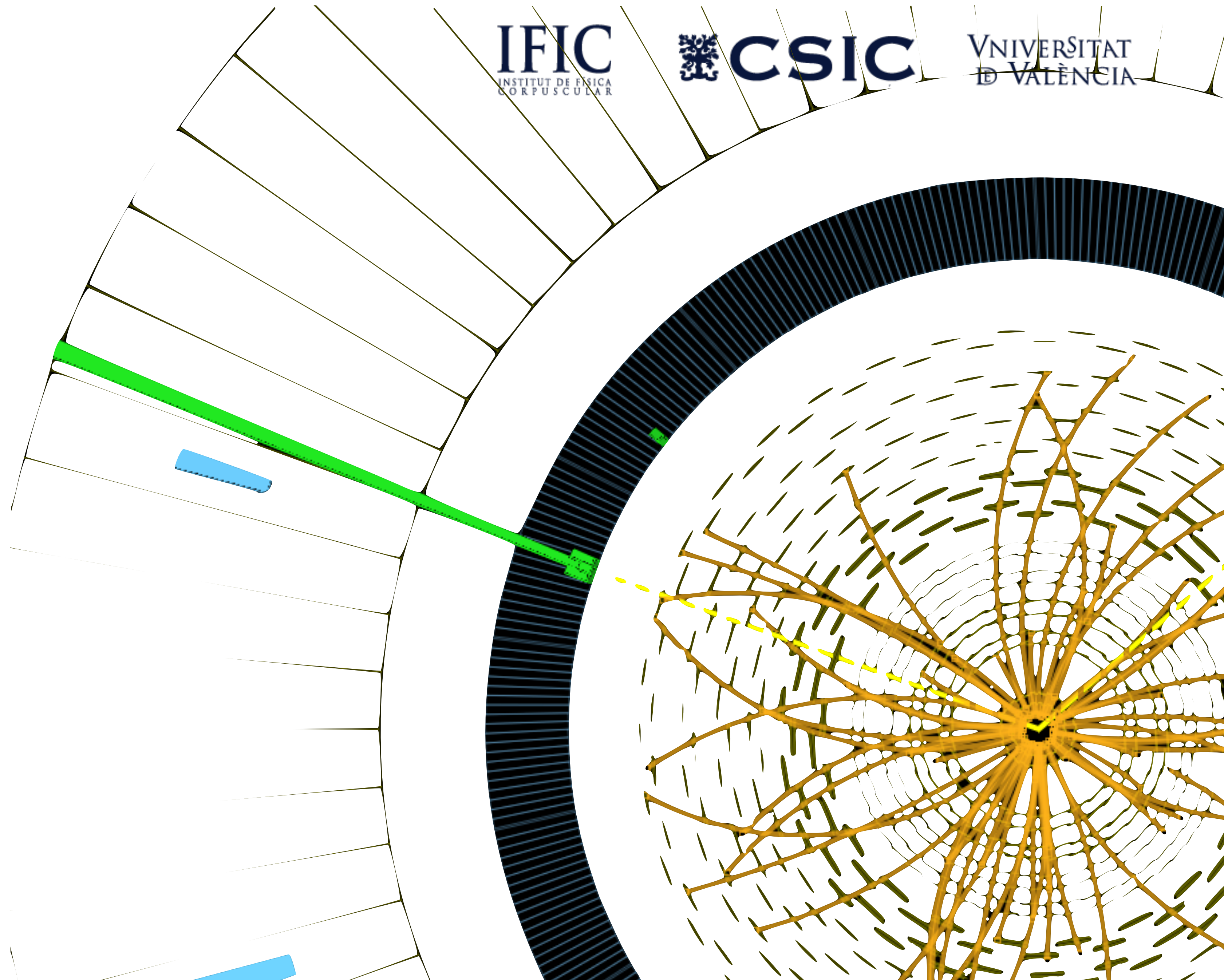
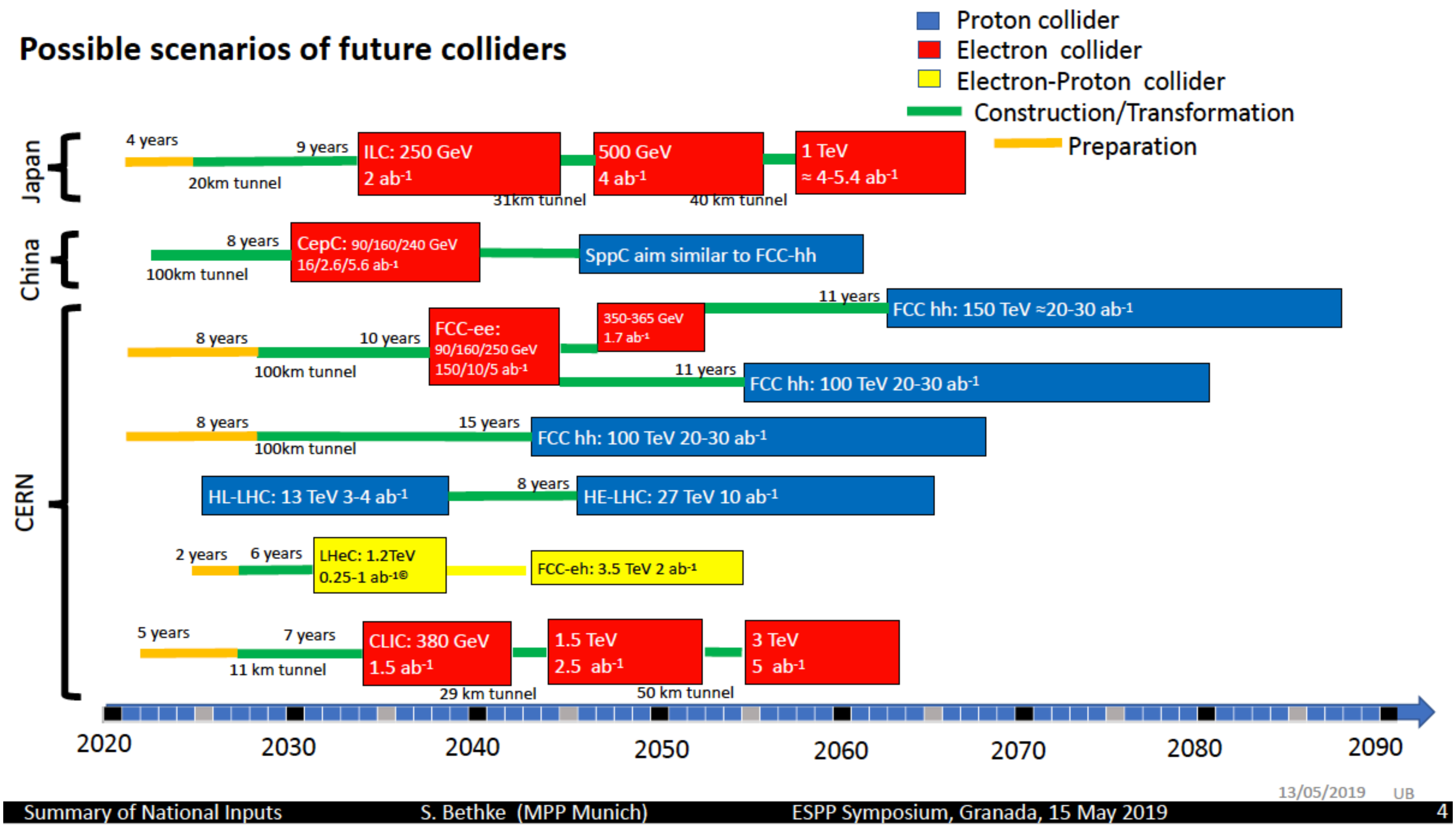
Open loops to trees
in four space-time dimensions

Germán Rodrigo

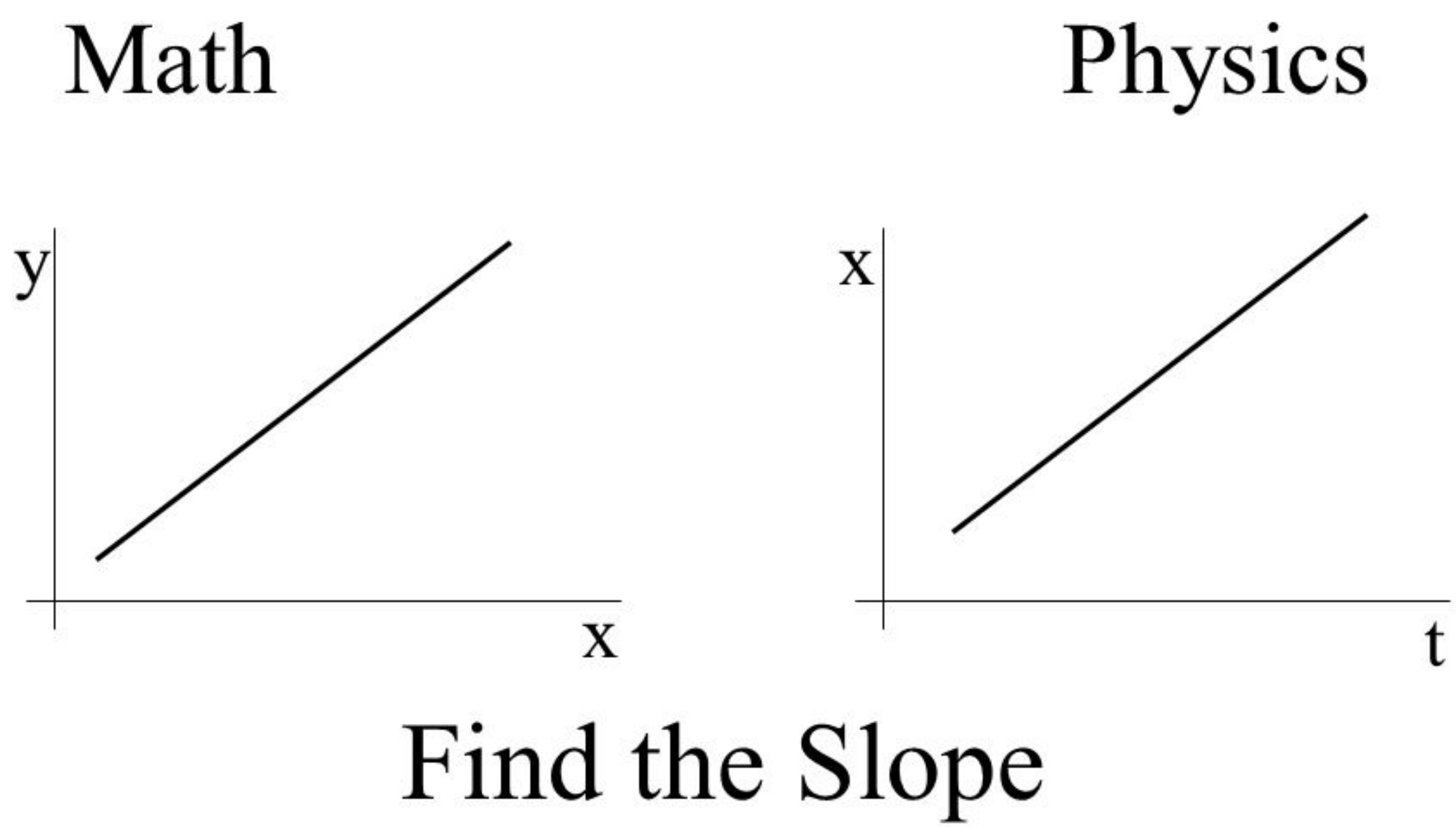


5 June 2020



- all options aimed at **attobarn^{-1} physics**
- requires to go **far beyond NNLO for theory**
- even conservative estimates **not reachable with current techniques**

The difficulties to reach higher orders arise because we have defined Quantum Field Theory **not in the optimal way**



* "Mathematics is the language of the Universe", attributed to **Galileo Galilei** 1564-1642

QFT = Quantum Mechanics + space-time

- Loops encode quantum fluctuations at **infinite energy** (**zero distance**): **SM/BSM** extrapolated at energies $\gg M_{\rm Plank}$

- QED/QCD massless gauge bosons/quarks: quantum state with N partons $\neq$ quantum state with **zero energy emission (infinite distance)** of extra partons

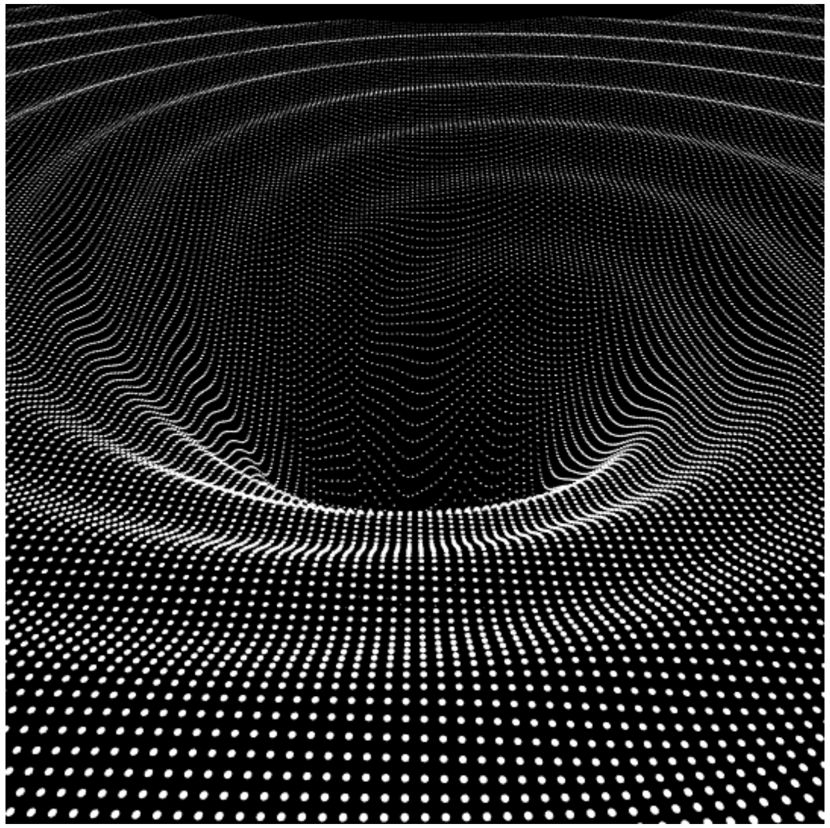
- Partons can be emitted in **exactly the same direction (zero distance)**

['t Hooft, Veltmann 1972]
[Giambiagi, Bollini 1972]

Dimensional Regularization **DREG**

$$d = 4 - 2\epsilon$$

- Qualitative interpretation: A way to give some **extra space** to the colliding particles
- Intrinsic infinities appear as **poles in the extra dimensions**: $1/\epsilon$ after integration over loop momenta and phase-space
- different quantum fluctuations should contribute with poles of opposite sign, such that theoretical predictions for physical observables remain finite
- Mathematically well-defined but **unphysical**, and difficult to implement at higher orders



https://www.pinterest.es/pin/311733605450189454/

ASYMPTOTIC EXPANSIONS

LTD works in the **Euclidean** space of the loop three-momenta where the hierarchy of scales is well defined

LOOP-TREE DUALITY (LTD)

open loop amplitudes to **non-disjoint trees** by introducing a number of on-shell conditions equal to the number of loops

