Infrared Singularities and the Precision Frontier

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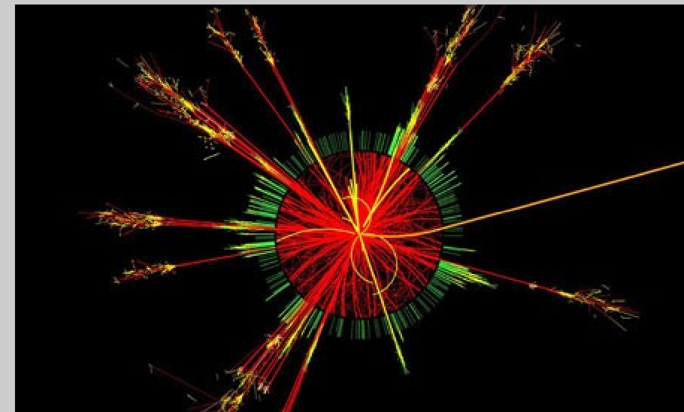
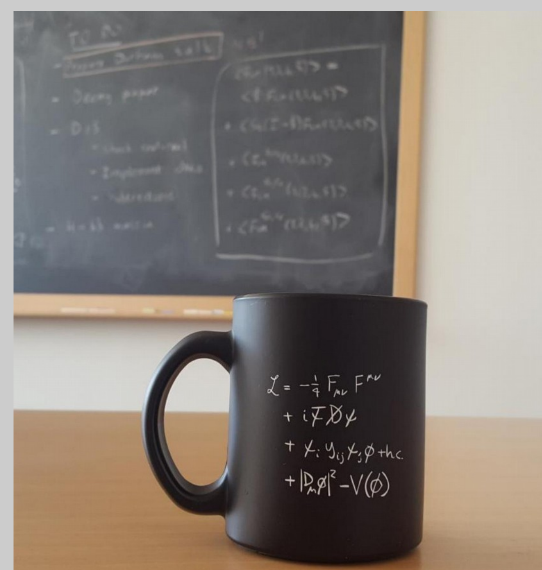
Introduction

- Study behavior of particles in controlled environments: colliders.
- E.g. **L**arge **H**adron **C**ollider.



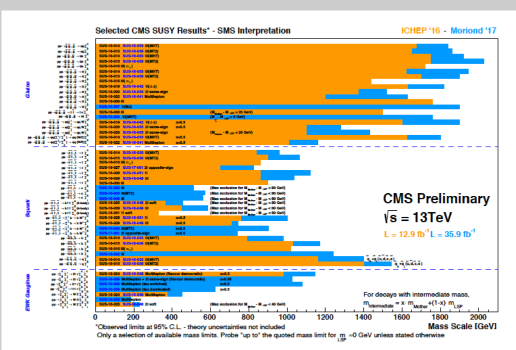
- **S**tandard **M**odel: an elegant and mathematically consistent description of particle behavior.



The LHC Thus Far

- The Expected: the Higgs boson.

- The Unexpected: no physics **B**eyond the **SM**.



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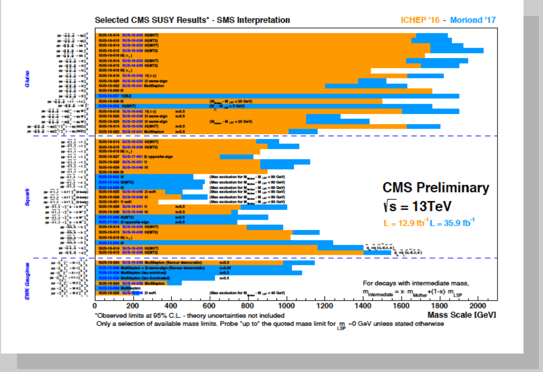

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Understand EWSB.

Discover or constrain BSM physics.

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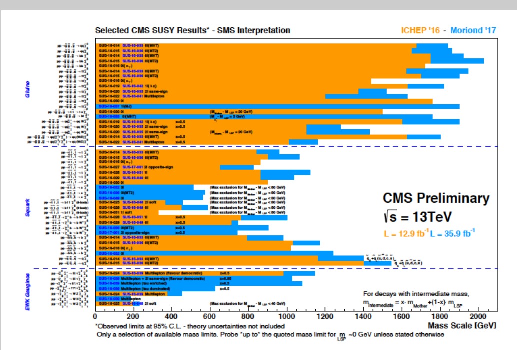

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HIGH-PRECISION PROGRAMME

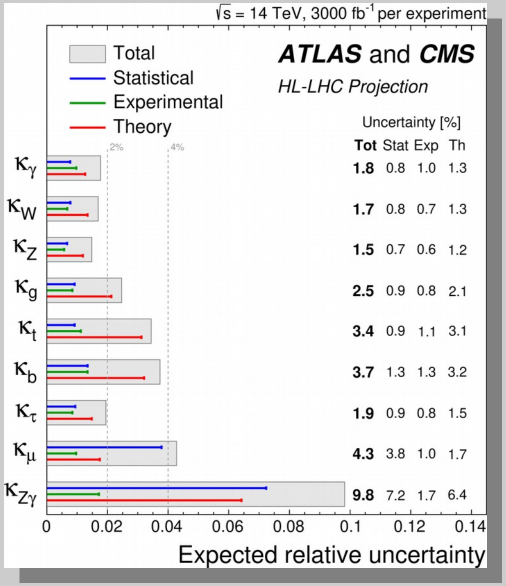
PRECISION FRONTIER

- How precise do we need to be?
  - Suppose we have BSM physics at scale $\Lambda_{NP} \sim 1 - \text{ few TeV}$
  - Difficult to produce directly at LHC, but have indirect impact.
  - Simple scaling argument:

$$\frac{Q^2}{\Lambda_{\rm NP}^2} \sim \left(\frac{100 \text{ GeV}}{1 \text{ TeV}}\right)^2 \sim \text{few}\%$$

- Is this achievable experimentally?
  - Yes!
  - "Except for rare decays, the overall uncertainties will be dominated by the theoretical systematics, with a precision close to percent level."

    - Report on *Physics Potential of the HL-LHC,* submitted to CERN Council



Surviving At The Precision Frontier

- Is this achievable theoretically?
  - Factorization theorem:

$$\mathrm{d}\sigma = \sum_{ij} \int \mathrm{d}x_1 \mathrm{d}x_2 f_i(x_1,\mu^2) f_j(x_2,\mu^2) \mathrm{d}\hat{\sigma}_{ij}(\{p_i\},\mu^2,x_1,x_2) \left(1 + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)\right)$$

  - Potential showstopper: (uncontrolled) nonperturbative effects in hadron collision.
  - Nonperturbative effects appear at $\Lambda_{\mathrm{QCD}}/Q \sim (300\ \mathrm{MeV})/(30\ \mathrm{GeV}) \sim 1\%$
  - So it is feasible theoretically...
  - ... but challenging!

The LHC Precision Programme

Requires progress in every aspect of collider physics.

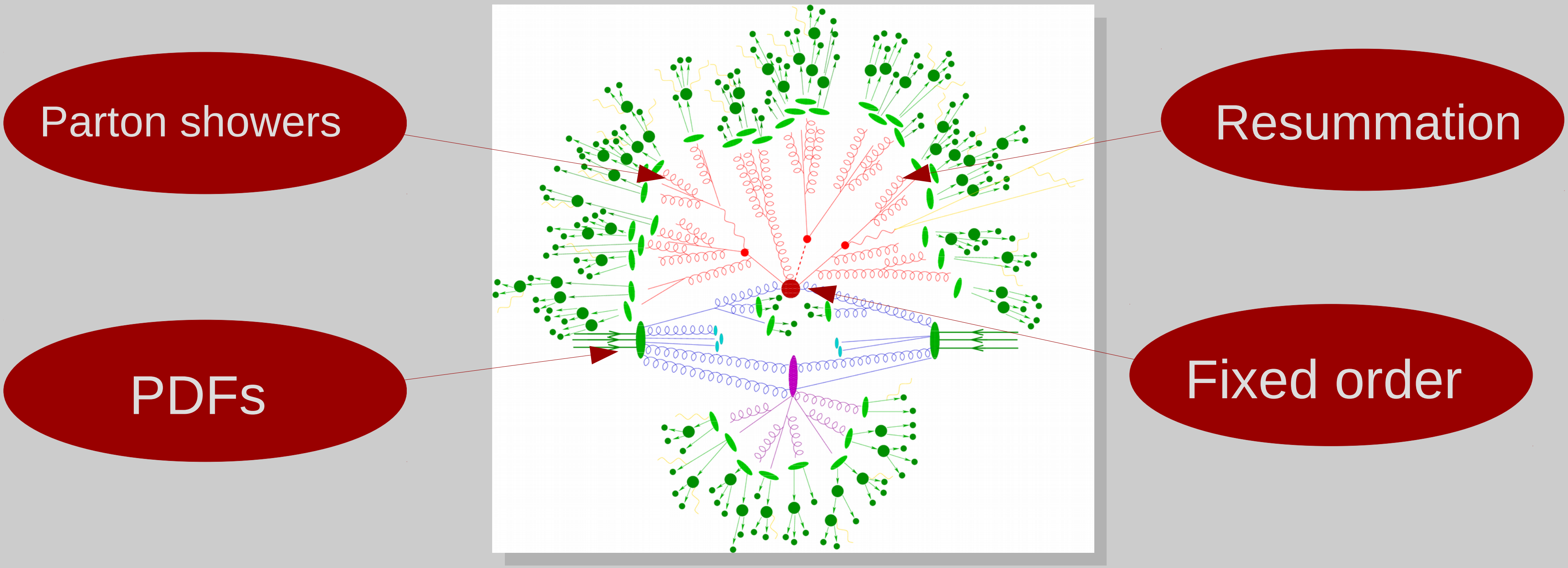


Image: S. Höche

- Technical challenges: deep understanding of QFTs of SM:
  - Interesting on purely theoretical as well as phenomenological grounds.

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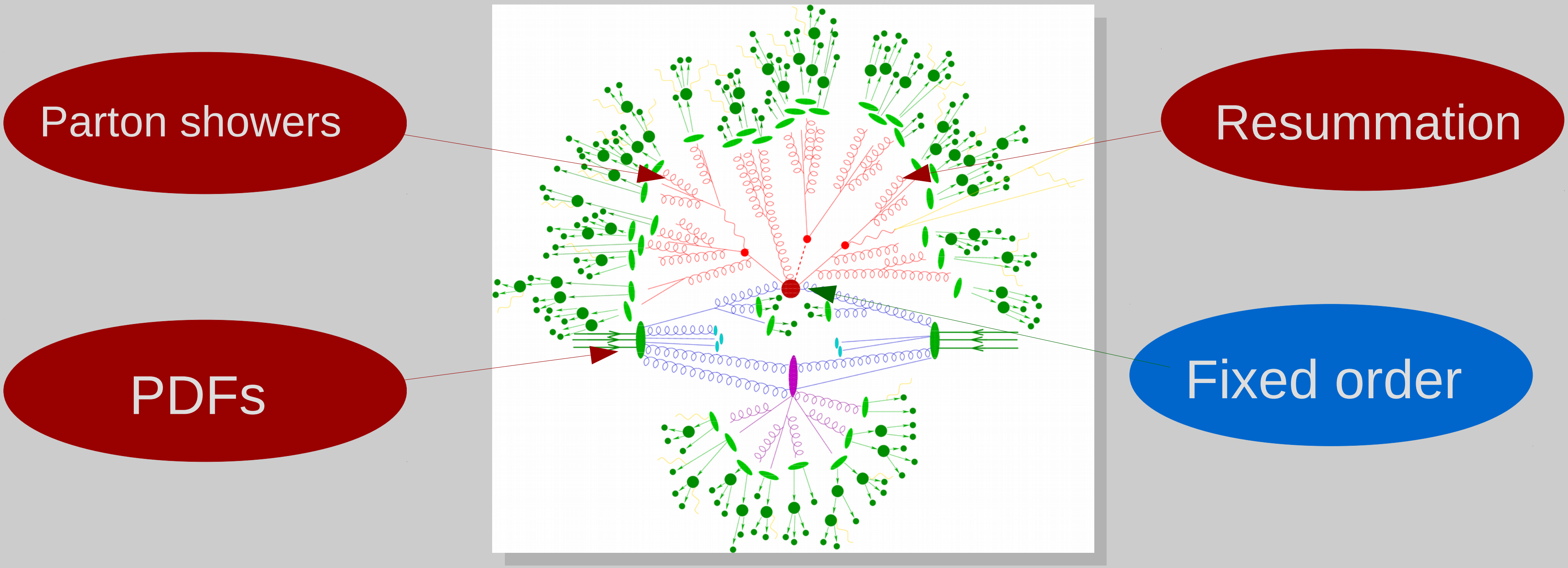

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- Technical challenges: deep understanding of QFTs of SM:
  - Interesting on purely theoretical as well as phenomenological grounds.

Fixed-order calculations

- Expansion in strong coupling of QCD $\alpha_s \sim 0.1$ and in electroweak (EW) coupling $\alpha \sim 0.01$

$$\hat{\sigma} ij = \hat{\sigma}_{ij}^{(0,0)} + \alpha_s \hat{\sigma}_{ij}^{(1,0)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2,0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(3,0)} + \ldots + \alpha \, \hat{\sigma}_{ij}^{(0,1)} + \alpha_s \alpha \, \hat{\sigma}_{ij}^{(1,1)} + \ldots$$

LO	NLO QCD	NNLO QCD	N3LO QCD	NLO EW	NLO QCD-NLO EW
Order of magntiude	~10%	~1%	Special processes	~1%	Special processes/high energies

- Two challenges:
  - Multiloop calculations;
  - Treating infrared (IR) singularities.

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LO	NLO QCD	NNLO QCD	N3LO QCD	NLO EW	NLO QCD-NLO EW
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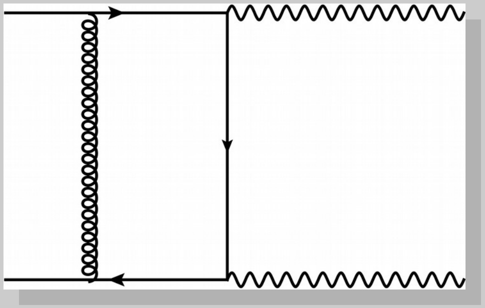
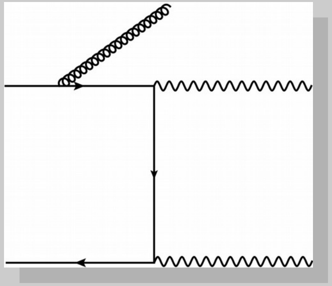
Infrared Singularities



Virtual corrections



Real corrections

- Virtual and real corrections separately have IR divergences (soft and/or collinear radiation).
- Unphysical, will cancel when combining real and virtuals (Kinoshita-Lee-Nauenberg).
- Virtual corrections: divergences as explicit poles in amplitudes.
- Real corrections: divergences from integrating over phase space.
- **But:** then lose information on kinematic observables.
  - Needed for fully differential calculations.
- Need to extract singularities from real emissions without losing kinematical information on radiated parton.

