Precise strong coupling determinations from tau decays and $e^+e^- \to \mathrm{hadrons}$

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- DB, M. Golterman, K. Maltman, S. Peris, M.V. Rodrigues and W. Schaaf, arXiv:2012.10440 , PRD (2021).

- DB.V. Mateu, arXiv:1912:06237 PLB (2020), 2001:11041 JHEP (2020)

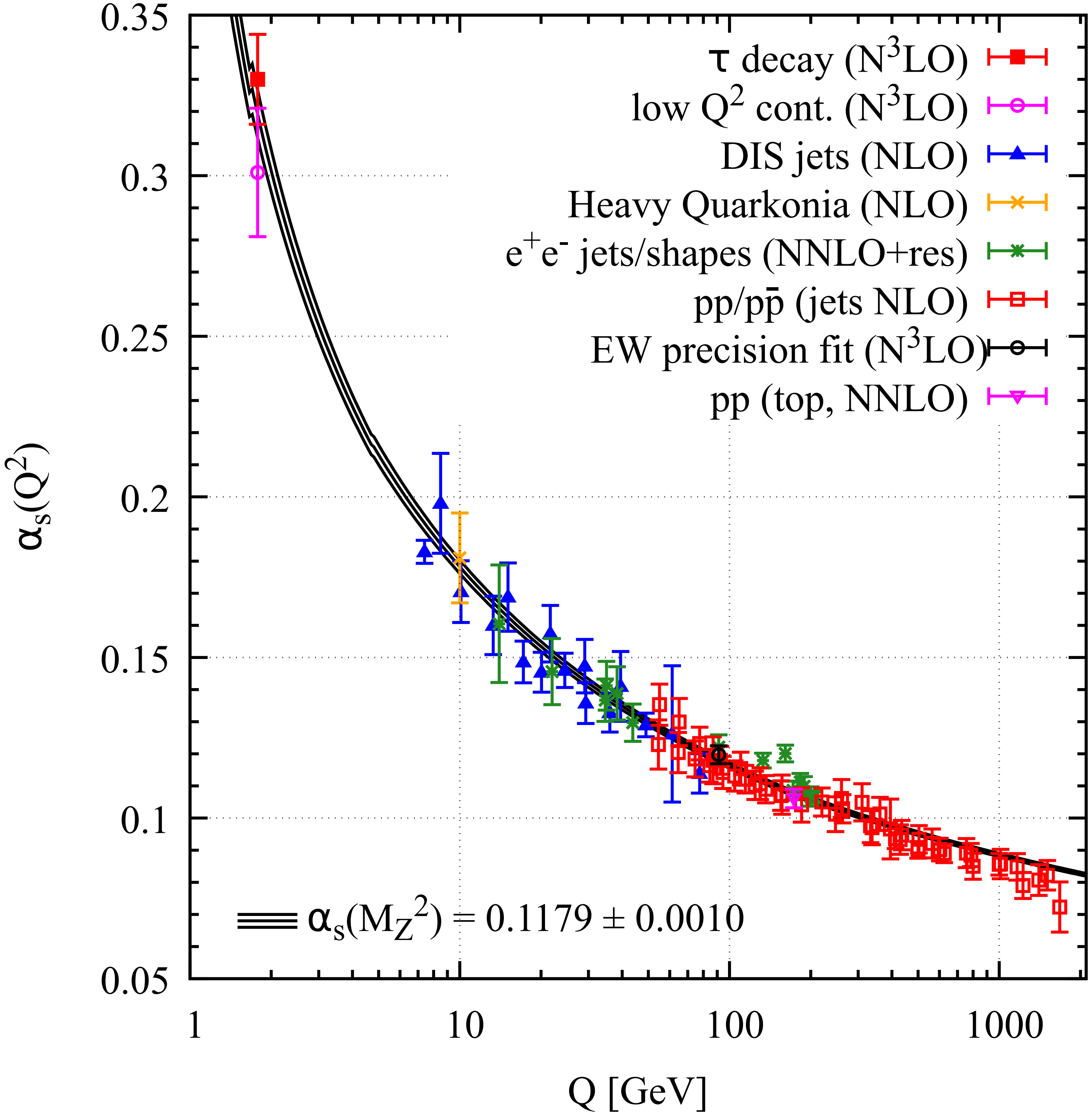
- DB.V Mateu and MV Rodrigues, in preparation

Vienna, March 2021

α_s in 2021

The strong coupling in 2021

PDG 2019

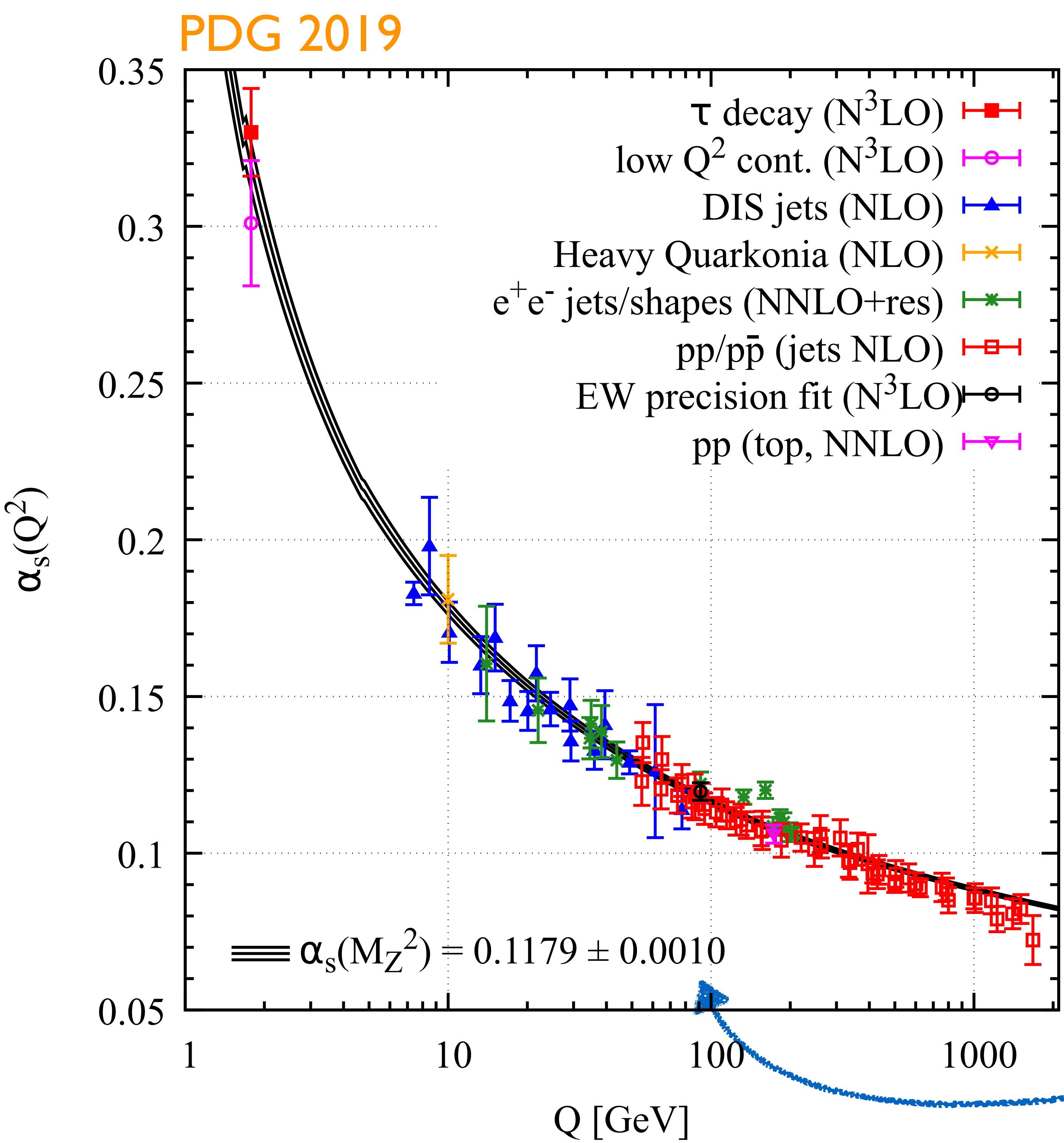


Overall picture is very consistent.

Discrepancies persist:
uncertainty has been *enlarged.*

Precision: cross-sections for the LHC, top-quark observables, SM vacuum…

The strong coupling in 2021



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Discrepancies persist: uncertainty has been *enlarged.*

The PDG uncertainty was +/- 0.0007 in 2014

Precision: cross-sections for the LHC, top-quark observables, SM vacuum…

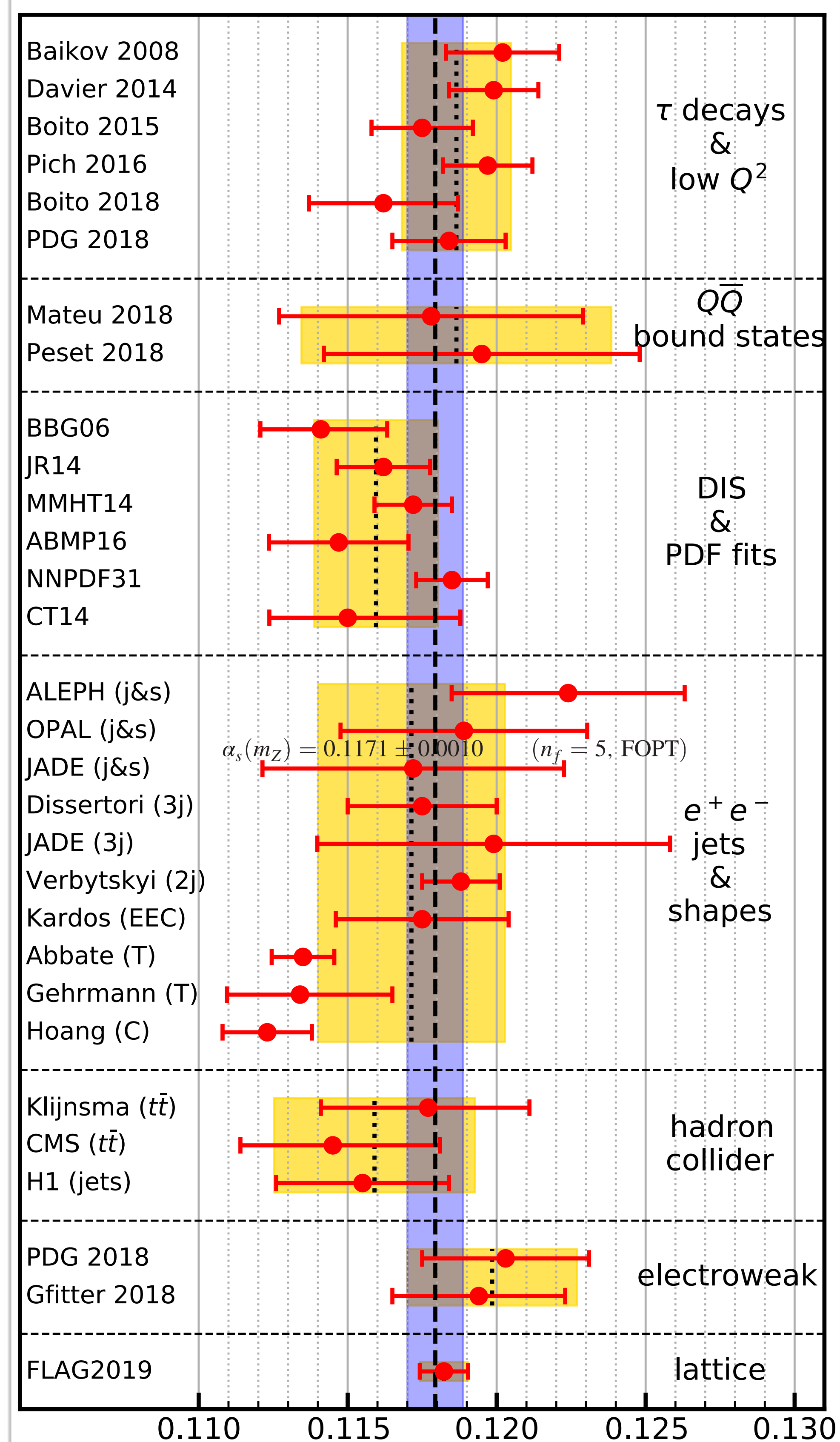
The strong coupling in 2021

PDG 2019

Tensions in determinations from same data

Event shapes give systematically lower results

Starting to be dominated by lattice (some uncertainties should still be scrutinised?)



$$\alpha_s^{(\mathrm{PDG})}(m_Z) = 0.1179 \pm 0.0010$$

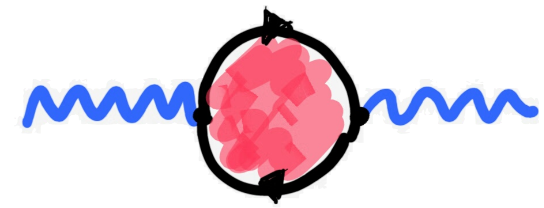
Strong coupling from quarkonium sum rules

- DB.V. Mateu, arXiv:1912:06237 PLB (2020), 2001:11041 JHEP (2020)
- DB.V Mateu and MV Rodrigues, in preparation

Sum rules

$$j^{\mu}(x) = \bar{q}(x)\gamma^{\mu}q(x)$$

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle 0|T\{J_\mu(x)J_\nu(0)^\dagger\}|0\rangle$$



Correlators with massive quarks: expansion around $q^2 = 0$

$$R_{q\bar{q}}(s) = \frac{\sigma_{e^+e^-\to q\bar{q}+X}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$ (for *q=c, b*)

Using analyticity and unitarity (dispersion relation): sum rules

Experiment Theory

$$M_q^{V,n} = \int \frac{\mathrm{d}s}{s^{n+1}} R_{q\bar{q}}(s) = \frac{12\pi^2 Q_q^2}{n!}\frac{\mathrm{d}^n}{\mathrm{d}s^n}\Pi_q^V(s)\Big|_{s=0}$$

Shifman, Vainshtein, Zakharov '79

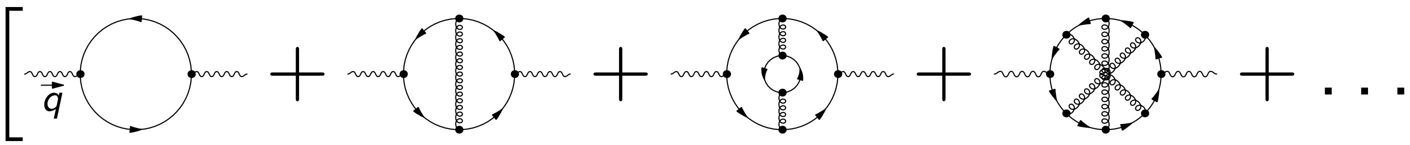
We restrict the sum rules to $n \leq 4$. Typical scale m_q/n.

Relativistic sum rules

Theory input

A Maier

$$M_q^{V,n} = \frac{12\pi^2 Q_q^2}{n!} \left(\frac{d}{dq^2}\right)^n \Big[\cdots \Big]_{q^2=0}$$



Perturbative expansion

$$\hat{M}_q^{X,n} = \frac{1}{(2\,\overline{m}_q)^{2n}} \sum_{i=0} \left[\frac{\alpha_s(\overline{m}_q)}{\pi}\right]^i c_i^{X,n}$$

(summing logs)

Known up to $\mathcal{O}(\alpha_s^3)$ for $n \leq 4$

Chetyrkin, Kühn, Sturm ’06; Boughezal, Czakon, Schutzmeier ’06
Maier, Maierhöfer, Smirnov ’08/’09; Maier and Marquard '17

General expansion in terms of the two scales (using RG)

$$M_q^{(n)} = \frac{1}{[2\,\overline{m}_b(\mu_m)]^{2n}} \sum_{i=0} \left[\frac{\alpha_s^{(n_f)}(\mu_\alpha)}{\pi}\right]^i \sum_{a=0}^{i} \sum_{b=0}^{[i-1]} c_{i,a,b}^{(n)}(n_f)\, \ln^a\left(\frac{\mu_m}{\overline{m}_b(\mu_m)}\right) \ln^b\left(\frac{\mu_\alpha}{\overline{m}_b(\mu_m)}\right)$$

Highly sensitive to the mass, ideal for quark-mass determinations

Dehnadi, Hoang, Mateu, Zebarjad ’11, Dehnadi, Hoang, Mateu ‘15
Chetyrkin, Kühn, Steinhauser, Sturm, Maier…

Ratios of moments: strong coupling extraction

We consider dimensionless ratios of moments

$$R_q^{X,n} \equiv \frac{\left(M_q^{X,n}\right)^{\frac{1}{n}}}{\left(M_q^{X,n+1}\right)^{\frac{1}{n+1}}}$$

Central object of this part

…similar to the ones used in lattice studies of the PS correlators

Maezawa, Petreczky '16

Perturbative expansion

$$R_b^{V,n} = \sum_{i=0}\left[\frac{\alpha_s(\mu_\alpha)}{\pi}\right]^i \sum_{k=0}^{[i-1]}\sum_{j=0}^{[i-2]} r_{i,j,k}^{(n)} \ln^j\left(\frac{\mu_m}{\overline{m}_b(\mu_m)}\right)\ln^k\left(\frac{\mu_\alpha}{\overline{m}_b(\mu_m)}\right)$$

Example

$$a_s = \frac{\alpha_s(\mu_\alpha)}{\pi}$$

$$R_c^{V,2} = 1.0449\big[1 + 0.57448\, a_s + (0.32576 + 2.3937\, L_\alpha)\, a_s^2$$
$$- \left(2.1093 + 4.7873 L_m - 6.4009 L_\alpha - 9.9736 L_\alpha^2\right) a_s^3 + \mathcal{O}\left(\alpha_s^4\right)\big]$$

DB, Mateu '19

Almost insensitive to the quark mass (only through logs at $\mathcal{O}(\alpha_s^2)$)
Sensitive to the coupling.
Available at N^{3}LO up to $R_q^{V,3}$
Can be accurately determined from data.

Ratios of moments: strong coupling extraction

Perturbative expansion

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Typical size of pt. corrections: 13%, 7%, and 5% (for charm with *n*=1,2,3)

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Typical size of pt. corrections: 13%, 7%, and 5% (for charm with n=1,2,3)

Non-perturbative contributions: gluon-condensate contri. known to NLO.

$$\Delta M_n^{X,\langle G^2\rangle} = \frac{1}{(4M_q^2)^{n+2}}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm RGI}\left[[a_X(n_f)]_n^0 + \frac{\alpha_s^{(n_f)}(\mu_\alpha)}{\pi}\, [a_X(n_f)]_n^1 \right]$$

Added as an estimate of non-perturbative uncertainties.

Completely irrelevant for the bottom-quark case.

Theory errors: scale variation

$$R_b^{V,n} = \sum_{i=0} \left[\frac{\alpha_s(\mu_\alpha)}{\pi}\right]^i \sum_{k=0}^{[i-1]} \sum_{j=0}^{[i-2]} r_{i,j,k}^{(n)} \ln^j\left(\frac{\mu_m}{\overline{m}_b(\mu_m)}\right) \ln^k\left(\frac{\mu_\alpha}{\overline{m}_b(\mu_m)}\right)$$

Independent scale variation important for conservative error estimate

$\overline{m}_q \leq \mu_\alpha, \mu_m \leq \mu_{\max}$ With $\mu_{\max} = 4\,(15)\,\mathrm{GeV}$ for charm (bottom)

Dehnadi, Hoang, Mateu '15

With the following constraint

$$1/\xi \leq (\mu_m/\mu_\alpha) \leq \xi$$

With $\xi = 2$ our (canonical) choice

Aways checking order-by-order convergence.

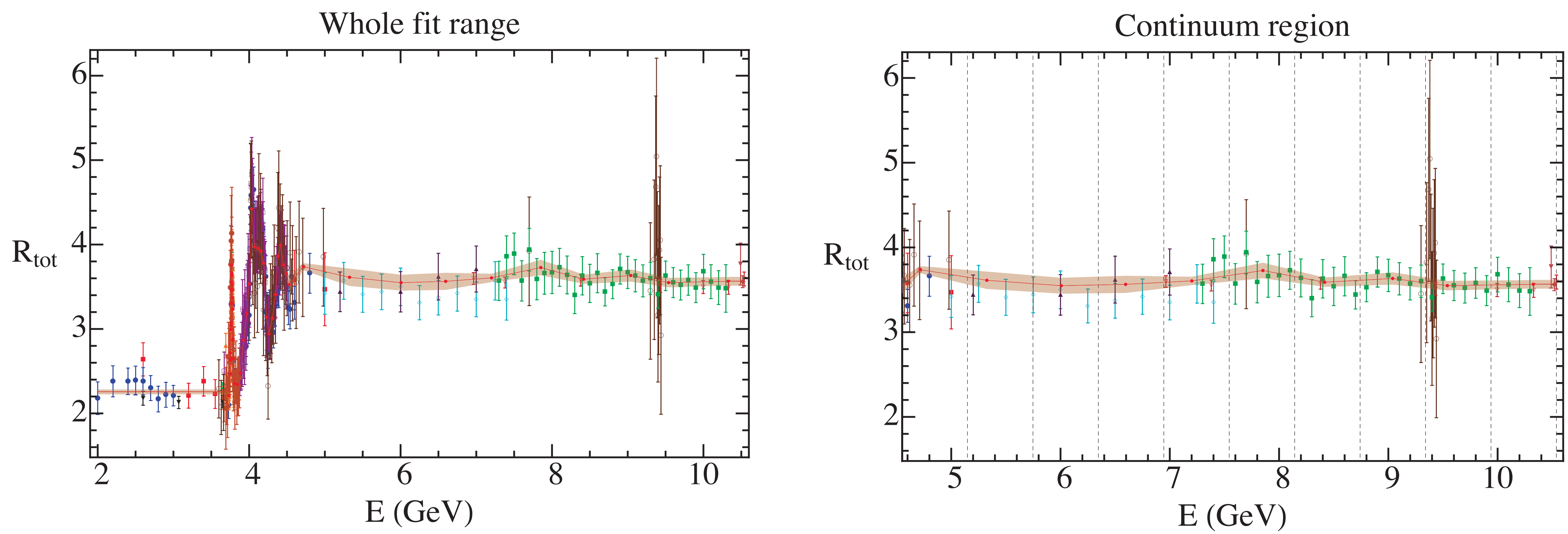
Experimental ratios of moments

Resonance data

Pt. continuum (theory)

$$M_q^{V,n} = \int \frac{\mathrm{d}s}{s^{n+1}} R_{q\bar{q}}(s) = \text{(resonan.)} + \int_{s_{\rm th}}^{s_{\rm max}} \frac{ds}{s^{n+1}} R_{q\bar{q}}(s) + \int_{s_{\rm max}}^{\infty} \frac{ds}{s^{n+1}} R_{q\bar{q}}(s)$$

Combined R data



Dehnadi, Hoang, Mateu, Zebarjad '11, Dehnadi, Hoang, Mateu '15

Exp moments determined from resonances and combined *R* data.

Correlations must be taken into account in the procedure.

