QCD resummation in the large-β_0 limit

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Outline

We seek closed expressions for QCD perturbative series

Problem 1

QCD series are in general divergent

Problem 2

We don't know their infinitely many terms!

1. Asymptotic series and the Borel transform
2. The large-β_0 limit of QCD

In these conditions

Develop the resummation formalism for...

3. Series without cusp-anomalous dimension
4. Series with cusp-anomalous dimension

Apply it

5. Phenomenology I: short-distance mass schemes
6. Phenomenology II: jets from massless and massive quarks

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Divergent series

Perturbative series in QFT can be

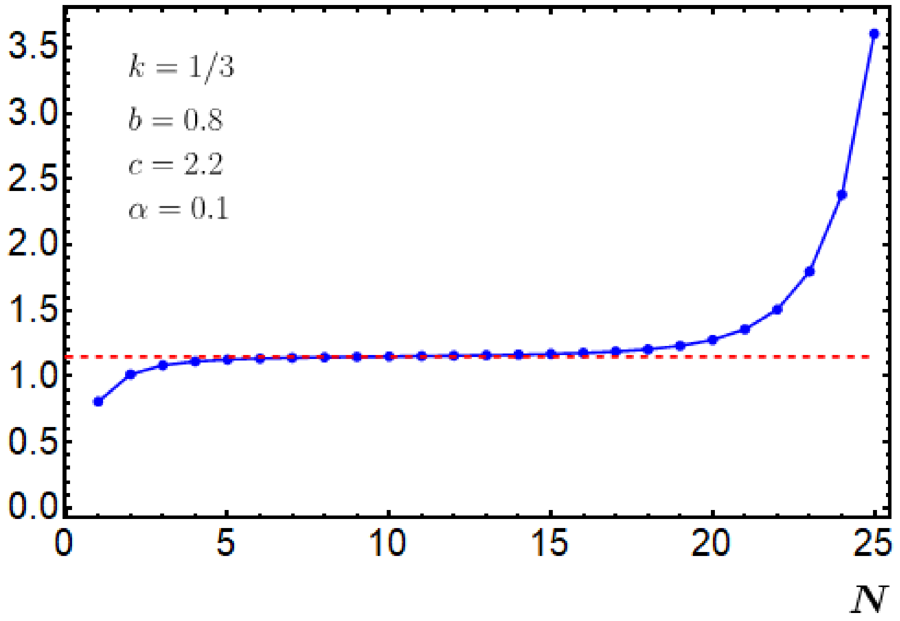
Convergent → Finite convergence radius

Asymptotic (divergent!) → Zero convergence radius

Example: $\sum_{n=0}^{\infty} a_n \alpha^n \,, \quad a_n \sim k\, b^n \Gamma(n+1+c)$

Seems to converge at low n but diverges after $n \sim 20$

$S_N \equiv \sum_{n=0}^{N} a_n \alpha^n$

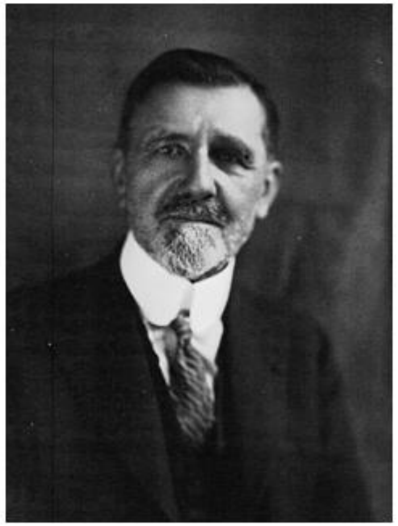


Borel transform

We seek closed expressions for perturbative series (resummation).

Is there any way to resum asymptotic series?

Borel resummation



MÉMOIRE

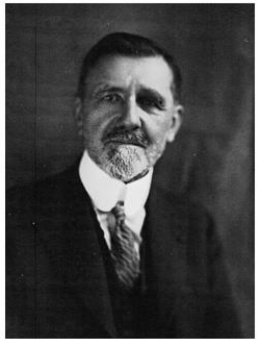
SUR

LES SÉRIES DIVERGENTES,

PAR M. ÉMILE BOREL.

1899

Borel transform



Borel transform improves convergence: $\alpha_s^{n+1} \mapsto \frac{u^n}{n!}$

$$A(\alpha_s) = \sum_{n=0}^{\infty} a_n \alpha_s^n \quad \longmapsto \quad \boxed{\mathcal{B}[A(\alpha_s)](u) = a_0 + \sum_{n=0}^{\infty} a_n \frac{u^n}{n!}}$$

Sum up (Borel sum)

Invert

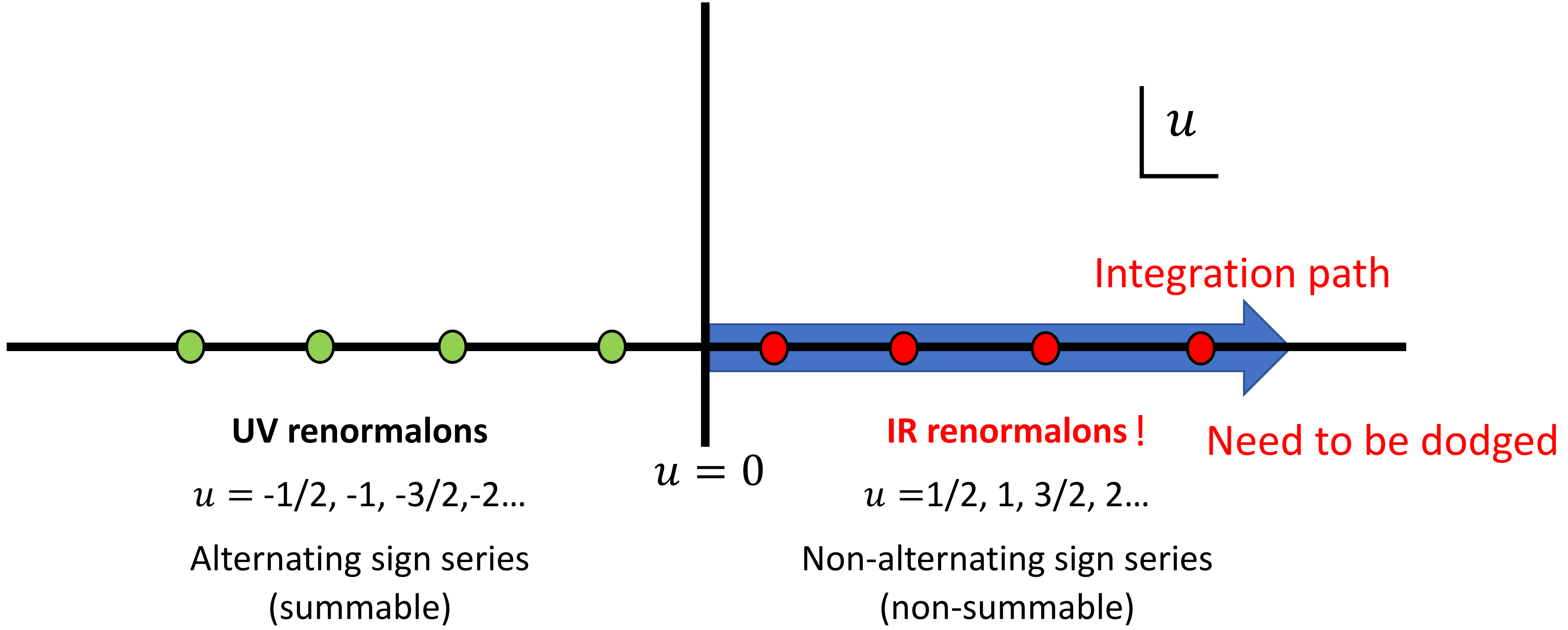
The trick!

$$\boxed{A(\alpha_s)} = a_0 + \int_0^{\infty} \mathrm{d}u e^{-\alpha_s/u} \mathcal{B}[A(\alpha_s)](u)$$

Resummed

- Convergent series $\longrightarrow$ Finite integral
- Asymptotic series $\longrightarrow \mathcal{B}[A(\alpha_s)](u)$ has poles

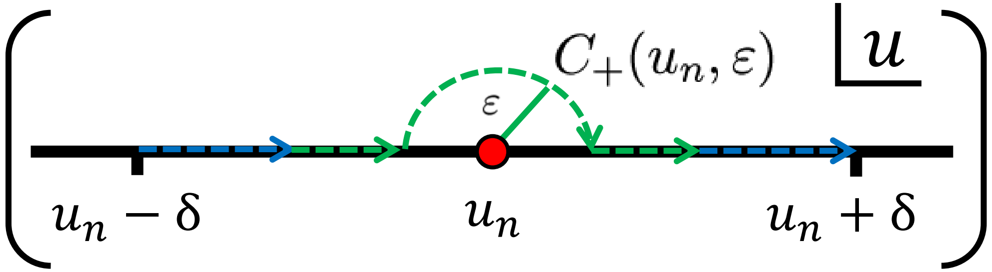
For asymptotic series, the poles of $\mathcal{B}[A(\alpha_s)](u)$ in the complex u-plane are known as *renormalons*



Principal value prescription (P.V.)

Dodge poles by infinitesimally deforming the integration contour

$$\lim_{\varepsilon\mapsto 0}\Big(\;\cdots\;\Big)\equiv \mathrm{P.V.}_{+}$$



$$\mathrm{P.V.}_{\pm}\left\{\int_{x_n-\delta}^{x_n+\delta}\mathrm{d}x\frac{1}{(x-x_n)^k}\right\}=\begin{cases}\boxed{\mp\mathrm{i}\pi, \qquad k=1}\\ \dfrac{-2}{(k-1)\delta^{k-1}}, \quad k \text{ even}\\ 0, \qquad k \text{ odd}, \quad k>1\end{cases}$$

Simple poles add imaginary part

Ambiguity

$$\boxed{\mathrm{P.V.}\{A\}\equiv\frac{1}{2}[\mathrm{P.V.}_+\{\mathcal{B}[A]\}+\mathrm{P.V.}_-\{\mathcal{B}[A]\}]}$$

Estimated resummed value of the series

$$\boxed{\delta_A\equiv\frac{1}{2\pi\mathrm{i}}[\mathrm{P.V.}_+\{\mathcal{B}[A]\}-\mathrm{P.V.}_-\{\mathcal{B}[A]\}]}$$

Estimated resummation error (~ minimal term). Given by sum of residues of IR renormalons.

*This definition ensures $\mathrm{P.V.}\{f\}\in\mathbb{R}$ if $f\in\mathbb{R}$ and $\mathrm{P.V.}\{f\}$ zero for odd f along simmetric interval.

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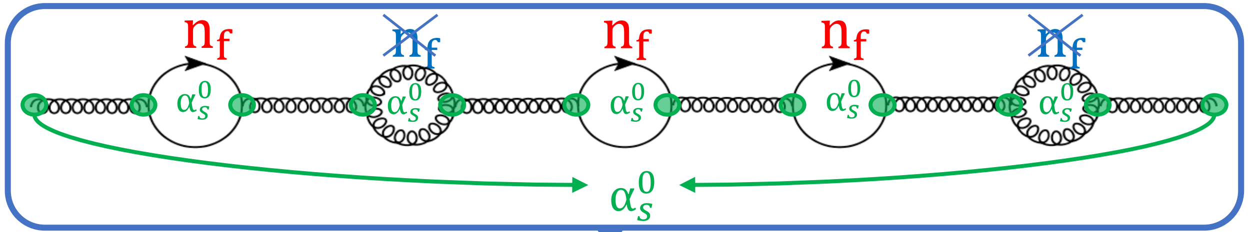
The large-β_0 limit

We start with a series in the bare formalism



$$A_0 = 1 + \sum_{l=1}^{\infty} \left(\frac{g_0^2}{(4\pi)^2} \right)^l \boxed{\sum_{n=0}^{l-1} a_{l,n} n_f^n}$$

Large-β_0 counting: keep track of powers of n_f by $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$

$$A_0 = 1 + \sum_{l=1}^{\infty} \left(\frac{g_0^2}{(4\pi)^2} \right)^l \sum_{n=0}^{l-1} b_{l,n} \beta_0^n \qquad b_{l,i} \equiv \sum_{n=i}^{l-1} \binom{n}{i} (-1)^i \frac{a_{l,n}}{\left(\frac{4}{3} T_F\right)^n} \left(\frac{11}{3} C_A \right)^{n-i}$$

