

Recoil-sensitive jet angularity distributions using SCET

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in collaboration with

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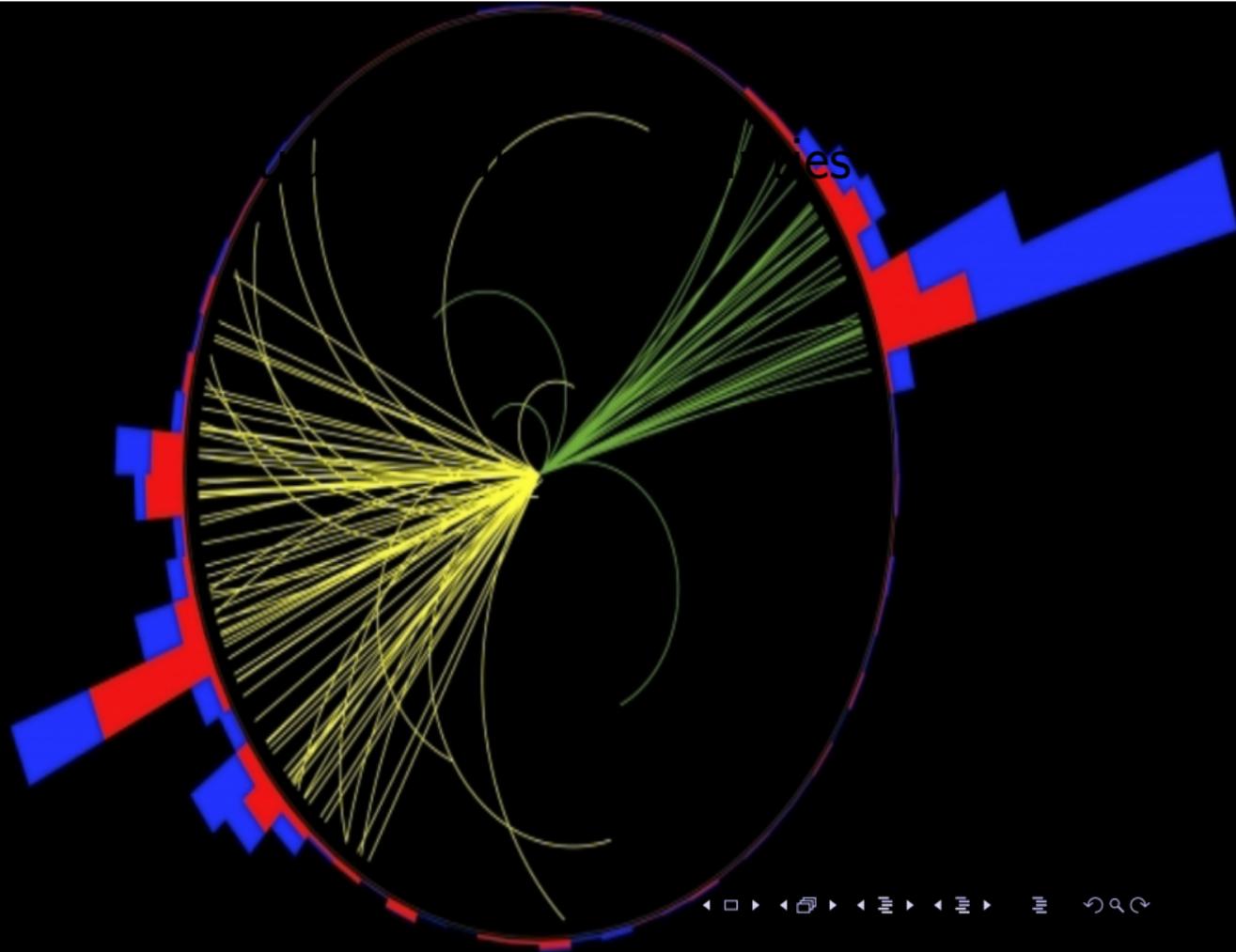
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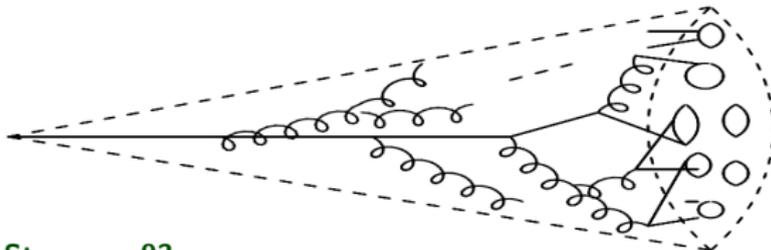
(based on : One-loop Angularity Distribution with Recoil using SCET,
arXiv: 1903.11087 and ongoing work)

1. Jet Angularities
2. Factorization theorem for recoil-sensitive angularities
3. The Zero-Bin and Coupled Distributions
4. Fixed order results
5. Preliminary NLL results
6. Summary



Jet Angularities

- Jet substructure techniques are a set of tools to study the radiation pattern inside a jet



- **Berger, Kucs, Sterman, 03**

$$\tau_b = \frac{1}{Q} \sum_{i \in X} |\vec{p}_{ti}| e^{-b|\eta_i|} ; \quad b > -1$$

- includes

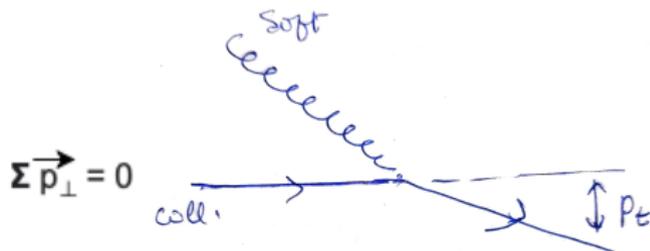
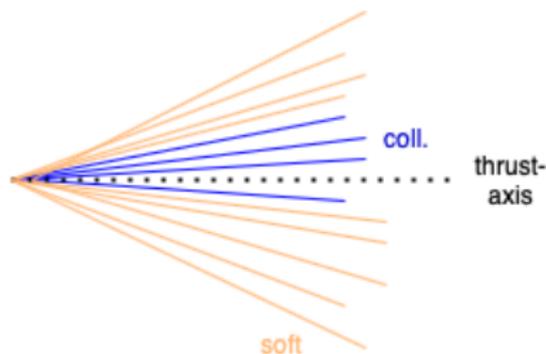
$$\text{Thrust: } \tau = \frac{1}{Q} \sum_{i \in X} |\vec{p}_{ti}| e^{-|\eta_i|} \quad (b = 1)$$

$$\text{Broadening: } e = \frac{1}{Q} \sum_{i \in X} |\vec{p}_{ti}| \quad (b = 0)$$

- Varying 'b' changes the sensitivity to the splitting angle of a collinear radiation in the jet.