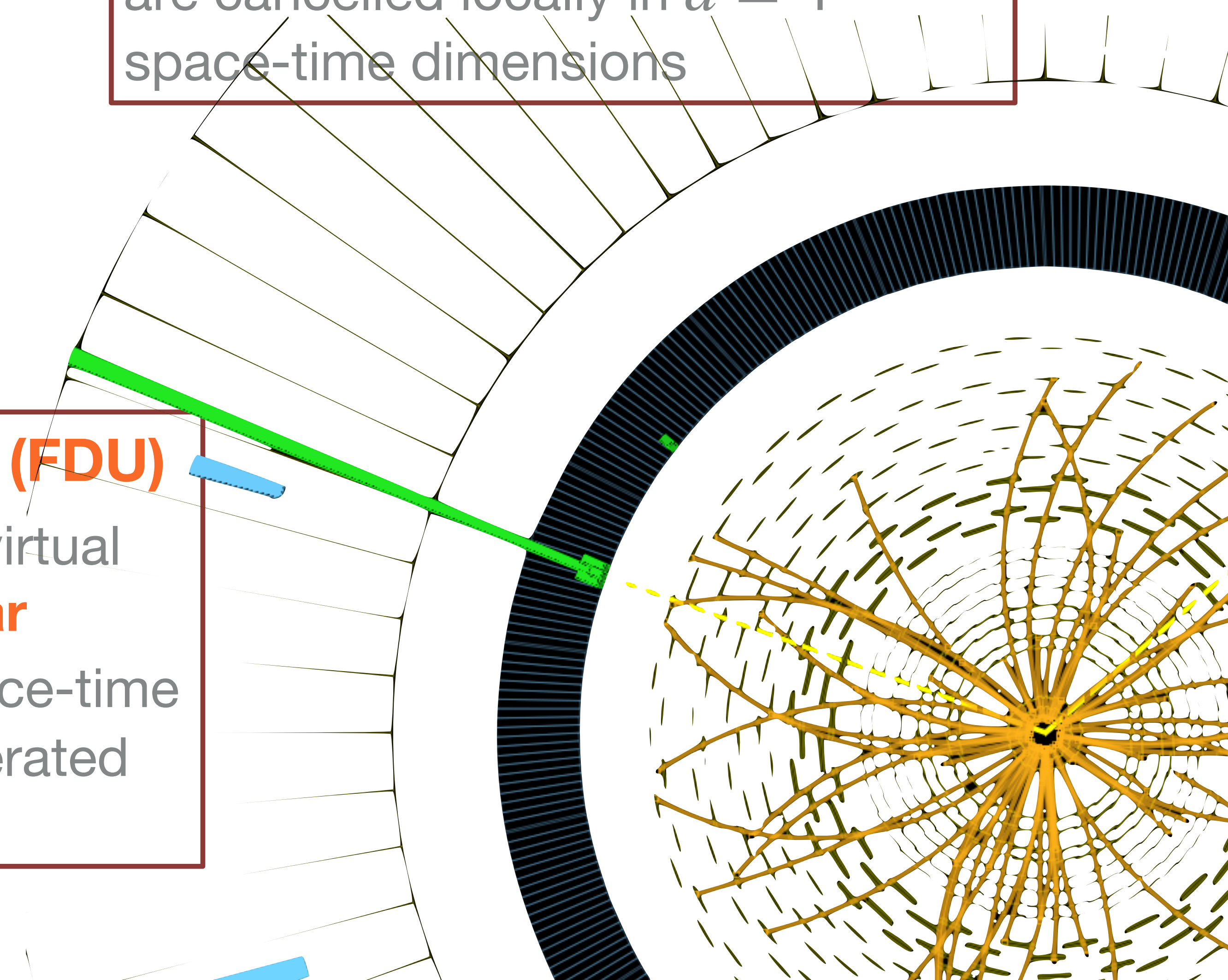
$\neq$ Generalized Unitarity

LOCAL RENORMALISATION

Suppress bad behavior at very high energies such that **UV singularities** are cancelled locally in $d = 4$ space-time dimensions

FOUR DIMENSIONAL UNSUBTRACTION (FDU)

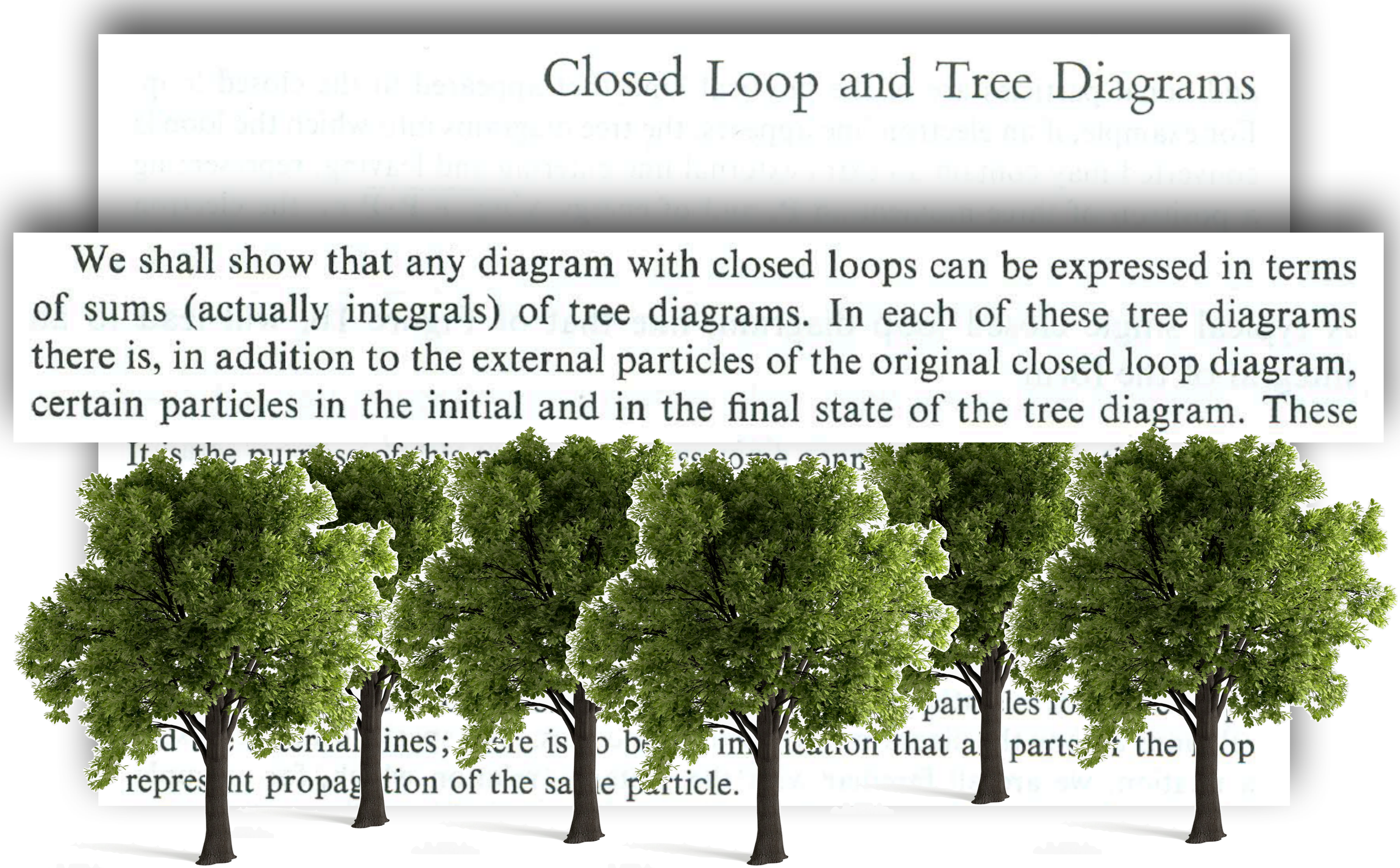
Introduce mappings of momenta between the virtual and real kinematics such that **soft and collinear singularities** are cancelled locally in $d = 4$ space-time dimensions. Real and virtual contributions generated simultaneously



Gell-Mann suggested to study massless Yang-Mills theory

R. P. Feynman, Closed Loop And Tree Diagrams. (talk),

Magic Without Magic: J. A. Wheeler, A Collection of Essays in Honor of his Sixtieth Birthday.
Edited by John R. Klauder, 1972, p.355
In *Brown, L.M. (ed.): Selected papers of Richard Feynman* 867-887



PROPAGATORS

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

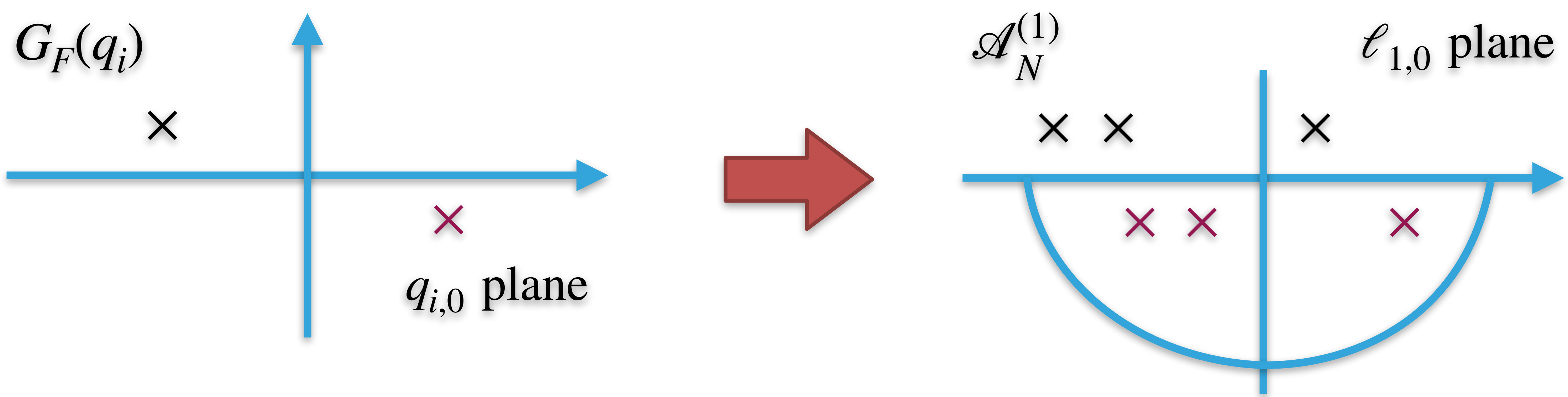
- MATH: the +i0 is a small quantity usually ignored, assuming that the **analytical continuation** to the physical kinematics is well defined
- PHYSICS: the +i0 encodes **CAUSALITY | positive frequencies are propagated forward in time, and negative backward**

[Catani et al. JHEP **0809**, 065]

THE LOOP-TREE DUALITY (LTD)

Cauchy residue theorem

in the loop energy complex plane



Feynman Propagator +i0:
positive frequencies are propagated forward in time, and negative backward

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

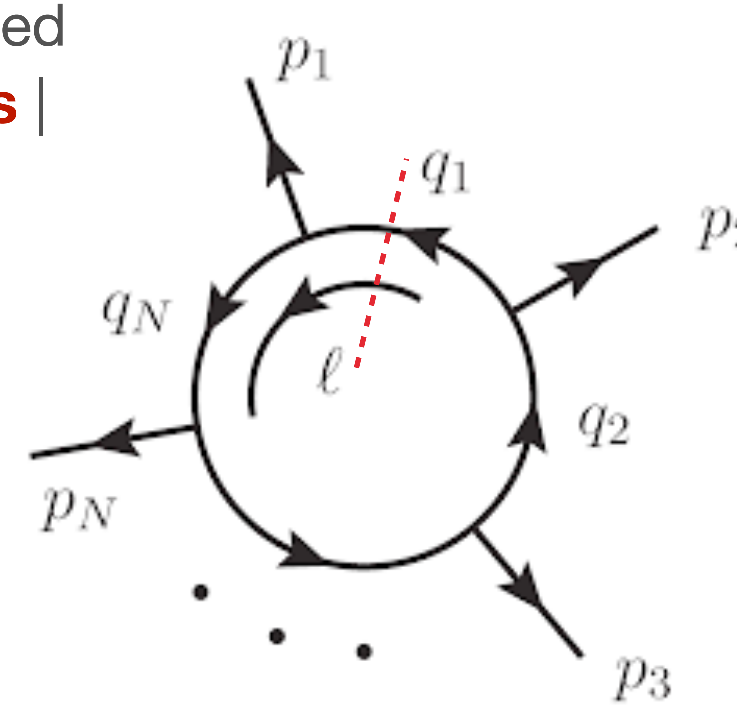
selects residues with definite **positive energy and negative imaginary part**

in arbitrary coordinate systems: reduce the dimension of the integration domain by one unit

[Catani et al. JHEP **0809**, 065]

THE LOOP–TREE DUALITY (LTD)

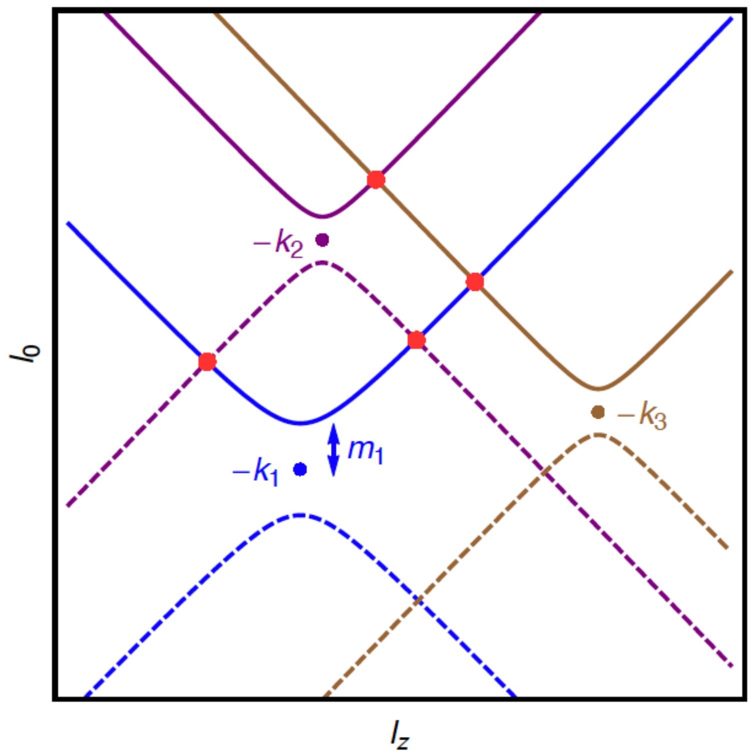
One-loop amplitudes in any relativistic, local and unitary QFT represented as a linear combination of *N* **single-cut phase-space/dual amplitudes** | **non-disjoint trees** (at higher orders: number of cuts equal to the number of loops)

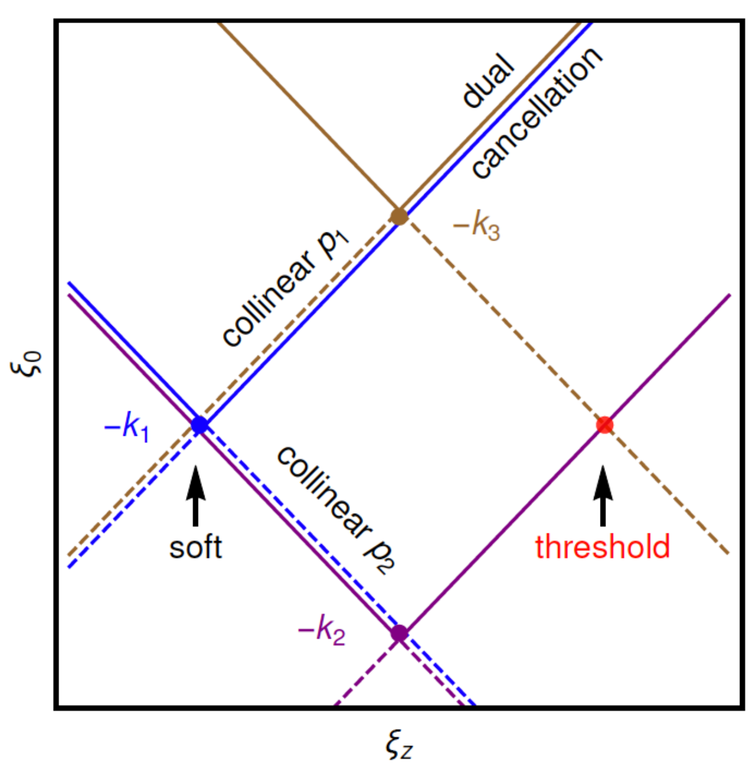


$$\int_{\ell_1} \mathcal{N}(\ell_1) \prod G_F(q_i) = - \int_{\ell_1} \mathcal{N}(\ell_1) \otimes \sum \tilde{\delta}(q_i) \prod_{i \neq j} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = \imath 2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \dfrac{1}{q_j^2 - m_j^2 - \imath 0\, \eta k_{ji}}$ **dual propagator** $k_{ji} = q_j - q_i$ $q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}$
- LTD realised by **modifying the customary +i0 prescription** of the Feynman propagators (only the sign matters), it encodes in a compact way the effect of **multiple-cut** contributions that appear in the **Feynman's Tree Theorem**
- **Lorentz invariant**, best choice $\eta^\mu = (1, \mathbf{0})$: energy component integrated out, remaining integration in **Euclidean space**

THE DUAL FOREST | CAUSALITY



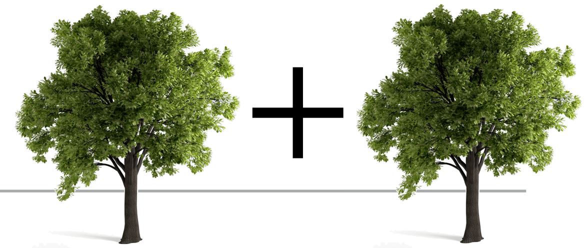


- **LTD** is equivalent to integrate along the forward on-shell hyperboloids (light-cones for massless) or positive energy modes
- The dual integrand becomes singular when a second propagator gets eventually on-shell
- The location of singularities is determined by a linear identity in the on-shell energies

