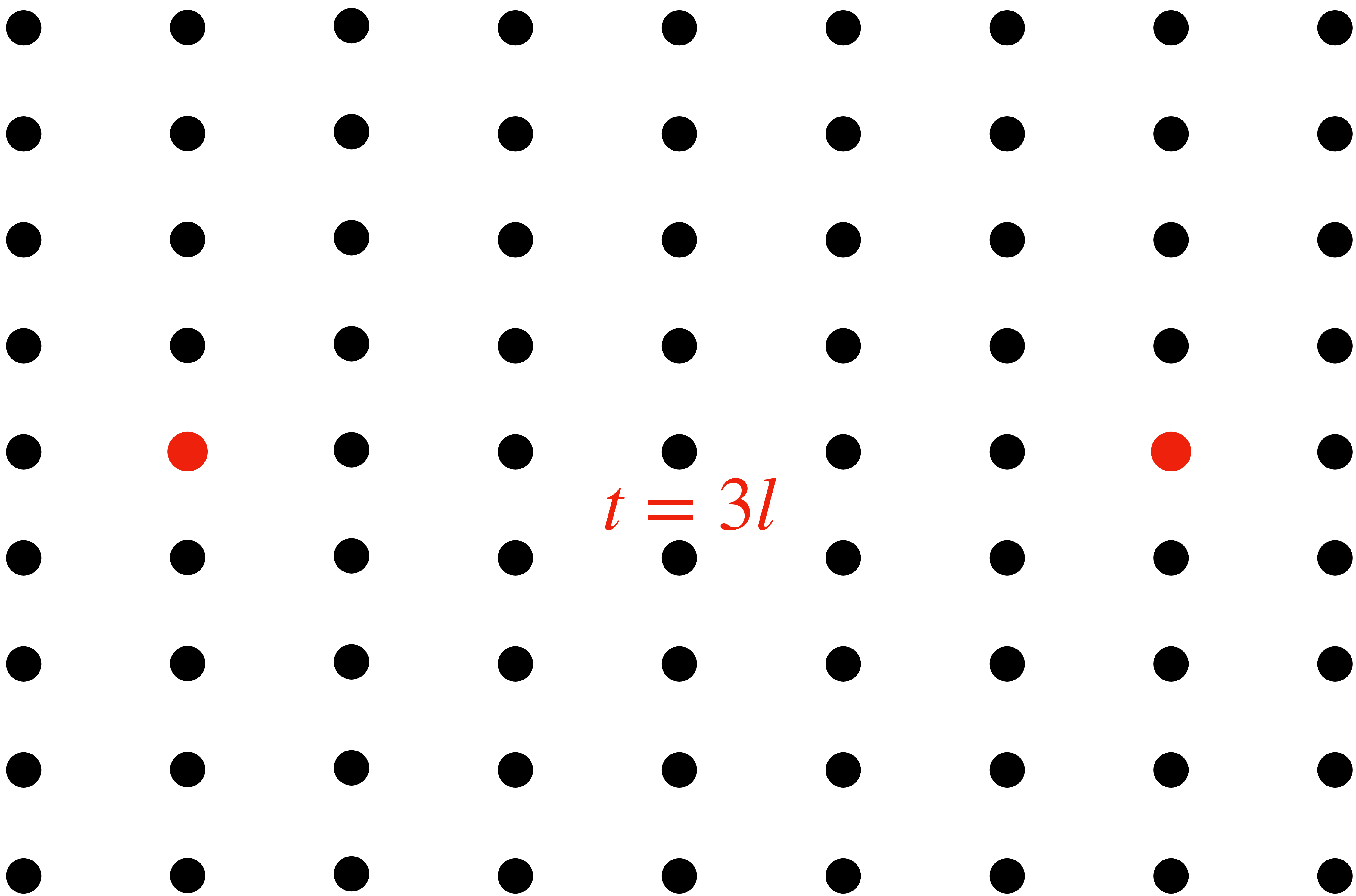

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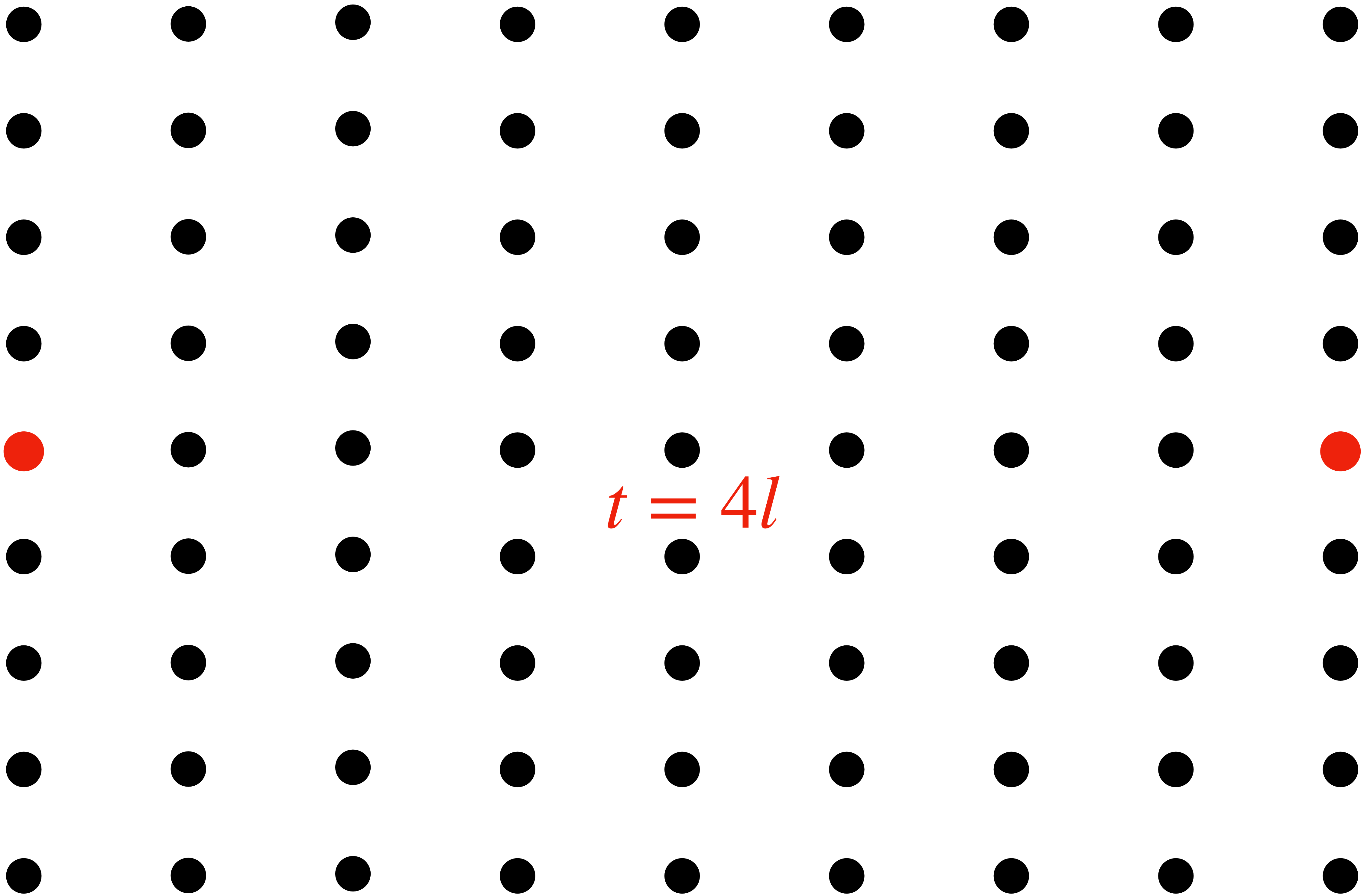


