Two-pion contributions to the anomalous magnetic moment of the muon

Peter Stoffer
Faculty of Physics, University of Vienna

October 13, 2020
Seminar on Particle Physics
University of Vienna
1. Introduction

2. Standard Model prediction for the muon $g - 2$

3. Hadronic light-by-light scattering

4. Hadronic vacuum polarization

5. Summary and outlook
Overview

1. Introduction
2. Standard Model prediction for the muon $g - 2$
3. Hadronic light-by-light scattering
4. Hadronic vacuum polarization
5. Summary and outlook
Magnetic moment

- relation of spin and magnetic moment of a lepton:

\[ \vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s} \]

- $g_\ell$: Landé factor, gyromagnetic ratio
- Dirac’s prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics
- today: probing not only QED but entire SM
Electron vs. muon magnetic moments

- influence of heavier virtual particles of mass $M$
scales as

$$\frac{\Delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2}$$

- $a_e$ used to determine $\alpha_{\text{QED}}$

- $(m_\mu/m_e)^2 \approx 4 \times 10^4 \Rightarrow$ muon is much more sensitive
to new physics, but also to EW and hadronic contributions

- $a_\tau$ experimentally not yet known precisely enough
Muon anomalous magnetic moment $(g - 2)_{\mu}$

experimental progress in near future:

- FNAL expected to improve precision by a **factor of 4**
- theory needs to reduce **SM uncertainty**!

![Photo: Glukicov (License: CC-BY-SA-4.0)](image)

**muon $g - 2$ discrepancy**


Brookhaven E821

$10^{11} \times (a_{\mu} - a_{\mu}^{SM})$
Introduction

Muon anomalous magnetic moment \((g - 2)_\mu\)

Experimental progress in near future:

- FNAL expected to improve precision by a **factor of 4**
- Theory needs to reduce **SM uncertainty**!

![Photo: Glukicov (License: CC-BY-SA-4.0)](image)

\[
10^{11} \times (a_\mu - a_\mu^{SM})
\]

Brookhaven E821
FNAL E989 projected

6.1σ??
\[(g - 2)_\mu: \text{theory vs. experiment}\]

- discrepancy between SM and experiment 3.7\(\sigma\)
- hint to new physics?
- size of discrepancy points at \textit{electroweak scale}
  \(\Rightarrow\) heavy new physics needs some enhancement mechanism
- theory error completely dominated by \textit{hadronic effects}
Overview

1. Introduction

2. Standard Model prediction for the muon $g - 2$
   - QED and Electroweak Contribution
   - Hadronic contributions

3. Hadronic light-by-light scattering

4. Hadronic vacuum polarization

5. Summary and outlook
SM theory white paper

→ T. Aoyama et al. (Muon $g - 2$ Theory Initiative)

• community white paper on current status of SM calculation

• new consensus on SM prediction, ready for comparison with upcoming FNAL result

• many improvements on hadronic contributions
QED and electroweak contributions

- full $O(\alpha^5)$ calculation by Kinoshita et al. 2012 (involves 12672 diagrams!)
- EW contributions (EW gauge bosons, Higgs) calculated to two loops (three-loop terms negligible)

<table>
<thead>
<tr>
<th></th>
<th>$10^{11} \cdot a_\mu$</th>
<th>$10^{11} \cdot \Delta a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED total</td>
<td>116 584 718.931</td>
<td>0.104</td>
</tr>
<tr>
<td>EW</td>
<td>153.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Theory total</td>
<td>116 591 810</td>
<td>43</td>
</tr>
</tbody>
</table>
Hadronic contributions

- quantum corrections due to the strong nuclear force
- much smaller than QED, but dominate uncertainty

- hadronic vacuum polarization (HVP)
  \[ a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11} \]

- hadronic light-by-light scattering (HLbL)
  \[ a_\mu^{\text{HLbL}} = 92(18) \times 10^{-11} \]
Hadronic vacuum polarization (HVP)

- at present evaluated via dispersion relations and cross-section input from $e^+e^- \rightarrow \text{hadrons}$
- lattice QCD making fast progress
- intriguing discrepancies between $e^+e^-$ experiments
- $2.3\sigma$ discrepancy between dispersion relations and latest lattice results → S. Borsanyi et al., arXiv:2002.12347 [hep-lat]
Hadronic vacuum polarization (HVP)

photon HVP function:

\[
\Pi(q^2) = i(q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi(q^2)
\]