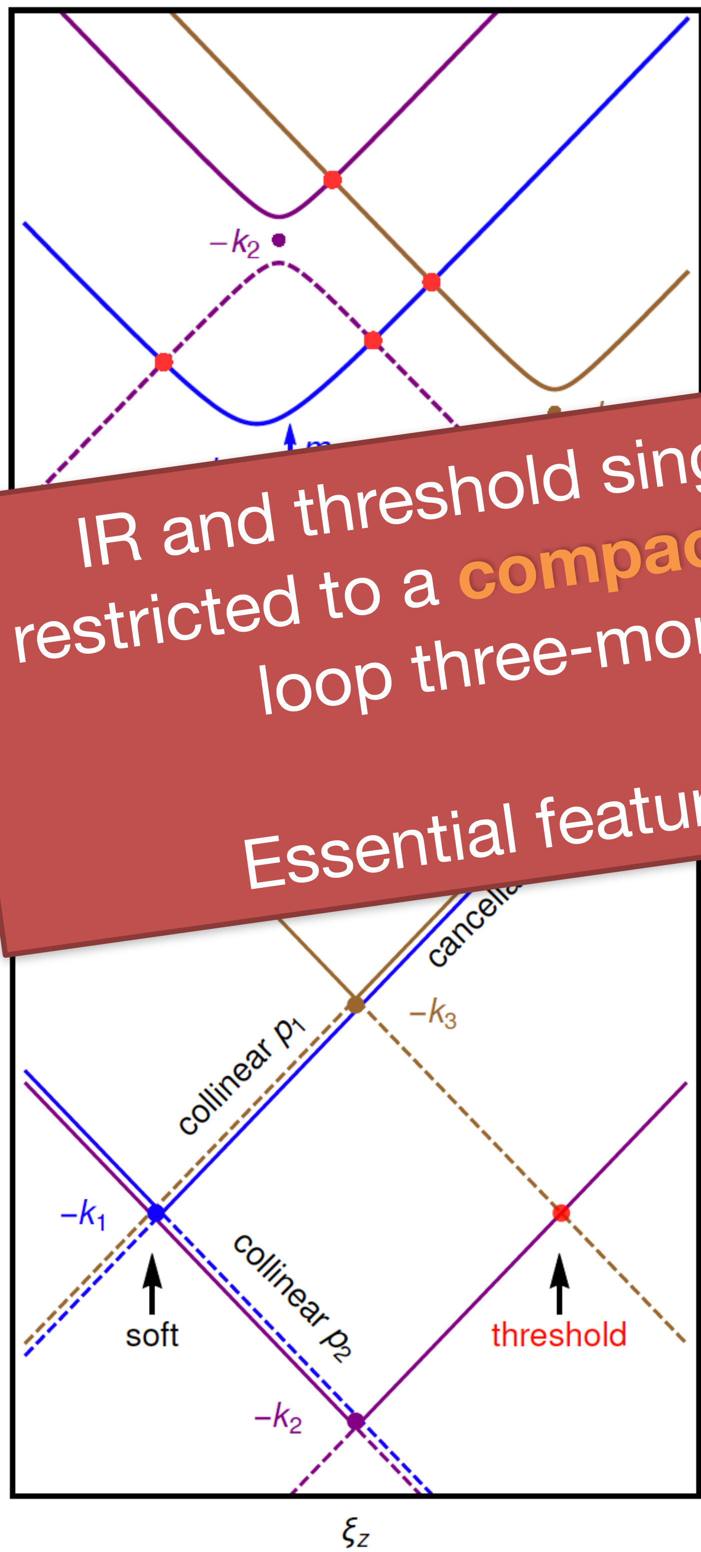
$$\lambda_{ij}^{\pm\pm} = \pm\, q_{i,0}^{(+)} \pm q_{j,0}^{(+)} + k_{ji,0} \to 0$$

where

$$q_{r,0}^{(+)} = \sqrt{\boldsymbol{q}_r^2 + m_r^2 - \imath 0} \qquad k_{ji} = q_j - q_i$$



THE DUAL FOREST | CAUSALITY



IR and threshold singularities are restricted to a **compact region** of the loop three-momentum

Essential feature for FDU

$$\mathcal{S}_{ij}^{(1)} = (2\pi\imath)^{-1}\, G_D(q_i;q_j)\,\tilde{\delta}(q_i) + (i \leftrightarrow j)$$

- **Time-like distance (causally connected):** generates physical threshold singularities: always **+i0**

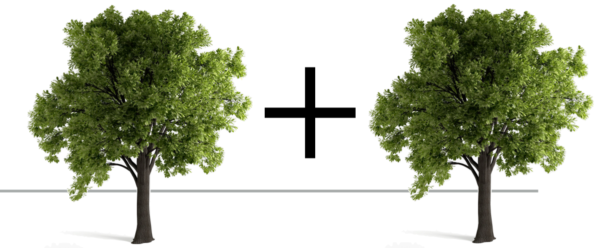
$$= \frac{\theta(-k_{ji,0})\,\theta(k_{ji}^2-(m_i+m_j)^2)}{x_{ij}\,(-\lambda_{ij}^{++} - \imath 0\, k_{ji,0})} + \mathcal{O}\left((\lambda_{ij}^{++})^0\right)$$

$$x_{ij} = 4\, q_{i,0}^{(+)} q_{j,0}^{(+)} \qquad +i0$$

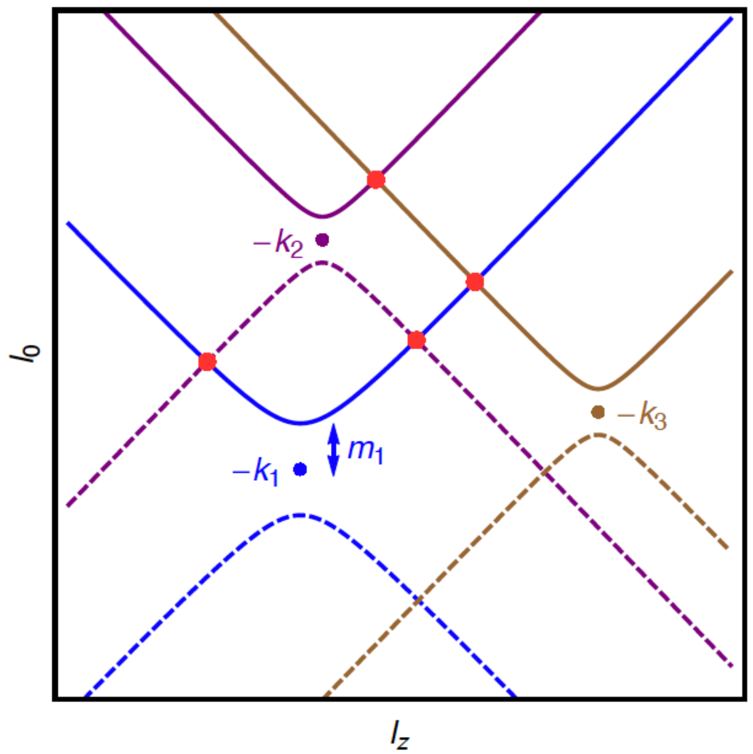
- **Space-like distance**: there is a perfect cancellation of singularities, due to the dual **+i0** prescription

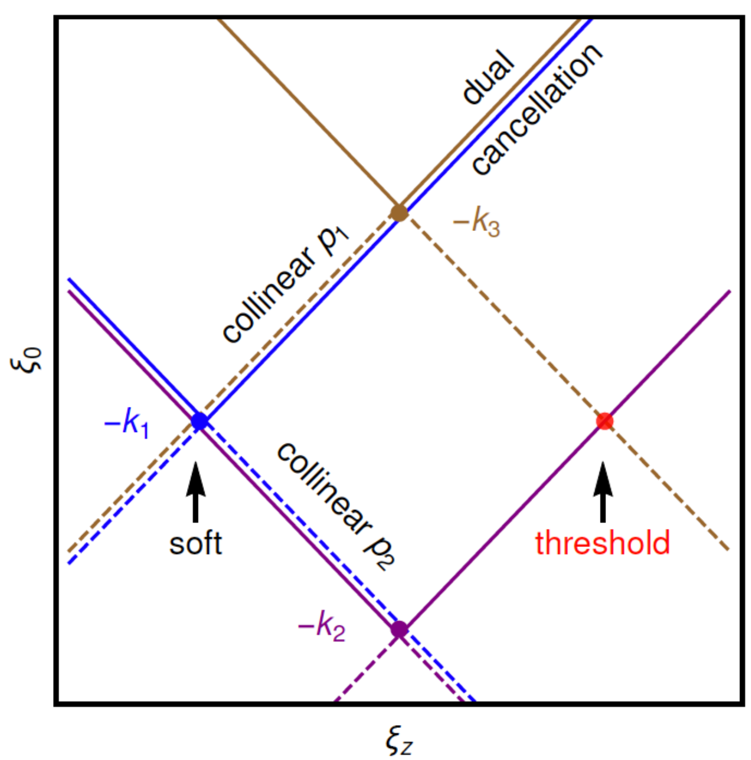
$$\lim_{\lambda_{ij}^{+-}\to 0} \mathcal{S}_{ij}^{(1)} = \mathcal{O}\left((\lambda_{ij}^{+-})^0\right) \qquad k_{ji}^2 - (m_j - m_i)^2 \leq 0$$

- **Light-like distance:** both singular configurations, partial cancellation, IR singularities remain in a compact region



THE FEYNMAN'S FOREST | CAUSALITY





FTT: $$\mathcal{F}_{ij}^{(1)} = (2\pi\imath)^{-1}\, G_F(q_j)\, \tilde\delta(q_i) + (i \leftrightarrow j)$$

- **Time-like distance (causally connected):** physics does not depend on the FTT or LTD representation

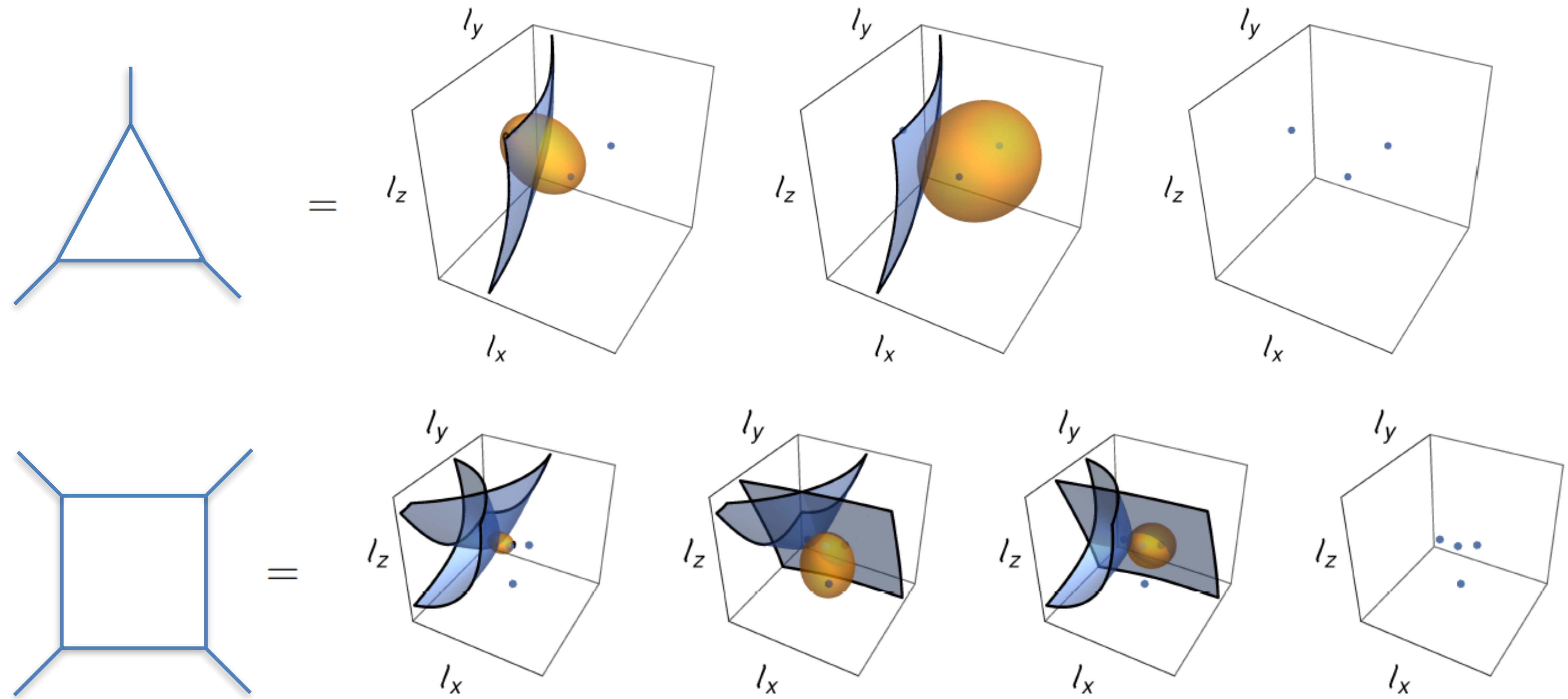
$$\lim_{\lambda_{ij}^{++}\to 0} \mathcal{F}_{ij}^{(1)} = \frac{\theta(-k_{ji,0})\,\theta(k_{ji}^2 - (m_i+m_j)^2)}{x_{ij}\,(-\lambda_{ij}^{++} + \imath 0)} + \mathcal{O}\left((\lambda_{ij}^{++})^0\right)$$

- **Space-like distance**: there is mismatch in the **+i0** prescription

$$\lim_{\lambda_{ij}^{+-}\to 0} \mathcal{F}_{ij}^{(1)} \sim \frac{1}{-\lambda_{ij}^{+-} + \imath 0} + \frac{1}{\lambda_{ij}^{+-} + \imath 0} + \mathcal{O}\left((\lambda_{ij}^{+-})^0\right)$$

- needs to be compensated by the contribution from **multiple cuts**

LTD SINGULAR STRUCTURE



- non-causal singularities (forward-forward in blue): undergo **dual cancellations** among dual pairs
- causal singularities (forward-backward in orange): bounded to a **compact region,** which is of the size of the **hard scale**, collapse to a finite segment for **infrared singularities (→ FDU)**
- Numerical integration in the **Euclidean** space of the loop three-momenta, CPU/GPU time do not scale significantly with the number of legs

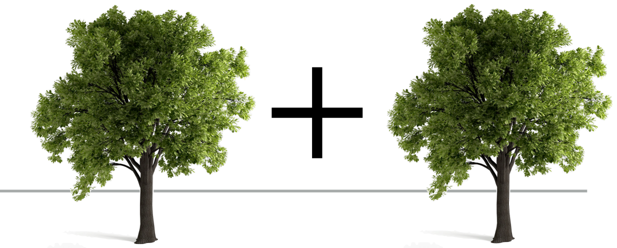
NUMERICAL INTEGRATION

	Rank	Tensor Hexagon	Real Part	Imaginary Part	Time[s]
P20	1	SecDec	$-1.21585(12) \times 10^{-15}$		36
		LTD	$-1.21552(354) \times 10^{-15}$		6
P21	3	SecDec	$4.46117(37) \times 10^{-9}$		5498
		LTD	$4.461369(3) \times 10^{-9}$		11
P22	1	SecDec	$1.01359(23) \times 10^{-15}$	$+i\ 2.68657(26) \times 10^{-15}$	33
		LTD	$1.01345(130) \times 10^{-15}$	$+i\ 2.68633(130) \times 10^{-15}$	72
P23	2	SecDec	$2.45315(24) \times 10^{-12}$	$-i\ 2.06087(20) \times 10^{-12}$	337
		LTD	$2.45273(727) \times 10^{-12}$	$-i\ 2.06202(727) \times 10^{-12}$	75
P24	3	SecDec	$-2.07531(19) \times 10^{-6}$	$+i\ 6.97158(56) \times 10^{-7}$	14280
		LTD	$-2.07526(8) \times 10^{-6}$	$+i\ 6.97192(8) \times 10^{-7}$	85

Table 5: Tensor hexagons involving numerators of rank one to three.

Propagator	Real Part	Imaginary Part
1	$2.530(4) \times 10^{-14}$	$+\ i\ 8.514(1) \times 10^{-14}$
$\ell.p_3 \times \ell.p_5$	$8.08(4) \times 10^{-15}$	$+\ i\ 6.144(5) \times 10^{-13}$

Table 1: Scalar and tensor decagon with all internal masses different.



