C-parameter hadronisation in the symmetric 3-jet limit and the strong coupling

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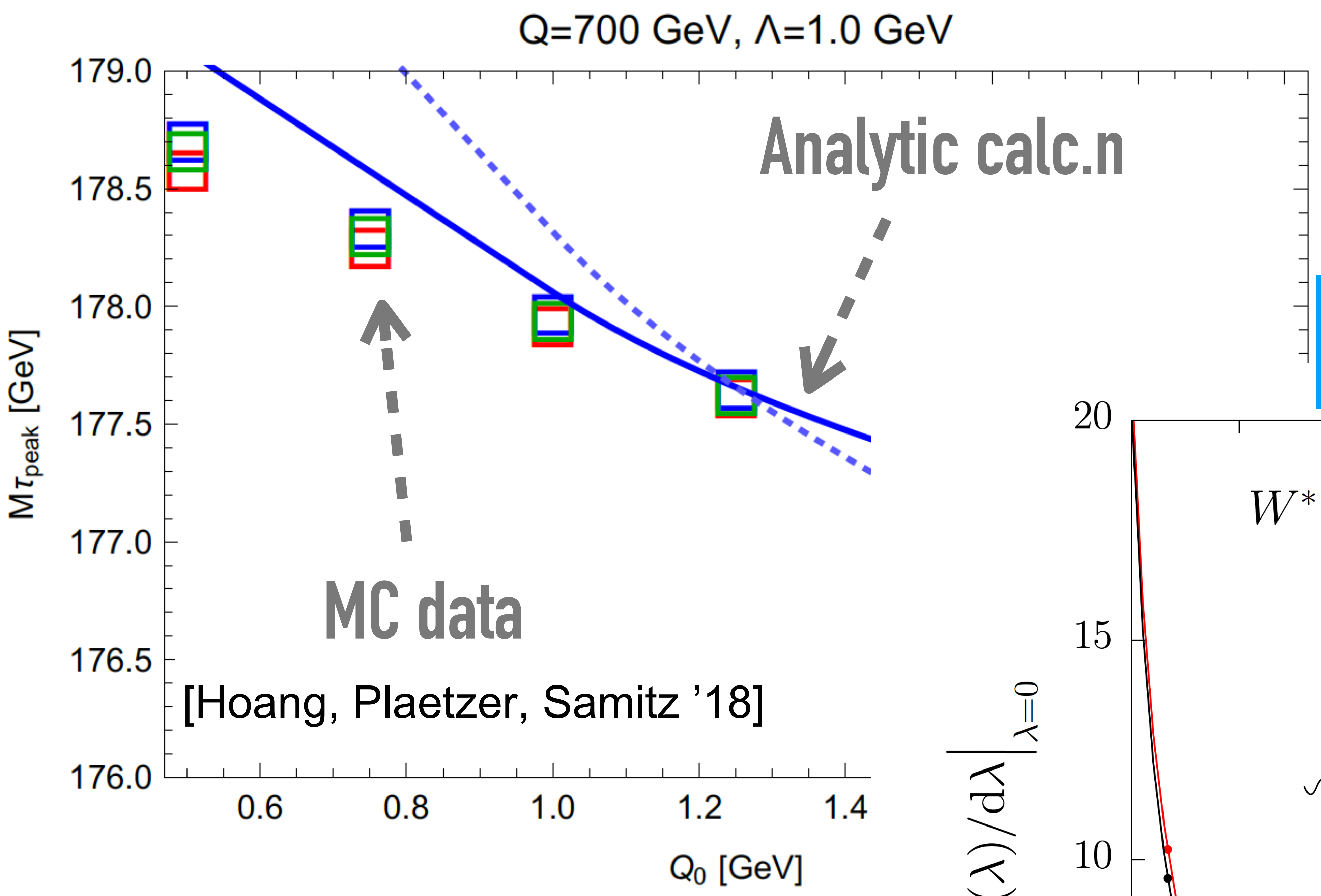
Beyond perturbation theory

- Outstanding precision reached in measurements challenges perturbative QFT computations at colliders - NP corrections are the bottleneck in several observables



Size of linear renormalon (in large-n_F model) for pole and MSbar mass

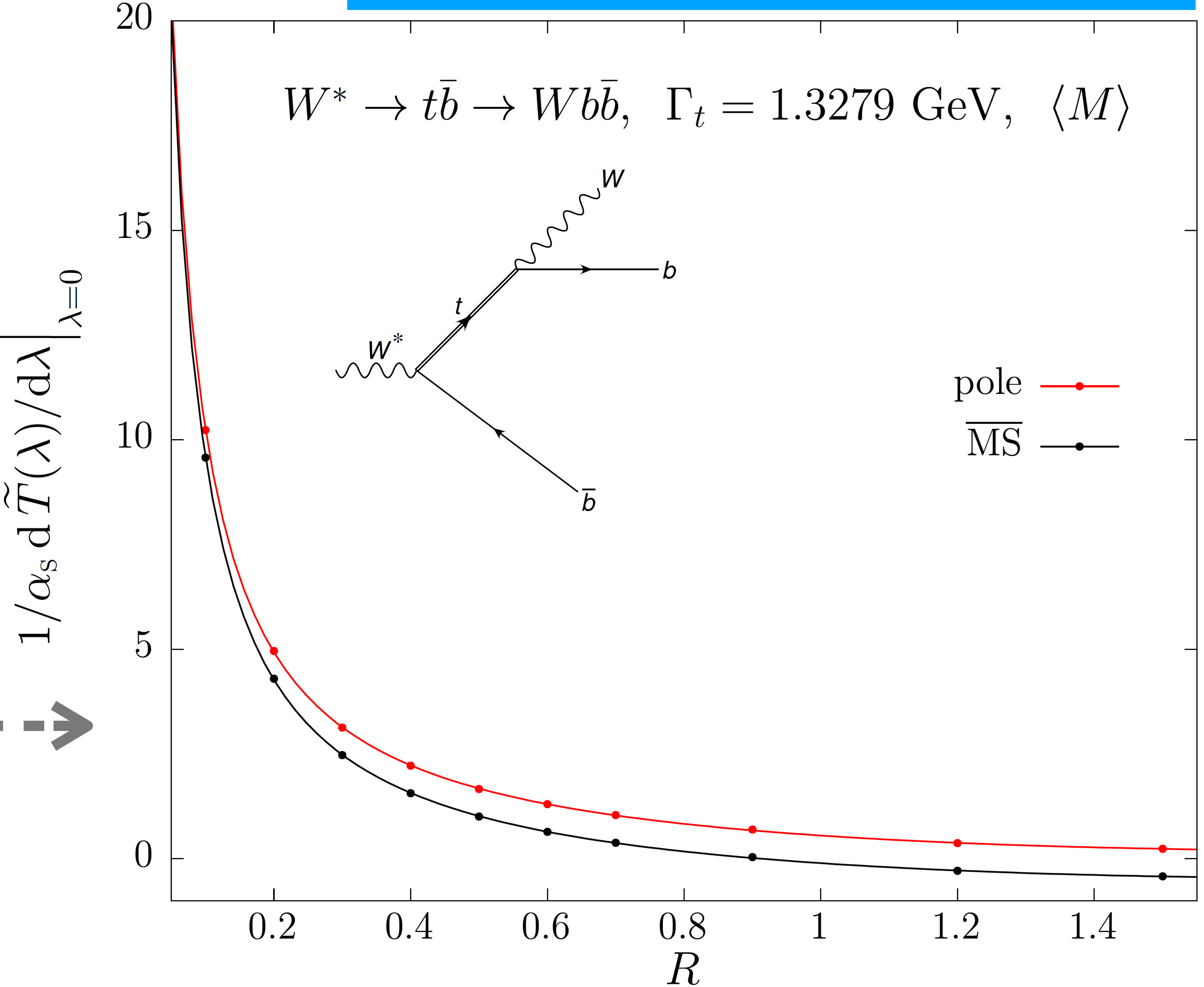
[Ferrario Ravasio, Nason, Oleari '18]

Estimates for the pole top mass in:

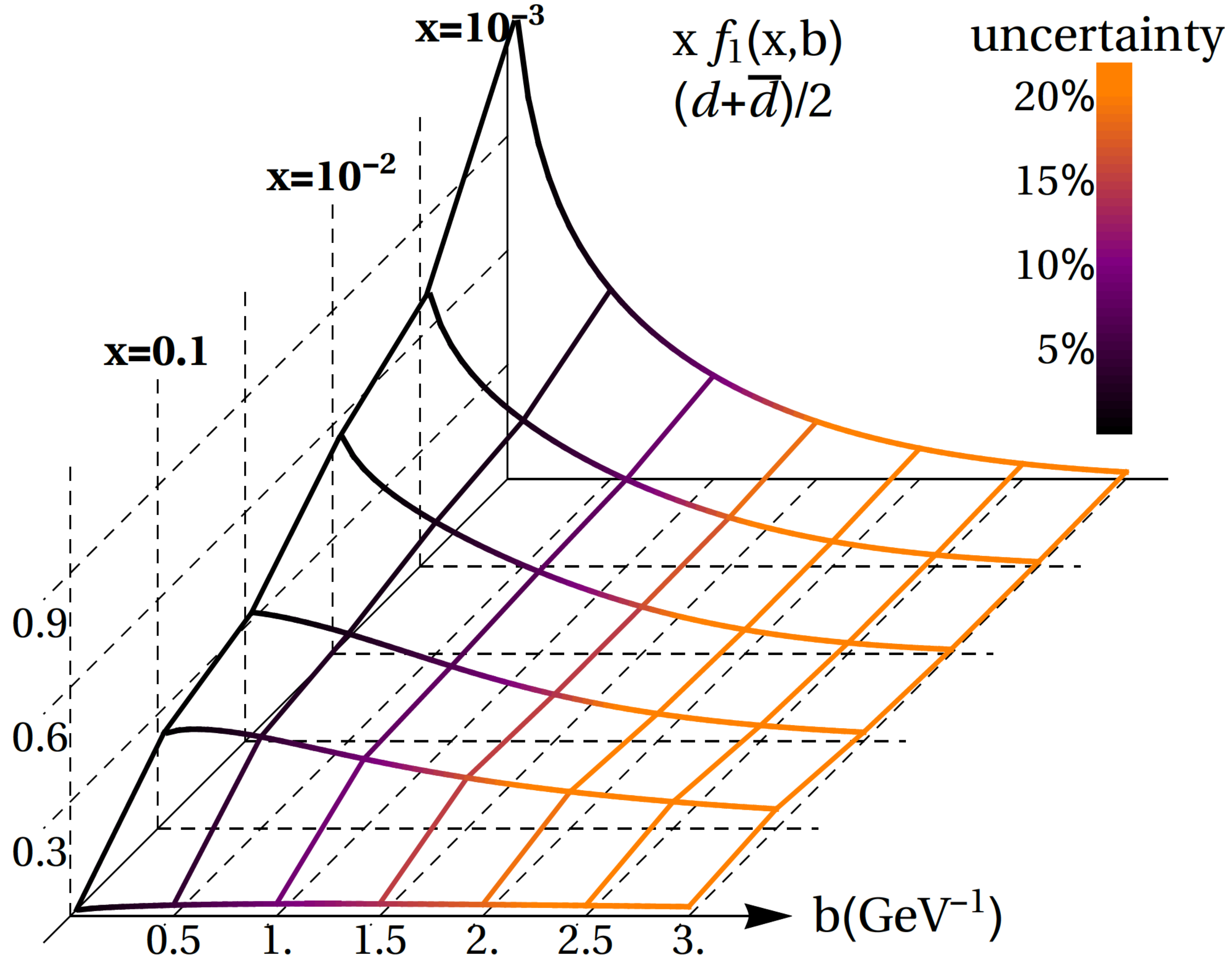
[Beneke, Marquard, Nason, Steinhauser '17]

