Semileptonic $b \rightarrow c$ decays: puzzles and opportunities

Paolo Gambino
Università di Torino & INFN, Torino

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A LONG-STANDING TENSION

Semileptonic B decays measure the magnitude of the CKM matrix elements $V_{cb}$ and $V_{ub}$.

Their determinations from inclusive and exclusive decays differ since many years.
Last few years: new analyses of B-factories data, new calculations of FFs by several lattice collaborations and with light-cone sum rules, rising to the challenges of a precision measurement
NEW PHYSICS?

Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data (Crivellin, Pokorski etc.)
Jung & Straub 1801.01112
The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT) $V_{cb}$ plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[ 1 + O(\lambda^2) \right]$$

where it often dominates the theoretical uncertainty.

$V_{ub}/V_{cb}$ constrains directly the UT

Our inability to determine precisely $V_{cb}$ hampers significantly NP searches
VIOLATION OF LFU WITH TAUS

Introduction: The tension with the SM

\[ R \left( D^{(*)} \right) = \frac{\mathcal{B} \left( B \rightarrow D^{(*)} \tau \nu_\tau \right)}{\mathcal{B} \left( B \rightarrow D^{(*)} \ell \nu_\ell \right)} \]

\[
\begin{align*}
R(D) & = 0.299 \pm 0.005 \\
R(D^*) & = 0.258 \pm 0.005
\end{align*}
\]

\[ \Delta \chi^2 = 1.0 \text{ contours} \]

\(~3\sigma\)
**Simple idea:** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators.

The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: *double series in* \(\alpha_s, \Lambda/m_b\).

Lowest order: decay of a free \(b\), linear \(\Lambda/m_b\) absent. Depends on \(m_{b,c}\), 2 parameters at \(O(1/m_b^2)\), 2 more at \(O(1/m_b^3)\)...
INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in $\Lambda/m_b$ and $\alpha_s$

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 M_i^{(2)} + \left( M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)} \right) \frac{\mu_{\pi}^2}{m_b^2}$$

$$+ \left( M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)} \right) \frac{\mu_{G}^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(L_S,0)} \frac{\rho_{L_S}^3}{m_b^3} + \ldots$$

$$\mu_{\pi}^2(\mu) = \frac{1}{2M_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu \quad \mu_{G}^2(\mu) = \frac{1}{2M_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$$

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.

Current HFLAV kinetic scheme fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3), m_c$ and other constraints from sum rules/lattice
Global shape parameters (first moments of the distributions for various lower cut on $E_l$) tell us about $m_b, m_c$ and the B structure, total rate about $|V_{cb}|$.

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications (rare decays, $V_{ub}$,...)
New from Belle and Belle II

big potential for improvement @ Belle II

**Results:**
- measured BR compatible with PDG
  - slow-pion systematics expected to improve with updated correction factors
  - updates coming soon!

**Related upcoming analysis:**
- measurement of $q^2$ moments
  - alternative method based on arXiv:1812.07472
  - includes corrections up to $1/m_b^4$
  - see Belle YSF talk by Raynette van Tonder

⇒ brand new inclusive $V_{cb}$ measurement... coming soon!


**Inclusive $B \rightarrow X_c \ell \nu$: Hadronic Mass Moments**


**Publication in preparation**

**Preliminary Results: 1st moment**

Compare:
- measured moments with truth expectation
  - Dominated by modelling errors at lower cuts
  - Stat. & Sys. uncertainties $\times 10$

**Putting everything together**

$\langle q^2 \rangle^n$ for $n = 1 - 4$ for different cuts on $q^2$

For $n = 1 - 4$

$\langle q^2 \rangle = \langle M_X^n \rangle = \frac{\sum w_i(M_X) M_{X,\text{calib}}^n}{\sum w_i(M_X)} \times C_{\text{calib}} \times C_{\text{true}}$