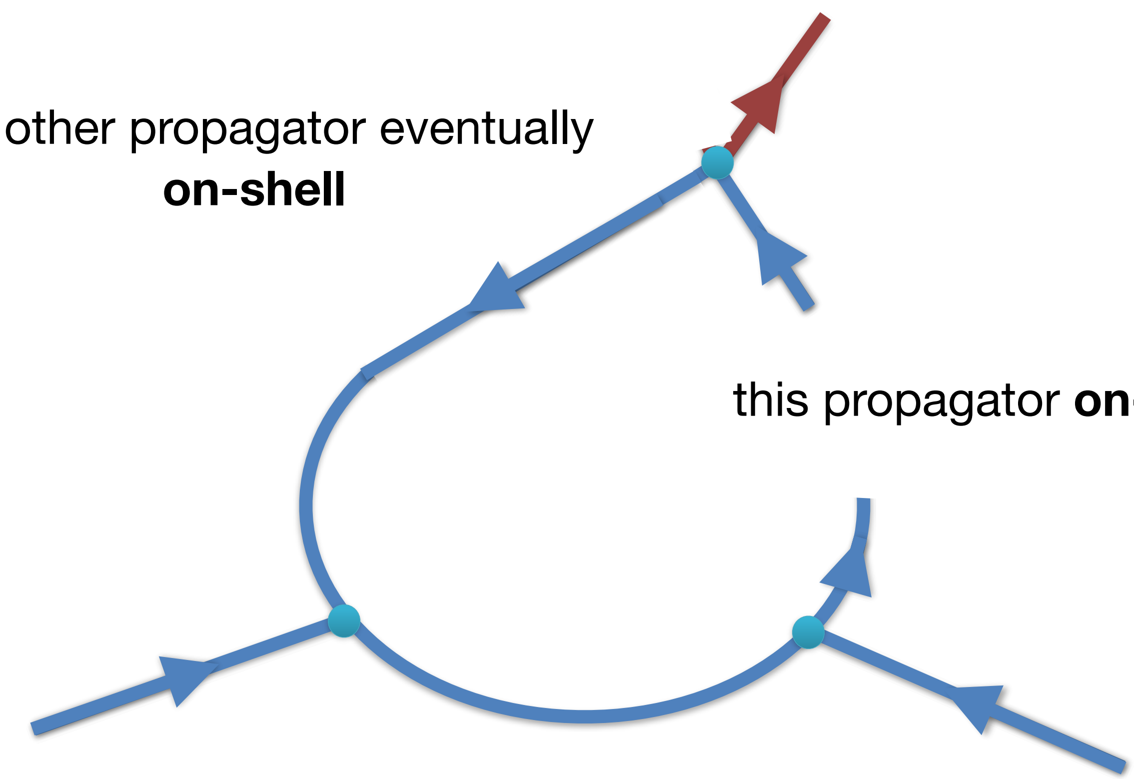
WHEN A BRANCHES GET BROKEN

energy of the **on-shell** propagator smaller than the energy of the emitted particles

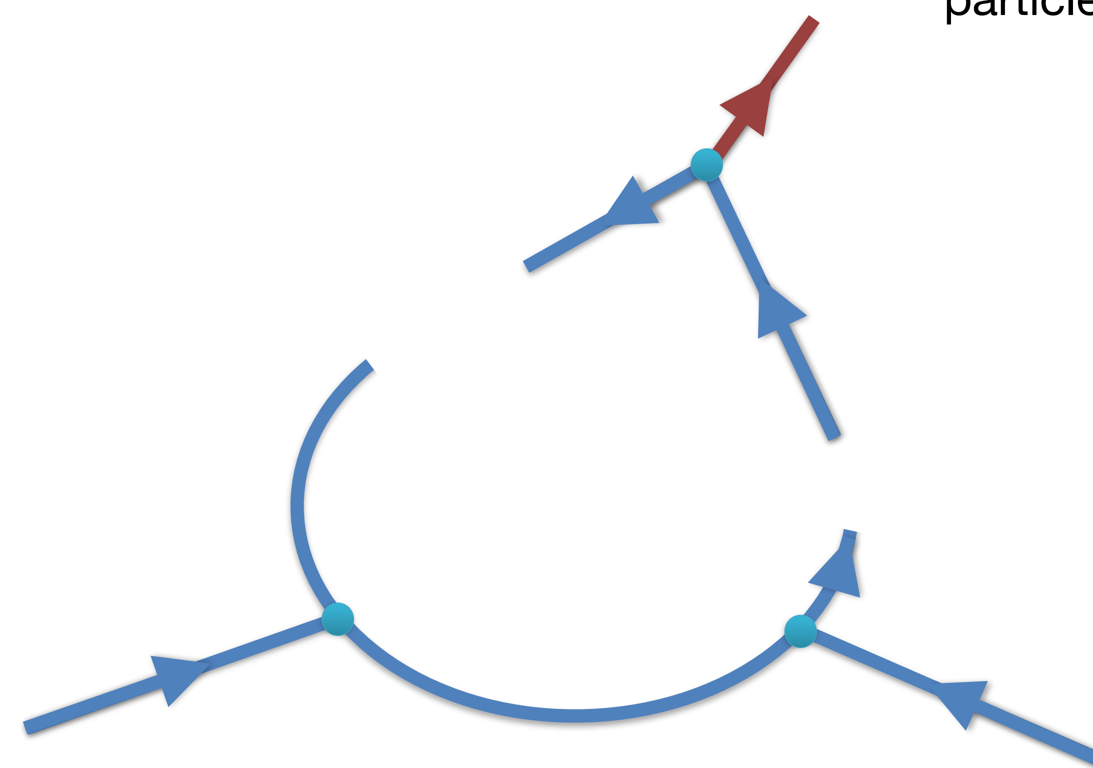
time-like or light-like

this other propagator eventually **on-shell**

this propagator **on-shell**



Space-like or light-like at energy of the **on-shell** propagator larger than the energy of the emitted particle(s)

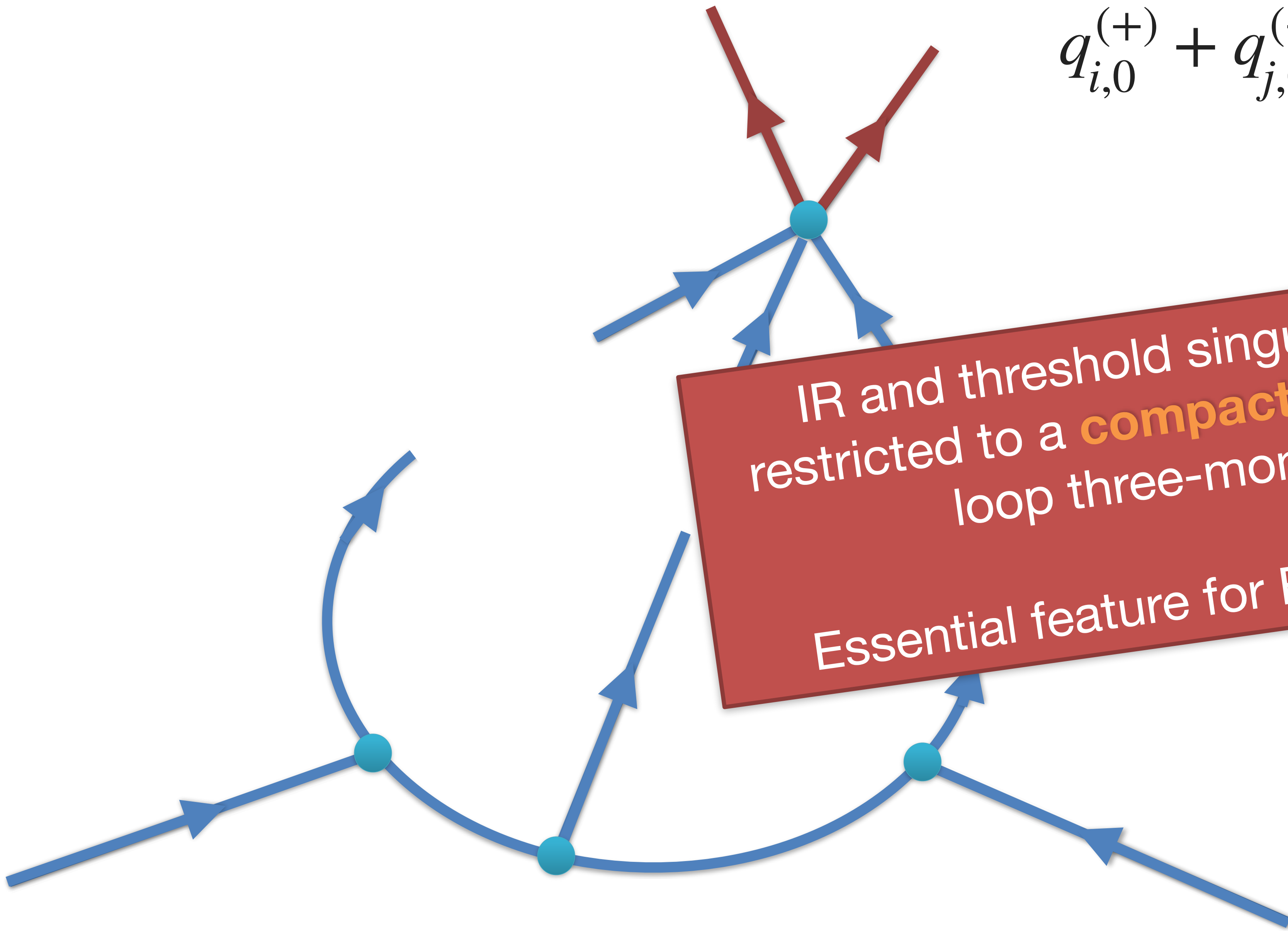


- **Threshold** singularities occur when a second propagator gets on-shell: consistent with **Cutkosky**
- It becomes **collinear (soft)** when a single massless particle is emitted
- **Causally connected**

- Virtual particle **emitted and absorbed on-shell**
- Potential **threshold and IR singularities cancel** in the sum of single-cut trees: **non-causal**
- Non-singular configurations at very large energies **(UV)** expected to be **suppressed.** If not sufficiently suppressed, **renormalise**

UNITARITY THRESHOLD / TRIPLE COLLINEAR

$$q_{i,0}^{(+)} + q_{j,0}^{(+)} + q_{k,0}^{(+)} - k_{ijk,0} = 0$$



IR and threshold singularities are restricted to a **compact region** of the loop three-momenta

Essential feature for FDU at NNLO

LOCAL UV RENORMALISATION

- Expand propagators and numerators around a UV propagator [Reuschle et al., similar to Pittau's FDR in the UV]

$$G_F(q_{\rm UV}) = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \qquad \{\ell_j^2 \mid \ell_j \cdot k_i\} \to \{\lambda^2 q_{\rm UV}^2 + (1-\lambda^2)\,\mu_{\rm UV}^2 \mid \lambda\, q_{\rm UV}\cdot k_i\}$$

- and adjust **subleading** terms, $c_{\rm UV}$, to subtract only the pole ($\overline{MS}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2} \left(1 + c_{\rm UV} \frac{\mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0} \right)$$

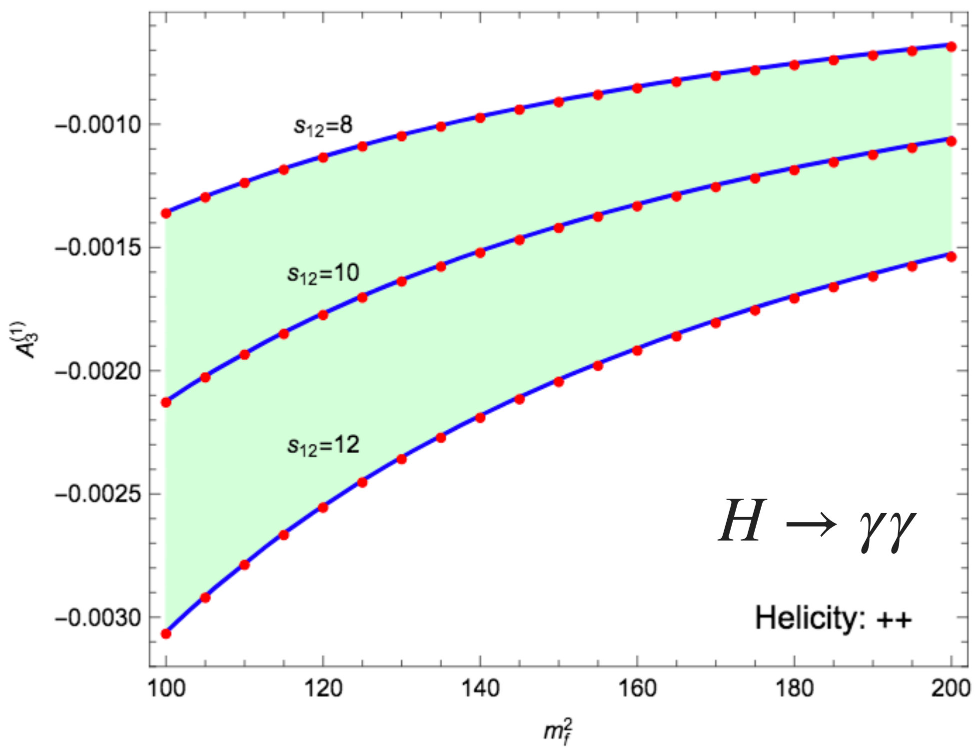
- dual representation needs to deal with **multiple poles** [Bierenbaum et al.]

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2\left(q_{\rm UV,0}^{(+)}\right)^2} \left(1 - \frac{3\, c_{\rm UV}\, \mu_{\rm UV}^2}{4\left(q_{\rm UV,0}^{(+)}\right)^2} \right) \qquad q_{\rm UV,0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

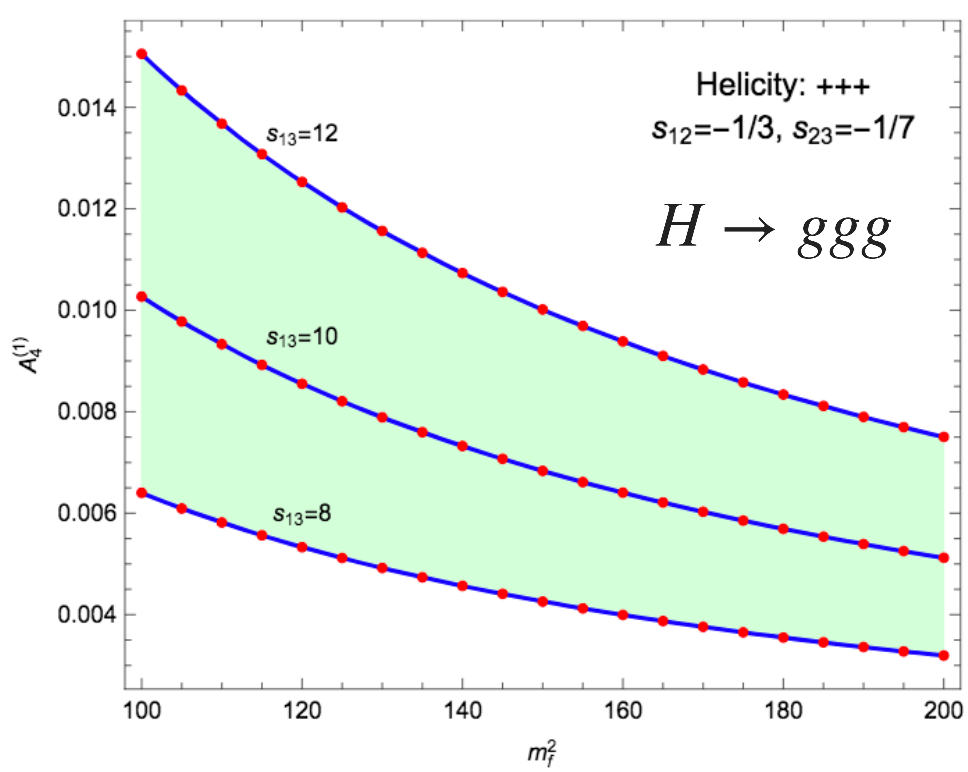
Hernández-Pinto, Sborlini, GR, JHEP **1602,** 044