[Hoang, Lepenik, Preisser '17]

E.g. Top mass from template fits



E.g. W mass from Z (lepton) p_T distribution



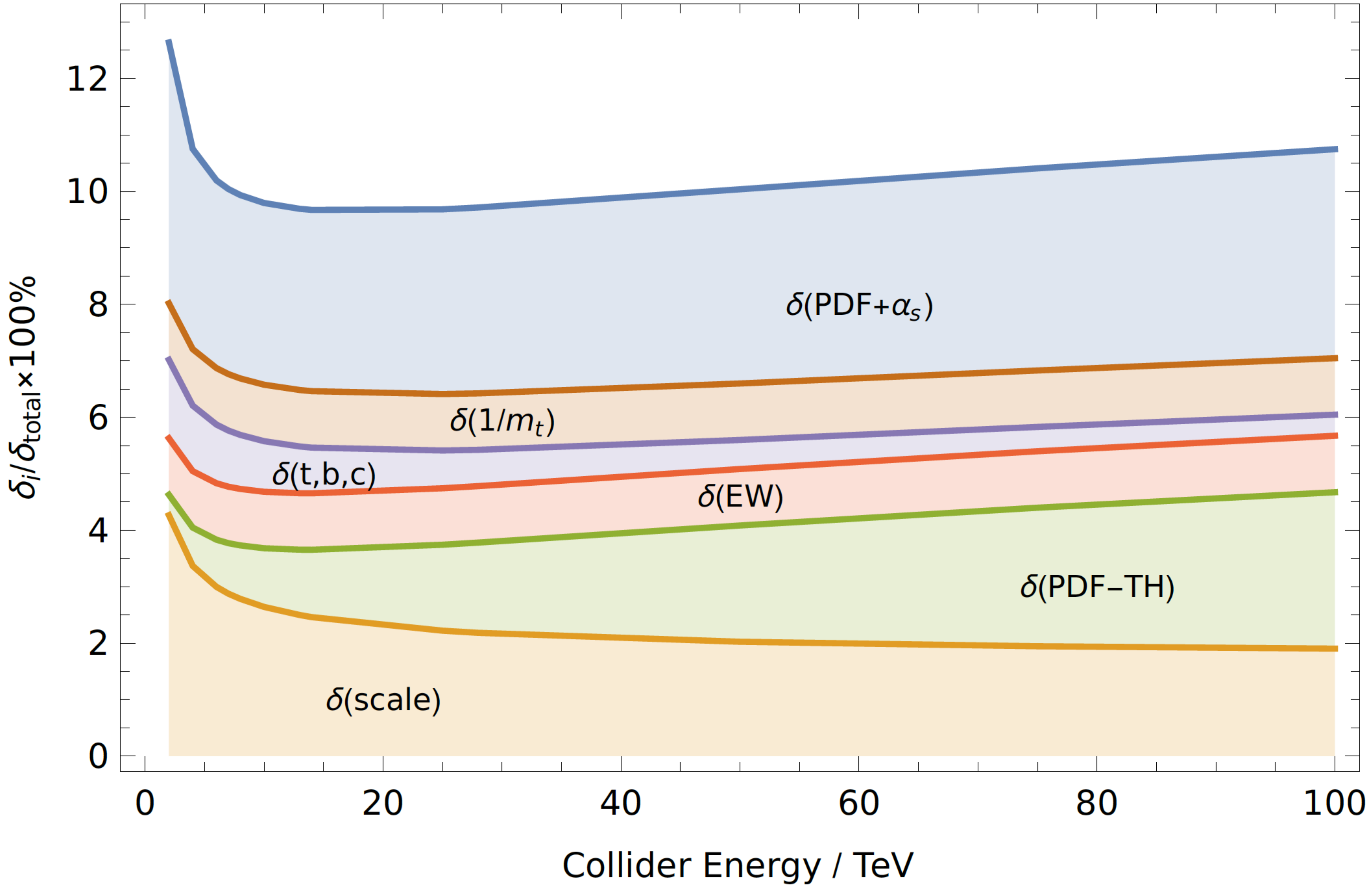
Fit of TMD pdfs from (small p_T) DY data

[Bertone, Scimemi, Vladimirov '19]

Strong coupling constant: status

- Least precisely known SM coupling, reduction of its uncertainty crucial for HL-LHC and FCC

- E.g. impact of current World Average unc. on Higgs prod^n in gg fusion



ggF cross section at N³LO QCD

$\sqrt{s}$	σ	$\delta(\text{theory})$	$\delta(\text{PDF})$	$\delta(\alpha_s)$
13 TeV	48.61 pb	$^{+2.08\text{pb}}_{-3.15\text{pb}}\left(^{+4.27\%}_{-6.49\%}\right)$	± 0.89 pb $(\pm 1.85\%)$	$^{+1.24\text{pb}}_{-1.26\text{pb}}\left(^{+2.59\%}_{-2.62\%}\right)$
14 TeV	54.72 pb	$^{+2.35\text{pb}}_{-3.54\text{pb}}\left(^{+4.28\%}_{-6.46\%}\right)$	± 1.00 pb $(\pm 1.85\%)$	$^{+1.40\text{pb}}_{-1.41\text{pb}}\left(^{+2.60\%}_{-2.62\%}\right)$
27 TeV	146.65 pb	$^{+6.65\text{pb}}_{-9.44\text{pb}}\left(^{+4.53\%}_{-6.43\%}\right)$	± 2.81 pb $(\pm 1.95\%)$	$^{+3.88\text{pb}}_{-3.82\text{pb}}\left(^{+2.69\%}_{-2.64\%}\right)$

[HE/HL-LHC Yellow Report '19]

Strong coupling constant: status

- World Average $\Longrightarrow$ spread between determinations
- E.g. significant tension between:
  - e^+e^- fits from 2j rate & T/C-parameter
  - $\langle$lattice$\rangle$ vs. e^+e^- fits T/C-parameter

Analytic hadronisation models lead to smaller couplings

Fit from C parameter w/ N^{3}LL+NNLO perturbation theory (+mass effects and analytic NP corrections)

[Hoang, Kolodrubetz, Mateu, Stewart '15]

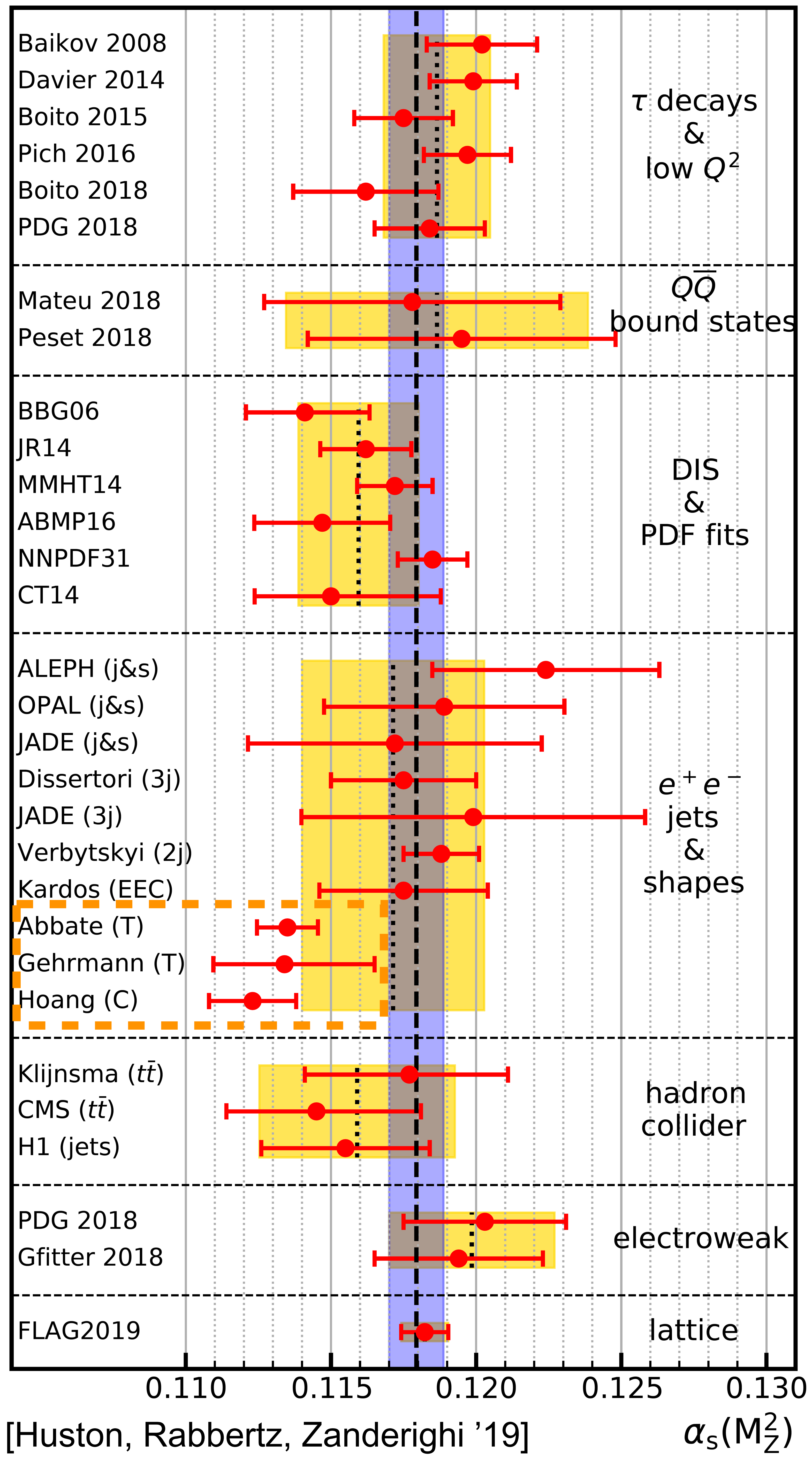
$$\alpha_s(M_Z^2) = 0.1123 \pm 0.0015$$

~ 3.35 σ

Flavour Lattice Averaging Group 2019

[Aoki et al. '19]

$$\alpha_s(M_Z^2) = 0.1182 \pm 0.0008$$



[Huston, Rabbertz, Zanderighi '19]

