TMDs and Jets

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NWO

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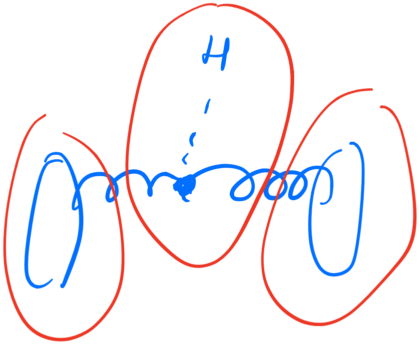
Factorization



$p^\mu = (E, 0, 0, E)$

$p_s^\mu = x\, p^\mu + p_\perp$

- All predictions rely on factorizing hard scattering from universal nonperturbative parton distributions $f_i(x)$ of incoming protons:

$$\sigma \sim \sum_{i,j} \int \mathrm{d}x_1 \int \mathrm{d}x_2\, \hat{\sigma}_{ij}(Q, x_1, x_2) f_i(x_1) f_j(x_2)$$

- Probe intrinsic $p_T \sim \Lambda_{\mathrm{QCD}}$ of partons in proton, $f_i(x, p_T)$.
- Key process is $ep \to ehX$. **Can we replace hadron h by jet?**

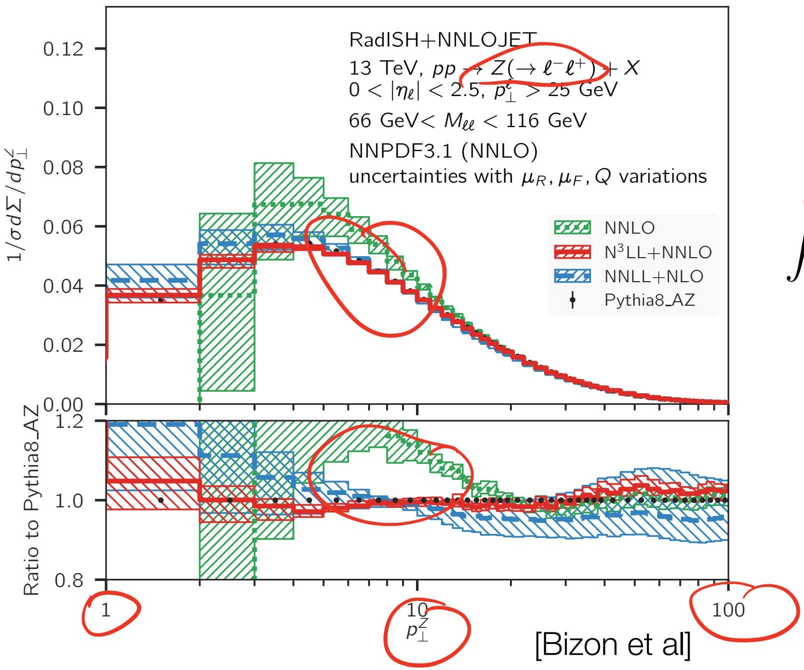
Outline

1. Introduction:
   A. Transverse momentum factorization
   B. Jets and Winner-Take-All axis
2. Processes:
   A. $e^+e^- \to JJX$
   B. $ep \to eJX$

   $\Big\}$ [Gutierrez-Reyes, Scimemi, WW, Zoppi]

   C. $pp \to VJX$ [Chien, Rahn, Schrijnder van Velzen, Shao, WW, Wu]
3. Conclusions

1A. Example: Z-boson q_T spectrum



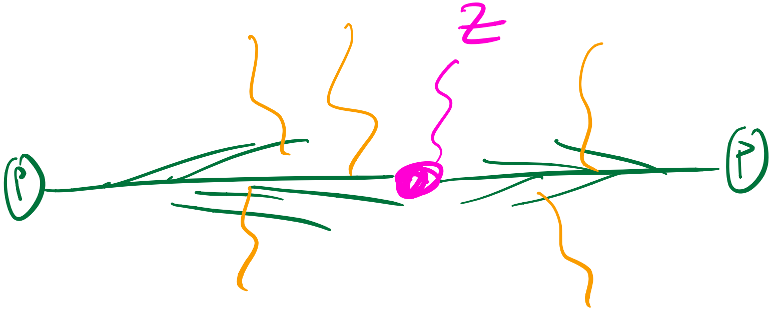
[Bizon et al]

$$\int_0^{q_T} \mathrm{d}q_T \frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \begin{array}{l} \# \\ + \alpha_s(\#L^2 + \#L + \#) \\ + \alpha_s^2(\#L^4 + \#L^3 + \cdots) \\ + \alpha_s^3(\#L^6 + \#L^5 + \cdots) \\ \qquad \vdots \qquad\qquad \vdots \end{array}$$

- Cross section dominated by small q_T, where it suffers from large logarithms $L = \ln(Q/q_T)$

$Q = (p_e + p_{\bar{e}})^2$

1A. Transverse momentum factorization



- For $q_T \ll Q$, radiation is collinear or soft.
- Factorize physics at scales Q, q_T

[Collins, Soper, Sterman; Becher, Neubert; Chiu, Jain, Neill, Rothstein; Echevarria, Idilbi, Scimemi, …]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}\vec{q}_T} = H(Q) \int \mathrm{d}\vec{p}_{T,a}\, B_a(\vec{p}_{T,a}, x_a) \int \mathrm{d}\vec{p}_{T,b}\, B_b(\vec{p}_{T,b}, x_b)$$

$$\times \int \mathrm{d}\vec{p}_{T,s},\, S(\vec{p}_{T,s})\, \delta(\vec{q}_T - \vec{p}_{T,a} - \vec{p}_{T,b} - \vec{p}_{T,s})$$

1A. Renormalization group and resummation

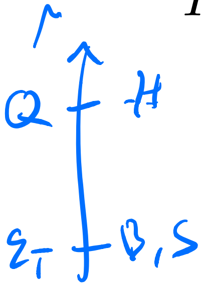
- Each ingredient in factorization involves single scale. E.g.

$$H(Q,\mu) \propto 1 + \frac{\alpha_s C_F}{4\pi}\left[-\cancel{\ln^2 \frac{Q^2}{\mu^2}} + \cancel{3\ln\frac{Q^2}{\mu^2}} - 8 + \frac{7\pi^2}{6}\right]$$

- Resum large logarithms by evaluating ingredients at their natural scale and evolving them to a common scale:

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}\ln\mu} H(Q,\mu) = \gamma_H(Q,\mu) H(Q,\mu)}$$

$$H(Q,\mu) = \exp\left[-\frac{\alpha_s C_F}{4\pi}\ln^2\frac{Q^2}{\mu^2} + \dots\right] H(Q,Q)$$



1A. Rapidity logarithms

- There is a mismatch of logarithms:

$$\ln^2(Q/q_T) \overset{?}{=} \ln^2(Q/\mu) + \ln^2(q_T/\mu)$$

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$$\ln^2(Q/q_T) = \ln^2(Q/\mu) - \ln^2(q_T/\mu) - 2\ln(Q/q_T)\ln(q_T/\mu)$$

- Beam and soft function require rapidity regularization:

$$S^{(1)}(\vec{p}_T) \propto \alpha_s \frac{\mu^{2\epsilon}}{(\vec{p}_T^{\,2})^{1+\epsilon}} \int \mathrm{d}y$$

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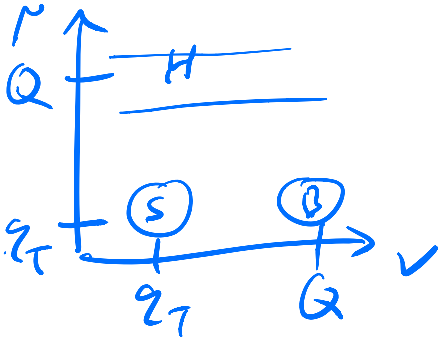
- Beam and soft function require rapidity regularization:

$$S^{(1)}(\vec{p}_T) \propto \alpha_s \frac{\mu^{2\epsilon}\,\nu^\eta}{(\vec{p}_T^{\,2})^{1+\epsilon+\eta/2}} \int \mathrm{d}y\, |2\sinh y|^{-\eta}$$