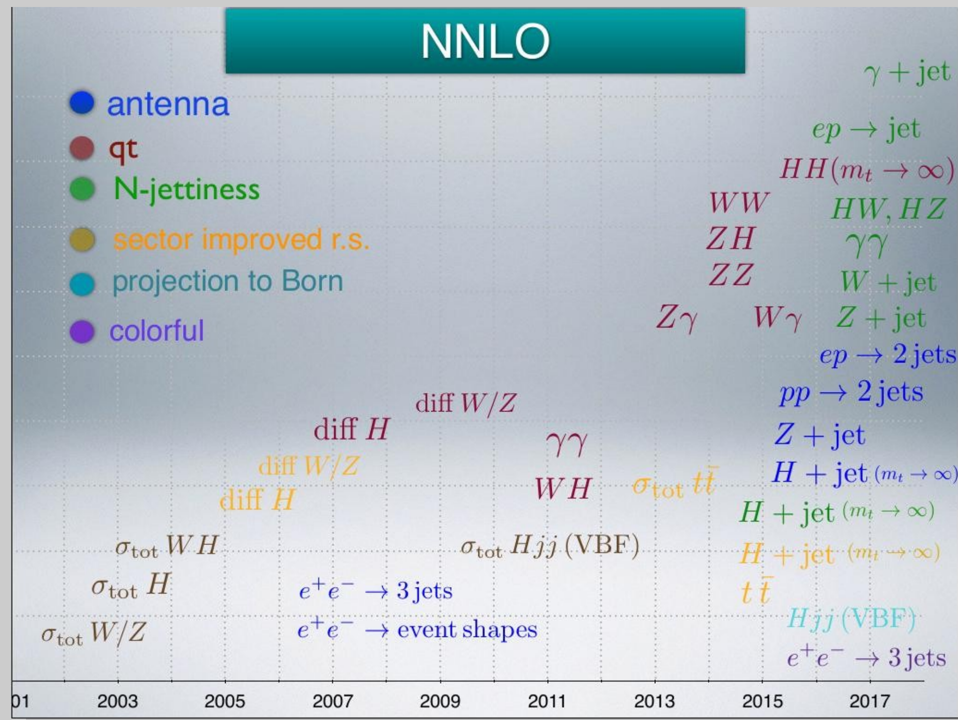
  ➡ **Subtraction scheme.**

Subraction Schemes

- Fully solved at NLO. [Catani, Seymour ‘96; Frixione, Kunszt, Signer ‘96]
- NNLO: greatly complicated by convoluted singularity structure (“overlapping” singularities).

- Several proposed solutions; successful phenomenological applications for 2 → 2 processes

  ➡ *“the NNLO revolution”*



Slide from G. Heinrich

Subtraction Schemes at NNLO

None of the NNLO subtraction schemes are completely satisfactory:

- Fully local;
- Fully analytic;
- Applicable to any process at the LHC:
  - Initial state partons;
  - Final state parton

	Analytic	Local	FS Colour	IS Colour
Antenna	✓	✗	✓	✓
STRIPPER	✗	✓	✓	✓
CoLoRFul	~	✓	✓	✗

Inspired by Nigel Glover, *Amplitudes 2015*

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Can we construct a method that ticks all the boxes?

- Efficient subtractions for richer phenomenology in higher multiplicity processes.
- Theoretical clarity by identifying singular structures.
- Eventual automation of NNLO calculations.
- Connections with electroweak corrections, matching of NNLO to parton showers, ...

Taming Infrared Singularities

20 years in the making:

1. Singular limits needed for NNLO calculations.

   [Catani, Grazzini, de Florian, Campbell, Glover, Bern, Dixon, Dunbar, Kosower, Uwer, Del Duca, Frizzo, Maltoni, Kilgore, Schmidt,..., ~ Y2K]

2. Phase space partitioning.

   [Frixione, Kunszt, Signer '96]

3. Parametrization of radiative phase space.

   [Anastasiou, Melnikov, Petriello '03, '04]

4. Sector decomposition of phase space.

   [Czakon '10, '11]

   ➡ STRIPPER

- Problem: underlying physics obscured.
  - Energies and angles treated on same footing.
- How can we construct something with greater clarity?

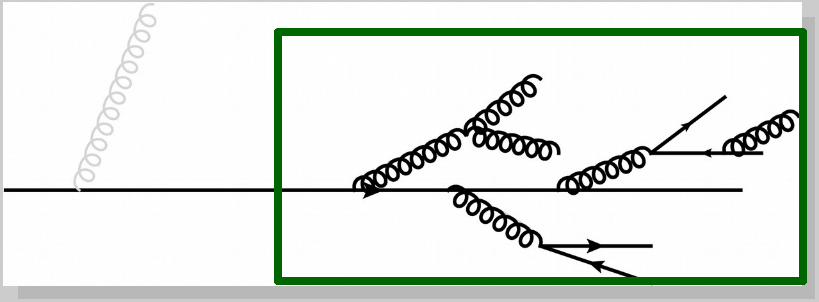
Taming Infrared Singularities

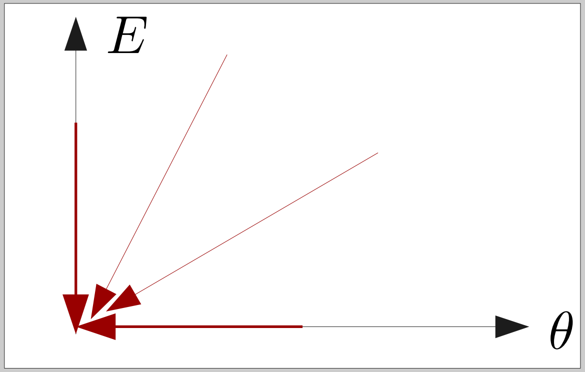
Use the fact that soft and collinear limits commute!

Not true for propagators: have behavior $f(x,y) = \frac{x}{x+y}$ so limits do not commute.

Is true for physical (gauge invariant, onshell) QCD amplitudes.

- Colour coherence:
  - Used in resummation/PS, not manifest in subtraction schemes.
  - Soft gluon cannot resolve details of collinear splitting.
  - Only sensitive to **overall** colour charge.





- No overlap between soft and collinear limits

➡ can be treated independently.

- Energies and angles decouple.

Nested Soft-Collinear Subtractions

Using colour coherence leads to dramatic simplifications.

- Colour coherence + sector decomposition of the phase space:

  ➡ **nested soft-collinear subtraction scheme.**

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Regular Article - Theoretical Physics

Nested soft-collinear subtractions in NNLO QCD computations

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- ✔ Can handle initial state and final state partons.

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- ✓ Fully local.
- ✓ Analytic.
- ✓ Can handle initial state and final state partons.

- Minimal structure with clear physical origin of singularities.
- Final result remarkably simple.
- Handles massive partons.
- Natural alpha-parameters.
- Two kinds of collinear limit: small & large rapidity separations.

  ➡ resummation/parton showers?

Building Towards Generality

Aim: formula for fully local, fully analytic subtraction of IR singularities in arbitrary production process.

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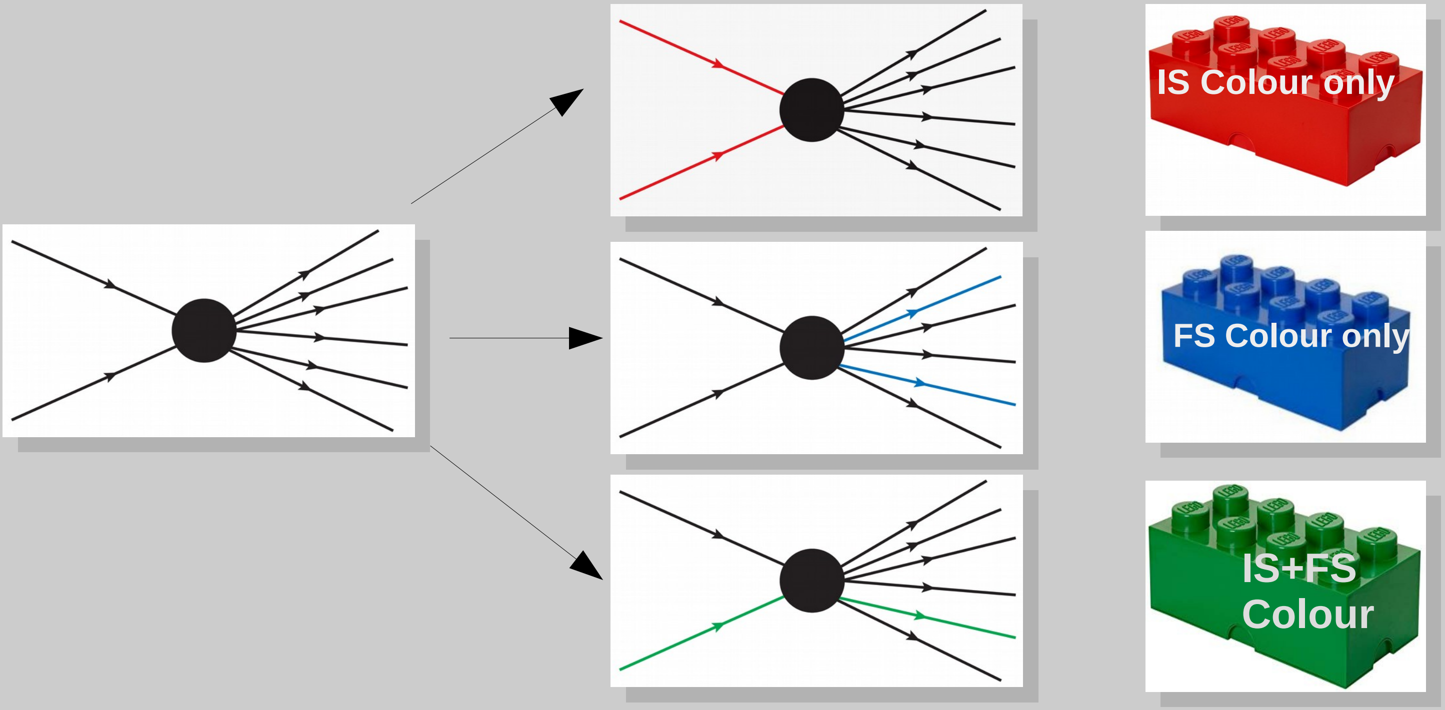

- Complete control on each block, then put them together.

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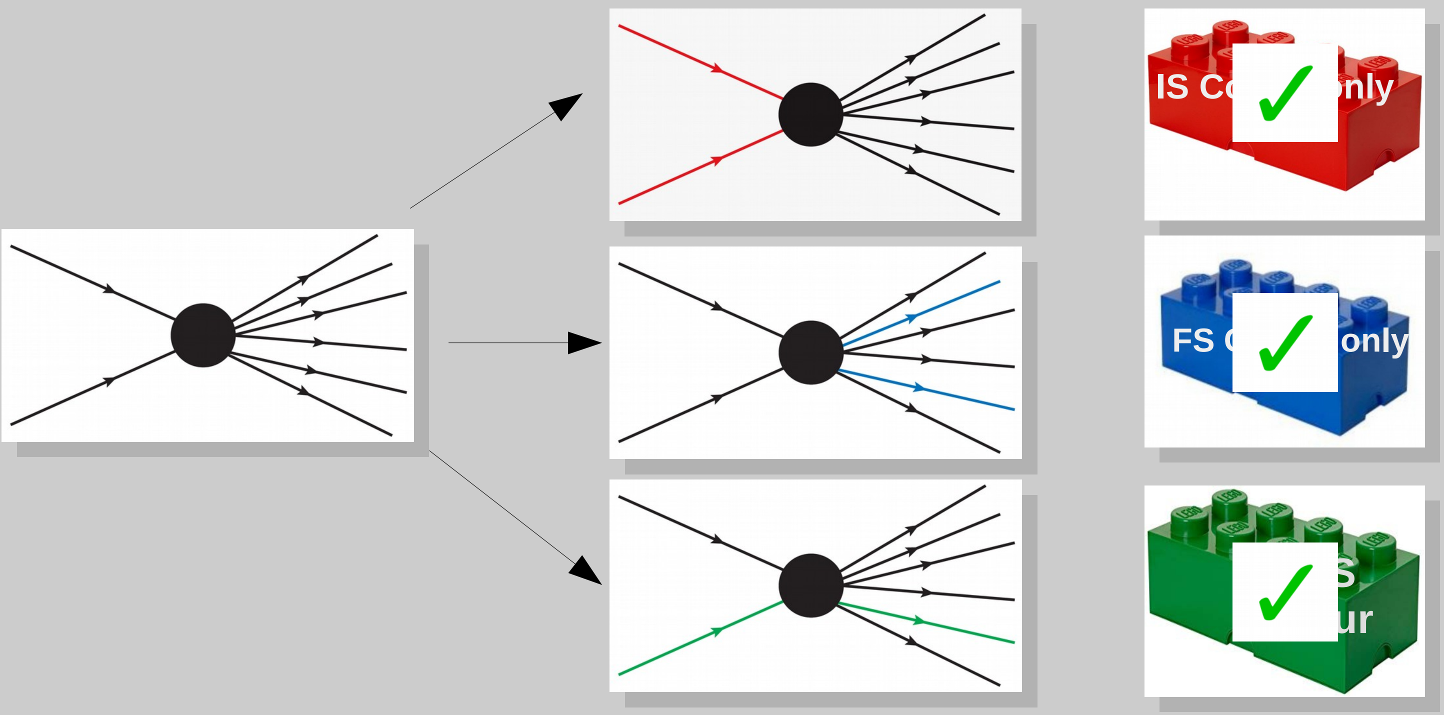


[Caola, Melnikov, R.R. hep-ph/1902.02081]

[Caola, Melnikov, R.R., hep-ph/1907.05398]

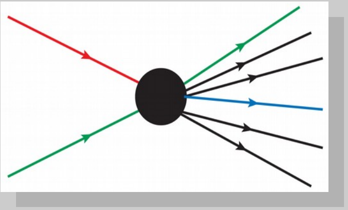
[Asteriadis, Caola, Melnikov, R.R.,

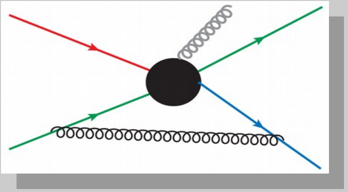
hep-ph/1910.13761]

See also [Caola, Delto, Frellesvig, Melnikov ‘18]; [Delto, Melnikov ‘19] – double-soft and triple-collinear counterterms.