Large-β_0 expansion: expand assuming $O(\alpha_s \beta_0) \sim 1$

Large-β_0 expansion: expand assuming $O(\alpha_s\beta_0) \sim 1$

The renormalization of α_s is simple when β_0 is large since from $\beta_n \sim O(\beta_0^n)$ for $n > 0$ one has

$$Z_\alpha = \frac{1}{1+\alpha_s\beta_0/(4\pi\epsilon)} \longrightarrow \frac{g_0^2}{4\pi} = \frac{\mu^{2\epsilon}}{1+\alpha_s\beta_0/(4\pi\epsilon)}\alpha_s$$

Performing this substitution and expanding in $1/\beta_0$

$$\beta \equiv \frac{\alpha_s\beta_0}{4\pi}$$

We take this as our new modified coupling constant

$$A_0 = 1 + \frac{1}{\beta_0}\sum_{i=1}^{\infty} c_i \left(\frac{\alpha_s\beta_0}{4\pi}\right)^i + O\left(\frac{1}{\beta_0^2}\right)$$

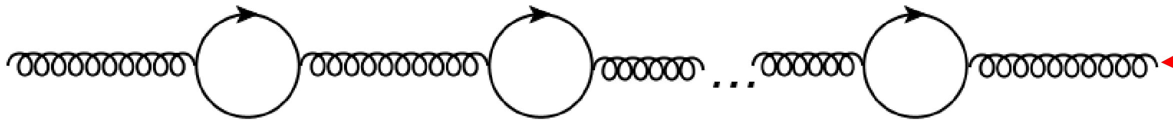
$$c_i = (-1)^i\epsilon^{-i}\sum_{l=1}^{i}\frac{(-1)^l(l)_{i-l}}{(i-l)!}\epsilon^l\,\tilde{\mu}^{2l\epsilon}b_{l,l-1}$$

Coefficients of highest n_f power

Only corrections from gluon propagator corrected with quark loops

The large-β_0 limit

$$A_0 = 1 + \frac{1}{\beta_0}\sum_{i=1}^{\infty} c_i \left(\frac{\alpha_s \beta_0}{4\pi}\right)^i + O\left(\frac{1}{\beta_0^2}\right) \qquad c_i = (-1)^i \epsilon^{-i} \sum_{l=1}^{i} \frac{(-1)^l (l)_{i-l}}{(i-l)!} \epsilon^l \, \tilde{\mu}^{2l\epsilon} b_{l,l-1}$$



The effective gluon propagator can be computed in arbitrary gauge with $d = 4 - 2\epsilon$ and naive non-abelianization $n_f \mapsto -\frac{3}{2}\beta_0$:

$$\Delta_n^{\mu\nu ab}(k) \equiv \left[\frac{g_0^2}{(4\pi)^{2-\epsilon}} n_f T_F P_B(\epsilon)\right]^n (-\mathrm{i}\delta^{ab}) \left[\frac{g^{\mu\nu}}{(k^2)^{1+n\epsilon}} - \frac{k^\mu k^\nu}{(k^2)^{2+n\epsilon}}(1 - \xi\delta_{n0})\right]$$

$$P_B(\epsilon) \equiv \frac{(-1)^{-\epsilon} 4(1-\epsilon)}{(2\epsilon - 3)} \frac{\Gamma(\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)}$$

Gluon propagator with momentum shifted by $h \equiv n\epsilon$

In practice the insertion of $\Delta_n^{\mu\nu ab}(k)$ is a (shifted) 1-loop computation which we split as:

$$D_{\mathrm{sh}}(h) = \left[\frac{g_0^2}{(4\pi)^{2-\epsilon}} n_f T_F P_B(\epsilon)\right]^n \left(\frac{g_0}{4\pi}\right)^2 a(h, \epsilon)$$

The function $a(h,\epsilon)$ generates the infinitely many $1/\beta_0$ terms.

The large-β_0 limit

All-in all, the expression for our series in the large-β_0 limit is (defining $\delta A_0 \equiv A_0 - 1$):

$$\beta_0 A_0 = \sum_{i=1}^{\infty} \beta^i (-1)^i \epsilon^{-i} \sum_{l=1}^{i} \frac{(-1)^l (l)_{i-l}}{(i-l)!} \frac{F^\mu(\epsilon, l\epsilon)}{l}$$

$$F^\mu(\epsilon, u) = \frac{(\mu^2 e^{\gamma_E})^u u}{(4\pi)^\epsilon} \left[-\frac{3}{4} \epsilon P_B(\epsilon) \right]^{\frac{u}{\epsilon}-1} a(u-\epsilon, \epsilon) \qquad F^\mu(\epsilon, u) = \left(\frac{\mu^2}{\omega^2}\right)^u F(\epsilon, u)$$

where ω is an external energy-scale (ex. mass or CM energy).

From here we distinguish when $u \mapsto 0$

- Finite series (no renormalization) $\longrightarrow F(\epsilon, u)$ starts at $O(u)$.
- Non-cusp series (1-loop divergences start as $1/\epsilon$) $\longrightarrow F(\epsilon, u)$ starts at $O(1)$
- Cusp series (1-loop divergences start as $1/\epsilon^2$) $\longrightarrow F(\epsilon, u)$ starts at $O(1/u)$

in all cases $F(\epsilon, u)$ starts at $O(1)$ when $\epsilon \mapsto 0$.

We found closed expressions for all three cases!

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Perturbative form

For non-cusp series we have regular $F(\epsilon, u)$:

$$F^{\mu}(\epsilon, u) \equiv \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \epsilon^i u^j F^{\mu}_{i,j}, \qquad F^{\mu}_{i,0} \equiv F_{i,0}$$

Plugging it back and after several boring manipulations…

μ-dependent term

μ-independent terms

$$\beta_0 \delta A_0 = \sum_{i=1}^{\infty} \beta^i \left[\Gamma(i) F^{\mu}_{0,i} - \frac{(-1)^i F_{i,0}}{i} \right] - \sum_{j=1}^{\infty} \frac{1}{\epsilon^j} \sum_{i=0}^{\infty} (-\beta)^{i+j} \frac{F_{i,0}}{i+j}$$

Finite term: efficient way of computing all the coefficients of the renormalized series

UV divergences have been separated for simple removal (we discarded $j > 0$ terms)

*In the large-β_0 limit, multiplicative renormalization reduces to addition:

$$A_0 = Z_A A = 1 + \delta Z_A + \delta A + O(1/\beta_0^2)$$

Perturbative form

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Plugging it back and after several boring manipulations…

$$\beta_0 \delta A_0 = \boxed{\sum_{i=1}^{\infty}\beta^i\left[\Gamma(i)F^{\mu}_{0,i} - \frac{(-1)^i F_{i,0}}{i}\right]} - \sum_{j=1}^{\infty}\boxed{\frac{1}{\epsilon^j}\sum_{i=0}^{\infty}(-\beta)^{i+j}\frac{F_{i,0}}{i+j}}$$

Nice but still perturbative

Each of these terms admits a closed form

*In the large-β_0 limit, multiplicative renormalization reduces to addition:

$$A_0 = Z_A A = 1 + \delta Z_A + \delta A + O(1/\beta_0^2)$$

Closed forms

The magic occurs when one realizes each term in the perturbative sum admits a closed integral form:

Integral tricks   Regularity condtion (backwards)

$$\sum_{i=1}^{\infty}\beta^i\Gamma(i)F^{\mu}_{0,i} = \sum_{i=1}^{\infty}F^{\mu}_{0,i}\int_0^{\infty}\mathrm{d}\tau\,\tau^{i-1}\mathrm{e}^{-\tau/\beta} = \int_0^{\infty}\mathrm{d}\tau\,\mathrm{e}^{-\tau/\beta}\frac{F^{\mu}(0,\tau)-F(0,0)}{\tau},$$

$$-\sum_{i=1}^{\infty}\frac{(-\beta)^i}{i}F_{i,0} = \sum_{i=1}^{\infty}F_{i,0}\int_{-\beta}^{0}\mathrm{d}\tau\,\tau^{i-1} = \int_{-\beta}^{0}\mathrm{d}\tau\,\frac{F(\tau,0)-F(0,0)}{\tau},$$

$$-\sum_{i=0}^{\infty}\frac{(-\beta)^{i+j}}{i+j}F_{i,0} = \sum_{i=0}^{\infty}F_{i,0}\int_{-\beta}^{0}\mathrm{d}\tau\,\tau^{i+j-1} = \int_{-\beta}^{0}\mathrm{d}\tau\,\tau^{j-1}F(\tau,0),$$

Closed forms

The magic occurs when one realizes each term in the perturbative sum admits a closed integral form:

Series = Borel-like, ambiguous integral + Unambiguous integral, finite convergence radius

$$\beta_0 \delta A = \int_0^\infty d\tau\, e^{-\tau/\beta} \frac{F^\mu(0,\tau) - F(0,0)}{\tau} + \int_{-\beta}^0 d\tau \frac{F(\tau,0) - F(0,0)}{\tau}$$

Renormalized series

Do poles and ambiguities depend on μ?!

UV substractions

$$Z_A^{\overline{\mathrm{MS}}} = \sum_{j=1}^\infty \frac{1}{\epsilon^j} \int_{-\beta}^0 d\tau\, \tau^{j-1} F(\tau,0)$$

Renormalization factor

$$\gamma_A(\beta) = \frac{2\beta}{\beta_0} \sum_{i=0}^\infty \beta^i (-1)^i F_{i,0} = \underbrace{\frac{2\beta}{\beta_0} F(-\beta,0)}$$

Anomalous dimension

No UV substractions

Closed, non-integral form ⟶ unambiguous

Do poles and ambiguities depend on μ?!

Removing the μ dependence from the Borel integral

In the large-β_0 limit, the β_{QCD} function acquires a simple form ($\epsilon = 0$):

$$\beta_{\mathrm{QCD}} = \frac{-\alpha_s^2 \beta_0}{2\pi}$$

This can be used to solve the running of $\beta \equiv \frac{\alpha_s \beta_0}{4\pi}$ $\longrightarrow$

$$\beta(\mu) = \frac{\beta_{\mu_0}}{1 + 2\beta_{\mu_0} \log\left(\frac{\mu}{\mu_0}\right)} = \frac{1}{2\log\left(\frac{\mu}{\Lambda_{\mathrm{QCD}}}\right)}$$

where the Landau pole is at the μ-independent position $\Lambda_{\mathrm{QCD}} \equiv \mu e^{-\frac{1}{2\beta(\mu)}}$

$$\int_0^\infty \mathrm{d}\tau \, \frac{\mathrm{e}^{-\tau/\beta}\left[\left(\frac{\mu^2}{\omega^2}\right)^\tau F(0,\tau) - F(0,0)\right]}{\tau} = \int_0^\infty \mathrm{d}\tau \, \mathrm{e}^{-\tau/\beta}\left(\frac{\mu^2}{\omega^2}\right)^\tau \frac{F(0,\tau) - F(0,0)}{\tau} + F(0,0) \underbrace{\int_0^\infty \mathrm{d}\tau \, \frac{\mathrm{e}^{-\tau/\beta}}{\tau}\left[\left(\frac{\mu^2}{\omega^2}\right)^\tau - 1\right]}_{\text{Taylor-expand and integrate}}$$

$$\mathrm{e}^{-\tau/\beta}\left(\frac{\mu}{\omega}\right)^{2\tau} = \left(\frac{\Lambda_{\mathrm{QCD}}}{\omega}\right)^{2\tau}$$

μ-independent!

Alternative closed expression

$$\beta_0\delta A \;=\; F(0,0)\log\left(\frac{\beta_\omega}{\beta}\right)+\int_0^\infty \mathrm{d}\tau \left(\frac{\Lambda_{\mathrm{QCD}}}{\omega}\right)^{2\tau}\frac{F(0,\tau)-F(0,0)}{\tau}+\int_{-\beta}^{0}\mathrm{d}\tau\,\frac{F(\tau,0)-F(0,0)}{\tau}$$

Multiplies the residues of $F(0,\tau)$

Renormalized series, alternative expression

The entire μ-dependence of the series is contained in the unambiguous terms through $\beta = \beta(\mu)$.

The IR renormalons are the poles of $F(0,\tau)$ and neither their position nor their residues deppend on the unphysical scale μ, but the latter does deppend on ω.

Each pole's ambiguity is enhanced by $\left(\Lambda_{QCD}/\omega\right)^{2\tau}$, with τ being the pole's position in the positive real axis.

The explicit presence of Λ_{QCD} indicates δ_A estimates the size of non-perturbative (power) corrections.