- Rapidity divergences $1/\eta$ lead to rapidity renormalization → corresponding ν-evolution resums rapidity logarithms.
[Chiu, Jain, Neill, Rothstein; see also Collins, Soper; Collins; Becher, Neubert, …]

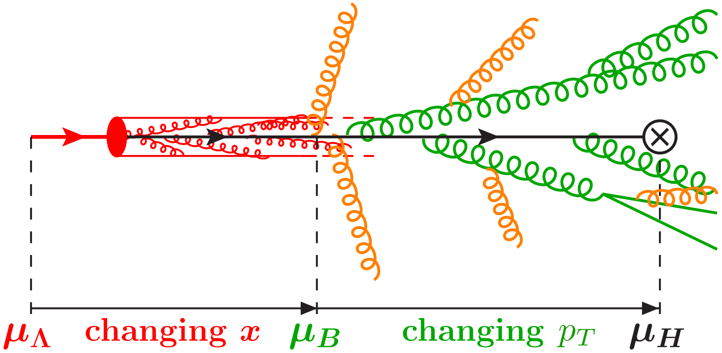


1A. Transverse momentum distributions (TMDs)

- Beam functions can be matched onto PDFs for $p_T \gg \Lambda_{\rm QCD}$

$$B_i(\vec{p}_T, x, \mu, \nu) = \sum_j \int \frac{\mathrm{d}x'}{x'}\, I_{ij}\Big(\vec{p}_T, \frac{x}{x'}, \mu, \nu\Big) f_j(x', \mu)$$

[Collins, Soper; Stewart, Tackmann, WW]



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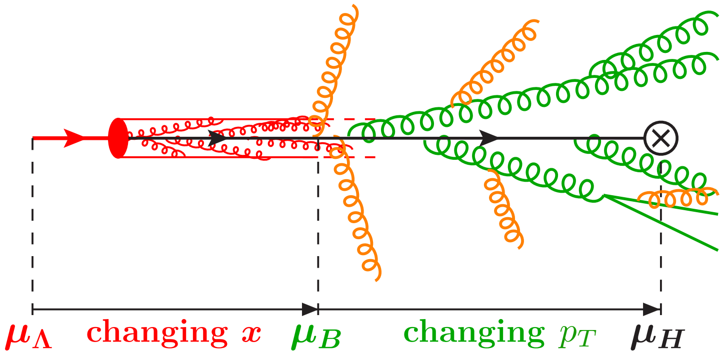


- For $p_T \sim \Lambda_{\rm QCD}$, the beam functions describes the intrinsic transverse momenta of partons in the proton (TMD PDFs).
- Often one absorbs the soft function into the beam functions.

$$BBS = \overset{B}{(B\sqrt{S})}\overset{B}{(B\sqrt{S})}$$

1A. Polarization

- Gluon TMDs can be linearly polarized for unpolarized protons:

$$B_g^{\alpha\beta}(\vec{b}_T) \sim \langle P|A_\perp^\alpha A_\perp^\beta|P\rangle$$

$$= -\frac{g_\perp^{\alpha\beta}}{d-2}B_g + \left(\frac{g_\perp^{\alpha\beta}}{d-2} + \frac{b_T^\alpha b_T^\beta}{\vec{b}_T^2}\right)B_g^L$$

- Many more possibilities with polarization. E.g. quark TMDs:

quark pol.

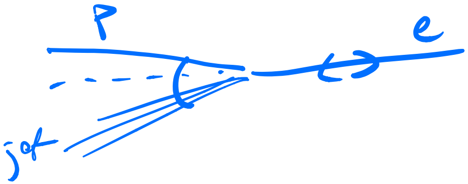
nucleon pol.

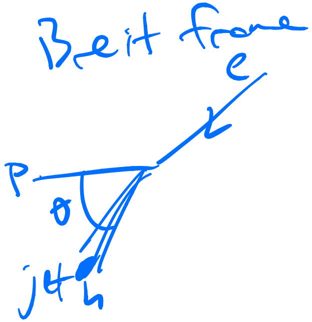
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

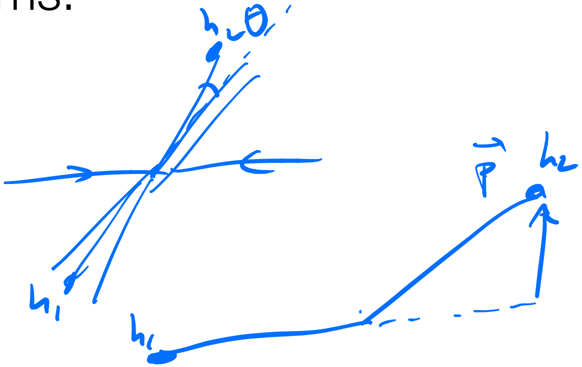
- $g_{1T}, h_{1L}^\perp, h_{1T}^\perp$: integral over p_T is zero, match onto twist 2.
- $f_{1T}^\perp, h_1^\perp$: T-odd, match onto twist 3.

Goal: TMDs and Jets

- Three standard TMD factorization theorems:







$$\frac{\mathrm{d}\sigma(pp \to VX)}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}\vec{q}_T} = H_{q\bar{q}\to V}(Q) B_q(\vec{q}_T, x_1) \otimes B_{\bar{q}}(\vec{q}_T, x_2)$$

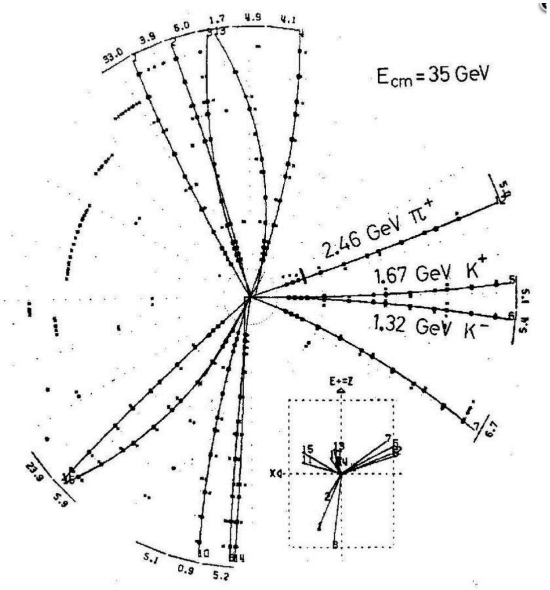
$$\frac{\mathrm{d}\sigma(ep \to ehX)}{\mathrm{d}Q\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\vec{q}_T} = H_{eq\to eq}(Q) B_q(\vec{q}_T, x) \otimes D_{q\to h}(\vec{q}_T, z)$$

$$\frac{\mathrm{d}\sigma(e^+e^- \to h_1 h_2 X)}{\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}\vec{q}_T} = H_{e^+e^-\to q\bar{q}}(Q) D_{q\to h_1}(\vec{q}_T, z_1) \otimes D_{q\to h_2}(\vec{q}_T, z_2)$$

- **Question 1:** Can we replace h by a jet?
- **Question 2:** TMDs for processes with >2 collinear directions?