...Related upcoming analysis:
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  - alternative method based on arXiv:1812.07472
  - includes corrections up to $1/m_b^4$
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⇒ brand new inclusive $V_{cb}$ measurement... coming soon!
FIT RESULTS

\[
\begin{array}{cccccccccc}
\text{mass}_{b}^{\text{kin}} & \overline{m}_c(3\text{ GeV}) & \mu_\pi^2 & \rho_D^3 & \mu_G^2 & \rho_{LS}^3 & \text{BR}_{c\ell\nu} & 10^3|V_{cb}| \\
4.553 & 0.987 & 0.465 & 0.170 & 0.332 & -0.150 & 10.65 & 42.21 \\
0.020 & 0.013 & 0.068 & 0.038 & 0.062 & 0.096 & 0.16 & 0.78 \\
\end{array}
\]

Without mass constraints \( m_{b}^{\text{kin}}(1\text{GeV}) - 0.85 \overline{m}_c(3\text{GeV}) = 3.714 \pm 0.018 \text{ GeV} \)

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of \( V_{cb} \)
- 20-30% determination of the OPE parameters
- \( b \) mass determination in agreement with recent lattice and sum rules results
HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8
In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$

Lowest Lying State Saturation Approx (LLSA) truncating

\[ \langle B|O_1O_2|B \rangle = \sum_n \langle B|O_1|n \rangle \langle n|O_2|B \rangle \]

and relating higher dimensional to lower dimensional matrix elements, e.g.

\[ \rho_D^3 = \epsilon \mu_\pi^2 \quad \rho_{LS}^3 = -\epsilon \mu_G^2 \quad \epsilon \sim 0.4 \text{GeV} \]

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers. The rest of the fit is unchanged, with slightly smaller theoretical errors

\[ |V_{cb}| = 42.00(64) \times 10^{-3} \]

Healy, Turzcyk, PG 1606.06174
PROSPECTS for INCLUSIVE $V_{cb}$

- Theoretical uncertainties generally larger than experimental ones
- $O(\alpha_s/m_b^3)$ calculation completed for width (Mannel, Pivovarov) in progress for the moments (S. Nandi, PG)
- 3loop relation between MSbar and kin scheme recently completed in 2005.06487. It can be used to improve the precision of the $m_b$ input
- $O(\alpha_s^3)$ corrections to total width just completed by Fael, Schoenwald, Steinhauser 2011.13654: towards 1% uncertainty
- Electroweak (QED) corrections require checks, in progress
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now, $q^2$ moments (Fael, Mannel, Vos: preliminary $V_{cb}=41.7(1.2) \times 10^{-3}$, useful check of higher powers
- Lattice QCD is the next frontier
MESON MASSES FROM ETMC

\[ M_{HQ} = m_Q + \bar{\Lambda} + \frac{\mu^2_\pi - a_H \mu^2_G}{2m_Q} + \ldots \]

- on the lattice one can compute mesons masses for arbitrary quark masses
  see also Kronfeld & Simone hep-ph/0006345, 1802.04248
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, \( a=0.62-0.89 \) fm, \( m_{\pi}=210-450 \) MeV, heavy masses from \( m_c \) to \( 3m_c \), ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1 GeV, good sensitivity up to \( 1/m^3 \) corrections
- Results consistent with s.l. fits, improvements under way, also following new 3loop calculation of pole-kinetic mass relation
INCLUSIVE DECAYS ON THE LATTICE

- Inclusive processes nearly impossible to treat directly on the lattice
- However, vacuum current correlators can be computed in euclidean space-time and related to $e^+e^- \rightarrow \text{hadrons}$ or $\tau$ decay via analyticity.
- In our case the correlators have to be computed in the B meson. Hashimoto 1703.01881
- Analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.
- While the calculation of the spectral density of hadronic correlators is an ill-posed problem, it is accessible after smearing, as provided by phase-space integration. Hansen, Meyer; Robaina, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa
A NEW APPROACH