What's interesting about Angularities ?

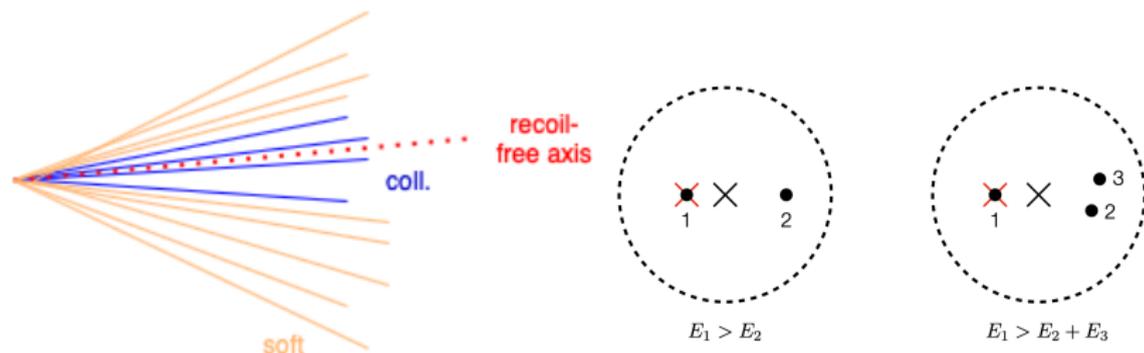
- Possibility to tune b exposes us to a wealth of information than its special limits of $b = 0$ and $b = 1$ separately.
- If measured relative to thrust axis, angularities close to $b = 0$ sensitive to recoil while when $b \gtrsim 1$, recoil becomes power suppressed.



What's interesting about Angularities ?

- Possibility to tune b exposes us to a wealth of information than its special limits of $b = 0$ and $b = 1$ separately.
- If measured relative to thrust axis, angularities close to $b = 0$ sensitive to recoil while when $b \gtrsim 1$, recoil becomes power suppressed.
- If measured relative to broadening axis, all angularity exponents become recoil-free.