1B. Jets

- First evidence for gluons from 3-jet event:



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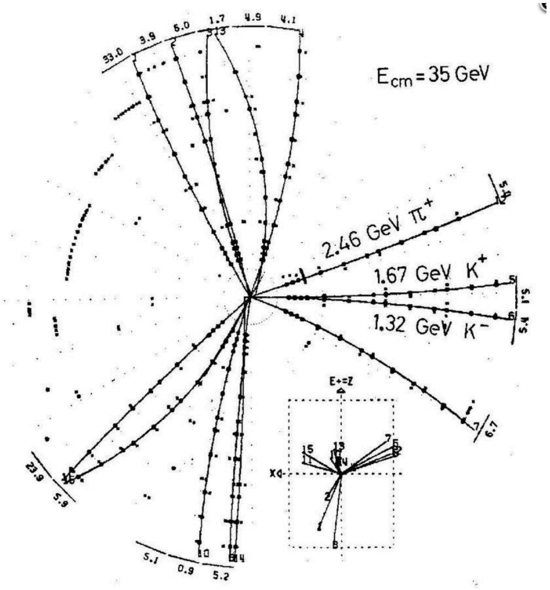
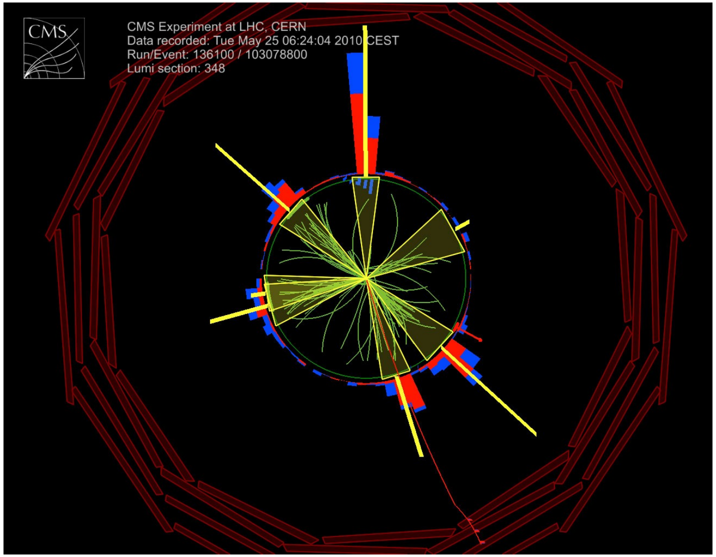


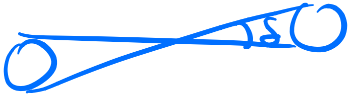


- Jets at the LHC: How should we define them?

1B. Cone algorithms

- Sterman-Weinberg jets: back-to-back cones with angle δ, containing all but fraction ε of event energy.

$E_{out} < \varepsilon E$



- Extension to multiple jets hard: Seeds for initial jet directions? How to handle overlapping cones? IR safety?

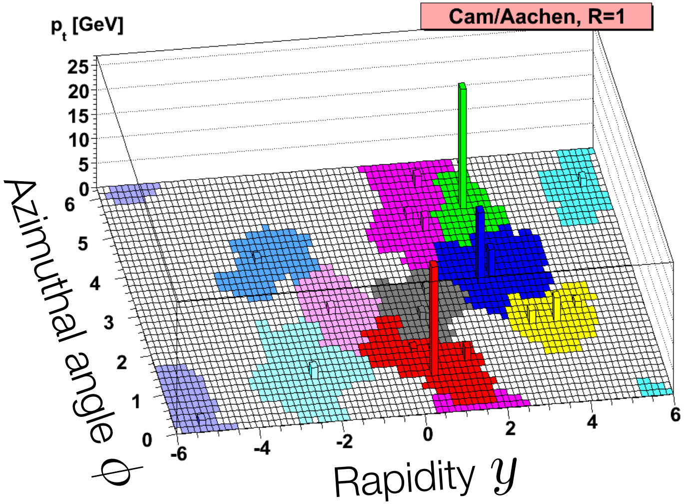
stable cone

1B. Clustering algorithms

- Determine distances between “particles”. E.g. Cam/Aachen:

$$d_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$$

- Combine nearest “particles”: $p_i^\mu, p_j^\mu \to p_i^\mu + p_j^\mu$
- Repeat until distances larger than jet radius parameter R.



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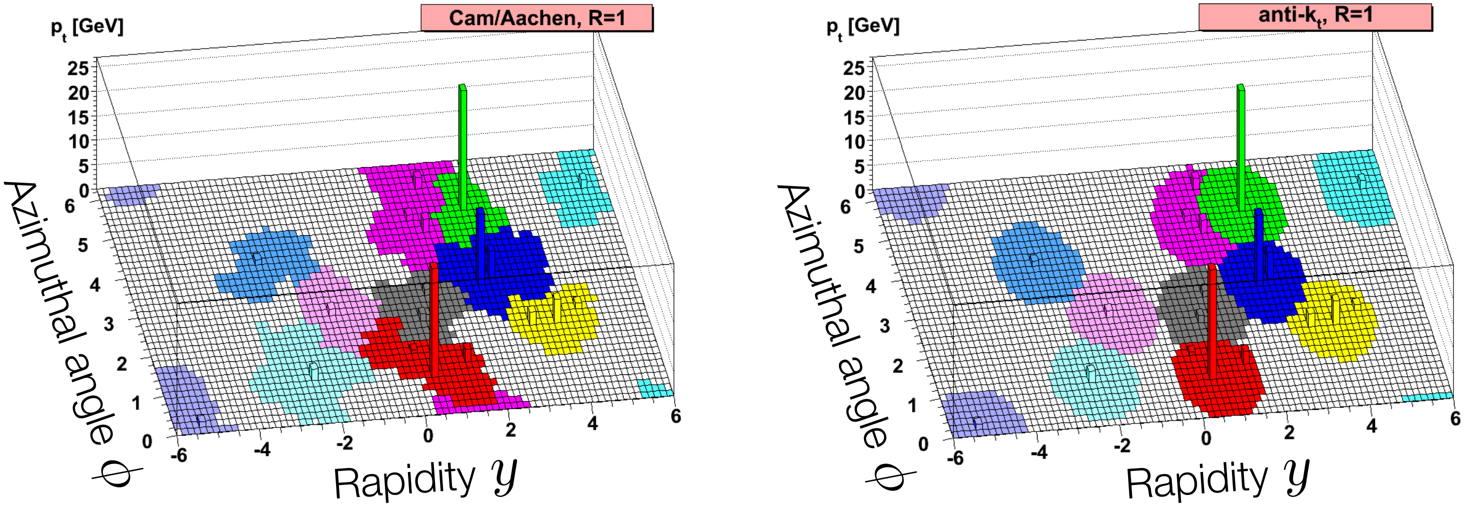
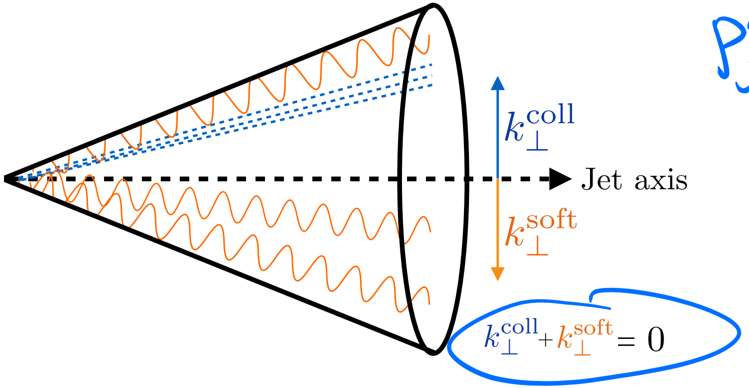


[Cacciari, Salam, Soyez]

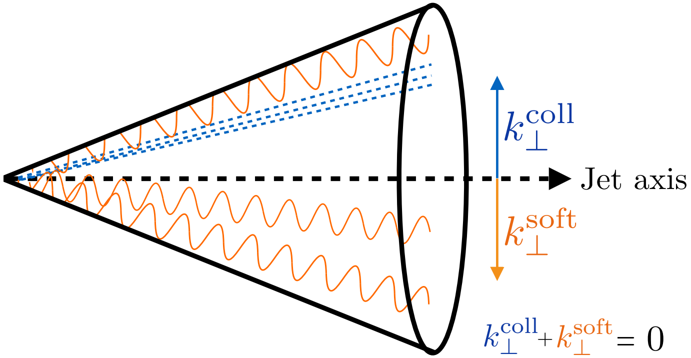
1B. Winner-Take-All axis



$\vec{p}_{jet} = \sum_{i \in jet} \vec{p}_i$

- Jet axis along jet momentum: recoiled by soft radiation in jet → Theory: non-global logarithms, Exp: contamination.