A Wilson line is a relatively simple object on a lattice

Wilson line can be easily discretized on the lattice

$$Y_n = \mathrm{P}\exp\left[ig\,\delta x \sum_{i=n_0}^{2n_0} \phi_{x_i}(t = x_i - n_0)\right]$$

$$Y_{\bar{n}}^\dagger = \mathrm{P}\exp\left[-ig\,\delta x \sum_{i=0}^{n_0} \phi_{x_i}(t = n_0 - x_i)\right]$$

Use time evolution to change the time at each lattice point

$$\mathrm{T}[Y_n\, Y_{\bar{n}}^\dagger] = e^{-iH\, n_0 \delta x} \exp\left[ig\,\delta x\left(\phi_{x_{2n_0}} - \phi_{x_0}\right)\right] \times e^{iH\delta x} \exp\left[ig\,\delta x\left(\phi_{x_{2n_0-1}} - \phi_{x_1}\right)\right]$$
$$\times \cdots \times e^{iH\delta x} \exp\left[ig\,\delta x\left(\phi_{x_{n_0}} - \phi_{x_{n_0}}\right)\right].$$

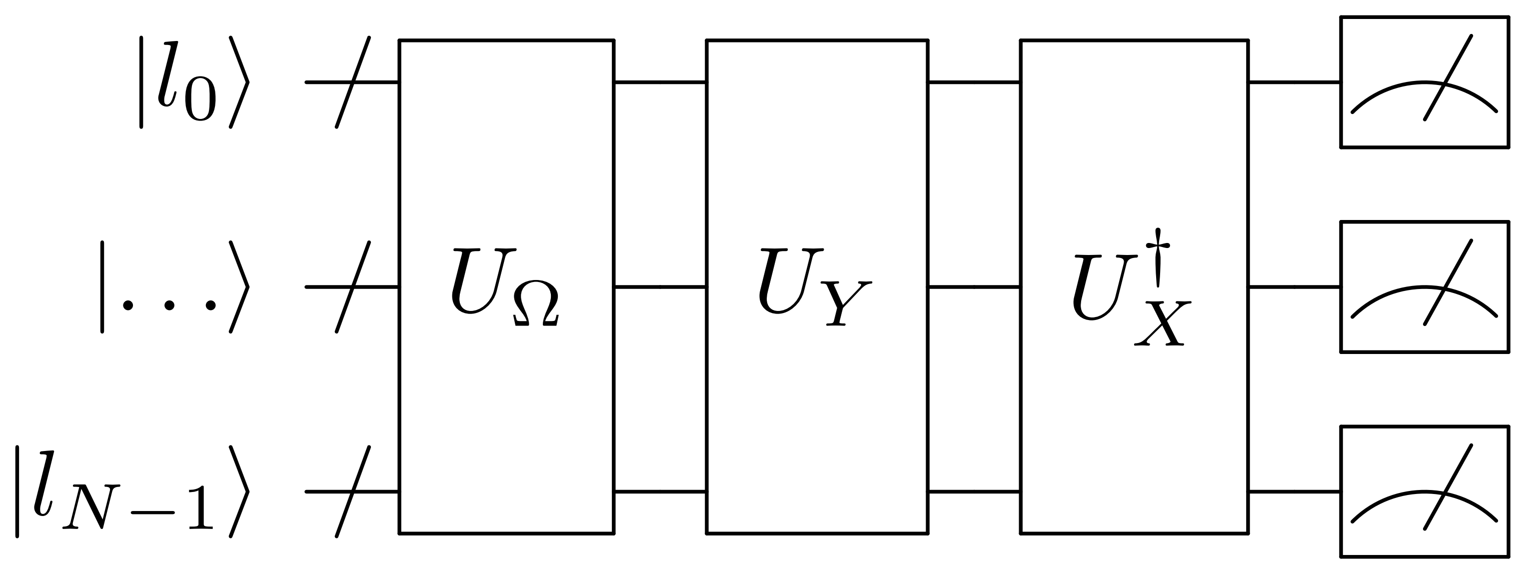
Alternate between exponential of field operator and Hamiltonian evolution

Soft function is the expectation value of a “Wilson line” operator between initial and final state

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

Have worked out quantum circuit to create vacuum state $|\Omega\rangle$, circuit for $T[Y_n Y_{\bar{n}}^\dagger]$ and circuit to measure final state $|X\rangle$

[CWB, Freytsis, Nachman, PRL 127 (2021), 212001]



Constructing the relevant circuit is relatively straightforward

Hamiltonian Evolution

Jordan, Lee, Preskill ('12)
Somma ('16)
Macridin et al ('18)
Savage, Klco ('19)

Crucial simplification: this problem only requires Hamiltonian of free field theory

$$H = H_\phi + H_\pi \qquad H_\phi = \hat{\phi}^2/2\,, \quad H_\pi = \hat{\pi}^2/2$$

Can move between ϕ and π basis via QFT

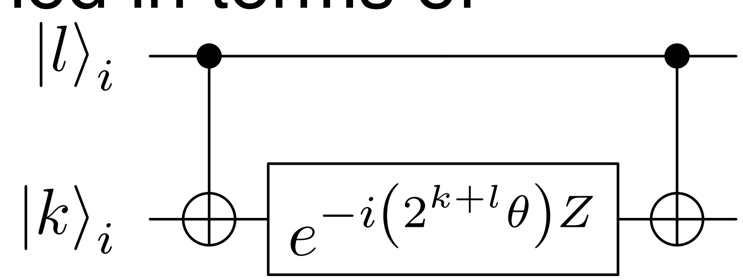
$$e^{iH_\pi t} = \mathrm{QFT}^{-1}\, e^{i\delta x\, t\phi_i^2}\, \mathrm{QFT}$$

and express ϕ operator through Z operators

$$\hat{\phi}_i = \sum_{j=0}^{n_Q-1} 2^j \hat{\sigma}_{z,i}^{(j)}$$

Entire Hamiltonian therefore determined in terms of

$$\exp\left[i\theta\hat{\phi}_i\hat{\phi}_j\right] = \prod_{l=0}^{n_Q-1}\prod_{k=0}^{n_Q-1}\exp\left[i\,2^{(l+k)}\theta\,\sigma_{z,i}^{(l)}\sigma_{z,j}^{(k)}\right] \quad =$$