**unitarity** of the $S$-matrix implies the optical theorem:

\[
\text{Im}\Pi(s) = \frac{s}{e(s)^2} \sigma(e^+e^- \rightarrow \text{hadrons})
\]
Dispersion relation

causality implies **analyticity**:

\[
\Pi(s) = \frac{1}{2\pi i} \oint_{\gamma} \frac{\Pi(s')}{s' - s} ds'
\]

deform integration path:

\[
\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M^2_\pi}^{\infty} \frac{\text{Im}\Pi(s')}{(s' - s - i\epsilon)s'} ds'
\]
HVP contribution to \((g - 2)_\mu\)

\[
a^\text{HVP}_\mu = \frac{m^2_\mu}{12\pi^3} \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s} \sigma(e^+e^- \rightarrow \text{hadrons})
\]

- basic principles: unitarity and analyticity
- direct **relation to data**: total hadronic cross section \(\sigma(e^+e^- \rightarrow \text{hadrons})\)
- dedicated **e^+e^-** program (BaBar, Belle, BESIII, CMD3, KLOE, SND)
Hadronic light-by-light (HLbL)

- dominating contributions evaluated with dispersion relations
- hadronic models for subdominant contributions
- matching to asymptotic constraints
- lattice-QCD result compatible, but larger uncertainty
  ⇒ weighted average
### Theory vs. experiment

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$10^{11} \cdot a_\mu$</th>
<th>$10^{11} \cdot \Delta a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED total</td>
<td>116,584,718.931</td>
<td>0.104</td>
</tr>
<tr>
<td>EW</td>
<td>153.6</td>
<td>1.0</td>
</tr>
<tr>
<td>HVP</td>
<td>6,845</td>
<td>40</td>
</tr>
<tr>
<td>HLbL</td>
<td>92</td>
<td>18</td>
</tr>
<tr>
<td><strong>SM total</strong></td>
<td>116,591,810</td>
<td>43</td>
</tr>
<tr>
<td><strong>experiment (E821)</strong></td>
<td>116,592,089</td>
<td>63</td>
</tr>
<tr>
<td><strong>difference theory—exp</strong></td>
<td>279</td>
<td>76</td>
</tr>
</tbody>
</table>
Overview

1. Introduction

2. Standard Model prediction for the muon $g - 2$

3. Hadronic light-by-light scattering

4. Hadronic vacuum polarization

5. Summary and outlook
Hadronic light-by-light scattering

• previously based only on hadronic models
• first lattice-QCD results \( \rightarrow \) T. Blum et al., PRL 124 (2020) 132002

\[ a_{\mu}^{\text{HLbL, lattice}} = 79(35) \times 10^{-11} \]

• our work: **dispersive framework**, replacing hadronic models step by step
• dispersion relations + hadronic models

\[ a_{\mu}^{\text{HLbL, pheno}} = 94(19) \times 10^{-11} \]
BTT Lorentz decomposition

→ Colangelo, Hoferichter, Procura, Stoffer, JHEP 09 (2015) 074

Lorentz decomposition of the HLbL tensor:

→ Bardeen, Tung (1968) and Tarrach (1975)

\[
\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_i T_{i}^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)
\]

- Lorentz structures manifestly **gauge invariant**
- scalar functions \( \Pi_i \) **free of kinematic singularities**
  \( \Rightarrow \) dispersion relation in the Mandelstam variables
Dispersive representation

- write down a double-spectral (Mandelstam) representation for the HLbL tensor
- split the HLbL tensor according to the sum over intermediate (on-shell) states in unitarity relations

\[
\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \Pi_{\mu\nu\lambda\sigma}^{\pi\pi} + \ldots
\]
Dispersive representation

- write down a double-spectral (Mandelstam) representation for the HLbL tensor
- split the HLbL tensor according to the sum over intermediate (on-shell) states in unitarity relations

\[
\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0-\text{pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \Pi_{\mu\nu\lambda\sigma}^{\pi\pi} + \ldots
\]

one-pion intermediate state
Dispersive representation

- write down a double-spectral (Mandelstam) representation for the HLbL tensor
- split the HLbL tensor according to the sum over intermediate (on-shell) states in unitarity relations

\[ \Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \Pi_{\mu\nu\lambda\sigma}^{\pi\pi} + \ldots \]

two-pion intermediate state in both channels
Dispersive representation

- write down a double-spectral (Mandelstam) representation for the HLbL tensor
- split the HLbL tensor according to the sum over intermediate (on-shell) states in unitarity relations

\[ \Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \Pi_{\mu\nu\lambda\sigma}^{\pi\pi} + \ldots \]

two-pion intermediate state in first channel
Dispersive representation

- write down a double-spectral (Mandelstam) representation for the HLbL tensor
- split the HLbL tensor according to the sum over intermediate (on-shell) states in unitarity relations

\[ \Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\pi}_{\mu\nu\lambda\sigma} + \cdots \]

higher intermediate states
Hadronic light-by-light scattering

Pion pole

\[ \Pi^\pi_1 \text{-pole} = \frac{\mathcal{F}^{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) \mathcal{F}^{\pi^0\gamma^*\gamma}(q_3^2, 0)}{q_3^2 - M^2_{\pi}} \]

\[ \Pi^\pi_2 \text{-pole} \text{ via crossing symmetry} \]