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Perturbative expression

We present an extension of the previous formalism to series with cusp-anomalous dimension

For such series $F(\epsilon,u)$ starts at $O(1/u)$ so we define $G(\epsilon,u)\equiv uF(\epsilon,u)$

$$\beta_0 A_0=\sum_{i=1}^{\infty}\beta^i(-1)^i\epsilon^{-i}\sum_{l=1}^{i}\frac{(-1)^l(l)_{i-l}}{(i-l)!}\boxed{\frac{F^\mu(\epsilon,l\epsilon)}{l}}$$

$$\beta_0 \delta A_0=\sum_{i=1}^{\infty}\beta^i(-1)^i\epsilon^{-i}\sum_{l=1}^{i}\frac{(-1)^l(l)_{i-l}}{(i-l)!}\boxed{\frac{G^\mu(\epsilon,l\epsilon)}{l^2\epsilon}}$$

$$G^\mu(\epsilon,u)\equiv\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}G^\mu_{i,j}\epsilon^i u^j$$

$G(\epsilon,u)$ is regular

$$G^\mu_{i,0}\equiv G_{i,0}$$

The presence of an extra l^2 complicates manipulations, but we do not despair and find…

Perturbative coefficients

This term provides the anomalous dimension

$$\beta_0\delta A_0 = \sum_{i=1}^{\infty}(-\beta)^i\left[G^\mu_{0,i+1}(-1)^i\Gamma(i)-\frac{H_i G_{i+1,0}}{i}-\frac{G^\mu_{i,1}}{i}\right]-\frac{1}{\epsilon}\sum_{i=1}^{\infty}\frac{(-\beta)^i}{i}\left(H_i G_{i,0}+G^\mu_{i-1,1}\right)$$
$$-\sum_{j=2}^{\infty}\frac{1}{\epsilon^j}\sum_{i=0}^{\infty}\left[(-\beta)^{i+j-1}\frac{H_{i+j-1}G_{i,0}}{i+j-1}+(-\beta)^{i+j}\frac{G^\mu_{i,1}}{i+j}\right]$$

New nuisance! Harmonic numbers

Again each term admits a closed integral form but it is worth saying this time is much harder to find due to H_i…

$$(-\beta)^i H_i = \int_{-\beta}^{0} \mathrm{d}\tau \frac{\tau^i - (-\beta)^i}{\beta + \tau} \qquad \frac{(-\beta)^i}{i} H_i = \int_{-\beta}^{0} \mathrm{d}\tau \tau^{i-1} \log\left(1 + \frac{\tau}{\beta}\right)$$

Spliting the renormalization equation in a cusp and non-cusp part as

$$\delta A_0 = \delta A + \delta Z_A^{\mathrm{nc}} + \log\left(\frac{\mu^2}{\omega^2}\right) \delta Z_A^{\mathrm{cusp}}$$

$$\begin{aligned}\delta Z_A^{\mathrm{nc}} &= \frac{1}{\epsilon} \int_{-\beta}^{0} \mathrm{d}\tau \left[\left.\frac{\mathrm{d}G(\tau,s)}{\mathrm{d}s}\right|_{s=0} - \log\left(1 + \frac{\tau}{\beta}\right) \frac{G(\tau,0) - G(0,0)}{\tau} \right] \\ &+ \sum_{j=2}^{\infty} \frac{1}{\epsilon^j} \int_{-\beta}^{0} \mathrm{d}\tau \tau^{j-1} \left[\left.\frac{\mathrm{d}G(\tau,s)}{\mathrm{d}s}\right|_{s=0} - G(\tau,0) \log\left(1 + \frac{\tau}{\beta}\right) \right]\end{aligned}$$

Renormalization factor (non-cusp part)

$$\delta Z_A^{\mathrm{cusp}} = \sum_{j=1}^{\infty} \frac{1}{\epsilon^j} \int_{-\beta}^{0} \mathrm{d}\tau \tau^{j-1} G(\tau,0)$$

Renormalization factor (cusp part)

Non-cusp Z factor term needs UV substraction

Closed forms

The renormalized cusp-series also has an apparently μ-dependent ambiguous integral that turns out to be μ-independent

Series = Ambiguous + Finite convergence radius

Renormalized cusp series

$$\begin{aligned}\beta_0\delta A &= \log\left(\frac{\mu^2}{\omega^2}\right)G_{0,0}+\left[\frac{G_{0,0}}{\beta_\omega}-G_{0,1}\right]\log\left(\frac{\beta}{\beta_\omega}\right)\\ &+\int_0^\infty \mathrm{d}\tau\left(\frac{\Lambda_{\mathrm{QCD}}}{\omega}\right)^{2\tau}\left[\frac{G(0,\tau)-G(0,0)}{\tau^2}-\frac{1}{\tau}\frac{\mathrm{d}G(0,s)}{\mathrm{d}s}\bigg|_{s=0}\right]\\ &+\int_{-\beta}^0 \mathrm{d}\tau\left\{\frac{1}{\tau}\frac{\mathrm{d}[G(\tau,s)-G(0,s)]}{\mathrm{d}s}\bigg|_{s=0}+\frac{G(\tau,0)-G(0,0)}{\tau}\log\left(\frac{\mu^2}{\omega^2}\right)\right.\\ &\left.-\log\left(1+\frac{\tau}{\beta}\right)\left[\frac{G(\tau,0)-G(0,0)}{\tau^2}-\frac{1}{\tau}\frac{\mathrm{d}G(s,0)}{\mathrm{d}s}\bigg|_{s=0}\right]\right\}\end{aligned}$$

UV substractions: some terms need two of them

And finally we have closed forms for the anomalous dimension

$$\mu\frac{\mathrm{d}A}{\mathrm{d}\mu}=\gamma_A+\log\left(\frac{\mu^2}{\omega^2}\right)\Gamma_A$$

Cusp and non-cusp anomalous dimensions

$$\begin{aligned}\gamma_A(\beta) &= \frac{2}{\beta_0}\int_{-\beta}^0 \mathrm{d}\tau\frac{G(\tau,0)-G(-\beta,0)}{\beta+\tau}+\frac{2\beta}{\beta_0}\frac{\mathrm{d}G(-\beta,s)}{\mathrm{d}s}\bigg|_{s=0}\\ \Gamma_A(\beta) &= \frac{2\beta}{\beta_0}G(-\beta,0)\end{aligned}$$

Non-cusp needs UV substraction

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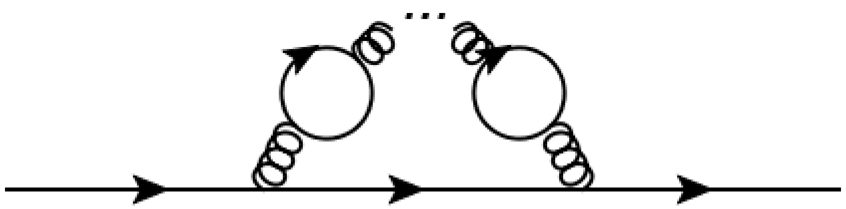
Massive quark self-energy



We compute the massive quark self-energy with the effective gluon propagator.

$$F_{Z_m^{\mathrm{OS}}}(\epsilon,u) = 2C_F e^{\gamma_E u}\frac{(u-1)(3-2\epsilon)\Gamma(1+u)\Gamma(1-2u)}{\Gamma(3-u-\epsilon)}\left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

Where the energy scale is given by $\omega = \bar{m}(\mu)$

This one function generates the large-β_0 series for the mass renormalization factor in the on-shell scheme, which allows us to comptue…

- Relation between pole and $\overline{\mathrm{MS}}$ masses $\longrightarrow$ $\overline{\mathrm{MS}}$ mass anomalous dimension and running
- MSR mass and its R-anomalous dimension

… all of these perturbatively and in closed form (PV value and ambiguities).

Pole- $\overline{\text{MS}}$ mass relation

$$\delta\bar{m} = m_p - \bar{m} = -\delta Z_m^{\rm OS}|_{\rm finite} = \frac{\bar{m}}{\beta_0}\sum_{i=1}^{\infty}\beta^i\sum_{j=0}^{i} a_{i,j}^{\overline{\rm MS}}\log^j\left(\frac{\mu}{\bar{m}}\right)$$

$$= -\frac{\bar{m}}{\beta_0}\left\{F_{0,0}^{Z_m^{\rm OS}}\log\left(\frac{\beta_{\bar{m}}}{\beta}\right) + \int_0^\infty d\tau\left(\frac{\Lambda_{\rm QCD}}{\bar{m}}\right)^{2\tau}\frac{F_{Z_m^{\rm OS}}(0,\tau) - F_{0,0}^{Z_m^{\rm OS}}}{\tau} + \int_{-\beta}^{0} d\tau\,\frac{F_{Z_m^{\rm OS}}(\tau,0) - F_{0,0}^{Z_m^{\rm OS}}}{\tau}\right\}$$

	Poles	Order	Crossed
$\boldsymbol{F_{Z_m^{\rm OS}}(\epsilon,0)}$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$\boldsymbol{F_{Z_m^{\rm OS}}(0,u)}$	$(2n+1)/2, \quad n=0,1,2\ldots$	simple	Yes
	$-n, \quad n=1,2,3\ldots$	simple	No
	2	simple	Yes

Indeed we don't cross the poles of $F(\epsilon, 0)$

There are poles at all positive half integers.
The most severe renormalon lays at $u = 1/2$

The ambiguity δ_{m_p} of the pole mass is the sum of the residues and with extra effort it can be also resummed

$$\delta_{m_p} = -\frac{C_F}{2\beta_0}e^{5/6}\Lambda_{\rm QCD}\left[x^3 + (2-x^2)\sqrt{4+x^2}\right] \overset{u=1/2}{\underset{\downarrow}{=}} -\frac{2C_F}{\beta_0}e^{5/6}\Lambda_{\rm QCD} + O(\Lambda_{\rm QCD}^3) \qquad x \equiv \frac{e^{5/6}\Lambda_{\rm QCD}}{\bar{m}}$$

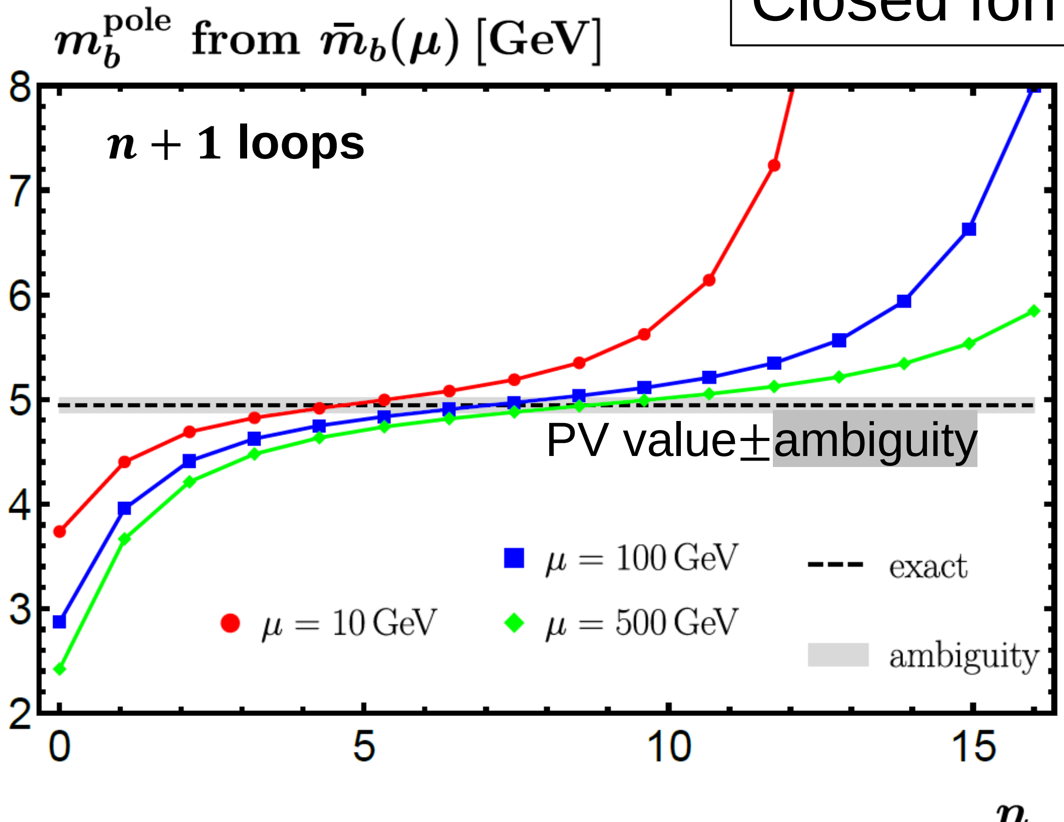
The leading power ambiguity does not deppend on $\bar{m}$

Pole- $\overline{\rm MS}$ mass relation

$$\delta\bar{m} = m_p - \bar{m} = -\delta Z_m^{\rm OS}|_{\rm finite} = \frac{\bar{m}}{\beta_0}\sum_{i=1}^{\infty}\beta^i\sum_{j=0}^{i}a_{i,j}^{\overline{\rm MS}}\log^j\left(\frac{\mu}{\bar{m}}\right)$$

$$= -\frac{\bar{m}}{\beta_0}\left\{F_{0,0}^{Z_m^{\rm OS}}\log\left(\frac{\beta_{\bar{m}}}{\beta}\right) + \int_0^{\infty}{\rm d}\tau\left(\frac{\Lambda_{\rm QCD}}{\bar{m}}\right)^{2\tau}\frac{F_{Z_m^{\rm OS}}(0,\tau) - F_{0,0}^{Z_m^{\rm OS}}}{\tau} + \int_{-\beta}^{0}{\rm d}\tau\,\frac{F_{Z_m^{\rm OS}}(\tau,0) - F_{0,0}^{Z_m^{\rm OS}}}{\tau}\right\}$$

Closed form results vs partial sum of perturbative series



As series, m_p is clearly asymptotic

The PV prescription value is μ-independent and agrees (within ambiguity) with the “convergent” value of the series.