Larkoski, Neill, Thaler, 2014

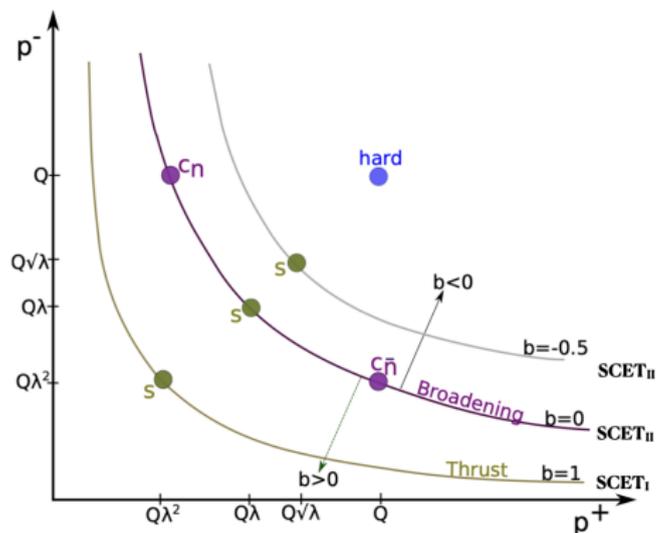


Modes and Scalings

$$p_{c(n)} \sim Q(\lambda^2, 1, \lambda)$$

$$p_{c(\bar{n})} \sim Q(1, \lambda^2, \lambda)$$

$$p_s \sim Q(\lambda^{1+b}, \lambda^{1+b}, \lambda^{1+b})$$

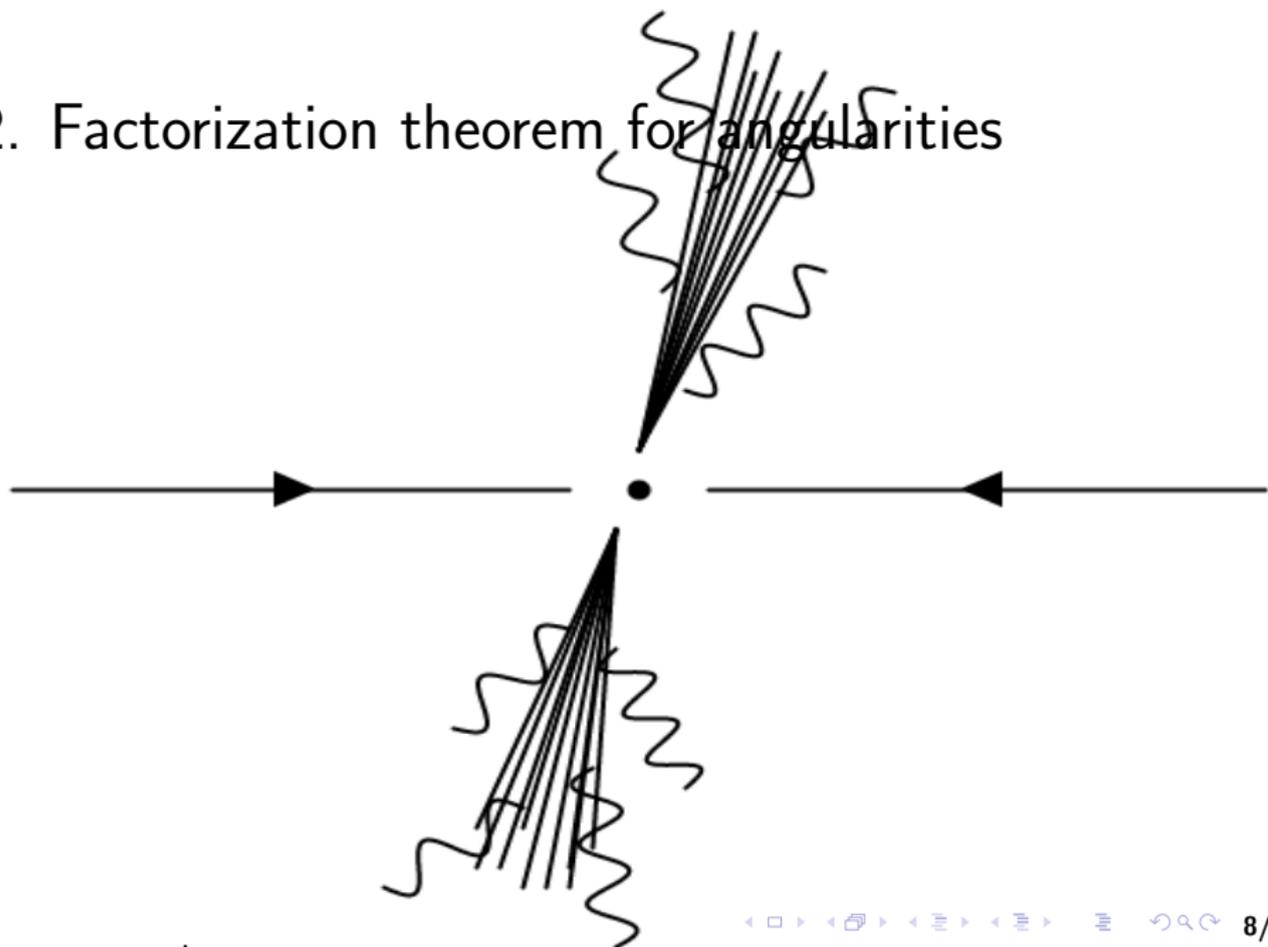


Angularities allow to study the transition between SCET_I and SCET_{II} theories.

Case Study: $e^+ e^-$ Angularity Distributions relative to thrust-axis

- Only collinear and soft modes contribute. Hard mode is the same as for any $e^+ e^- \rightarrow$ di-jet observables.
- SCET_I has been used for all $b \gtrsim 1$ angularities to NNLL' accuracy but fails as b approaches 0. Hornig, Lee, Ovanesyan, 09; Bell, Hornig, Lee, Talbert, 18
- For $b = 0$, SCET_{II} framework has been applied and the results are available to NNLL accuracy. Becher, Bell, Neubert, 11; Chiu, Jain, Neill, Rothstein, 11; Becher and Bell, 12
- I will describe a novel framework based on SCET_{II} that can be applied to the whole range of angularity exponents.
- No hadronization corrections are included in this talk.

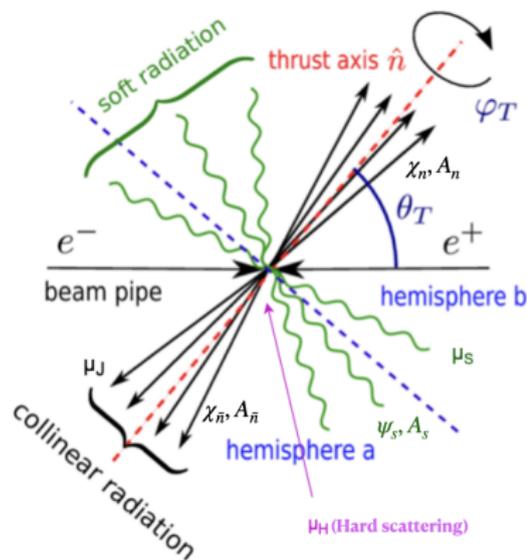
2. Factorization theorem for angularities



Factorization Theorem for Angularities

- Adapting a broadening-like factorization theorem for all angularity exponents

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{d\tau_L d\tau_R} = H(Q; \mu) \cdot [\mathcal{J}_n \otimes \mathcal{J}_{\bar{n}} \otimes \mathcal{S}](\tau, \vec{p}_\perp)$$



- All fields and measurement operators are b -dependent.
- $A_S \sim (\lambda^{1+b}, \lambda^{1+b}, \lambda^{1+b})$ rather than $A_S \sim (\lambda, \lambda, \lambda)$ specific to broadening.

Why does it work ?

- Turning to the generalized soft function for angularities

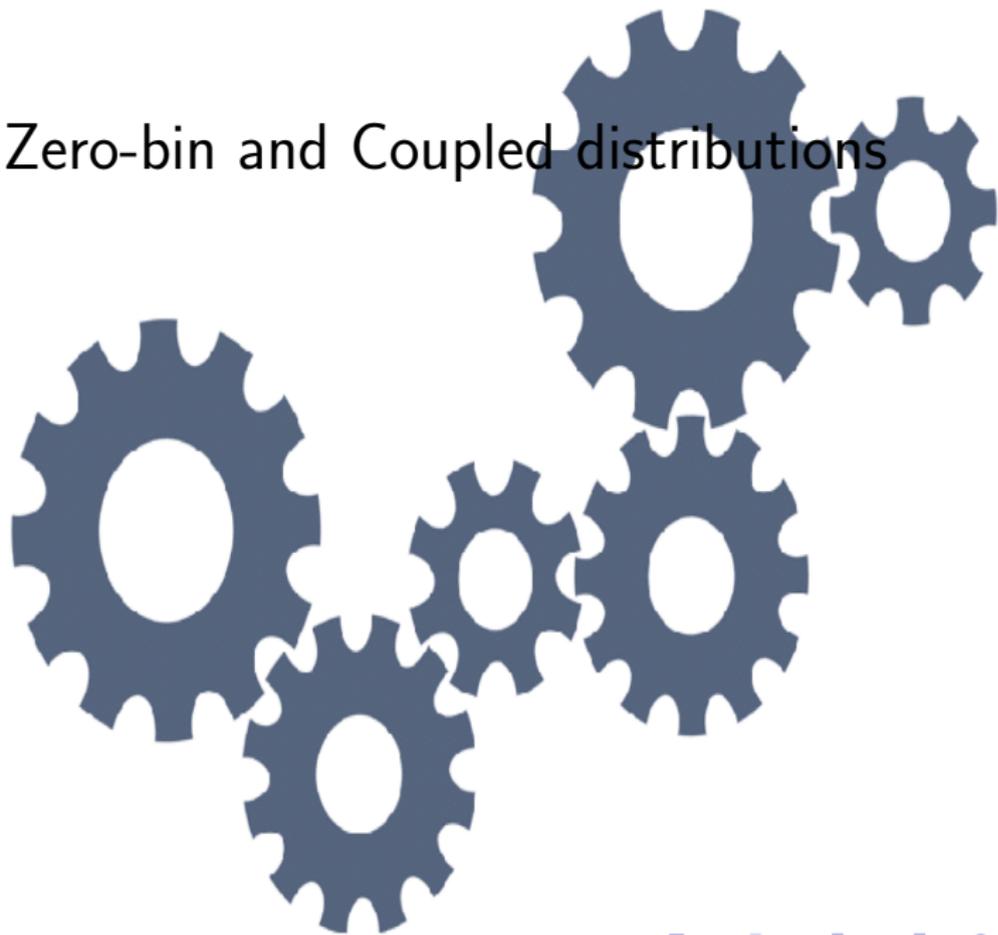
$$\mathcal{S}(\tau_n^s, \tau_{\bar{n}}^s, \vec{p}_\perp^2, \vec{k}_\perp^2) = \# \text{tr} \langle 0 | S_{\bar{n}}^\dagger S_n \delta^{(2-2\epsilon)}(\vec{p}_\perp + \mathbb{P}_{n\perp}) \delta(\tau_n^s - \hat{\tau}_{\bar{n}}^s) \delta^{(2-2\epsilon)}(\vec{k}_\perp + \bar{\mathbb{P}}_{n\perp}) \delta(\tau_{\bar{n}}^s - \hat{\tau}_n^s) S_n^\dagger S_{\bar{n}} | 0 \rangle .$$