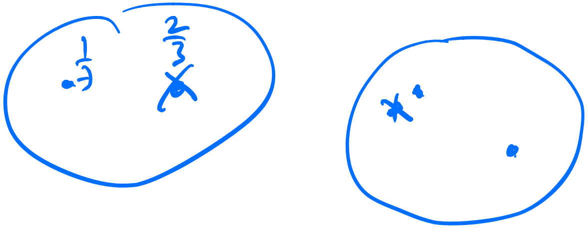
1B. Winner-Take-All axis



- Jet axis along jet momentum: recoiled by soft radiation in jet → Theory: non-global logarithms, Exp: contamination.
- Absent for Winner-Take-All (WTA) recombination scheme:

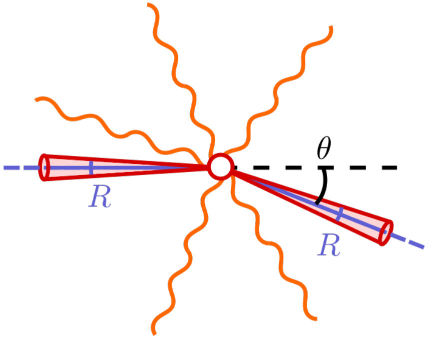
$$E_r = E_1 + E_2$$

$$\hat{n}_r = \begin{cases} \hat{n}_1 & \text{if } E_1 > E_2 \\ \hat{n}_2 & \text{if } E_2 > E_1 \end{cases}$$



[Salam; Bertolini, Chan, Thaler]

2A. $e^+e^- \to JJX$: regimes



$$\frac{\mathrm{d}\sigma(e^+e^- \to JJX)}{\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}\vec{q}_T} = \sum_q H_{e^+e^-\to q\bar{q}}(Q) \int \frac{\mathrm{d}\vec{b}_T}{(2\pi)^2}\, e^{-\mathrm{i}\vec{b}_T\cdot\vec{q}_T}\, J_q(\vec{b}_T, z_1, QR)\, J_q(\vec{b}_T, z_2, QR)$$

- Angular decorrelation related to transverse momentum $\theta \approx \dfrac{2q_T}{Q}$
- $\theta \gg R$: no jet axis dependence, match onto coll. jet functions.

$\ln R$

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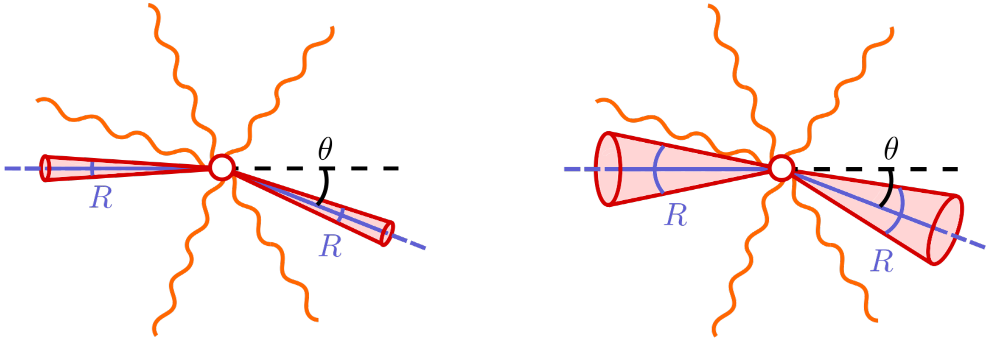

$$\frac{d\sigma(e^+e^- \to JJX)}{dz_1\, dz_2\, d\vec{q}_T} = \sum_q H_{\underline{e^+e^- \to q\bar{q}}}(Q) \int \frac{d\vec{b}_T}{(2\pi)^2}\, e^{-i\vec{b}_T\cdot\vec{q}_T}\, J_q(\vec{b}_{\underline{T}}, \underline{z_1}, QR)\, J_q(\vec{b}_T, \underline{z_2}, QR)$$

- Angular decorrelation related to transverse momentum $\theta \approx \dfrac{2q_T}{Q}$
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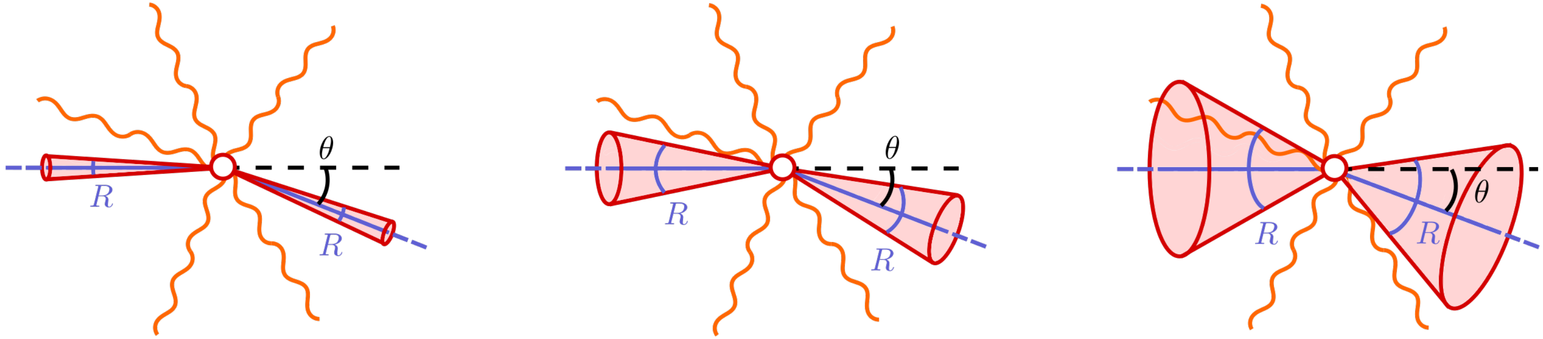

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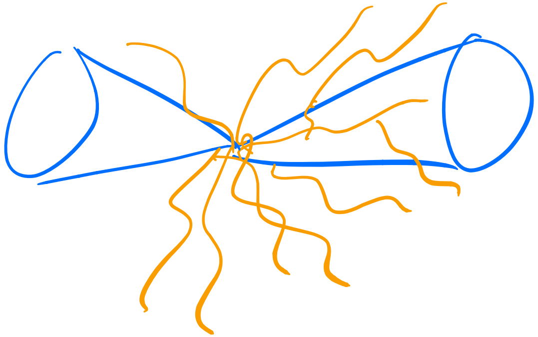
- Angular decorrelation related to transverse momentum $\theta \approx \dfrac{2q_T}{Q}$
- $\theta \gg R$: no jet axis dependence, match onto coll. jet functions.
- $\theta \sim R$: finite terms in jet function depend on axis choice.
- $\theta \ll R$: factorization only holds for Winner-Take-All axis.

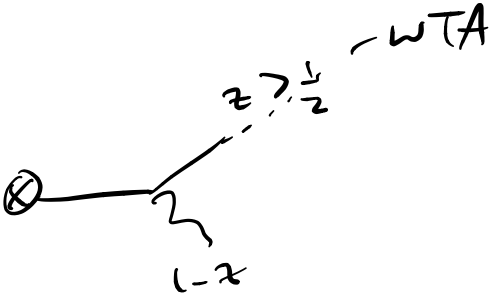
2A. Soft function



$p^{\sim}$ $\quad \zeta_T(\overset{p^+}{1}, \overset{p^-}{1}, \overset{p_\perp}{1})$

- WTA insensitive to soft radiation → no difference for radiation in/outside jet. Total recoil from standard TMD soft function.
[Echevarria, Scimemi, Vladimirov; Luebbert, Oredsson, Stahlhofen; Li, Zhu]
- SJA only sensitive to soft radiation outside the jet. →
For $\theta \gtrsim R$, wide-angle soft does not resolve narrow jet.
For $\theta \ll R$, TMD factorization breaks due to nonglobal logs.
[Dasgupta, Salam; see also Banfi, Dasgupta, Delenda]