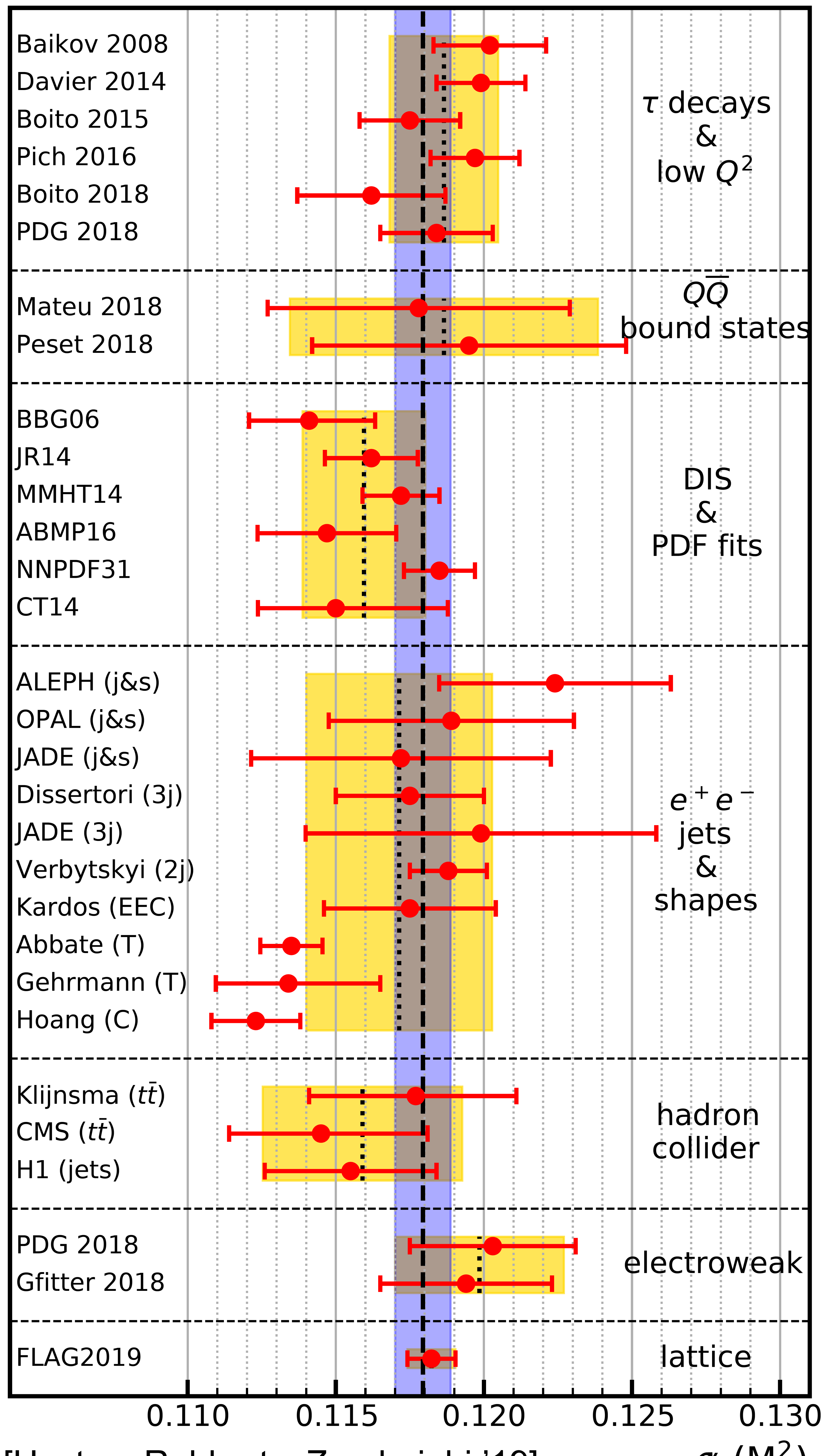
Strong coupling constant: status

- World Average $\Longrightarrow$ spread between determinations

- Optimistic prospects from lattice QCD to reach high precision in the coming decade see e.g. [Dalla Brida, Ramos '19]

- At future colliders, high precision extractions possible from EWPO, e.g. ratio of $\sigma_{e+e-\to had}/\sigma_{e+e-\to\mu+\mu-}$

However, pinning down the above tensions crucial for:
Control of systematics in combination of fits;
Understand reliability of hadronisation models; ...



[Huston, Rabbertz, Zanderighi '19]

Hadronisation: MC models

- MC models: control over event kinematics

Drawback: tuned with MC generator with low PT accuracy: "NP Corrections" might be under/over estimated

$$\mathcal{M}_{ij}(d\sigma^{\rm parton}_{\rm MC}(\mathcal{O}) \to d\sigma^{\rm hadron}_{\rm MC}(\mathcal{O}))$$

- E.g. migration matrices w/ unitarity constr^{n} for jet rates

$$R_2^{(\rm p)} + R_3^{(\rm p)} + R_{\geq 4}^{(\rm p)} = 1$$

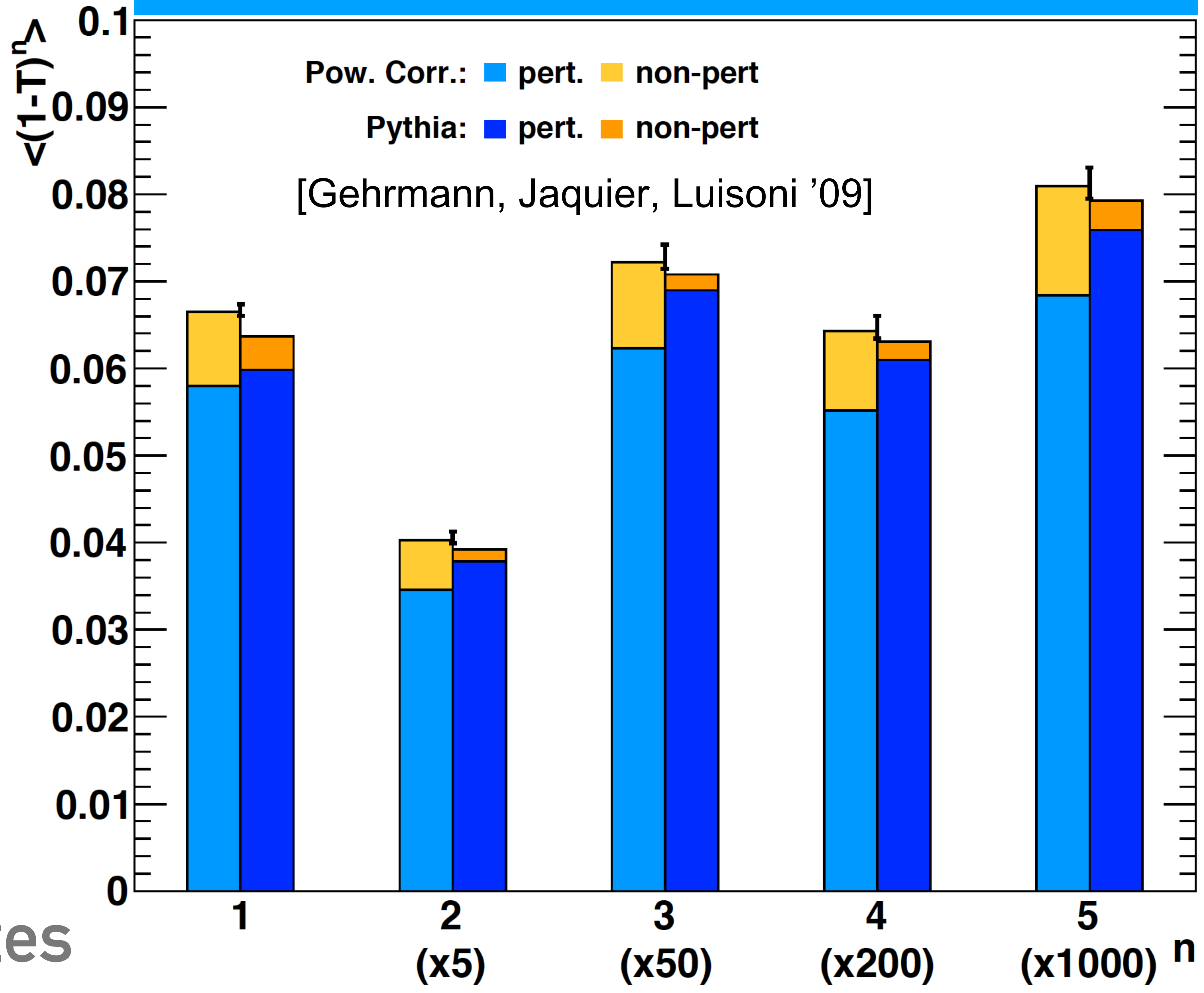
partonic hadronic

$$R_2^{(\rm h)} = \cos^2(\xi_1 + \delta\xi_1)\,, \quad R_3^{(\rm h)} = \sin^2(\xi_1 + \delta\xi_1)\cos^2(\xi_2 + \delta\xi_2)$$

$$R_{\geq 4}^{(\rm h)} = \sin^2(\xi_1 + \delta\xi_1)\sin^2(\xi_2 + \delta\xi_2)$$

[Verbytskyi, Banfi, Kardos, PM, Kluth, Somogyi, Szor, Trocsanyi, Tulipant, Zanderighi '19]

Difference in Thrust moments



R_2 @ N^{3}LO+NNLL:

$$\alpha_s(M_Z) = 0.11881 \pm 0.00131$$

Hadronisation: Analytic models

Vast literature: dispersive model, shape function, dressed gluon exponentiation, large-n_F, . . .

- Analytic models: in principle usable with state of the art perturbative calculations

Drawback: full dynamics complex; expand in powers of 1/Q and extract the leading term
[often only near logarithmic singularities]

$$d\sigma^{\text{hadron}}(\mathcal{O}) \simeq d\sigma^{\text{parton}}(\mathcal{O} - \frac{\alpha_0}{Q}\Delta(\mathcal{O}))$$

E.g. dispersive model: correction due to radiation of a gluer with (effective) coupling

[Dokshitzer, Marchesini, Webber '95]

$$\tilde{\alpha}_s^{\text{NP}}(k_t^2) = \tilde{\alpha}_s(k_t^2) - \tilde{\alpha}_s^{\text{PT}}(k_t^2)$$

$$\tilde{\alpha}_s(\mu^2) \equiv \tilde{\alpha}_s(0) + \int_0^\infty \mathrm{d}m^2 \frac{\mu^2}{m^2+\mu^2}\frac{\mathrm{d}\alpha_{\text{eff}}(m^2)}{\mathrm{d}m^2}$$

Dispersive representation of QCD coupling
[i.e. prod^n of a system with gluon quantum numbers]

[Dokshitzer, Marchesini, Webber '95]

$$\tilde{\alpha}_s^{\text{PT}}(\mu^2) = \alpha_s\left(1 + \frac{\alpha_s}{2\pi}K^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 K^{(2)} + \mathcal{O}(\alpha_s^3)\right)$$

Perturbative counterpart
[i.e. soft physical coupling scheme]

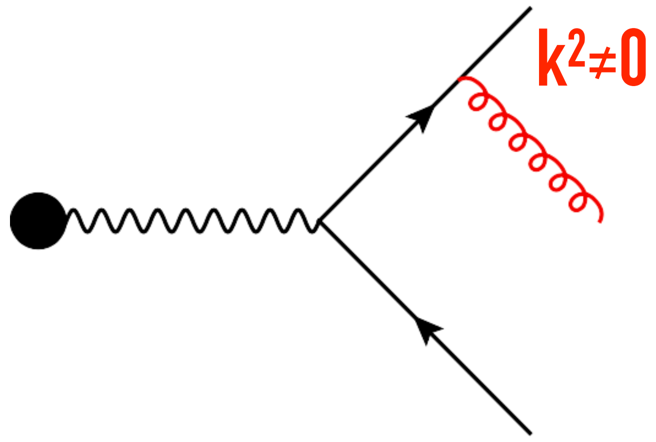
[Catani, Marchesini, Webber '91]
$K^{(2)}$ in [Banfi, El-Menoufi, PM '18; Catani, De Florian, Grazzini '19]