Hashimoto, PG 2005.13730

$$\frac{d\Gamma}{dq^2 dq^0 dE_\ell} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}$$

triple diff distribution $B_s$ decays

$$W^{\mu\nu} \sim \sum_{X_c} \frac{1}{2E_{B_s}} \langle \tilde{B}_s(p)|J^{\mu\dagger}|X_c(r)\rangle \langle X_c(r)|J^{\nu}|\tilde{B}_s(p)\rangle \sim \text{Im} \int d^4x e^{-iq.x} \langle B_s | T J^{\mu\dagger}(x) J^{\nu}(0) | B_s \rangle$$

after integration over $E_i$

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_{q^2_{\text{max}}}^{q^2} dq^2 \sqrt{q^2} \bar{X} \quad \bar{X} \equiv \int_{m_{B_s} - \sqrt{q^2}}^{\sqrt{m_{B_s}^2 + q^2}} X d\omega = \int K(\omega, q)_{\mu\nu} W^{\mu\nu} d\omega$$

where $\omega$ hadr. energy, $X^{(l)}$ linear combinations of $W^{\mu\nu}$.

4point functions on the lattice are related to the hadronic tensor in euclidean

$$B \sim \langle B_s | J^{\mu\dagger}(x, t) J^{\nu}(0,0) | B_s \rangle$$
Quantum Mechanics in a Box

\[ C(t) = \langle 0 \| \pi(t) \pi(0) \| 0 \rangle \]

\[ C(t) = \sum_{n} |Z_n|^2 e^{-E_n t} \]

Laplace transform

\[ C(\omega) = \sum_{n} |Z_n|^2 \delta(\omega - E_n) \]

\[ = \langle 0 \| O \delta(\hat{H} - \omega) O \| 0 \rangle \]

\[ = \langle \psi \| \delta(\hat{H} - \omega) \| \psi \rangle \]

Or for inclusive B-decays:

\[ |\psi\rangle = J O \| 0 \rangle \]

W. Jay @Snowmass workshop
Quantum Mechanics in a Box

- What about hadronic tensor $W(\omega, q)$?
- Elastic channel: $\propto \delta(\omega - M)$
- Inelastic thresholds: $\propto \Theta(\omega - E_{\text{thresh}}) \times \text{(phase space)}$

W. Jay @Snowmass workshop

needs smearing!
A NEW APPROACH

\[ \sum_{x} e^{i q \cdot x} \frac{1}{2m_{B_s}} \langle B_s(0) | J_{\mu}^{\dagger}(x, t) J_{\nu}(0, 0) | B_s(0) \rangle \sim \langle B_s(0) | \tilde{J}_{\mu}^{\dagger}(-q) e^{-\hat{H} t} \tilde{J}_{\nu}(q) | B(0) \rangle \]

\[ \tilde{J} \text{ FT of } J \]

Integral over \( \omega \) becomes

\[ \int_{0}^{\infty} d\omega K(\omega, q) \langle B_s(0) | \tilde{J}_{\mu}^{\dagger}(-q) \delta(\hat{H} - \omega) \tilde{J}_{\nu}(q) | B_s(0) \rangle \]

\[ = \langle B_s(0) | \tilde{J}_{\mu}^{\dagger}(-q) K(\hat{H}, q) \tilde{J}_{\nu}(q) | B_s(0) \rangle \]

\[ K \text{ approximated by polynomials} \]

\[ K(\hat{H}, q) = k_0(q) + k_1(q) e^{-\hat{H}} + \cdots + k_N(q) e^{-N\hat{H}} \]

\( K \) has a sharp hedge: sigmoid \( 1/(1 + e^{x/\sigma}) \) used to replace kinematic \( \theta(x) \) for \( \sigma \to 0 \)

Larger number \( N \) of Chebyshev polynomials needed for small \( \sigma \)
A PILOT NUMERICAL STUDY
Hashimoto, PG 2005.13730