Building Towards Generality

- These building blocks allow NNLO corrections to $\boxed{pp \to V + j}$ (*V*=any color singlet).
- Full generality (2 or more jets): colour-correlations.
- New structures in e.g. real-virtual limits:





$$f_{abc}\langle\mathcal{M}^{(0)}(\{p\})|T_k^a T_i^b T_j^c|\mathcal{M}^{(0)(\{p\}}\rangle$$

[Catani, Grazzini, '00]

- Colour-correlations in double-soft eikonal have a simpler structure: [Catani, Grazzini, '99]

$$\frac{1}{2}\sum_{i,j,k,l} S_{ij}(q_1)S_{kl}(q_2)|\langle\mathcal{M}(\{p\})|\{\vec{T}_i\cdot\vec{T}_j,\vec{T}_k\cdot\vec{T}_l\}|\mathcal{M}(\{p\})\rangle|^2$$
$$-C_A\sum_{i,j} S_{ij}(q_1,q_2)|\langle\mathcal{M}(\{p\})|\vec{T}_i\cdot\vec{T}_j|\mathcal{M}(\{p\})\rangle|^2$$

Double-soft counterterms almost all known:

- arbitrarily many massless hard partons; [Delto, Caola, Frellesvig, Melnikov '18]
- Back-to-back massive hard partons [Behring, Bizon '19]

Don't expect any insurmountable obstacles from colour-correlated limits!

Towards collider phenomenology

Brief recap:

- Constructed the nested soft-collinear subtraction scheme using color coherence + sector decomposition of radiatve phase space.
- Identify building blocks to generalize to arbitrary production processes at LHC:
  - Initial-initial;
  - Final-final;
  - Initial-final.

Absolute control on each block:

- < per mille agreement with analytic results.

Excellent numerical performance so far:

- Initial-initial processes (Drell-Yan, Higgs production): per mille accuracy on NNLO cross section in ~ 1 CPU hour.
- By comparison: `MATRIX` takes ~ 20 CPU days [Grazzini, Kallweit, Wiesemann, '18]

Towards collider phenomenology

Used these results for collider phenomenology:

- Mixed scalar-pseudoscalar Higgs to NNLO (initial-initial)

  [Jaquier, RR, '19]

- Mixed QCD-EW in Drell-Yan (abelianized initial-initial)

  [Delto, Jaquier, Melnikov, RR, '19]

- $WH(\to b\bar{b})$ to NNLO (initial-initial + final-final)

  [Caola, Luisoni, Melnikov, RR, '17]

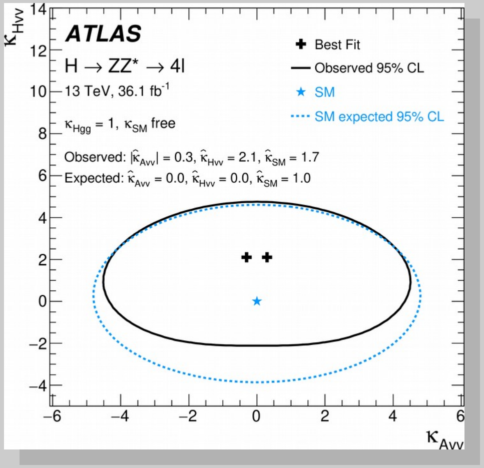
Future plans:

- Initial-final: DIS $\to$ VBF;
- Heavy flavor production;
- Excellent numerical performance ⇨ $2 \to 3$ processes;
- N3LO using slicing or Projection-to-Born.

Mixed Scalar-Pseudoscalar Higgs

Fundamental property of the Higgs: parity.

Pseudoscalar state ruled out, but may be admixture of scalar and pseudoscalar.



Framework: Higgs Characterization model: [Artoisenet *et al.* '13]

- Effective Lagrangian for spin-0 particle coupling to gluons and (neutral) EW vector bosons.

$$
\begin{aligned}
\mathcal{L}_{\rm eff} \supset \Bigg\{ & c_\alpha \kappa_{\rm SM} \frac{1}{2} g_{HZZ} Z_\mu Z^\mu - \frac{1}{4}\left[c_\alpha \kappa_{H\gamma\gamma} g_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{A\gamma\gamma} g_{A\gamma\gamma} A_{\mu\nu} \tilde{A}^{\mu\nu}\right] \\
& - \frac{1}{2}\left[c_\alpha \kappa_{HZ\gamma} g_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{AZ\gamma} g_{AZ\gamma} Z_{\mu\nu} \tilde{A}^{\mu\nu}\right] - \frac{1}{4}\left[c_\alpha \kappa_{Hgg} g_{Hgg} G_{\mu\nu} G^{\mu\nu} + s_\alpha \kappa_{Agg} g_{Agg} G_{\mu\nu} \tilde{G}^{\mu\nu}\right] \\
& - \frac{1}{4}\frac{1}{\Lambda}\left[c_\alpha \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{AZZ} Z_{\mu\nu} \tilde{Z}^{\mu\nu}\right] - \frac{1}{\Lambda} c_\alpha \left[\kappa_{H\partial\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\partial Z} Z_\nu \partial_\mu Z^{\mu\nu}\right] \Bigg\} H.
\end{aligned}
$$

Parity of Higgs controlled by $\boxed{c_\alpha = \cos(\alpha); s_\alpha = \sin(\alpha)}$

Mixed Scalar-Pseudoscalar Higgs

- Theory predictions:
  - Differential, include mixing but LO or NLO;

    [Gao *et al.* ‘10; Artoisenet *et al.* ‘13; Maltoni, Mawatari, Zaro ‘14]
  - NNLO but inclusive and no mixing.

    [Harlander, Kilgore ‘02, Anastasiou, Melnikov ‘03]
- Amplitudes required for NNLO known.

  [Ravindran, Smith, van Neerven ‘05; Ahmed *et al.* ‘15; Berger, Del Duca, Dixon ‘07; Badger, Glover ‘06; Glover, Mastrolia, Williams ‘08; Dixon, Glover, Khoze ‘04; Dixon, Sofianatos ‘09]
- Scalar-pseudoscalar interference at NNLO?
- Impact of NNLO corrections on distributions?

Mixed Scalar-Pseudoscalar Higgs

[Jacquier, R.R., '19]

- Inclusive results (undecayed Higgs) ⟶ No scalar-pseudoscalar interference in inclusive cross section at NNLO.

	σ^{LO} [pb]	σ^{NLO} [pb]	σ^{NNLO} [pb]
Pure scalar: $\cos(\alpha) = 1$	$15.13^{-14\%}_{+16\%}$	$34.81^{-14\%}_{+20\%}$	$43.85^{-9\%}_{+9\%}$
Pure pseudoscalar: $\cos(\alpha) = 0$	$34.04^{-14\%}_{+16\%}$	$79.01^{-15\%}_{+20\%}$	$99.46^{-9\%}_{+9\%}$
Equal admixture: $\cos(\alpha) = \sqrt{1/2}$	$24.59^{-14\%}_{+16\%}$	$56.91^{-15\%}_{+20\%}$	$71.66^{-9\%}_{+9\%}$

Average of these two

- With decay $H \to ZZ^* \to e^- e^+ \mu^- \mu^+$ and cuts on leptons

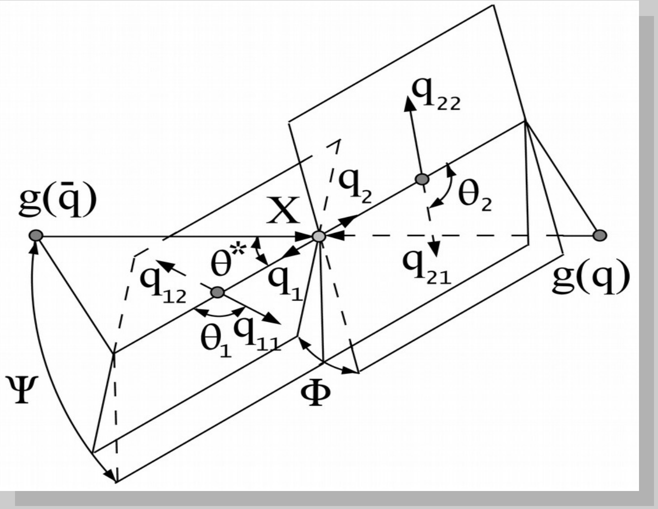
	σ^{LO} [ab]	σ^{NLO} [ab]	σ^{NNLO} [ab]
$\cos(\alpha) = 1$	$10.6^{-14\%}_{+15\%}$	$23.5^{-14\%}_{+19\%}$	$29.1^{-8\%}_{+8\%}$
$\cos(\alpha) = 0$	$0.0151^{-14\%}_{+15\%}$	$0.0344^{-14\%}_{+19\%}$	$0.0428^{-8\%}_{+8\%}$
$\cos(\alpha) = \sqrt{1/2}$	$8.61^{-14\%}_{+15\%}$	$19.2^{-14\%}_{+19\%}$	$23.7^{-8\%}_{+8\%}$
$\cos(\alpha) = 0.6$	$9.95^{-14\%}_{+15\%}$	$22.4^{-14\%}_{+19\%}$	$27.7^{+8\%}_{-8\%}$

NOT average of these two

- Pseudoscalar decays are suppressed.
- Scalar-pseudoscalar interference between production and decay.
- Scale uncertainties & NNLO corrections largely independent of scalar-pseudoscalar mixing.

Mixed Scalar-Pseudoscalar Higgs

Angular distribution sensitive to the parity of the Higgs:



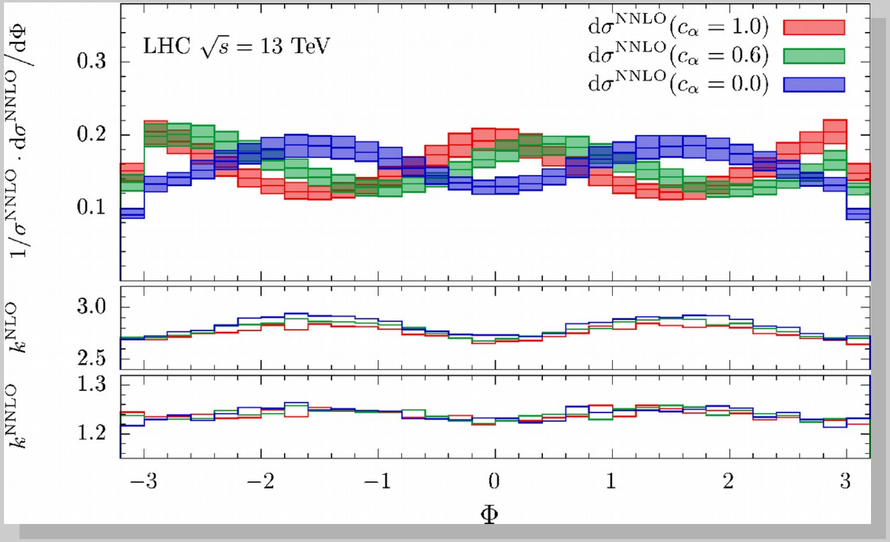
$$\Phi = \frac{\vec{q}_1 \cdot (\hat{n}_1 \times \hat{n}_2)}{|\vec{q}_1 \cdot (\hat{n}_1 \times \hat{n}_2)\,|} \times \arccos\left(-\hat{n}_1 \cdot \hat{n}_2\right)$$

$\vec{q}_1$: Three-momentum of Z_1 in Higgs rest frame.

$\vec{q}_{i1(2)}$: Three-momentum of lepton(antilepton) from decay of Z_1 in Higgs rest frame.

$$\hat{n}_i = \frac{\vec{q}_{i1} \times \vec{q}_{i2}}{|\vec{q}_{i1} \times \vec{q}_{i2}|}$$

[Gao *et al.* '10]



- NLO corrections have mild dependence on observable (due to cuts).
- NNLO corrections flat.
- Corrections largely independent of scalar-pseudoscalar mixing.

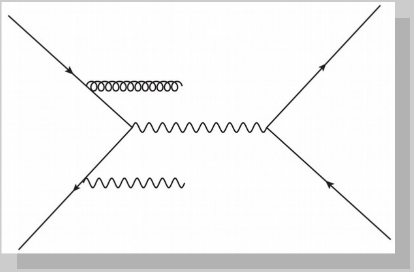
Mixed QCD-EW corrections in DY

- Since $\alpha \sim \alpha_s^2 \sim 0.01$, usually NNLO QCD and NLO EW is sufficient for percent precision.
- Need mixed QCD-EW corrections for:
  - Ultra-high precision observables;
  - High energy processes.

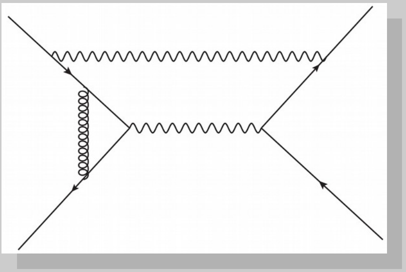
    [Alioli *et al.*, '16]
  - E.g. W-mass measurements: target accuracy is 10 MeV; QCD-EW corrections lead to shifts ~ 10 MeV

    [Dittmaier, Huss, Schwinn '15; Carloni Calame *et al.*, '17]

- Challenges:



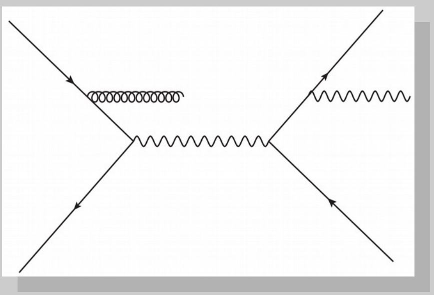
Simultaneous treatment of IR QCD and QED divergences.



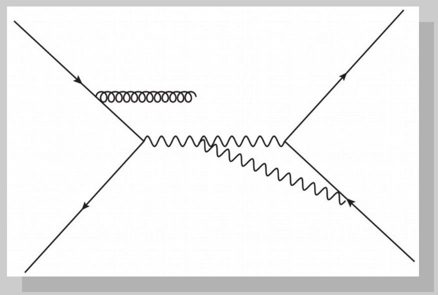
Massive two-loop amplitudes

Mixed QCD-EW corrections

Pole approximation: Systematic means of factorizing corrections in production and decay in resonance region. [Dittmaier, Huss, Schwinn, ‘14]



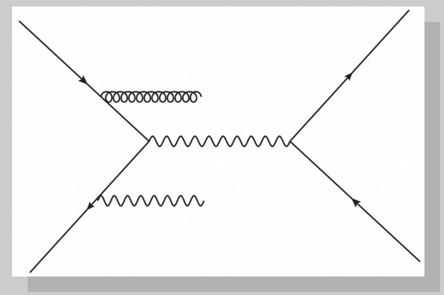
Prod. x Decay



Charged-current

- Dominant contributions.
- Differential results:

  [Dittmaier, Huss, Schwinn, ‘15]

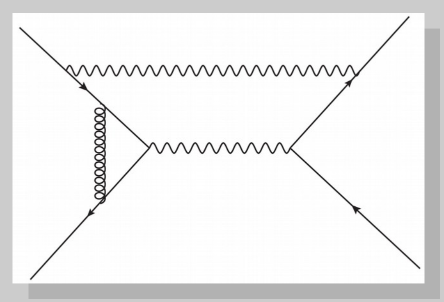


Prod. x Prod.

- Argued to be subleading

  [Dittmaier, Huss, Schwinn, ‘14].
- Inclusive results:

  [De Florian, Der, Fabre, ‘18] – comparable to NNLO QCD.