Hadronisation: Analytic models

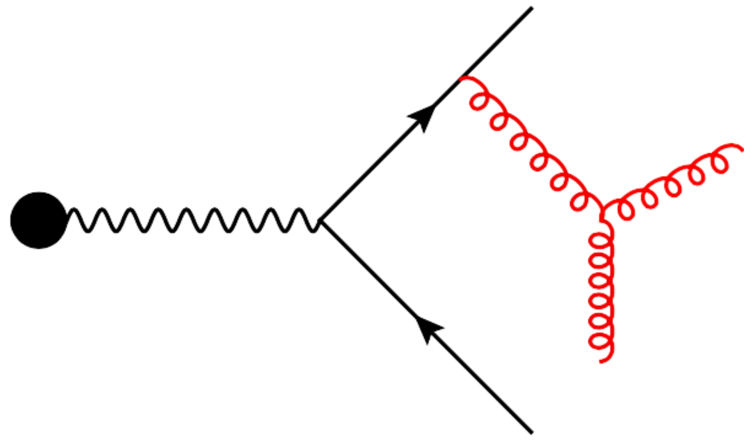
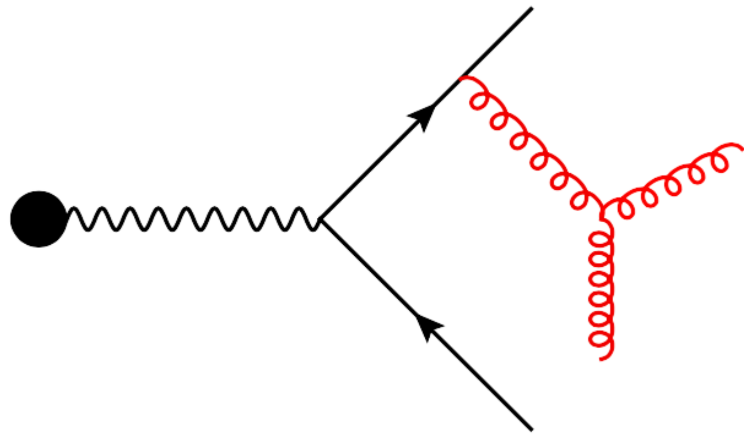
- E.g. dispersive model: O(1/Q) => shift in the average value of the observable

<u>C parameter in the 2-jet limit</u>

$$\Sigma^{\text{hadr.}}(C) = \Sigma^{\text{pert.}}(C - \langle \delta C \rangle (C))$$



$O(\{p\}, k)$ +

+  $O(\{p\}, k_1, k_2)$ -  $O(\{p\}, k_1 + k_2)$ + ...

Radiative correction universal for additive observable [Milan factor]

[Dokshitzer, Lucenti, Marchesini, Salam ’97-’98]

Shift in 2-jet limit naturally emerges from structure of log resummation

Hadronisation: Analytic models

- E.g. dispersive model: O(1/Q) => shift in the average value of the observable

C parameter in the 2-jet limit

Second order corr^{ns} (Milan factor)

First moment of physical coupling

$$\langle \delta C \rangle \simeq \zeta(C)\,\mathcal{M}\,\frac{\mu_I}{Q}\,\frac{4\,C_F}{\pi^2}\Bigg[\alpha_0(\mu_I^2) - \alpha_s(\mu_R^2) - \alpha_s^2(\mu_R^2)\,\frac{\beta_0}{\pi}\left(2\ln\frac{\mu_R}{\mu_I} + \frac{K^{(1)}}{2\beta_0} + 2\right)$$
$$- \alpha_s^3(\mu_R^2)\,\frac{\beta_0^2}{\pi^2}\left(4\ln^2\frac{\mu_R}{\mu_I} + 4\left(\ln\frac{\mu_R}{\mu_I} + 1\right)\times\left(2 + \frac{\beta_1}{2\beta_0^2} + \frac{K^{(1)}}{2\beta_0}\right) + \frac{K^{(2)}}{4\beta_0^2}\right)\Bigg]$$

for derivation see e.g. [Davison, Webber ’08; Gehrmann, Luisoni, PM ’12] (Thrust case)

Obs. dependent coefficient

$$\Delta C(k) = \frac{k_t}{Q}\,\frac{3}{\cosh(\eta)} + \mathcal{O}\left(\frac{k_t^2}{Q^2}\right) \;\Rightarrow\; \zeta(0) = \int_{-\infty}^{\infty} \mathrm{d}\eta\,\frac{3}{\cosh(\eta)} = 3\pi$$

$$\alpha_0(\mu_I^2) \equiv \frac{1}{\mu_I}\int_0^{\mu_I} \mathrm{d}\mu\,\tilde{\alpha}_s(\mu^2)$$

Hadronisation: Analytic models

- E.g. dispersive model: O(1/Q) => shift in the average value of the observable

C parameter in the 2-jet limit

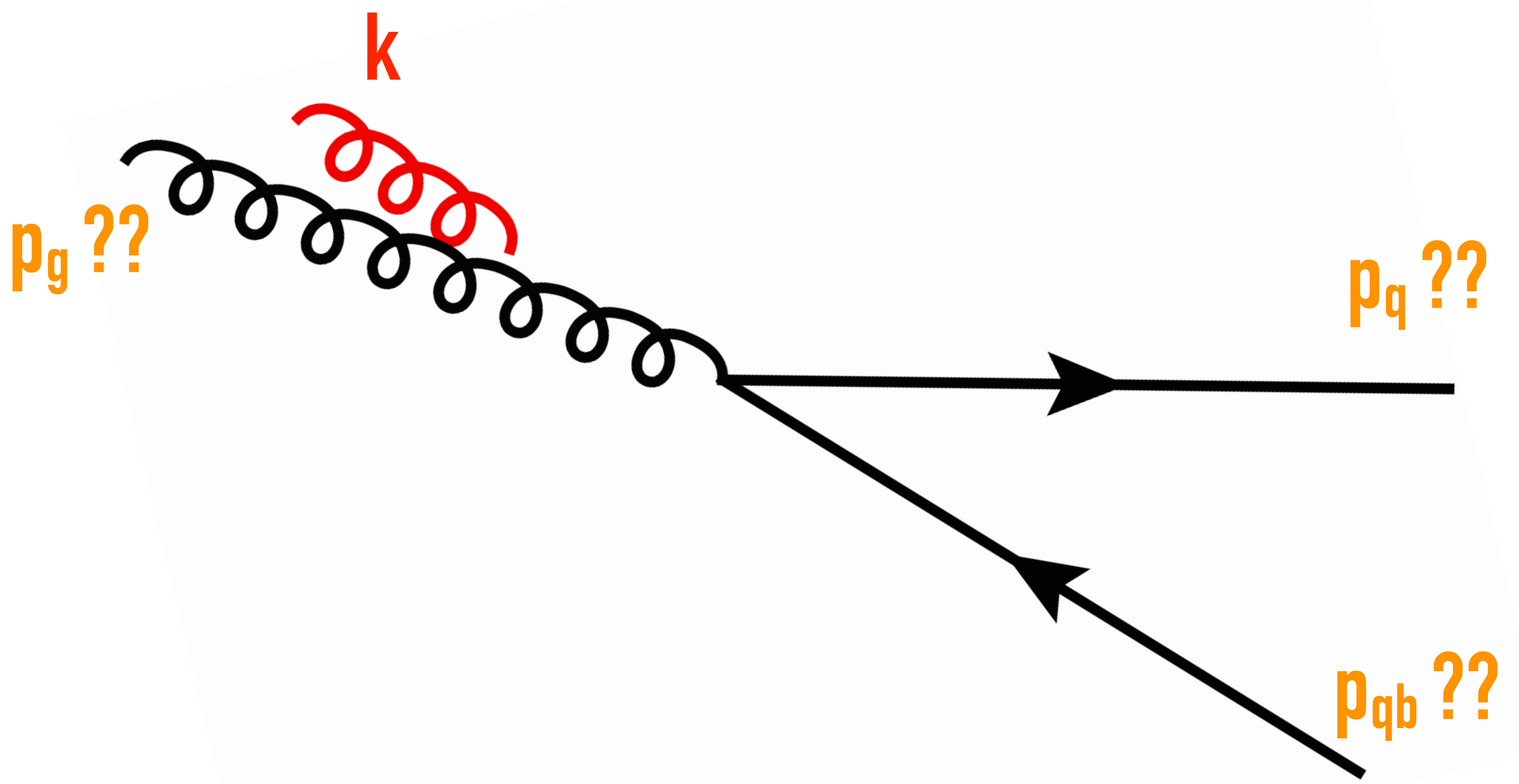
$$\langle \delta C \rangle \simeq \zeta(C)\,\mathcal{M}\,\frac{\mu_I}{Q}\frac{4\,C_F}{\pi^2}\Bigg[\alpha_0(\mu_I^2)\boxed{-\,\alpha_s(\mu_R^2) - \alpha_s^2(\mu_R^2)\frac{\beta_0}{\pi}\left(2\ln\frac{\mu_R}{\mu_I} + \frac{K^{(1)}}{2\beta_0} + 2\right)}$$
$$\boxed{-\,\alpha_s^3(\mu_R^2)\frac{\beta_0^2}{\pi^2}\left(4\ln^2\frac{\mu_R}{\mu_I} + 4\left(\ln\frac{\mu_R}{\mu_I} + 1\right)\times\left(2 + \frac{\beta_1}{2\beta_0^2} + \frac{K^{(1)}}{2\beta_0}\right) + \frac{K^{(2)}}{4\beta_0^2}\right)\Bigg]}$$

Subtraction of perturbative component

$$\alpha_0(\mu_I^2) \equiv \frac{1}{\mu_I}\int_0^{\mu_I} \mathrm{d}\mu\,\tilde{\alpha}_s(\mu^2)$$

Kinematic dependence across the spectrum

- Shift extrapolated from the 2-jet limit, <u>scaling across the spectrum neglected</u>!
- 2-jet limit is special in that dependence of δC on hard partons recoil is quadratic, i.e. <u>linear NP shift is independent of kinematic recoil map</u>
- In general, away from this limit, δC depends linearly on recoil scheme