Constructing the relevant circuit is relatively straightforward

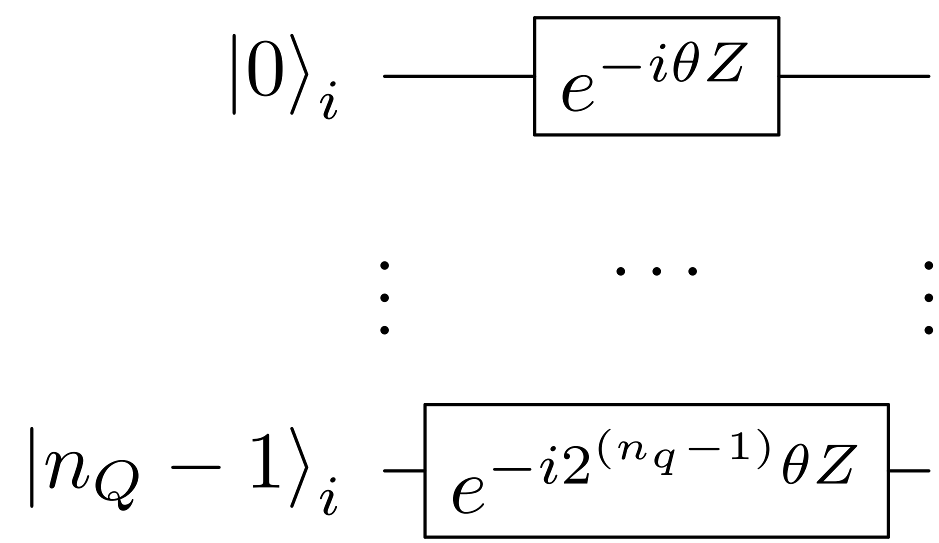
Exponential of field operator

CWB, Freytsis, Nachman, PRL 127 (2021), 212001

Much simpler to implement, using similar technique as for Hamiltonian

$$\exp[i\theta\hat{\phi}_i] = \prod_{j=0}^{n_Q-1} \exp\left[i2^j\theta\sigma_{z,i}^{(j)}\right]$$

=



Put together, allows to implement the whole Wilson line operator

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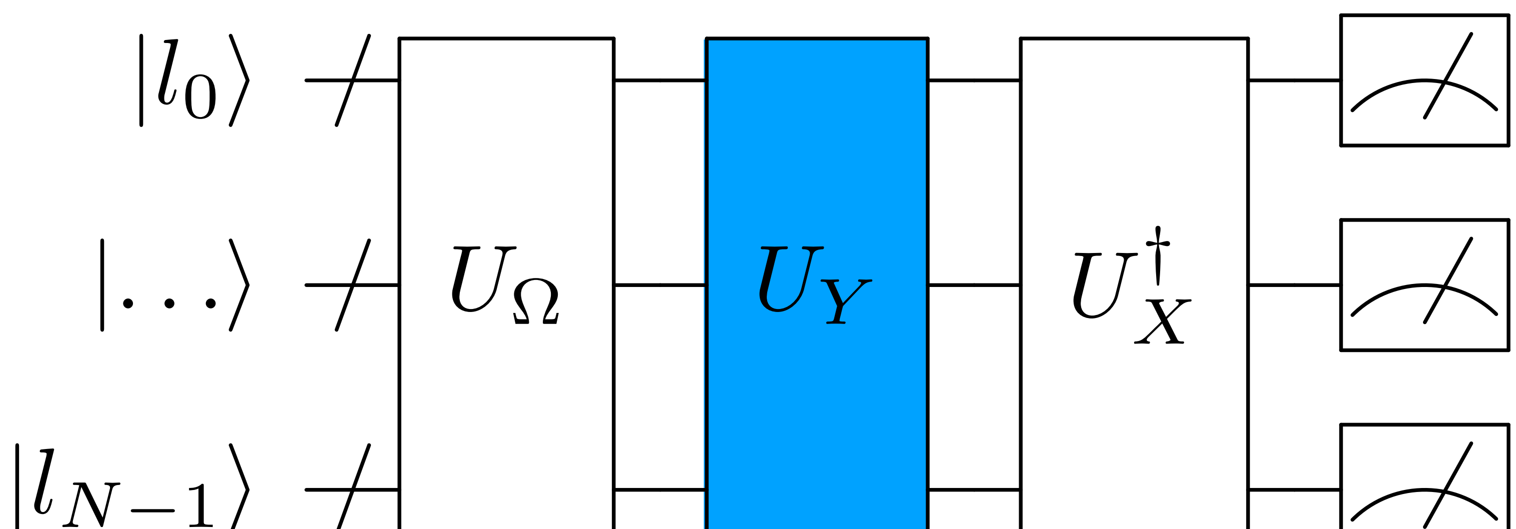


Constructing the relevant circuit is relatively straightforward

Ground state preparation

Kitaev, Webb ('08)

CWB, Deliyannis, Freytsis, Nachman (2109.10918)

Ground state of scalar field theory given by multivariate Gaussian

$$|\Psi\rangle = \exp\left[-\frac{1}{2}\hat{\phi}_i G_{ij} \hat{\phi}_j\right] |k_0\rangle \cdots |k\rangle_n$$

The covariance matrix G_{ij} can be diagonalized

$G = MDM^T$, where D is diagonal and M upper triangle matrix

General process is therefore to proceed in two steps

1. Prepare set of uncorrelated Gaussians with widths determined by D
2. Switch basis by applying M (a shearing operation)

Constructing the relevant circuit is relatively straightforward

Ground state preparation

Kitaev, Webb ('08)

CWB, Deliyannis, Freytsis, Nachman (2109.10918)