- input: doubly-virtual and singly-virtual pion transition form factors \( \mathcal{F}^{\gamma^*\gamma^*\pi^0} \) and \( \mathcal{F}^{\gamma^*\gamma\pi^0} \)

- dispersive analysis of transition form factor:
  \[ a_{\mu}^{\pi^0\text{-pole}} = 62.6^{+3.0}_{-2.5} \times 10^{-11} \]

Pion-box contribution

→ Colangelo, Hoferichter, Procura, Stoffer, JHEP 04 (2017) 161

- simultaneous two-pion cuts in two channels
- Mandelstam representation explicitly constructed
- $q^2$-dependence: pion VFF $F^V_{\pi}(q_i^2)$ for each off-shell photon factor out
- Wick rotation: integrate over space-like momenta
- dominated by low energies $\leq 1$ GeV
- result: $\alpha_{\mu}^{\pi\text{-box}} = -15.9(2) \times 10^{-11}$
Rescattering contribution

- expansion into partial waves
- unitarity gives imaginary parts in terms of helicity amplitudes for $\gamma^*\gamma^{(*)} \rightarrow \pi\pi$:

$$\text{Im}_{\pi\pi} h^J_{\lambda_1\lambda_2,\lambda_3\lambda_4}(s) \propto \sigma_{\pi}(s) h_{J,\lambda_1\lambda_2}(s) h^*_{J,\lambda_3\lambda_4}(s)$$

- resummation of PW expansion reproduces full result: checked for pion box
Topologies in the rescattering contribution

our $S$-wave solution for $\gamma^*\gamma^* \rightarrow \pi\pi$:

\[
\begin{align*}
\text{recursive} & \quad \text{PWE, no LHC}
\end{align*}
\]

two-pion contributions to HLbL:

\[
\begin{align*}
\text{pion box} & \quad \text{rescattering contribution}
\end{align*}
\]
$S$-wave rescattering contribution

→ Colangelo, Hoferichter, Procura, Stoffer, JHEP 04 (2017) 161

- pion-pole approximation to left-hand cut
  $\Rightarrow q^2$-dependence given by $F^V_\pi$
- phase shifts based on modified inverse-amplitude method ($f_0(500)$ parameters accurately reproduced)
- result for $S$-waves:
  $$a_{\mu,J=0}^{\pi\pi,\pi\text{-pole LHC}} = -8(1) \times 10^{-11}$$
- extension to $f_0(980)$ in progress → Danilkin, Hoferichter, Stoffer
Extension to $D$-waves

* Hoferichter, Stoffer, JHEP 07 (2019) 073

- inclusion of resonance LHC
- unitarization with Omnès methods

![Graph showing cross section as a function of $\sqrt{s}$ and $|\cos \theta|$]
### HLbL overview

<table>
<thead>
<tr>
<th>Term</th>
<th>$10^{11} \cdot a_\mu$</th>
<th>$10^{11} \cdot \Delta a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$, $\eta$, $\eta'$-poles</td>
<td>93.8</td>
<td>4.0</td>
</tr>
<tr>
<td>pion/kaon box</td>
<td>-16.4</td>
<td>0.2</td>
</tr>
<tr>
<td>$S$-wave $\pi\pi$ rescattering</td>
<td>-8</td>
<td>1</td>
</tr>
<tr>
<td>scalars, tensors</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>axials</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>light quarks, short distance</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>$c$-loop</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td><strong>HLbL total (LO)</strong></td>
<td>92</td>
<td>19</td>
</tr>
</tbody>
</table>
Overview

1. Introduction
2. Standard Model prediction for the muon $g - 2$
3. Hadronic light-by-light scattering
4. Hadronic vacuum polarization
5. Summary and outlook
Hadronic vacuum polarization

- **final white paper number: data-driven evaluation**

\[ a_{\mu}^{\text{LO HVP, pheno}} = 6.931(40) \times 10^{-11} \]

- **average of published lattice-QCD results**

\[ a_{\mu}^{\text{LO HVP, lattice average}} = 7.116(184) \times 10^{-11} \]