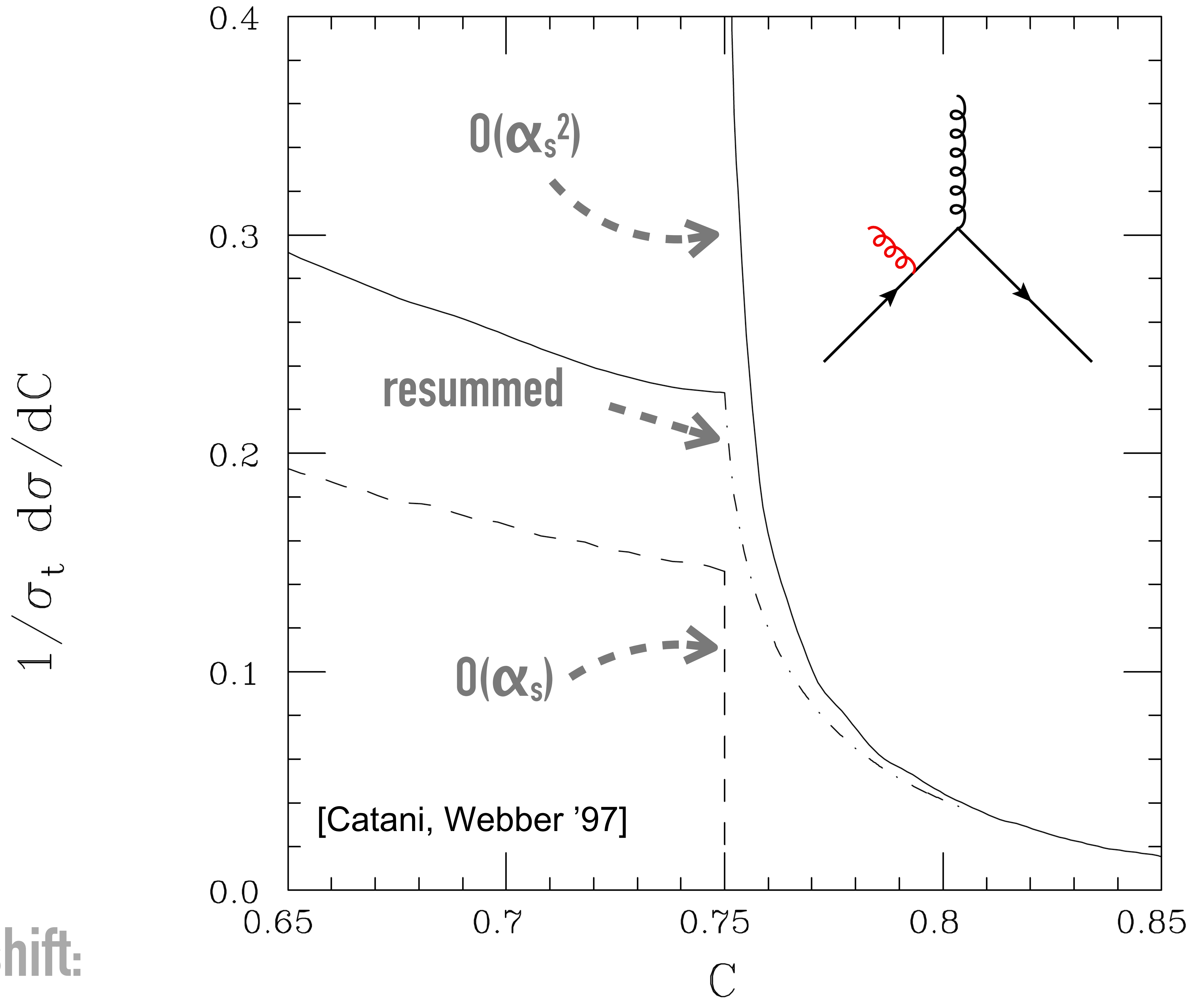


Possible to gain any insight on the scaling across the spectrum ?

Sudakov shoulder & recoil

$$C = \frac{3}{4} - \frac{81}{16}\left(\epsilon_q^2 + \epsilon_q\epsilon_{\bar q} + \epsilon_{\bar q}^2\right) + \mathcal{O}\left(\epsilon^3\right)$$

Quadratic dependence on the energies of the hard partons at the shoulder [linear corr^n insensitive to recoil]



Insensitivity to recoil map allows for the computation of the NP shift:

$$\Delta C(k) = \frac{3\sqrt{3}}{2}\,\frac{\sin^2(\phi)}{2\cosh(\eta) - \cos(\phi)}\,\frac{k_t}{Q} + \mathcal{O}\left(\frac{k_t^2}{Q^2}\right) \quad \Rightarrow \quad \zeta(3/4) = \frac{3\sqrt{3}}{4}\,\frac{C_A + 2C_F}{C_F}\left(4\,E(1/4) - 3\,K(1/4)\right)$$

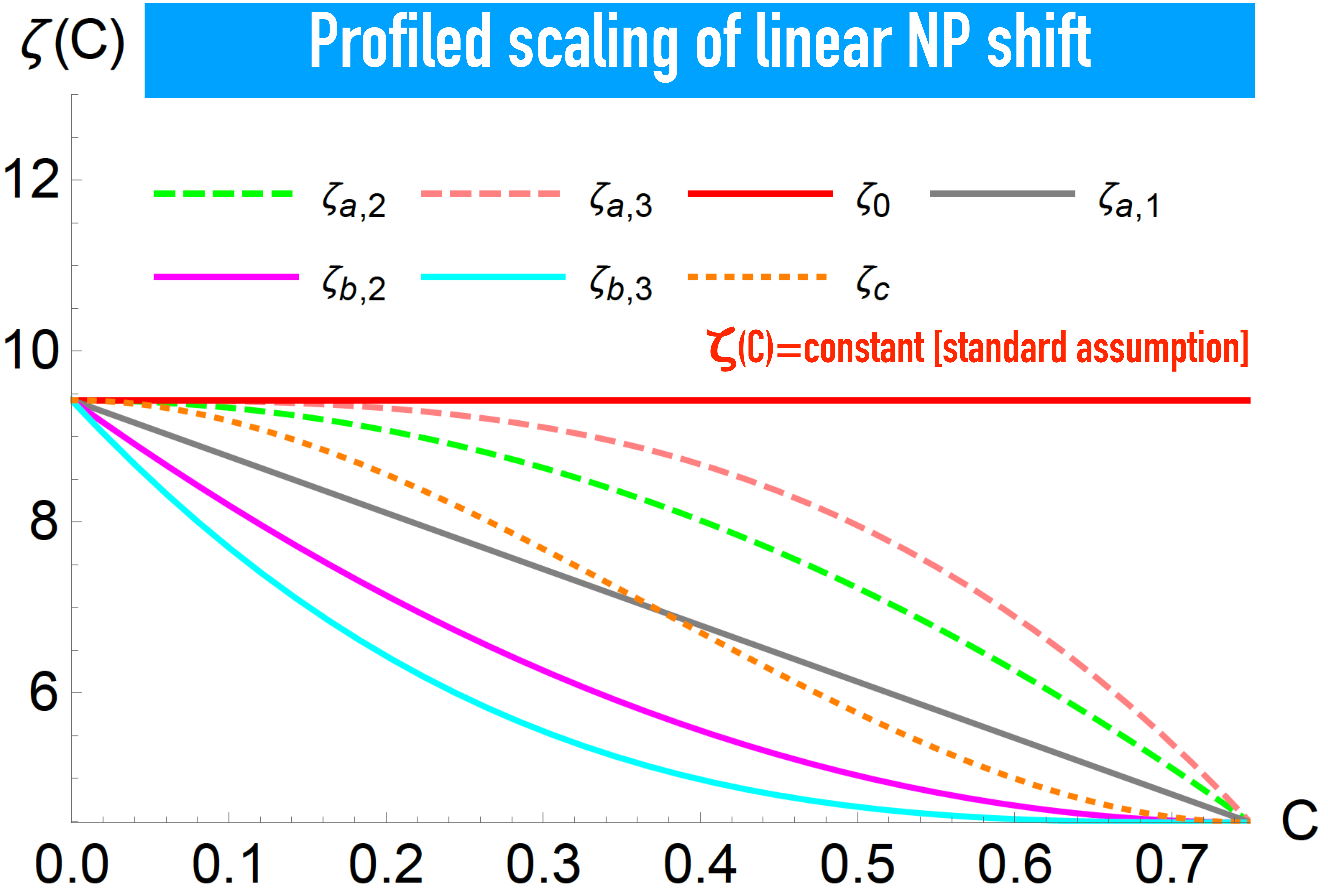
[Luisoni, PM, Salam '20]

$$\zeta(0)/\zeta(3/4) \simeq 2.1$$

Non-inclusive corrections (i.e. Milan factor) as in 2-jet limit

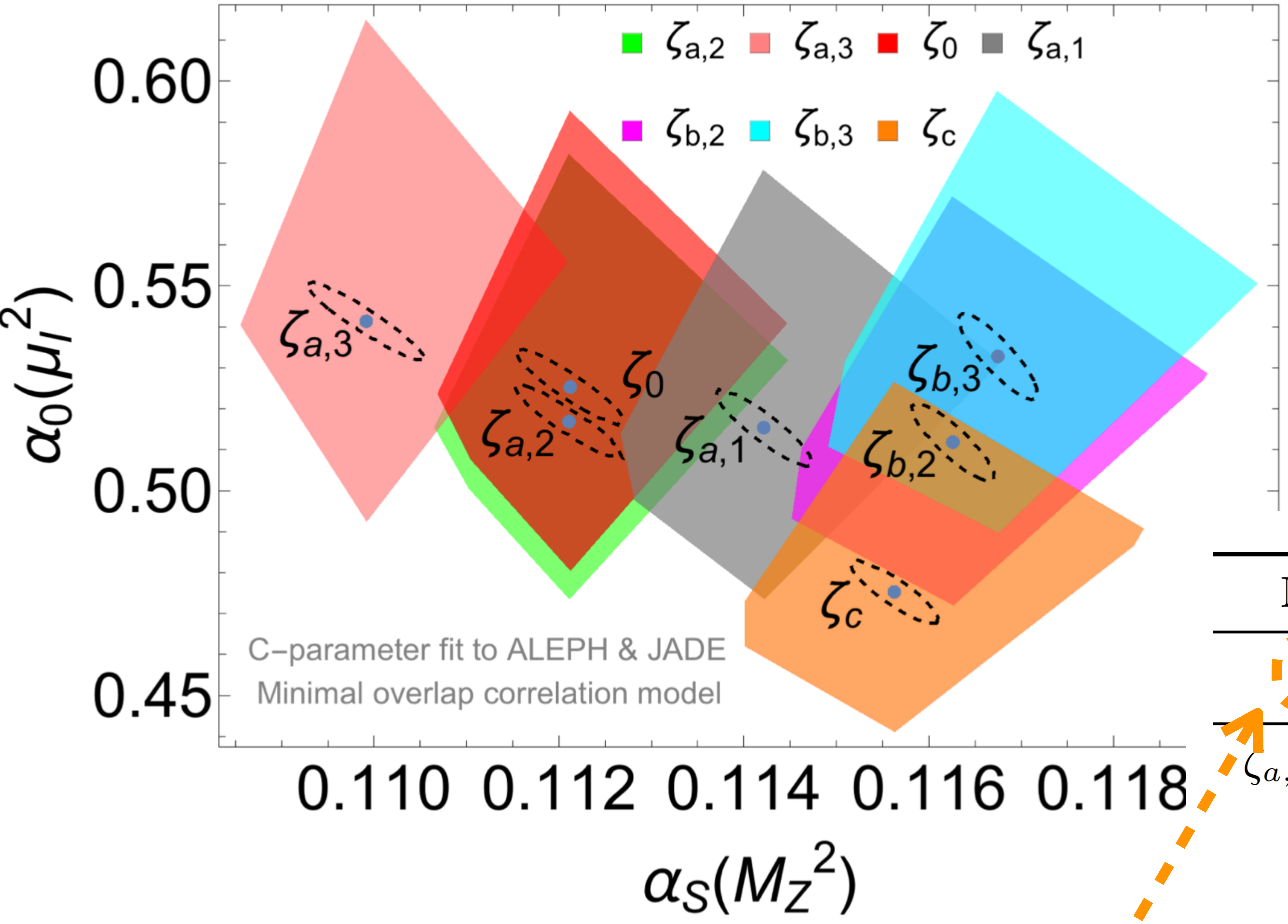
Impact on strong coupling fit

Toy model: Assume different power scaling templates in 0 < C < 3/4. Assess impact on fit from experimental data from ALEPH & JADE data* [backup]



*Use minimal overlap model for EXP systematics
Moderate impact of systematic correlation model [backup]

Impact on strong coupling fit



Fit based on NNLL+NNLO perturbation theory.
Theory systematic uncertainty includes :
[TH uncertainty≡scales (μ_R, μ_{RES}) ⊕ matching ⊕ Milan factor]