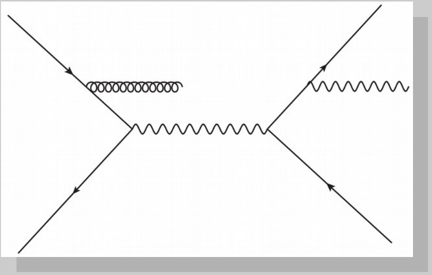


Non-factorizable

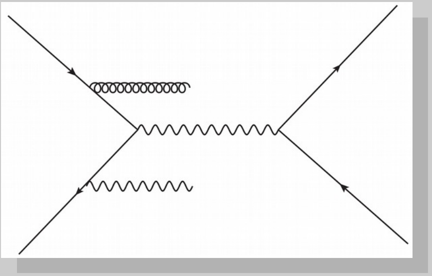
- Negligible. [Dittmaier, Huss, Schwinn, ‘14]

Mixed QCD-EW corrections

- Goal: Assess impact of QCD-EW correction on distributions in charged-current (CC) and neutral-current (NC) DY production near resonance.
  - ✔ Mixed factorizable **QCD-QED** corrections to **NC** DY.



Prod. x Decay ~ NLO x NLO.



Prod. x Prod: Abelianize NNLO QCD corrections

Inclusive: [De Florian, Der, Fabre, ‘18]

Differential: [Delto, Jaquier, Melnikov, RR, ‘19]

  - ➢ Mixed factorizable **QCD-EW** corrections to **NC** DY (work in progress).
    - ◆ Only need QCD-EW amplitudes, cf. [Kilgore, Sturm ‘11; Bonciani *et al.* ‘17, Ajjath *et al.* ‘19]
  - ➢ Mixed factorizable **QCD-EW** correction to **CC** DY (work in progress).
    - ◆ Charged-current corrections.

Mixed QCD-EW corrections

Onshell Z-boson decaying to leptons:

Expand

$$\mathrm{d}\sigma_{pp\to Z\to\ell^+\ell^-} = \frac{\mathrm{d}\sigma_{pp\to Z}\mathrm{d}\Gamma_{Z\to\ell^+\ell^-}}{\Gamma_Z} = \mathrm{Br}_{Z\to\ell^+\ell^-} \times \mathrm{d}\sigma_{pp\to Z} \times \frac{\mathrm{d}\Gamma_{Z\to\ell^+\ell^-}}{\Gamma_{Z\to\ell^+\ell^-}}$$

$$\mathrm{d}\sigma_{\mathrm{LO}} = \mathrm{Br}_{Z\to\ell^+\ell^-} \times \mathrm{d}\sigma^{(0,0)}_{pp\to Z} \times \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}} \,,$$

$$\mathrm{d}\sigma^{(\alpha_s)}_{\mathrm{NLO}} = \mathrm{Br}_{Z\to\ell^+\ell^-} \times \mathrm{d}\sigma^{(1,0)}_{pp\to Z} \times \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}}$$

,

$$\mathrm{d}\sigma^{(\alpha)}_{\mathrm{NLO}} = \mathrm{Br}_{Z\to\ell^+\ell^-} \times \left[\mathrm{d}\sigma^{(0,1)}_{pp\to Z} \times \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}} + \mathrm{d}\sigma^{(0,0)}_{pp\to Z} \times \left(\frac{\mathrm{d}\Gamma^{(1)}}{\Gamma^{(0)}} - \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}} \times \frac{3\alpha}{4\pi}\right)\right] \,,$$

$$\mathrm{d}\sigma^{(\alpha_s^2)}_{\mathrm{NNLO}} = \mathrm{Br}_{Z\to\ell^+\ell^-} \times \mathrm{d}\sigma^{(2,0)}_{pp\to Z} \times \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}} \,,$$

$$\mathrm{d}\sigma^{(\alpha_s\alpha)}_{\mathrm{NNLO}} = \mathrm{Br}_{Z\to\ell^+\ell^-} \times \left[\mathrm{d}\sigma^{(1,1)}_{pp\to Z} \times \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}} + \mathrm{d}\sigma^{(1,0)}_{pp\to Z} \times \left(\frac{\mathrm{d}\Gamma^{(1)}}{\Gamma^{(0)}} - \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}} \times \frac{3\alpha}{4\pi}\right)\ \right] .$$

NNLO QCD: $$\Delta_{\alpha_s^2} = \frac{\sigma^{(\alpha_s^2)}_{\mathrm{NNLO}}}{\sigma_{\mathrm{LO}} + \sigma^{(\alpha_s)}_{\mathrm{NLO}}}$$

QCD-EW: $$\Delta_{\alpha_s\alpha} = \frac{\sigma^{(\alpha_s\alpha)}_{\mathrm{NNLO}}}{\sigma_{\mathrm{LO}} + \sigma^{(\alpha_s)}_{\mathrm{NLO}}}$$

NLO EW: $$\Delta_{\alpha} = \frac{\sigma^{(\alpha)}_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}} + \sigma^{(\alpha_s)}_{\mathrm{NLO}}}$$

Mixed QCD-EW corrections

Fiducial results*

$$\Delta_{\alpha_s^2} = -(1.2 \cdot 10^{-2}) \ ,$$
$$\Delta_{\alpha} = (3.0 \cdot 10^{-3})_P - (7.2 \cdot 10^{-3})_D - (1.6 \cdot 10^{-3})_W \ ,$$
$$\Delta_{\alpha_s \alpha} = -(1.5 \cdot 10^{-4})_{P\otimes P} - (4.9 \cdot 10^{-3})_{P\otimes D} - (0.3 \cdot 10^{-3})_{P\otimes W} \ .$$

- NLO EW and QCD-EW corrections smaller than NNLO QCD corrections.
- QCD-EW Prod. x Decay relatively large compared to NLO EW – selection criteria affected by QCD radiation.
- Prod. x Prod corrections < per mille
  - Large cancellation among partonic channels.
- Photon-induced contributions ~ 15% of Prod. x Prod.

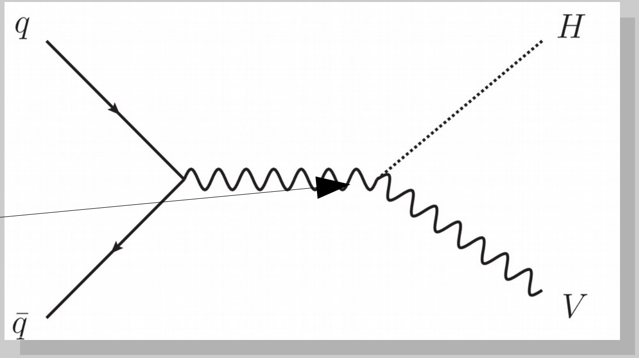
*Lepton definition following [Alioli *et al.* ‘16]:
combine photon and lepton if

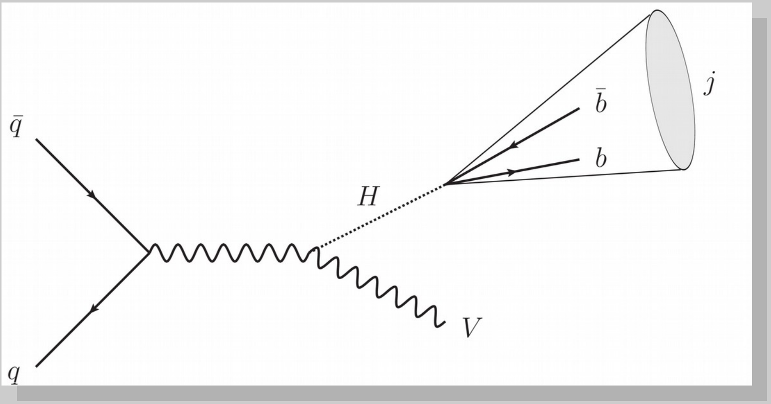
$$R_{\ell^\pm\gamma} = \sqrt{(y_{\ell^\pm} - y_\gamma)^2 + (\varphi_{\ell^\pm} - \varphi_\gamma)^2} < 0.1$$

$$p_{\perp,\ell_1} > 24 \text{ GeV} \ , \quad p_{\perp,\ell_2} > 16 \text{ GeV} \ , \quad |y_{\ell_i}| < 2.4$$
$$50 \text{ GeV} < m_{\ell\bar{\ell}} < 120 \text{ GeV}$$

Associated Production $WH(\rightarrow b\bar{b})$

- Associated production ("*Higgsstrahlung*") has 3rd and 4th highest production cross section.
- Direct access to *VVH* couplings.





- $H\rightarrow b\bar{b}$ gives direct access to Yukawa coupling of *b*-quark.
- **Most common decay mode** but difficult to measure (large QCD background).
- Can be measured in boosted Higgs kinematics – both *b*-quarks inside fat jet.

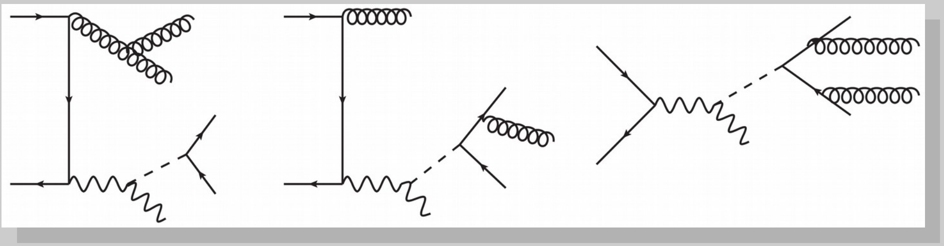
  [Butterworth, Davison, Rubin, Salam '08]
- Requires Higgs production with associated jet or EW vector boson.

Quite complicated final state (boost, jet properties, etc)

➡ need **reliable** theoretical predictions for signal and background.

Associated Production $WH(\to b\bar{b})$

- NNLO corrections in narrow-width approximation [Caola, Luisoni, Melnikov, R.R. '17]

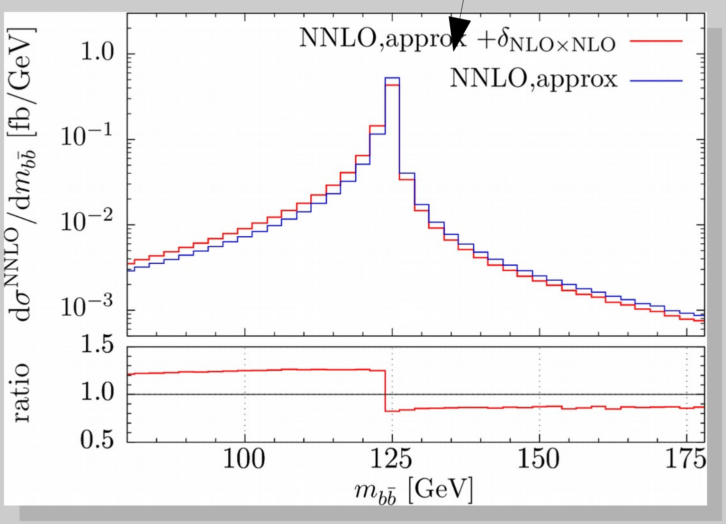


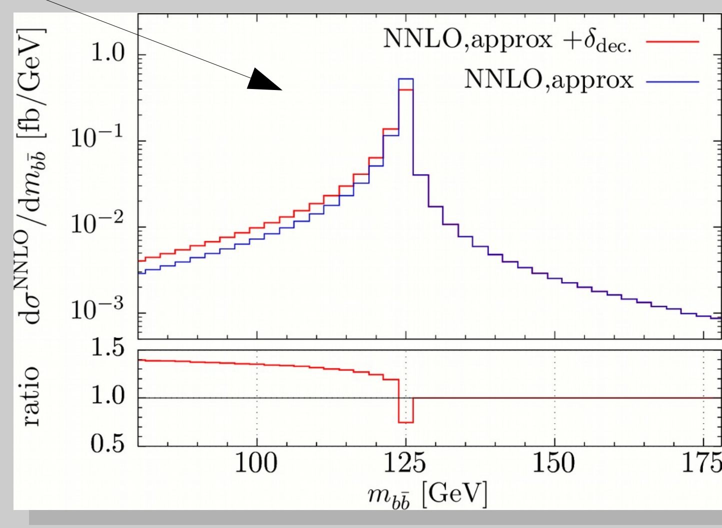
- Flavour-kT algorithm to define b-jets with massless bottom quarks.
  [Banfi, Salam, Zanderighi '06]

NNLO prod.
= NNLO approx.

NLO x NLO

NNLO decay

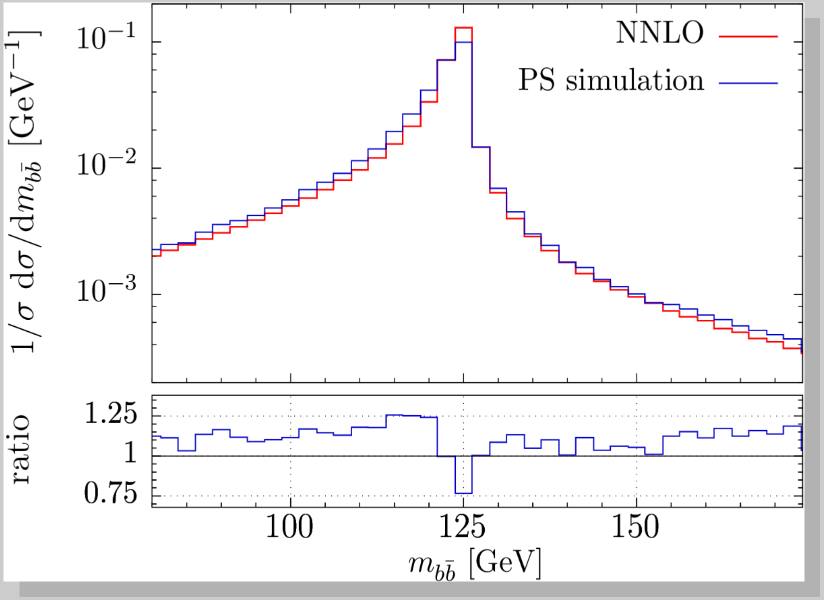




- Confirmed results of [Ferrera, Somogyi, Tramontano '17]
  (see also [cf. Astill, Bizon, Re, Zanderighi '18])

Associated Production $WH(\to b\bar{b})$

- Parton showers* capture NNLO effects to **~10%.**
- Discrepancy:
  - Higher logs captured by parton showers.
  - Hard effects captured by fixed order.
  - **Different jet algorithms:**
  - NNLO: **flavor-kT –** only option with massless *b*-quarks.
  - PS: **anti-kT** and MC truth – similar to experiments.



*`HWJ` generator from `POWHEG-Box` with `MiNLO`; $H\to b\bar{b}$ through `PYTHIA`.