- **newest lattice-QCD result**

\[ a_{\mu}^{\text{LO HVP, lattice}} = 7.087(53) \times 10^{-11} \]

Two-pion contribution to HVP

- $\pi\pi$ contribution amounts to more than 70% of HVP contribution
- responsible for a similar fraction of HVP uncertainty
Unitarity and analyticity

implications of unitarity (two-pion intermediate states):

1. $\pi \pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi \pi$ scattering
3. $\pi \pi$ scattering—$\pi \pi$ scattering

\[ \sigma(e^+e^- \rightarrow \pi^+\pi^-) \propto |F_V^\pi(s)|^2 \]

analyticity $\Rightarrow$ dispersion relation for HVP contribution
Unitarity and analyticity

implications of unitarity (two-pion intermediate states):

1. $\pi\pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi\pi$ scattering
3. $\pi\pi$ scattering—$\pi\pi$ scattering

\[ F^V_\pi(s) = |F^V_\pi(s)|e^{i\delta_1(s)} + \ldots \]

analyticity $\Rightarrow$ dispersion relation for pion VFF
Unitarity and analyticity

implications of unitarity (two-pion intermediate states):

1. $\pi\pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi\pi$ scattering
3. $\pi\pi$ scattering—$\pi\pi$ scattering

\[ \text{analyticity, crossing, PW expansion} \Rightarrow \text{Roy equations} \]
**Dispersive representation of pion VFF**

\[ F^V_\pi(s) = \Omega^1_1(s) \times G_\omega(s) \times G^N_{\text{in}}(s) \]

- Omnès function with elastic \( \pi \pi \)-scattering \( P \)-wave phase shift \( \delta^1_1(s) \) as input:

\[
\Omega^1_1(s) = \exp \left\{ \frac{s}{\pi} \int_{4M_{\pi}^2}^{\infty} ds' \frac{\delta^1_1(s')}{s'(s' - s)} \right\}
\]
Hadronic vacuum polarization

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP 02 (2019) 006

\[ F^V_\pi(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s) \]

- isospin-breaking \(3\pi\) intermediate state: negligible apart from \(\omega\) resonance (\(\rho-\omega\) interference effect)

\[
G_\omega(s) = 1 + \frac{s}{\pi} \int_{9 M^2_\pi}^{\infty} ds' \frac{\text{Im} g_\omega(s')} {s' (s' - s)} \left( 1 - \frac{9 M^2_\pi} {s'} \right)^4, \\
g_\omega(s) = 1 + \epsilon_\omega \frac{s} {(M_\omega - \frac{i}{2} \Gamma_\omega)^2 - s}
\]
Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP 02 (2019) 006

\[ F^V_\pi(s) = \Omega_1^1(s) \times G_\omega(s) \times G^{N}_{\text{in}}(s) \]

- heavier intermediate states: \( 4\pi \) (mainly \( \pi^0\omega \)), \( \bar{K}K \), \ldots
- described in terms of a conformal polynomial with cut starting at \( \pi^0\omega \) threshold

\[ G^{N}_{\text{in}}(s) = 1 + \sum_{k=1}^{N} c_k(z^k(s) - z^k(0)) \]

- correct \( P \)-wave threshold behavior imposed
Fit result for the VFF $|F^V_\pi(s)|^2$

![Graph showing fit result for the VFF $|F^V_\pi(s)|^2$](image)

- Total error
- Fit error
- NA7
- SND
- CMD-2
- BaBar
- KLOE08
- KLOE10
- KLOE12

$s [\text{GeV}^2]$
Fit result for the VFF $|F^V_\pi(s)|^2$

- **Total error**
- **Fit error**
- **NA7**
Hadronic vacuum polarization

Relative difference between data sets and fit result

<table>
<thead>
<tr>
<th>Total error</th>
<th>BaBar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit error</td>
<td>KLOE08</td>
</tr>
<tr>
<td>SN</td>
<td>KLOE10</td>
</tr>
<tr>
<td>CMD-2</td>
<td>KLOE12</td>
</tr>
</tbody>
</table>

\[ \frac{|F_{\pi}(s)|_{\text{fit}}^2}{|F_{\pi}(s)|_{\text{data}}^2} - 1 \]

\[ \sqrt{s} \text{ [GeV]} \]
Hadronic vacuum polarization

Result for $a_{\mu}^{HVP, \pi\pi}$ below 1 GeV

- SND
- CMD-2
- BaBar
- KLOE”

Energy scan

- All $e^+e^-$
- All $e^+e^-$, NA7

$10^{10} \times a_{\mu}^{\pi\pi} \mid_{\leq 1 \text{ GeV}}$
Contribution to $(g - 2)_\mu$