Fit ALEPH+JADE. uncertainties: EXP$^{+\mathrm{TH}}_{-\mathrm{TH}}$

Model	$\alpha_s(M_Z^2)$	$\alpha_0(\mu_I^2)$	χ^2/d.o.f
ζ_0	$0.1121 \pm 0.0006^{+0.0023}_{-0.0014}$	$0.53 \pm 0.01^{+0.07}_{-0.04}$	1.076
$\zeta_{a,1} \equiv \zeta_{b,1}$	$0.1142 \pm 0.0005^{+0.0026}_{-0.0015}$	$0.52 \pm 0.01^{+0.06}_{-0.04}$	1.045
$\zeta_{a,2}$	$0.1121 \pm 0.0006^{+0.0024}_{-0.0015}$	$0.52 \pm 0.01^{+0.07}_{-0.04}$	1.033
$\zeta_{a,3}$	$0.1099 \pm 0.0007^{+0.0022}_{-0.0014}$	$0.54 \pm 0.01^{+0.07}_{-0.05}$	1.116
$\zeta_{b,2}$	$0.1163 \pm 0.0005^{+0.0028}_{-0.0017}$	$0.51 \pm 0.01^{+0.06}_{-0.04}$	1.079
$\zeta_{b,3}$	$0.1167 \pm 0.0004^{+0.0028}_{-0.0018}$	$0.53 \pm 0.01^{+0.06}_{-0.04}$	1.143
ζ_c	$0.1156 \pm 0.0005^{+0.0027}_{-0.0016}$	$0.48 \pm 0.01^{+0.05}_{-0.03}$	1.074

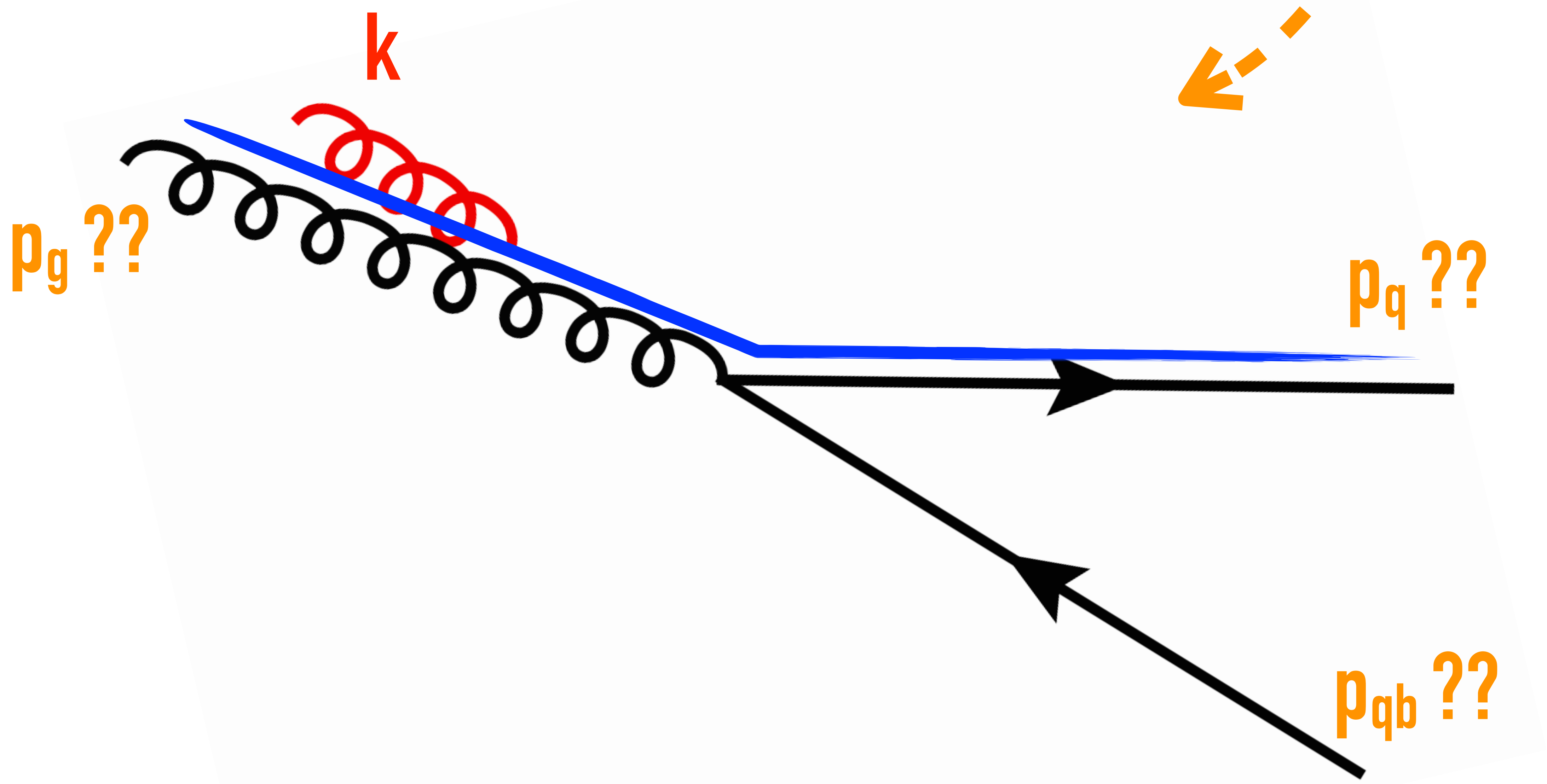
Spread of values at the **3-4% level** with a similar fit quality
[scheme (a,2) shows a lower χ^2 but disfavoured from fixed order checks (next slide)]

Fixed order study of scaling (first order)

Examine different recoil maps away from singular points

$$\zeta(C) = \lim_{\epsilon\to 0} \frac{\pi Q}{2\alpha_s C_F} \int [dk]\, M^2(k)\, \Delta C(\{p_i'\}, \{p_i\}; k)\, \delta(k_t - \epsilon)$$

$$\zeta(C) = \frac{1}{\mathcal{N}} \lim_{\epsilon\to 0} \frac{\pi Q}{2\alpha_s C_F} \int d\Phi_{q\bar{q}g}\, [dk]\, M^2_{q\bar{q}g}(\{p_i\})\, M^2(k) \Delta C(\boxed{\{p_i'\}}, \{p_i\}; k) \delta\left(C - C(\{p_i\})\right)\, \delta(k_t - \epsilon)$$



Explore different recoil schemes

- Catani-Seymour (CS) scheme: fully local [NB: we partition at equal angles in the event frame!] [Catani, Seymour '96]
- PanLocal scheme: fully local
- PanGlobal scheme: longitudinal d.o.f. local ⊕ transverse global [Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]
- Forshaw-Holguin-Plaetzer (FHP): only one longitudinal d.o.f. local ⊕ rest global [Forshaw, Holguin, Plaetzer '20]

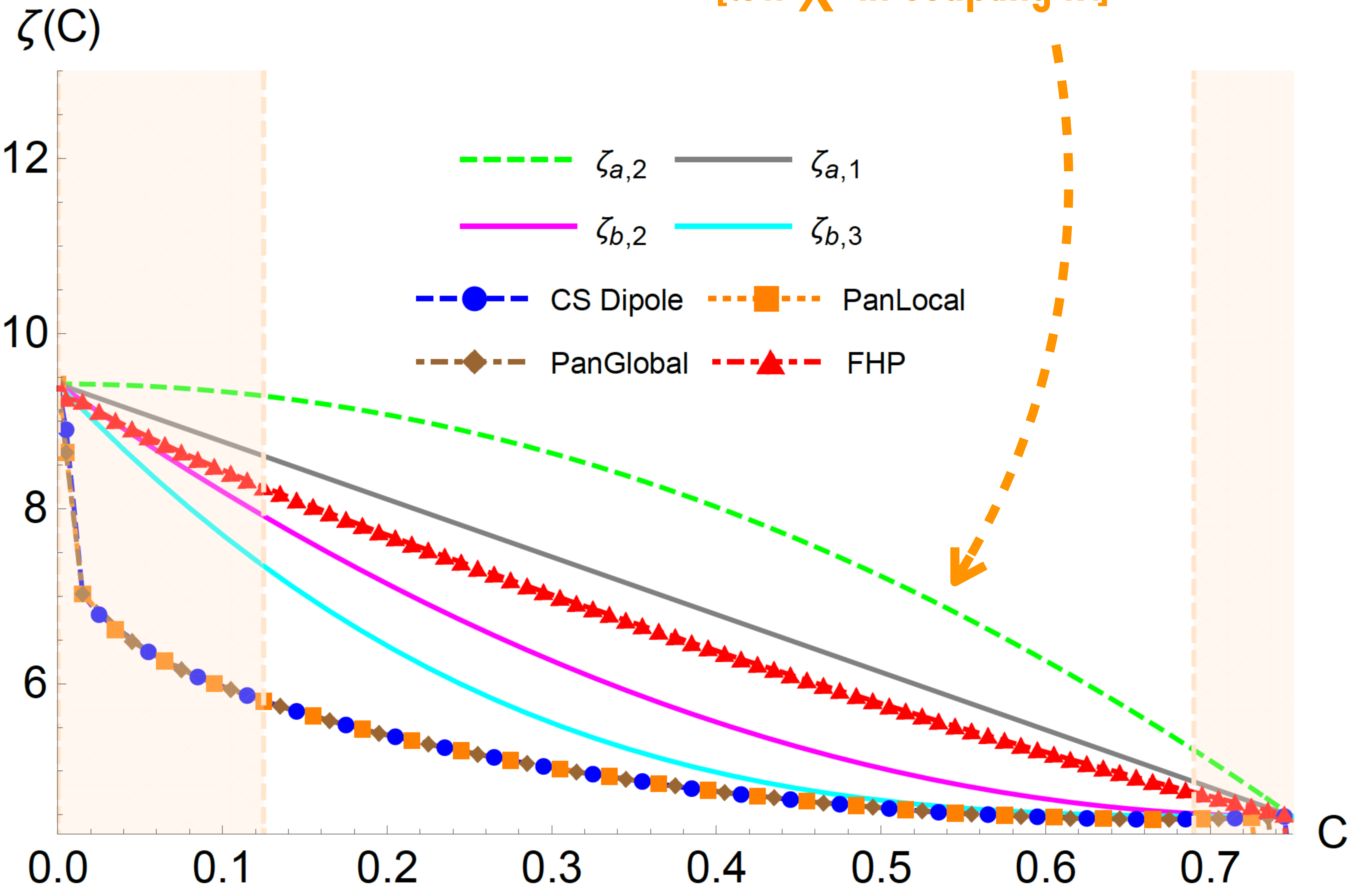
Fixed order study of scaling (first order)

Scheme (a,2) seems to be disfavoured

[low χ^2 in coupling fit]

Fixed (first) order scaling insensitive to assignment of transverse recoil; dependence on the longitudinal d.o.f. of the map.