Smeared spectral functions can be computed on the lattice in JLQCD setup, see 1704.08993

2+1 flavours of Moebius domain wall fermions with $1/a=3.610(9)$GeV on $48^3 \times 96$ $M_{Bs}=3.45$ GeV, i.e. $m_b^{\text{kin}}(1\text{GeV}) \approx 2.70$GeV physical charm mass $m_c^{\text{MS}}(3\text{GeV}) = 1.00$GeV $m_b-m_c \sim 1.7$GeV only, $q^{\text{max}} \sim 1.16$GeV NB $m_b^{\text{lat}} = 2.44 m_c^{\text{lat}}$ : we don’t know it precisely...

Extrapolation to $\sigma \to 0$ possible, but error due to finite $N$ must be estimated
COMPARISON WITH OPE

OPE matrix elements from fits, sizeable power and pert corrections!

\[ \Gamma / |V_{cb}|^2 = 4.5(6) \times 10^{-13} \text{ GeV} \]
\[ \Gamma / |V_{cb}|^2 = 5.4(8) \times 10^{-13} \text{ GeV} \]

Lattice

OPE including \( O(\alpha_s^2, 1/m_b^3) \)

OPE uncertainty: “b” mass error (dominant), higher orders, matrix elements
Comparison of data with JLQCD $B \rightarrow D^{(*)}$ form factors. $D^{(**)}$ and continuum strongly suppressed.

This is due to approx heavy quark symmetry and is stronger for unphysically light $m_b$
LEPTONIC MOMENTS

$\langle E_\ell \rangle$ for fixed $q^2$

- LO OPE
- $\bar{Y}_{1}^{AA,\parallel}$ at $\mathcal{O}(1/m_0^2, \alpha_s^0)$ and $\mathcal{O}(1/m_0^3, \alpha_s)$
- $\bar{Y}_{1}^{AA,\perp}$ at $\mathcal{O}(1/m_0^2, \alpha_s^0)$ and $\mathcal{O}(1/m_0^3, \alpha_s)$

Hashimoto, Kaneko, Maechler, PG in progress
WHAT NEXT?

- Leptonic, hadronic energy moments, SV sum rules with existing data
- D inclusive semileptonic decays vs Cleo-c data for widths and lepton spectra (validation of the method, study of lattice systematics such as finite volume effects and disconnected diagrams, …)
- Towards the physical $b$ mass (ratio method, step scaling, …): large recoil momentum $q$ problematic
- Smooth cuts on experimental and OPE side?
- $B \rightarrow X_u \ell \nu, B \rightarrow X_s \ell^+ \ell^-$: kinematic cuts can in principle be implemented
- Extension of the method to low energy $l$-$N$ inelastic scattering
  Hashimoto et al., 2010.01253 [hep-lat]
EXCLUSIVE DECAYS

There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

\[
\langle D(k) | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle = \left[ (p + k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] f_{+B \to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_{0B \to D}(q^2)
\]

Information on FFs from LQCD (at high \(q^2\)), LCSR (at low \(q^2\)), HQE, exp…
A model independent parametrization is necessary

BGL (Boyd, Grinstein, Lebed 1995-7) is based on crossing & analyticity, power series in \( z(q^2) \) with coefficients \( a_i \) that satisfy unitarity bounds \( \Sigma_i a_i^2 \leq 1 \)

HQET for \( B^{(*)} \to D^{(*)} \) form factors:

\[
F_i(w) = \xi(w) \left[ 1 + c_i^b \frac{\alpha_s}{\pi} + c_i^b \epsilon_b + c_i^c \epsilon_c + \ldots \right] \quad \epsilon_{b,c} = \frac{\Lambda}{2m_{b,c}}
\]

\( c_{b,c} \) can be computed using subleading Isgur-Wise functions from QCD sumrules Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