The fixed order expresion is μ-dependent and μ plays a role in the asymptotic behavior: for lower values the series “converges” faster but the divergent behavior is more pronounced

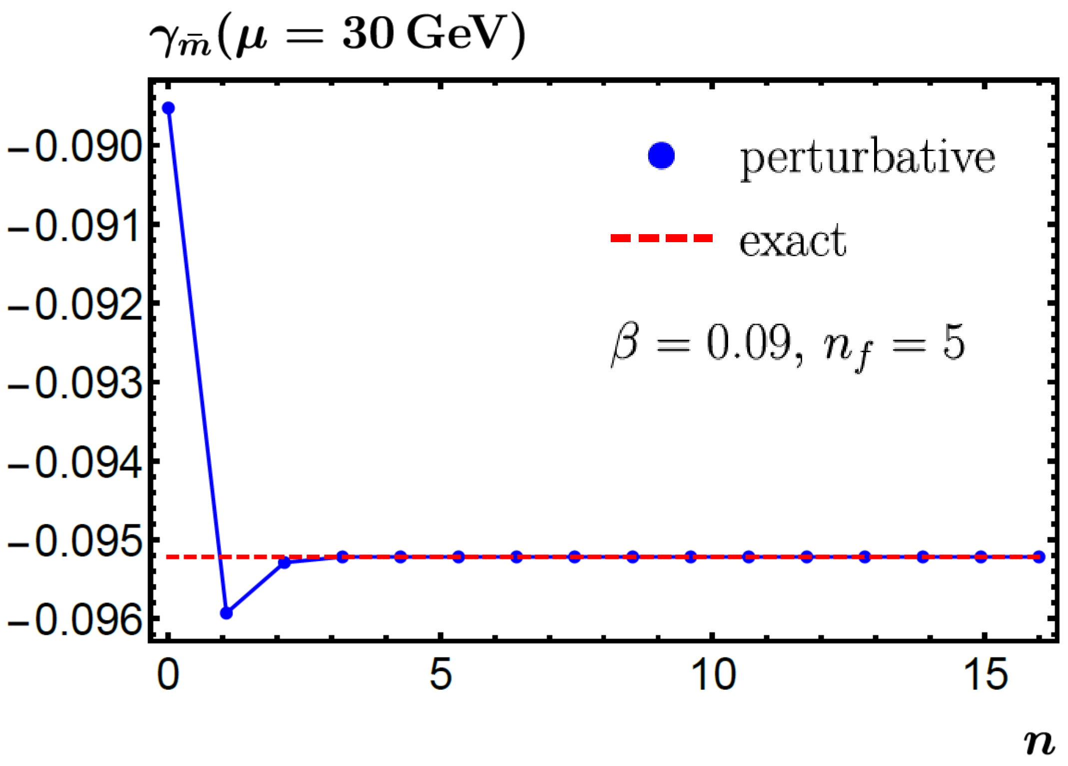
$\overline{\text{MS}}$ anomalous dimension

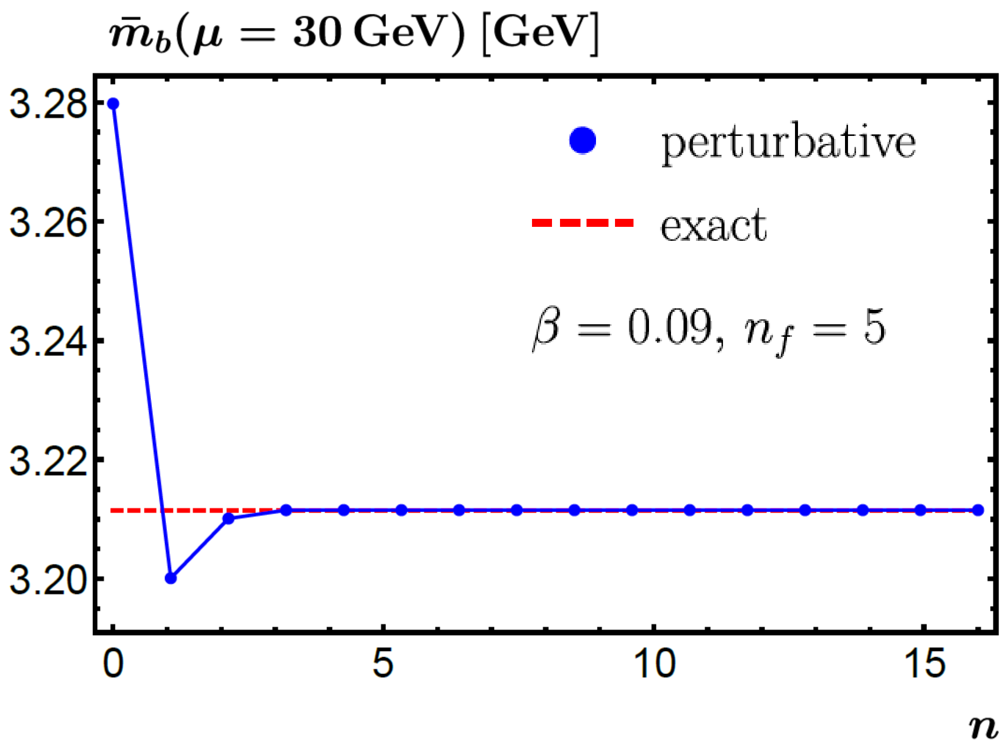
The $\overline{\text{MS}}$ anomalous dimension is unambiguous (the μ derivative cancels the Borel integral)

(agrees with the derivation of Palanques-Mestre and Pascual, Grozin)

$$\gamma_{\bar{m}}(\beta) = -\frac{2\beta}{\beta_0} F_{Z_m^{\text{OS}}}(-\beta, 0)$$
$$= -\frac{C_F \beta(3+2\beta)\Gamma(4-2\beta)}{3\beta_0(2+\beta)\Gamma(1-\beta)\Gamma(2+\beta)^3}$$

Convergence radius of $\beta = 2.5$





MSR mass

The MSR mass is obtained from the pole-$\overline{\rm MS}$ relation

$$n_f = n_l + \cancel{n_h} \quad (n_h \to 0)$$

$$\delta m^{\rm MSR} = m_p - m^{\rm MSR} = \frac{R}{\beta_0}\sum_{i=1}^{\infty} a_{i,0}^{\overline{\rm MS}}\beta_R^i$$

$$= -\frac{R}{\beta_0}\left\{\int_0^\infty {\rm d}\tau\left(\frac{\Lambda_{\rm QCD}}{R}\right)^{2\tau}\frac{F_{Z_m^{\rm OS}}(0,\tau)-F_{Z_m^{\rm OS}}(0,0)}{\tau} + \int_{-\beta_R}^0 {\rm d}\tau\,\frac{F_{Z_m^{\rm OS}}(\tau,0)-F_{Z_m^{\rm OS}}(0,0)}{\tau}\right\}$$

and presents the same leading renormalon at 1/2

MSR anomalous dimension

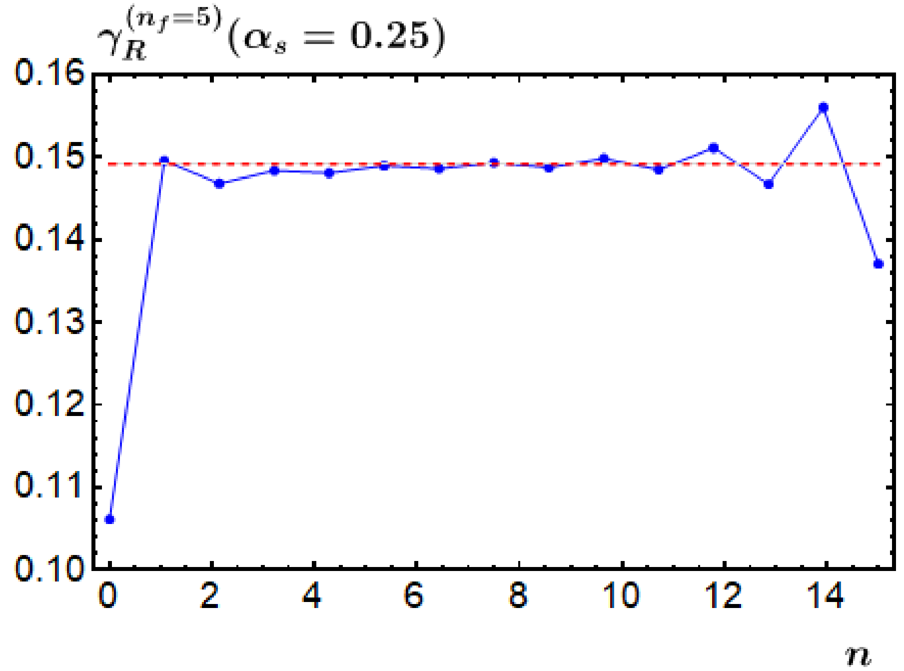
The R derivative does not cancel the Borel integral, however the 1/2 renormalon does cancel (higher order ones stay) $\longrightarrow$ R-evolution is (a bit) ambiguous

$$\beta_0\gamma_R(\beta_R) = -\frac{\rm d}{{\rm d}R}[\beta_0 m^{\rm MSR}(R)]$$

$$= -\int_0^\infty {\rm d}\tau\left(\frac{\Lambda_{\rm QCD}}{R}\right)^{2\tau}\boxed{(1-2\tau)}\frac{F_{Z_m^{\rm OS}}(0,\tau)-F_{Z_m^{\rm OS}}(0,0)}{\tau}$$

$$-\int_{-\beta_R}^0 {\rm d}\tau\,\frac{F_{Z_m^{\rm OS}}(\tau,0)-F_{Z_m^{\rm OS}}(0,0)}{\tau}$$

$$-2\beta_R[F_{Z_m^{\rm OS}}(-\beta_R,0)-F_{Z_m^{\rm OS}}(0,0)]$$



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The MSR mass is obtained from the pole-$\overline{\rm MS}$ relation

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$$= -\frac{R}{\beta_0}\left\{\int_0^\infty {\rm d}\tau \left(\frac{\Lambda_{\rm QCD}}{R}\right)^{2\tau} \frac{F_{Z_m^{\rm OS}}(0,\tau) - F_{Z_m^{\rm OS}}(0,0)}{\tau} + \int_{-\beta_R}^{0} {\rm d}\tau\, \frac{F_{Z_m^{\rm OS}}(\tau,0) - F_{Z_m^{\rm OS}}(0,0)}{\tau}\right\}$$

and presents the same leading renormalon at 1/2

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The R derivative does not cancel the Borel integral, however the 1/2 renormalon does cancel (higher order ones stay) $\longrightarrow$ R-evolution is (a bit) ambiguous