2A. TMD jet function at one loop



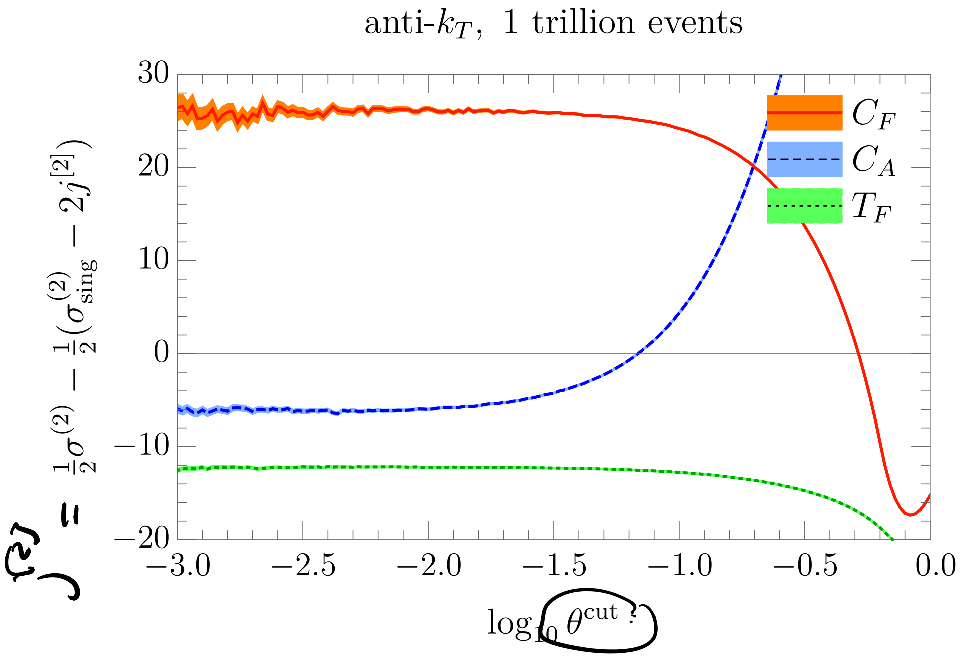
- At one-loop: WTA axis along most energetic particle.
- Standard jet axis and WTA agree for $\theta \gg R$.
- Very different behavior for $\theta \ll R$:

$$J_i^{\mathrm{WTA}}(\vec{q}_T, z, QR) \to \delta(1-z)\mathcal{J}_i(\vec{q}_T)$$

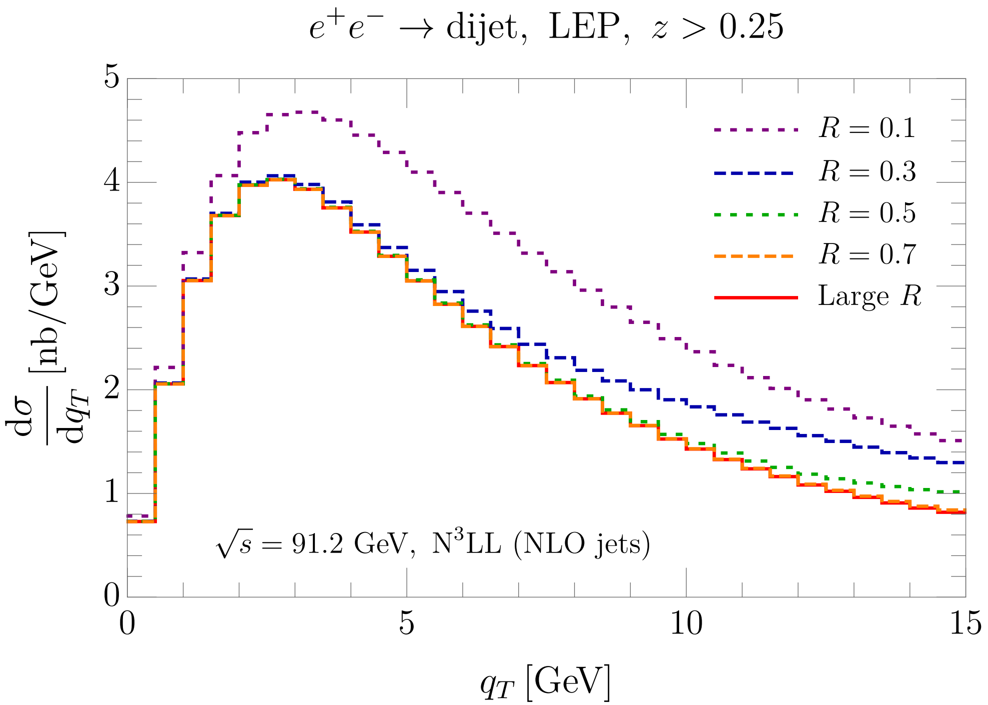
$$J_i^{\mathrm{SJA}}(\vec{q}_T, z, QR) \to \delta(1-z)\delta^2(\vec{q}_T)J_i(QR)$$

2A. Large-R jet function at two loops

- Jet function for $\theta \ll R$ determined by anomalous dimensions, apart from constant → extract at NNLO using EVENT2.
  [Catani, Seymour]