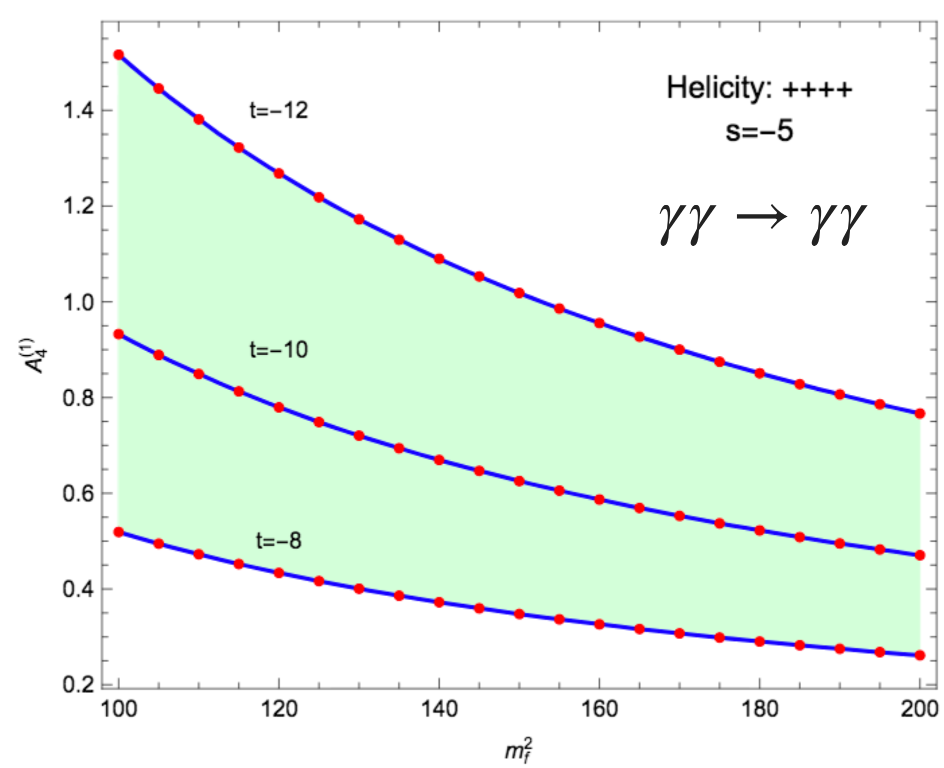
- Integration on the UV on-shell hyperboloid: loop three-momentum unconstrained, but **loop contributions suppressed** for loop energies larger than $\mu_{\rm UV}$

LOCAL UV RENORMALIZATION



- **UV finite** helicity amplitudes, but unintegrated amplitudes locally singular
- $\mathscr{A}^{(L)}_{\rm R} = \mathscr{A}^{(L)} - \mathscr{A}^{(L)}_{\rm UV}\Big|_{d=4}$ $\qquad \mathscr{A}^{(L)}_{\rm UV}\Big|_{d} = 0$
- Subtract not only logarithmic UV singularities, but also linear and quadratic
- Disentangle the UV from the IR behaviour in scaleless integrals (e.g. self-energies)





LOCAL UV RENORMALISATION: MULTILOOP

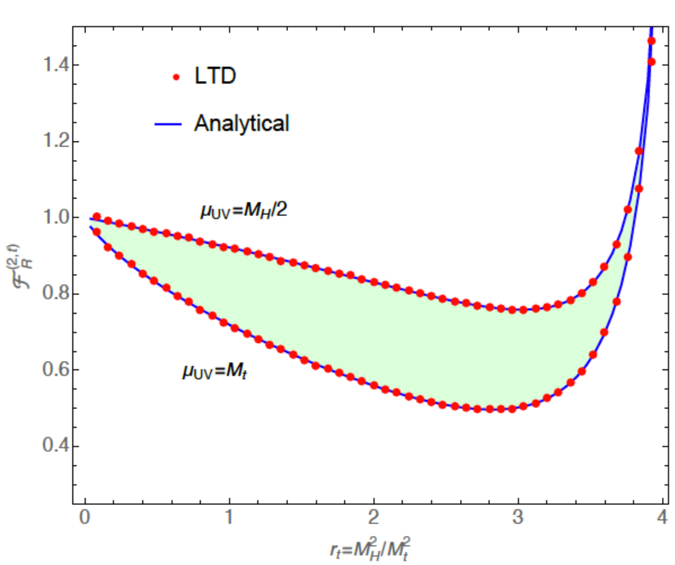
$$\begin{cases} |\boldsymbol{\ell}_1| \to \infty \\ |\boldsymbol{\ell}_2| \ \text{fixed} \end{cases}, \quad \begin{cases} |\boldsymbol{\ell}_1| \ \text{fixed} \\ |\boldsymbol{\ell}_2| \to \infty \end{cases}, \quad \begin{cases} |\boldsymbol{\ell}_1| \to \infty \\ |\boldsymbol{\ell}_2| \to \infty \end{cases}.$$

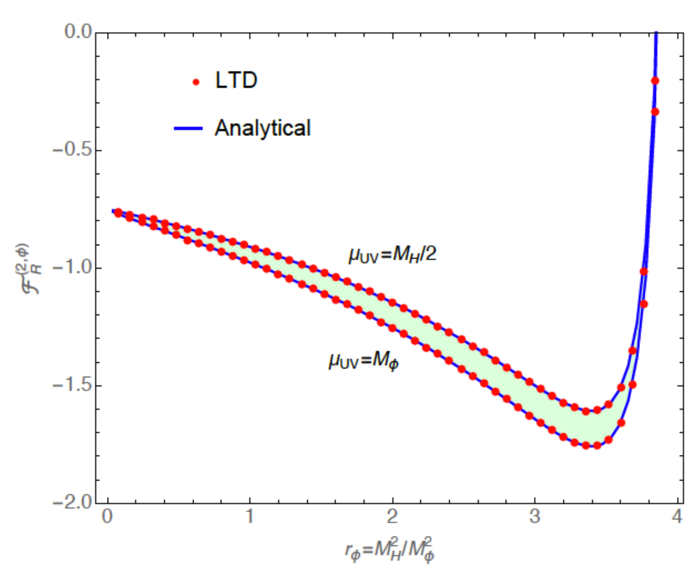
- Multiple UV limit

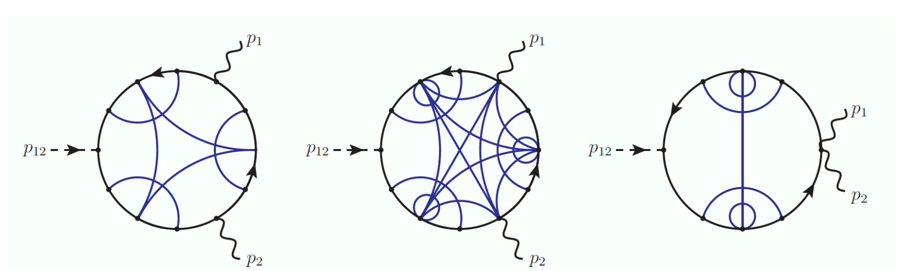
$$\mathrm{UV}^2 : \{\ell_j^2 \mid \ell_j \cdot \ell_k \mid \ell_j \cdot k_i\}$$
$$\to \{\lambda^2 q_{j,\mathrm{UV}}^2 + (1-\lambda^2)\,\mu_{\mathrm{UV}}^2 \mid \lambda^2 q_{j,\mathrm{UV}} \cdot q_{k,\mathrm{UV}} + (1-\lambda^2)\,\mu_{\mathrm{UV}}^2/2 \mid \lambda\, q_{j,\mathrm{UV}} \cdot k_i\}$$

- Most subtle steep the adjustment of the **subleading** terms, d_{UV2}, to be in agreement with e.g. the $\overline{MS}$ **scheme**

$$\left(\mathscr{A}^{(L)} - \mathscr{A}^{(L)}_{1,\mathrm{UV}} - \mathscr{A}^{(L)}_{2,\mathrm{UV}}\right)_{\mathrm{UV}^2} - d_{\mathrm{UV2}}\,\mu_{\mathrm{UV}}^4 \int_{\ell_1 \ell_2} \left(G_F(q_{1,\mathrm{UV}})\right)^3 \left(G_F(q_{2,\mathrm{UV}})\right)^3$$







$H \to \gamma\gamma$ **at two-loops**

[Analytic expressions from Aglietti, Bonciani, Degrassi, Vicini, JHEP **0701** (2007) 021]

FOUR-DIMENSIONAL UNSUBTRATION (FDU) @ NLO

- The **LTD representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_N d\sigma_{\rm V}^{(1,{\rm R})} = \int_N \int_{\vec{\ell}_1} 2\,{\rm Re}\,\langle \mathcal{M}_N^{(0)} | \left(\sum_i \mathcal{M}_N^{(1)}(\tilde{\delta}(q_i)) \right) - \mathcal{M}_{\rm UV}^{(1)}(\tilde{\delta}(q_{\rm UV}))\rangle$$

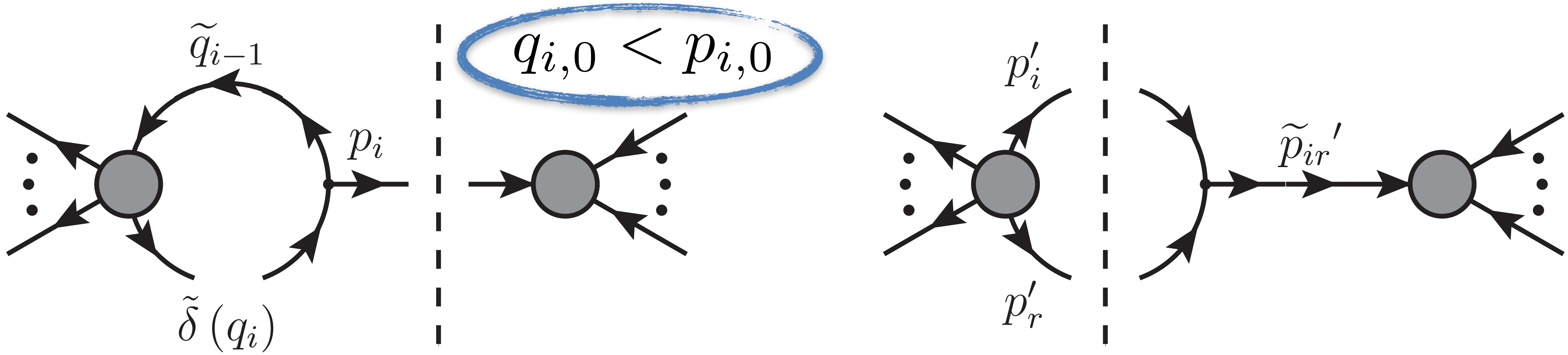
- A **partition** of the real phase-space

$$\sum_i \mathcal{R}_i(\{p'_j\}_{N+1}) = 1$$

- The real contribution **mapped** to the Born kinematics + loop three-momentum

$$\int_{N+1} d\sigma_{\rm R}^{(1)} = \int_N \int_{\vec{\ell}_1} \sum_i \mathcal{J}_i(q_i)\,\mathcal{R}_i(\{p'_j\})\,|\mathcal{M}_{N+1}^{(0)}(\{p'_j\})|^2 \Big|_{\{p'_j\}_{N+1}\to(q_i,\{p_k\}_N)}$$

MOMENTUM MAPPING: MULTI-LEG



- Motivated by the **factorisation properties of QCD**: assuming q_i^μ on-shell, and close to collinear with p_i^μ,we define the momentum mapping

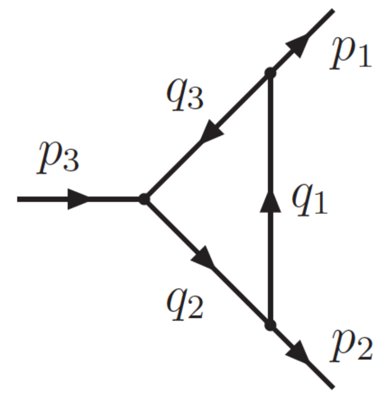
$$p_r'^\mu = q_i^\mu \ ,$$

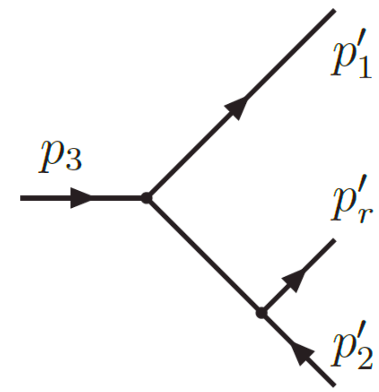
$$p_i'^\mu = p_i^\mu - q_i^\mu + \alpha_i \, p_j^\mu \ , \qquad \alpha_i = \frac{(q_i - p_i)^2}{2p_j \cdot (q_i - p_i)} \ ,$$

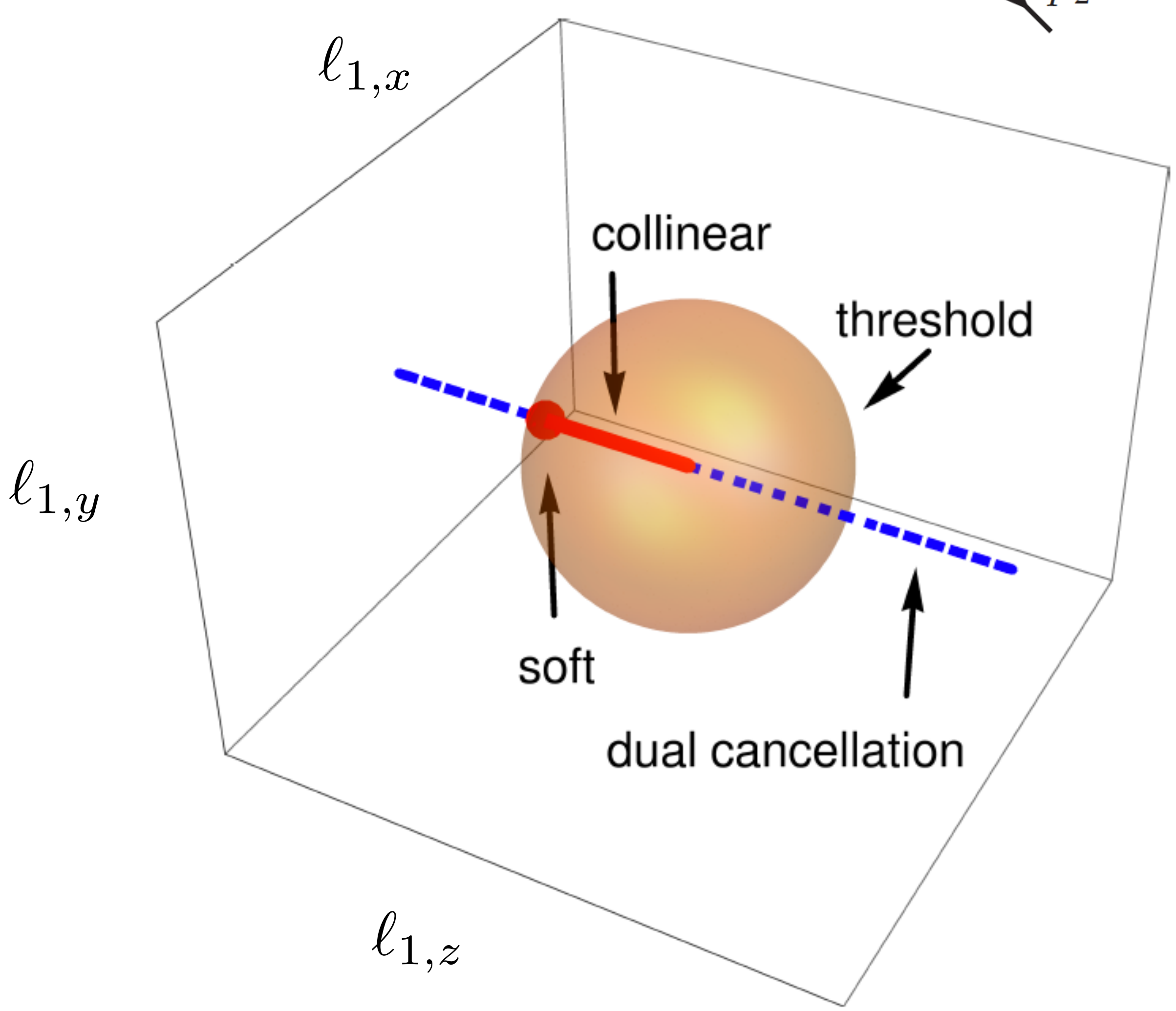
$$p_j'^\mu = (1 - \alpha_i) \, p_j^\mu \ , \qquad p_k'^\mu = p_k^\mu \ , \qquad k \neq i, j$$