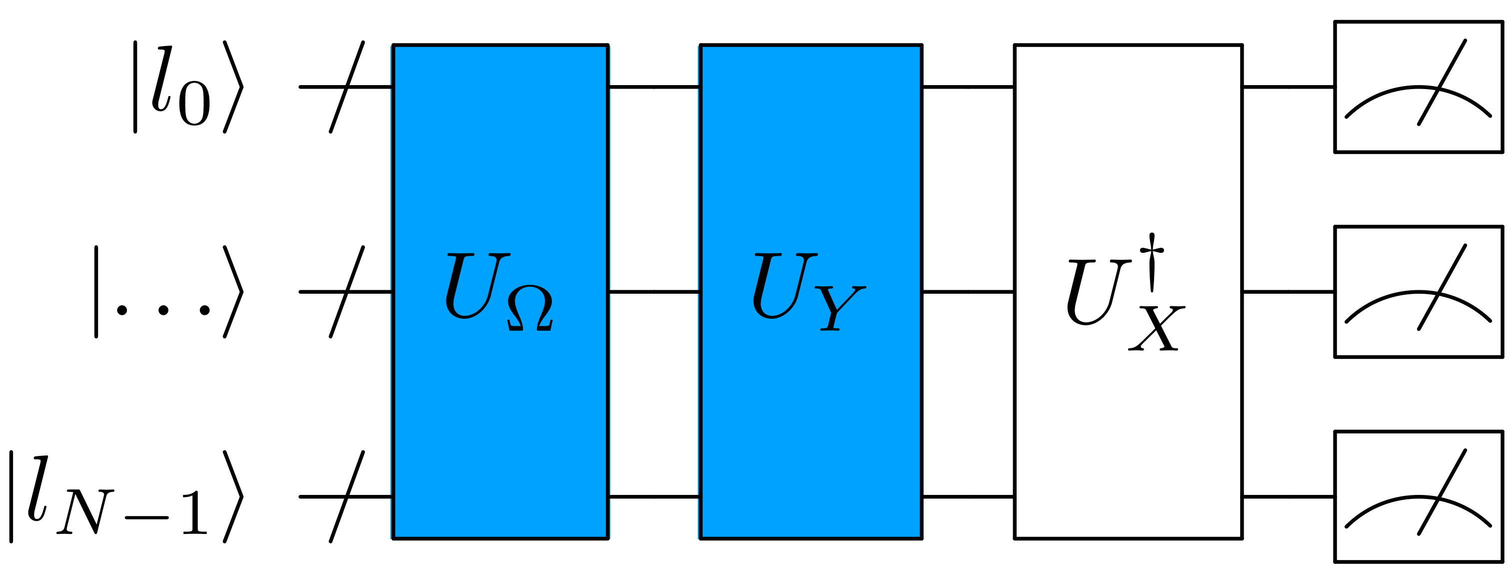
1. Prepare set of uncorrelated Gaussians with widths determined by D
   - Classical complexity scales as $N \exp(n_\phi)$
   - Quantum algorithm exists that has polynomial scaling $Np(n_\phi)$
   - Requires to perform relatively complicated quantum arithmetic
   - Since n_ϕ typically not very large, might be most efficient to simply create classically computed state

2. Switch basis by applying M (a shearing operation)
   - Classical complexity scales as $\exp(Nn_\phi)$
   - Quantum algorithm exists that has polynomial scaling $p(Nn_\phi)$
   - Since N typically large, imperative to use much more efficient quantum algorithm

Soft function is the expectation value of a “Wilson line” operator between initial and final state

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

Have worked out quantum circuit to create vacuum state $|\Omega\rangle$, circuit for $T[Y_n Y_{\bar{n}}^\dagger]$ and circuit to measure final state $|X\rangle$



Constructing the relevant circuit is relatively straightforward

Excited state preparation

Jordan, Lee, Preskill ('12)

1. Given the ground state of the theory, can obtain excited state by acting with creation operator.
2. Not a unitary operation, but can be implemented using ancillary quit
3. Complexity scales as $p(Nn_\phi)$

Soft function is the expectation value of a "Wilson line" operator between initial and final state

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

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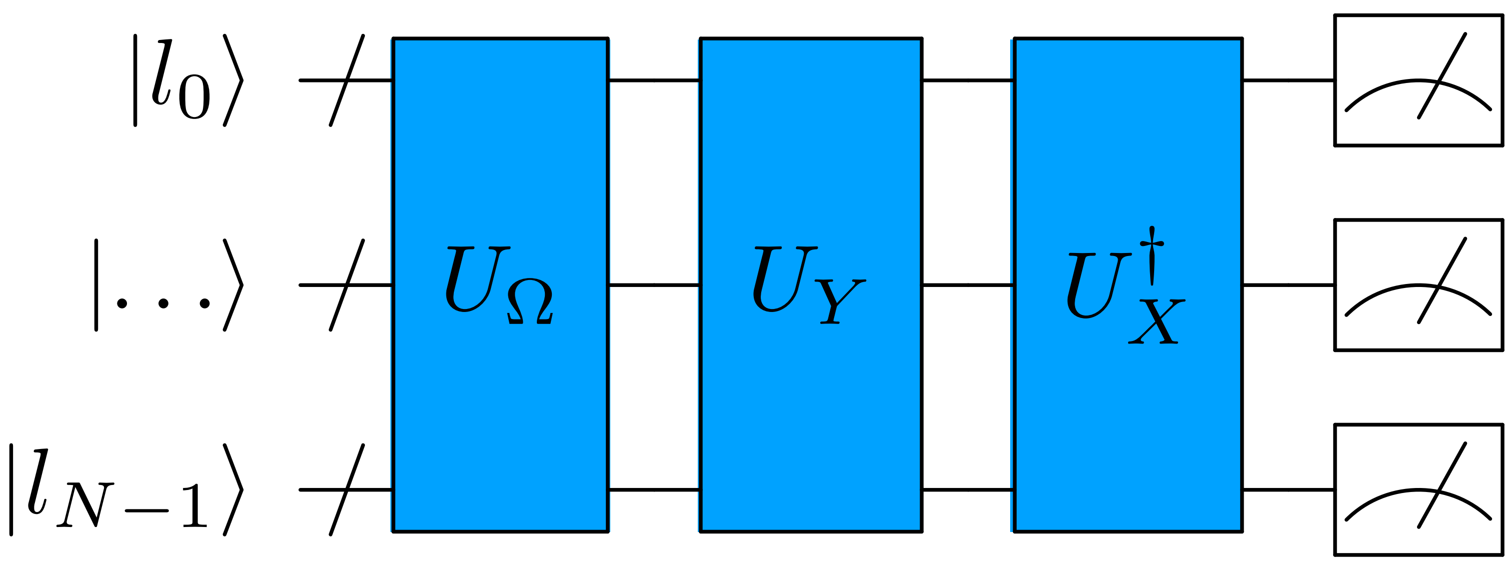
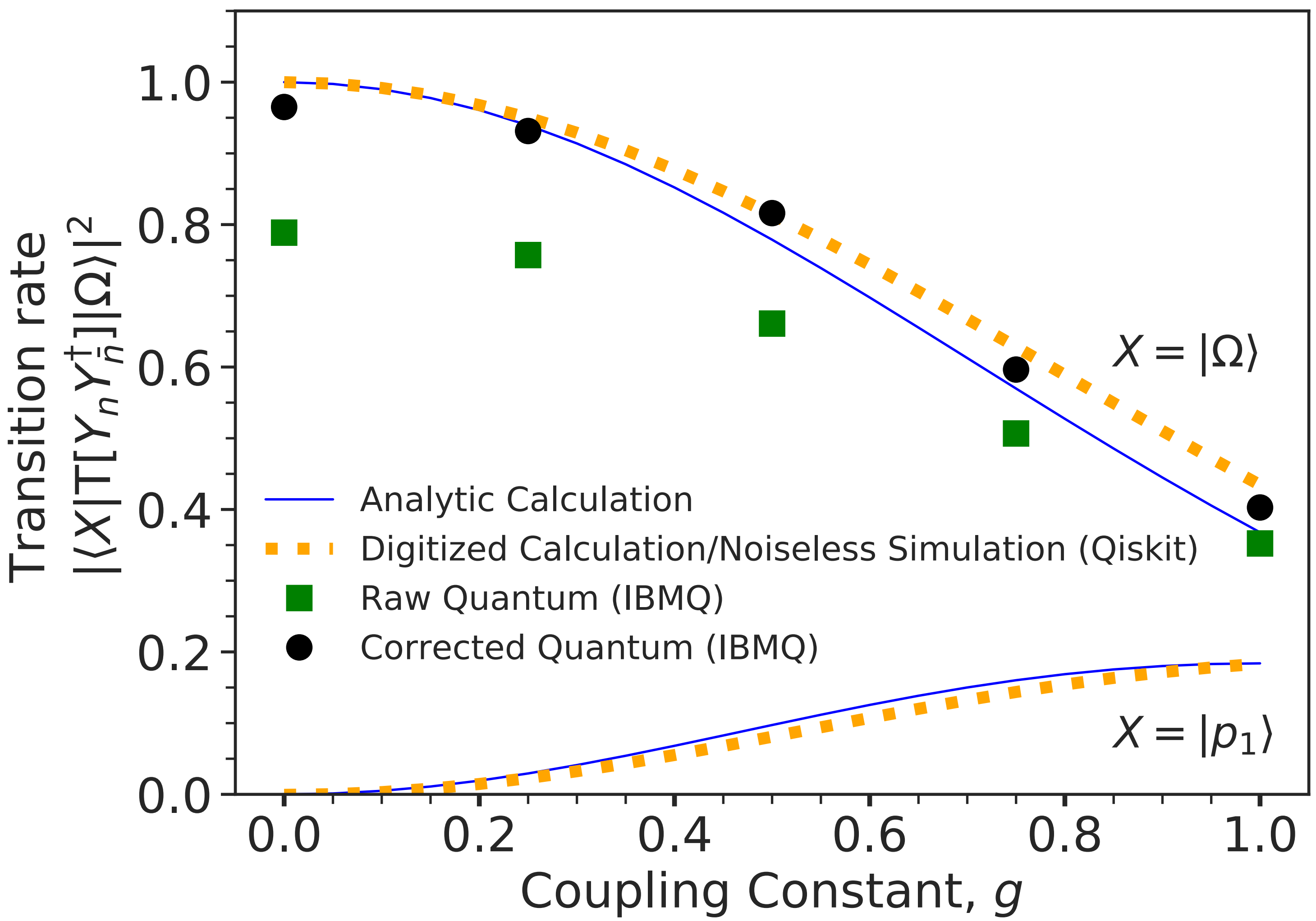