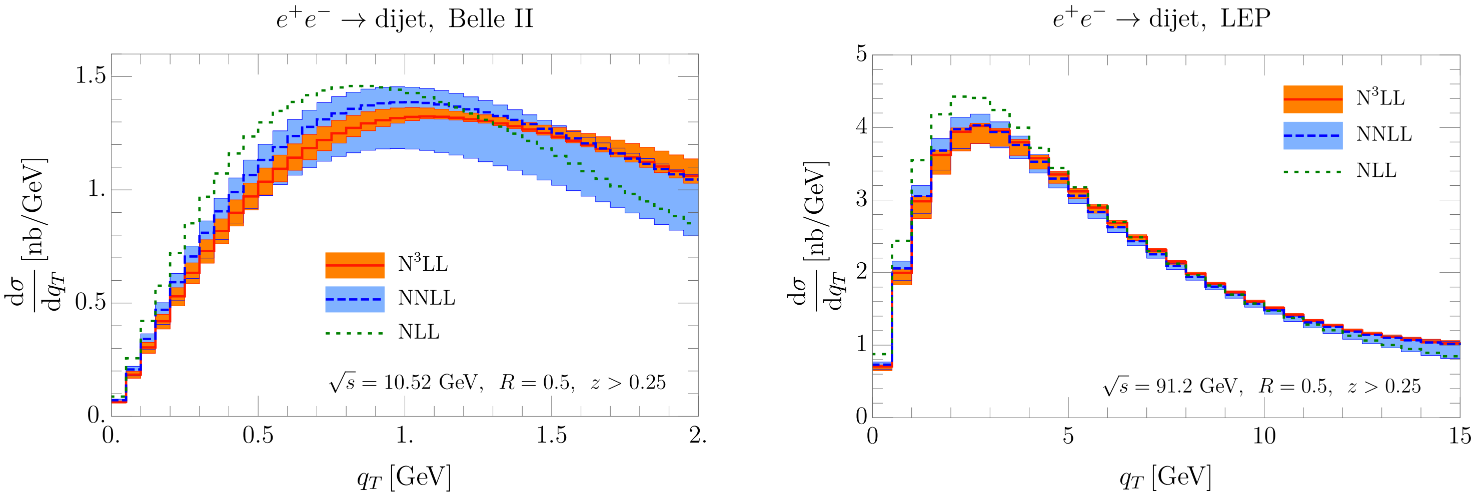


2A. Jet radius dependence



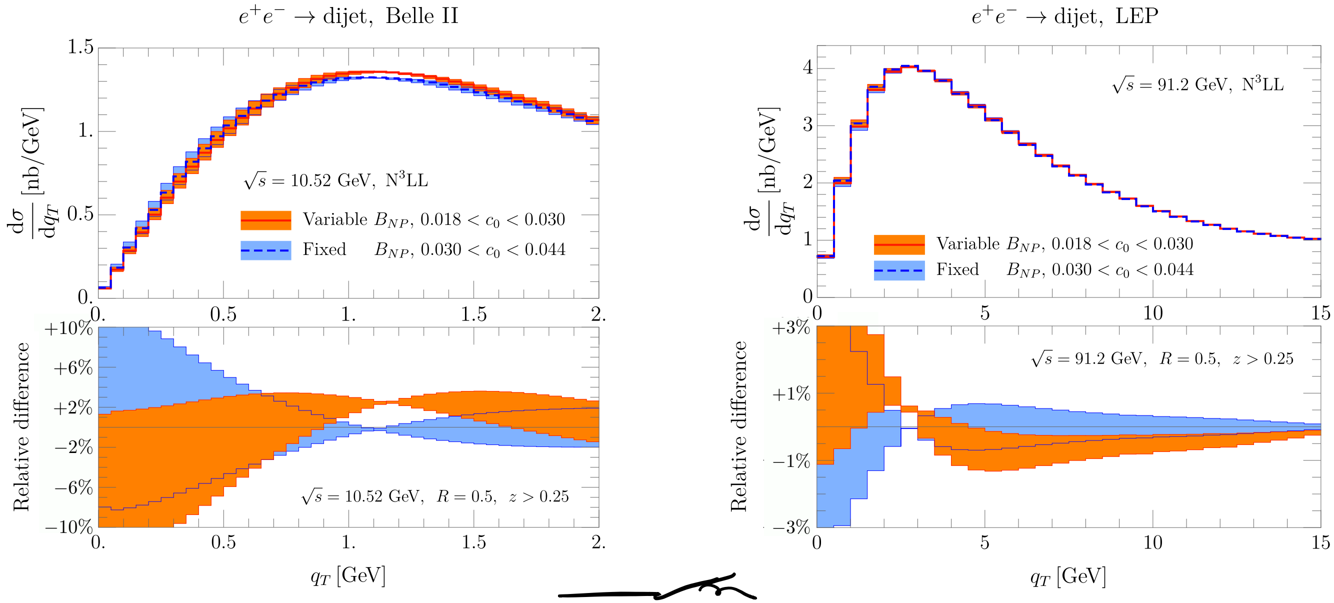
- Even for medium values of R, can use large-R jet function.
- Large-R jet function known at two loops → N^{3}LL.

2A. $e^+e^- \to JJX$: results



- Implemented in arTeMiDe. [Scimemi, Vladimirov]
- Good convergence of resummed perturbation theory. Uncertainty band artificially small at NLL (not shown).

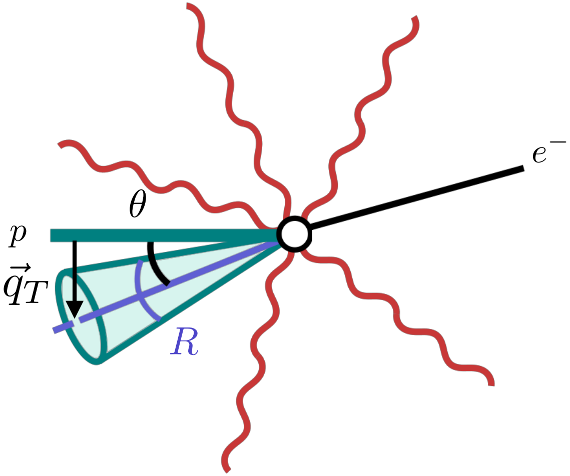
2A. $e^+e^- \to JJX$: nonperturbative sensitivity



- Dominant dependence on nonperturbative contribution to rapidity anomalous dimension.
- Vary within current uncertainty from Drell-Yan extraction.
  [Bertone, Scimemi, Vladimirov]

$J(\underset{z}{q_T})$ $\underline{D_1(q_T, \boxed{z})}$

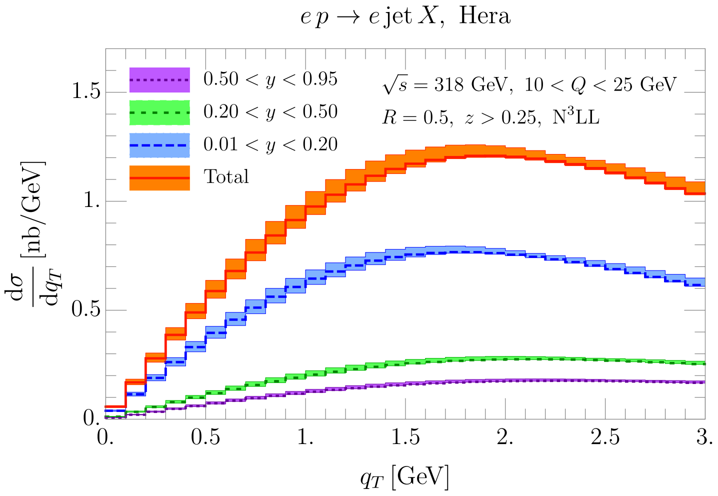
2B. $ep \to eJX$: factorization

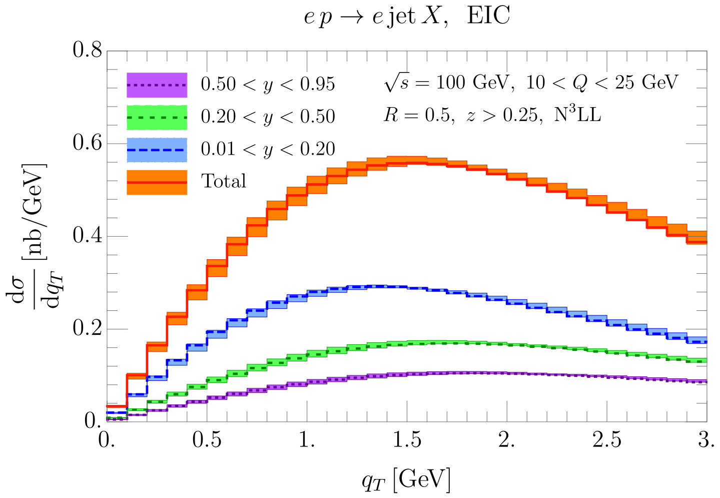


$$\frac{\mathrm{d}\sigma(ep \to eJX)}{\mathrm{d}Q\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\vec{q}_T} = \sum_q H_{eq\to eq}(Q) \int \frac{\mathrm{d}\vec{b}_T}{(2\pi)^2}\, e^{-\mathrm{i}\vec{b}_T\cdot\vec{q}_T}\, B_q(\vec{b}_T, x)\, J_q(\vec{b}_T, z, QR)$$

- Breit frame: virtual photon has $q^0 = 0$.
- Replace TMD fragmentation function by TMD jet function.