→ Colangelo, Hoferichter, Stoffer, JHEP 02 (2019) 006

- low-energy $\pi\pi$ contribution:

$$a_{\mu,\pi\pi}^{\text{HVP},\pi\pi}|_{\leq 0.63 \text{ GeV}} = 132.8(0.4)(1.0) \times 10^{-10}$$

- $\pi\pi$ contribution up to 1 GeV:

$$a_{\mu,\pi\pi}^{\text{HVP},\pi\pi}|_{\leq 1 \text{ GeV}} = 495.0(1.5)(2.1) \times 10^{-10}$$

- enters the white-paper value in a conservative merging with direct cross-section integration
Tension with lattice QCD


• implications of changing HVP?

• modifications at high energies affect hadronic running of $\alpha_{\text{QED}}^{\text{eff}} \Rightarrow$ clash with global EW fits


• lattice studies point at region $< 2 \text{ GeV}$

• $\pi\pi$ channel dominates

• relative changes in other channels would be prohibitively large
Hadronic vacuum polarization

Tension with lattice QCD


• force a different HVP contribution in VFF fits by including “lattice” datum with tiny uncertainty

• three different scenarios:
  • “low-energy” physics: $\pi\pi$ phase shifts
  • “high-energy” physics: inelastic effects, $c_k$
  • all parameters free

• study effects on pion charge radius, hadronic running of $\alpha_{\text{QED}}^{\text{eff}}$, phase shifts, cross sections
Hadronic vacuum polarization

Modifying $a^{\pi\pi}_\mu \leq 1 \text{ GeV}$


• “low-energy” scenario requires local changes in the cross section of $\sim 8\%$ in the $\rho$ region

• “high-energy” scenario has an impact on pion charge radius and the space-like VFF $\Rightarrow$ chance for independent lattice-QCD checks
Modifying $\alpha_{\mu}^{\pi\pi}|_{\leq 1 \text{ GeV}}$

correlations between $\alpha_{\mu}^{\pi\pi}$ and $\Delta\alpha_{\pi\pi}^{(5)}(M_Z^2)$

- phase shifts
  - $c_k, N - 1 = 3$
  - $c_k, N - 1 = 4$
- all parameters ---
Hadronic vacuum polarization

Modifying $a_{\mu}^{\pi\pi}|_{\leq 1 \text{ GeV}}$

correlations between $a_{\mu}^{\pi\pi}$ and $\langle r_{\pi}^2 \rangle$

phase shifts
$c_k, N - 1 = 3$
$c_k, N - 1 = 4$
all parameters

$\langle r_{\pi}^2 \rangle$ [fm$^2$]

$10^{10} \times a_{\mu}^{\pi\pi}|_{\leq 1 \text{ GeV}}$

480 490 500 510 520 530
Hadronic vacuum polarization

Modifying $\alpha_{\mu \pi \pi} \leq 1$ GeV

results for the VFF $|F^V_{\pi}(s)|^2$

- total error
- fit error
- NA7
- JLab
- phase shifts changed
- $c_k$ changed, $N - 1 = 4$
- all parameters changed
Modifying $a_{\mu}^{\pi\pi}|_{\leq 1 \text{ GeV}}$

The relative difference to the fit result is shown in the graph below, with various error bars representing different scenarios such as total error, fit error, and phase shifts changed. The plot includes data points from experiments like BaBar, KLOE08, KLOE10, KLOE12, SND, and CMD-2, with specific changes noted, such as $c_k$ changed and $N-1=4$. The graph covers the root mean square energy ($\sqrt{s}$) range from 0.6 to 1 GeV.
Overview

1. Introduction
2. Standard Model prediction for the muon $g - 2$
3. Hadronic light-by-light scattering
4. Hadronic vacuum polarization
5. Summary and outlook
Summary

• both lattice-QCD and dispersive methods making progress on hadronic contributions to $(g - 2)_\mu$ ⇒ white paper

• **achieved precision adequate** for first FNAL results

• final FNAL precision goal calls for **further improvement** in HLbL and HVP
Summary: HLbL

- precise *dispersive evaluations* of dominant contributions
- models reduced to sub-dominant contributions, but dominate uncertainty
Summary: HVP

- long-standing discrepancy between BaBar/KLOE $\Rightarrow$ wait for new $e^+e^-$ data
- intriguing tension with lattice-QCD $\Rightarrow$ unitarity/analyticity enable independent checks via pion VFF and $\langle r_\pi^2 \rangle$, in addition to further direct lattice results on HVP