Parametrize the continuum contribution (highly linear dependence on the coupling) (including mass corrections)

Experimental ratios of moments

Resonance data — Pt. continuum (theory)

$$M_q^{V,n} = \int \frac{\mathrm{d}s}{s^{n+1}} R_{q\bar{q}}(s) = (\text{resonan.}) + \int_{s_{\rm th}}^{s_{\max}} \frac{ds}{s^{n+1}} R_{q\bar{q}}(s) + \int_{s_{\max}}^{\infty} \frac{ds}{s^{n+1}} R_{q\bar{q}}(s)$$

Combined R data

Slightly update as compared with the original works. Dehnadi, Hoang, Mateu, Zebarjad '11, Dehnadi, Hoang, Mateu '15
Cross checked with other R-data combinations Keshavarzi, Nomura, Teubner '18

For the charm quark ratios we have

$$\Delta_\alpha = 0.1181 - \alpha_s$$

$$R_c^{V,1} = (1.770 - 0.705\,\Delta_\alpha) \pm 0.017, \qquad [\sigma_{\rm rel} = 0.98\%]$$

$$R_c^{V,2} = (1.1173 - 0.1330\,\Delta_\alpha) \pm 0.0022, \qquad [\sigma_{\rm rel} = 0.22\%]$$

$$R_c^{V,3} = (1.03535 - 0.04376\,\Delta_\alpha) \pm 0.00084. \qquad [\sigma_{\rm rel} = 0.104\%]$$

Continuum contribution smaller for higher n

Small uncertainties partially due to positive correlation among the moments Mn.

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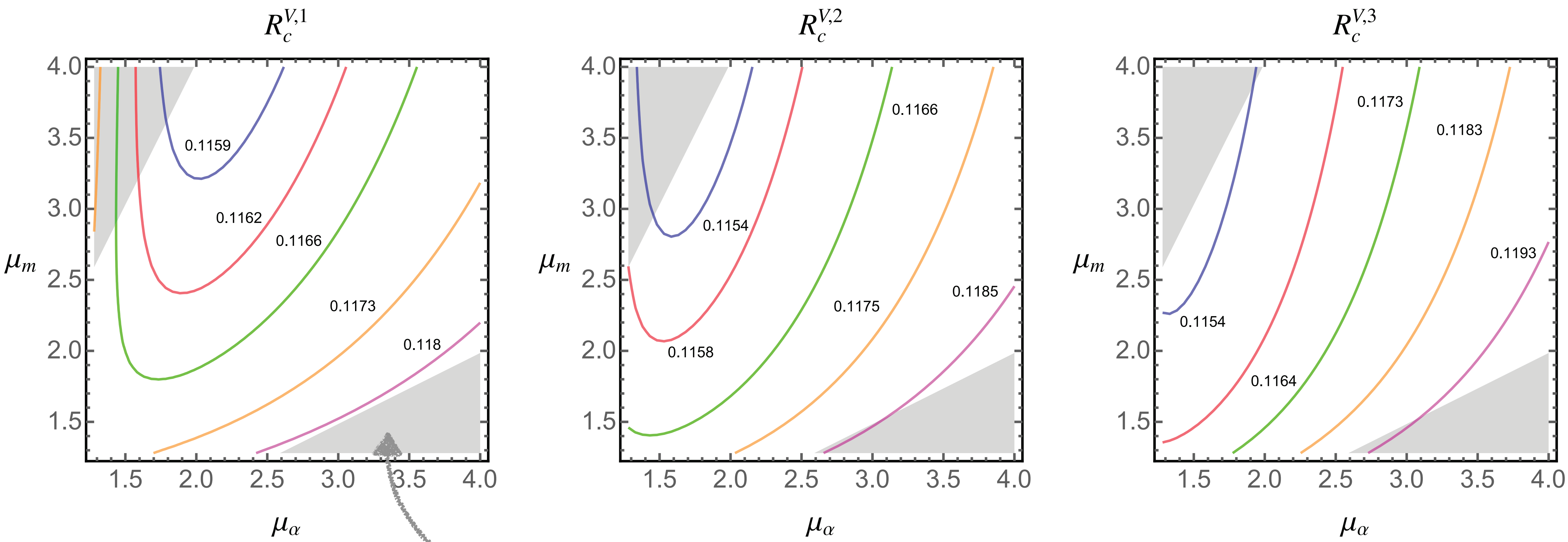
Results for charmonium sum rules

α_s with $n_f = 4$ and $R_c^{V,n}$ with $n = 1$, 2, and 3

Perturbative error analysis



$$1/\xi \leq (\mu_m/\mu_\alpha) \leq \xi$$

The gray areas are not included in the analysis

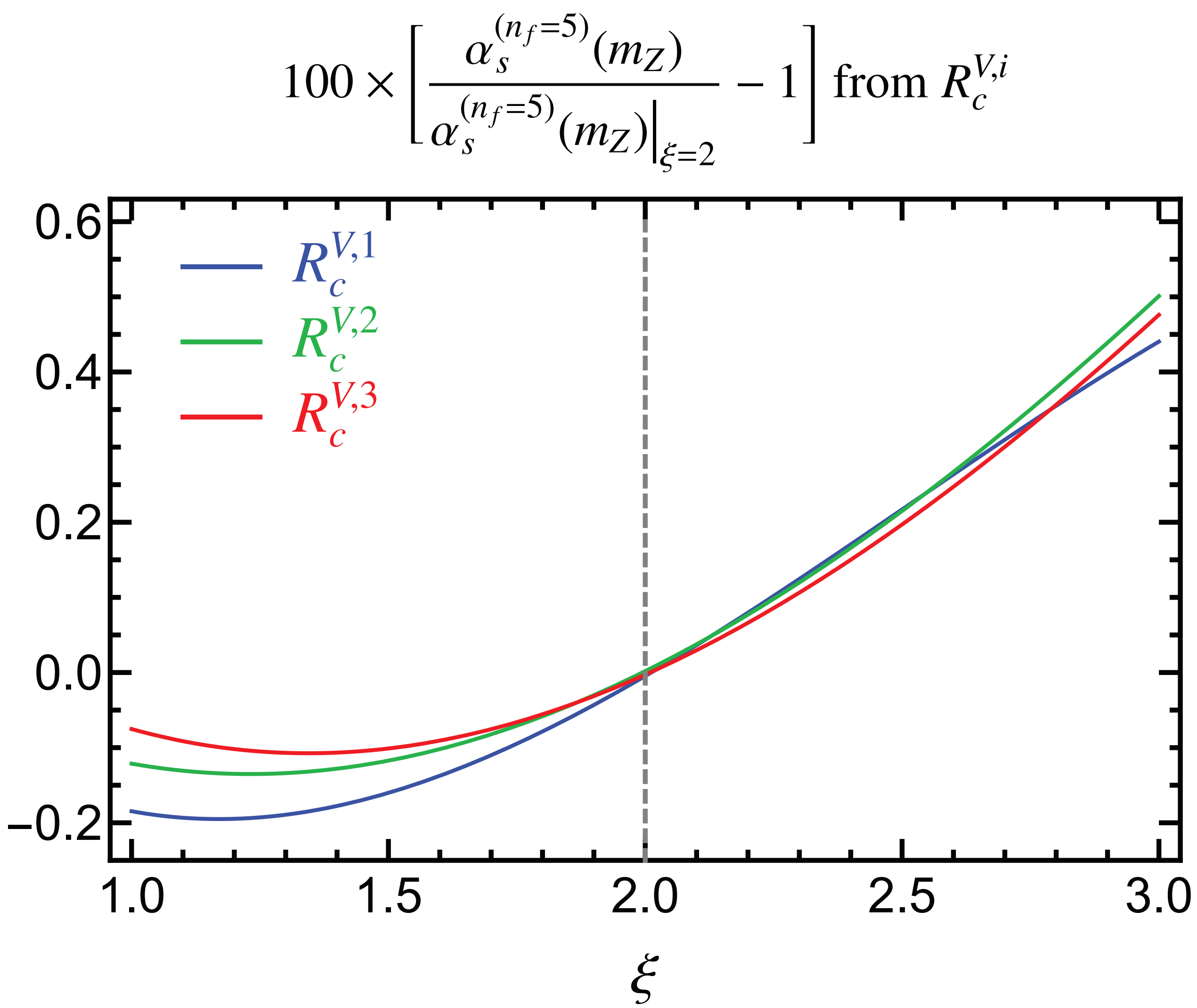
$$\xi = 2$$

Perturbative error analysis

$$1/\xi \leq (\mu_m/\mu_\alpha) \leq \xi$$

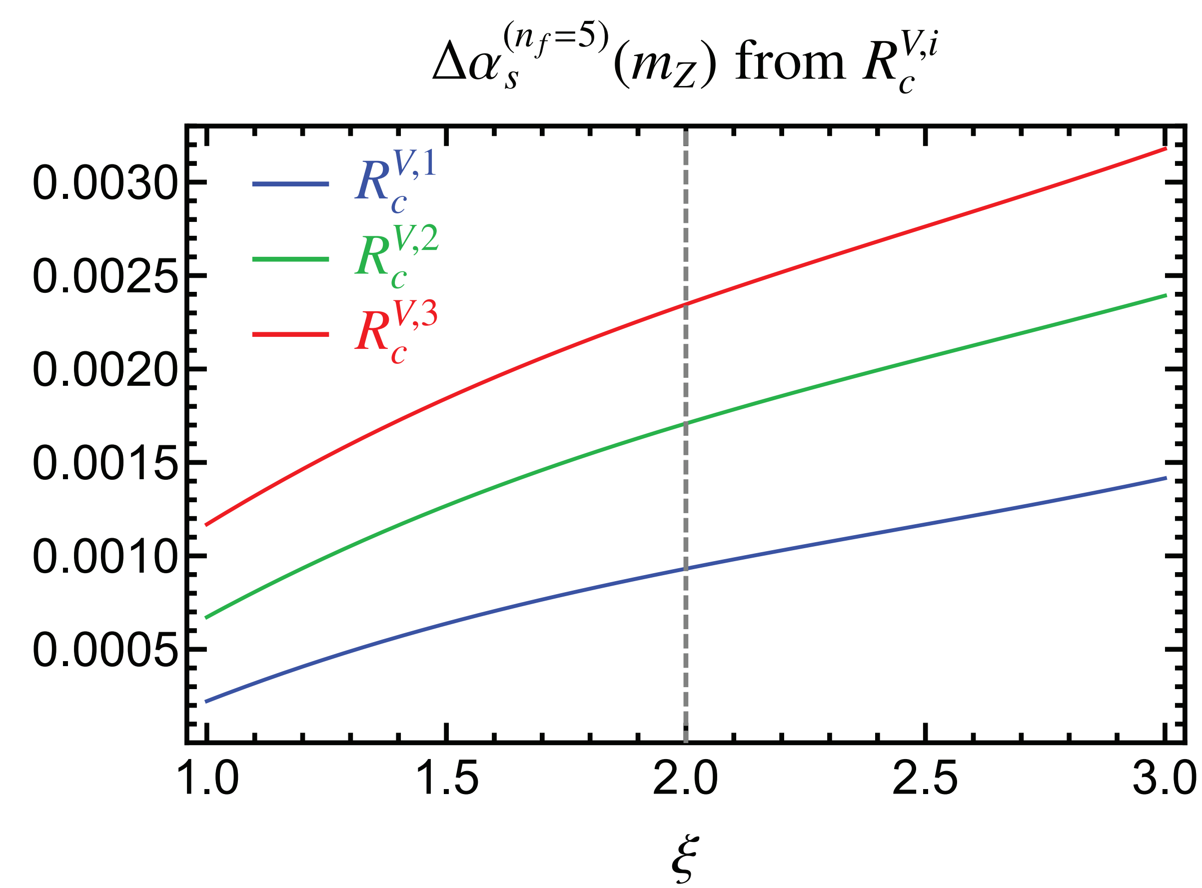
$\xi = 1 \rightarrow \mu_m = \mu_\alpha$

$\xi = 2$ our (canonical) choice



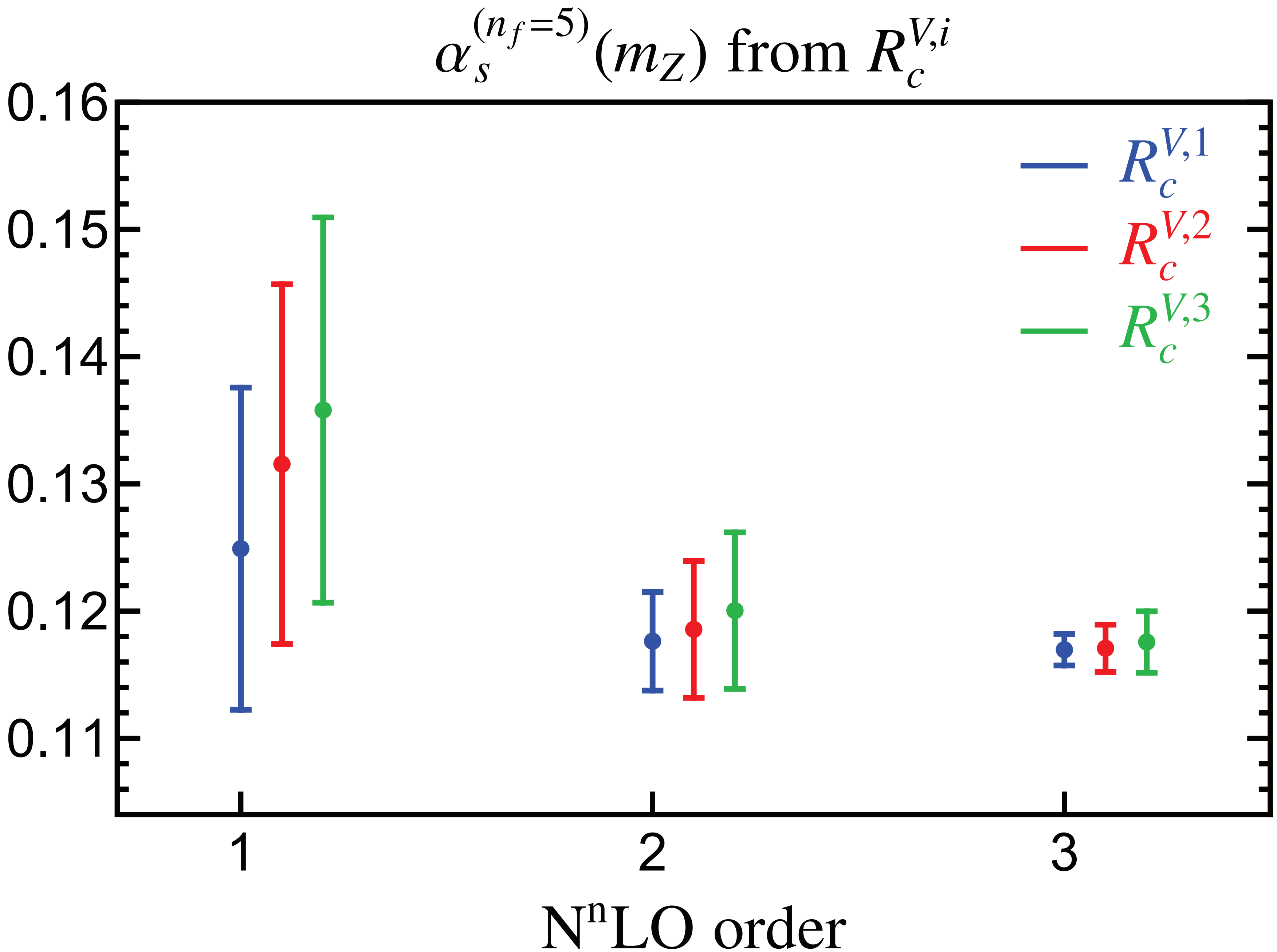
Small variations in the central values (~0.5%)

Diagonal variation: errors underestimated by a factor of up to ~2.0



Perturbative error analysis

Order by order convergence



Results

flavor	n	$\alpha_s^{(n_f=5)}(m_Z)$	σ_{pert}	σ_{exp}	σ_{m_q}	σ_{np}	σ_{total}
bottom	1	0.1183	0.0011	0.0089	0.0002	0.0000	0.0090
	2	0.1186	0.0011	0.0046	0.0001	0.0000	0.0048
	3	0.1194	0.0013	0.0029	0.0001	0.0000	0.0032
charm	1	0.1168	0.0010	0.0028	0.0003	0.0006	0.0030
	2	0.1168	0.0015	0.0009	0.0003	0.0007	0.0019
	3	0.1173	0.0020	0.0005	0.0003	0.0006	0.0022

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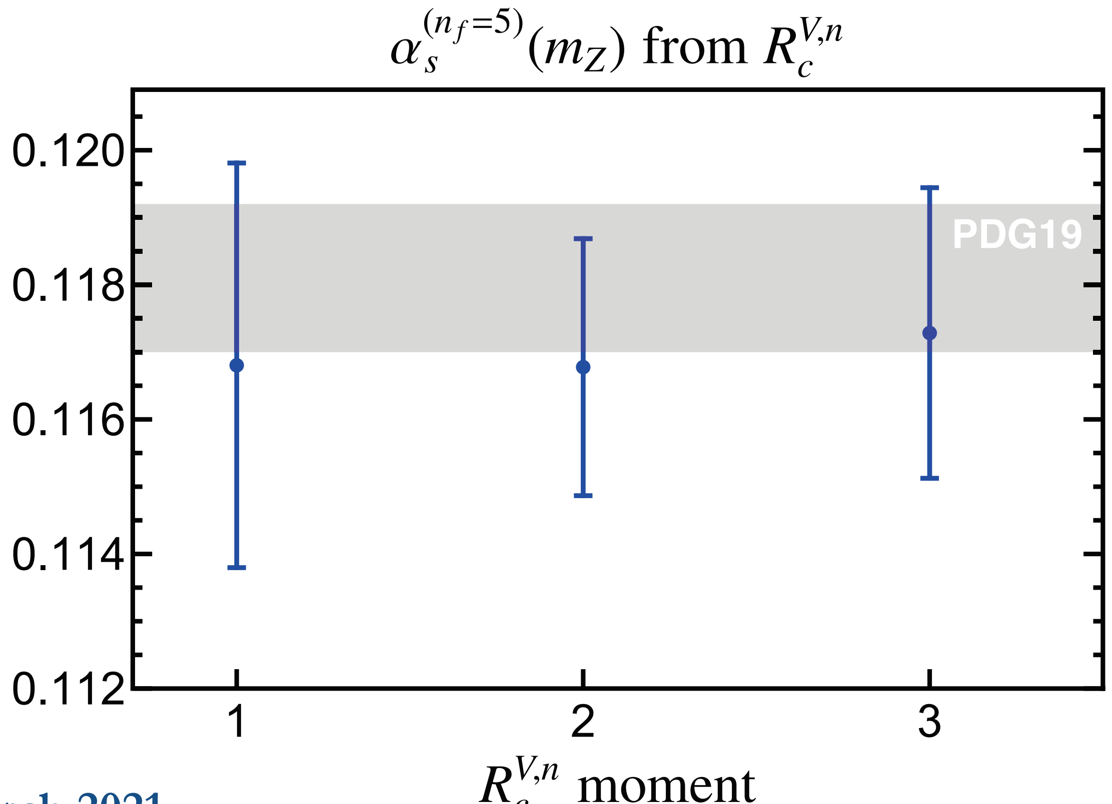

$$\alpha_s(m_Z) = 0.1168 \pm 0.0019$$

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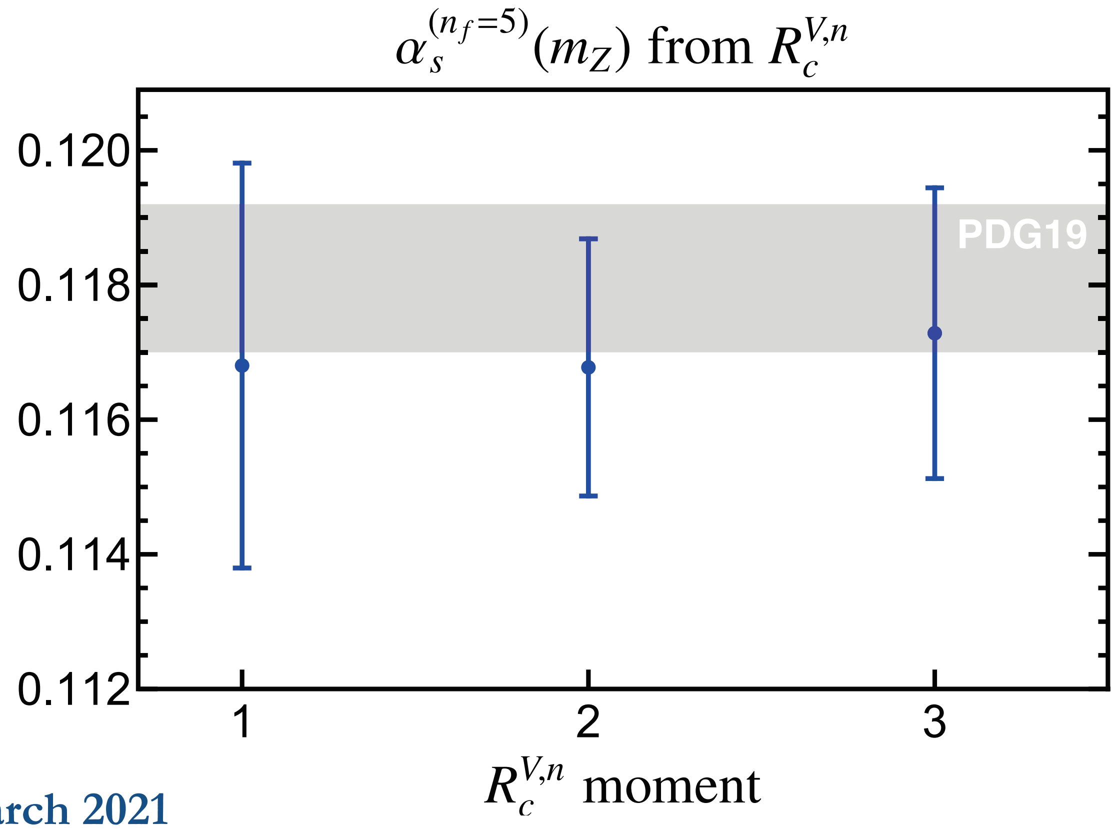
Trend to larger values



$$\alpha_s(m_Z) = 0.1168 \pm 0.0019$$

(Re)analysis of lattice data for pseudo-scalar charm-quark moments

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Results from lattice correlators

Data for moments of the pseudo-scalar corrents are available from the lattice

moment	[6]	[9]	[10]	[11]	[12]
$M_c^{P,0}$	1.708(7)	1.709(5)	1.699(9)	1.705(5)	–
$R_c^{P,1}$	–	–	1.199(4)	1.1886(13)	1.188(5)
$R_c^{P,2}$	–	–	1.0344(13)	1.0324(16)	1.0341(19)

[6] HPQCD, Allison et al, Phys. Rev. D (2008)

[9] McNeile et al., Phys. Rev. D (2010)

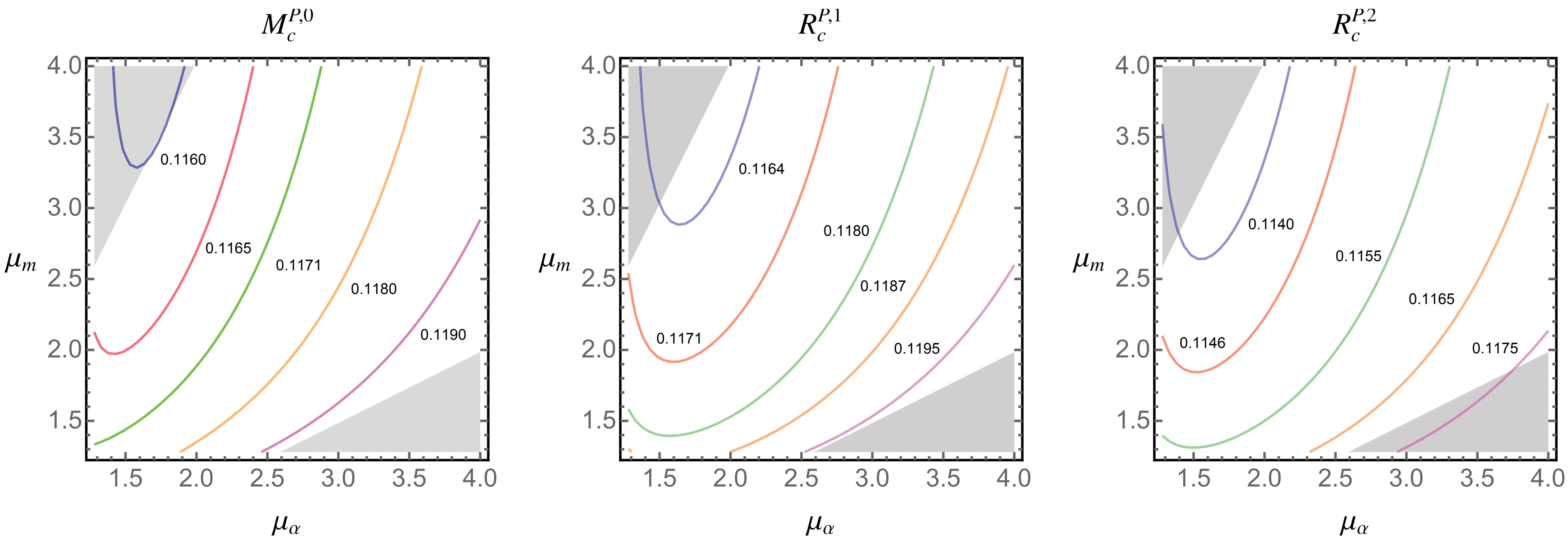
[10] Maezawa and Petreczky, Phys. Rev. D (2016)