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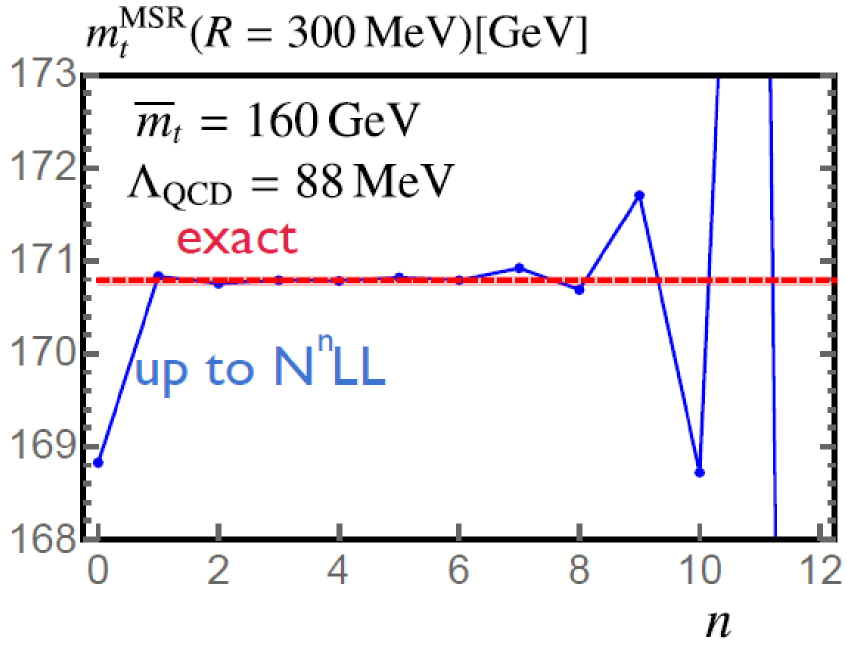
$$= -\int_0^\infty {\rm d}\tau \left(\frac{\Lambda_{\rm QCD}}{R}\right)^{2\tau} (1-2\tau) \frac{F_{Z_m^{\rm OS}}(0,\tau) - F_{Z_m^{\rm OS}}(0,0)}{\tau}$$

$$- \int_{-\beta_R}^{0} {\rm d}\tau\, \frac{F_{Z_m^{\rm OS}}(\tau,0) - F_{Z_m^{\rm OS}}(0,0)}{\tau}$$

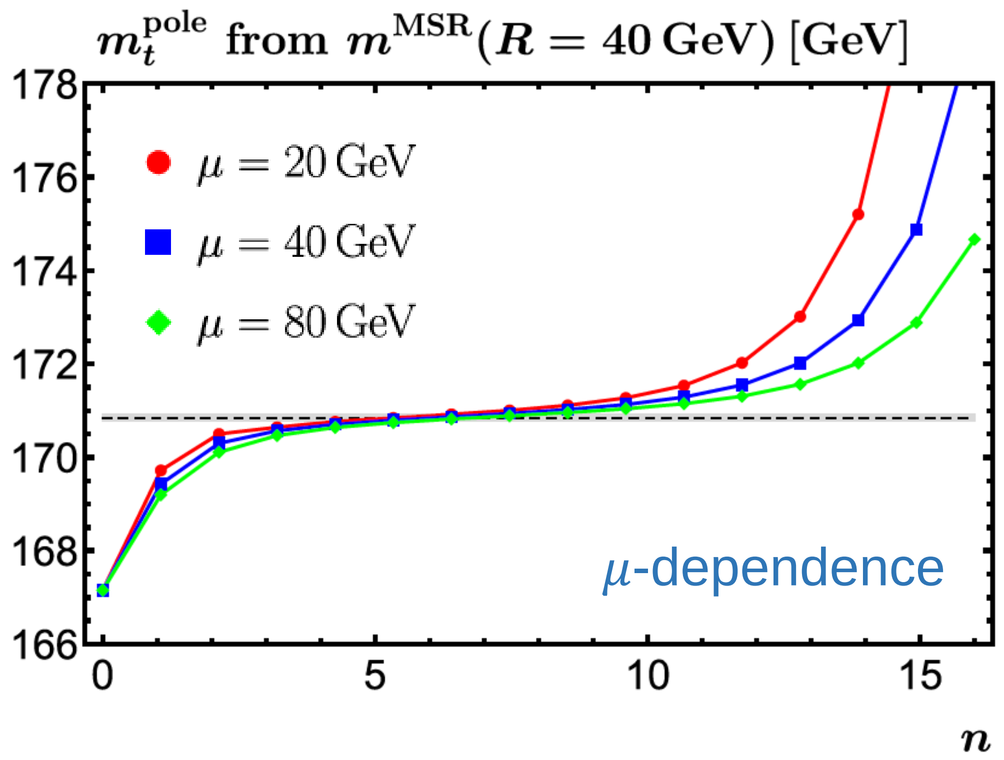
$$-2\beta_R[F_{Z_m^{\rm OS}}(-\beta_R,0) - F_{Z_m^{\rm OS}}(0,0)]$$

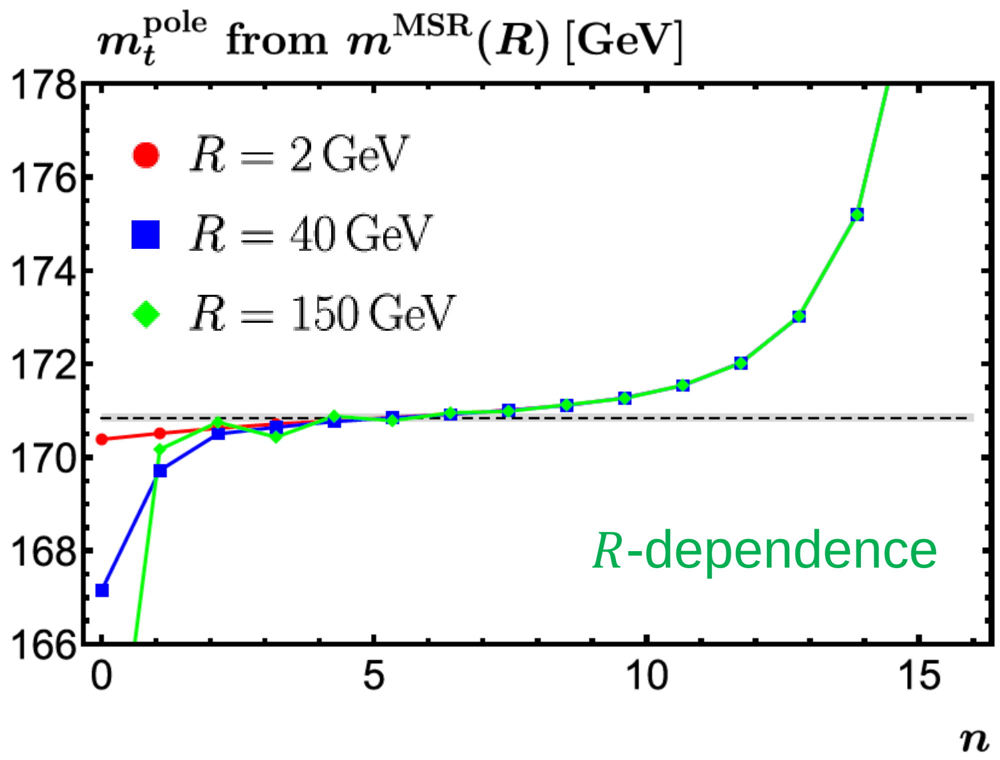


Pole mass from MSR mass

When expanded in terms of $\alpha_s(\mu)$, the MSR mass aqquires powers of $\log(\mu/R)$ and μ dictates the asymptotic behavior, while R doesn't.

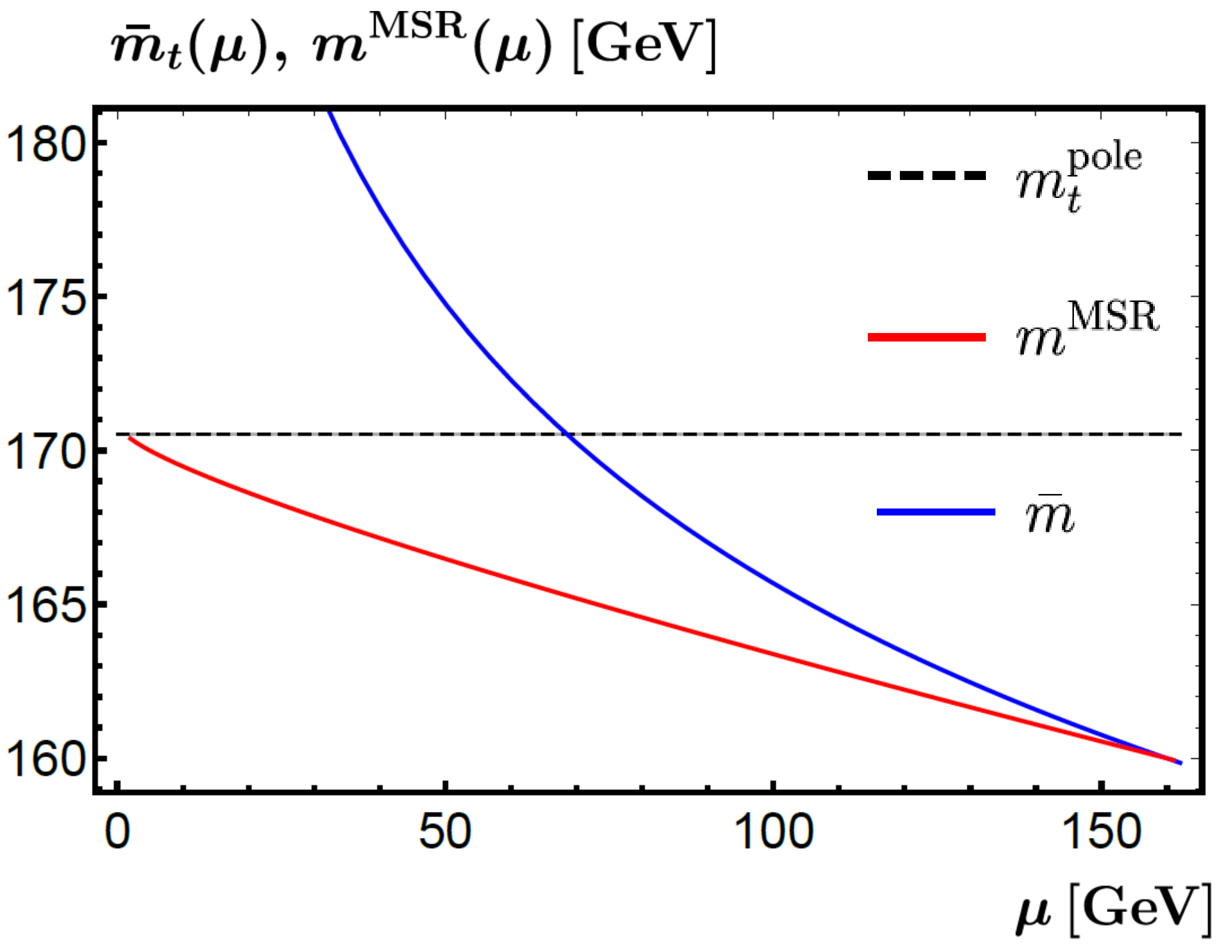
$$\delta m^{\mathrm{MSR}} = \frac{R}{\beta_0}\sum_{i=1}^{\infty}\beta_\mu^i \sum_{j=0}^{i-1} a_{i,j}^{\mathrm{MSR}} \log^j\left(\frac{\mu}{R}\right)$$





Therefore for a renormalon cancelation between two series both must be expanded in terms of the same $\alpha_s(\mu)$.

Comparison of short-distance mass schemes



The pole mass is computed as $m_p = \bar{m} + \mathrm{P.V.}\{\delta\bar{m}\}$ and its ambiguity is too small to be clearly seen.

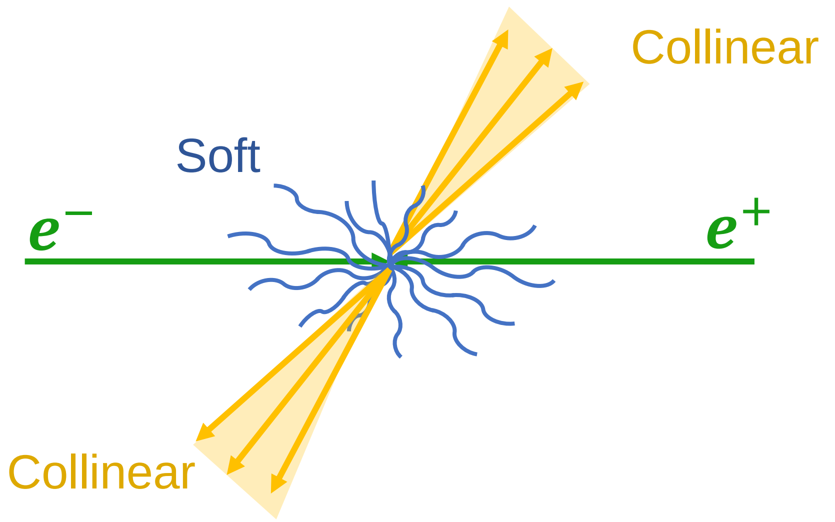
The $\overline{\mathrm{MS}}$ mass is computed through $\gamma_{\bar{m}}(\alpha_s)$. It grows for $\mu < \bar{m}$ and becomes larger tan agrees m_p at $\mu < \bar{m}/2$. It is not ambiguous.