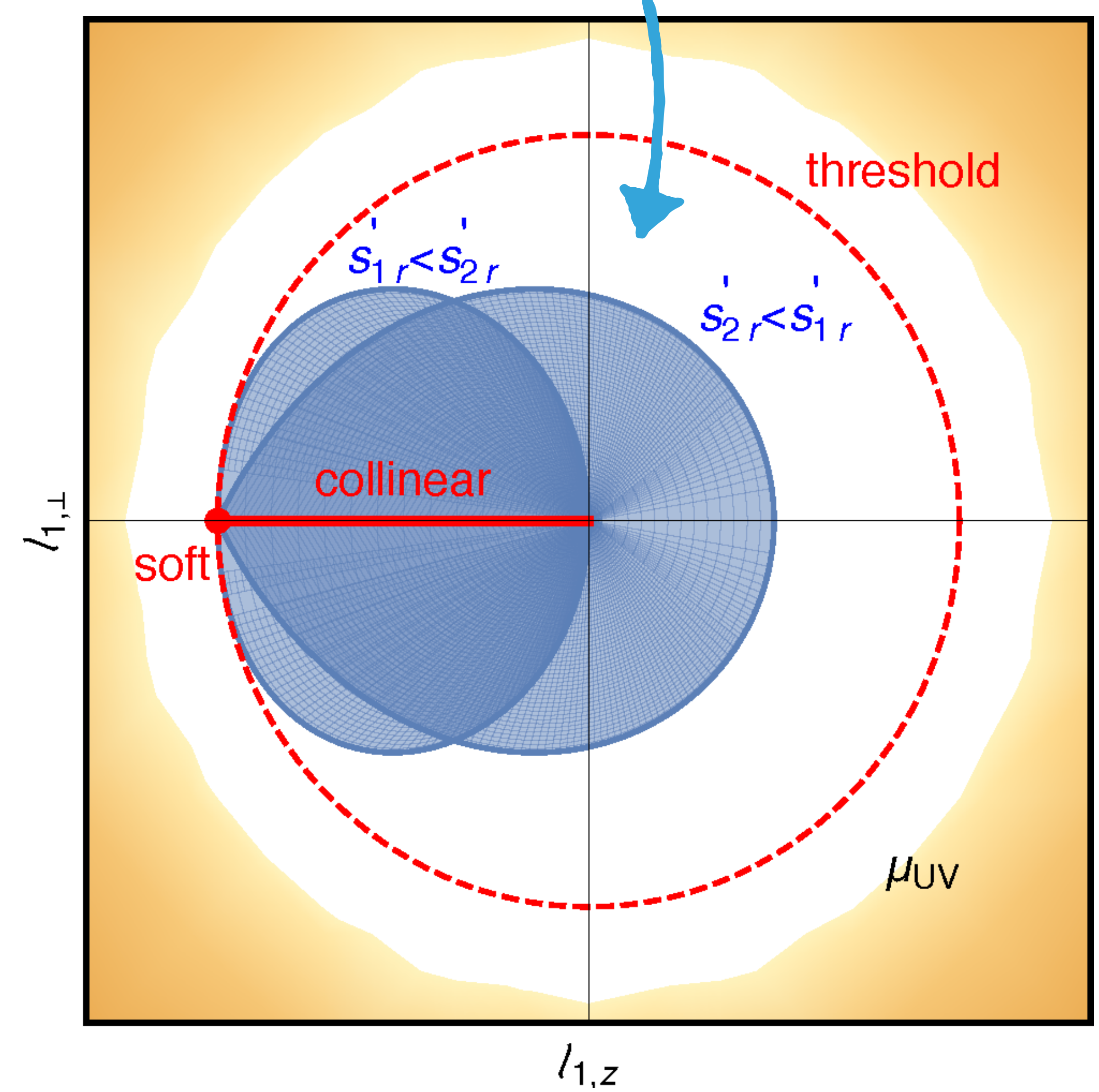
- All the primed momenta (real process) on-shell and momentum conservation: p_i^μ is the **emitter**, p_j^μ the **spectator** needed to absorb momentum recoil
- **Quasi-collinear configurations** can also be conveniently mapped such that the **massless limit is smooth** [Sborlini, Driencourt-Mangin, GR, JHEP **1610**, 162]

LTD / FDU IN THE LOOP THREE-MOMENTUM SPACE



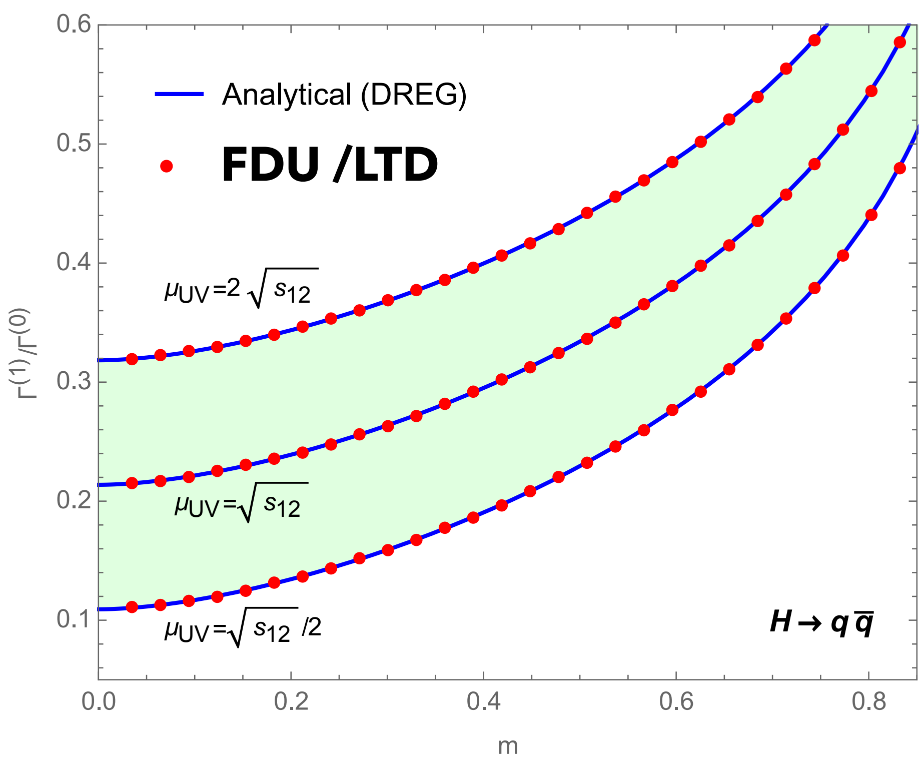


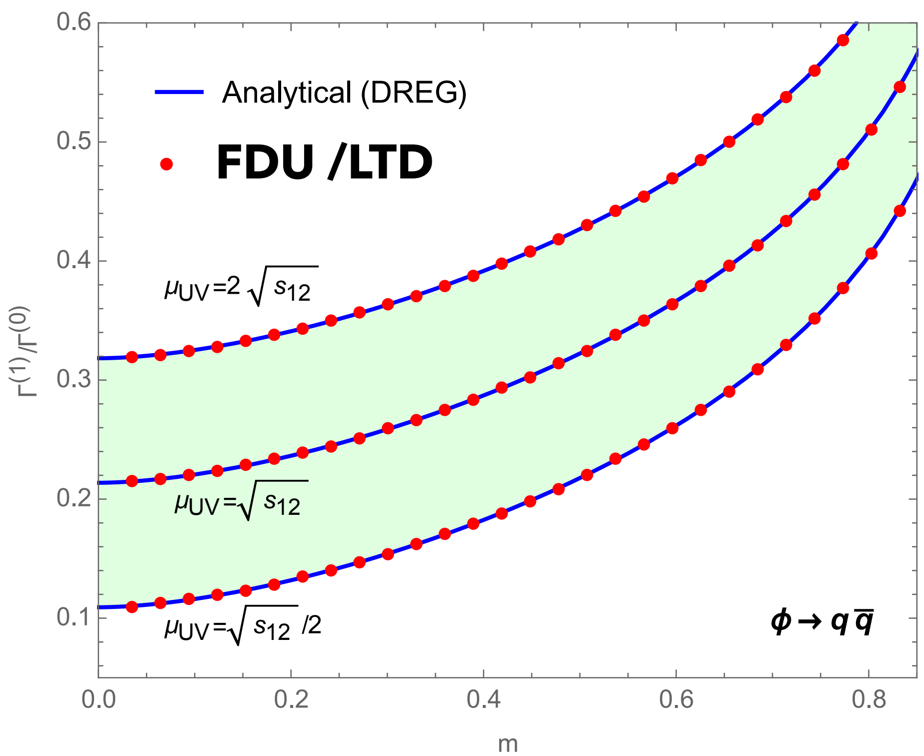


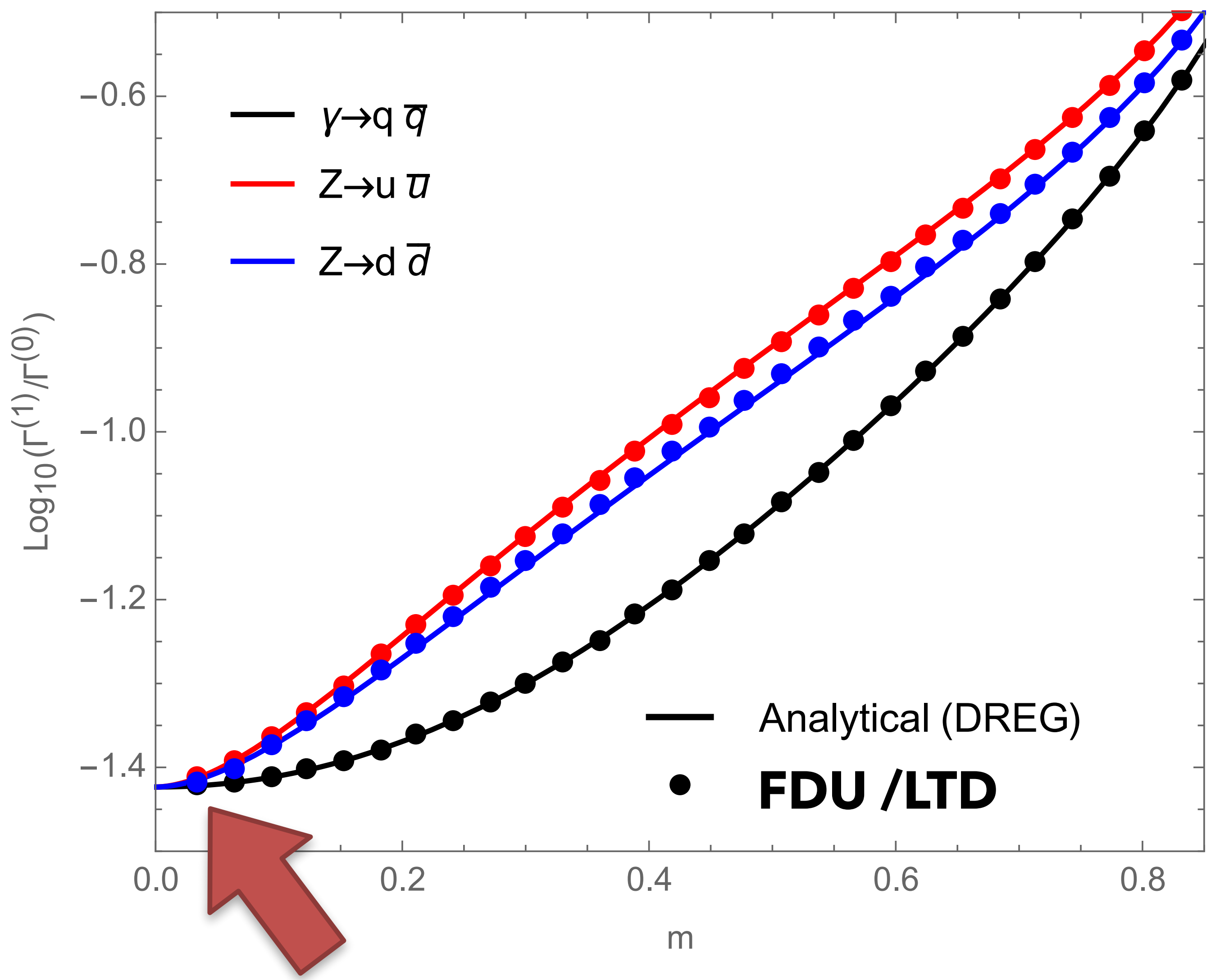


- **The bulk of the physics** is in the **"low" energy** region of the loop momentum

BENCHMARK APPLICATION: $A^* \to q\bar{q}(g)$







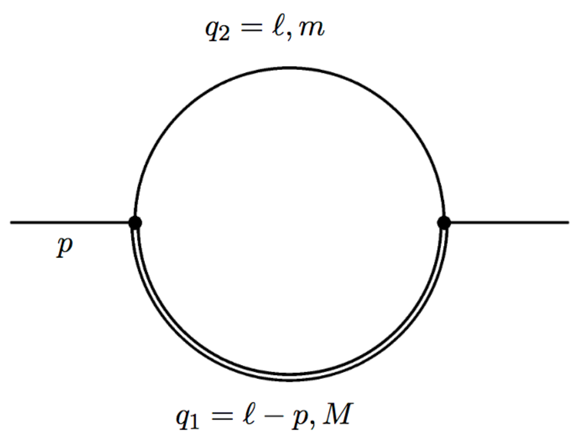
- Excellent agreement with analytic DREG
- Efficient numerical implementation
- **Smooth massless limit**

Driencourt, Sborlini, GR, EPJC **78** (2018) 231
Plenter, GR, arXiv:2005.02119

ASYMPTOTIC EXPANSIONS

- Expansion of **dual propagators**

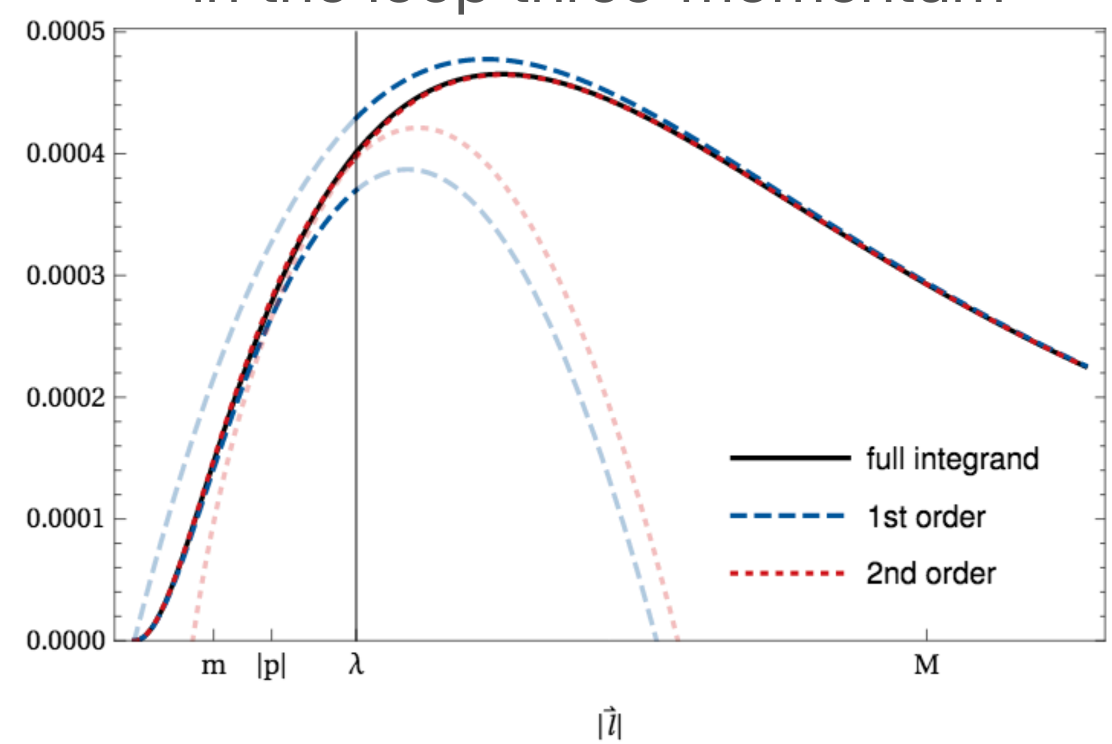
$$G_D(q_i;q_j) = \frac{1}{2q_i\cdot k_{ji} + \Gamma_{ij} + \Delta_{ij} - \imath 0\eta\cdot k_{ji}}\Bigg|_{\Delta_{ij}\ \mathrm{small}} = \sum_{n=0} \frac{(-\Delta_{ij})^n}{(2q_i\cdot k_{ji} + \Gamma_{ij} - \imath 0\eta\cdot k_{ji})^{n+1}}$$



- wrt **Expansion by Regions** [Smirnov, Beneke]: it does not mix UV with IR. Only the first terms might need local renormalisation

	$M^2 \gg \{m^2, p^2\}$	$p^2 \gg \{m^2, M^2\}$	$p^2 = (m+M)^2(1-\beta)\ ,\quad \beta\to 0^\pm$
$G_D(q_1;\ell)$			
Γ_{12}	M^2+p^2	p^2+M^2	$2Mp\cosh\left(\sqrt{-\frac{m\beta}{M}-\imath 0}\right)$
Δ_{12}	$-m^2$	$-m^2$	$p^2+M^2-m^2-\Gamma_{12}$
r_{12}	$\frac{\sqrt{p^2}}{M}$	$\frac{M}{\sqrt{p^2}}$	$\exp\left(\sqrt{-\frac{m\beta}{M}-\imath 0}\right)$
Q_1^2	M^2	p^2	$M\,p\exp\left(-\sqrt{-\frac{m\beta}{M}-\imath 0}\right)$
$G_D(\ell;q_1)$			
Γ_{21}	$-M^2-\frac{m^2p^2}{M^2}$	p^2+m^2	$2m\,p\cosh\left(\sqrt{-\frac{M\beta}{m}+\imath 0}\right)$
Δ_{21}	$p^2+m^2+\frac{m^2p^2}{M^2}$	$-M^2$	$p^2+m^2-M^2-\Gamma_{21}$
r_{21}	$\frac{m\sqrt{p^2}}{M^2}$	$-\frac{m}{\sqrt{p^2}}+\imath 0$	$-\exp\left(\sqrt{-\frac{M\beta}{m}+\imath 0}\right)$
Q_2^2	$-M^2$	p^2	$m\,p\exp\left(-\sqrt{-\frac{M\beta}{m}+\imath 0}\right)$

- Expansion by **dual regions** in the loop three-momentum



LTD BEYOND ONE-LOOP

R. P. Feynman, Closed Loop And Tree Diagrams. (talk 1972),
In *Brown, L.M. (ed.): Selected papers of Richard Feynman* 867-887

FIGURE 7.
Opening a double ring.