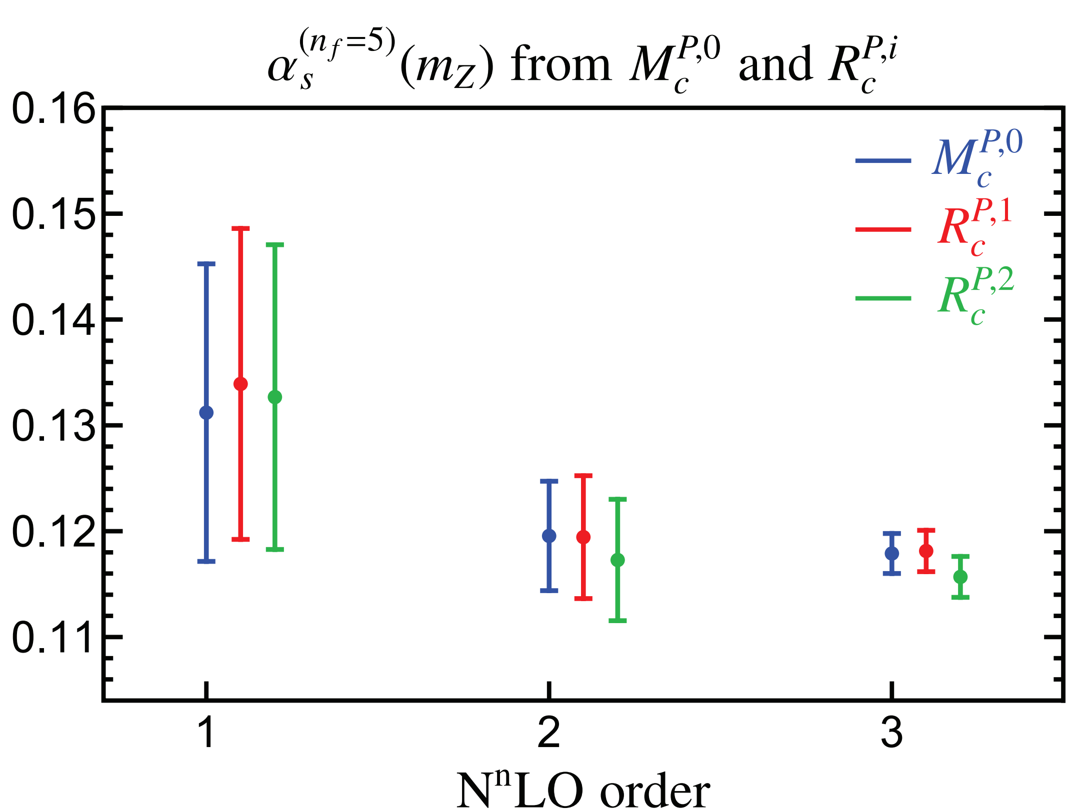
[11] Petreczky and Weber, Phys. Rev. D (2019)

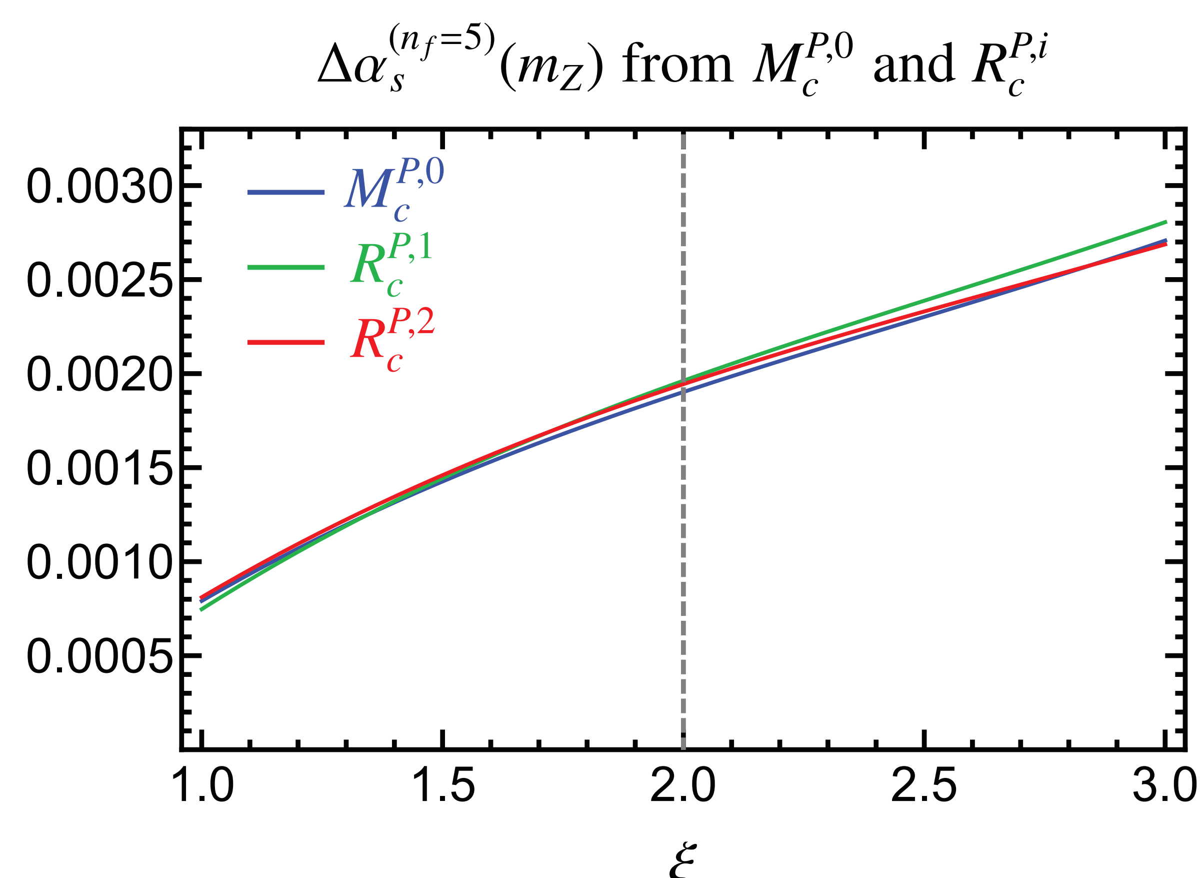
[12] Nakayama, Fahy, Hashimoto, Phys. Rev. D (2016)

We use the ratios and the 0-th moment to extract alpha_s and reassess pt. errors

Results from lattice correlators



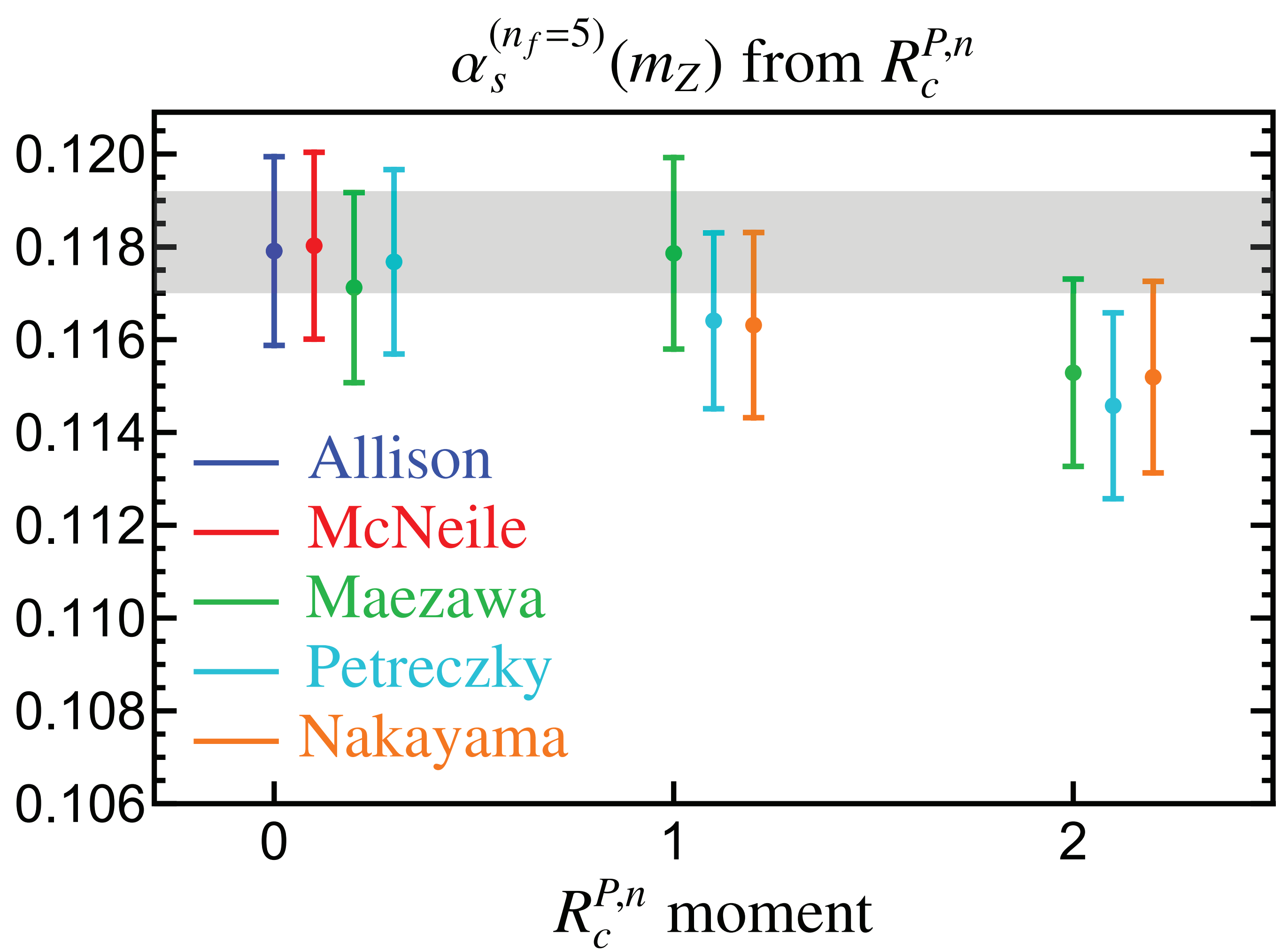




Results from lattice correlators

Larger errors due to more conservative pt. theory errors

Ref.	$\alpha_s^{(n_f=5)}(m_Z)$	$\sigma_{\rm pert}$	$\sigma_{\rm lattice}$	σ_{m_c}	$\sigma_{\rm NP}$	$\sigma_{\rm total}$
Allison et al. [6]	0.1179	0.0019	0.0006	0.0003	0.0004	0.0020
McNeile et al. [9]	0.1180	0.0019	0.0005	0.0003	0.0004	0.0020
Maezawa et al. [10]	0.1171	0.0018	0.0008	0.0003	0.0004	0.0020
Petreczky et al. [11]	0.1177	0.0019	0.0005	0.0003	0.0004	0.0020



Main result

Main result

Extraction from ***charm-quark vector-current moment ratios***:

Larger errors

$$\alpha_s(m_Z) = 0.1168(10)_{\rm pt}(28)_{\rm exp}(6)_{\rm np} = 0.1168(30)\ [R_c^{V,1}],$$
$$\alpha_s(m_Z) = 0.1168(15)_{\rm pt}(9)_{\rm exp}(7)_{\rm np} = 0.1168(19)\ [R_c^{V,2}],$$
$$\alpha_s(m_Z) = 0.1173(20)_{\rm pt}(5)_{\rm exp}(6)_{\rm np} = 0.1173(22)\ [R_c^{V,3}],$$

Large values of n

Very *conservative errors* (with diagonal scale variation error would be +/-0.0013)

Continuum contribution *treated self-consistently* (fixing it would give smaller errors).

Main result

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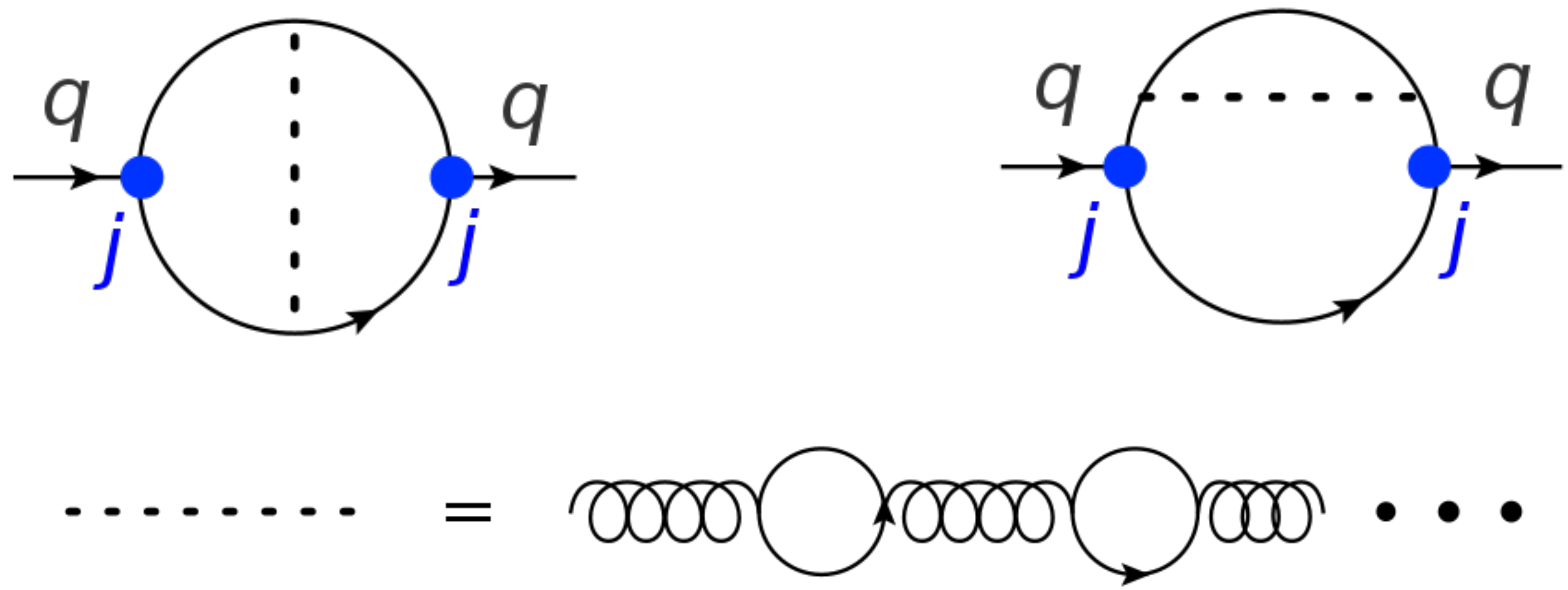
Continuum contribution *treated self-consistently* (fixing it would give smaller errors).

Perturbative behaviour and renormalons

large-β_0 results

Large-β_0 calculation of heavy-quark current correlators



Results available in the literature only for the moments of the *vector current*

Grozin & Sturm '04

$$j^V_\mu = \bar{\psi}\gamma_\mu\psi, \quad j^A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi, \quad j^S = \bar{\psi}\psi \quad \text{and} \quad j^P = i\bar{\psi}\gamma_5\psi.$$

We have calculated for the first time the corresponding result for A, S and PS cases

DB, V Mateu, M.V. Rodrigues, in preparation

large-β_0 results

one-loop normalizations

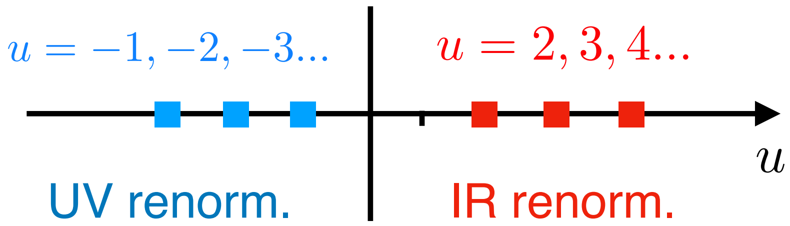
$$M_n^V = \left[12\pi^2 Q_q^2 \frac{3}{16\pi^2}\right] \frac{g_n^V(0)}{(4m^2(\mu))^n} A_n^V(\mu)$$

After renormalization and expressing everything in terms of the $\overline{\rm MS}$ Mass

$$\hat{A}_n^\delta = 1 + \frac{1}{\beta_0}\int_0^\infty {\rm d}u\, e^{-u/\beta(\alpha_s(\mu_0))}\, S_n^\delta(u) + \mathcal{O}\left(\frac{1}{\beta_0^2}\right)$$

General structure of the Borel transform of the moments

$$S_n^V(u) = \frac{8n}{u} + \left(\frac{e^{5/3}\mu_0^2}{m^2}\right)^u \frac{{\rm Csc}(\pi u)\Gamma(n+u)}{4^u\Gamma(3/2+n+u)}\pi^{3/2}(-1+u)(u+1+n)N_n^V(u)$$



Non-trivial polynomials in u for each value of n

$$N_1^V(u) = \frac{u^3}{9} + \frac{29u^2}{27} + \frac{92u}{27} + 3,$$

$$N_2^V(u) = \frac{u^5}{96} + \frac{7u^4}{54} + \frac{2887u^3}{2592} + \frac{7393u^2}{1296} + \frac{2095u}{162} + 10$$

...

Similar results for A, S, PS cases

large-β_0 results

Ratios of moments

$$R_n^V = \frac{(M_n^V)^{\frac{1}{n}}}{(M_{n+1}^V)^{\frac{1}{n+1}}}$$

Borel representation of the ratios of moments

$$R_n^V = \left(\frac{9}{4}Q_q^2\right)^{\frac{1}{n(n+1)}} \frac{(g_n^V(0))^{\frac{1}{n}}}{(g_{n+1}^V(0))^{\frac{1}{n+1}}} \left[1 + \frac{1}{\beta_0}\int_0^\infty du e^{-u/\hat{a}(\mu)} B_n^V(u) + \mathcal{O}\left(\frac{1}{\beta_0^2}\right)\right]$$

DB,V Mateu, M.V. Rodrigues, in preparation

Borel transform

$$B_n^V(u) = \frac{S_n^V(u)}{n} - \frac{S_{n+1}^V(u)}{n+1}$$

Explicitly

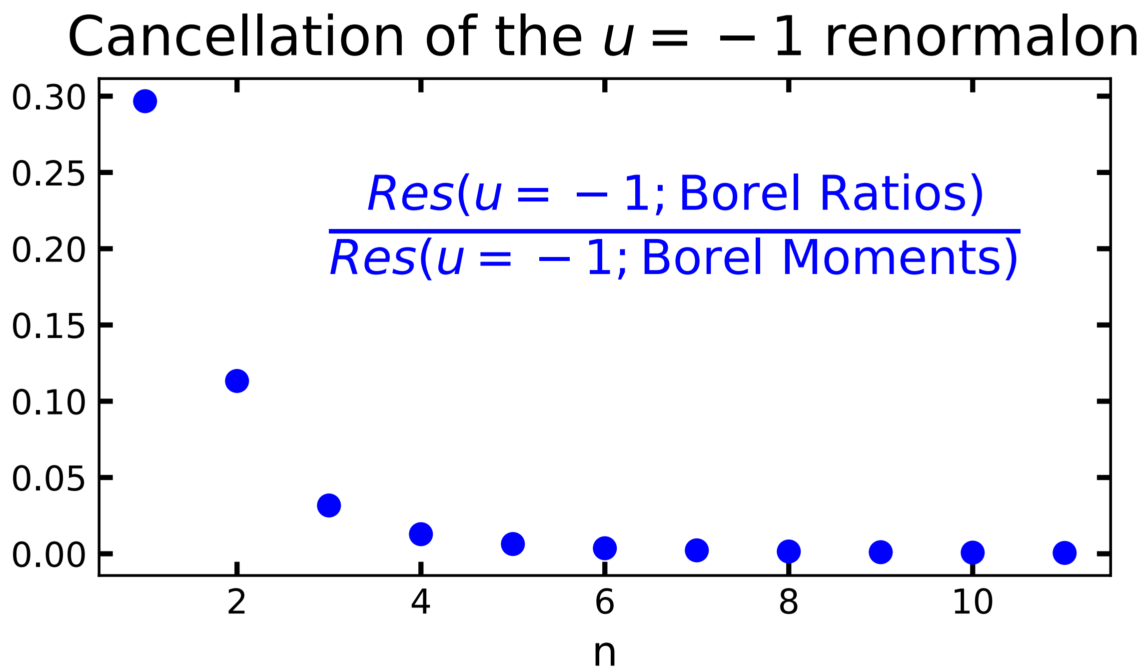
$$B_n^V(u) = \left(\frac{e^{5/3}\mu^2}{m^2}\right)^u \frac{\mathrm{Csc}(\pi u)\pi^{3/2}(-1+u)}{4^u}$$

$$\left[\frac{\Gamma(n+u)(u+1+n)N_n^V(u)}{n\,\Gamma(3/2+n+u)} - \frac{\Gamma(n+u+1)(u+2+n)N_{n+1}^V(u)}{(n+1)\,\Gamma(5/2+n+u)}\right]$$

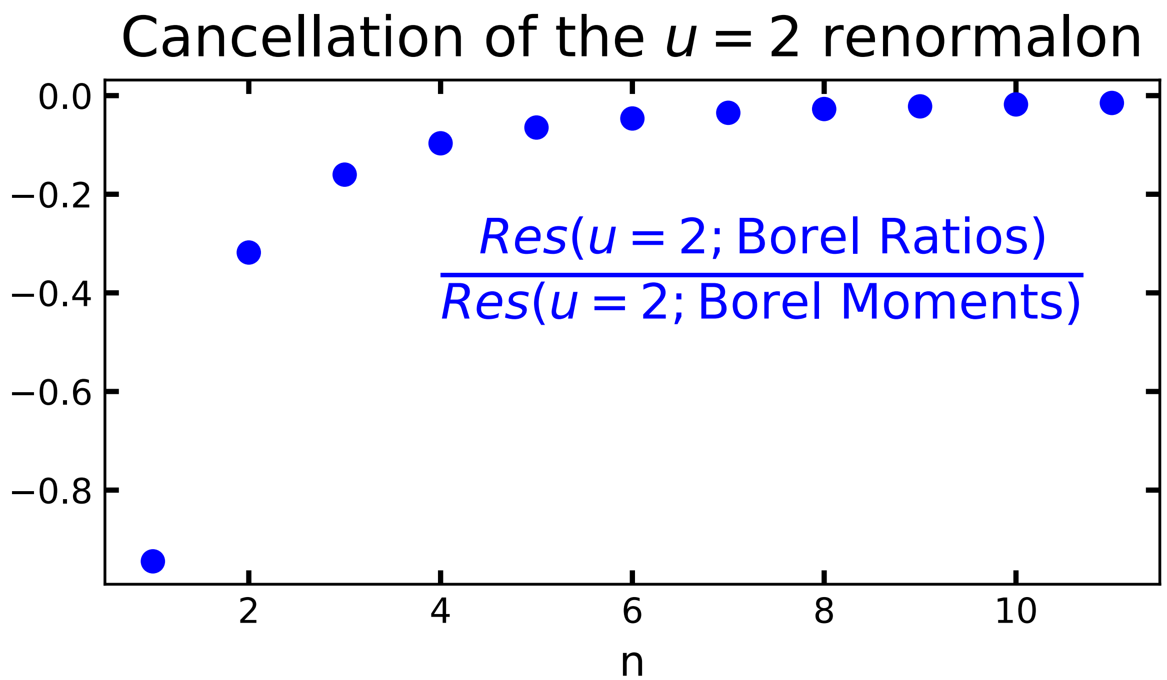
(partial) renormalon cancelations

large-β_0 results

Renormalon cancelation in the R_n ratios (vector case)