Ratios are free of IW function: can be used to get strong unitarity bounds but \( 1/m_c^2 \) corrections can be significant as shown by lattice calculations

Caprini-Lellouch-Neubert (CLN, 1998) parametrization is simple and has few parameters, but relies on NLO HQET. All exp analyses before 2017 were based on CLN.
$|V_{cb}| = 40.5(1.0) \times 10^{-3}$, $R(D) = 0.299(3)$

- Babar 2009
- Belle 2015
- MILC-FNAL
- HPQCD

$BGL \ N=4$
$\chi^2/dof = 19/22$

Lattice determines slopes, exp data shown at fitted $V_{cb}$

R(D) 1.3σ from exp

FLAG has very similar results

CLN cannot fit both ff missing higher orders!
$|V_{cb}|$ from $B \rightarrow D^{*}l\nu$ according to HFLAV (2019)

LQCD provides only light lepton FF at zero recoil, $w=1$, where rate vanishes. Experimental results must therefore be extrapolated to zero-recoil.

*Exp error only $\sim 1.1\%$*  
\[ F(1)\eta_{ew}|V_{cb}| = 35.27(38) \times 10^{-3} \]

Beware: HFLAV extrapolate with CLN (w/o error)

Two unquenched lattice calculations

\[ F(1) = 0.906(13) \quad \text{Bailey et al 1403.0635 (FNAL/MILC)} \]
\[ F(1) = 0.895(26) \quad \text{Harrison et al 1711.11013 (HPQCD)} \]

Using their average $0.904(12)$:

\[ |V_{cb}| = 38.8(7) \times 10^{-3} \]

\~ $3.4\sigma$ or $\sim 8\%$ from inclusive determination

**Heavy quark sum rules**  
$F(1) < 0.925$ and estimate of inelastic contribution $F(1) \approx 0.86$  
Mannel, Uraltsev, PG, 2012
2017 tagged Belle analysis (still preliminary!) 1702.01521

$w$ and angular deconvoluted distributions (independent of parameterization). All previous analyses are CLN based and do NOT provide diff distributions.

Bands show two parametrizations both fitting data well, with 6% different $V_{cb}$

$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$

see also Kobach & Grinstein
EXTRAPOLATING TO ZERO RECOIL

![Graph showing fit results with different parametrizations.](image)

+30 additional angular bins

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**Figure 1.** Comparison of fit results with different parametrizations.

Full Belle dataset, most precise study to date; provides data in a way that can be reanalysed with different assumptions. Central values obtained without systematics.

- CLN and BGL\(^{(102)}\) analysis lead to very similar results, suggesting low \( |V_{cb}| = 38.4(0.9) \times 10^{-3} \).

- We used BGL\(^{(222)}\) to fit the data, taking into account D'Agostini effect and got \( |V_{cb}| = 39.1(+1.5-1.3) \times 10^{-3} \).
A GLOBAL FIT TO 2017 & 2018 DATA

Jung, Schacht, PG 1905.08209

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
BGL^{(222)} & Data + lattice (weak) & Data + lattice (strong) \\
\hline
$\chi^2$/dof & 80.1/72 & 80.1/72 \\
$|V_{cb}|10^3$ & 39.6 ($\pm1.1$) & 39.6 ($\pm1.1$) \\
$a_0^f$ & 0.01221 (16) & 0.01221 (16) \\
$a_1^f$ & 0.006 ($^{+32}_{-45}$) & 0.006 ($^{+20}_{-32}$) \\
$a_2^f$ & $-0.2$ ($^{+12}_{-8}$) & $-0.2$ ($^{+7}_{-3}$) \\
$F_1^f$ & 0.0042 ($^{+22}_{-22}$) & 0.0042 ($^{+19}_{-22}$) \\
$F_1^g$ & $-0.069$ ($^{+41}_{-37}$) & $-0.068$ ($^{+41}_{-30}$) \\
$a_0^g$ & $0.024$ ($^{+21}_{-9}$) & $0.024$ ($^{+12}_{-4}$) \\
$a_1^g$ & $0.05$ ($^{+39}_{-72}$) & $0.05$ ($^{+21}_{-41}$) \\
$a_2^g$ & $1.0$ ($^{+0}_{-20}$) & $0.9$ ($^{+0}_{-18}$) \\
\hline
\end{tabular}
\end{table}