Soft function is the expectation value of a "Wilson line" operator between initial and final state

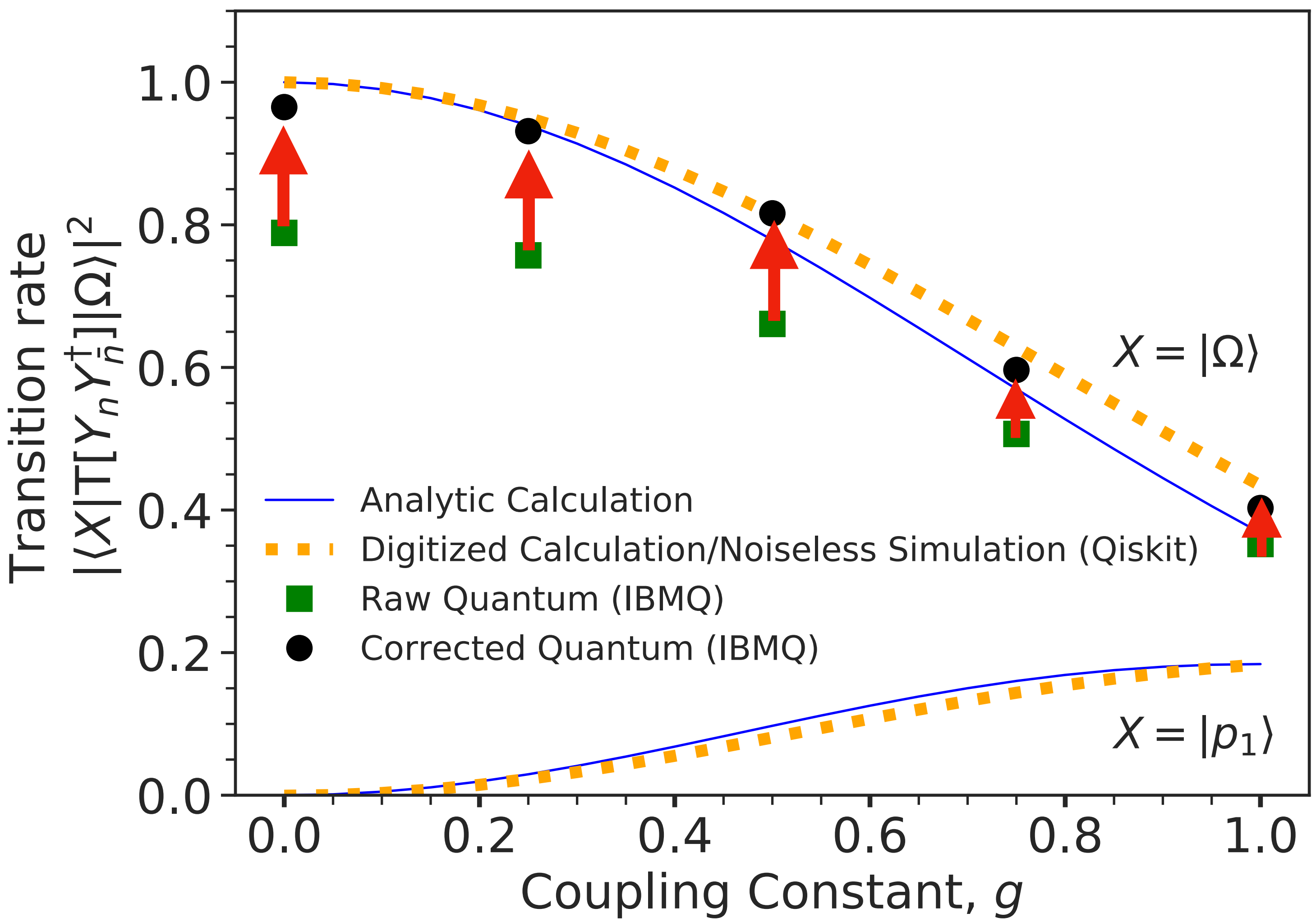
CWB, Freytsis, Nachman, accepted by PRL



Quantum computer gives a good description of the analytical result

Soft function is the expectation value of a “Wilson line” operator between initial and final state

Not that good out of the box … need error mitigation!



Quantum computer gives a good description of the analytical result

Dealing with errors in quantum computers

There are two types of errors in quantum computation: Readout errors and gate errors

Readout (measurement errors)

Readout (measurement) errors only happen when qubits are measured.