If there is more than one loop in the original diagram, the loops may be opened in succession. Choose any one loop; that is, integration over any one

- **After the first LTD round the position of the poles in the complex plane is momentum dependent**

(1) Use a **general identity** to transform into Feynman propagators the dual propagators that enter the successive LTD rounds [Bierenbaum et al., 2010]

- First **full two-loop** calculation ($H \to \gamma\gamma$) with local UV renormalization [Driencourt et al., 2019]
- **Analytic proof** of the dual cancellation of unphysical (non-causal) singularities, causal and anomalous thresholds as well as infrared in a compact region (→ FDU) [Aguilera et al., 2019]

(2) **Average** over all possible momentum flows [Runkel et al., 2019]: cumbersome symmetry factors

(3) Keep track of the position of the poles and close the Cauchy contour either **from above or from below** to cancel that dependence [Capatti et al., 2019]

- **Numerical test** of dual cancellations

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, GR, Sborlini, Torres, Tracz, PRL124 (2020) 21, 211602

LTD TO ALL ORDERS AND POWERS

- **Multi-loop scattering amplitude:** n sets of momenta that depend on L loop momenta or a linear combination

$$\mathscr{A}_N^{(L)}(1,\ldots,n) = \int_{\ell_1\cdots\ell_L} \mathscr{N}(\{\ell_i\}_L, \{p_j\}_N)\, G_F(1,\ldots,n)$$

$$= \prod_{i\in 1\cup\ldots\cup n} \left(G_F(q_i)\right)^{a_i}$$

- The **dual function** involving two sets that depend on the same loop momentum: momenta in the set t remain off-shell

$$G_D(s;t) = -2\pi\imath \sum_{i_s\in s} \text{Res}\left(G_F(s,t), \text{Im}(\eta q_{i_s}) < 0\right)$$

- **Cauchy contour** always from **below** the real axis
- valid for arbitrary powers and **Lorentz invariant** [Catani et al. JHEP **0809**, 065]
- reverse momenta, if necessary, to keep a coherent momentum flow
$t \to \bar{t} \quad (q_{i_t} \to -q_{i_t})$

- The **nested residue** involving several sets

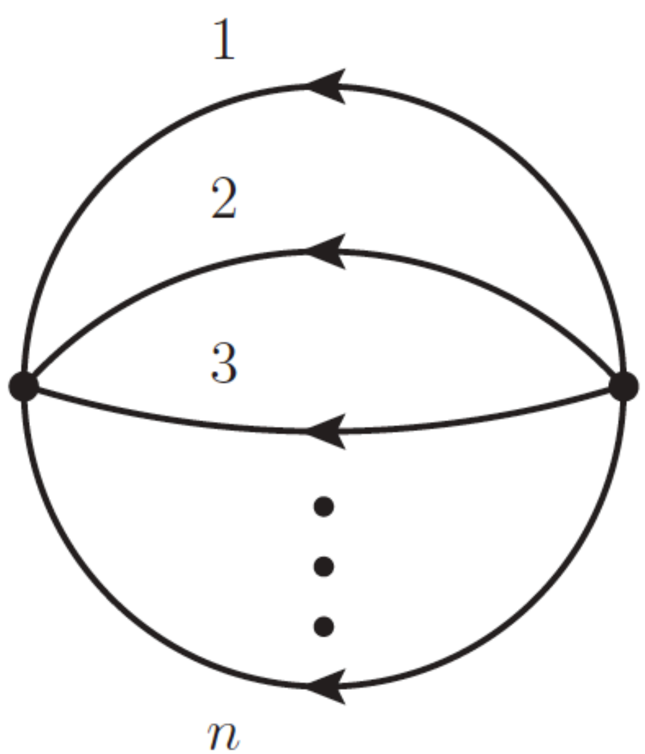
$$G_D(1,\ldots,r;n) = -2\pi\imath \sum_{i_r\in r} \text{Res}\left(G_D(1,\ldots,r-1;r,n), \text{Im}(\eta q_{i_r}) < 0\right)$$

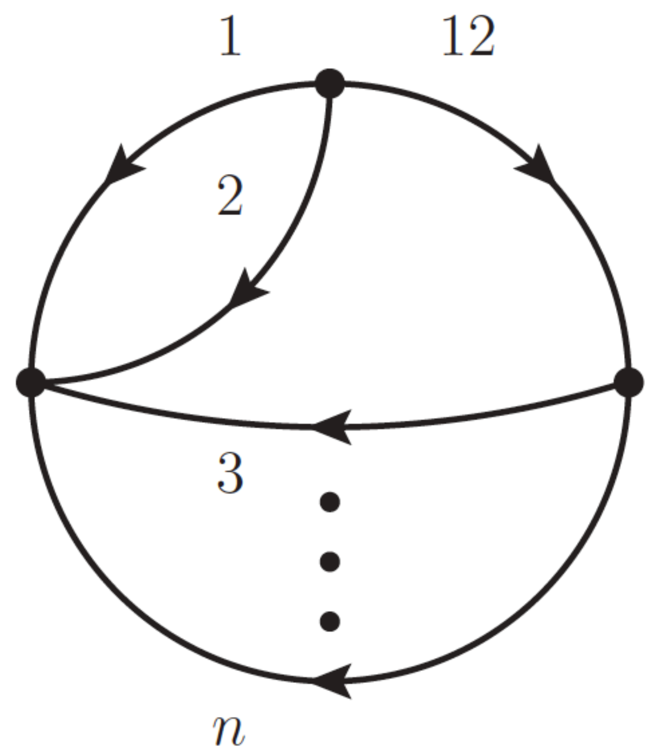
Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, GR, Sborlini, Torres, Tracz, PRL124 (2020) 21, 211602

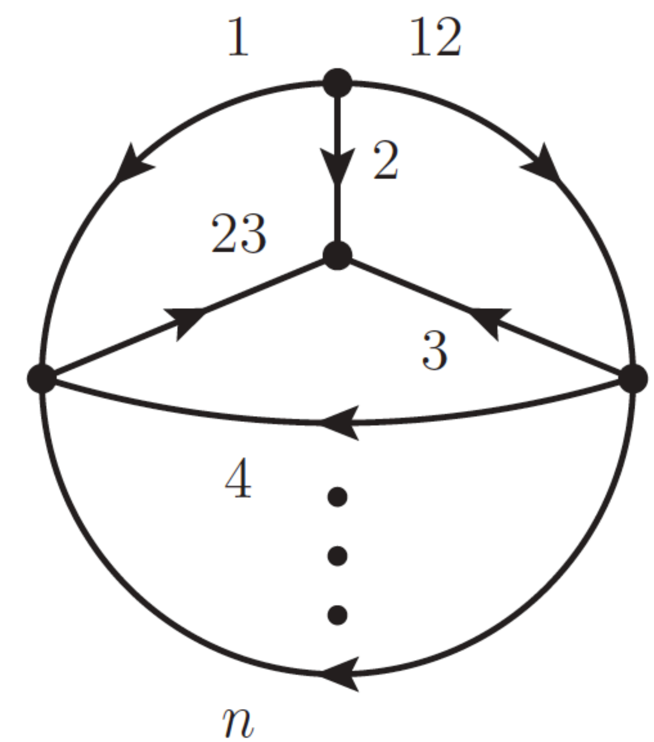
MULTILOOP TOPOLOGIES

arbitrary number of external legs attached to each line

$q_{i_s} = \ell_s + k_{i_s}, \quad q_{i_{12}} = -\ell_1 - \ell_2 + k_{i_{12}}$







MLT

Maximal Loop Topology

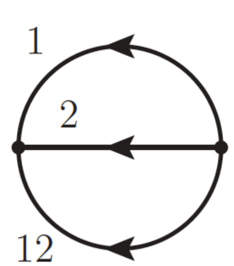
NMLT

Next-to-maximal Loop Topology

N2MLT

Next-to-next-to-maximal Loop Topology

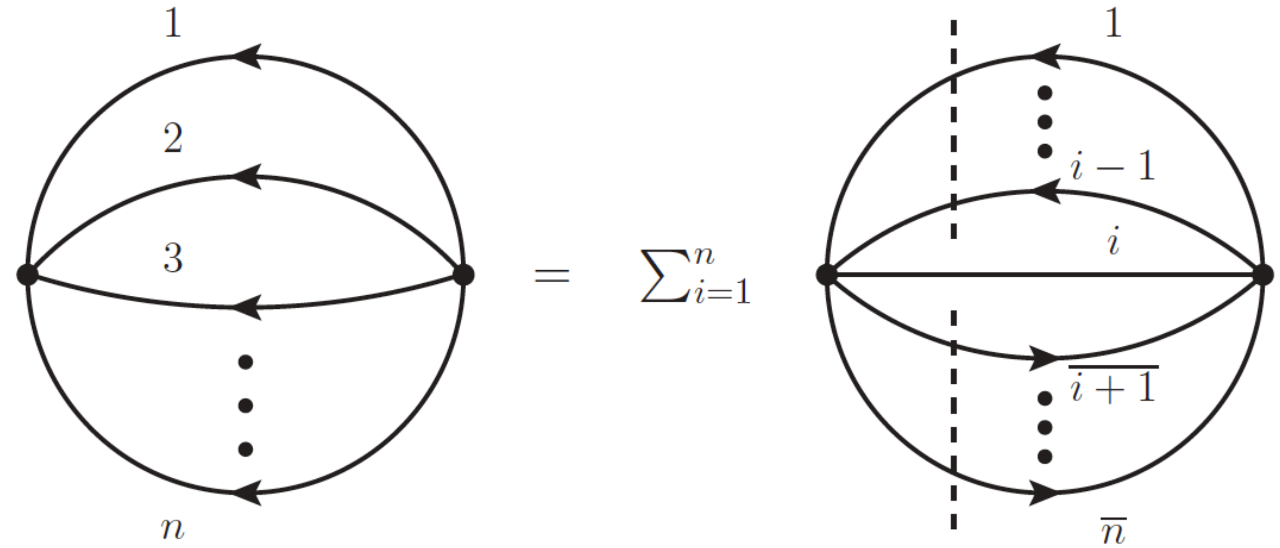
unique topology at two loops



both topologies starting at three loops

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, GR, Sborlini, Torres, Tracz, PRL124 (2020) 21, 211602

MAXIMAL LOOP TOPOLOGY



- **extremely simple and symmetric** LTD representation, proven by induction and directly **independent of the position of the poles** in the complex plane

$$\mathcal{A}^{(L)}_{\rm MLT}(1,\ldots,n) = \int_{\ell_1\cdots\ell_L} \sum_{i=1}^{n} \mathcal{A}^{(L)}_{D}(1,\ldots,i-1,\overline{i+1},\ldots,\overline{n};i)$$

- **causal singularities** when on-shell momenta get aligned [Aguilera et al. JHEP **1912**, 163]

$$\mathcal{A}^{(L)}_{D}(1,\ldots,n-1;n)\,|_{n \text{ onshell}} \to \mathcal{A}^{(L)}_{D}(1,2,\ldots,n) \qquad | \qquad \mathcal{A}^{(L)}_{D}(\overline{2},\ldots,\overline{n};1)\,|_{\overline{1} \text{ onshell}} \to \mathcal{A}^{(L)}_{D}(\overline{1},\overline{2},\ldots,\overline{n})$$

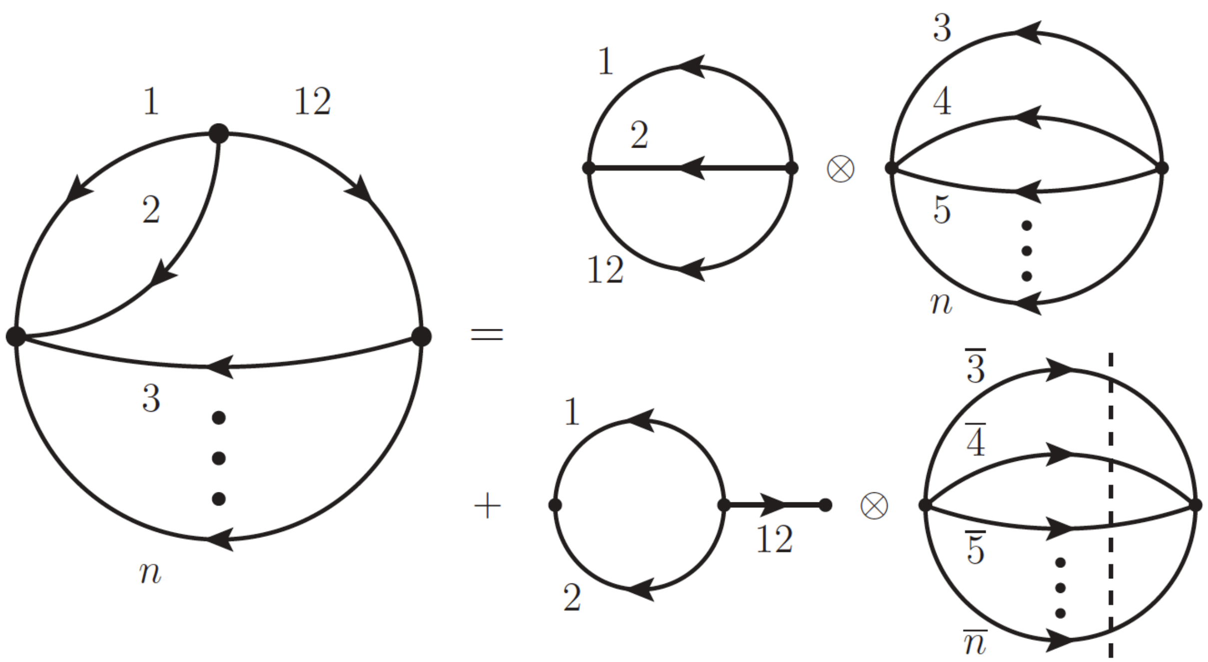
- **non-causal singularities** (unphysical) entangled among dual pairs, they cancel

$$\mathcal{A}^{(L)}_{D}(\overline{2},\overline{3},\ldots,\overline{n};1) + \mathcal{A}^{(L)}_{D}(1,\overline{3},\ldots,\overline{n};2) \to \mathcal{A}^{(L)}_{D}(1,\overline{2},\overline{3},\ldots,\overline{n}) - \mathcal{A}^{(L)}_{D}(1,\overline{2},\overline{3},\ldots,\overline{n})$$