• No parametrization dependence (CLN and BGL give ~same central value)

• About 1 $\sigma$ higher than HFLAV, larger uncertainty on firmer ground, p-value

~24% $1.9\sigma$ from inclusive, consistent with B to Dlv

• We truncate the BGL series when additional terms do not change the fit
(no overfitting!). Fit stable. Strong constraints irrelevant.
\[ R_1(w) = \frac{V_4(w)}{A_1(w)} = 1 + O(\alpha_s, 1/m) \]

\[ R_2(w) = \frac{w - r}{w - 1} \left(1 - \frac{1 - r}{w - r} A_5(w)\right) \]

Comparison of \( R_{1,2} \) from BGL fit to 2017+2018 data vs NLO HQET+QCDSR predictions (with parametric + 15% th uncertainty)

Very good agreement in both cases.
SEMITALUONIC DECAYS

Decays with tau require pseudoscalar FF unconstrained from fit, no lattice calculation yet. We use kinematic constraint at $q^2=0$ and HQET with conservative uncertainty.

**Table 2.3**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$R(D^*)$</th>
<th>Exp. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[32]</td>
<td>$0.257(3)$</td>
<td>$3.3\sigma$</td>
</tr>
<tr>
<td>[13, 36]</td>
<td>$0.254(7)$</td>
<td>$3.2\sigma$</td>
</tr>
<tr>
<td>[34]</td>
<td>$0.257(5)$</td>
<td>$3.1\sigma$</td>
</tr>
<tr>
<td>[37]</td>
<td>$0.250(3)$</td>
<td>$3.7\sigma$</td>
</tr>
</tbody>
</table>

**Figure 3. Form factor ratios $P_1(z)$**

$P_1(w_{max}) = A_5(w_{max})$

- **Endpoint constraint**
  - $P_1(w_{max}) = A_5(w_{max})$

- **Formulas**
  - $R(D^*) = 0.254^{+0.007}_{-0.006}$, $2.8\sigma$ from exp
  - $P_\tau = -0.476^{+0.037}_{-0.034}$, $1.4\sigma$ from exp
  - $F^D_L = 0.476^{+0.015}_{-0.014}$

**Note:**
- NLO HQET+QCDSR ±15%
- Normalized to $V_i$
- Normalized to $A_i$
NEW PRELIMINARY LATTICE RESULT

\( R(D^*) \) results in context

**No constraint** \( w_{\text{Max}} \):  
\[
\begin{align*}
R(D^*)_{\text{Lat}} &= 0.266(14) \\
R(D^*)_{\text{Lat}+\text{Exp}} &= 0.2484(13)
\end{align*}
\]

**W/ constraint** \( w_{\text{Max}} \):  
\[
\begin{align*}
R(D^*)_{\text{Lat}} &= 0.274(10) \\
R(D^*)_{\text{Lat}+\text{Exp}} &= 0.2492(12)
\end{align*}
\]


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Alejandro Vaquero (University of Utah)  

\( B \to D^* \ell \nu \) at non-zero recoil  

April 22nd, 2021
Preliminary lattice calculations (M. Jung)

$R_1(w)$: FNAL slope surprising, compatible at 1-2σ

$R_2(1)$: Discrepancy FNAL (1.12 ± 0.06) vs. (HQE fit, experiment)!
HQE@1/$m^2$: 0.78$^{+0.10}_{-0.06}$, BGL: 0.81 ± 0.11, HFLAV: 0.852 ± 0.018
Flavour universality in $B \to D^*(e, \mu)\nu$ (M.Jung)