The MSR mass is computed through $\gamma_R(\alpha_s)$. It agrees with the $\overline{\mathrm{MS}}$ mass at $\mu = \bar{m}$ and smoothly aproaches m_p for $\mu = 0$. It's ambiguity is too small to be shown.

Outline

1. Asymptotic series and the Borel transform
2. The large-β_0 limit of QCD
3. Series without cusp-anomalous dimension
4. Series with cusp-anomalous dimension
5. Phenomenology I: short-distance mass schemes
6. Phenomenology II: jets from massless and massive quarks

SCET computations



Often, $e^+e^- \to$ **Hadrons** collisions at high energies adopt di-jet configurations:

- High energetic (collinear) radiation travels together in two jets
- Low-energy, soft radiation populates the space between the jets.

In Soft Collinear Effective Theory (SCET) there is a factorization theorem for the event-shape differential cross section

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_n \otimes J_{\bar{n}} \otimes S$$

(Fleming, Hoang, Mantry, Stewart)

(Bauer, Fleming, Lee, Sterman)

We present the computation in the large-β_0 limit of the

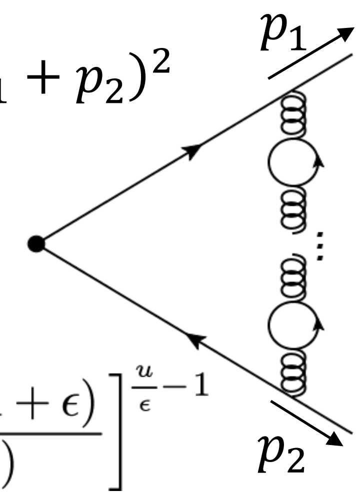
Hard function (universal QCD mathing onto SCET) and the

SCET jet function (for hemisphere mass and trust event shapes)

Hard function

The hard function $H_Q(Q,\mu)$ is the modulus squared of the QCD matching coefficient onto SCET, $C_H(Q^2+i0^+,\mu)$

$Q^2 = (p_1+p_2)^2$



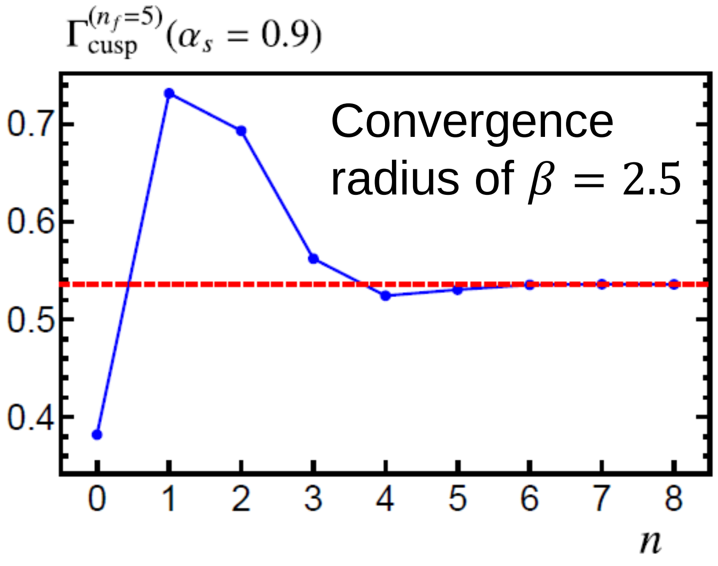
on-shell scheme and dim.reg. $\longrightarrow$ all self-energy and SCET diagrams vanish

$\longrightarrow$ C_H= massless quark vector form factor

$$G_{C_H}(\epsilon,u) \;=\; 2C_F e^{\gamma_E u}[(u-1)\epsilon^2+(u^2-2u+3)\epsilon-2]\frac{\Gamma(1+u)\Gamma^2(1-u)}{\Gamma(3-u-\epsilon)}\left[\frac{3(\epsilon-1)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{(2\epsilon-3)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

$\omega^2 = -Q^2$

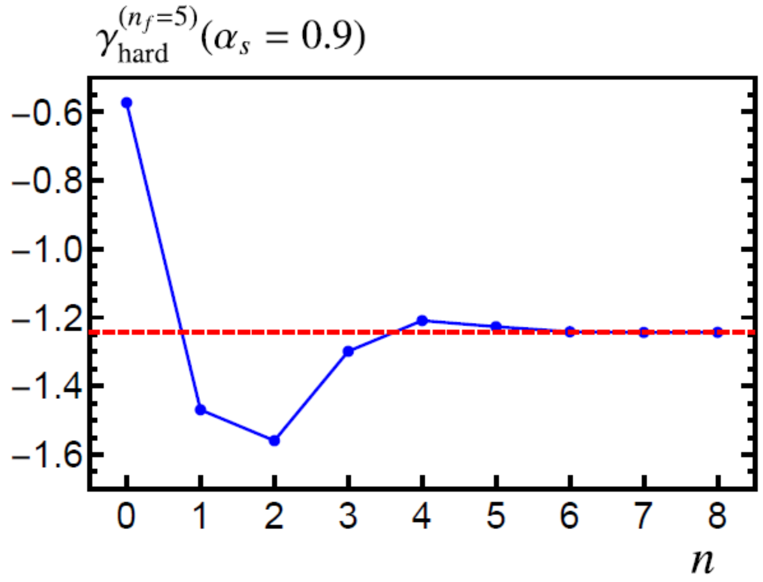
On the way to the hard function we can use G_{C_H} to compute the anomaolus dimension



$$\Gamma_{\mathrm{cusp}}(\beta) = \frac{2C_F}{3\pi}\,\frac{\sin(\pi\beta)\Gamma(4+2\beta)}{\beta_0\Gamma(2+\beta)^2}$$

(agrees with Scimemi and Vladimirov)

Our results reporoduce the leading flavour known results in full QCD up to $O(\alpha_s^4)$



Hard function

The hard function $H_Q(Q,\mu)$ is the modulus squared of the QCD matching coefficient $C_H(Q^2+i0^+,\mu)$

$$G_H(\epsilon,u) \;=\; 2\cos(\pi u) G_{C_H}(\epsilon,u)$$

	Poles	Order	Crossed
$\boldsymbol{G_H(\epsilon,0)}$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$\boldsymbol{G_H(0,u)}$	$-n, \quad n=2,3,4\ldots$	simple	No
	$1,2$	double and simple	Yes
	$n, \quad n=3,4,5\ldots$	simple	Yes

Again we don't cross the poles of $G_H(\epsilon,0)$

This time the two first renormalons at u =1 and 2 are two double poles

The ambiguity can again be resumed

We observe logaritmic enhancement in the ambiguities corresponding to double poles

$$\delta_{H_Q} = \frac{C_F}{\beta_0}\left\{\frac{8}{3}\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2\left[6\log\left(\frac{\Lambda_{\rm QCD}}{Q}\right)+5\right]-\frac{2}{3}\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^4\left[12\log\left(\frac{\Lambda_{\rm QCD}}{Q}\right)+1\right]\right.$$
$$\left.+4\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2-6\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^4+4\left[\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2-1\right]^2\log\left[1-\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2\right]\right\}$$

Double poles signal anomalous dimension with n_f dependence at leading order for dimensión 2 and 4 operators in OPE.

The hard function $H_Q(Q,\mu)$ is the modulus squared of the QCD matching coefficient $C_H(Q^2+i0^+,\mu)$

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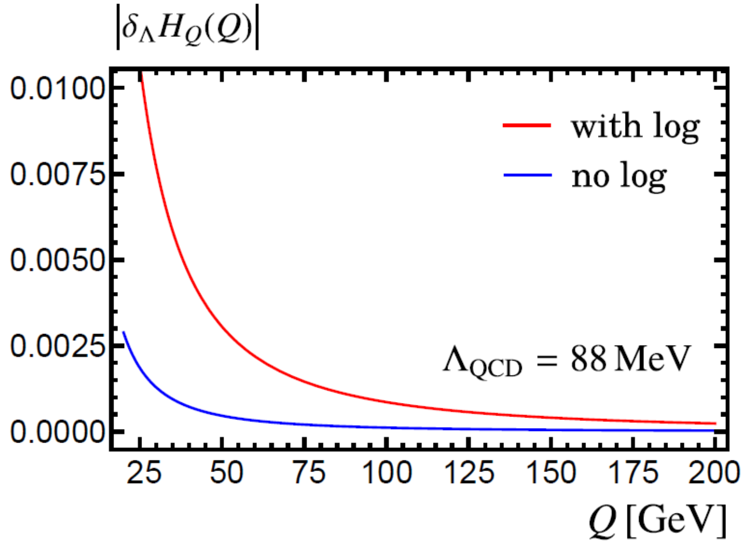

Double poles signal anomalous dimension with n_f dependence at leading order for dimensión 2 and 4 operators in OPE.

Although it is small even for the smallest Q aplicable in SCET.

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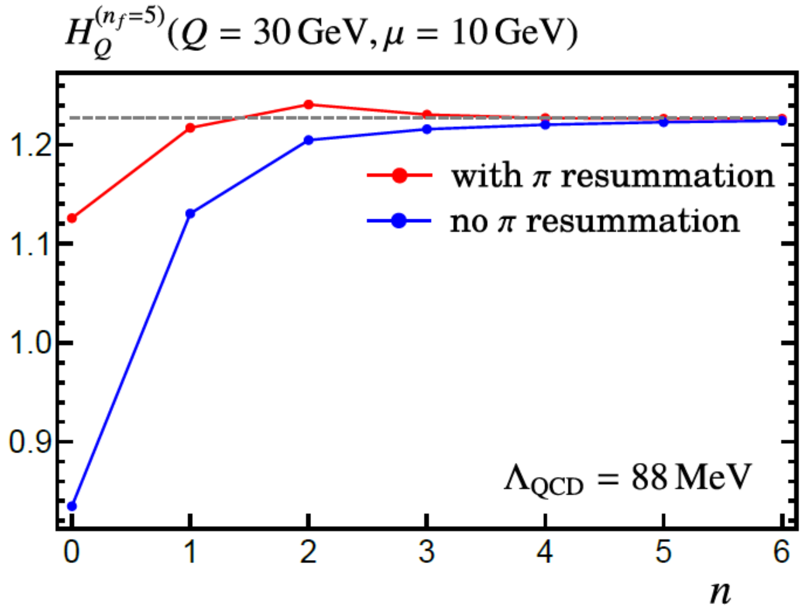
Again we don't cross the poles of $G_H(\epsilon,0)$

This time the two first renormalons at u =1 and 2 are two double poles

Pertuvatively one can decide wheter to sum up factors of π.

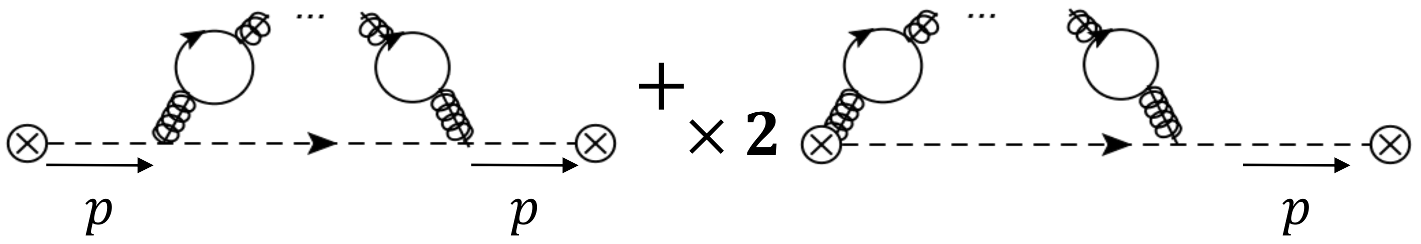
$$\mu_0 = -iQ \qquad \mu_0 = Q$$

We observe faster convergence faster convergence when π-resummation is included.