[Luisoni, Nason, Oleari, Tramontano ‘13];

[Nason ‘04]; [Frixione, Nason, Oleari ‘07]; [Alioli, Nason, Oleari, Re ‘10];

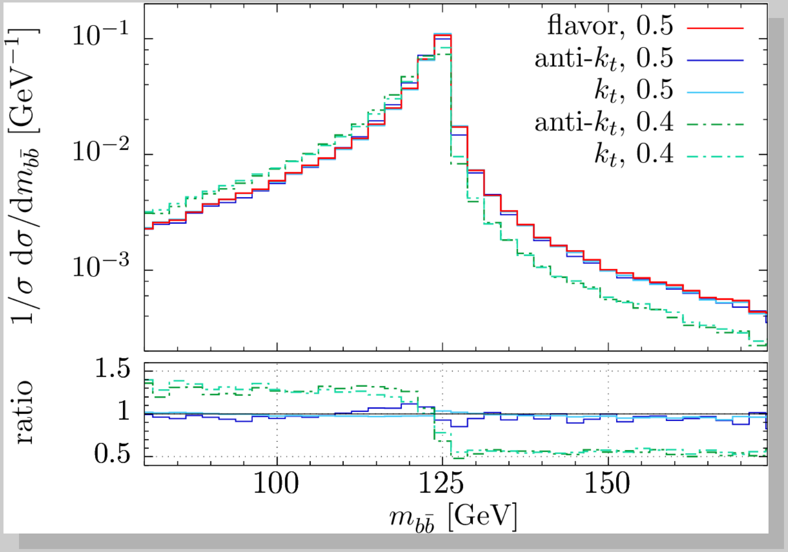
[Hamilton, Nason, Zanderighi ‘12]; [Hamilton, Nason, Oleari, Zanderighi ‘13]

[Sjostrand, Mrenna, Skands ‘08]

Associated Production $WH(\to b\bar{b})$

- Results insensitive to jet algorithm.
- Sensitive to jet radius.

  [cf. Astill, Bizon, Re, Zanderighi '18]



- Want NNLO calculation with jets **as done in experiments.**
- Using conventional jet algorithms will require calculation with massive b-quarks (work in progress).
- Recent $H\to b\bar{b}$ with massive b-quarks in NSCS scheme [Behring, Bizon '19]
  - Resolved a theoretical issue relating to the interference with $H\to gg$ decay.

Looking to the future

- Mixed QCD-EW corrections to DY.
- $WH(\to b\bar{b})$ with bottom quark masses.
- VBF @ NNLO.
- Mass effects in heavy flavour (bottom & charm quark) production:
  - EW boson + heavy flavour quark(s).
- $2 \to 3$ production at NNLO (once two-loop amplitudes available):
  - Trijet production (strong coupling, jet substructure);
  - $H+2j$ production (disentangle GF and VBF production modes);
  - $V+b\bar{b}$ production (background to associated production, understanding of heavy flavor production);
  - $t\bar{t}H$ production (direct measurement of top Yukawa).
  - ...
- N3LO:
  - DY;
  - Higgs;
  - VBF.
  - ...
- Match to PS?
- ...

CONCLUSION

- Demands of high precision programme require the utmost in theoretical advances.
- Despite successes of IR subtraction schemes, ultimate scheme yet to be developed.
- Proposed nested soft-collinear subtraction scheme:
  - ✔ Fully local, fully analytic, remarkably straightforward.
  - ✔ Building block for subtraction for arbitrary processes.
  - ✔ Huge phenomenological potential – enable precision to maximize discovery potential of the LHC.

THANK YOU FOR YOUR ATTENTION

BACKUP SLIDES

NESTED SOFT-COLLINEAR SUBTRACTION

FKS subtraction at NLO: Notation

Consider real corrections to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4) :$$

$$\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}g_4] F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4)\rangle.$$

$$F_{LM}(1,2,4) = \mathrm{dLips}_V \; |\mathcal{M}(1,2,4,V)|^2 \; \mathcal{F}_{\mathrm{kin}}(1,2,4,V)$$

$$[\mathrm{d}g_4] = \frac{\mathrm{d}^{d-1}p_4}{(2\pi)^d 2E_4}\theta(E_{\max} - E_4)$$

Lorentz-inv. Phase space for V **(incl. delta-fn)**

Matrix-element sq.

IR-safe observable

Integration in partonic CoM frame

Define soft and collinear operators:

$$S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A \qquad \rho_{ij} = 1 - \cos\theta_{ij}$$

FKS subtraction at NLO: Subtraction

Remove singular limits and add back as subtraction terms:

$$\begin{aligned}\langle F_{LM}(1,2,4)\rangle = &\langle (I - C_{41} - C_{42})(I - S_4) F_{LM}(1,2,4)\rangle + \\ &\langle S_4 F_{LM}(1,2,4)\rangle + \\ &\langle (C_{41} + C_{42})(I - S_4) F_{LM}(1,2,4)\rangle\end{aligned}$$

- First term: finite, can be integrated numerically in 4-dimensions.
- Second term: soft subtraction term – gluon decouples completely (need upper bound: E_{max}).
- Third term: collinear and soft+collinear subtraction terms – gluon decouples partially or completely.
- Singularities made explicit by integrating subtraction terms over phase space of unresolved gluon.

FKS subtraction at NLO: finite result

- Combining with virtual corrections and pdf renormalization → cancel poles.
- Take $\epsilon \to 0$ limit to get finite remainder – NLO correction:

$$2s \cdot \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \left\langle F_{LV}^{\mathrm{fin}}(1,2) + \frac{\alpha_s(\mu)}{2\pi}\left[\frac{2}{3}\pi^2 C_F - 2\gamma_q \log\left(\frac{\mu^2}{s}\right)\right] F_{LM}(1,2) \right\rangle$$
$$- \frac{\alpha_s(\mu)}{2\pi} \int_0^1 \mathrm{d}z \left[\hat{P}_{qq,R}^{(0)}(z) \ln\left(\frac{\mu^2}{s}\right) + \mathcal{P}'_{qq}(z)\right] \left\langle \frac{F_{LM}(z\cdot 1,2)}{z} + \frac{F_{LM}(1,z\cdot 2)}{z} \right\rangle$$
$$+ \langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4) \rangle.$$

$$\hat{O}_{\mathrm{NLO}} = (I - C_{41} - C_{42})(I - S_4)$$

$$\hat{P}_{qq,R}^{(0)} = C_F\left(2D_0(z) - (1+z)\right)$$ (AP splitting function without delta function)

$$\mathcal{P}'_{qq}(z) = -C_F\left[-4D_1(z) - (1-z) + 2(1+z)\log(1-z)\right]$$

$$\gamma_q = 3/2 C_F$$

FKS subtraction at NLO: finite result

- Combining with virtual corrections and pdf renormalization → cancel poles.
- Take $\epsilon \to 0$ limit to get finite remainder – NLO correction:

$$2s \cdot \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \left\langle F_{LV}^{\mathrm{fin}}(1,2) + \frac{\alpha_s(\mu)}{2\pi}\left[\frac{2}{3}\pi^2 C_F - 2\gamma_q \log\left(\frac{\mu^2}{s}\right)\right] F_{LM}(1,2)\right\rangle$$

$$- \frac{\alpha_s(\mu)}{2\pi}\int_0^1 \mathrm{d}z \left[\hat{P}_{qq,R}^{(0)}(z)\ln\left(\frac{\mu^2}{s}\right) + \mathcal{P}'_{qq}(z)\right]\left\langle \frac{F_{LM}(z\cdot 1,2)}{z} + \frac{F_{LM}(1,z\cdot 2)}{z}\right\rangle$$

$$+ \left\langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4)\right\rangle.$$

Sum of:

- **LO-like terms,** with or without convolutions with splitting functions.
- Real emission term, with singular configurations removed by iterated subtraction.
- Finite remainder of virtual corrections.

Real-real subtractions at NNLO

Aim to replicate NLO results as much as possible at NNLO.

Consider real-real correction to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4) + g(p_5):$$

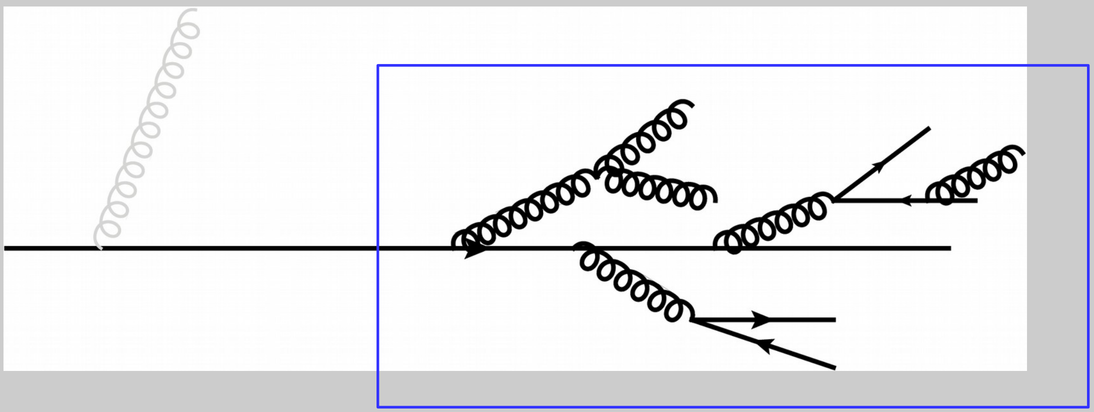
$$\mathrm{d}\sigma^{\mathrm{RR}} = \frac{1}{2s}\int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5)$$

IR singularities from

- g_4 and/or g_5 → soft.
- g_4 or g_5 → collinear to initial state partons.
- g_4 or g_5 → collinear to each other.
- g_4 and g_5 collinear to same initial state parton (triple collinear limit).

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Used in resummation & parton showers; not manifest in subtractions.
- Soft gluon cannot resolve details of collinear splittings; only sensitive to total color charge.



➡ No overlap between soft and collinear limits -- can be treated independently:

- Regularize soft singularities first, then collinear singularities.
- Energies and angles decouple.

Treatment of real-real singularities

- **Step 1:** Limit operators.
  - Recall $S_i A = \lim\limits_{E_i \to 0} A$ $C_{ij} A = \lim\limits_{\rho_{ij} \to 0} A.$ $(\rho_{ij} = 1 - \cos\theta_{ij})$
  - NNLO-like:

$$\mathbb{S} A = \lim_{E_4, E_5 \to 0} A, \text{ at fixed } E_5/E_4,$$
$$\mathbb{C}_i A = \lim_{\rho_{4i}, \rho_{5i} \to 0} A, \text{ with non vanishing } \rho_{4i}/\rho_{5i}, \rho_{45}/\rho_{4i}, \rho_{45}/\rho_{5i}.$$

- **Step 2:** Order gluon energies $E_4 > E_5.$

$$2\text{ s} \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5]\theta(E_4 - E_5) F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4,5) \rangle.$$

  - Gluon energies bounded by $E_{\max}.$
  - Energies defined in CoM frame.
  - Soft singularities: either double soft or g_5 soft.

Soft singularities

- **Step 3:** Regulate the *soft* singularities:

$$\left\langle F_{LM}(1,2,4,5)\right\rangle = \left\langle S\!\!\!S F_{LM}(1,2,4,5)\right\rangle + \left\langle S_5(I - S\!\!\!S)F_{LM}(1,2,4,5)\right\rangle$$
$$+ \left\langle (I - S_5)(I - S\!\!\!S)F_{LM}(1,2,4,5)\right\rangle.$$

- First term: both g_4 and g_5 soft.
- Second term: g_5 soft, soft singularities in g_4 are regulated.
- Third term: regulated against all soft singularities,
- All three terms contain **(potentially overlapping)** collinear singularities.

Phase-space partitioning

- **Step 4:** Introduce **phase-space partitions**

$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

with

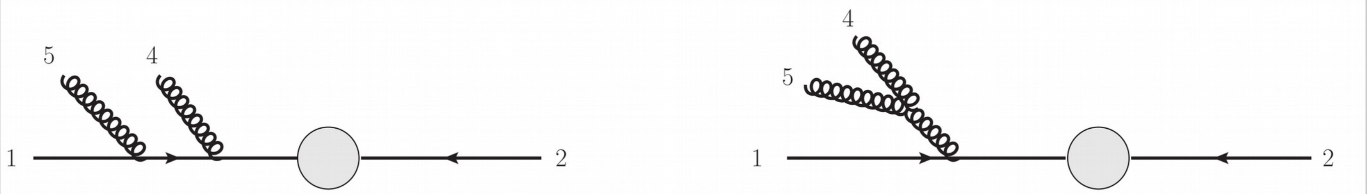
$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0$$
$$C_{41} w^{24,25} = C_{51} w^{24,25} = 0$$

➡

$w^{14,15}$ contains $C_{41},\ C_{51},\ C_{45}$

$w^{24,25}$ contains $C_{42},\ C_{52},\ C_{45}$

Triple collinear partition



and

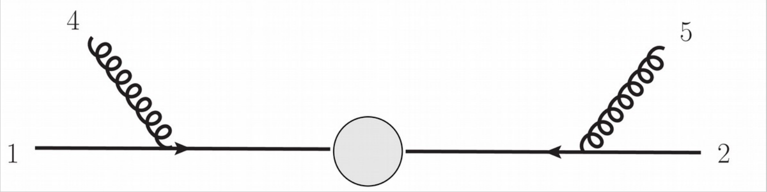
$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0$$
$$C_{41} w^{15,24} = C_{52} w^{15,24} = C_{45} w^{15,24} = 0$$

➡

$w^{14,25}$ contains $C_{41},\ C_{52}$

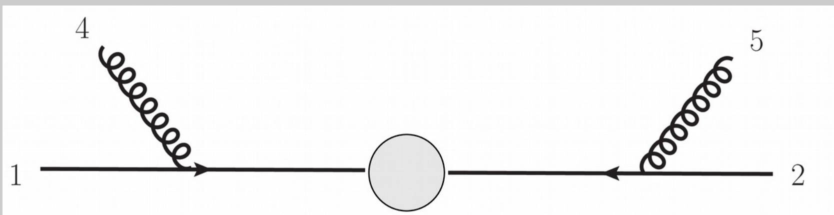
$w^{15,24}$ contains $C_{42},\ C_{51}$

Double collinear partition



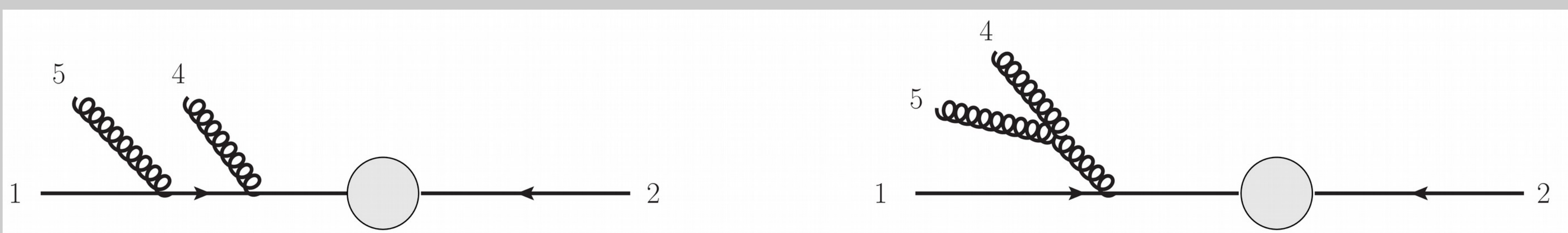
Phase-space partitioning

- Double collinear partition – large rapidity difference.



~ NLO x NLO ➡ simple

- Triple collinear partition – large/small rapidity difference.



Overlapping singularities remain – need one last step to separate these.