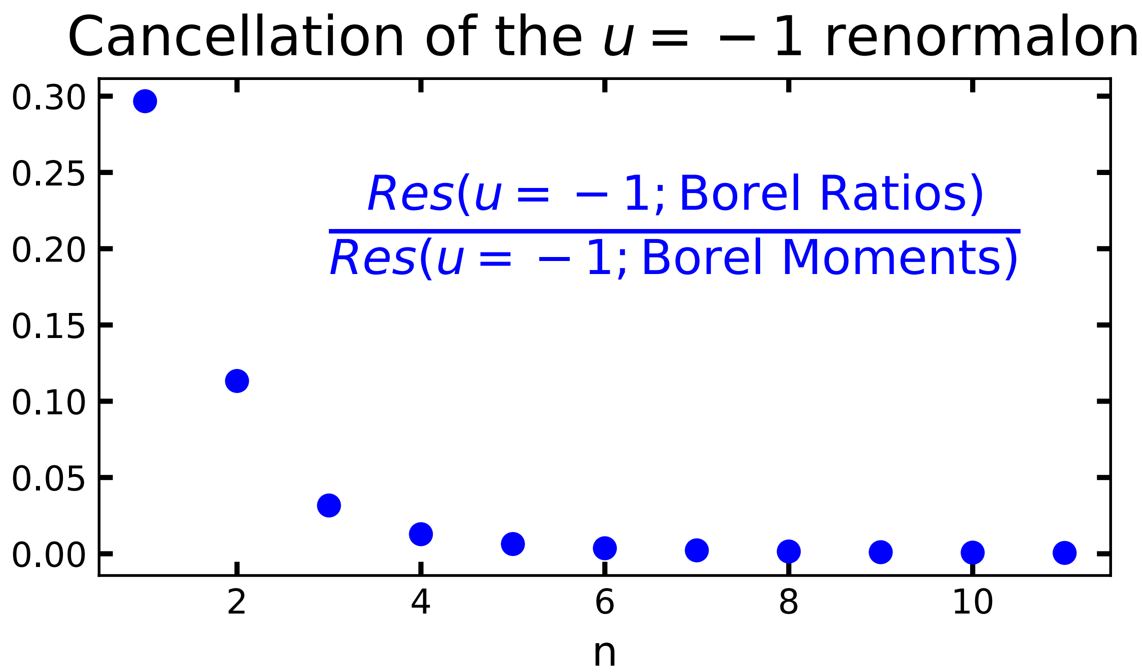
Preliminary

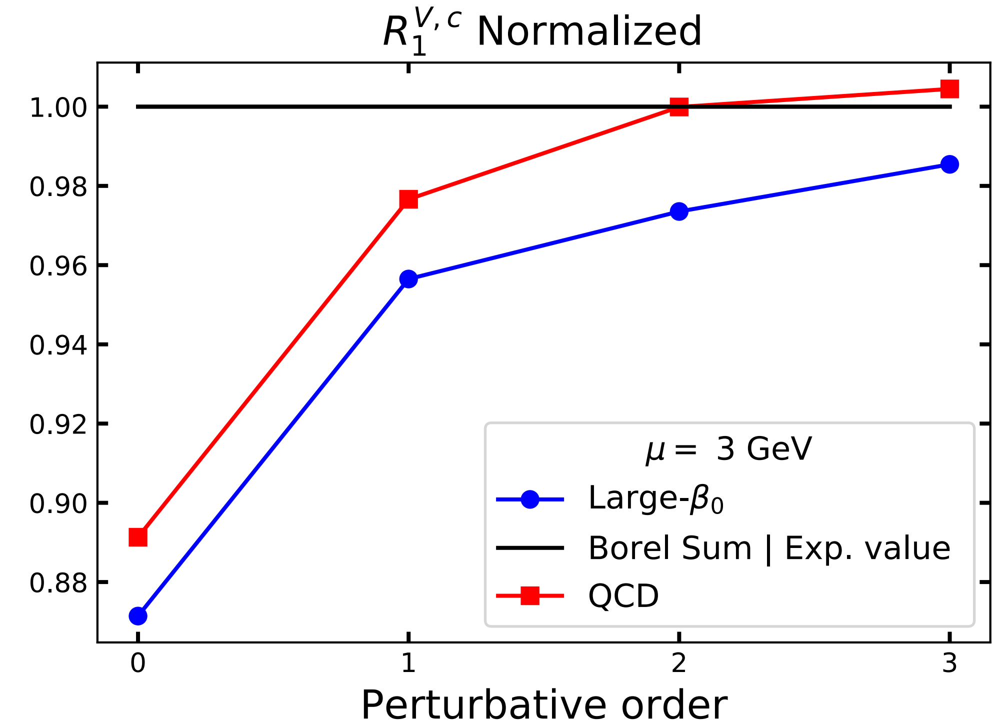


large-β_0 results

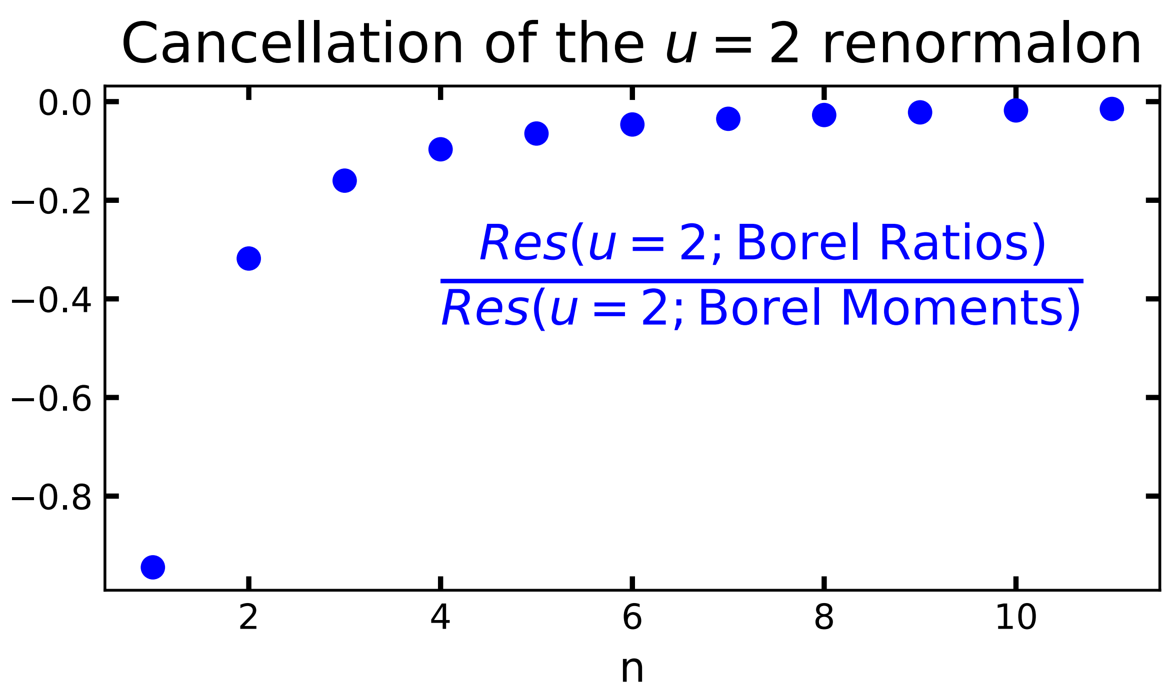
Renormalon cancelation in the R_n ratios (vector case)

QCD vs Large-beta 0

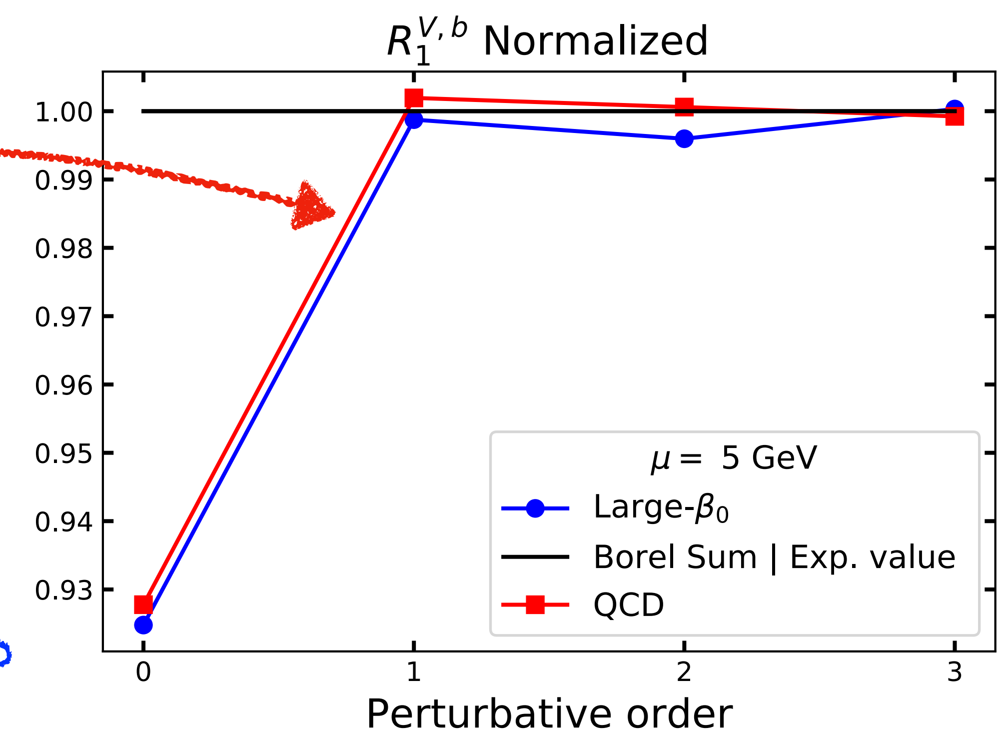




Preliminary



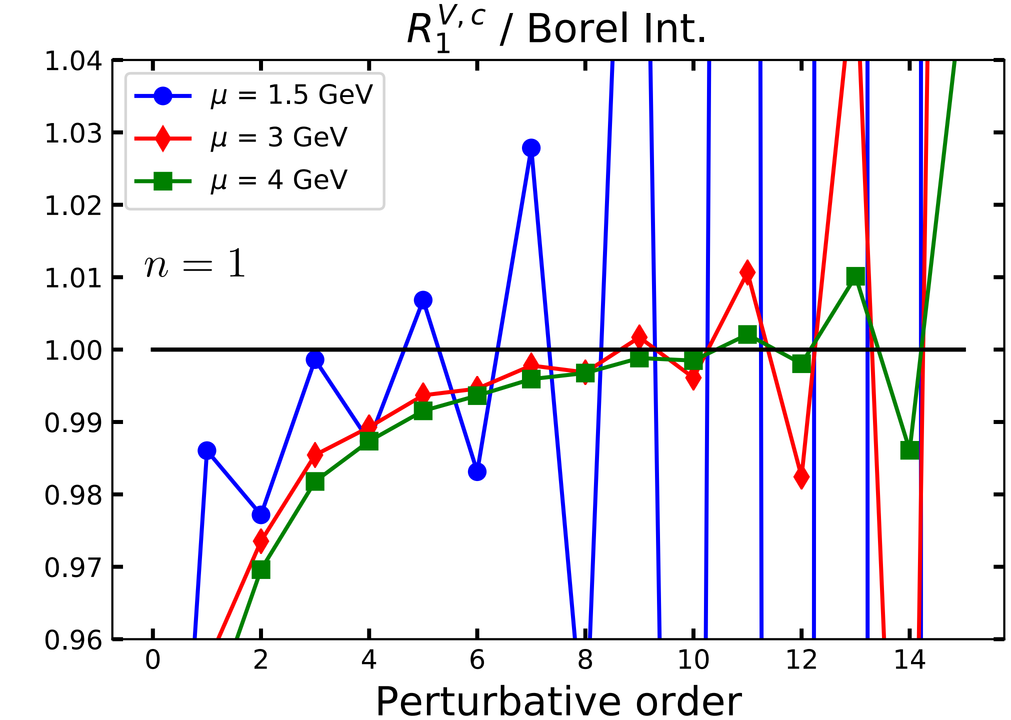
Similar behaviour but QCD is actually better than large-beta_0