- As $b \rightarrow 1$, $\mathbb{P}_{n\perp}$ and $\bar{\mathbb{P}}_{n\perp}$ measure transverse momentum of ultrasoft modes which scales as λ^2 .
- Factoring out $\delta^{(2-2\epsilon)}(\vec{p}_\perp) \delta^{(2-2\epsilon)}(\vec{k}_\perp)$ gives jet ans soft function for thrust and the factorization theorem reduces to

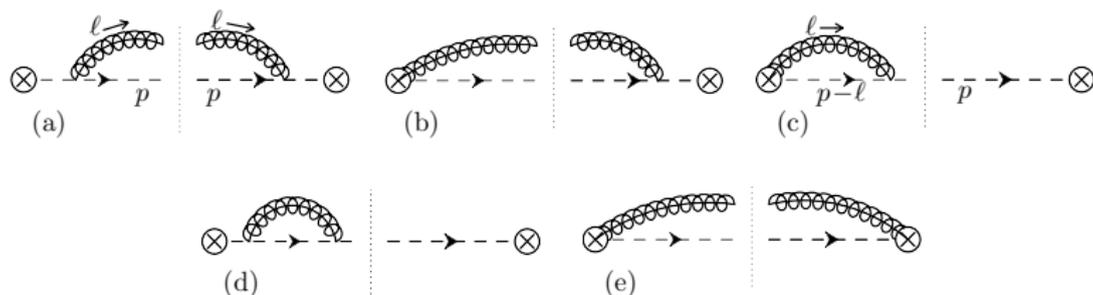
$$\frac{1}{\sigma_0} \frac{d\sigma}{d\tau_L d\tau_R} = H(Q; \mu) \cdot [\mathcal{J}_n \otimes \mathcal{J}_{\bar{n}} \otimes \mathcal{S}](\tau)$$

- We keep the transverse momentum convolutions for all b values and show that recoil gives only power suppressed terms when b approaches 1.

3. The Zero-bin and Coupled distributions



The one-loop jet function



- Only diagram (b) exhibits a zero-bin. The unsubtracted result is given as

$$\mathcal{J}_{\text{b,unsub}}^{(1)}(\tau, 0) = \frac{2}{1+b} \frac{\alpha_s(\mu) C_F}{\pi} \frac{e^{\epsilon\gamma_E} w^2}{\Gamma(1-\epsilon)} \left(\frac{\mu}{Q}\right)^{2\epsilon} \left(\frac{\nu}{Q}\right)^\eta \frac{1}{\tau^{1+\frac{2\epsilon}{1+b}}} \int_0^1 dx [(1-x)^{-b} + x^{-b}]^{\frac{2\epsilon}{1+b}} \frac{(1-x)}{x^{1+\eta}}$$

with $x = l^-/Q$.

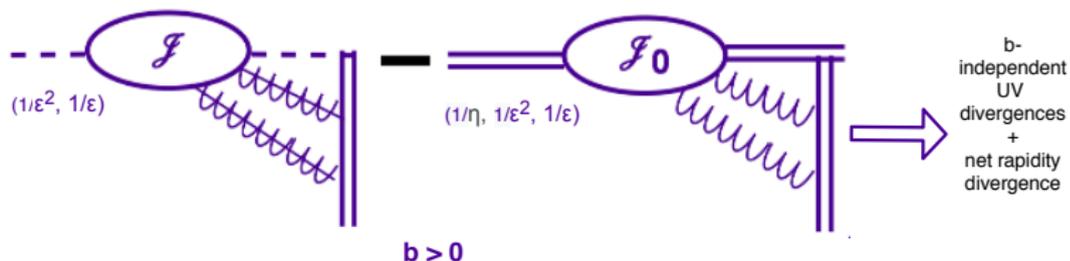
- η regulates the divergence at $x \rightarrow 0$, only when $b \leq 0$.

The Zero-bin of the jet function

- The zero-bin becomes non-trivial for generalized angularities. The same observation was also made recently in [2012.09212](#) for recoil-free version of angularities.
- The zero-bin for generalized angularities gives

$$\mathcal{J}_{b,\text{unsub}}^{(1)}(\tau, 0) = \frac{2}{1+b} \frac{\alpha_s(\mu) C_F}{\pi} \frac{e^{\epsilon\gamma_E} w^2}{\Gamma(1-\epsilon)} \left(\frac{\mu}{Q}\right)^{2\epsilon} \left(\frac{\nu}{Q}\right)^\eta \frac{1}{\tau^{1+\frac{2\epsilon}{1+b}}} \int_0^\infty dx [1+x^{-b}]^{\frac{2\epsilon}{1+b}} \frac{1}{x^{1+\eta}}$$

- Zero-bin gives η -divergence for all b exponents.