- **What if we reorder the loop lines? Do we get a different representation?**

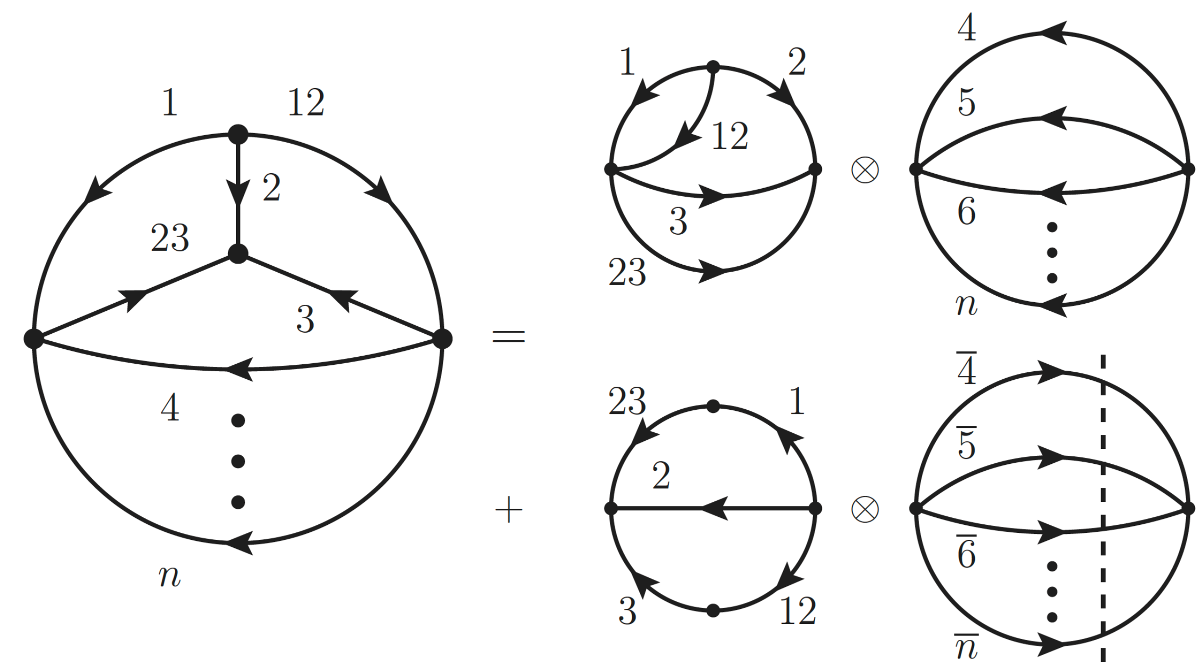
Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, GR, Sborlini, Torres, Tracz, PRL124 (2020) 21, 211602

NMLT AND N2MLT: CASCADE FACTORIZATION



$$\mathcal{A}^{(L)}_{\rm NMLT}(1,\ldots,n,12)$$
$$= \mathcal{A}^{(2)}_{\rm MLT}(1,2,12) \otimes \mathcal{A}^{(L-2)}_{\rm MLT}(3,\ldots,n)$$
$$+\mathcal{A}^{(1)}_{\rm MLT}(1,2) \otimes \mathcal{A}^{(0)}(12) \otimes \mathcal{A}^{(L-1)}_{\rm MLT}(\overline{3},\ldots,\overline{n})$$

- **causal singularities** determined by subtopologies



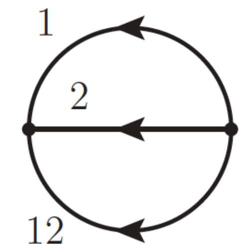
$$\mathcal{A}^{(L)}_{\rm N^2MLT}(1,\ldots,n,12,23)$$
$$= \mathcal{A}^{(3)}_{\rm NMLT}(1,2,3,12,23) \otimes \mathcal{A}^{(L-3)}_{\rm MLT}(4,\ldots,n)$$
$$+\mathcal{A}^{(2)}_{\rm MLT}(1 \cup 23, 2, 3 \cup 12) \otimes \mathcal{A}^{(L-2)}_{\rm MLT}(\overline{4},\ldots,\overline{n})$$

- **factorization** conjectured to hold to all potential topologies at higher orders with simpler topologies as building blocks

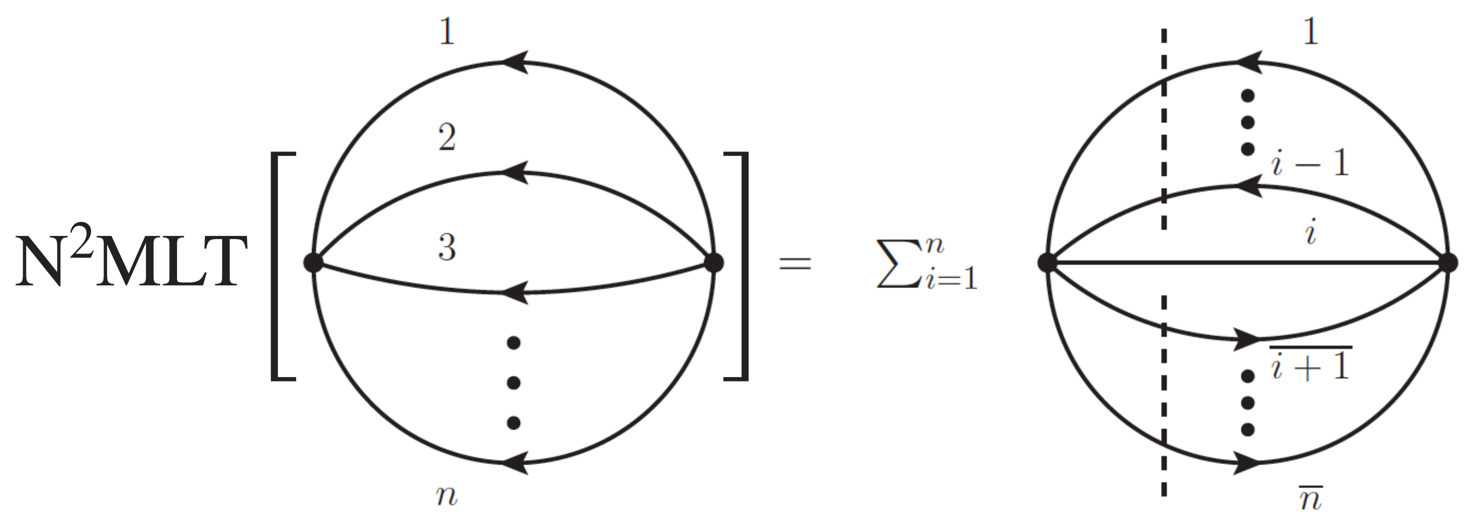
Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, GR, Sborlini, Torres, Tracz, PRL124 (2020) 21, 211602

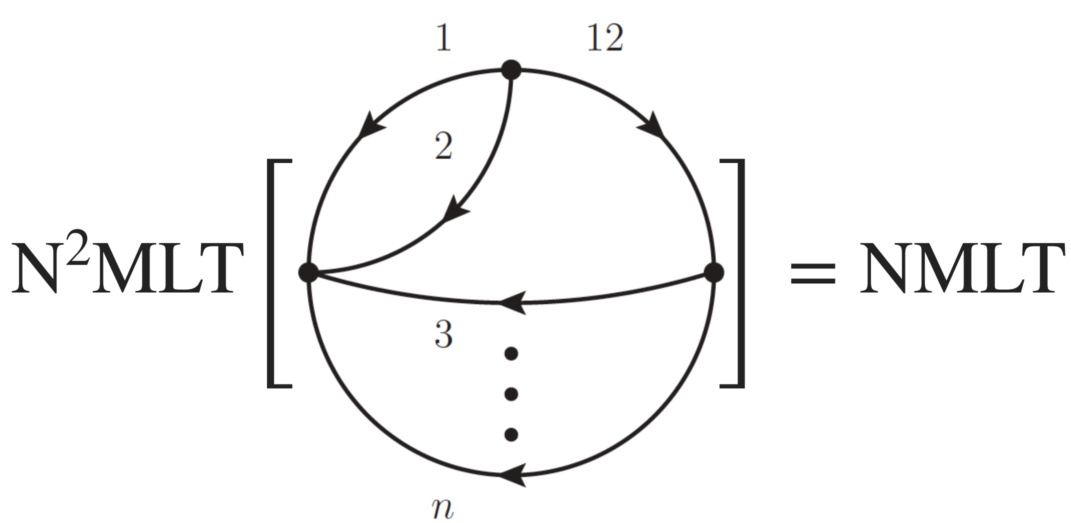
MASTER OPENING OF SCATTERING AMPLITUDES

- At **two loops** any scattering amplitude is opened as **MLT**



- At **three loops** the master opening is **N²MLT**

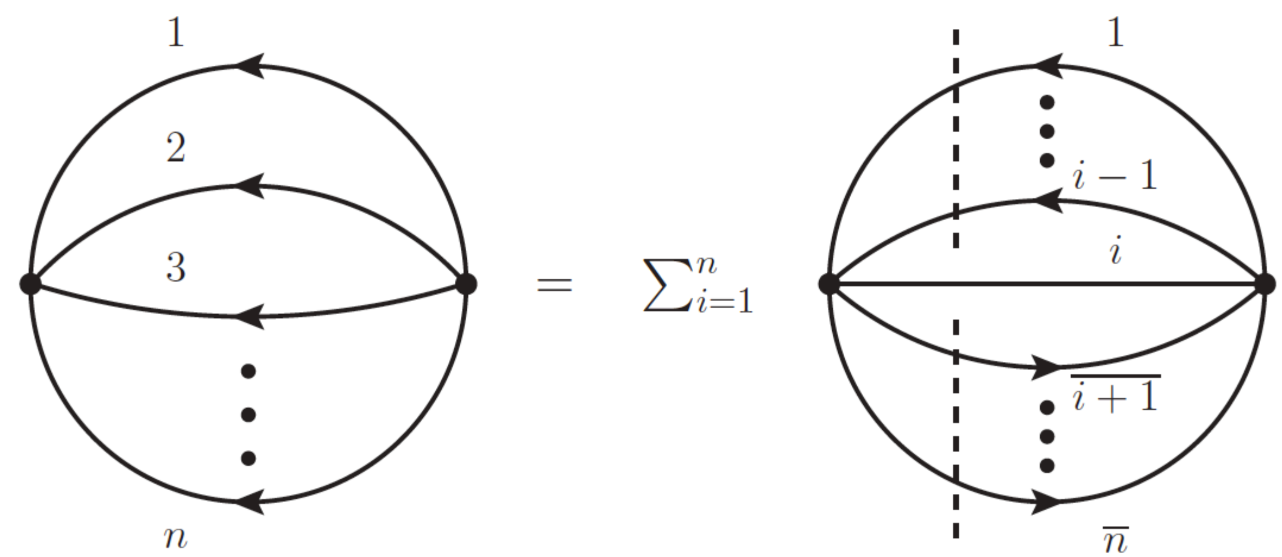




- direct and efficient application to **physical scattering processes**

CAUSAL REPRESENTATION

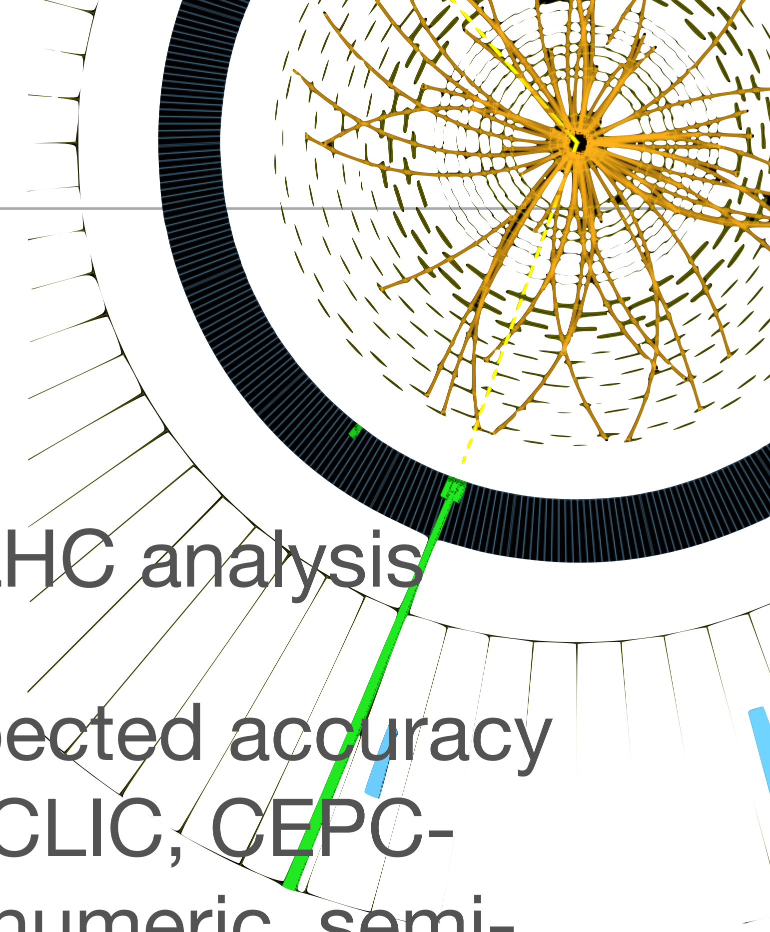
- **What if we reorder the loop lines? Do we get a different representation?**



- In fact no (math vs physics), e.g. scalar integral

$$\mathcal{A}^{(L)}_{\mathrm{MLT}}(1,\ldots,n) = \int_{\vec{\ell}_1\ldots\vec{\ell}_L} \frac{1}{\prod 2q^{(+)}_{i,0}} \left(\frac{1}{\lambda^{+}_{1,n}} + \frac{1}{\lambda^{-}_{1,n}} \right), \qquad \lambda^{\pm}_{1,n} = \sum q^{(+)}_{i,0} \pm k_{1n,0}$$

  - Independent of the initial momentum flow assignments
  - **Free of non-causal singularities:** conjectured for all topologies and internal configurations

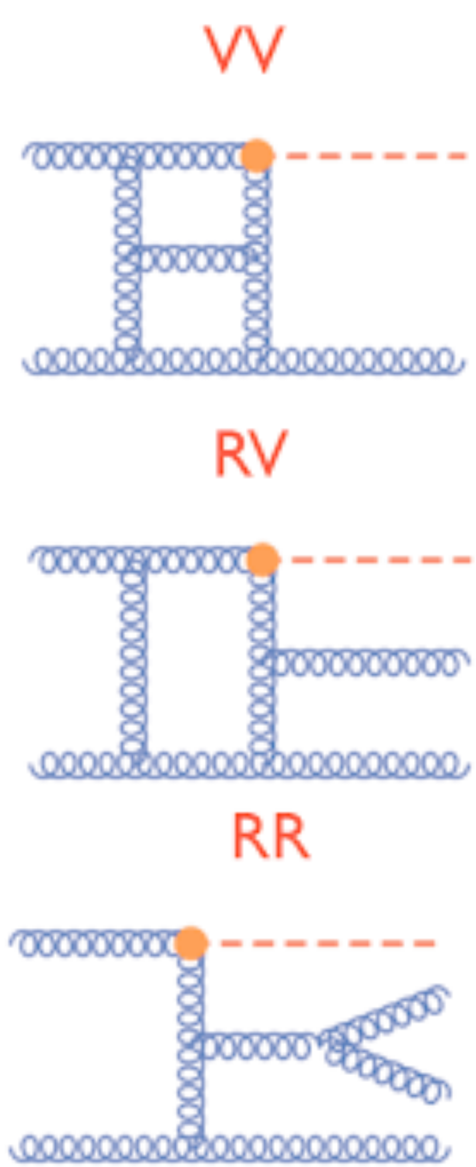


CONCLUSIONS

- **Theory is already the limiting factor** in many LHC analysis
- Current techniques insufficient to match the expected accuracy at future colliders (HL-LHC, FCC, HE-LHC, ILC/CLIC, CEPC-SPPC). New theoretical developments needed: numeric, semi-numeric or analytic
- Back to the physical four space-time dimensions and fully local
- **Loop-tree duality** powerful formalism to reveal intriguing properties of multi-loop scattering amplitudes: **manifest causal structure**. Reformulated to all orders in terms of the original Lorentz-invariant prescription.
- Still few phenomenological applications, more to come

CANCELLATION OF IR SINGULARITIES & DREG

- Cancellation of IR singularities at **NLO** is satisfactorily solved: efficient algorithms applicable to any process for which matrix elements are known (**Slicing** [Giele, Glover, …] vs **Subtraction**: dipole [Catani, Seymour], FKS [Frixione, Kunszt, Signer], NS [Nagy, Soper])
- At **NNLO** several working algorithms, successfully applied to specific processes with heavy computational costs



- Antennae Subtraction [Gehrmann et al.]
- ColourfulNNLO Subtraction [Del Duca et al.]
- Geometric Subtraction [Herzog]
- Leading Regions [Anastasiou, Sterman]
- Nested Soft-Collinear Subtraction [Caola et al.]
- N-Jettiness [Boughezal, Petriello et al., Gaunt et al.]
- Projection to Born [Bonciani et al.]
- q_T Substraction [Catani, Grazzini et al.]
- Stripper [Czakon et al.]

New strategy at d=4

- Four-dimensional Regularization (FDR) [Pittau et al.]
- Four-dimensional Unsubtraction (FDU) [Sborlini et al.]