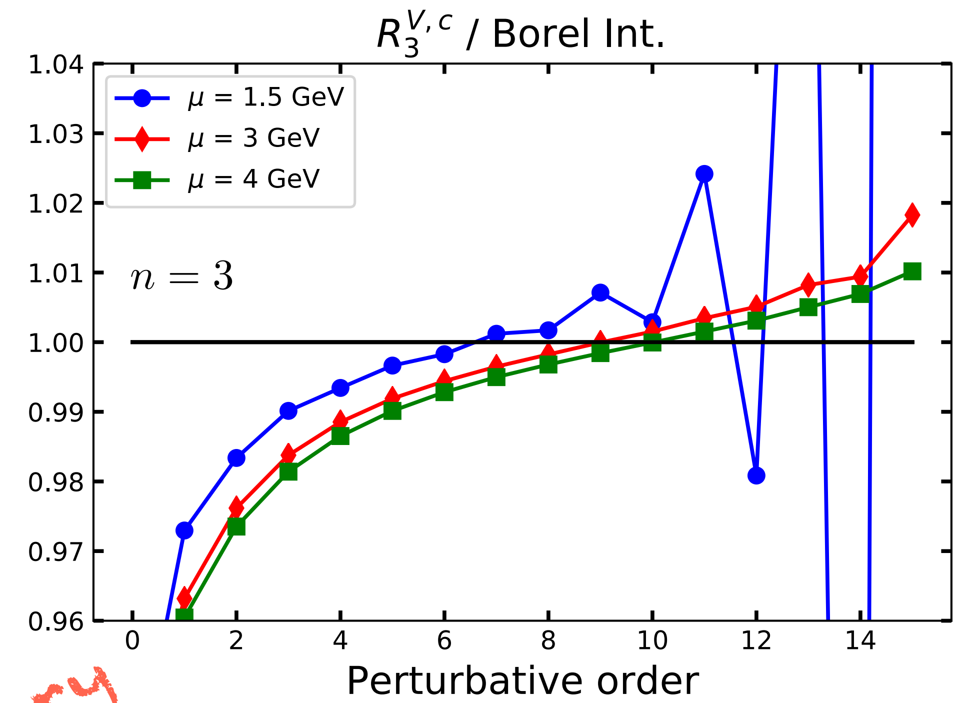


large-β_0 results

R_n^V

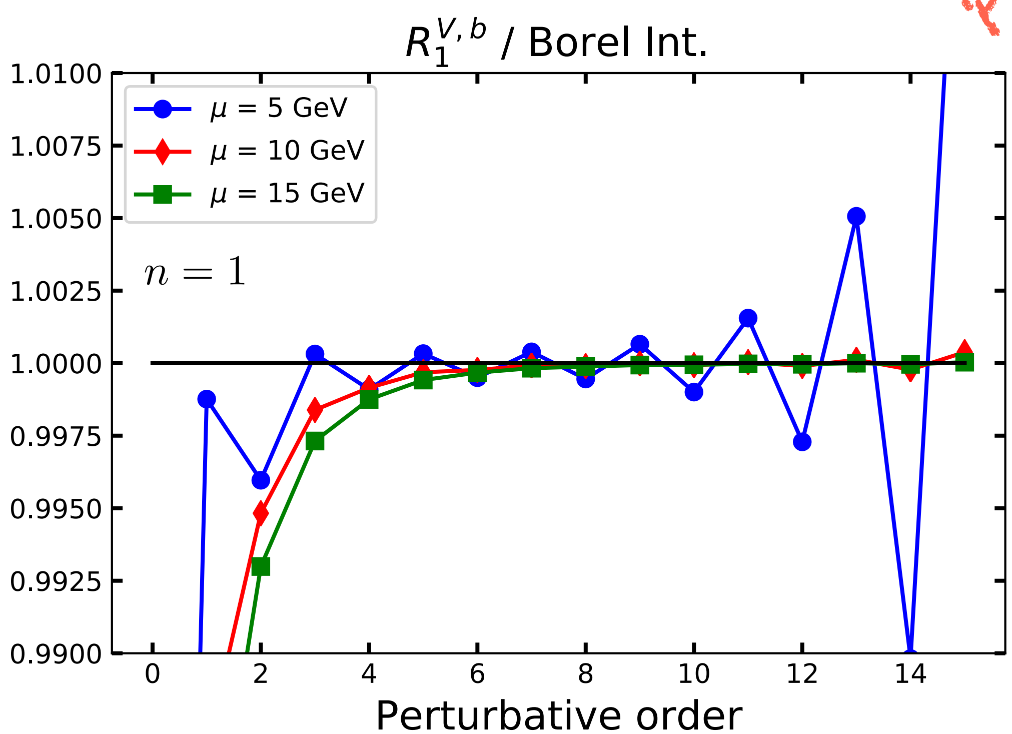
charm

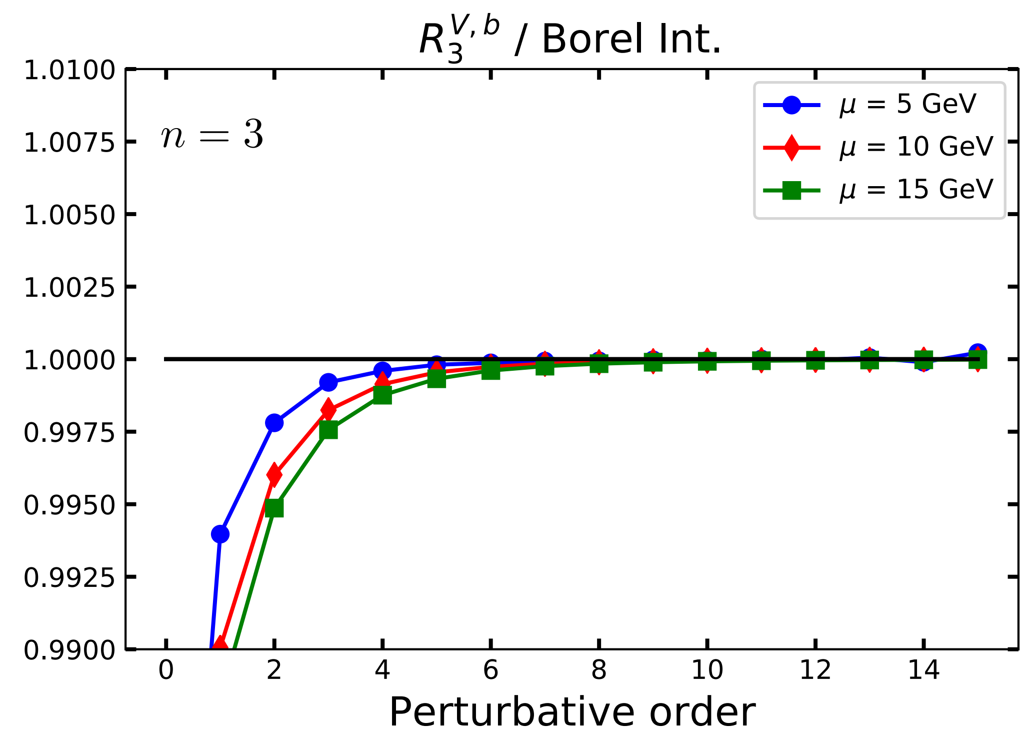




Preliminary

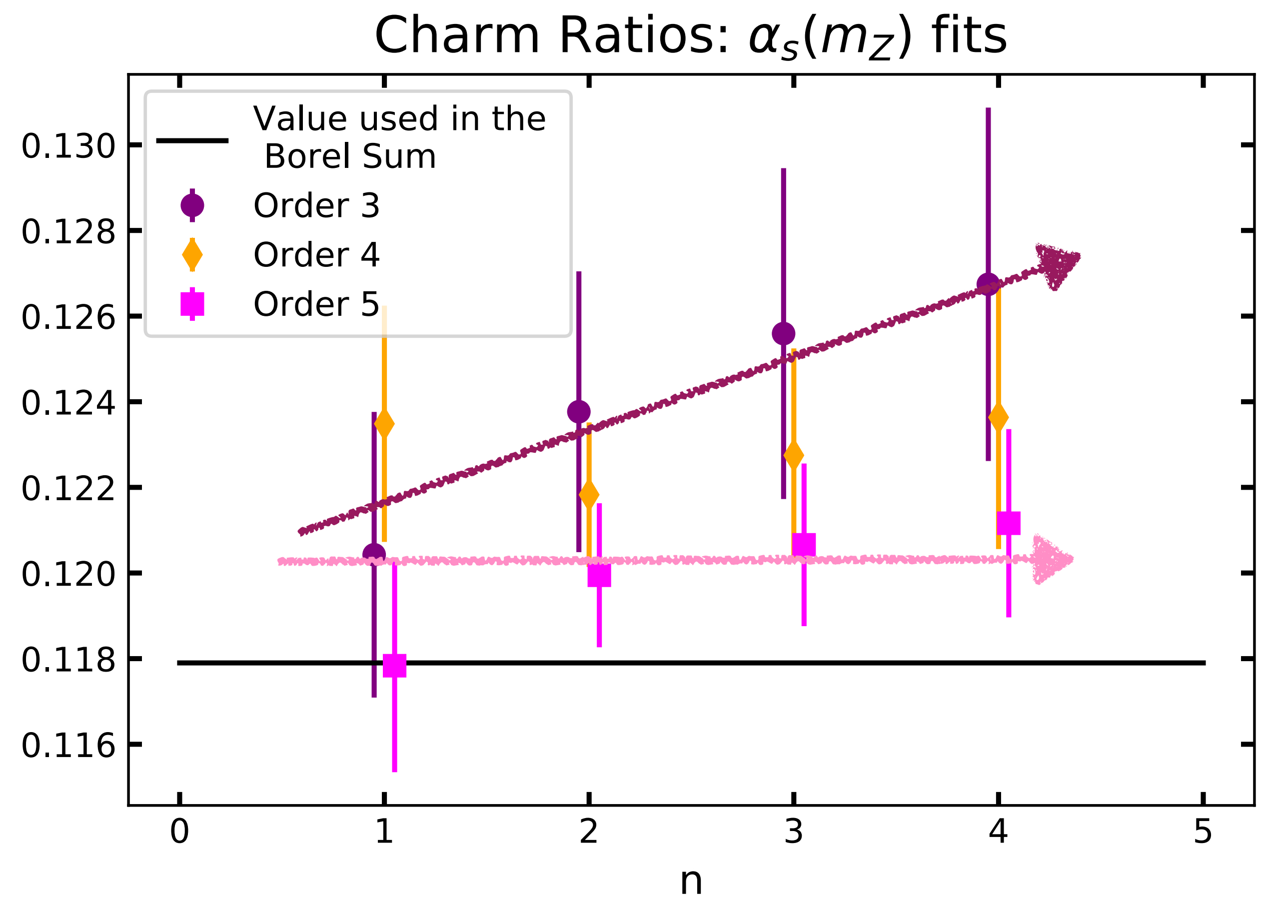
bottom

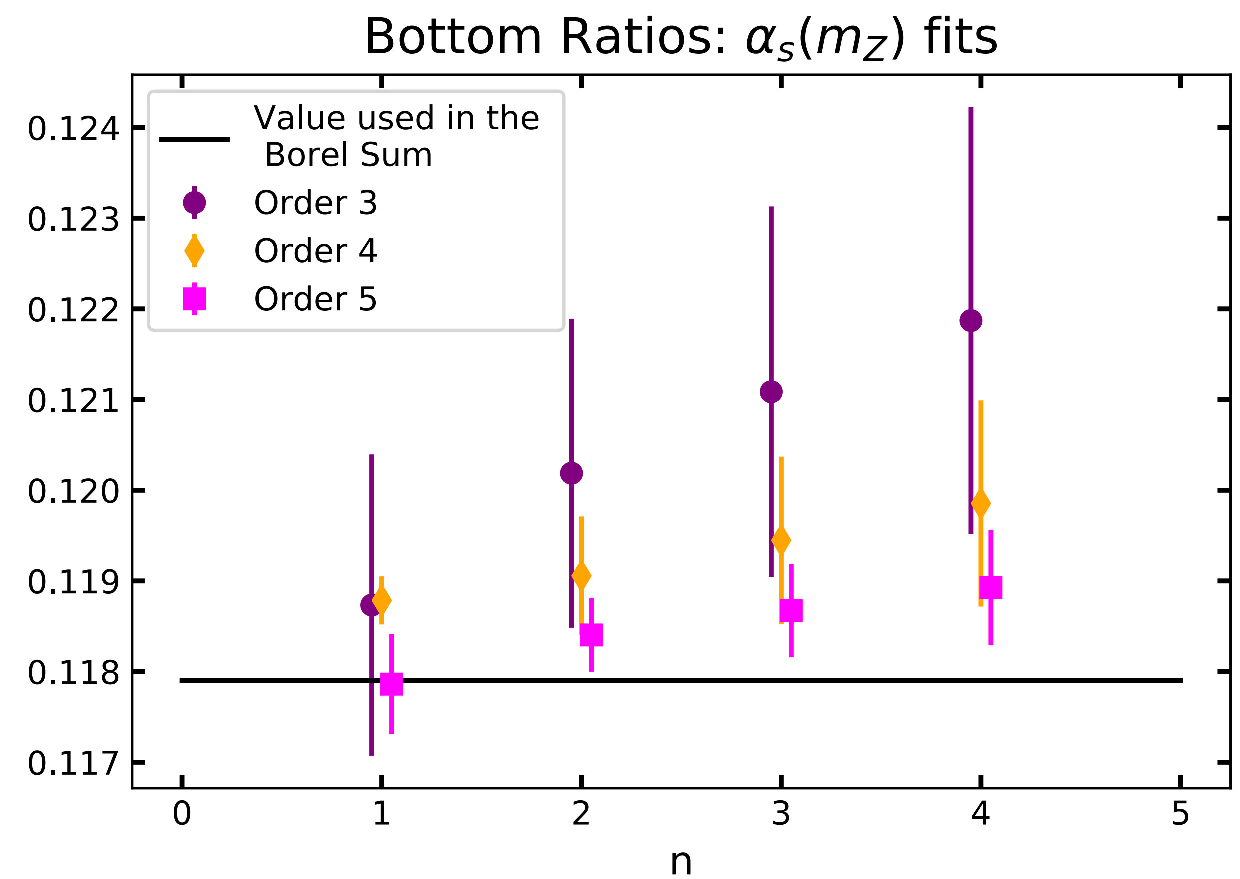




large-β_0 results

Toy extraction of α_s in large-β_0 with the Borel sum as "experiment"





Trends in alpha_s values qualitatively corroborated by large-beta0 results.

One order more in the pt. series should lead to more stable results.

Partial conclusions

Conclusions

α_s can be extracted reliably from *R* data with 4, and 5 active flavours.

Ratios of moments of bottomonium vector-current correlators ideal from the theory view point, but larger exp. errors.

Ratios tend to have good perturbative expansion (renormalon cancelations).

At present, best determination from charm ratio with *n*=2:

$$\alpha_s(m_Z) = 0.1168 \pm 0.0019$$

Our results are obtained with a conservative error estimate.

PS current moments (from lattice) give stable results but with larger uncertainty.

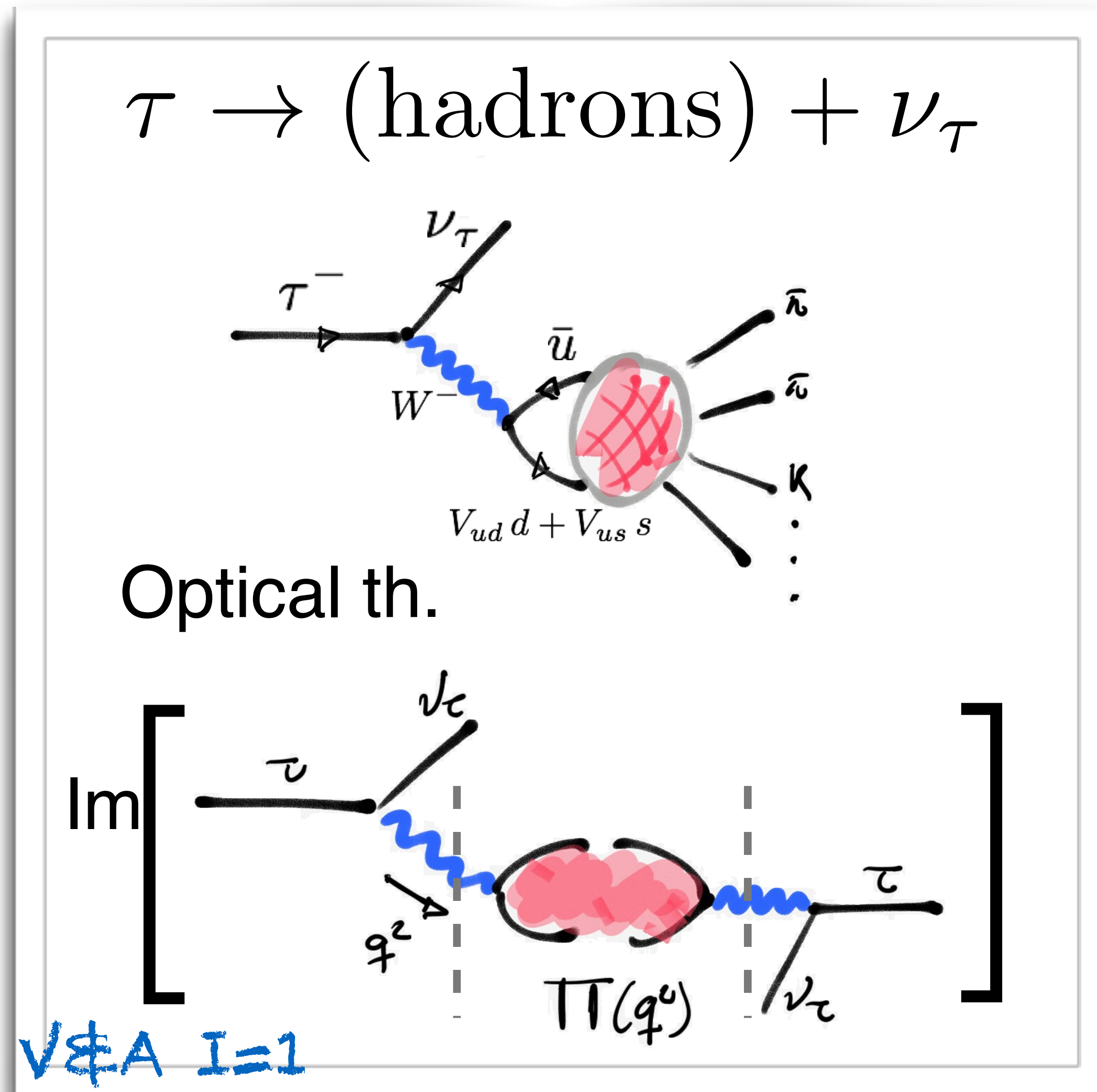
Our analysis of the perturbative error is more conservative than original studies

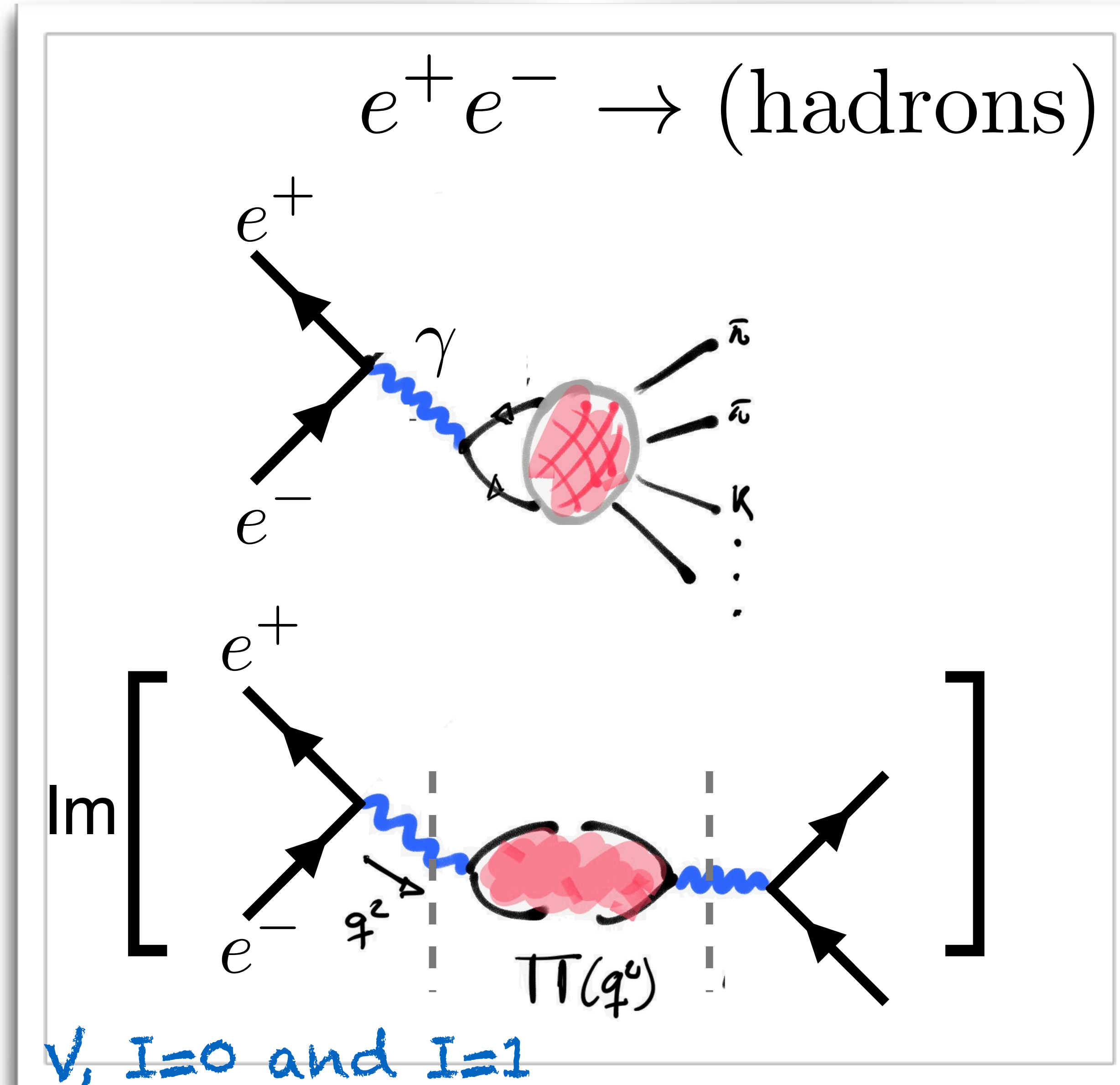
Strong coupling below charm ($n_f = 3$)

- DB, M. Golterman, A. Kheshavarzi, K. Maltman, D. Nomura, S. Peris, T. Teubner,
arXiv:1805.08176
Phys. Rev. D **98** n. 7 074030 (2018)

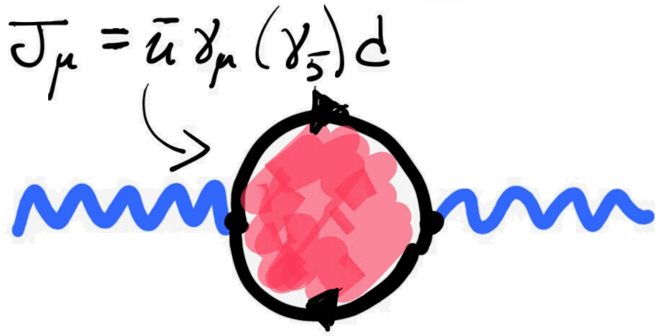
-DB, M. Golterman, K. Maltman, S. Peris, M.V. Rodrigues and W. Schaaf,
arXiv:2012.10440
Phys. Rev. D **103** 034028 (2021)

Inclusive process at low energies





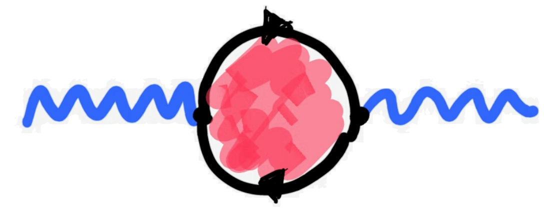
Massless (V&A) correlators



$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0|T\{J_\mu(x) J_\nu(0)^\dagger\}|\rangle$$

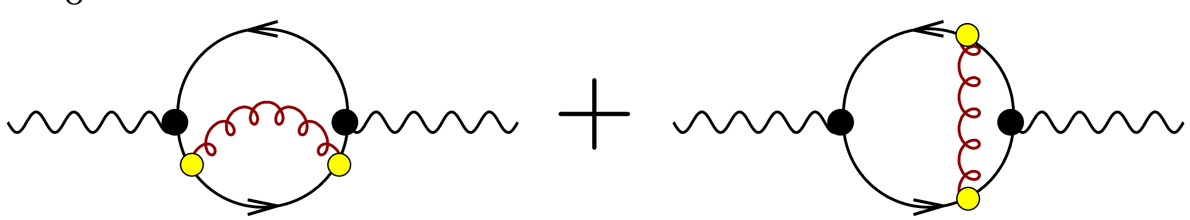
Theory input

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle 0|T\{J_\mu(x)J_\nu(0)^\dagger\}|0\rangle$$



Below charm one works with **massless** correlators.

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\,\tilde{\Pi}(z) \approx N_c(1+\delta^{(0)}+\delta_{\rm EW}+\delta_{\rm OPE}+\delta_{\rm DVs}) \qquad \delta^{(0)} = \sum_n^4 c_n\alpha_s^n$$



$$\frac{\alpha_s}{\pi} \approx \frac{0.3}{\pi} \sim 10\%$$

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91

Baikov, Chetyrkin, Kühn '08

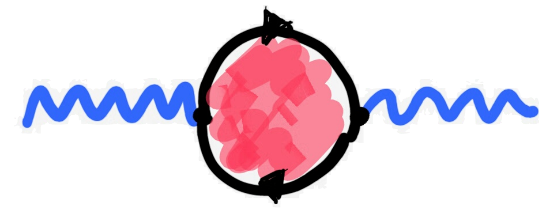
$$\begin{array}{ccccc} & \alpha_s^1 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 \\ \delta^{(0)}_{\rm FO} = & 0.1012 & + 0.0533 & + 0.0273 & + 0.0133 \; = \; 0.1952 \end{array}$$

pt. correction is ~20%

Moments dominated by perturbation theory

Theory input

$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0 | T\{J_\mu(x) J_\nu(0)^\dagger\} | 0 \rangle$$



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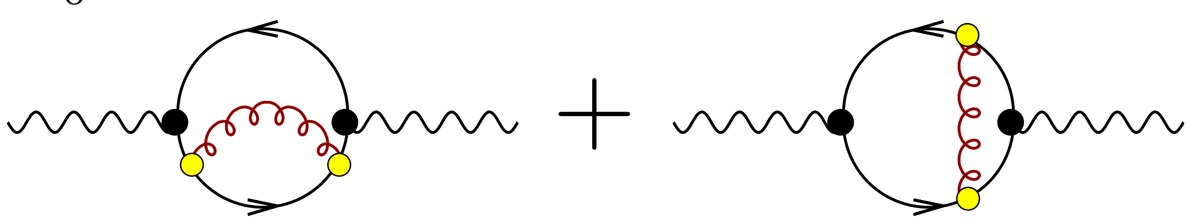

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Baikov, Chetyrkin, Kühn '08

$$\alpha_s^1 \quad \alpha_s^2 \quad \alpha_s^3 \quad \alpha_s^4$$

$$\delta^{(0)}_{\rm FO} = 0.1012 + 0.0533 + 0.0273 + 0.0133 = 0.1952$$

$$\delta^{(0)}_{\rm CI} = 0.1375 + 0.0262 + 0.0104 + 0.0072 = 0.1814$$

pt. correction is ~20%

theoretical uncertainty

Moments dominated by perturbation theory

Theory input

Discrepancy between FO and CIPT
linked to an incoherent treatment of the OPE
(previously assumed to be the same for both prescriptions)

Hoang and Regner '20

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91

Baikov, Chetyrkin, Kühn '08

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I will avoid results from CIPT because of potential inconsistent treatment of the OPE

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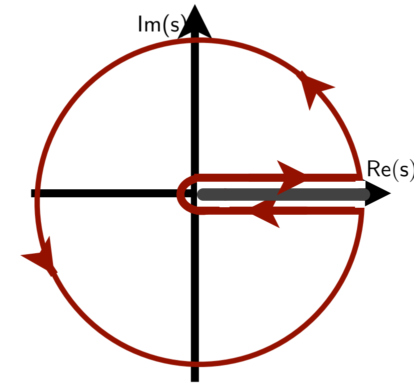
Strategy



Integrated moments → Sum rules

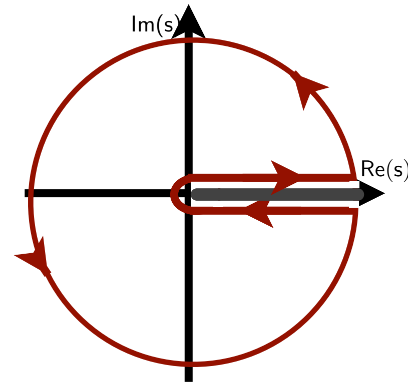
(using Cauchy's theorem)

experiment

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s/s_0)\frac{1}{\pi}\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(z)\Pi(z)$$

theory

Strategy



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theory

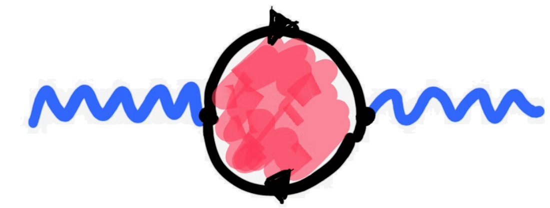
$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(\mathrm{s}) = \frac{1}{12\pi^2}R(s)$$

Spectral function

R ratio (in e+e-)

Theory input: summary

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iqx} \langle 0|T\{J_\mu(x) J_\nu(0)^\dagger\}|0\rangle$$



Perturbation theory + Condensates (OPE) →

$$\sum_{n=0}^{4} a_\mu^n \sum_{k=1}^{n+1} k \, c_{n,k} \left(\log \frac{-s}{\mu^2}\right)^{k-1} + \frac{C_4}{Q^4} + \frac{C_6}{Q^6} + \frac{C_8}{Q^8} + \cdots$$

Known up to α_s^4

Baikov, Chetyrkin, Kühn ‘08

Mass corrections →

$$\frac{m_s^2(\mu^2)}{6\pi^2} \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n \sum_{k=0}^{n} f_{nk} \left(\log \frac{-z}{\mu^2}\right)^k$$

EM and EW corrections →

$$\frac{1}{6\pi^2} c_{01} \to \frac{1}{6\pi^2} c_{01} \left(1 + \frac{\alpha}{4\pi}\right) \qquad S_{\rm EW}$$

Duality Violations (consistency check) →

$$\rho(s)_{\rm DV} = e^{-\delta - \gamma s} \sin(a + bs)$$

O. Catà, M. Golterman, S. Peris ’05, ’06, '08

DB, Caprini, Golterman, Maltman, Peris, PRD ’18

Theory input: moments and general strategy

$$\frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w(s/s_0) R(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(s/s_0)\Pi(z)$$

1. Good perturbative behaviour.
2. Small condensate contributions.
3. Suppression of DVs.

$w(y) \to 1$

$1 - y^2$

$(1-y)^2(1+2y)$

$(1-y^2)^2$

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\,\tilde{\Pi}(z) \approx N_c(1 + \delta^{(0)} + \delta_{\rm EM} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

Theory input: choice of weight functions

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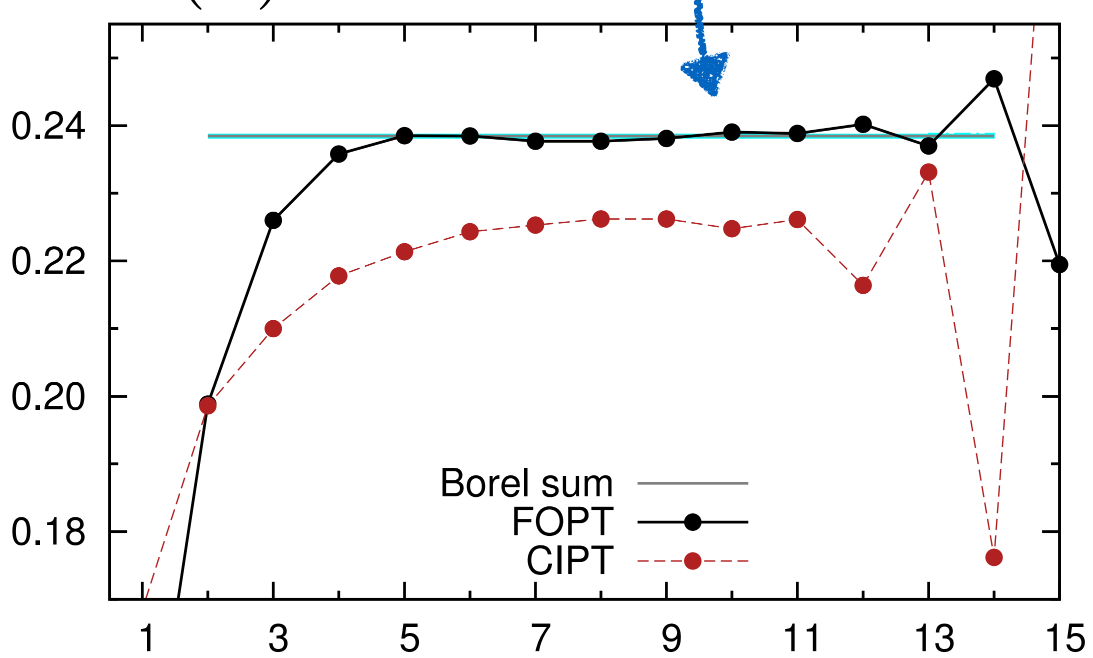
1. Good perturbative behaviour.

Reconstruction of the series at high orders.