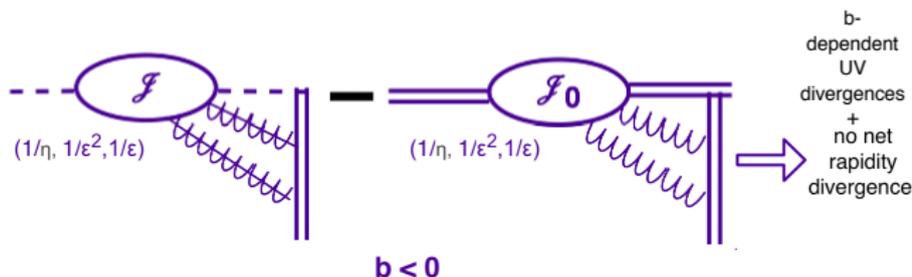


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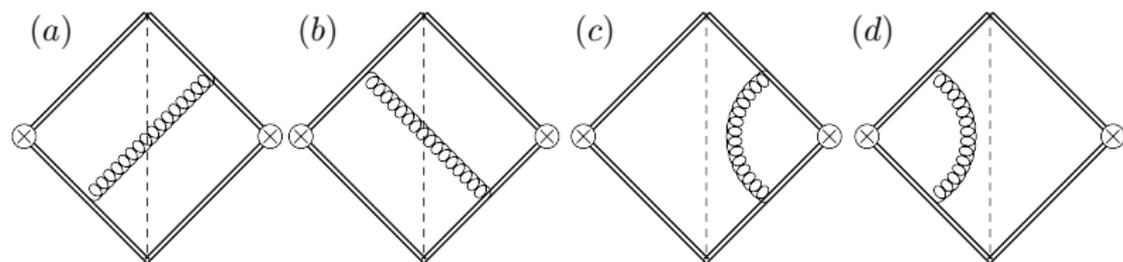
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- Zero-bin gives η -divergence for all b exponents.



The one-loop soft function



- At one-loop, need to compute only diagram (a). The one-loop soft function computes to

$$\begin{aligned}
 S^{(1)}(\tau_L, \tau_R, \vec{p}_t^2, \vec{k}_t^2) &= \frac{\alpha_s(\mu) C_F}{\pi} \frac{\mu^{2\epsilon} e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \nu^\eta w^2 Q \delta(\tau_L) \delta(\vec{k}_t^2) \theta\left(\left(\frac{|\vec{p}_t|}{Q\tau_R}\right)^{1/b} - 1\right) \\
 &\times (\vec{p}_t^2)^{-1-\epsilon-\frac{\eta+1}{2}} \frac{\left|1 - \left(\frac{Q\tau_R}{|\vec{p}_t|}\right)^{2/b}\right|^{-\eta}}{|b| \left(\frac{Q\tau_R}{|\vec{p}_t|}\right)^{1-\eta/b}} + \left\{ \begin{array}{l} \tau_L \leftrightarrow \tau_R \\ \vec{k}_t^2 \leftrightarrow \vec{p}_t^2 \end{array} \right\}.
 \end{aligned}$$

- This result holds for all angularity exponents.

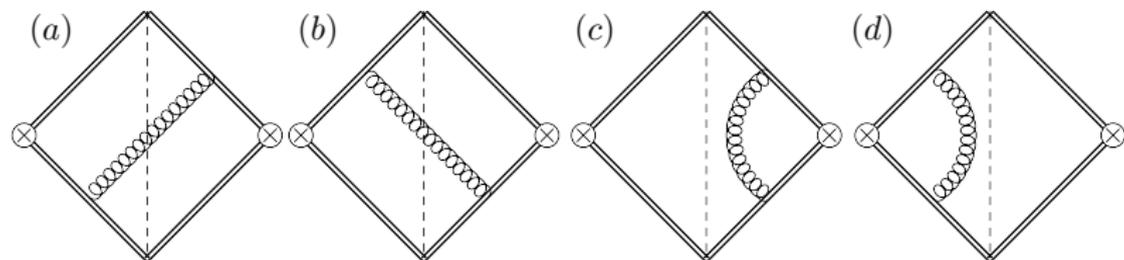
Dealing with Coupled Distributions

- To avoid coupled distributions in τ and \vec{p}_t , we perform a change of variables

$$v = \frac{Q \tau}{|\vec{p}_t|} \quad \text{for } b > 0; \quad \vec{u} = \frac{\vec{p}_t}{Q \tau} \quad \text{for } b < 0$$

- In these variables, one-loop soft contains a net η -divergence for $b > 0$ while no η -divergence for $b < 0$, consistent with the one-loop jet function.
- This leads to the appearance of a new kind of integral at the level of cross-section, which provides the correct recoil term at one-loop order.