SCET Jet function

The relevant diagrams to compute the jet function at $O(1/\beta_0)$ are



$$G_{\tilde{J}}(\epsilon,u) = 2C_F[(u-2)\epsilon - 3u + 4]\frac{\Gamma(2-\epsilon)\Gamma(1-u)}{\Gamma(1+u-\epsilon)\Gamma(3-u-\epsilon)} \left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

$$\omega = -\mathrm{i}e^{-\gamma_E}/y$$

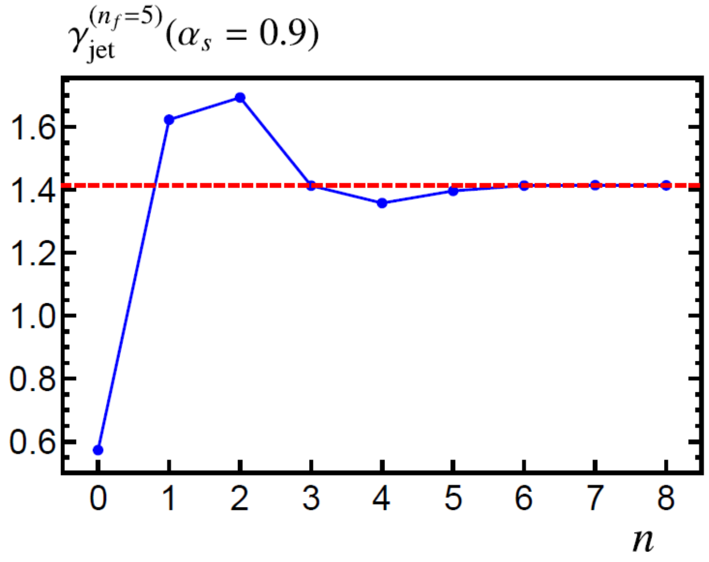
We take the Fourier transform w.r.t. $p^2 = s$ to avoid distributions

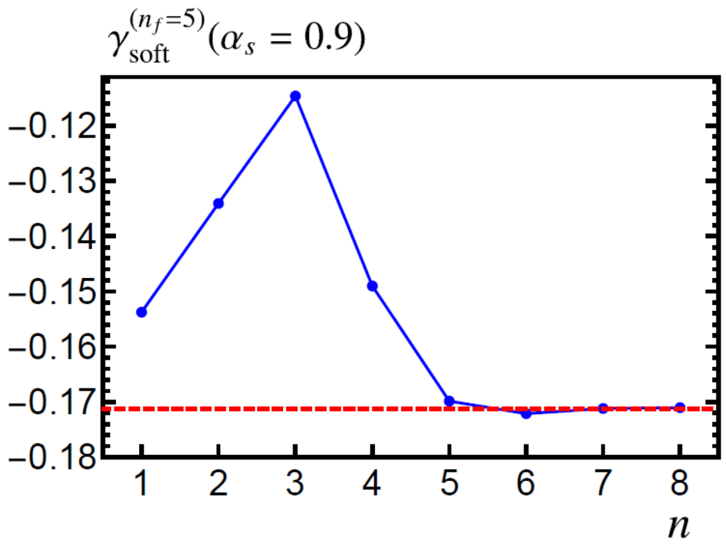
We recover the universal cusp anomalous dimension (cross-chek)

We compute $\gamma_J(\alpha_s)$,

And by consistency we predict $\gamma_S(\alpha_s)$

Perturbatively, we find agreement up to $O(\alpha_s^3)$ with the leading flavour structure in full QCD





SCET Jet function

$$G_{\tilde{J}}(\epsilon,u) \;=\; 2C_F[(u-2)\epsilon-3u+4]\frac{\Gamma(2-\epsilon)\Gamma(1-u)}{\Gamma(1+u-\epsilon)\Gamma(3-u-\epsilon)}\left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

	Poles	**Order**	**Crossed**
$\boldsymbol{G_{\tilde{J}}(\epsilon,0)}$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$\boldsymbol{G_{\tilde{J}}(0,u)}$	$1,2$	simple	Yes

Again we don't cross the poles of $G_{\tilde{J}}(\epsilon,0)$

There are only two (simple) poles at u =1 and 2

There are only two contributions to the ambiguity

For real y

$$\delta_{\tilde{J}} = -\frac{2C_F}{\beta_0}\left[\mathrm{i}ye^{5/3}\Lambda^2_{\mathrm{QCD}} + \left(\frac{1}{2}\mathrm{i}ye^{5/3}\Lambda^2_{\mathrm{QCD}}\right)^2\right]$$

$\mathrm{Re}[\tilde{J}]$ is free from $u=1$ renormalon

$\mathrm{Im}[\tilde{J}]$ is free from $u=2$ renormalon

SCET Jet function

$$G_{\tilde{J}}(\epsilon,u) \;=\; 2C_F[(u-2)\epsilon-3u+4]\frac{\Gamma(2-\epsilon)\Gamma(1-u)}{\Gamma(1+u-\epsilon)\Gamma(3-u-\epsilon)}\left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

	Poles	Order	Crossed
$G_{\tilde{J}}(\epsilon,0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
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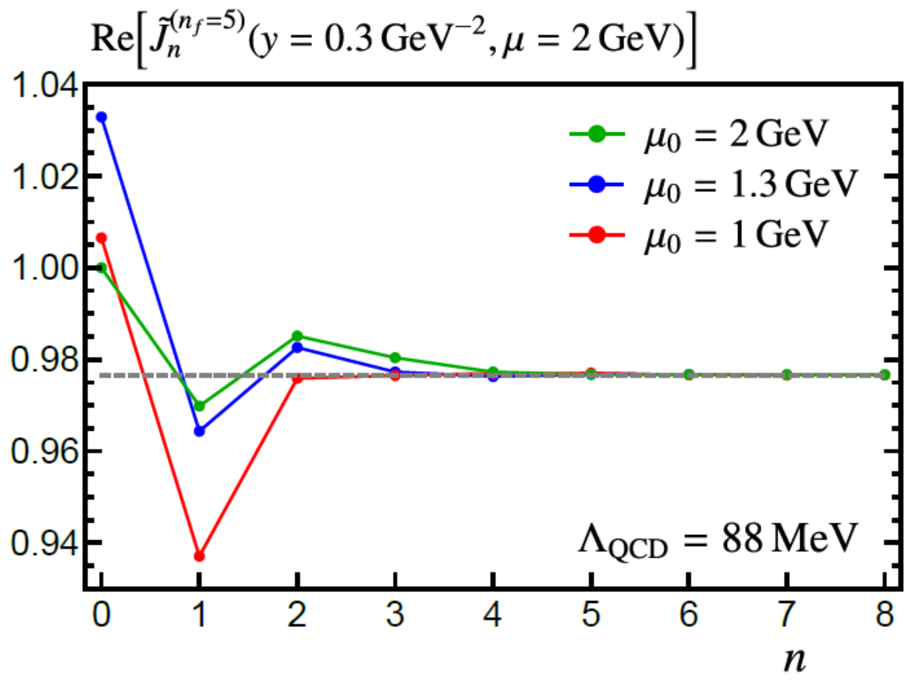
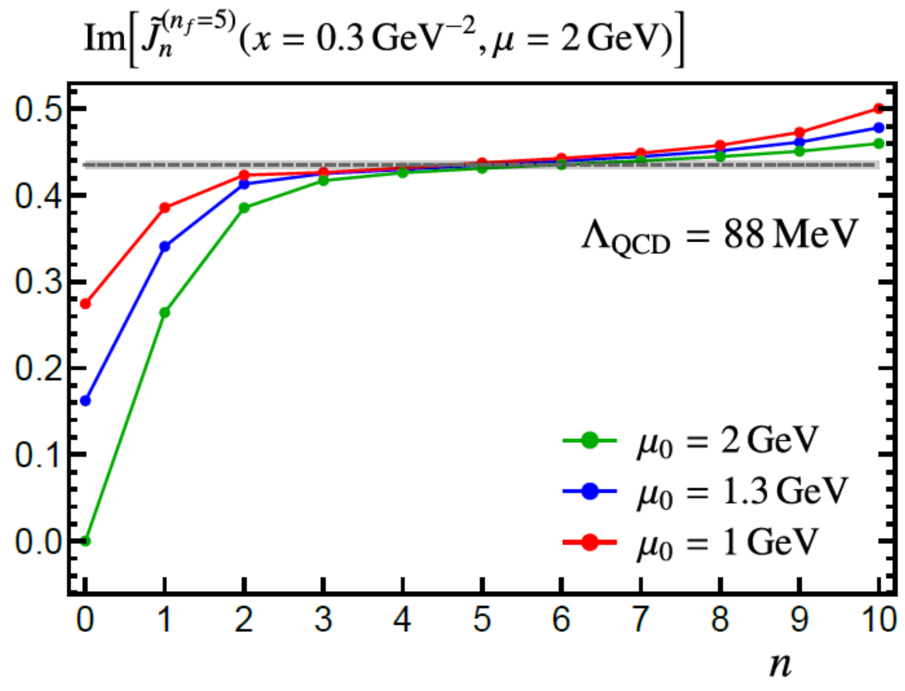




When jets are produced by heavy quarks there is an extra energy scale involved: the quark's mass m

In this case one can match SCET onto two copies of bHQET to sum up the new logs

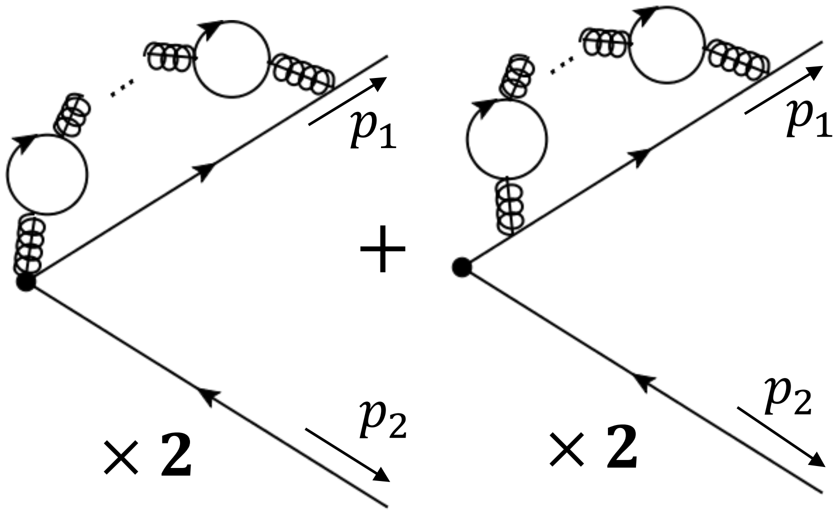
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times H_m \times B_n \otimes B_{\bar{n}} \otimes S$$

We present the computation of the additional mass-scale hard function and the bHQET jet function for hemisphere mass in the large-β_0 limit

Mass-scale hard function

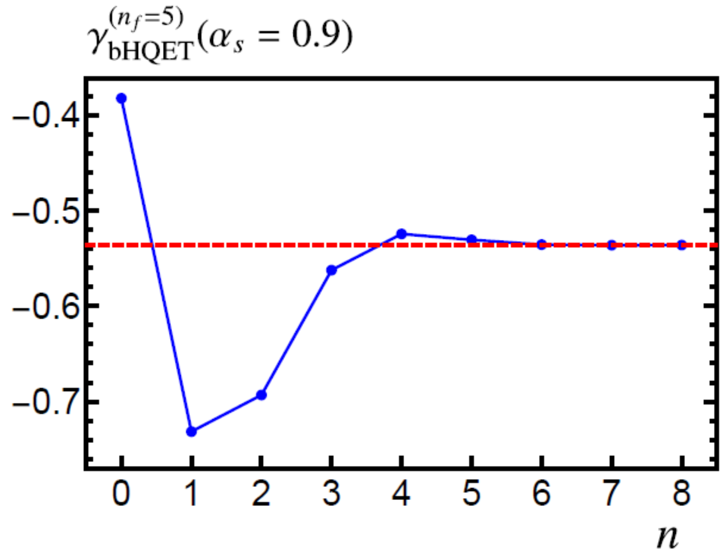
The mass-scale hard function $H_m(m,\mu)$ is the modulus squared of the massive SCET matching coefficient onto bHQET: $C_m(m,\mu)$

- on-shell scheme and dim.reg. $\longrightarrow$ all bHQET diagrams vanish
- For the self-energy diagram we need the wave-function renormalization $Z_\xi^{\rm OS}$, which we obtain from quark's self energy computation in QCD.



$$G_{C_m}(\epsilon,u) = 4C_F e^{\gamma_E u}u^2(1+u-\epsilon)[(2u^2-2u+1)\epsilon-3u^2+4u-2]\frac{\Gamma(u)\Gamma(-2u)}{\Gamma(3-u-\epsilon)}$$
$$\left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}.$$

Again we use this to compute the anomalous dimension



- Once more we recover the cusp anomalous dimension and find full agreement up to $O(\alpha_s^3)$ for the non-cusp part (unambiguous).