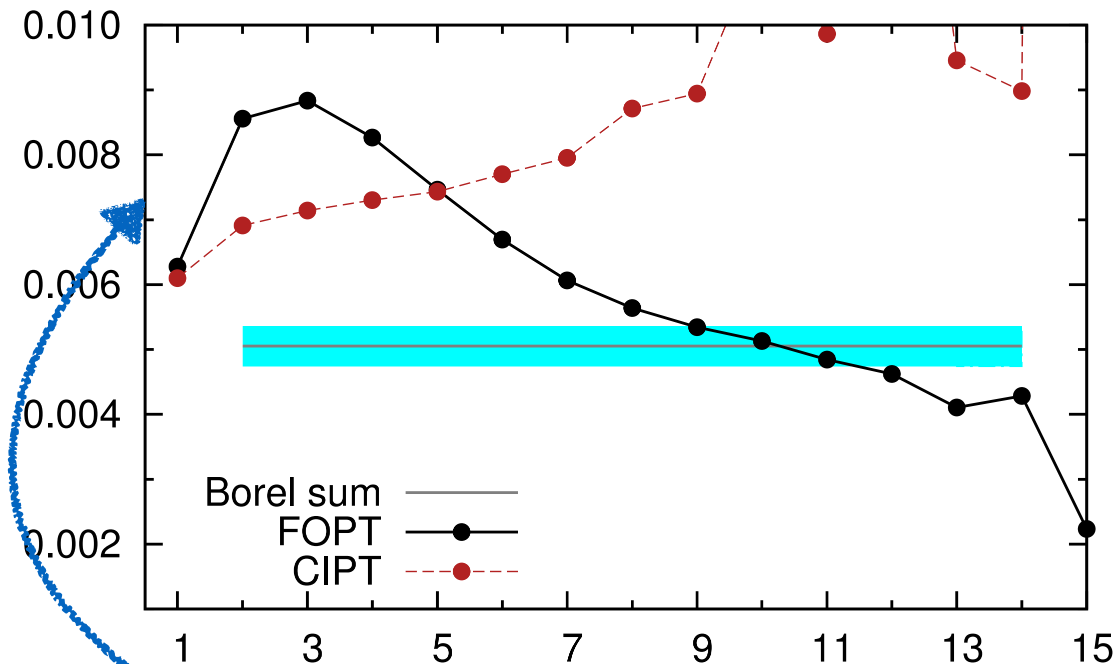
Beneke, Jamin '08
Beneke, DB, Jamin '12
DB, Masjuan, Oliani '18
DB, Oliani '20

Good pert behaviour

$w(x) = 1 - x^2$



$w(x) = (1-x)^3 x^2 (1+2x)$



Bad pert behaviour

Avoid moments with linear term in x (first IR renormalon)

Theory input: choice of weight functions

$$\frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w(s/s_0)R(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(s/s_0)\Pi(z)$$

2. Small condensate contributions.

Only non-zero for D=2(n+1)

$$\Pi^{(1+0)}_{\rm OPE} = \sum_{D=0,2,4\ldots}^{\infty} \frac{C_D(s)}{(-s)^{D/2}} \qquad w(x) = x^n \qquad \delta^{(D)}_{x^n,} = \frac{6\pi i}{(-s_0)^{D/2}} C_D \oint dx \frac{1}{x^{-n+D/2}}$$

$$w_0(y) = 1,$$ Tiny condensate contributions

$$w_2(y) = 1 - y^2,$$ Only D=6

$$w_3(y) = (1-y)^2(1+2y),$$ Only D=6 and 8 Tau kin. Moment

$$w_4(y) = (1-y^2)^2,$$ Only D=6 and 10

Avoid higher orders in the OPE! (Asymptotic expansion)

Theory input: choice of weight functions

$$\frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w(s/s_0)R(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(s/s_0)\Pi(z)$$

$w(y) \to 1$

$1 - y^2$

$(1-y)^2(1+2y)$

$(1-y^2)^2$

3. Suppression of duality violations.

DVs are the "OPE of the OPE". Perturbation theory is only asymptotic

$$R \sim \sum_n^{n^*} r_n \alpha_s^{n+1} + e^{-p/\alpha_s(Q^2)}$$

Theory input: choice of weight functions

$$\frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w(s/s_0)R(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(s/s_0)\Pi(z)$$

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Theory input: choice of weight functions

$$\frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w(s/s_0)R(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(s/s_0)\Pi(z)$$

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The OPE is also only asymptotic

These are the DVs

$$R \sim \sum_n^{n^*} r_n \alpha_s^{n+1} + \sum_k^{k^*} \frac{C_{2k}}{Q^{2k}} + e^{-\gamma\, q^2}\kappa \sin(\alpha + \beta q^2)$$

DB, I. Caprini, M. Golterman, K. Maltman, S. Peris '18

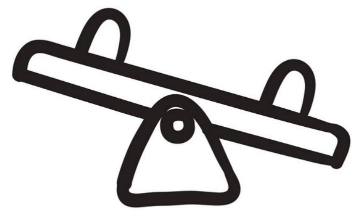
O. Catà, M. Golterman, S. Peris ’05, ’06, '08

DV suppression: higher energies and/or zeros in w(x) for x=1.

One of the main messages

Higher suppression of D.Vs comes with the price of additional (unknown) higher D contributions from the OPE.

$$w(y) = (1 - y^2)^n$$



DV strategy

DB, M. Golterman, K. Maltman, S. Peris, M. V. Rodrigues and W. Schaaf, 2012.10440

- Accept some D.Vs and have very little contamination on the OPE side.

Truncated OPE strategy

A Pich, A. Rodriguez-Sanchez 1605.06830

- Suppress D.Vs strongly but need to ignore the higher order contributions on the OPE side (too many parameters).

(Serious issues with the truncation of the OPE)

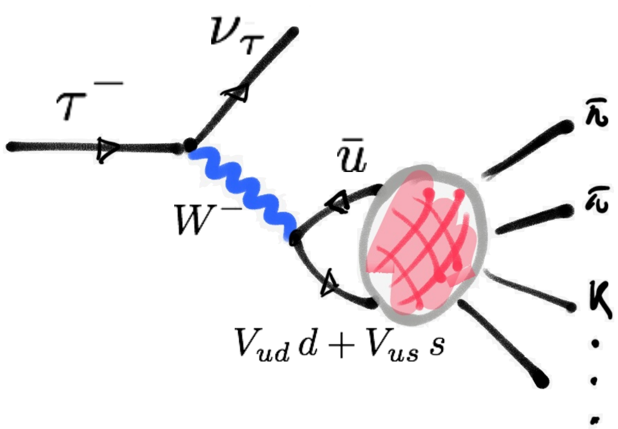
DB, M. Golterman, K. Maltman, S. Peris '16

Data

$$\tau \to (\text{hadrons}) + \nu_\tau$$

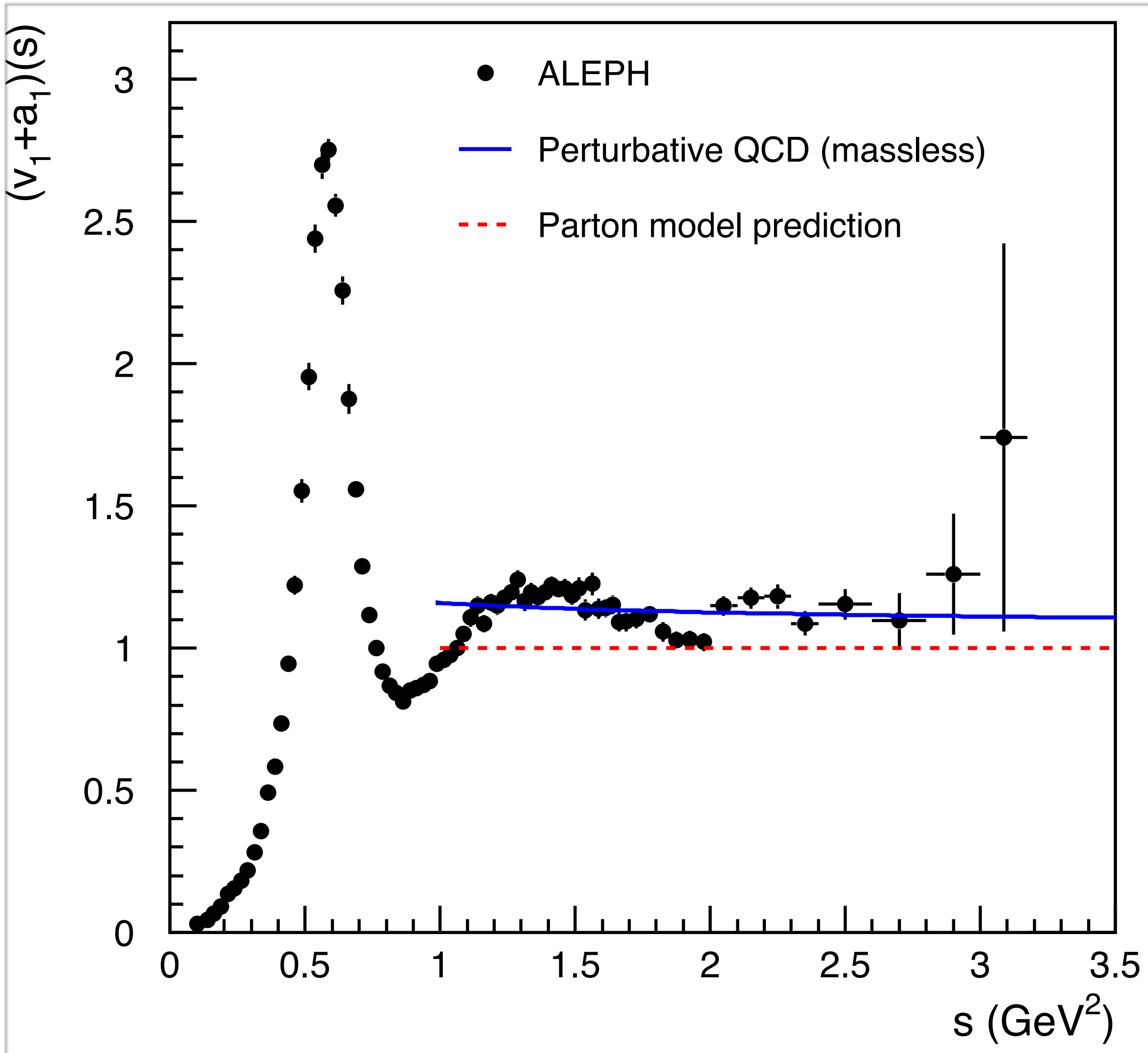
DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

$\tau \to (\text{hadrons}) + \nu_\tau$

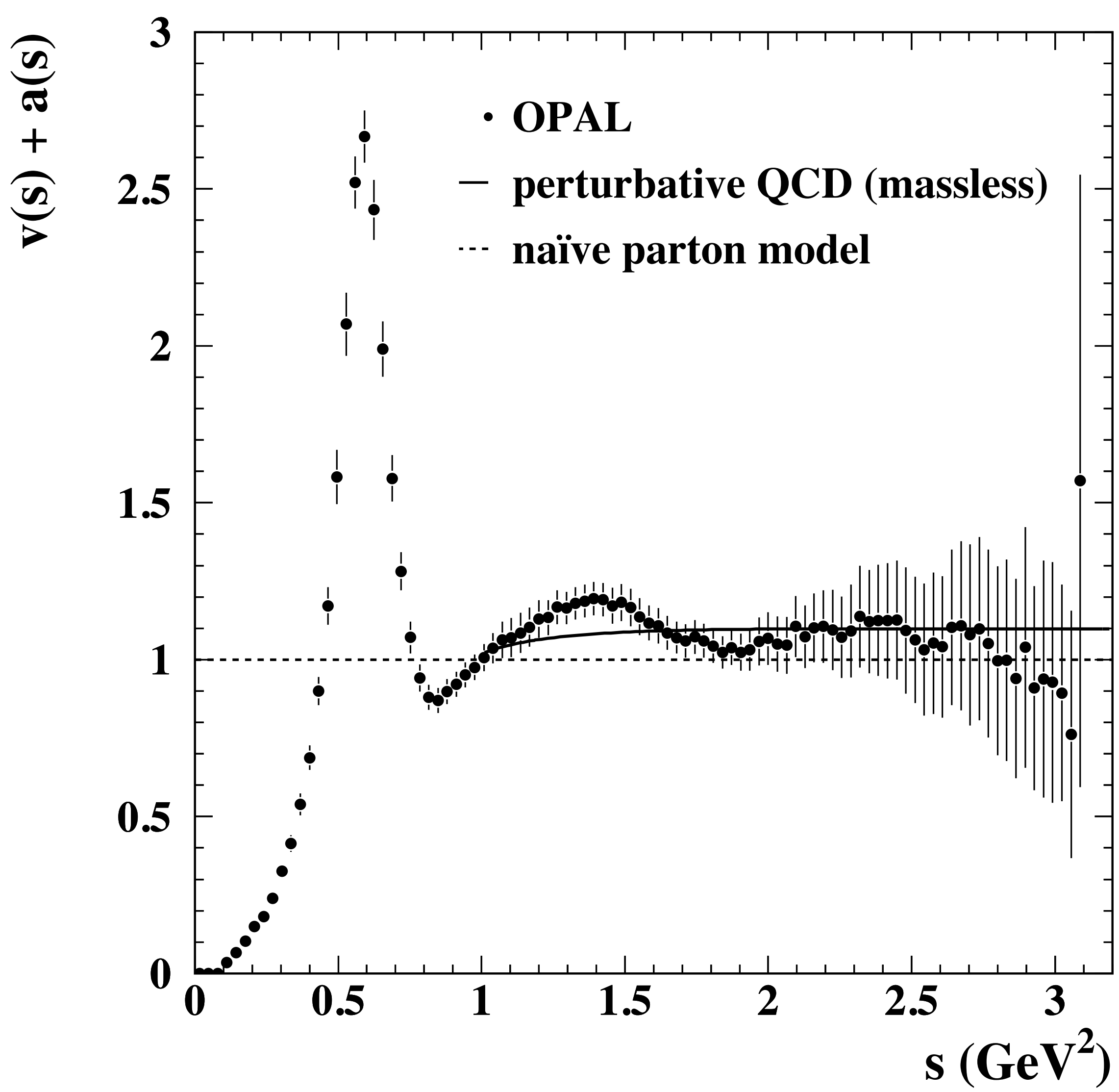


- V+A inclusive data from LEP
- V & A can be separated unambiguously for most channels (not all)
- Not all channels are measured (MC inputs)

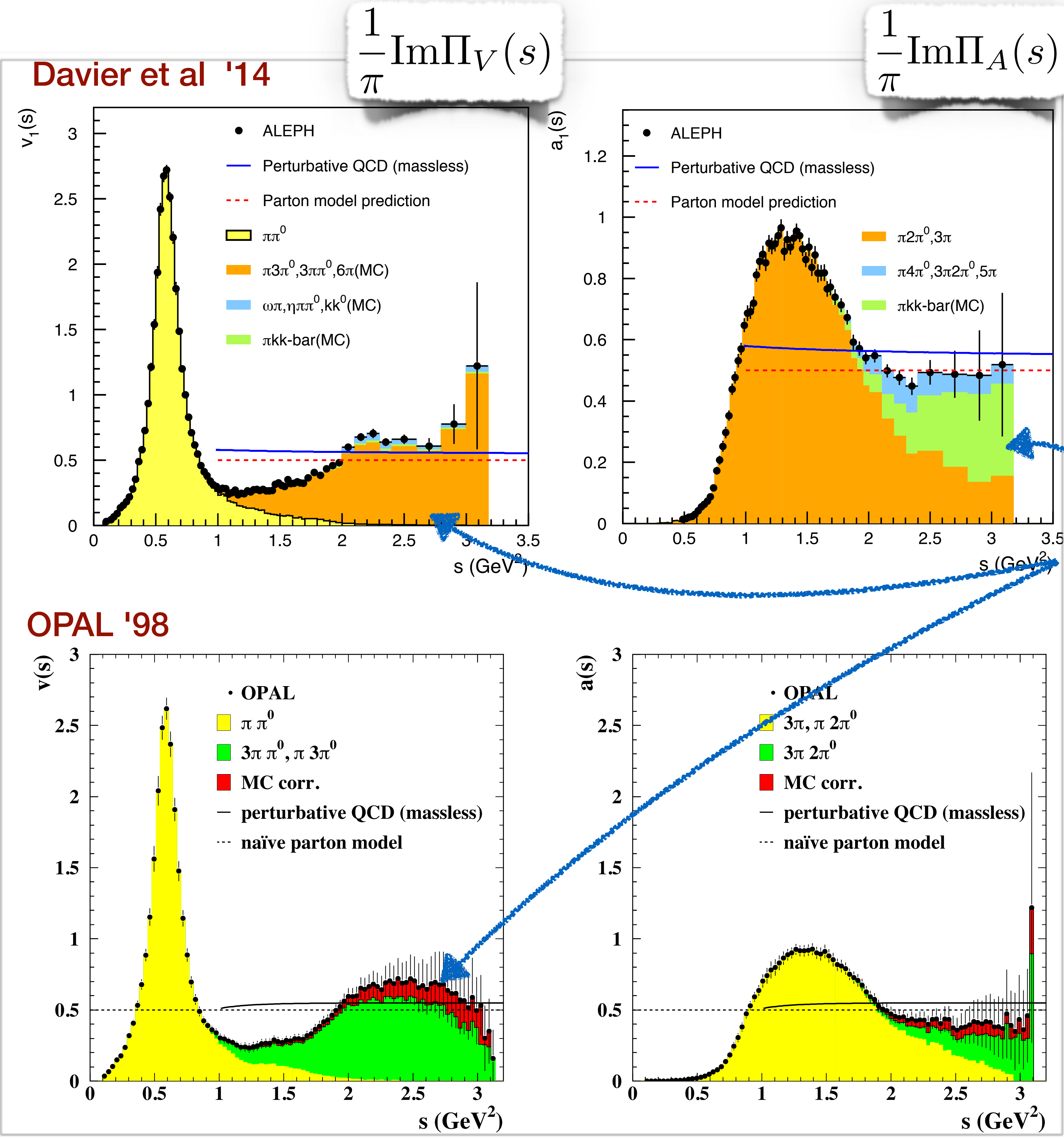
Davier et al '14



OPAL '98



$\tau \to (\text{hadrons}) + \nu_\tau$



- V channel dominated by $\tau \to 2\pi + \nu_\tau$ and $\tau \to 4\pi + \nu_\tau$
- "Residual" channels subdominant (but important!)
- MC inputs for several channels

Results were always based on OPAL or ALEPH (never combined)

New vector isovector spectral function

- Combined data for 2pi and 4pi channels from ALEPH & OPAL

  Data combination done with same algorithm used in R-data combination for the muon g-2. Keshavarzi, Nomura, Teubner '18

- Exp. data only: all residual channels extracted from recent cross-section measurement in e^+e^- No MC inputs

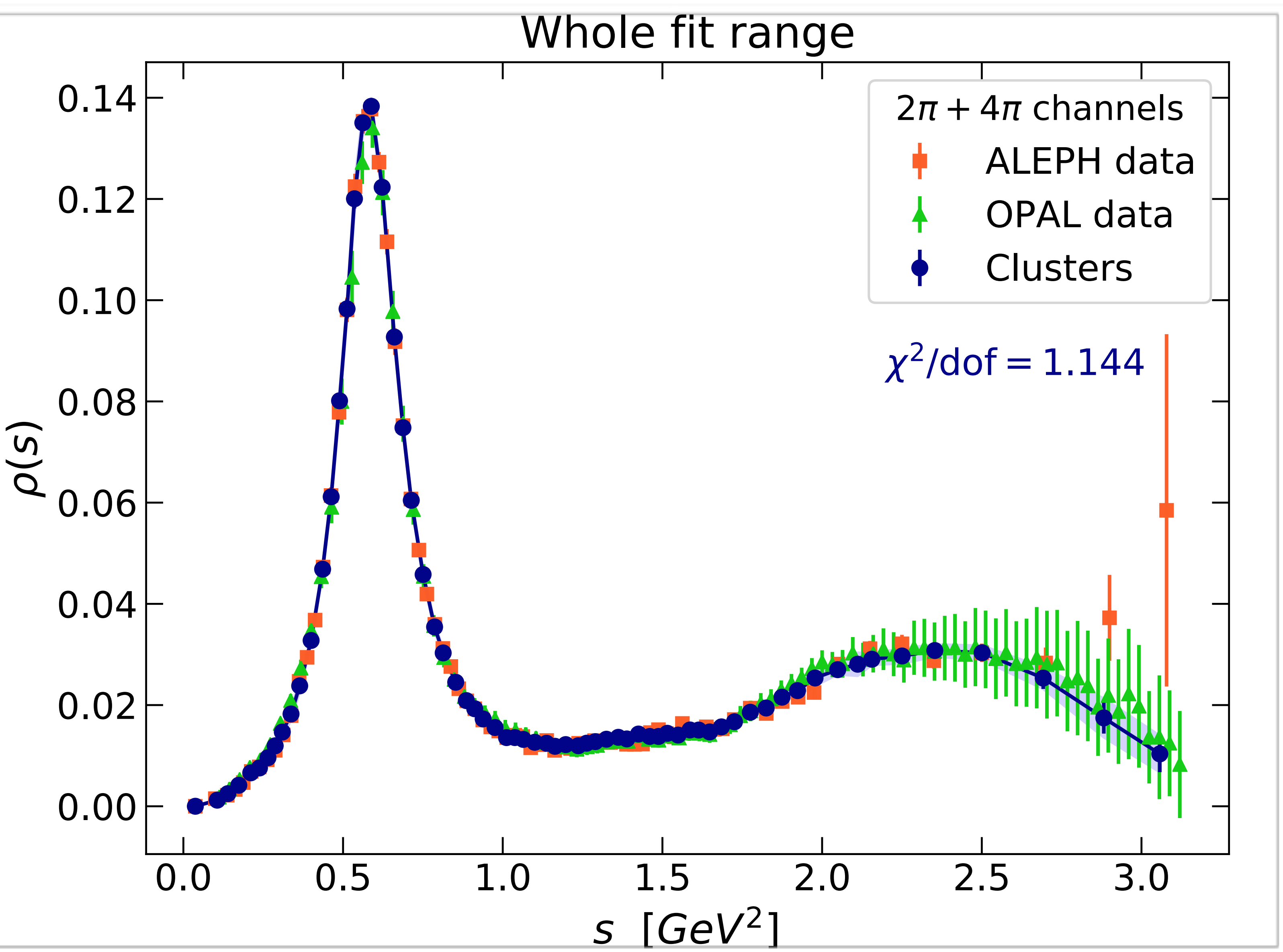
- All results updated for recent branching ratio measurements