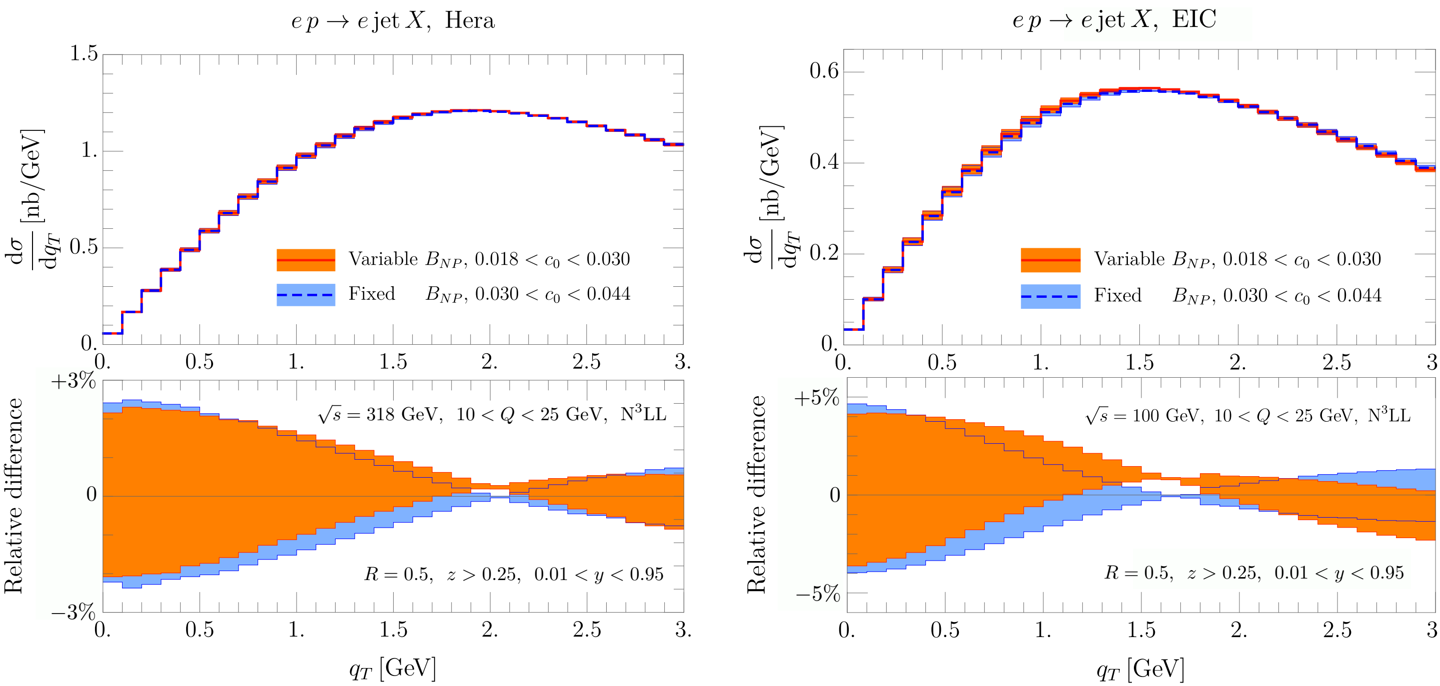
2B. $ep \to eJX$: cross section





- Results for Hera and EIC for different elasticity intervals.

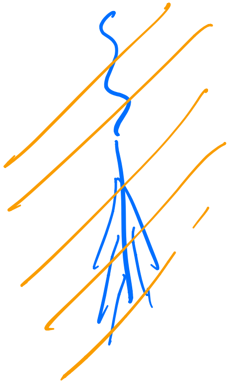
2B. $ep \to eJX$: nonperturbative sensitivity

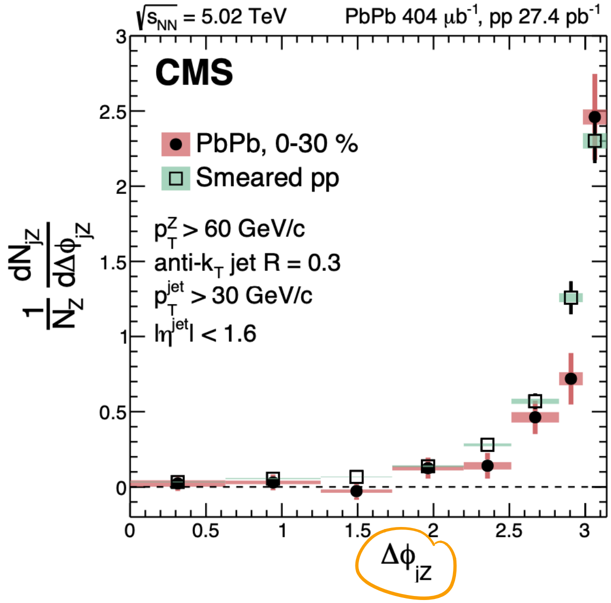


- Nonperturbative sensitivity on par with using hadrons.
- Pro: no sensitivity to nonperturbative fragmentation z.
- Con: jets based on calorimeters have worse angular resolution.

2C. $pp \to VJX$: motivation

- Important SM measurement and background.
- Probes medium in heavy-ion collisions.
- Study factorization violation (can also study $pp \to JJX$).

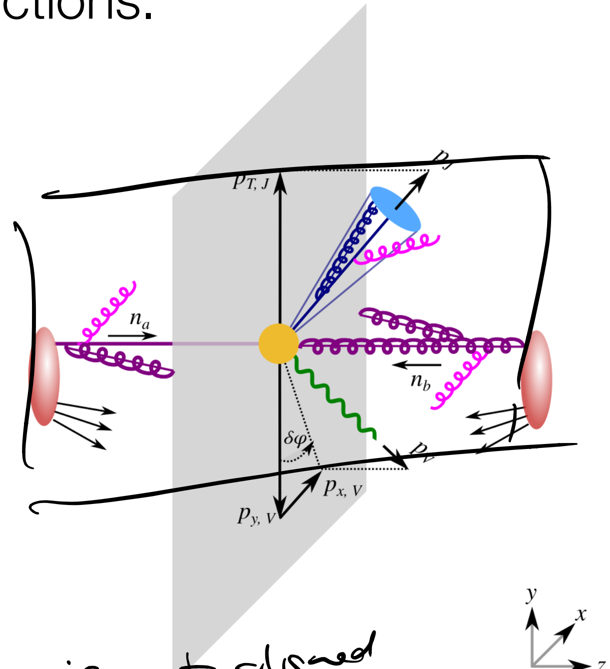




2C. $pp \to VJX$: factorization

- Azimuthal angle $\Delta\phi = \pi - \delta\phi$ with $\delta\phi \approx |p_{x,V}|/p_{T,V}$
- $p_{x,V}$ perpendicular to beam-jet plane.
  → Standard TMD beam and jet functions.

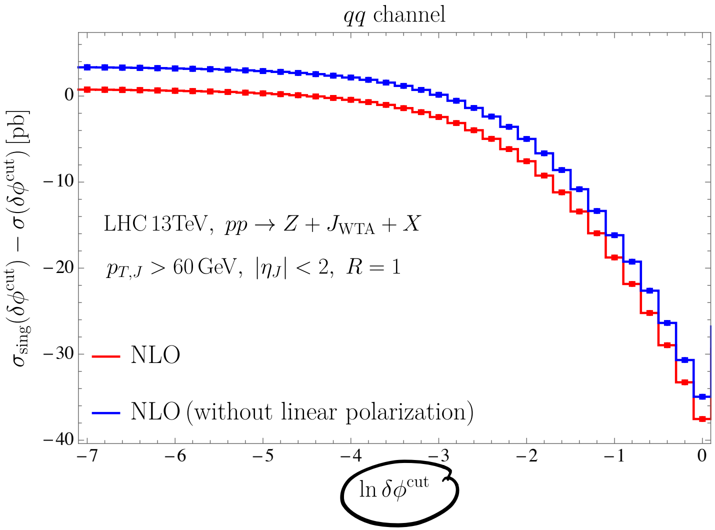
$$\frac{\mathrm{d}\sigma(pp \to VJX)}{\mathrm{d}p_{T,J}\,\mathrm{d}y_V\,\mathrm{d}\eta_J\,\mathrm{d}p_{x,V}} = \sum_{i,j,k} H_{ij\to Vk}(p_{T,J}, y_V - \eta_J) \times \int \frac{\mathrm{d}b_x}{2\pi}\, e^{-\mathrm{i}b_x\,p_{x,V}}\, S_{ijk}(b_x, \eta_J) \times B_i(\boxed{b_x}, x_1)\, B_j(\boxed{b_x}, x_2)\, \mathcal{J}_k(\boxed{b_x})$$