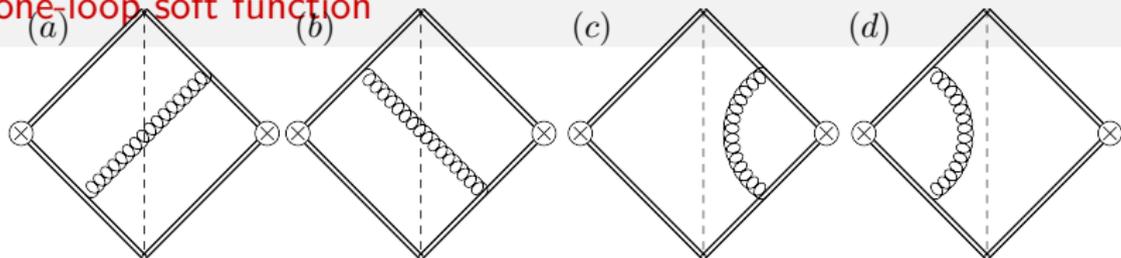
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 &\times (\vec{p}_t^2)^{-1-\epsilon-\frac{\eta+1}{2}} \frac{\left|1 - \left(\frac{Q\tau_R}{|\vec{p}_t|}\right)^{2/b}\right|^{-\eta}}{|b| \left(\frac{Q\tau_R}{|\vec{p}_t|}\right)^{1-\eta/b}} + \left\{ \begin{array}{l} \tau_L \leftrightarrow \tau_R \\ \vec{k}_t^2 \leftrightarrow \vec{p}_t^2 \end{array} \right\}.
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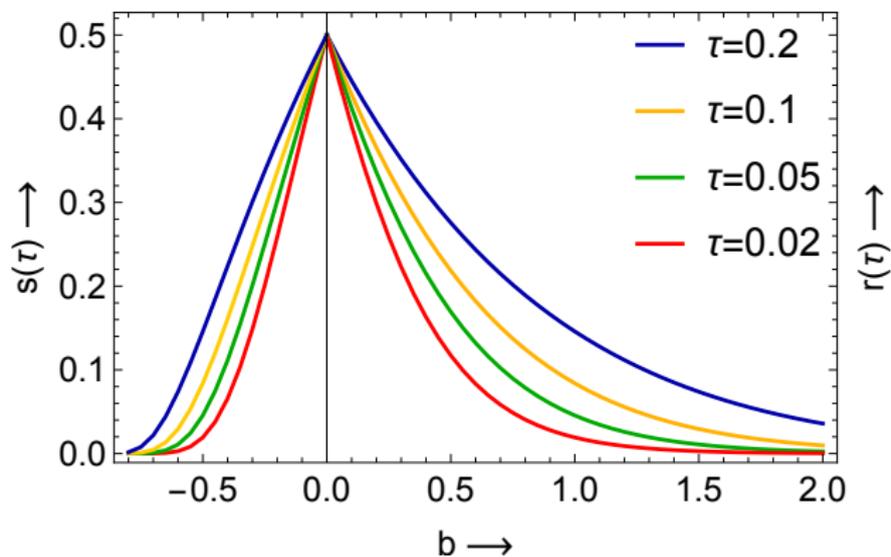
$$S^{(1+)} \simeq \# \frac{\mathcal{O}(1-v) |1-v^{2/b}|^{-\eta}}{|b| v^{1-\eta/b}} \times (\vec{p}_\perp^2)^{-1-\epsilon-\frac{\eta+1}{2}}$$

$$= \frac{1}{b} \left(\frac{b}{\eta} \delta(v) + \left[\frac{\mathcal{O}(1-v)}{v} \right]_+ + \mathcal{O}(\eta) \right) \times \left(2 \left[\frac{1}{(\vec{p}_\perp^2)^{1+\epsilon}} \right]_+ + \mathcal{O}(\eta) \right)$$

4. Fixed order results



Looking at the Recoil Contribution



Plotting $r(\tau)$ (for $b > 0$) and $s(\tau)$ (for $b < 0$) as a function of b for different τ values.

$b > 0$; small- τ and small- b limit

$$\left[\frac{1}{\sigma_0} \frac{d\sigma}{d\tau_b} \right]_{\tau_b \neq 0}^{b > 0} = \frac{\alpha_s C_F}{\pi} \left\{ -\frac{3}{1+b} \frac{1}{\tau_b} - \frac{4}{1+b} \frac{\ln \tau_b}{\tau_b} - \frac{4}{1+b} \frac{\ln(1-r)}{\tau_b} \right\}$$

$$\frac{r}{(1-r)^{1+b}} = \tau^b$$

- **Small- τ limit:**

$$r = a_1 \tau^b + a_2 \tau^{2b} + a_3 \tau^{3b} + a_4 \tau^{4b} + \dots$$

$$\begin{aligned} \Rightarrow \frac{\ln(1-r)}{\tau} &= \sum_{n=1}^{\infty} \frac{c_n}{\tau^{1-nb}} \\ &= \sum_{n=1}^{[1/b]-1} \frac{c_n}{\tau^{1-nb}} + \text{power - corrections} \end{aligned}$$

$b > 0$; small- τ and small- b limit

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$$\frac{r}{(1-r)^{1+b}} = \tau^b$$

• Small- b limit:

$$r = r_0 + b r_1 + b^2 r_2 + \dots \quad \text{where, } \frac{r_0}{1-r_0} = \tau^b$$

$$\Rightarrow \frac{\ln(1-r)}{\tau} = -\ln 2 \left[\frac{1}{\tau} \right]_+ - \frac{b}{2} \left[\frac{\ln \tau}{\tau} \right]_+ + \frac{b \ln 2}{2} \left[\frac{1}{\tau} \right]_+ + \mathcal{O}(b^2)$$

Numerical size of the recoil term compared to leading singular terms

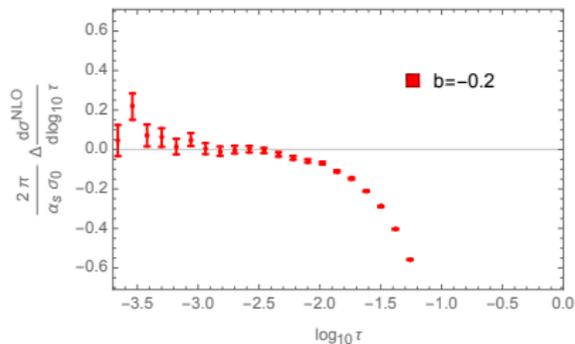
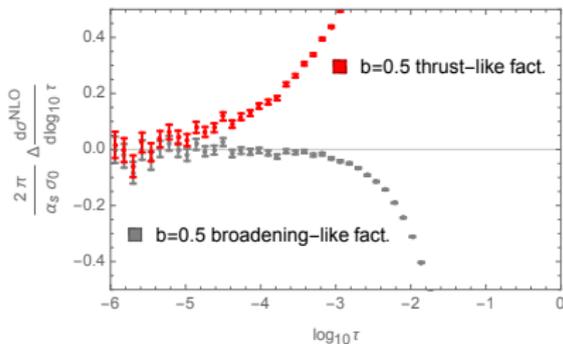
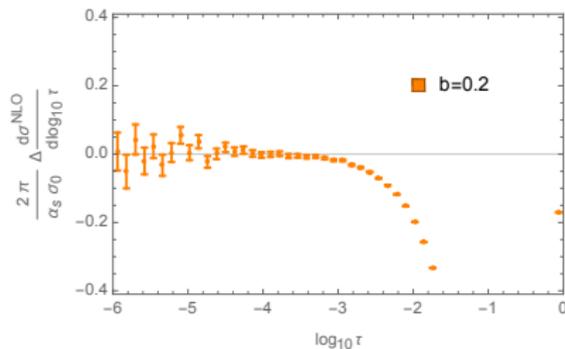
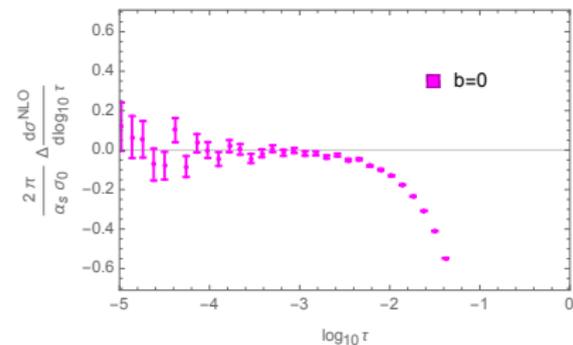
$$\left[\frac{1}{\sigma_0} \frac{d\sigma}{d\tau_b} \right]_{\tau_b \neq 0}^{b>0} = \frac{\alpha_s C_F}{\pi} \left\{ -\frac{3}{1+b} \frac{1}{\tau_b} - \frac{4}{1+b} \frac{\ln \tau_b}{\tau_b} - \frac{4}{1+b} \frac{\ln(1-r)}{\tau_b} \right\}$$