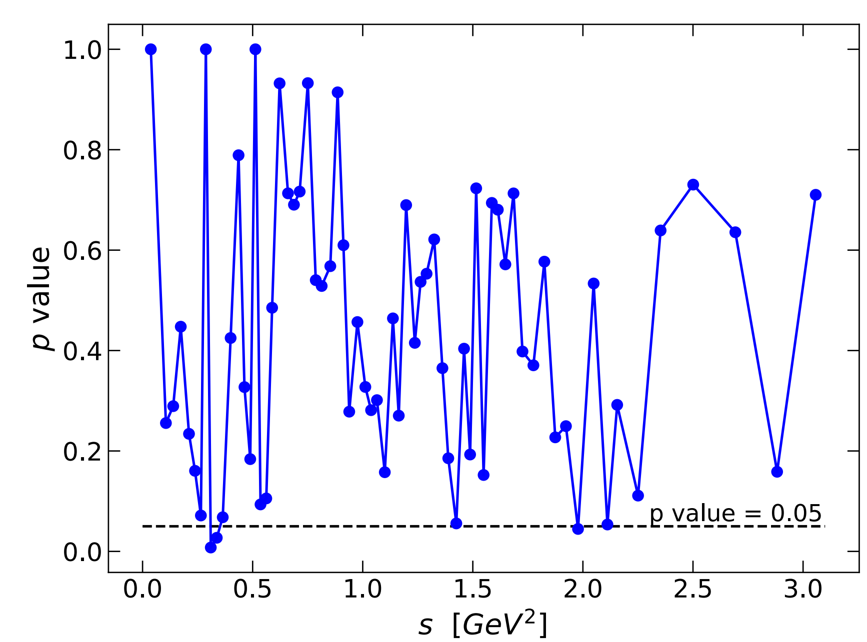
$\tau \to (\text{hadrons}) + \nu_\tau$

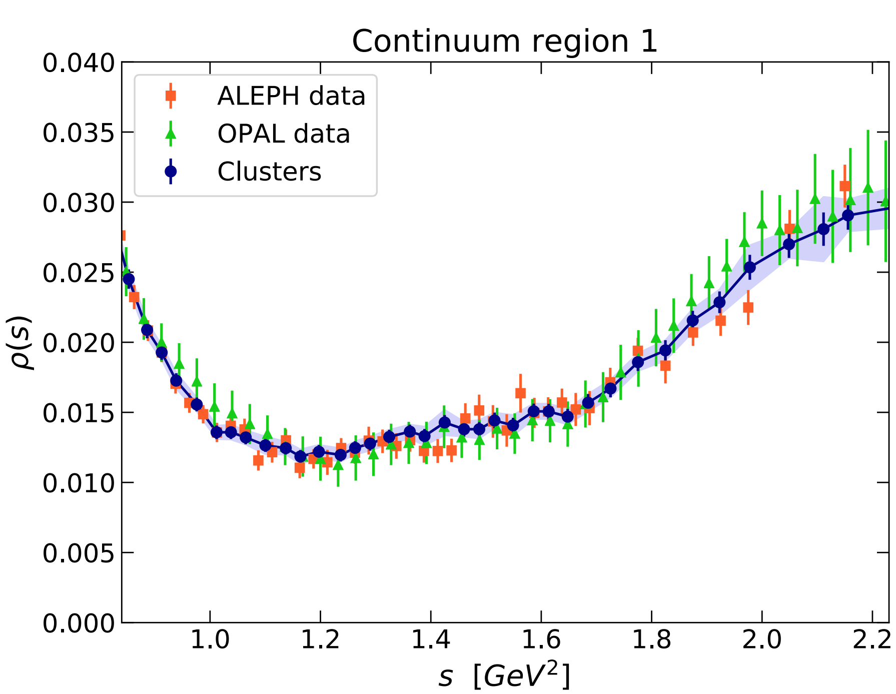
Combination of $2\pi + 4\pi$ channels

Good χ^2 both locally and globally, no χ^2 inflation needed

DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440





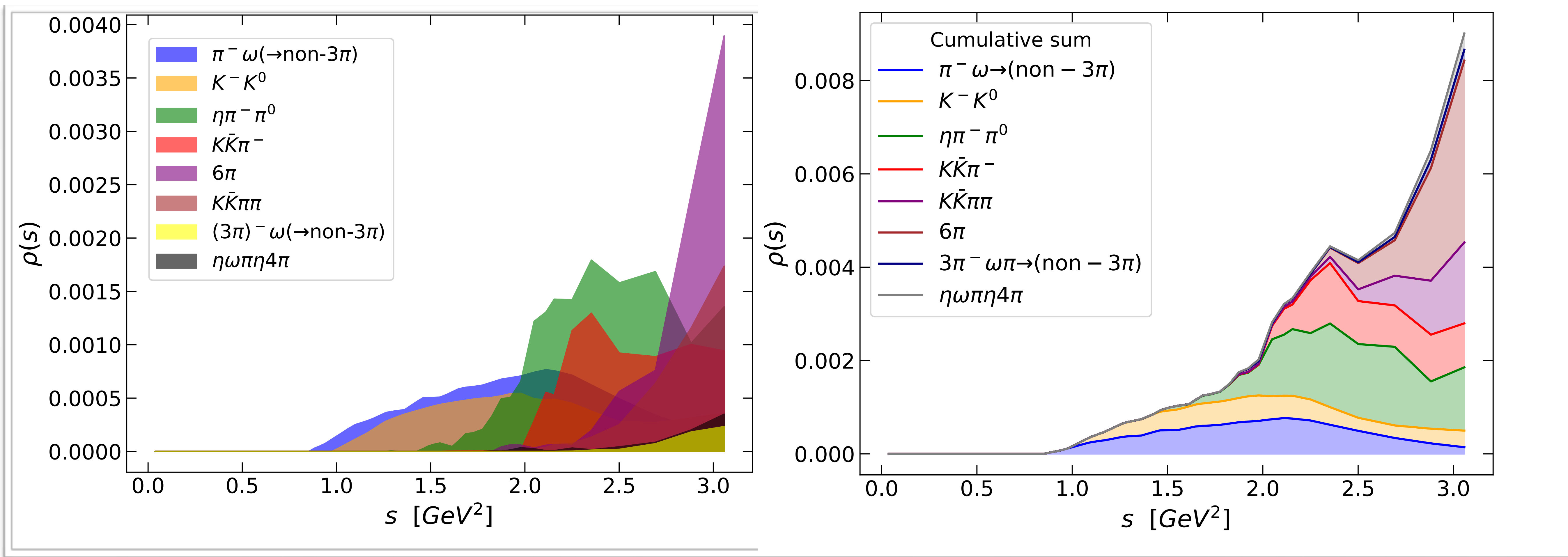


$\tau \to (\text{hadrons}) + \nu_\tau$

8 residual channels extracted from electroproduction data

Dramatic improvement for higher multiplicity modes (near end point)

No MC input.



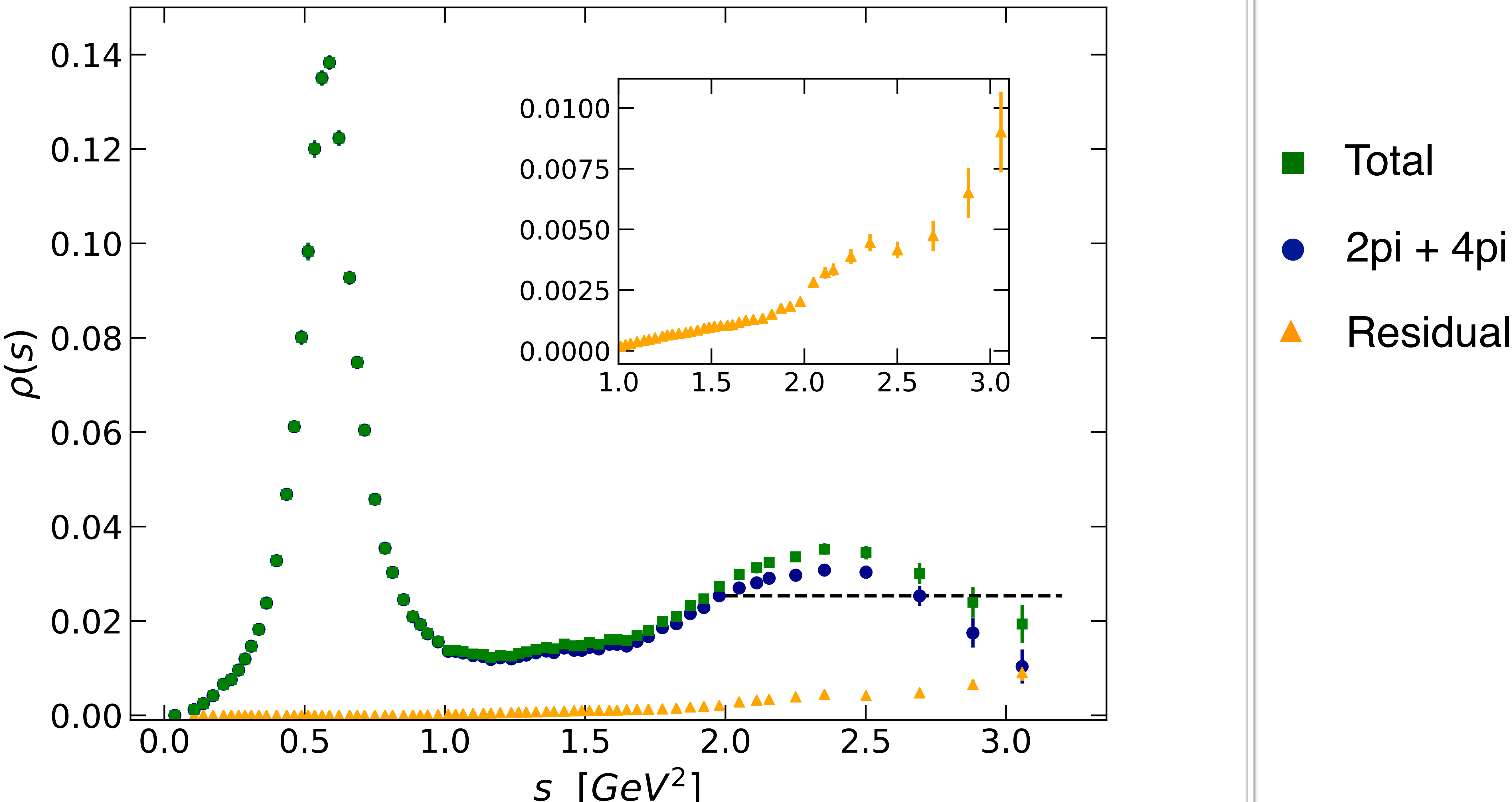
Original data sets from: BABAR, SND and CMD-3

DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

$\tau \to (\text{hadrons}) + \nu_\tau$

Final *new* vector-isovector spectral function

Combined 2pi + 4pi from ALEPH and OPAL + 8 residual channels from e^+e^-



DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

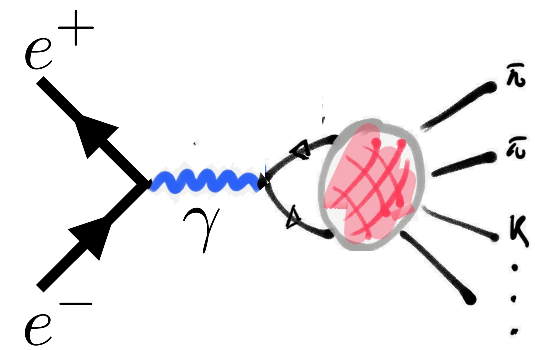
Data

$$e^+e^- \to \text{(hadrons)}$$

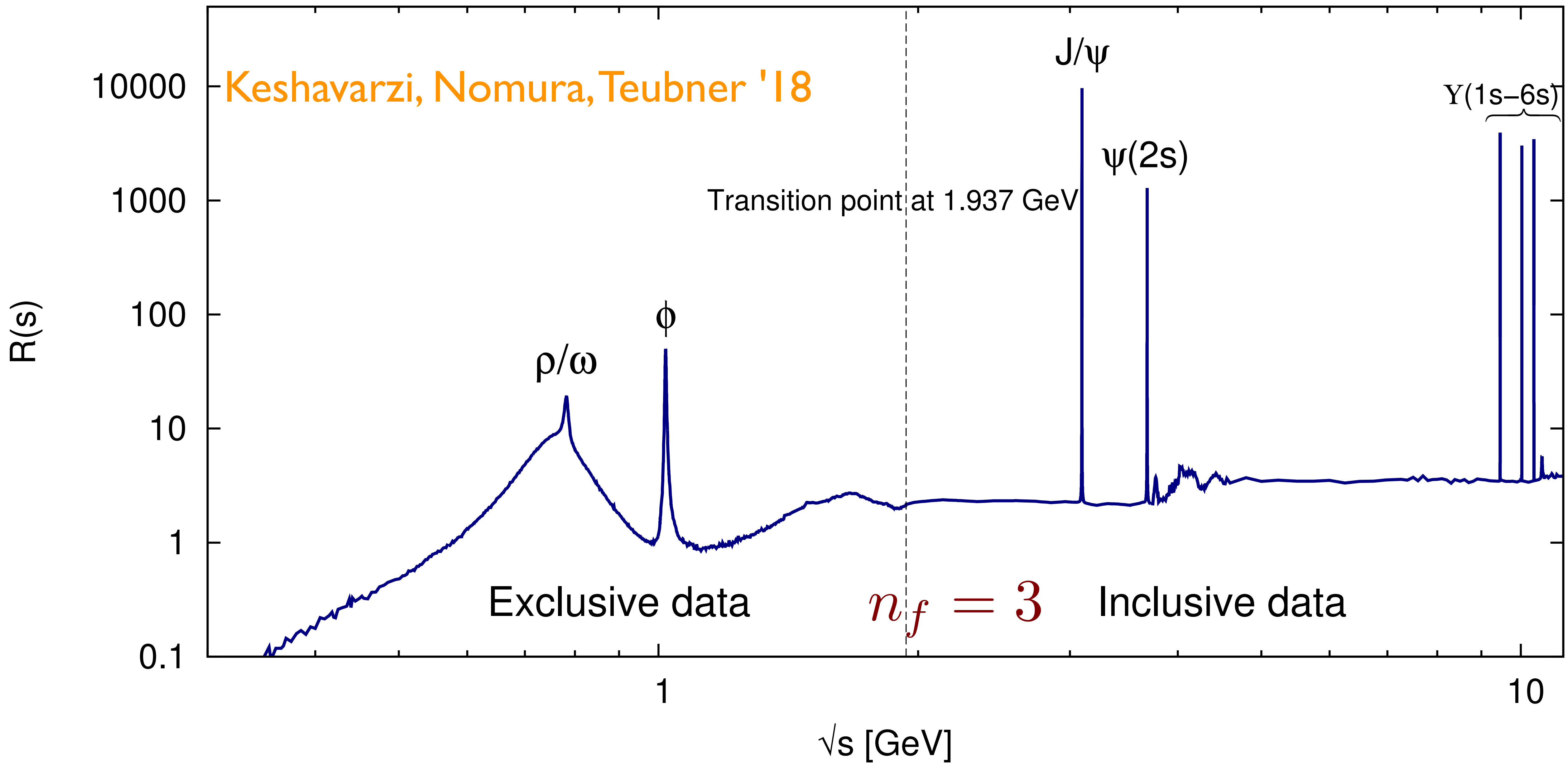
Keshavarzi, Nomura, Teubner '18

$e^+e^- \to$ hadrons

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

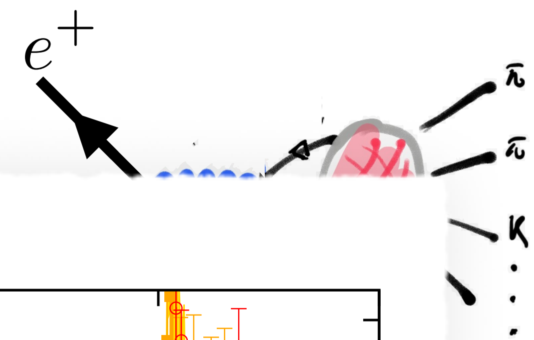


New compilation of *R*-ratio data (see also Davier et al '17, Jegerlehner '16)

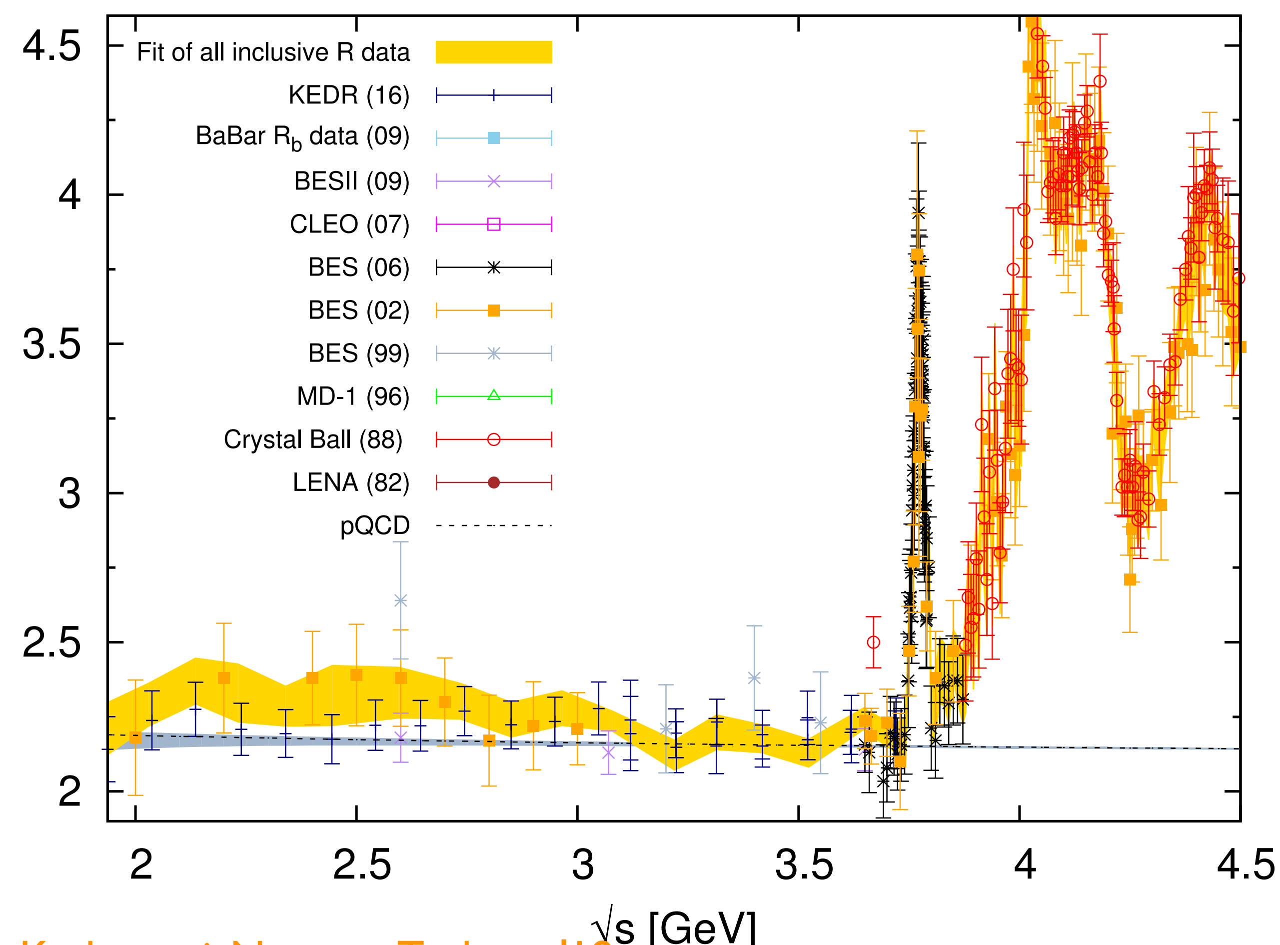


$e^+e^- \to$ hadrons

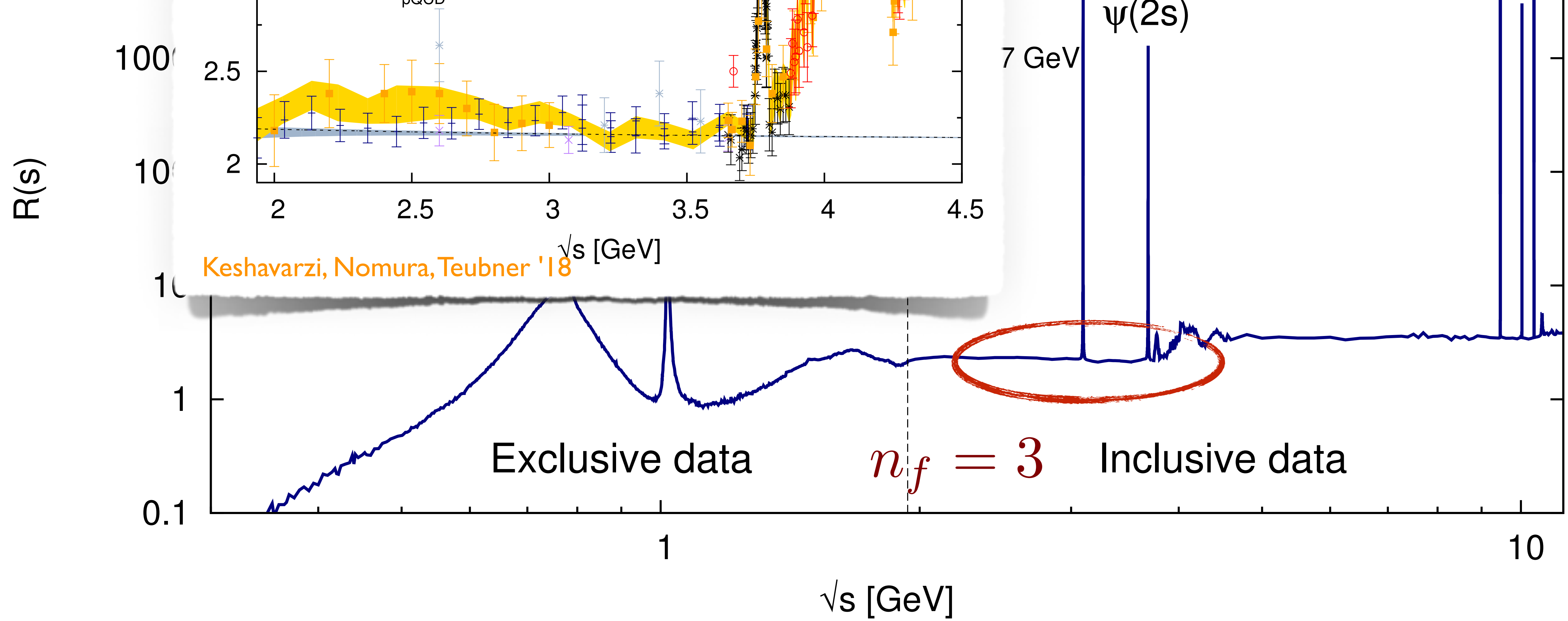
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma \ldots}$$



New co… see also Davier et al '17, Jegerlehner '16)



Keshavarzi, Nomura, Teubner '18



Integrated moments

$$\frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w(s/s_0)R(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(s/s_0)\Pi(z)$$