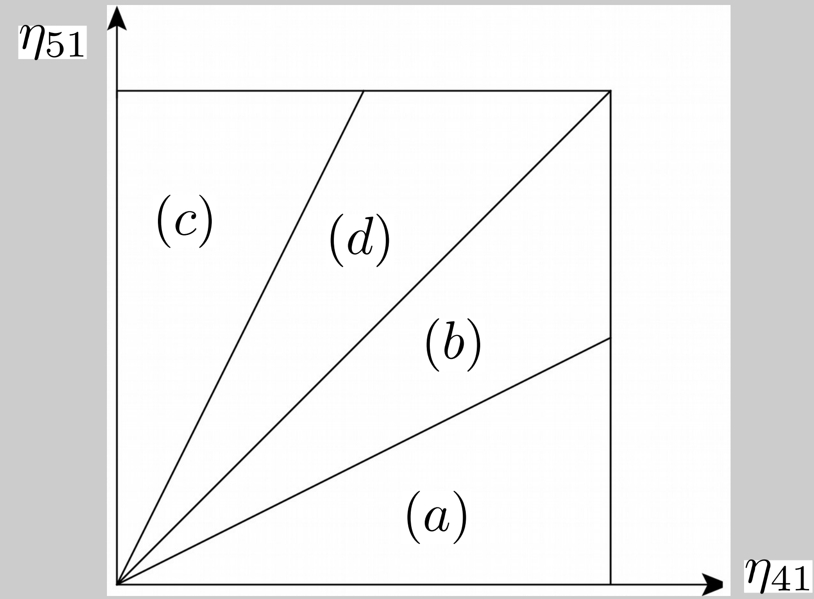
Sector Decomposition

- **Step 5:** **Sector decomposition:**

- Define angular ordering to separate singularities.

$\eta_{ij} = \rho_{ij}/2$

$$
\begin{aligned}
1 = &\, \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) \\
&+ \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \\
&\equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.
\end{aligned}
$$



- Thus the limits are

$$
\left.\begin{aligned}
\theta^{(a)} &: C_{51} \\
\theta^{(c)} &: C_{41}
\end{aligned}\right\} \text{Large rapidity difference}
$$

$$
\left.\begin{aligned}
\theta^{(b)} &: C_{45} \\
\theta^{(d)} &: C_{45}
\end{aligned}\right\} \text{Small rapidity difference}
$$

- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall energy ordering.
- Angular phase space parametrization [Czakon '10].

Removing collinear singularities

Then we can write soft-regulated term as

$$\Big\langle (I - S_5)(I - S\!\!\!S) F_{LM}(1,2,4,5) \Big\rangle = \Big\langle F_{LM}^{s_r c_s}(1,2,4,5) \Big\rangle + \Big\langle F_{LM}^{s_r c_t}(1,2,4,5) \Big\rangle + \Big\langle F_{LM}^{s_r c_r}(1,2,4,5) \Big\rangle,$$

$$\langle F_{LM}^{s_r c_r}(1,2,4,5) \rangle$$

- All singularities removed through nested subtractions – evaluated in 4-dimensions.
- Only term involving fully-resolved real-real matrix element.

$$\langle F_{LM}^{s_r c_{s,t}}(1,2,4,5) \rangle$$

- Contain (soft-regulated) single and triple collinear singularities.
- Matrix elements of lower multiplicity.
- Partitioning factors and sectors: one collinear singularity in each term.

Treating singular limits

We have four singular subtraction terms:

$\left\langle \mathbb{S}\!\!\!\!\!\;S F_{LM}(1,2,4,5) \right\rangle$ $\left\langle S_5 (I - \mathbb{S}\!\!\!\!\!\;S) F_{LM}(1,2,4,5) \right\rangle$ $\left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle$ $\left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$

We know how to treat them:

- Gluon(s) decouple partially or completely.
- Decouple completely:
  - Integrate over gluonic angles and energy.
- Decouple partially:
  - Integrate over gluonic angles.
  - Integral(s) over energy → integrals over splitting function in *z.*
- Analytic results for nontrivial integrals from double-soft and triple-collinear limits calculated in [Caola, Delto, Frellesvig, Melnikov ‘18; Delto, Melnikov ‘19].
- Significant analytic simplifications on <u>recombining sectors</u> after integration.

Treating singular limits

After integration: subtraction terms written as lower multiplicity terms:

- LO-like:

$$\langle F_{LM}(z\cdot 1,\bar{z}\cdot 2)\rangle\,,\ \langle F_{LM}(z\cdot 1,2)\rangle\,,\ \langle F_{LM}(1,z\cdot 2)\rangle\,,\ \langle F_{LM}(1,2)\rangle$$

(no final state partons).

- NLO-real-like (regulated by iterative subtraction):

$$\langle \mathcal{O}_{NLO}F_{LM}(z\cdot 1,2,4)\rangle\,,\ \langle \mathcal{O}_{NLO}F_{LM}(1,z\cdot 2,4)\rangle\,,\ \langle \mathcal{O}_{NLO}F_{LM}(1,2,4)\rangle$$

(maximum one final state parton).

convoluted with splitting functions with explicit singularities.

- Pole cancellation within each structure.

Finite remainders

- Relatively compact expressions for finite remainders for each lower-multiplicity structure.
- Familiar structures appear, e.g.

$$\mathrm{d}\sigma_{z1,2,4} = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 \mathrm{d}z \Bigg\{ \hat{P}^{(0)}_{qq,R}(z) \left\langle \log\frac{\rho_{41}}{4} \mathcal{O}_{\mathrm{NLO}} \left[\tilde{w}^{41,51}_{5||1} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \right] \right\rangle + \left[\mathcal{P}'_{qq}(z) - \hat{P}^{(0)}_{qq,R}(z) \log\left(\frac{\mu^2}{s}\right) \right] \mathcal{O}_{\mathrm{NLO}} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \Bigg\}$$

$$\mathrm{d}\sigma_{z1,\bar{z}2} = \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \int_0^1 \mathrm{d}z\mathrm{d}\bar{z} \left[\mathcal{P}'_{qq}(z) - \log\left(\frac{\mu^2}{s}\right) \hat{P}^{(0)}_{qq,R}(z) \right] \times \left[\mathcal{P}'_{qq}(\bar{z}) - \log\left(\frac{\mu^2}{s}\right)) \hat{P}^{(0)}_{qq,R}(z) \right] \frac{F_{LM}(z\cdot 1, \bar{z}\cdot 2)}{z\bar{z}}$$

- Same functions that appeared at NLO (as expected...)

Finite remainders

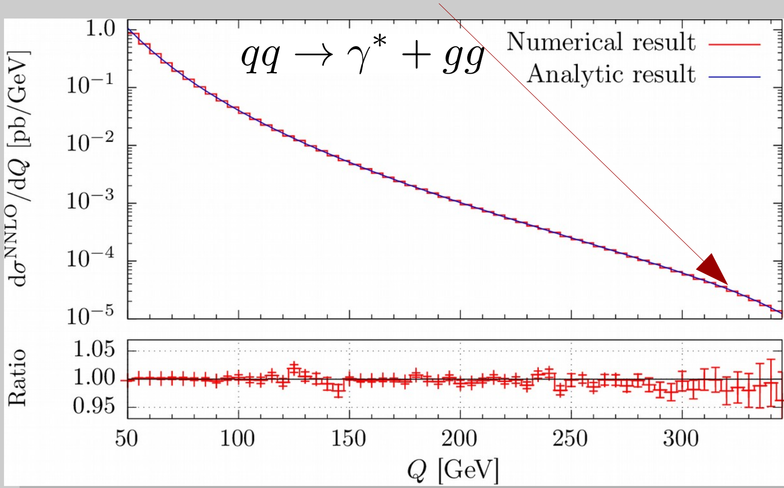
- New functions are relatively simple...
- Extension of NLO calculation to NNLO:
  - LO and NLO results convoluted with known functions.
  - Nested subtraction for real-real contribution.

$$
\begin{aligned}
&\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{F_{LM}(z\cdot 1,2)}(\mu^2 = s) = \\
&\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \int_0^1 \mathrm{d}z \Bigg\{ C_F^2 \Bigg[8\tilde{\mathcal{D}}_3(z) + 4\tilde{\mathcal{D}}_1(z)(1+\ln 2) + 4\tilde{\mathcal{D}}_0(z)\left[\frac{\pi^2}{3}\ln 2 + 4\zeta_3\right] \\
&+ \frac{5z-7}{2} + \frac{5-11z}{2}\ln z + (1-3z)\ln 2 \ln z + \ln(1-z)\left[\frac{3}{2}z - (5+11z)\ln z\right] \\
&\quad + 2(1-3z)\mathrm{Li}_2(1-z) \\
&+ (1-z)\bigg[\frac{4}{3}\pi^2 + \frac{7}{2}\ln^2 2 - 2\ln^2(1-z) + \ln 2\big[4\ln(1-z) - 6\big] + \ln^2 z \\
&\quad + \mathrm{Li}_2(1-z)\bigg] + (1+z)\bigg[-\frac{\pi^2}{3}\ln z - \frac{7}{4}\ln^2 2 \ln z - 2\ln 2 \ln(1-z)\ln z \\
&\quad + 4\ln^2(1-z)\ln z - \frac{\ln^3 z}{3} + \big[4\ln(1-z) - 2\ln 2\big]\mathrm{Li}_2(1-z)\bigg] \\
&\quad + \left[\frac{1+z^2}{1-z}\right]\ln(1-z)\big[3\mathrm{Li}_2(1-z) - 2\ln^2 z\big] - \frac{5-3z^2}{1-z}\mathrm{Li}_3(1-z) \\
&\quad + \frac{\ln z}{(1-z)}\left[12\ln(1-z) - \frac{3-5z^2}{2}\ln^2(1-z) - \frac{7+z^2}{2}\ln 2 \ln z\right]\Bigg] \\
&\quad + C_A C_F\bigg[-\frac{22}{3}\tilde{\mathcal{D}}_2(z) + \left(\frac{134}{9} - \frac{2}{3}\pi^2\right)\tilde{\mathcal{D}}_1(z) + \bigg[-\frac{802}{27} + \frac{11}{18}\pi^2 \\
&\quad + (2\pi^2 - 1)\frac{\ln 2}{3} + 11\ln^2 2 + 16\zeta_3\bigg]\tilde{\mathcal{D}}_0(z) + \frac{37-28z}{9} + \frac{1-4z}{3}\ln 2 \\
&\quad - \left(\frac{61}{9} + \frac{161}{18}z\right)\ln(1-z) + (1+z)\ln(1-z)\left[\frac{\pi^2}{3} - \frac{22}{3}\ln 2\right] \\
&\quad - (1-z)\left[\frac{\pi^2}{6} + \mathrm{Li}_2(1-z)\right] - \frac{2+11z^2}{3(1-z)}\ln 2 \ln z - \frac{1+z^2}{1-z}\mathrm{Li}_2(1-z) \times \\
&\quad \times \big[2\ln 2 + 3\ln(1-z)\big]\bigg] + R_+^{(c)}\mathcal{D}_0(z) + R^{(c)}(z)\Bigg\}\left\langle \frac{F_{LM}(z\cdot 1,2)}{z}\right\rangle.
\end{aligned}
$$

Validation of Results

- Exhaustively tested against analytic results for
  - ✔ Drell-Yan production [Hamberg, Matsuura, van Neerven ‘89]
  - ✔ Higgs production [Anastasiou, Melnikov ‘04]

- Good control in extreme kinematic regions.



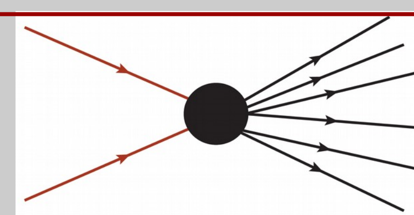
[Caola, Melnikov, R.R. ‘17]

- < per mille agreement for all NNLO *contributions*, including numerically tiny ones.

Channel	Color structures	Numerical result (nb)	Analytic result (nb)
$q_i\bar{q}_i \to gg$	–	8.351(1)	8.3516
$q_i\bar{q}_i \to q_j\bar{q}_j$	$C_F T_R n_{\rm up}$, $C_F T_R n_{\rm dn}$	-2.1378(5)	-2.1382
	$C_F(C_A - 2C_F)$	$-4.8048(3)\cdot 10^{-2}$	$-4.8048\cdot 10^{-2}$
	$C_F T_R$	$5.441(7)\cdot 10^{-2}$	$5.438\cdot 10^{-2}$
$q_i q_j \to q_i q_j$ $(i \neq -j)$	$C_F T_R$	0.4182(5)	0.4180
	$C_F(C_A - 2C_F)$	$-9.26(1)\cdot 10^{-4}$	$-9.26\cdot 10^{-4}$
$q_i g + g q_i$	–	-9.002(9)	-8.999
gg	–	1.0772(1)	1.0773

Table 1: Different contributions to the NNLO *coefficient* for on-shell Z production at the 13 TeV LHC with $\mu_R = \mu_F = 2m_Z$. All the color factors are included in the numerical results. The residual Monte-Carlo integration error is shown in brackets. See text for details.

[Caola, Melnikov, R.R. ‘19]

This building block  is reliable!

Validation of Results

Implies absolute control on physical results.

- Higgs production cross sections: per mille accuracy in ~ 1 CPU hr.

LHC. For this study, we set $\mu_R = \mu_F = m_H$. Running for less than an hour on a single core of a standard laptop, we obtain

$$\sigma_{\rm H}^{\rm LO} = 17.03(0)\ \text{pb}; \qquad \sigma_{\rm H}^{\rm NLO} = 30.25(1)\ \text{pb}; \qquad \sigma_{\rm H}^{\rm NNLO} = 39.96(2)\ \text{pb}. \qquad (5.1)$$

- Drell-Yan production *with symmetric cuts on final state leptons:* 2 per mille accuracy in ~1 CPU hr.