[Bobeth/Bordone/Gubernari/MJ/vDyk'21]

So far: Belle’18 data used in SM fits, flavour-averaged

However: Bins $40 \times 40$ covariances given separately for $\ell = e, \mu$

$\blacktriangleright$ Belle’18: $R_{e/\mu}(D^*) = 1.01 \pm 0.01 \pm 0.03$

$\blacktriangleright$ What can we learn about flavour-non-universality? $\rightarrow$ 2 issues:

1. $e - \mu$ correlations not given $\rightarrow$ constructable from Belle’18
2. 3 bins linearly dependent, but covariances not singular

Two-step analysis:

1. Extract $2 \times 4$ angular observables for $2 \times 30$ angular bins
   $\blacktriangleright$ Model-independent description including NP!

2. Compare with SM predictions, using FFs@1/$m_C^2$ [Bordone+’19]
   $\blacktriangleright$ $\sim 4\sigma$ discrepancy in $\Delta A_{FB} = A_{FB}^\mu - A_{FB}^e$. 

\[
\Delta x_2 \Delta x_3 \Delta x_4 \Delta x_5 \Delta x_6 \Delta x_7 \Delta x_8 \Delta x_9 \Delta x_{10}
\]
THE IMPORTANCE OF THE SLOPE

Blinding affects only marginally the slope of the ff F, which is key to $V_{cb}$.

Plot suggests well determined slope, $dF/dw|_{w=1}$.

If it were -1.40(7) the fit could still accommodate a high $V_{cb}$.

Here we use new improved LCSR results by Gubernari, Kokulu, van Dyk, 1811.00983 that improve upon 0809.0222.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>$10^3 V_{cb}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>40.8(0.8)</td>
<td>84.5/73</td>
</tr>
<tr>
<td>slope+LCSR</td>
<td>40.8(0.8)</td>
<td>88.0/76</td>
</tr>
</tbody>
</table>
Reanalysis of tagged B⁰ and B⁺ data, unbinned 4 dimensional fit with simplified BGL and CLN

About 6000 events

No data provided yet

No clear BGL⁽⁺⁻⁾/CLN difference

\( V_{cb} = 0.0384(9) \)

BGL form factors compared with CLN HFLAV
\[ V_{cb} \text{ FROM } B_s \text{ DECAYS} \]

Measurement of $|V_{cb}|$ with $B_s^0 \rightarrow D_s^{(*)-} \mu^+ \nu_\mu$ decays

\[
\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^- \mu^+ \nu_\mu)}, \\
\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)}
\]

\[
|V_{cb}|_{\text{CLN}} = (41.4 \pm 0.6 \text{ (stat)} \pm 0.9 \text{ (syst)} \pm 1.2 \text{ (ext)}) \times 10^{-3}, \\
|V_{cb}|_{\text{BGL}} = (42.3 \pm 0.8 \text{ (stat)} \pm 0.9 \text{ (syst)} \pm 1.2 \text{ (ext)}) \times 10^{-3},
\]

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL$^{222}$
CONCLUSIONS

- The **inclusive determination** appears in a good shape and can be improved by 3-loop calculations and studies on the lattice (virtual lab).

- Experimental situation for $B \rightarrow D^*$ confused, but **revisiting the exclusive $b \rightarrow c$ decays was useful**: uncertainties were underestimated and results possibly biased, several new experimental analyses appeared (Babar, LHCb) or are under way. The **practical CLN** must be abandoned and data reported model-independently.

- Several lattice coll. are computing all necessary FFs, but **progress is slow** and FNAL preliminary result problematic.

- Belle-II should improve both incl and excl measurements.

- **The semitauonic anomaly** persists and the SM predictions are robust.

- Inclusive/Exclusive tension remains, a bit weaker. Hopefully, it will disappear. Eventually, **LQCD will decide the fate of the $V_{cb}$ puzzle**.