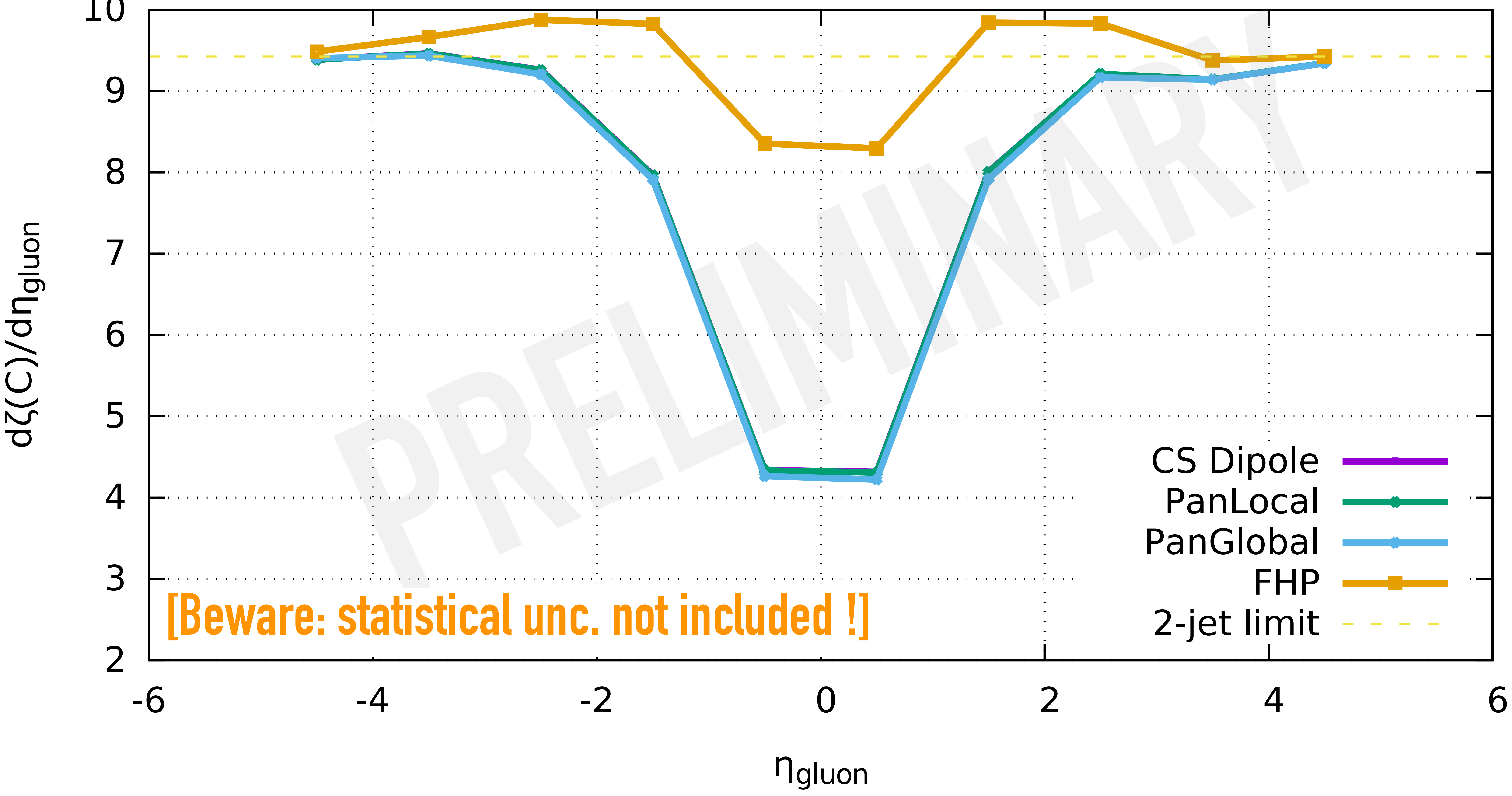
Spread associated with (b) schemes reflects uncertainty found at fixed order in the fit region



Dependence on kinematics of the hard event

Non-perturbative shift features a strong dependence on the kinematics of partons in the event. Additional soft radiation can potentially modify the scaling significantly.

E.g. NP shift vs. (perturbative) gluon rapidity near 2j limit ($C=10^{-5}$)

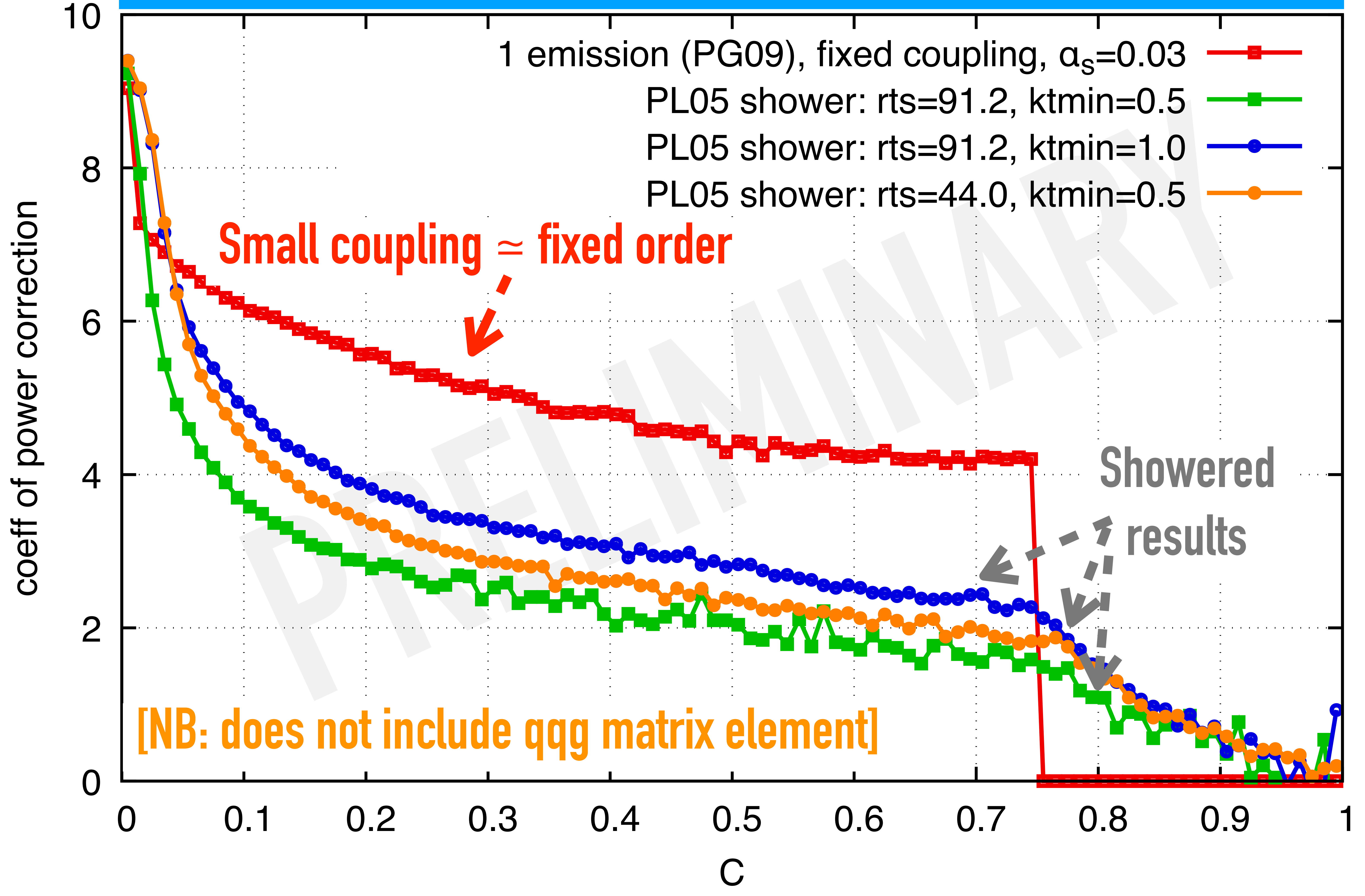


Effects of additional soft radiation

E.g. shower 2-jet event and assume gluer is softer than all emissions. Significant difference from fixed order near the Sudakov shoulder. Moderate dependence on shower kinematic evolution range [further studies necessary!]

Many thanks to Gavin Salam for the plot

E.g. Preliminary study with PanScales showers



PanScales showers formulated in
[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]

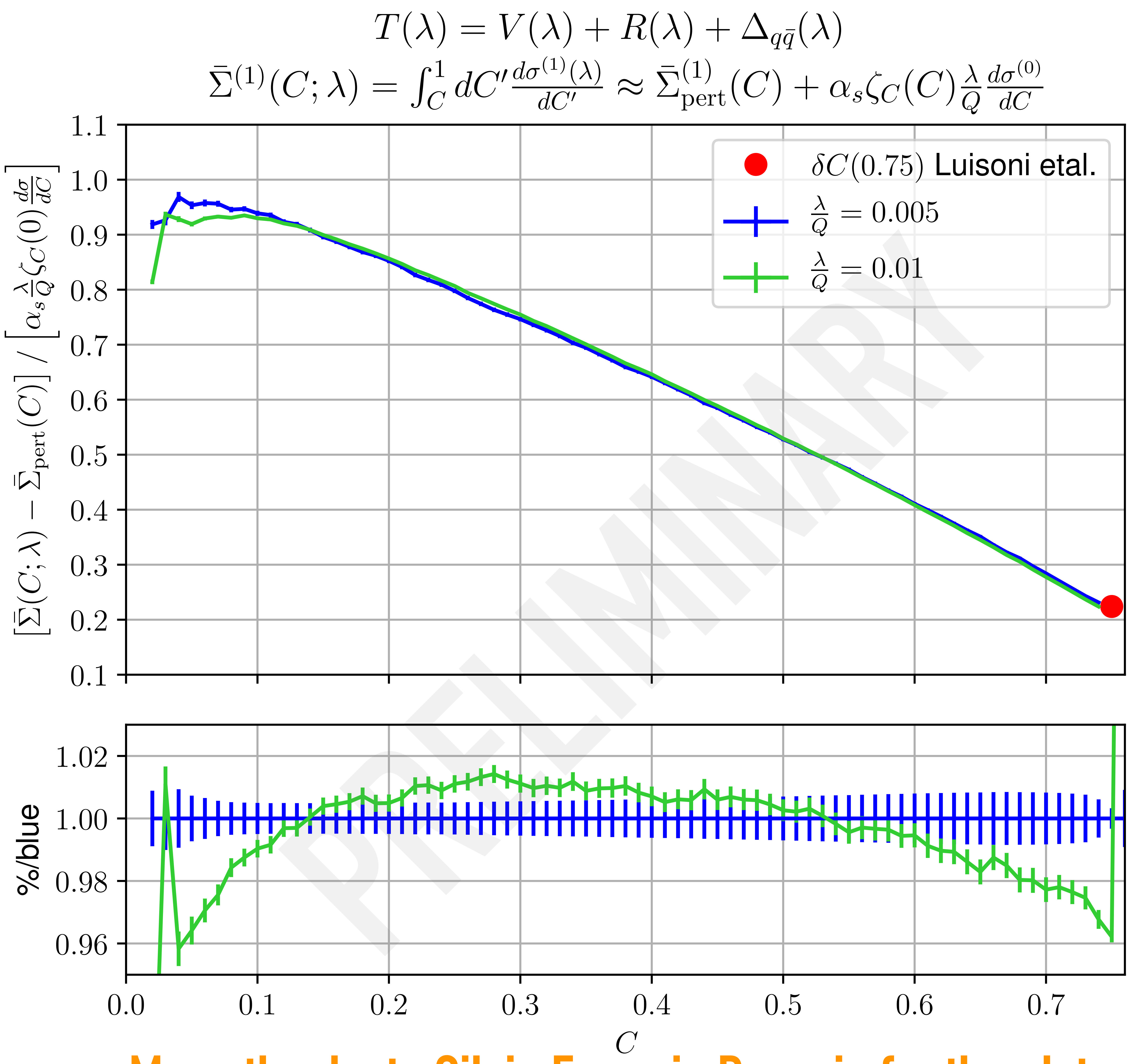
Comparison to large-n_F model: $\gamma^* \to q\bar{q}\gamma$+gluer

$$\bar{\Sigma}^{(1)}(O;\lambda) = \int_O \frac{d\sigma^{(1)}(\lambda)}{dO} = \bar{\Sigma}^{(1)}_{\rm pert}(O) + \alpha_{\rm S}\frac{\lambda}{Q}\zeta_O(O)\frac{d\sigma^{(0)}}{dO} + \mathcal{O}\left(\frac{\lambda^2}{Q^2}\right)$$

Direct calculation of linear power correction in a large-n_F theory + naive non-abelianization