Mass-scale hard function

We analize the poles of $G_{H_m}(\epsilon,u) \equiv 2G_{C_m}(\epsilon,u)$ and find…

	Poles	Order	Crossed
$G_{H_m}(\epsilon,0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_{H_m}(0,u)$	$(2n+1)/2, \quad n=0,1,2\ldots$	simple	Yes
	$-n, \quad n=2,3,4\ldots$	simple	No
	$1,2$	simple	Yes

Again we don't cross the poles of $G_{\tilde{J}}(\epsilon,0)$

There are poles at u =1 and 2 and all the positive half-integers

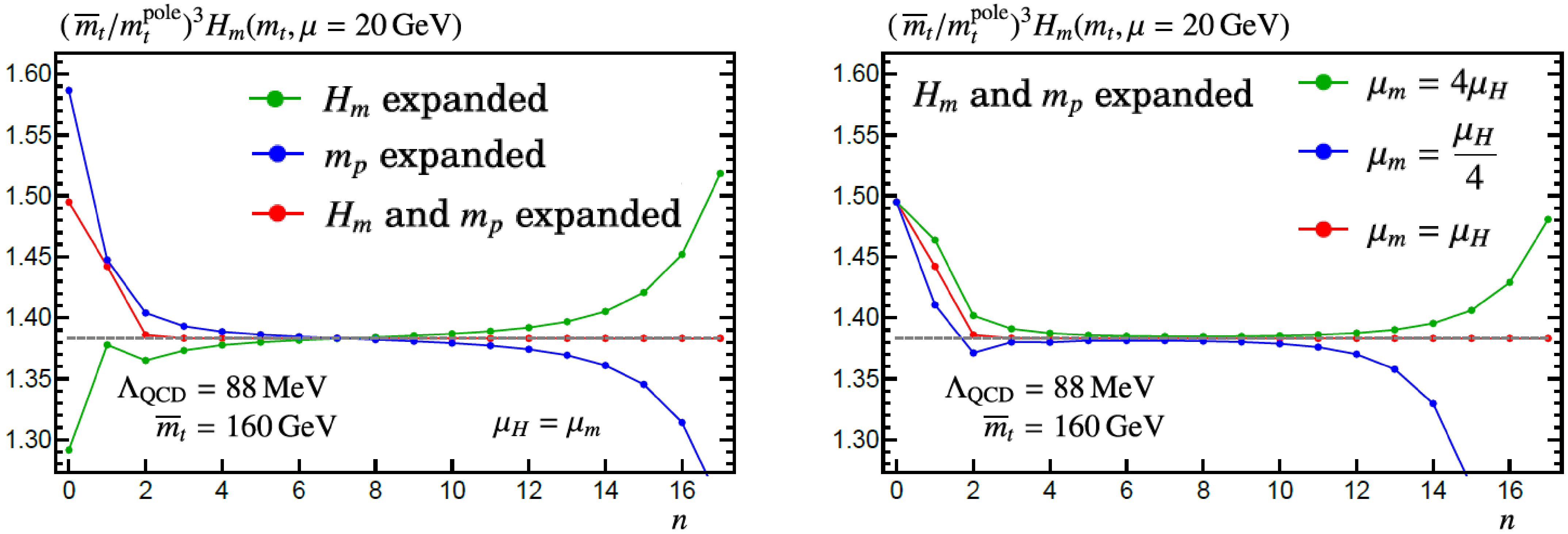
The leading renormalon for H_m lays then at u =1/2 and its ambiguity is

$$\delta_{H_m} = -\frac{6e^{\frac{5}{6}}C_F}{\beta_0}\frac{\Lambda_{\rm QCD}}{m}$$

This is three times higher tan the pole's mass ambiguity: $\delta_{m_p} = -\frac{2C_F}{\beta_0}e^{5/6}\Lambda_{\rm QCD}$

Therefore the combination H_m/m_p^3 is free from the leading ambiguity

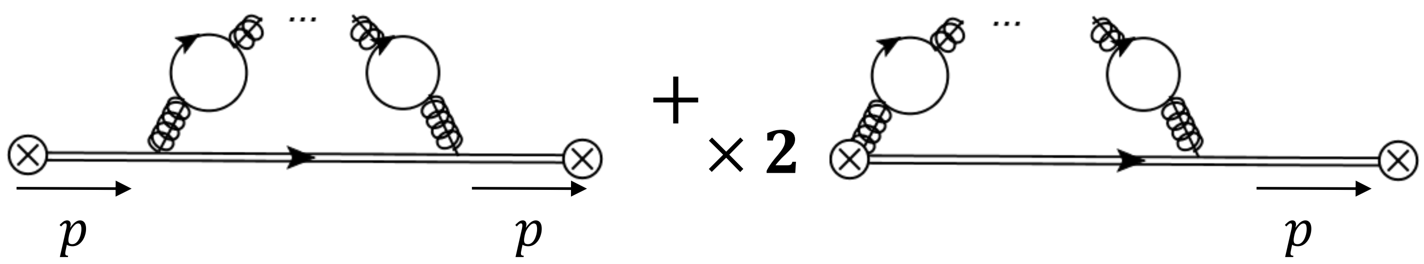
Therefore the combination H_m/m_p^3 is free from the leading ambiguity…



…when both series are expanded (left) in terms of the same $\alpha_s(\mu_m)$ (right)

Note: this renormalon affects the norm of the distribution and might lead to bad convergence of the distribution if not properly accounted for.

The relevant diagrams to compute the jet function for hemisphere masses at $O(1/\beta_0)$ are



We find

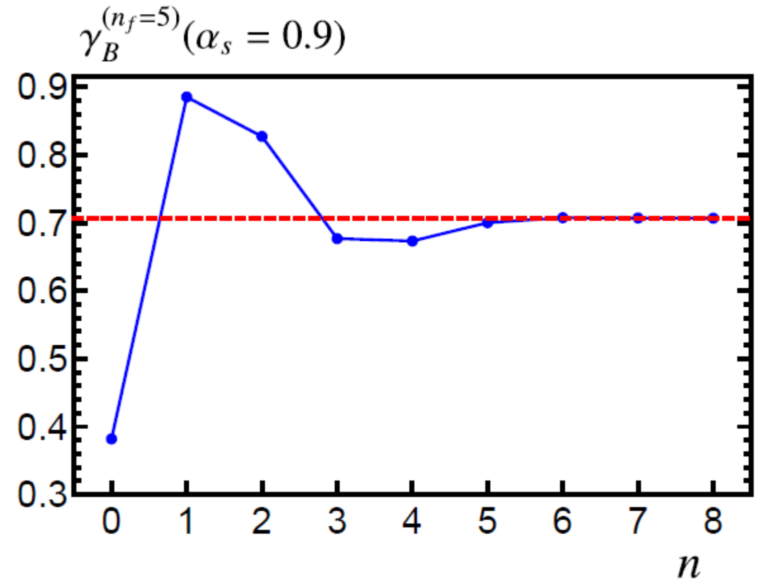
$$G_{\tilde{B}}(\epsilon, u) \;=\; 2C_F \frac{\mathrm{e}^{-u\gamma_E}(1-u)\Gamma(1-u)}{(1-2u)\Gamma(1+u-\epsilon)}\left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1} \qquad \omega = -\mathrm{i}e^{-\gamma_E}/x$$

where we have taken the Fourier transform w.r.t. $\hat{s} = (s - m^2)/m$ to avoid distributions

We recover the universal cusp anomalous dimension (cross-chek)

We compute the non-cusp, unambiguous anomalous dimension

Again, for both we find agreement up to $O(\alpha_s^3)$ with the leading flavour structure in full QCD



bHQET Jet function

$$G_{\tilde{B}}(\epsilon,u) \;=\; 2C_F\frac{\mathrm{e}^{-u\gamma_E}(1-u)\Gamma(1-u)}{(1-2u)\Gamma(1+u-\epsilon)}\left[\frac{3(1-\epsilon)\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

	Poles	Order	Crossed
$G_{\tilde{B}}(\epsilon,0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_{\tilde{B}}(0,u)$	$n, \quad n=2,3,4\ldots$ $1/2$	simple simple	Yes Yes

Again we don't cross the poles of $G_{\tilde{B}}(\epsilon,0)$

The leading renormalon lays at u =1/2

This time the leading ambiguity is twice that of the pole mass (except for a factor of $\mathrm{i}x$):

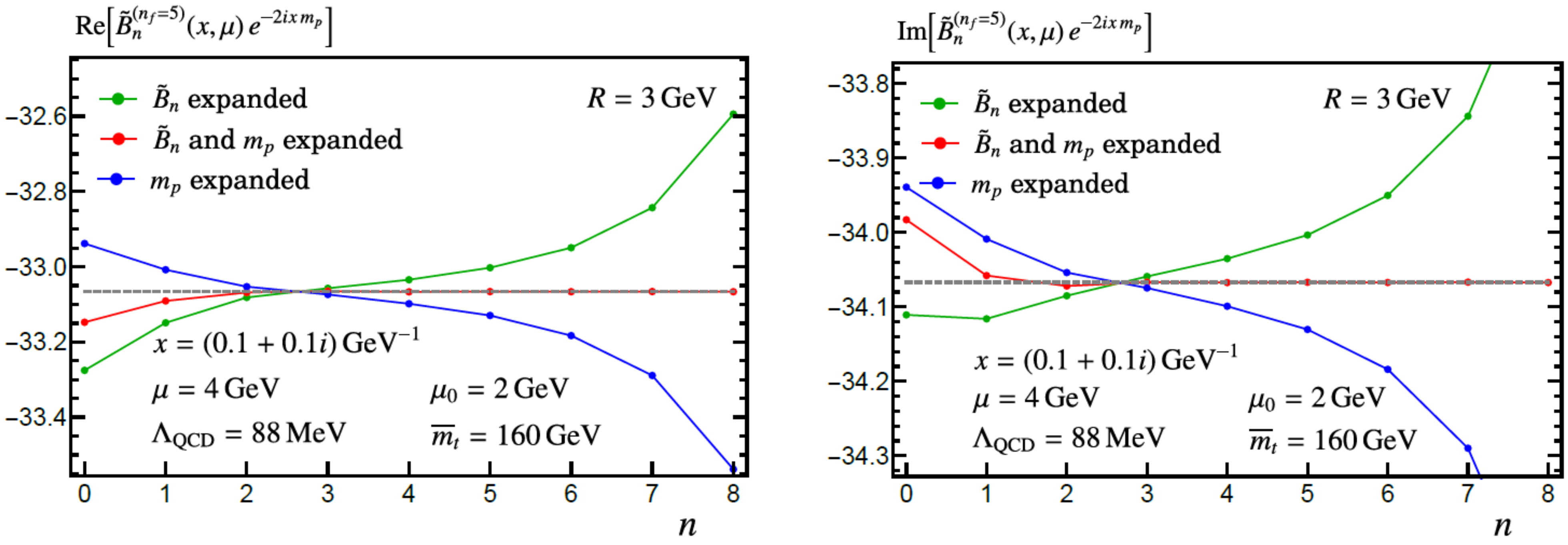
$$\delta_{\tilde{B}} = -\frac{4C_F\mathrm{e}^{5/6}}{\beta_0}\mathrm{i}x\Lambda_{\mathrm{QCD}} \qquad\qquad \delta_{m_p} = -\frac{2C_F}{\beta_0}e^{5/6}\Lambda_{\mathrm{QCD}}$$

Therefore the combination $\tilde{B}(x)e^{-2\mathrm{i}xm_p}$ is free from the leading renormalon

Therefore the combination $\tilde{B}(x)e^{-2\mathrm{i}xm_p}$ is free from the leading renormalon.

Expanding m_p in terms of the $\overline{\mathrm{MS}}$ mass breaks bHQET power counting since $\delta\overline{m} \propto \overline{m}$

Use instead the MSR mass in an expansion in powers of $\alpha_s(\mu)$ with $\mu \sim R \sim 1/x$ to avoid large logs



We used complex x since for real x the real part is free from the 1/2 renormalon

Conclussions

- In the large-β_0 limit we completely know QCD perturbative series, and we can resum them
  - We derived a formalism that recovers the known closed expressions for finite, non-cusp and extended it to cusp series and their anomalous dimensions
  - From these expressions we can study their asymptotic behaviour and estimate the size of non-perturbative power corrections
- We computed SCET and bHQET matrix elements (all divergent) and their anomalous dimensions (all finite)

Thanks for your attention