They are the largest errors on current devices [O(10%)]

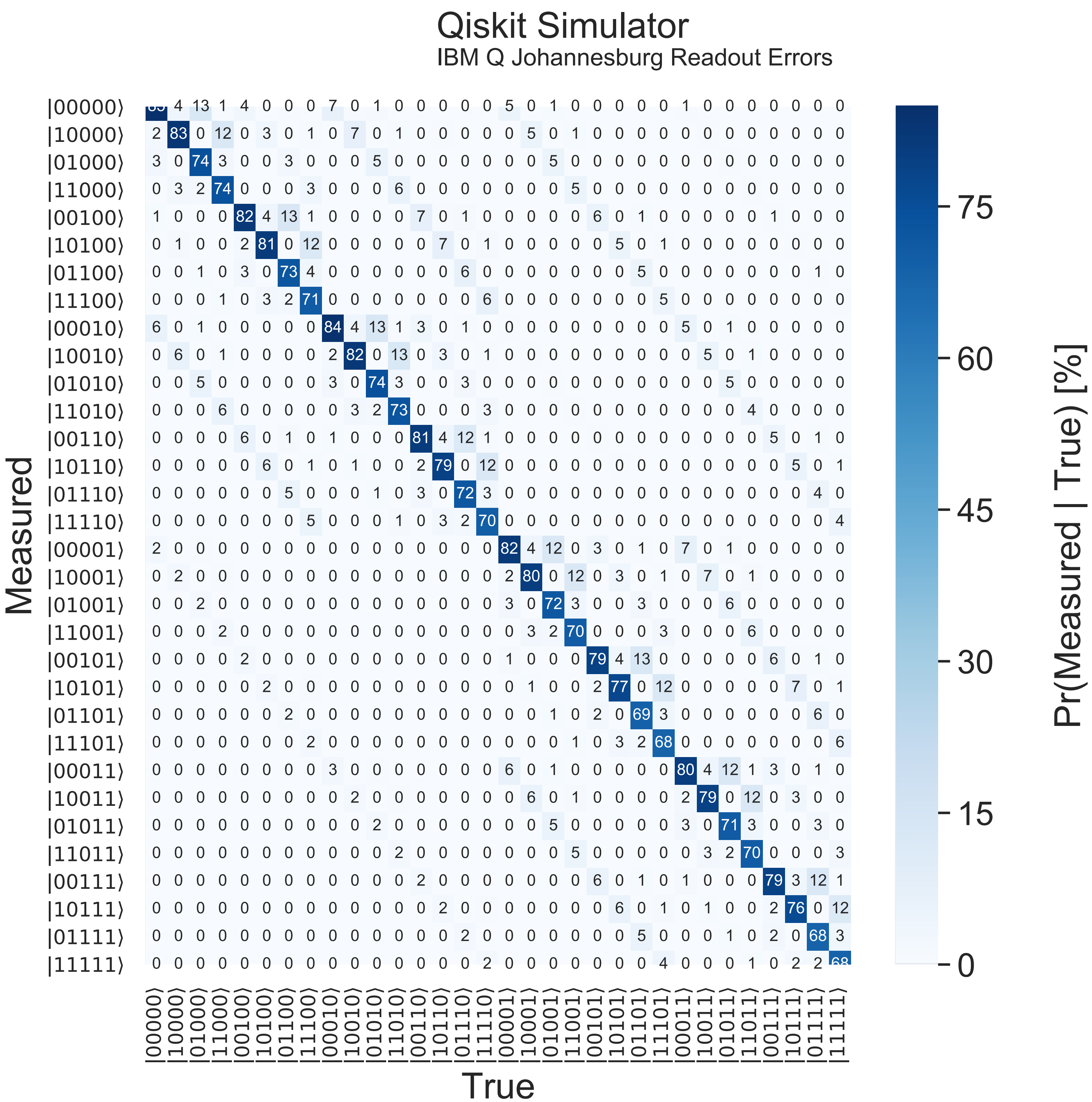
Gate errors

Gate errors happen whenever a gate is applied. The largest errors occur in entangling 2-qubit gates [O(few%)]

Gate errors accumulate and therefore limit the number of gates that can be used in a circuit

Readout errors arise from errors (for example decoherence) that arises during the measurement process

Nachman, Urbanek, de Jong, CWB, npj Quantum Information 6 (2020)



On a quantum computer, the state may be 1 but readout as a 0, etc.

For n qubits, there is a $2^n \times 2^n$ transition matrix.

Standard technique is to invert this readout matrix

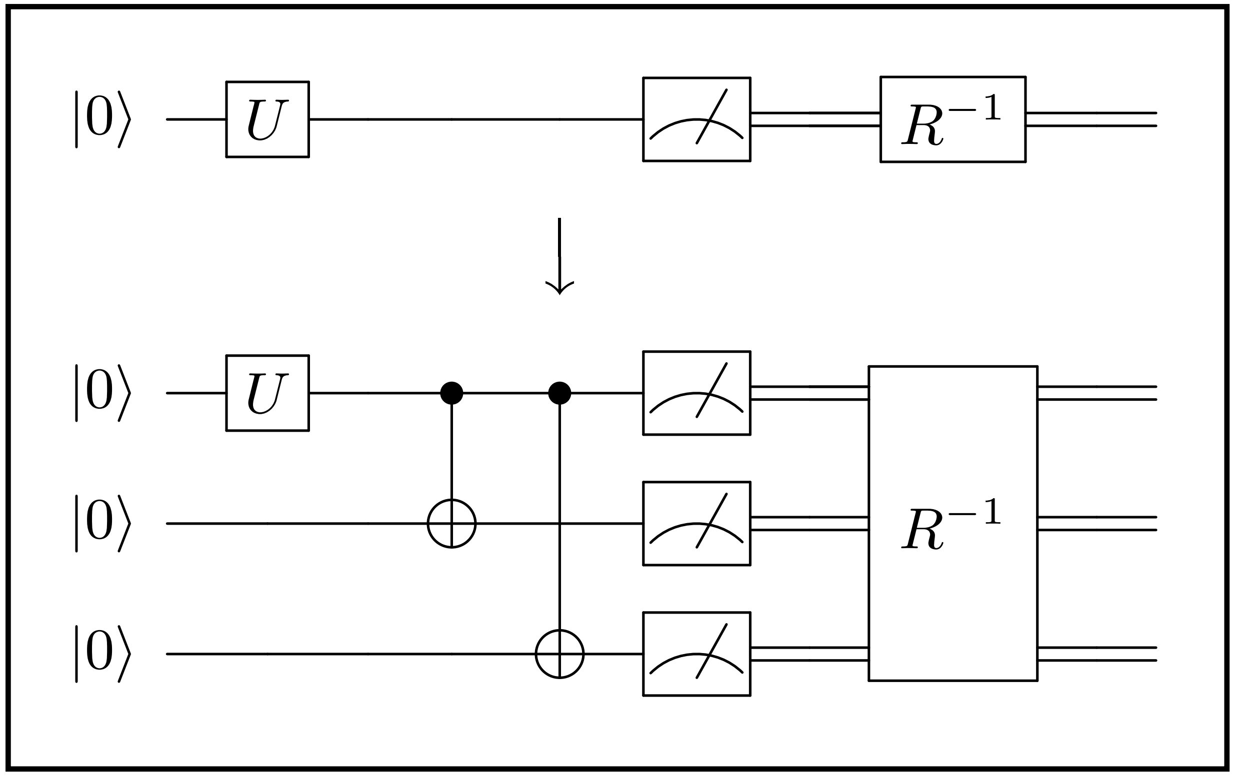
Can obtain more stable results using techniques borrowed from HEP detector unfolding