$$\left[\frac{1}{\sigma_0} \frac{d\sigma}{d\tau_b} \right]_{\tau_b \neq 0}^{b<0} = \frac{\alpha_s C_F}{\pi} \left\{ -\frac{3}{1+b} \frac{1}{\tau_b} - \frac{4}{(1+b)^2} \frac{\ln \tau_b}{\tau_b} - \frac{4}{(1+b)^2} \frac{\ln(1-s)}{\tau_b} \right\}$$

b	% correction for $\tau_b = 0.05$	% correction for $\tau_b = 0.1$	% correction for $\tau_b = 0.2$
1	2	6	18
0.5	8	16	38
0.25	16	26	54
0	31	45	80
-0.2	15	24	46
-0.5	2	5	13

Relative size of the extra singular contribution compared to the leading singular contribution in the peak region for the τ_b distribution, for various values of b . A 2 – 6% correction for $b = 1$ or -0.5 shows the typical size of the power corrections due to the additional term.

EVENT2 comparison



Difference between EVENT2 and our results from broadening-like factorization at NLO for $d\sigma/d\log_{10}\tau$ for different b values.

CAUTION

5. Preliminary NLL results



AREA UNDER CONSTRUCTION

A few Technicalities

- The convolutions in the factorization theorem are known to hold in the τ and \vec{p}_t variables.
- But the soft function cannot be well-defined in terms of independent distributions in these variables, for generalized angularities.

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- The convolutions in the factorization theorem are known to hold in the τ and \vec{p}_t variables.
- But the soft function cannot be well-defined in terms of independent distributions in these variables, for generalized angularities.
- One can use the exponentiation theorem to understand this subtlety.
- We have verified that the soft function in terms of the redefined still obeys the exponentiation theorem.
- This suggests that the RG equations can be equivalently written down for the new redefined variables as well.

Anomalous dimension for $b > 0$ in the conjugate space

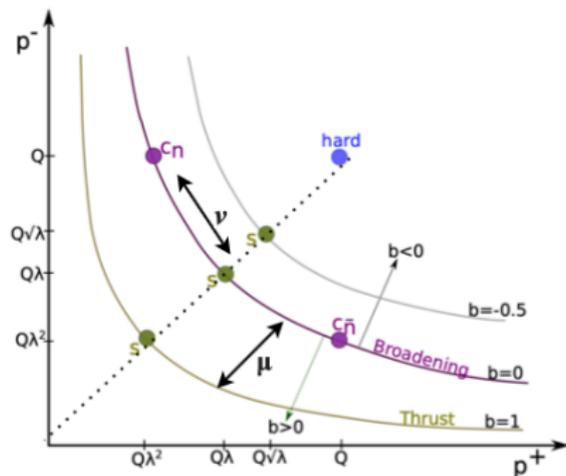
- The one-loop anomalous dimension for positive- b has the form

$$\gamma_{\mu}^{(1+)S} = -\frac{\alpha_s(\mu) C_F}{\pi} \left[2 \ln \frac{\nu}{\mu} + \frac{2}{b} (-\ln(s e^{\gamma_E}) - \Gamma(0, s)) \right]$$
$$\gamma_{\nu}^{(1+)S} = -\frac{\alpha_s(\mu) C_F}{\pi} \left[\ln \left(\frac{\vec{b}^2 \mu^2 e^{2\gamma_E}}{4} \right) \right]$$

where, s is Laplace conjugate of ν and \vec{b}^2 is the Fourier conjugate of \vec{p}_t^2 .

- This has some interesting features.
- The ν -anomalous dimension here has exactly the form as studied earlier for Higgs-pT resummation. [Chiu, Jain, Neill, Rothstein; 2012.](#)
- The **second term** in the μ -anomalous dimension has a $1/b$ dependence.

The μ -anomalous dimension



Integration limit proportional to b

- The recoil term arises from the phase space cut in the distribution and is lost if there is no such cut provided by the observable. Consistent with our observation at one-loop order.
- NLL result can only be computed numerically. (Work in progress)

Anomalous dimension for $b < 0$ angularities in conjugate space

- Only μ -anomalous dimension.

$$\gamma^{(1-)S} = -\frac{\alpha_s(\mu) C_F}{\pi} \left[\frac{\vec{b}^2}{4b^2} F_3\left(1, 1; 2, 2, 2; -\frac{\vec{b}^2}{4}\right) + \frac{2}{b} \ln\left(s e^{\gamma_E} \frac{\mu}{Q}\right) \right]$$

where, \vec{b} is Fourier conjugate to \vec{u} and s is Laplace conjugate to τ .

- Both the terms contain a $1/b$ singularity.
- As earlier, this is associated with the reduction in the range of integration and hence the resummed cross-section is finite even for $b = 0$.
- The reduction of this result to jet broadening result in the $b \rightarrow 0$ limit is highly non-trivial in this case. Can only be done at the level of cross-section.