In this case, we use $\mu_R = \mu_F = m_Z$. Running on a single core of a standard laptop for about an hour, we obtain

$$\sigma_{\rm DY}^{\rm LO} = 650.4 \pm 0.1\ \text{pb}; \qquad \sigma_{\rm DY}^{\rm NLO} = 700.2 \pm 0.3\ \text{pb}; \qquad \sigma_{\rm DY}^{\rm NNLO} = 734.8 \pm 1.4\ \text{pb}. \qquad (5.3)$$

- By comparison

Process (${process_id})	LO runtime (relative uncertainty)	NLO runtime (relative uncertainty)	NNLO runtime (relative uncertainty)	NNLO runtime estimate for 10^{-3} uncertainty
$pp \to H$ (pph21)	0 d 0 h 2 min (1.5×10^{-4})	0 d 0 h 12 min (2.7×10^{-4})	35 d 23 h 23 min (7.2×10^{-4})	19 d
$pp \to e^- e^+$ (ppeex02)	0 d 0 h 48 min (1.0×10^{-4})	0 d 2 h 24 min (2.8×10^{-4})	173 d 20 h 36 min (3.6×10^{-4})	22 d

[Grazzini, Kallweit, Wiesemann, 2018]

SCALAR-PSEUDOSCALAR HIGGS

Renormalization of operator

- Two (bare) effective operators on integrating top loop out:

$$\mathcal{O}_1^B = G_a^{\mu\nu}\tilde{G}_{a,\mu\nu} = \epsilon_{\mu\nu\rho\sigma}G_a^{\mu\nu}G_a^{\rho\sigma}\ , \qquad \mathcal{O}_2^B = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)\ ,$$

NNLO only

- Mix under renormalization:

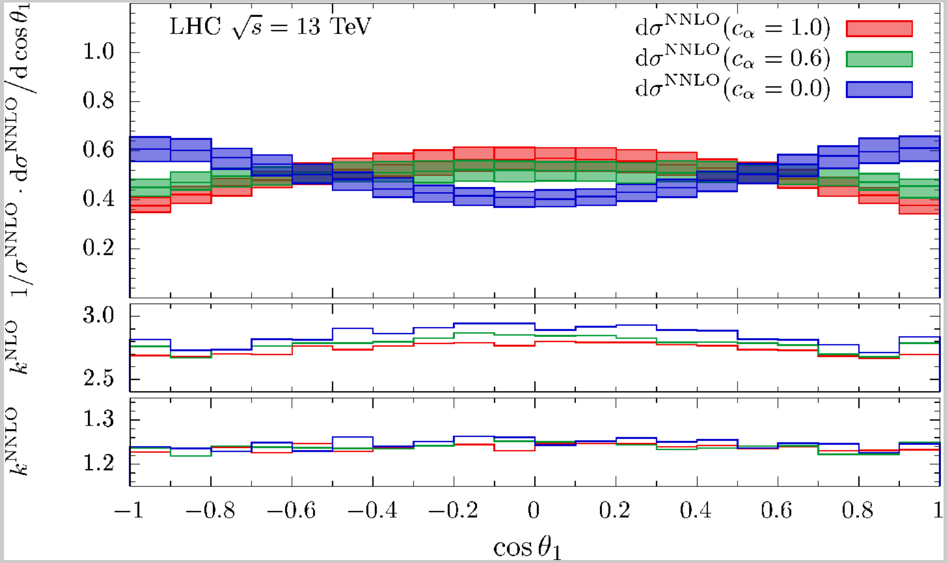
$$\mathcal{O}_i^R = \sum_{j=1}^{2} Z_{ij}\mathcal{O}_j^B; \qquad Z_{21} = 0$$

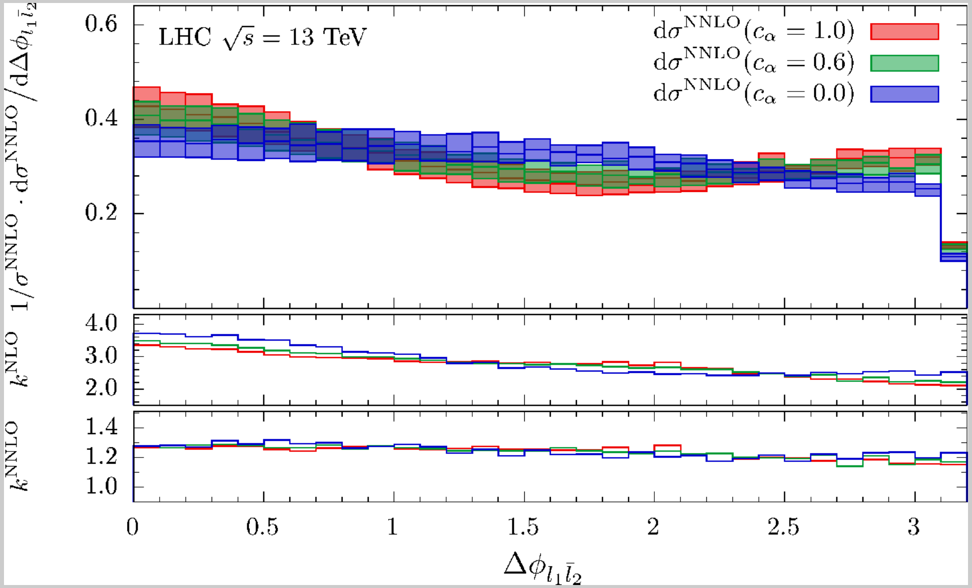
- No higher corrections to axial anomaly: $\mathcal{O}_2^R = \frac{\alpha_s}{\pi}\frac{n_f}{8}\mathcal{O}_1^R$

- Massless light quarks, only have contribution from $\mathcal{O}_1^B\mathcal{O}_2^B$

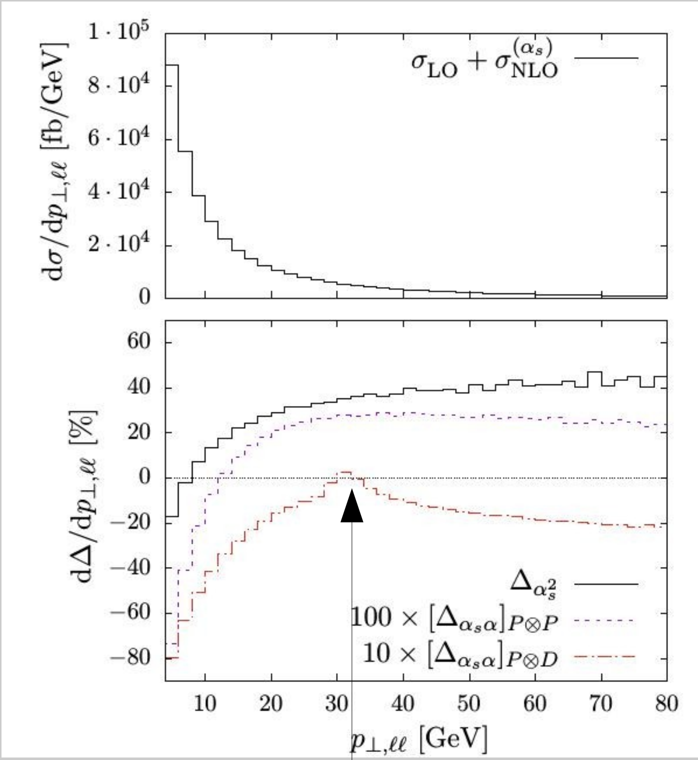
→ absorbed into $\mathcal{O}_2^R$

More distributions



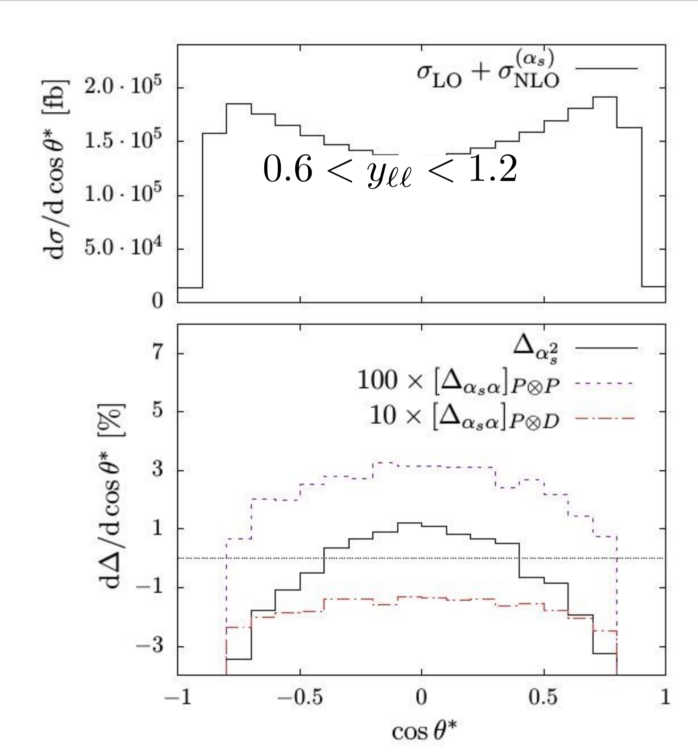


Mixed QCD-EW corrections



- Large NNLO QCD corrections (~NLO on Z+jet)
- Prod. x Prod ~ per mille
- Prod. x Decay ~ percent
  - Shape due to interplay of photon radiation from leptons, cuts and boost from QCD ISR

Prod. x Prod ~ Prod. x Decay here



- NNLO QCD ~ percent
- Prod. x Decay ~ per mille
- Prod. x Prod < per mille

$$WH(\to b\bar{b})$$

A Brief History of *VH* calculations

- **NLO QCD + EW:** [Han, Willenbrock ‘90]; [Baer, Bailey, Owens ‘93]; [Ohnemus, Stirling ‘93]; [Mrenna, Yuan ‘98]; [Spira ‘98]; [Djouadi, Spira ‘00]; [Ciccolini, Dittmaier, Kramer ‘03]; [Denner, Dittmaier, Kallweit, Muck ‘12].
- **NLO QCD + EW + PS:** [Frixione, Webber ‘05]; [Hamilton, Richardson, Tully ‘09]; [Granata, Lindert, Oleari, Pozzorini ‘17]; [Luisoni, Nason, Oleari, Tramontano ‘13].
- **NNLO cross section:** [Brein, Djouadi, Harlander ‘04] (after [Hamberg, v. Neerven, Vermaseren ‘91]); [Harlander, Kilgore ‘02]); [Brien, Harlander, Wiesemann, Zirke ‘12]; [VH@NNLO ‘13.]
- **NNLO differential:** [Ferrera, Grazzini, Tramontano ‘11].
- **NNLO + PS:** [Astill, Bizon, Re, Zanderighi ‘16].

A Brief History of *VH(→bb)* calculations

- **NNLO differential + H→bb (NLO, massless b-quarks):** [Ferrera, Grazzini, Tramontano ‘14].
- **NNLO differential + H→bb (NLO, massive b-quarks):** [Campbell, Ellis, Williams ‘16].
- **NNLO + PS:** [Astill, Bizon, Re, Zanderighi ‘18].
- **H → bb (NNLO, massless b-quarks):** [Del Duca, Duhr, Somogyi, Tramontano, Trócsányi ‘15].
- **NNLO differential + H→bb (NNLO, massless b-quarks)**: [Ferrera, Somogyi, Tramontano ‘17]; [Caola, Luisoni, Melnikov, R.R. ‘17].

Large QCD corrections observed!

[Ferrera, Somogyi, Tramontano 1705.10304]:

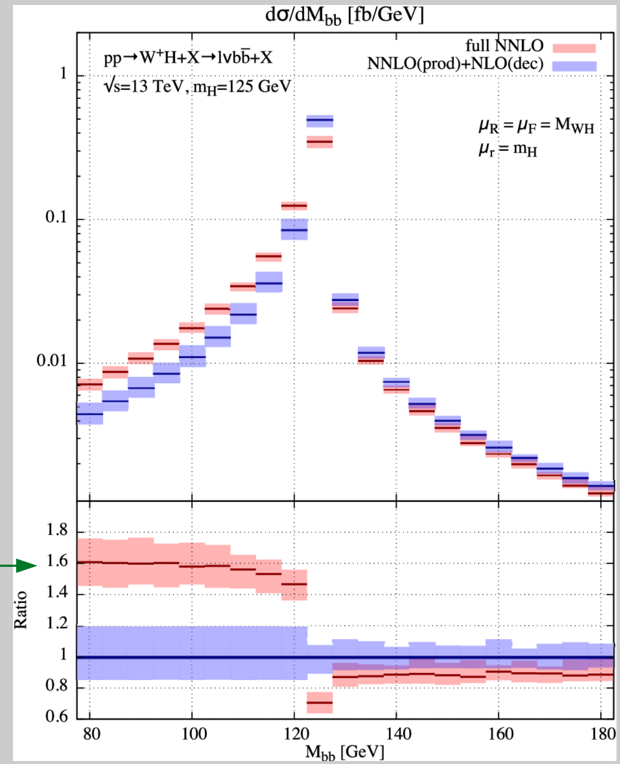
- Semi-boosted:

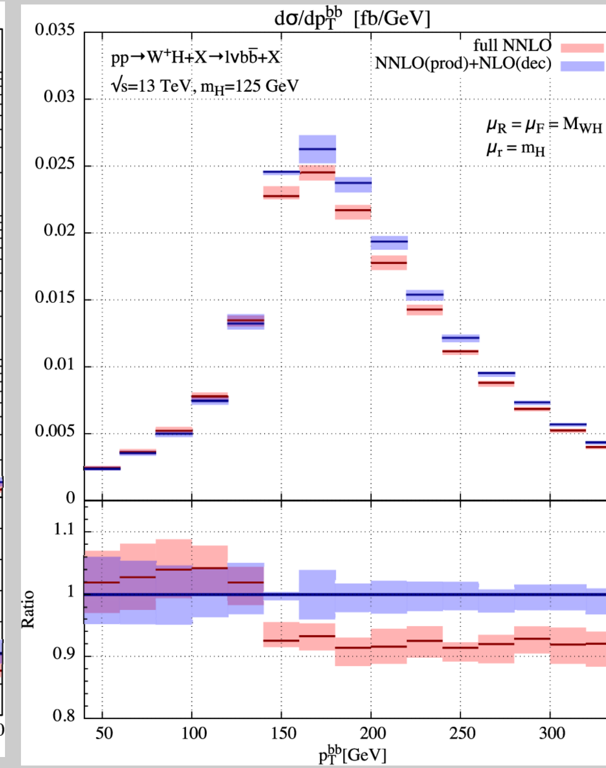
  $p_{T,W} > 150\ \mathrm{GeV}$

- ~ **7%** effect from corrections to *H→bb* decay on cross section.
- *b*-jets identified using flavor-kT algorithm

  [Banfi, Salam, Zanderighi '06]

- **Very large** (~60%) corrections in some regions.
- Strongly **phase-space dependent.**





QUESTIONS:

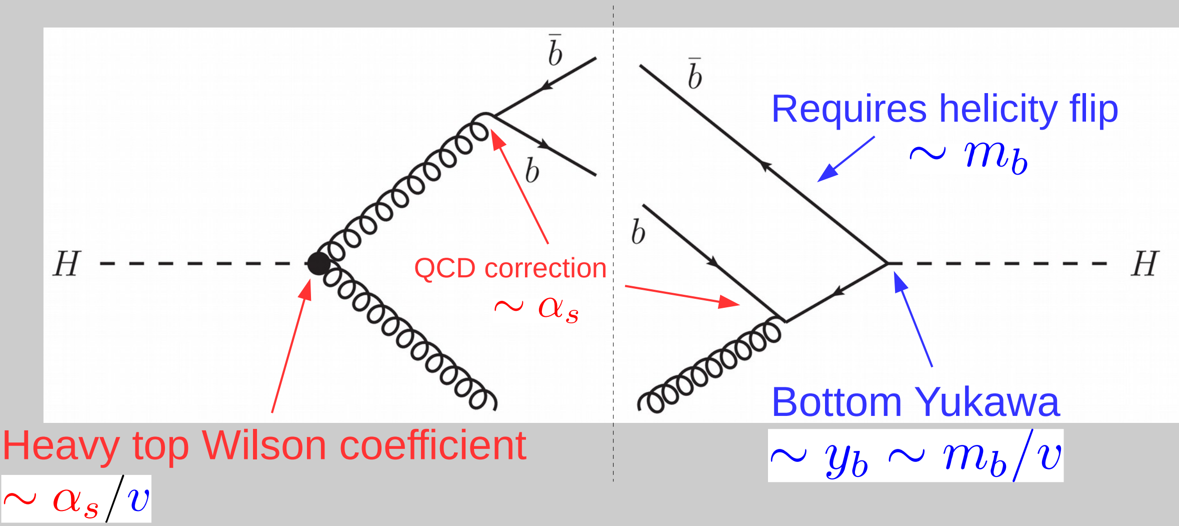
- Corrections due to NLO(prod) x NLO(dec) or LO(prod) x NNLO(dec)?
- Simulated using parton shower?