(e.g. used for linear renormalon estimate):

realistic model of scaling across the spectrum



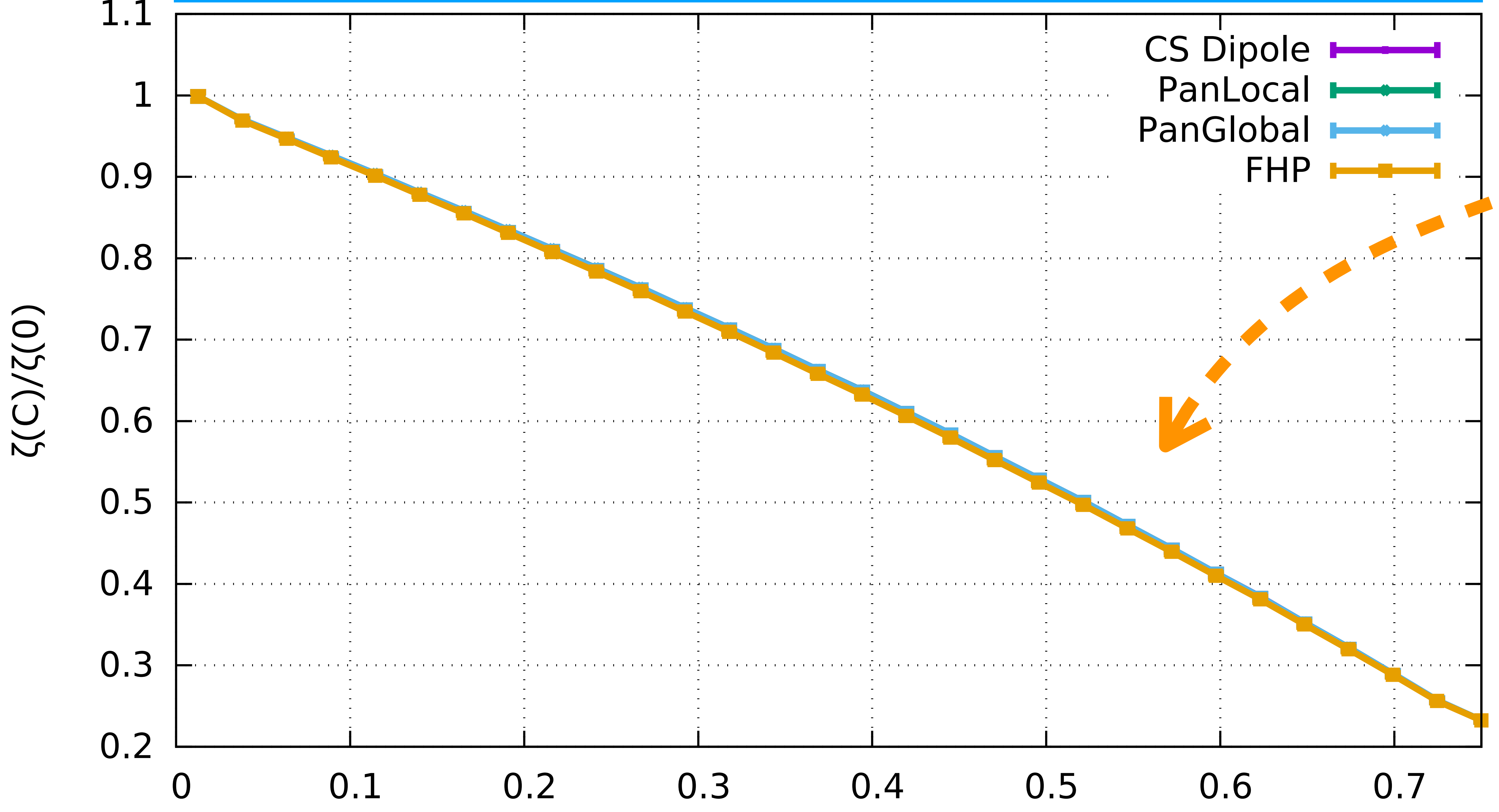
Many thanks to Silvia Ferrario Ravasio for the plot [talk @ KITP 2021]

Related work in
[Ferrario Ravasio, Nason, Oleari ’18; Ferrario Ravasio, Limatola, Nason ’20]

Comparison to large-n_F model: $\gamma^* \to q q \gamma$+gluer

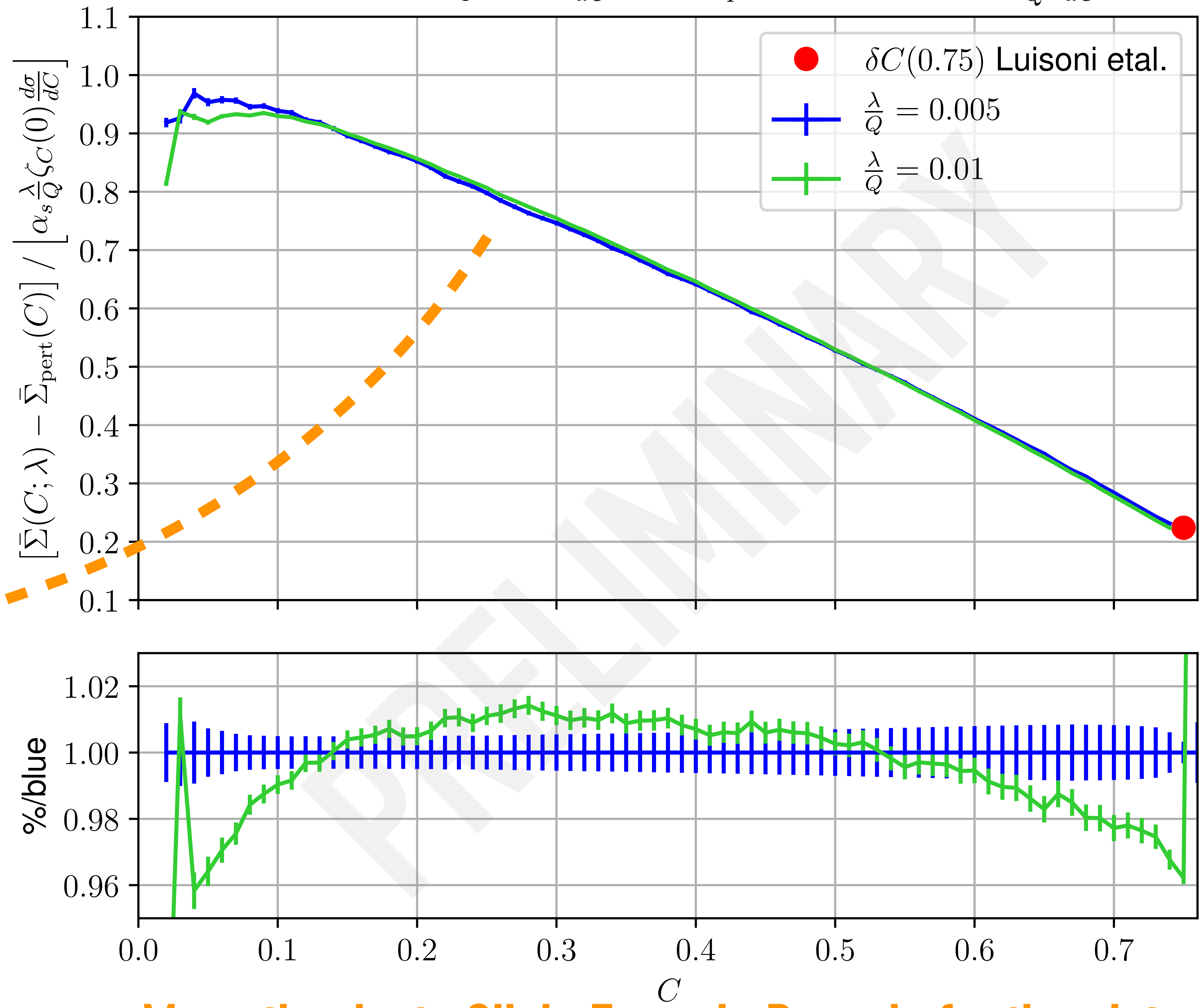
In the QED case we find no dependence on the recoil scheme, result agrees with large-n_F calculation ! (QCD case under study)

Ratio of 3-jet/2-jet NP shift [QED case]



$$T(\lambda) = V(\lambda) + R(\lambda) + \Delta_{q\bar{q}}(\lambda)$$

$$\bar{\Sigma}^{(1)}(C;\lambda) = \int_C^1 dC' \frac{d\sigma^{(1)}(\lambda)}{dC'} \approx \bar{\Sigma}^{(1)}_{\text{pert}}(C) + \alpha_s \zeta_C(C) \frac{\lambda}{Q} \frac{d\sigma^{(0)}}{dC}$$



Many thanks to Silvia Ferrario Ravasio for the plot [talk @ KITP 2021]

Summary

- Presence of Sudakov shoulder in the C parameter spectrum allows for the calculation of non-perturbative shift in the symmetric 3-jet configuration [~2 times smaller than in the 2-jet case]

- Extractions of the strong coupling from lepton-collider event shapes are potentially affected by non-perturbative corrections at the 3-4% level

- Future calculations in the large-n_F model can shed more light on the dependence of the leading (~1/Q) NP shift across the spectrum

- Important to figure out the role of all-order effects to carry out precision phenomenology

Backup material

Strong coupling fit: datasets

Exp.	Q (GeV)	Fit range	N. bins	Ref.
ALEPH	91.2	$0.27 < C < 0.69$	22	[49]
ALEPH	133.0	$0.20 < C < 0.675$	6	[49]
ALEPH	161.0	$0.16 < C < 0.675$	7	[49]
ALEPH	172.0	$0.16 < C < 0.675$	7	[49]
ALEPH	183.0	$0.16 < C < 0.675$	7	[49]
ALEPH	189.0	$0.16 < C < 0.675$	7	[49]
ALEPH	200.0	$0.125 < C < 0.675$	8	[49]
ALEPH	206.0	$0.125 < C < 0.675$	8	[49]
JADE	44.0	$0.61 < C < 0.68$	2	[50]

Strong coupling fit: correlation model

Assume uncorrelated EXP systematics instead of minimal overlap

Model	$\alpha_s(M_Z^2)$	$\alpha_0(\mu_I^2)$	$\chi^2/\text{d.o.f.}$
ζ_0	$0.1122 \pm 0.0007^{+0.0024}_{-0.0014}$	$0.52 \pm 0.01^{+0.07}_{-0.04}$	0.813
$\zeta_{a,1} \equiv \zeta_{b,1}$	$0.1142 \pm 0.0006^{+0.0026}_{-0.0015}$	$0.52 \pm 0.01^{+0.06}_{-0.04}$	0.796
$\zeta_{a,2}$	$0.1121 \pm 0.0007^{+0.0024}_{-0.0015}$	$0.52 \pm 0.01^{+0.07}_{-0.04}$	0.787
$\zeta_{a,3}$	$0.1100 \pm 0.0008^{+0.0022}_{-0.0014}$	$0.54 \pm 0.01^{+0.07}_{-0.05}$	0.845
$\zeta_{b,2}$	$0.1162 \pm 0.0005^{+0.0028}_{-0.0017}$	$0.51 \pm 0.01^{+0.06}_{-0.04}$	0.822
$\zeta_{b,3}$	$0.1167 \pm 0.0005^{+0.0028}_{-0.0018}$	$0.53 \pm 0.01^{+0.06}_{-0.04}$	0.870
ζ_c	$0.1156 \pm 0.0006^{+0.0027}_{-0.0016}$	$0.48 \pm 0.01^{+0.05}_{-0.03}$	0.807