Resummed Angularity cross-section in the $b \rightarrow 0^-$ limit

- $b \rightarrow 0^-$ limit

$$\frac{1}{\sigma_0} \frac{d^2 \sigma^{\text{NLL}}}{d\tau_L d\tau_R} = H^{\text{NLL}} \left[2 \frac{e^{\omega_0 \gamma_E - \frac{2}{b} K_0}}{\Gamma(-\omega_0)} \left(\frac{\mu}{Q}\right)^{\omega_0} \frac{1}{\tau_R^{1+\omega_0}} \int_0^1 dx \frac{1-x}{x^{3+\omega_0}} \int_0^\infty dz J_0\left(\frac{2\sqrt{z}(1-x)}{x}\right) \right. \\ \left. \times e^{-\omega_0 z} {}_2F_3(1,1;2,2,2;-z) \right] \times \{\tau_L \leftrightarrow \tau_R\}$$

where, $x = \frac{\tau_L^s}{\tau_R}$ and $z = \frac{b^2}{4}$.

- Broadening cross-section

Chiu, Jain, Neill, Rothstein; 2012

$$\frac{1}{\sigma_0} \frac{d^2 \sigma^{\text{NLL}}}{d e_L d e_R} = H^{\text{NLL}} \left[\left(\frac{\mu}{Q}\right)^{-\omega_S} \frac{e^{-\omega_S \gamma_E}}{\Gamma(\omega_S)} \frac{1}{e_R^{1-\omega_S}} \left(1 - \frac{\omega_S}{2-\omega_S} B_{1/2}(1+\omega_S, 0)\right) \right] \\ \times \{\tau_L \leftrightarrow \tau_R\}$$

The negative- b kernels in $b \rightarrow 0^-$ limit

- The two kernels appearing in the $b \rightarrow 0^-$ NLL cross-section expanded in powers of α_s can be written as

$$\omega_0 = -\frac{\alpha_s(\mu)}{2\pi b} \Gamma_0 \ln \frac{\mu}{\mu_0} - \frac{\alpha_s^2(\mu)}{8\pi^2 b} \Gamma_0 \left(\beta_0 \ln^2 \frac{\mu}{\mu_0} + \frac{\Gamma_1}{\Gamma_0} \ln \frac{\mu}{\mu_0} \right) + \mathcal{O}(\alpha_s^3)$$

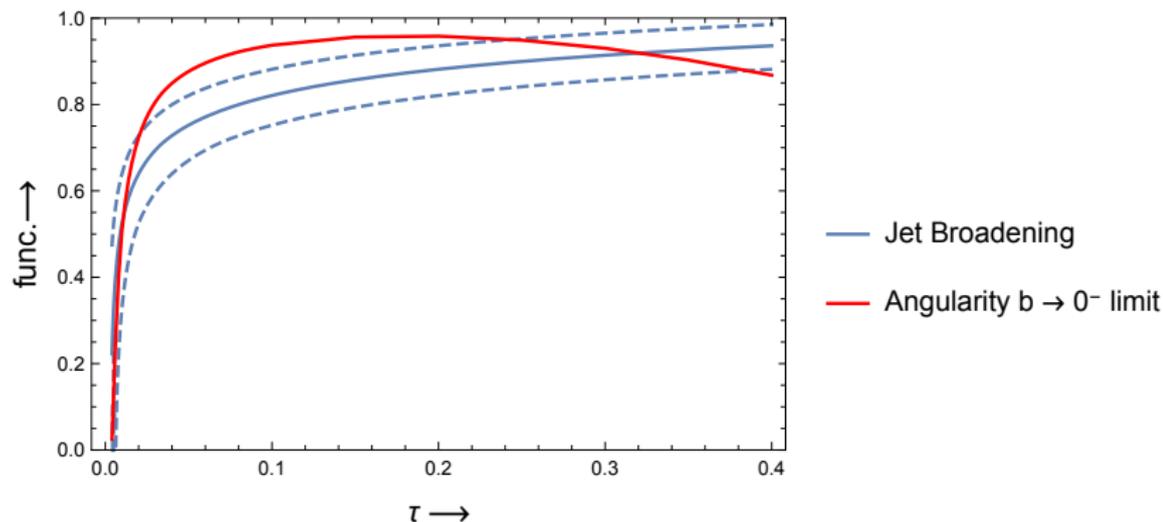
$$K_0 = -\frac{\alpha_s(\mu)}{4\pi b} \Gamma_0 \ln^2 \frac{\mu}{\mu_0} - \frac{\alpha_s^2(\mu)}{16\pi^2 b} \Gamma_0 \ln^2 \frac{\mu}{\mu_0} \left(\frac{2}{3} \beta_0 \ln \frac{\mu}{\mu_0} + \frac{\Gamma_1}{\Gamma_0} \right) + \mathcal{O}(\alpha_s^3)$$

- Making the canonical choice of scales, i.e. $\mu \sim Q\tau^{\frac{1}{1+b}}$ and $\mu_0 \sim Q\tau$.
-

$$\omega_0 = \frac{\alpha_s(\mu)}{2\pi} \Gamma_0 \ln \tau + \frac{\alpha_s^2(\mu)}{8\pi^2} \Gamma_1 \ln \tau + \mathcal{O}(\alpha_s^3) \xrightarrow{b \rightarrow 0} -\omega_S$$

$$K_0 \sim \mathcal{O}(b) \xrightarrow{b \rightarrow 0} 0$$

Comparison to $b \rightarrow 0$ (Preliminary)



Plotting the angularity (red) and broadening (blue) result after cancelling the common terms in the cross-section. The blue band is obtained by the simultaneous variation of μ and ν .

Summary

1. Jet angularities provide a novel way of looking into the substructure which remains unexposed while looking at a single event shape observable.
2. The presence of two kinematical scales ($Q\tau$ and \vec{p}_t), two types of divergences and renormalization scales and a continuous dependence on b , make this analysis very rich and complex to compute.
3. At NLO, a broadening-like factorization provides angularity distributions for all $b > -1$.
4. This analysis allows us to smoothly interpolate between the thrust and jet broadening limits.
5. The one-loop framework provides evidence that the NLO singular cross-section for recoil-sensitive angularities is not-differentiable at $b = 0$.
6. There is completely different kind and number of divergences for $b > 0$ and $b < 0$ angularities, hence requiring different RG structures.

References

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