↰ jet axis not aligned with momentum

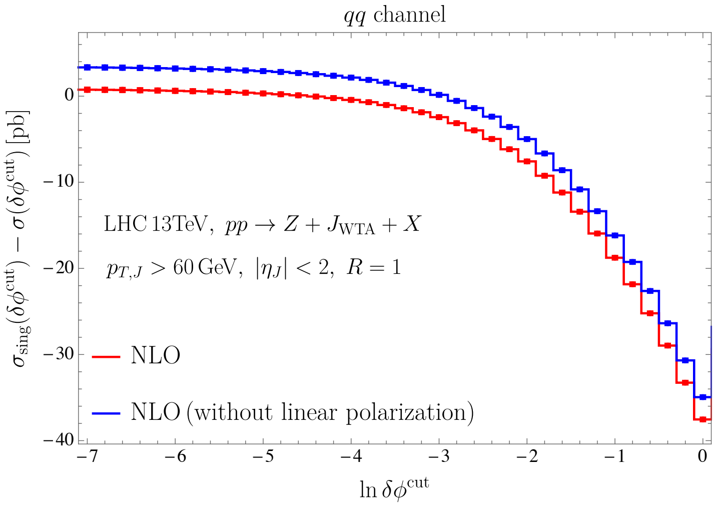
2C. Linear polarization

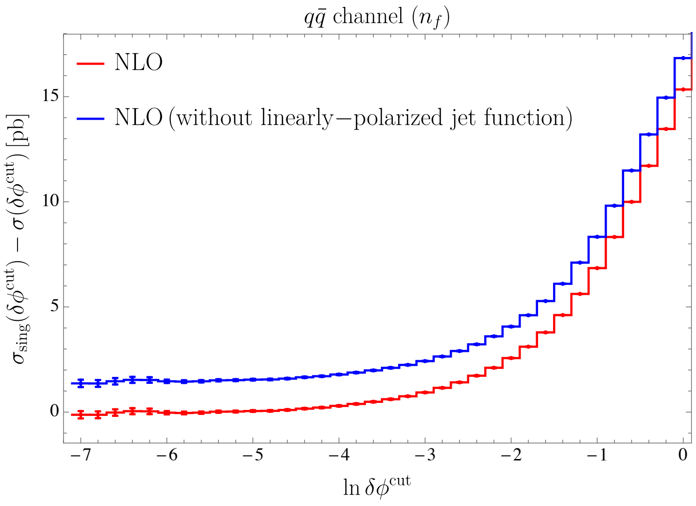
- Linearly-polarized gluon beam function contributes at NLO. (NNLO for Higgs production) Test using MCFM. [Campbell, Ellis et al]



2C. Linear polarization

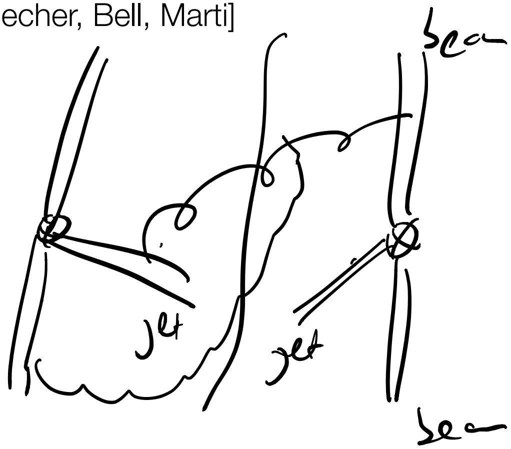
- Linearly-polarized gluon beam function contributes at NLO. (NNLO for Higgs production) Test using MCFM. [Campbell, Ellis et al]
- Linearly-polarized jet function: $\mathcal{J}_g^L(\vec{b}_\perp) = \frac{\alpha_s}{4\pi}\Big(-\frac{1}{3}C_A + \frac{2}{3}T_F n_f\Big)$





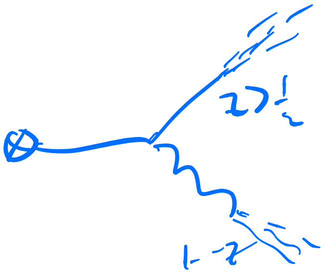
2C. Hard and soft function

- At NNLL need NLO Hard function. [Arnold, Reno; Becher, Lorentzen, Schwartz]
    - For linearly-polarized gluon, $H^L = |P^L_{\mu\nu} C^{\mu\nu}|^2$ starts at LO.
- Soft function contribution from two Wilson lines related to standard TMD soft function by boosting to back-to-back.
  [See Gao, Li, Moult, Zhu]
    - Three Wilson lines contribution vanishes at NNLO.
      [Becher, Bell, Marti]



2C. Track-based measurements

- Track-based measurement has superior angular resolution.
- Switching to tracks only modifies jet function.
  → Anomalous dimensions must be the same.
- Change in jet function constant calculable with track functions.
  [Chang, Procura, Thaler, WW]



$z \overset{?}{>} 1-z$

$x_q z > x_g (1-z)$

2C. Track-based measurements

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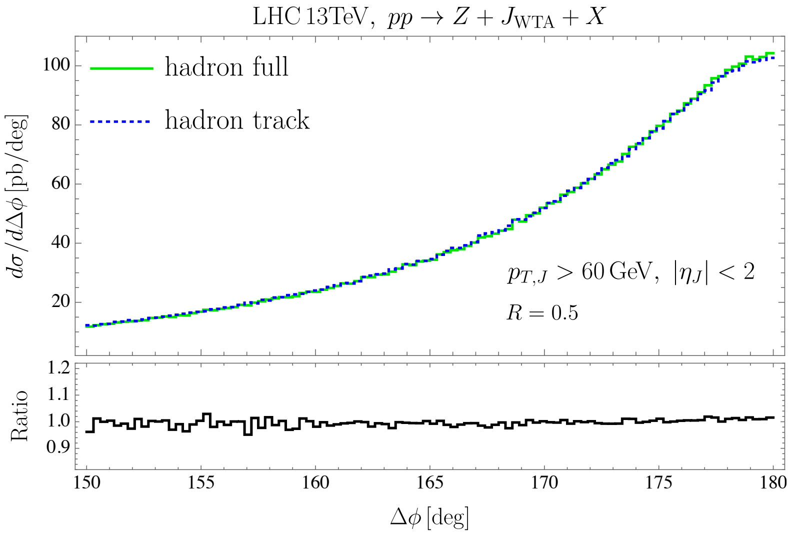
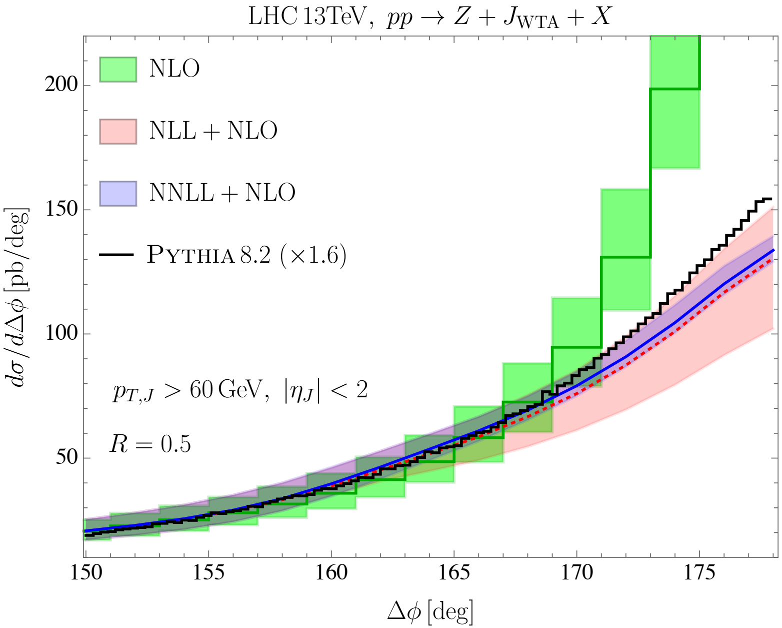
$\delta\phi \ll R$



2C. Results



- Small b_x: match to NLO using transition function.
  Large b_x: avoid Landau pole with b^* prescription. [Collins, Soper, Sterman]
- Good perturbative convergence.
- Pythia (with NLO K-factor) agrees reasonably well. [Sjostrand et al]

3. Conclusions

- Using the **WTA axis**, TMD fragmentation functions can be replaced by TMD jet functions in factorization theorems.
- Pro: Jets don't have nonperturbative momentum fraction.
- Con: limited angular resolution of calorimeters → tracks.
- $ep \to eJX$: constrain TMDs at EIC using jets!

TEEC

- $pp \to VJX$: TMDs with more than two directions.
  High precision is possible: most ingredients for N^{3}LL known.
  Interesting for medium modifications in heavy ion collisions and factorization violating effects.

$e^+e^- \to$ jets

Thank you!