One can also use active readout error correction, which uses techniques similar to create fault tolerant computers

Hicks, Kobrin, CWB, Nachman (2108.12432)

Has advantage of allowing event-by-event correction

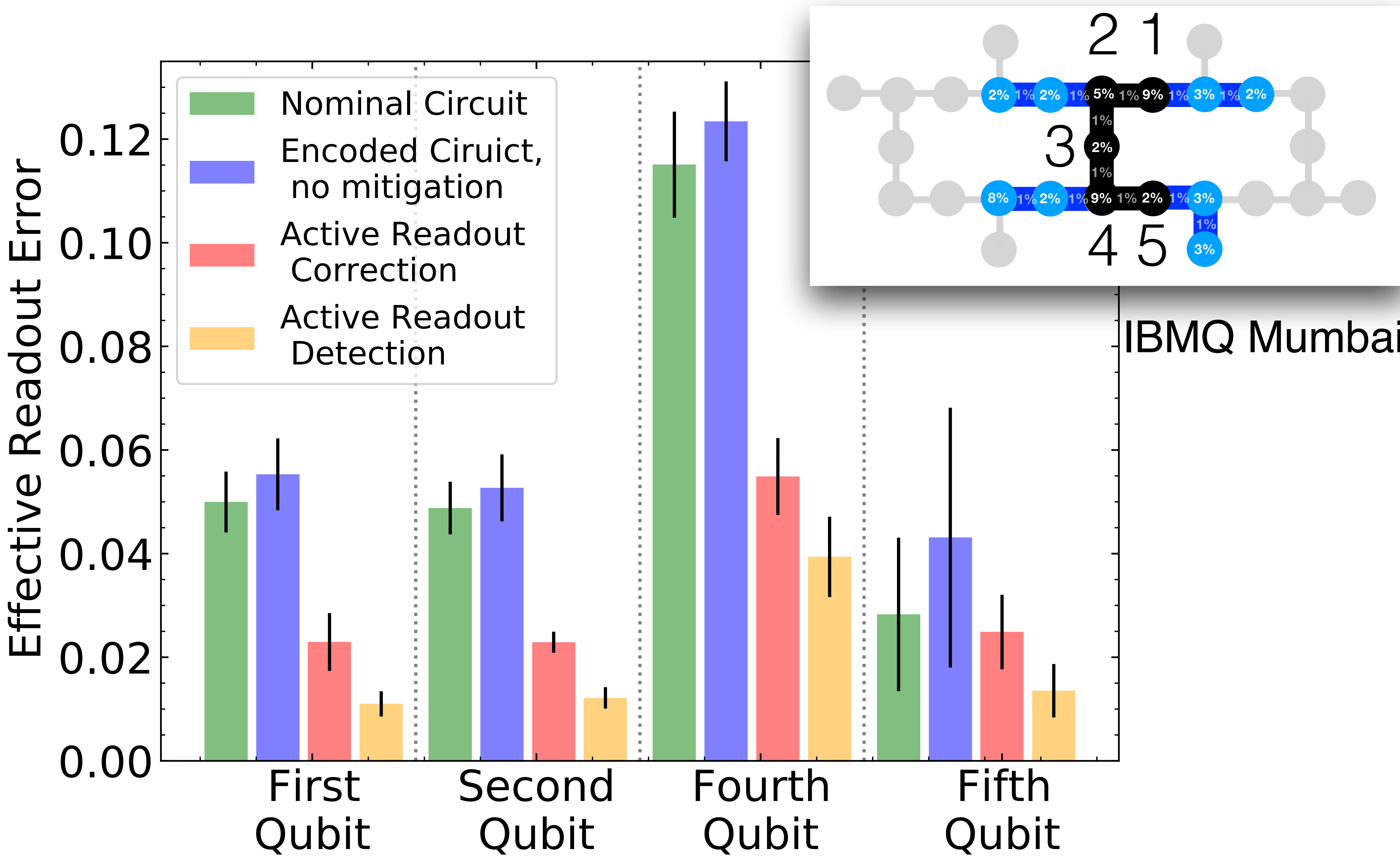
When readout errors are larger than gate errors (as is often the case), we can trade one for the other



We have developed a new protocol for exactly this purpose!

One can also use active readout error correction, which uses techniques similar to create fault tolerant computers

Hicks, Kobrin, CWB, Nachman (2108.12432)



The dominant gate errors one can find in quantum computers are occurring in entangling CNOT gates

Typical errors are O(%) for each CNOT gate.
Can not run circuits with more than O(10) CNOT gates without correction

Have worked to develop techniques based on “Zero Noise Extrapolation”