$w(y) \to 1$

$1-y^2$

$(1-y)^2(1+2y)$

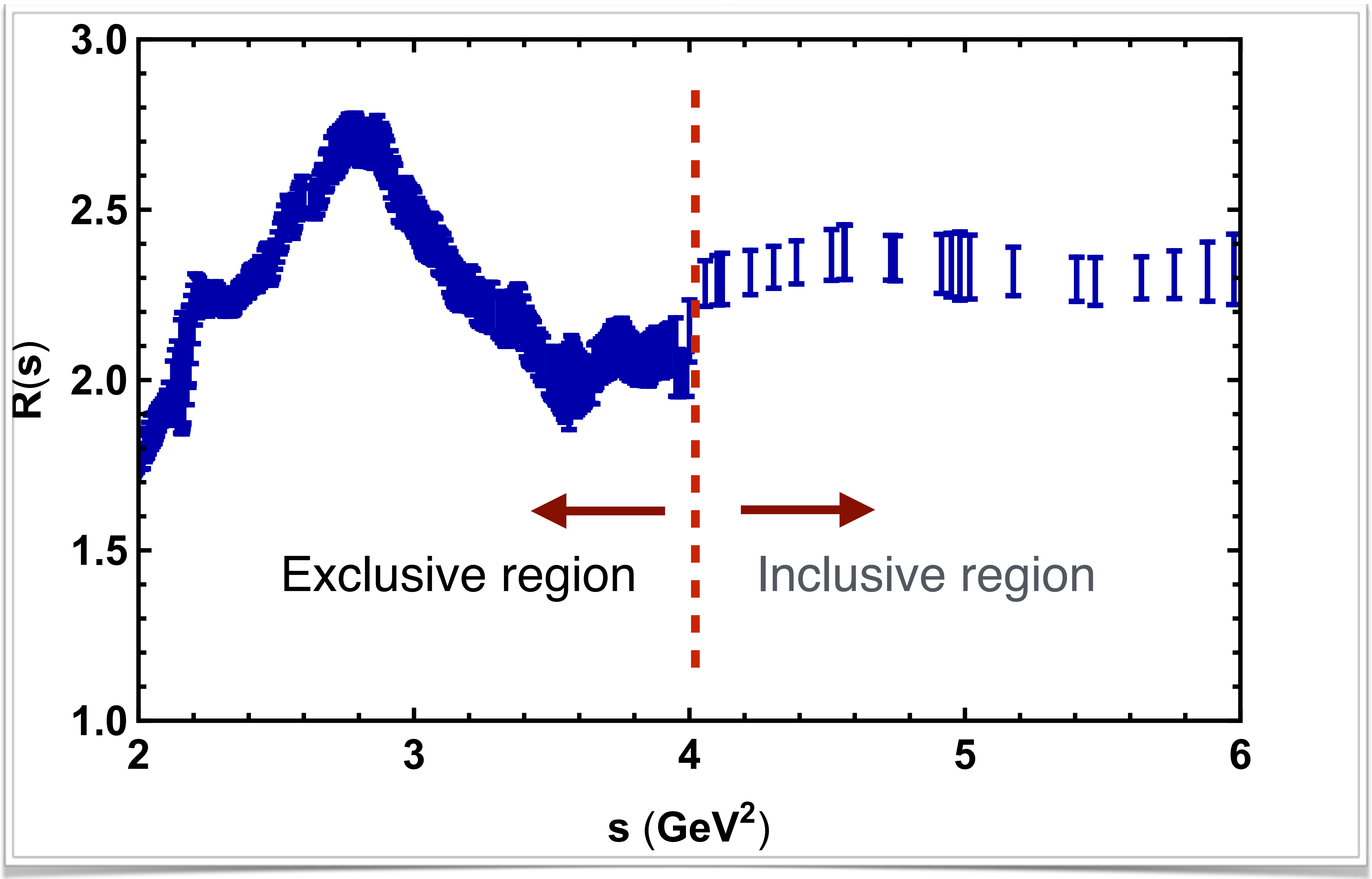
$(1-y^2)^2$

integrated moments have reduced errors

quantity	error at $s_0 = 4\ \mathrm{GeV}^2$	OPE coeff: $D = 2k$
$R(s_0)$	4.3%	–
$I^{(w=1)}(s_0)$	1.04%	$D = 2$
$I^{(w=1-y^2)}(s_0)$	0.73%	$D = 2, 6$
$I^{(w=(1-y)^2(1+2y))}(s_0)$	0.56%	$D = 2, 6, 8$
$I^{(w=(1-y^2)^2)}(s_0)$	0.59%	$D = 2, 6, 10$

Strategy

Much more data in the exclusive region



The precision of α_s is determined by the data in the exclusive region

Results

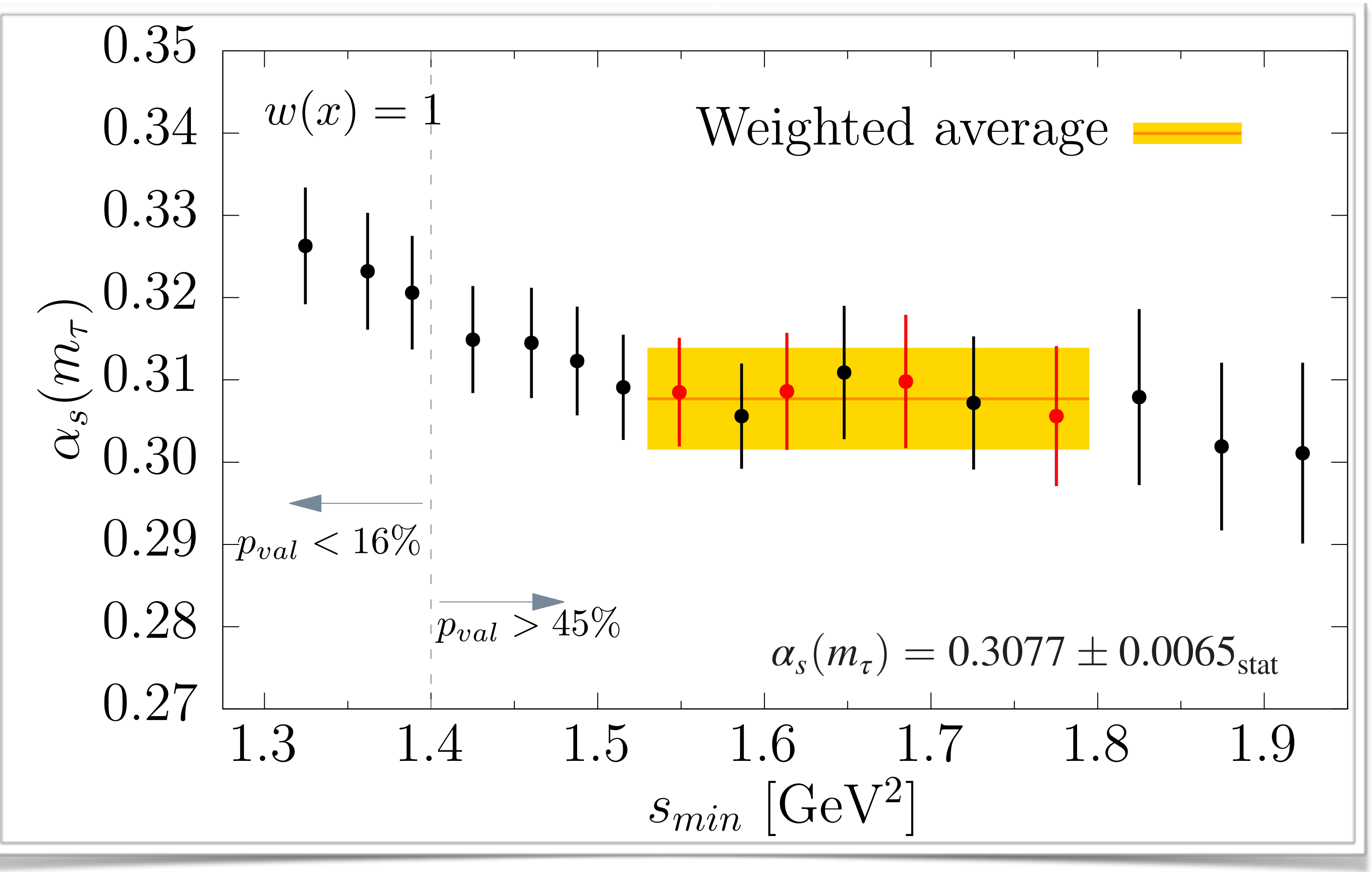
$\tau \to (\text{hadrons}) + \nu_\tau$

$$w_0(y) = 1,$$
$$w_2(y) = 1 - y^2,$$
$$w_3(y) = (1-y)^2(1+2y),$$
$$w_4(y) = (1-y^2)^2,$$

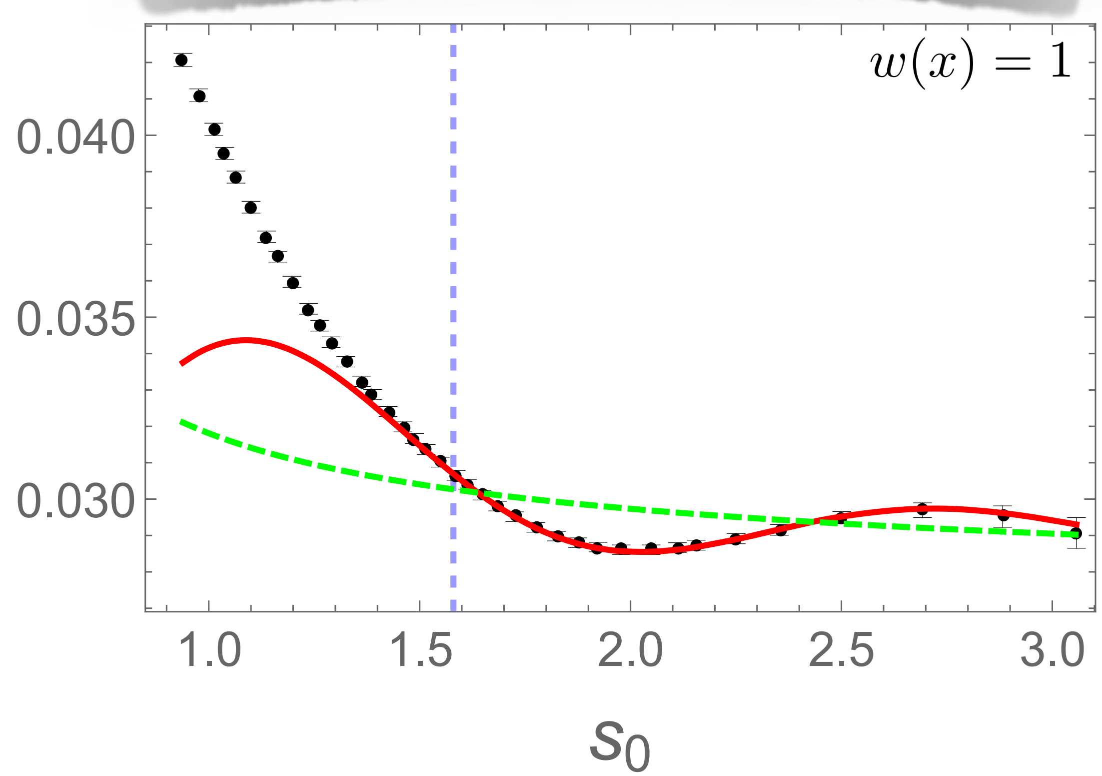
Several fits, single moments or in combination

Many fit windows: $[s_{\min} : m_\tau^2]$

Consistency between different fits (alpha_s, condensates DV parameters.)



$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s/s_0)\frac{1}{\pi}\text{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(z)\Pi(z)$$



$\tau \to (\text{hadrons}) + \nu_\tau$

$w_0(y) = 1,$
$w_2(y) = 1 - y^2,$
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$w_4(y) = (1-y^2)^2,$

Consistency between different fits (alpha_s, condensates DV parameters.)

	w_0	$w_0 \& w_2$	$w_0 \& w_3$	$w_0 \& w_4$
$\alpha_s(m_\tau) =$	$0.3077(65)$	$0.3091(69)$	$0.3080(70)$	$0.3079(70)$
$c_6 =$		$-0.0059(13)$	$-0.0070(12)$	$-0.0068(12)$ [GeV^6]

Final value

pt. series truncation, scale variation

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{\rm stat} \pm 0.0038_{\rm pert}$$
$$= 0.3077 \pm 0.0075 \qquad (n_f = 3,\ \text{FOPT})$$

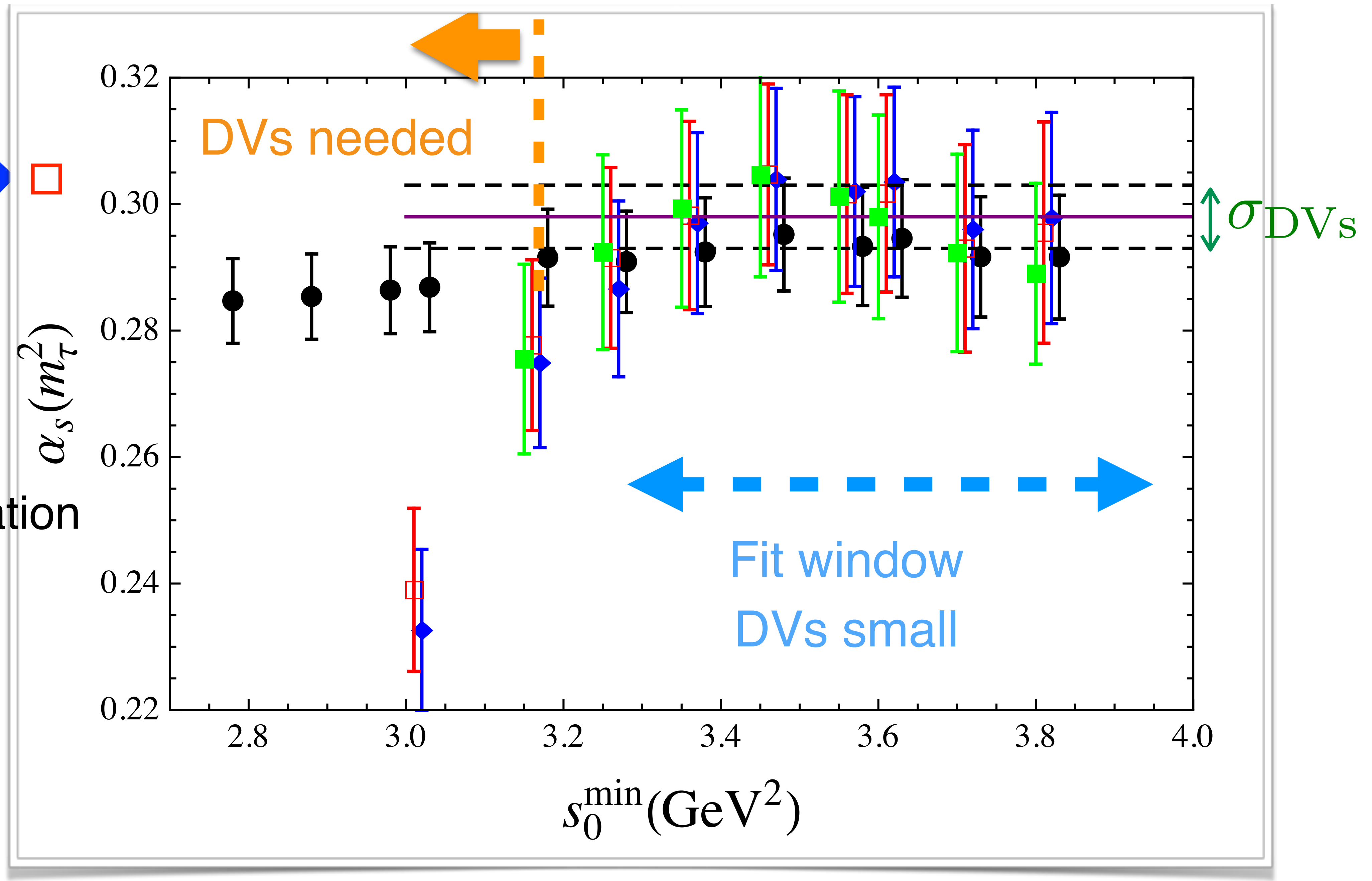
DVs in the analysis from $e^+e^- \to$ (hadrons)

- Consistency check: adding DVs is small effect above ~3.2 GeV²

Three moments, FOPT, no DVs

$w = 1$ with DVs

(our DVs use information from tau decays)



Final results (with three flavours)

Final results

Results evolved to m_Z

$$\alpha_s(m_Z)(\overline{\rm MS}, N_f = 5)$$

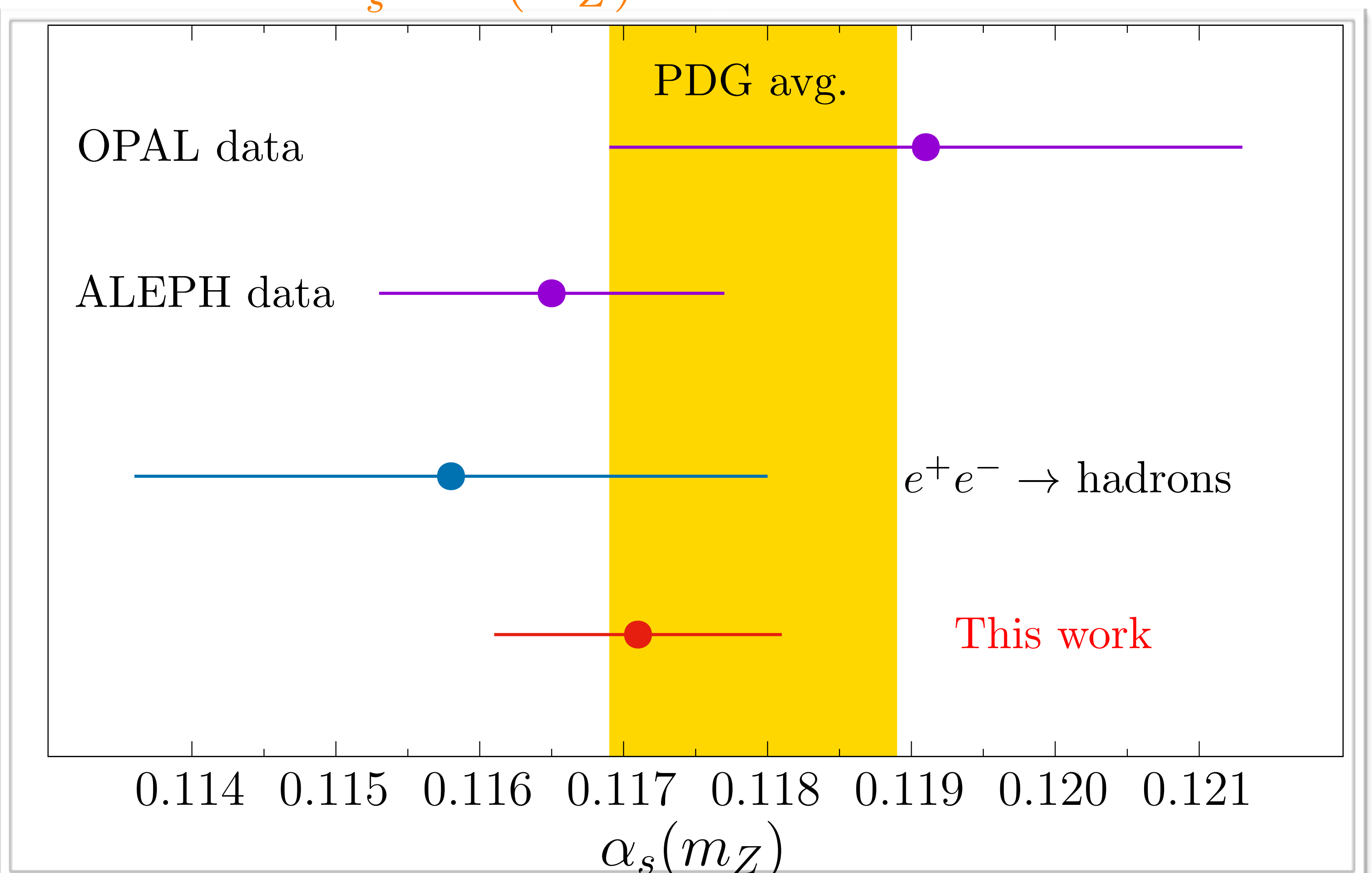
$$\tau \to (\text{hadrons}) + \nu_\tau$$

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

$$e^+e^- \to (\text{hadrons})$$

$$\alpha_s(m_Z) = 0.1158 \pm 0.0022$$

$$\alpha_s^{(\rm PDG)}(m_Z) = 0.1179 \pm 0.0010$$



Final results

Results evolved to m_Z

$$\alpha_s(m_Z)(\overline{\rm MS}, N_f = 5)$$

$\tau \to (\text{hadrons}) + \nu_\tau$

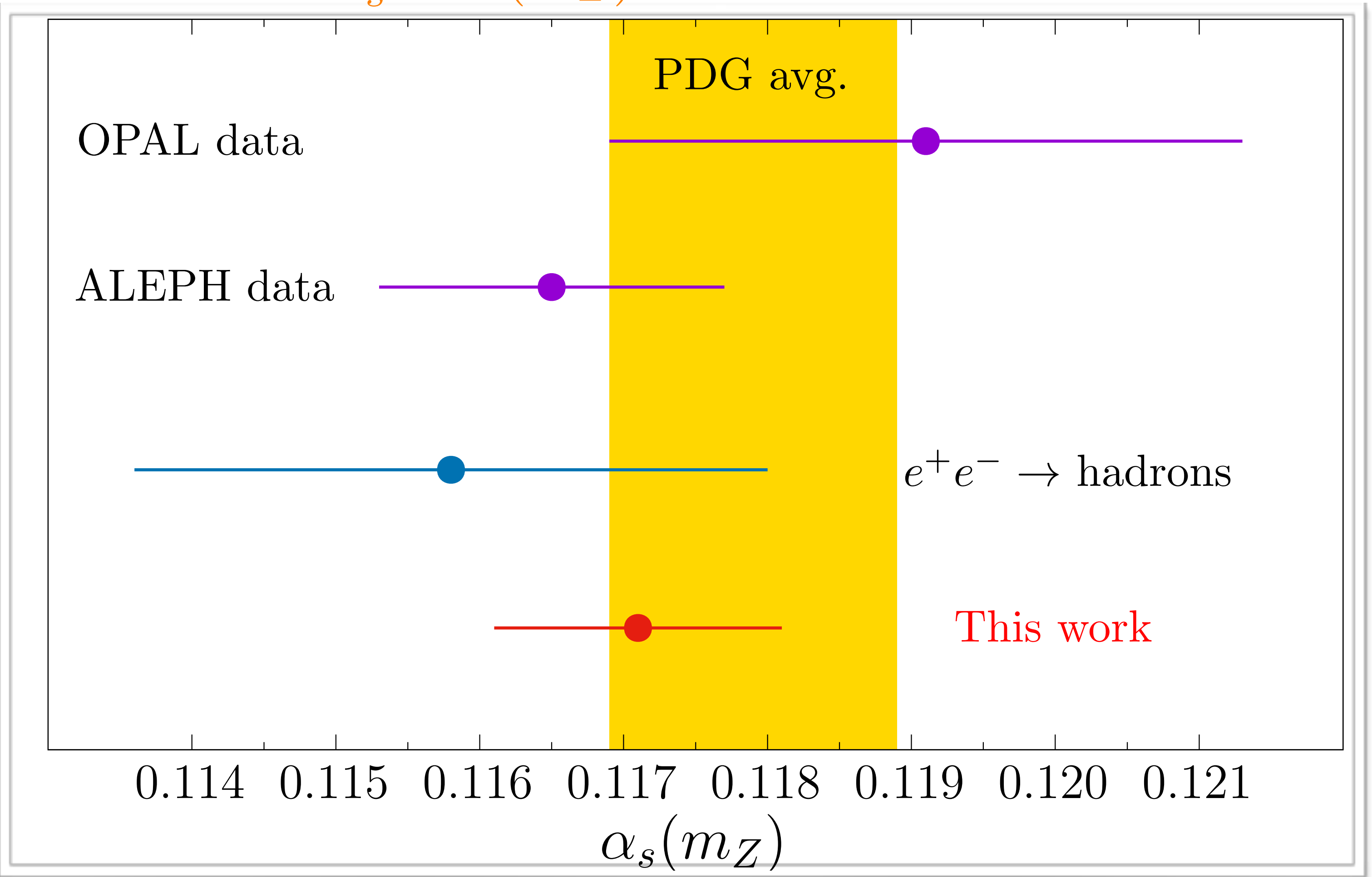
$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

$e^+e^- \to (\text{hadrons})$

$$\alpha_s(m_Z) = 0.1158 \pm 0.0022$$

$$\alpha_s^{(\text{CIPT})} = 0.1191$$

$$\alpha_s^{(\text{PDG})}(m_Z) = 0.1179 \pm 0.0010$$



Conclusions

α_s can be extracted reliably from *R* data with 3, 4, and 5 active flavours.

Smallest uncertainties from hadronic tau decays (~ 0.0010).

Previous results from CIPT were inconsistent (important reduction in theory error).

$$\tau \to (\text{hadrons}) + \nu_\tau$$

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

Ratio of V charm moments

$$\alpha_s(m_Z) = 0.1168 \pm 0.0019$$

Bottom vector moment ratios and purely *R*(s) extractions are theoretically clean, but the data is not precise enough (yet).

Conclusions

$\alpha_s^{(\mathrm{PDG})}(m_Z) = 0.1179 \pm 0.0010$