A. He, BPN, W. de Jong, C. Bauer, PRA 102 (2020) 012426

Basic idea

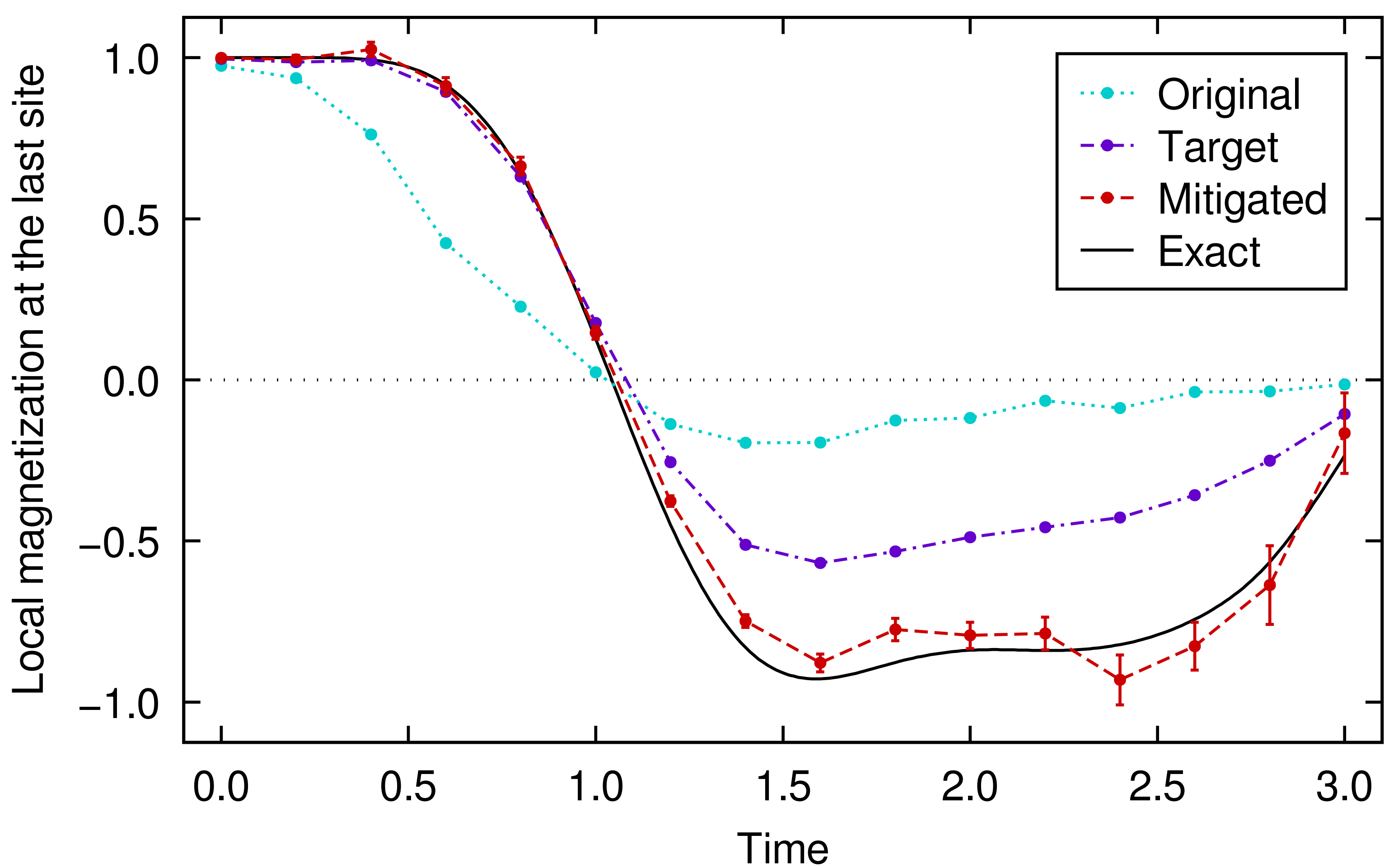
- Dependence on error rate is to first approximation linear
- Can increase error rate through extra insertion of gates
- Allows to extrapolate noise to zero

The dominant gate errors one can find in quantum computers are occurring in entangling CNOT gates

Typical errors are O(%) for each CNOT gate.
Can not run circuits with more than O(10) CNOT gates without correction

Second technique uses circuit to estimate and then correct noise

Urbanek, Nachman, Pascuzzi, He, CWB, deJong 2103.08591 (accepted in PRL)



Combining EFTs with quantum algorithms, can compute long distance physics from first principles

Using noise mitigation techniques, can use existing quantum computers to obtain stable results for simplest observables

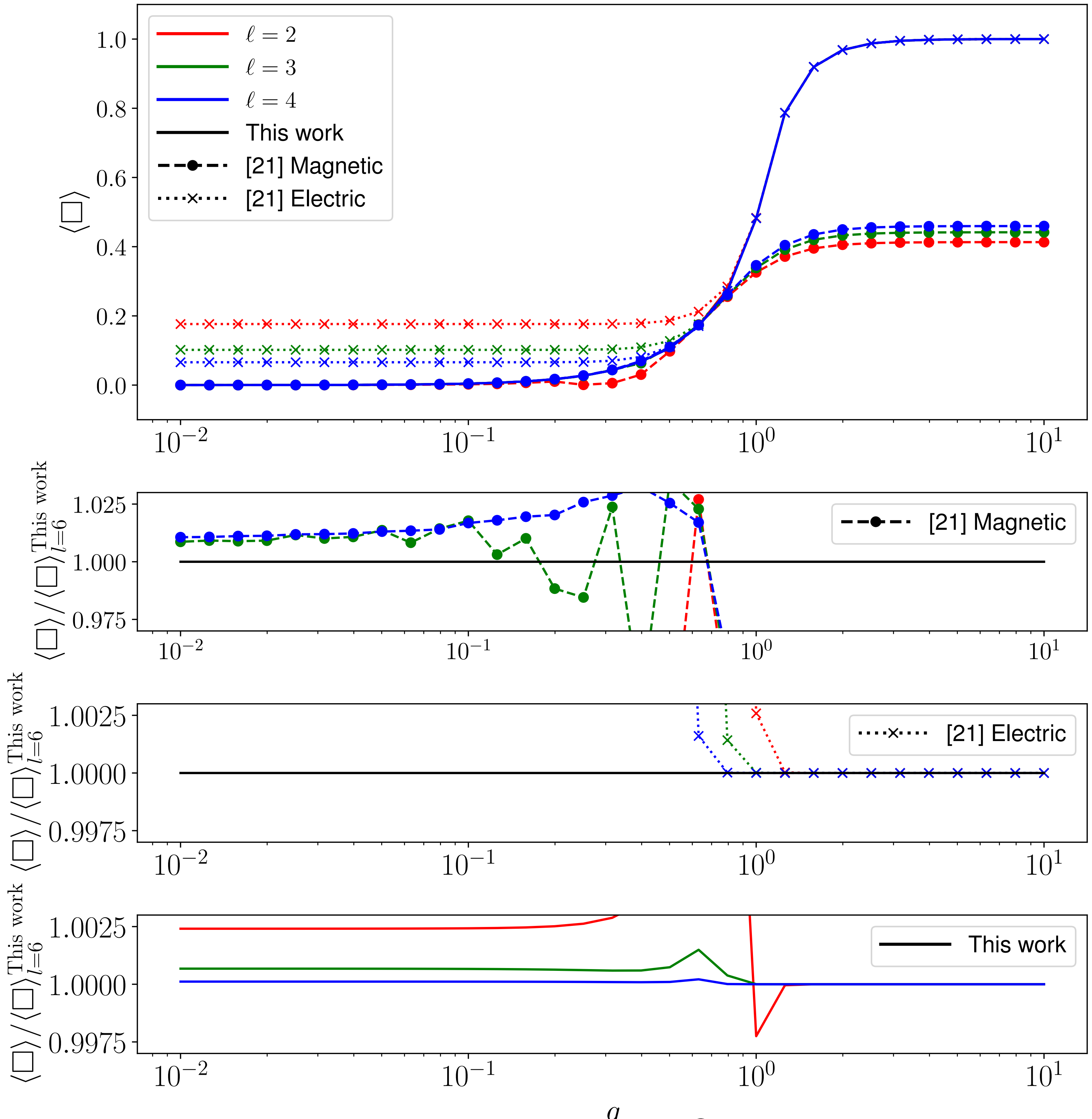
While this has shown that the relevant EFT calculations are possible, much more work required for real world applications

1. Calculation done for scalar field theory
   Implementation for gauge theories

2. Calculation done in bare theory:
   Think carefully about renormalization in EFT

First step to extend to gauge theories recently completed with careful study of lattice U(1) gauge theory

CWB, Grabowska, 2111.08015



Does significantly better than the previously best approach for all values of the coupling


QUESTIONS?