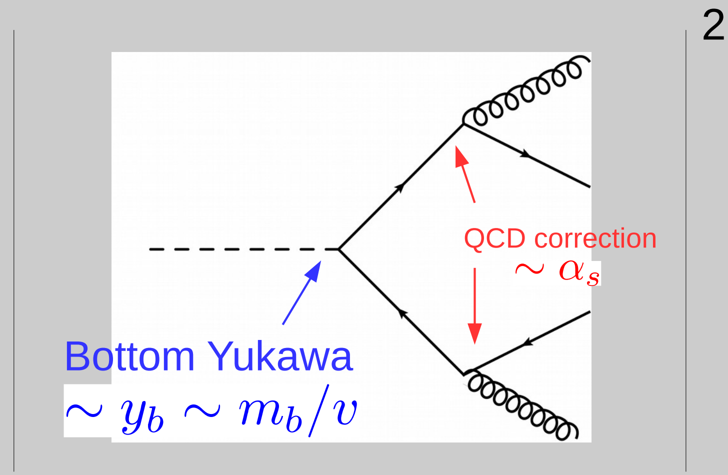
Bottom mass effects in $H \to bb$

- In $H \to bb$ decay, want massless b-quarks but non-zero y_b

$$m_b \ll m_H \Rightarrow \mathrm{d}\sigma \sim y_b^2 (A + B\, m_b^2/m_H^2 + \ldots) = A y_b^2$$

- Works at LO & NLO, but not at NNLO – **interference terms.**

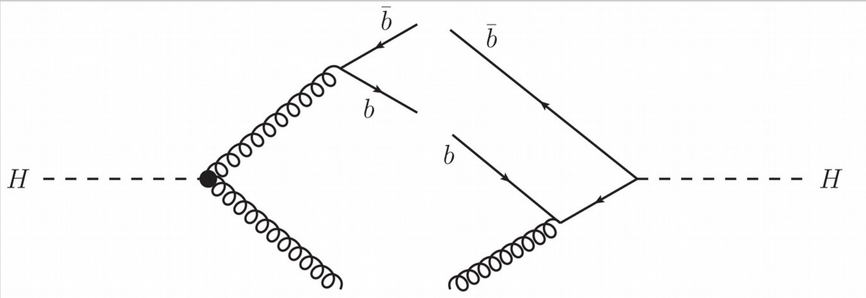




Top-loop interference contribution	“Regular” contribution squared
$\sim \alpha_s^2 m_b^2 / v^2 \sim \alpha_s^2 y_b^2$	$\sim \alpha_s^2 m_b^2 / v^2 \sim \alpha_s^2 y_b^2$

Interference contribution has **identical parametric scaling** to other NNLO corrections.

Bottom mass interference



Obvious strategy: factor out one power of m_b and then take $m_b = 0$

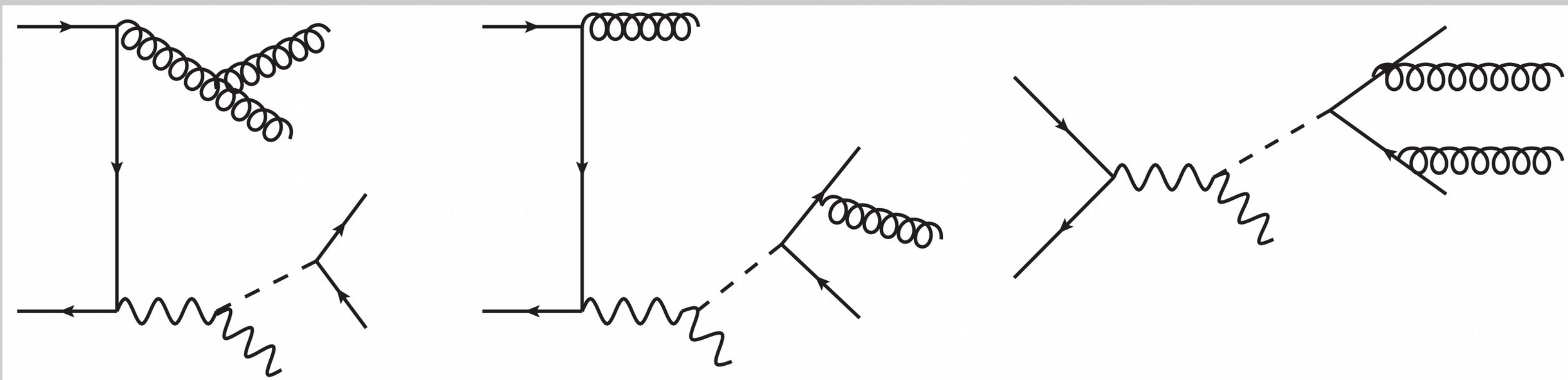
BUT:

- Reduced matrix elements have unusual IR behaviour: *subleading power singularities*, e.g. **soft singularities from quarks!**
- $\log(m_b/m_H)$ **don't cancel** between real and virtual interference terms – cannot take massless limit!
- **Cannot** be regulated using flavor-kT algorithm (doesn't regulate soft quark singularity).
- **Cannot** define an inclusive cross section for $H \to bb$ at NNLO with massless b-quarks.
- Calculation in double-log approx: **~ 30%** of NNLO corrections to $H \to bb$ decay.
  - Effect on kinematic distributions?
- Different dependence on bottom Yukawa – **different behavior in BSM models.**

NNLO calculation of $H \to bb$ to massive bottom quarks required.

Corrections to production and decay

α_s^2 corrections to *pp* → *WH(*→*bb):*



NNLO prod. x LO decay

= NNLO approx.

NLO prod. x NLO decay

LO prod. x NNLO decay

$$d\sigma_{WH(b\bar{b})} = d\sigma_{WH} \times \frac{d\Gamma_{bb}}{\Gamma_H} = \mathrm{Br}(H \to b\bar{b}) \times d\sigma_{WH} \times \frac{d\Gamma_{bb}}{\Gamma_{bb}}$$

Keep fixed

Expand in α_s

Corrections to production and decay

Define $\mathrm{d}\gamma^{(i)} = \frac{\sum_{j=0}^{i} \mathrm{d}\Gamma_{b\bar{b}}^{(j)}}{\sum_{j=0}^{i} \Gamma_{b\bar{b}}^{(j)}}$ so that over full decay phase space $\int \mathrm{d}\gamma^{(i)} = 1$

Physical cross sections at LO, NLO, NNLO are:

$$\mathrm{d}\sigma^{\mathrm{LO}}_{WH(b\bar{b})} = \mathrm{Br}(H \to b\bar{b})\ \mathrm{d}\sigma^{(0)}\ \mathrm{d}\gamma^{(0)}$$

$$\mathrm{d}\sigma^{\mathrm{NLO}}_{WH(b\bar{b})} = \mathrm{Br}(H \to b\bar{b}) \left[\mathrm{d}\sigma^{(0)}\ \mathrm{d}\gamma^{(1)} + \mathrm{d}\sigma^{(1)}\ \mathrm{d}\gamma^{(0)}\right]$$

$$\mathrm{d}\sigma^{\mathrm{NNLO}}_{WH(b\bar{b})} = \mathrm{Br}(H \to b\bar{b}) \left[\mathrm{d}\sigma^{(0)}\ \mathrm{d}\gamma^{(2)} + \mathrm{d}\sigma^{(1)}\ \mathrm{d}\gamma^{(1)} + \mathrm{d}\sigma^{(2)}\ \mathrm{d}\gamma^{(0)}\right]$$

$$\mathrm{d}\sigma^{\mathrm{NNLO,approx}}_{WH(b\bar{b})} = \mathrm{Br}(H \to b\bar{b}) \left[\mathrm{d}\sigma^{(0)}\ \mathrm{d}\gamma^{(1)} + \mathrm{d}\sigma^{(1)}\ \mathrm{d}\gamma^{(0)} + \mathrm{d}\sigma^{(2)}\ \mathrm{d}\gamma^{(0)}\right]$$

Expansion of denominator → separation into production and decay effects at different orders in pQCD **not straightforward!**

Parameters for calculation

- $pp \to W^-(\to e^- \bar{\nu})H(\to b\bar{b})$ at 13 TeV LHC.
- Reconstruct b-jets using flavor-kT algorithm with $\Delta R = 0.5.$
  - *b*-quarks from *H* → *bb* as well as NNLO corrections in production and decay.
  - Require at least one b-jet and one antib-jet with

    $$p_{T,j_b} > 25 \text{ GeV}, \qquad |\eta_{j_b}| < 2.5$$
- Leptonic cuts: $p_{T,e^-} > 15 \text{ GeV}, \qquad |\eta_{e^-}| < 2.5$
- NNPDF3.0 at LO, NLO, NNLO.
- Production scale $\mu_R^2 = \mu_F^2 = (p_W + p_H)^2$; decay $\mu = m_H.$
- Results with and without boosting cut $p_{T,W} > 150 \text{ GeV}.$

Fiducial cross sections (I)

- Without cut on $p_{T,W}$

$$\sigma_{\rm LO} = 15.50^{+0.44}_{-0.56}\ \text{fb}, \qquad \sigma_{\rm NLO} = 16.13^{-0.09}_{+0.20}\ \text{fb}$$
$$\sigma_{\rm NNLO} = 15.20^{-0.08}_{+0.11}\ \text{fb}, \qquad \sigma_{\rm NNLO,approx.} = 16.56^{-0.11}_{+0.16}\ \text{fb}.$$

- With cut on $p_{T,W}$

$$\sigma_{\rm LO} = 2.027^{-0.013}_{+0.006}\ \text{fb}, \qquad \sigma_{\rm NLO} = 2.381^{-0.041}_{+0.055}\ \text{fb}$$
$$\sigma_{\rm NNLO} = 2.357^{-0.026}_{+0.018}\ \text{fb} \qquad \sigma_{\rm NNLO,approx.} = 2.516^{-0.030}_{+0.025}\ \text{fb}.$$

- Scale uncertainty: vary *production* scale by {½,2}.
  - ~1% uncertainty – **underestimate** of theory uncertainty.
- **7-8 times** fewer events with $p_{T,W}$ cut.
  - Still **~ 7500 events** for HL-LHC.

Fiducial cross sections (II)

- Without cut on $p_{T,W}$

$$\sigma_{\text{LO}} = 15.50^{+0.44}_{-0.56}\ \text{fb}, \qquad \sigma_{\text{NLO}} = 16.13^{-0.09}_{+0.20}\ \text{fb}$$
$$\sigma_{\text{NNLO}} = 15.20^{-0.08}_{+0.11}\ \text{fb}, \qquad \sigma_{\text{NNLO,approx.}} = 16.56^{-0.11}_{+0.16}\ \text{fb}.$$

- With cut on $p_{T,W}$

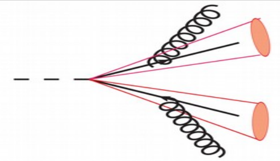
$$\sigma_{\text{LO}} = 2.027^{-0.013}_{+0.006}\ \text{fb}, \qquad \sigma_{\text{NLO}} = 2.381^{-0.041}_{+0.055}\ \text{fb}$$
$$\sigma_{\text{NNLO}} = 2.357^{-0.026}_{+0.018}\ \text{fb} \qquad \sigma_{\text{NNLO,approx.}} = 2.516^{-0.030}_{+0.025}\ \text{fb}.$$

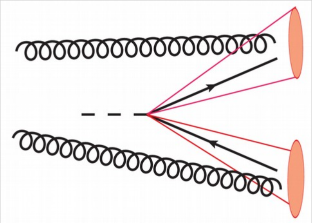
- NLO corrections ~4% without cut, ~ 17% with cut.
  - *W* **recoils against additional radiation** at NLO.
- NNLO corrections to production are similar: ~ 3% higher than NLO without cut, ~6% higher with cut.
- “NLO x NLO” and “NNLO decay” corrections decreases the cross section by ~ 9% without the cut, ~ 7% with the cut.
  - **Cancellations** between NNLO corrections in production and decay – **strongly dependent on cut.**

Invariant mass distribution (I)

Invariant mass of *b*-jets reconstructing the Higgs:

- LO: Delta function at Higgs mass.
- Corrections to decay **decrease** inv. mass.
- Corrections to production **increase** inv. mass.

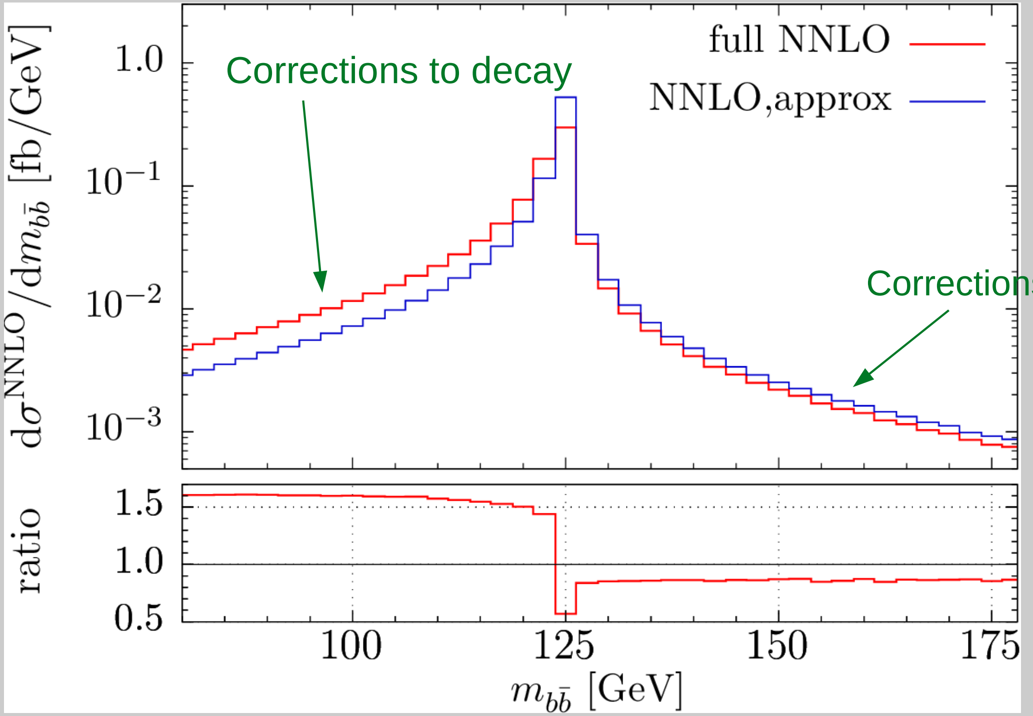




With cut on $p_{T,W}$

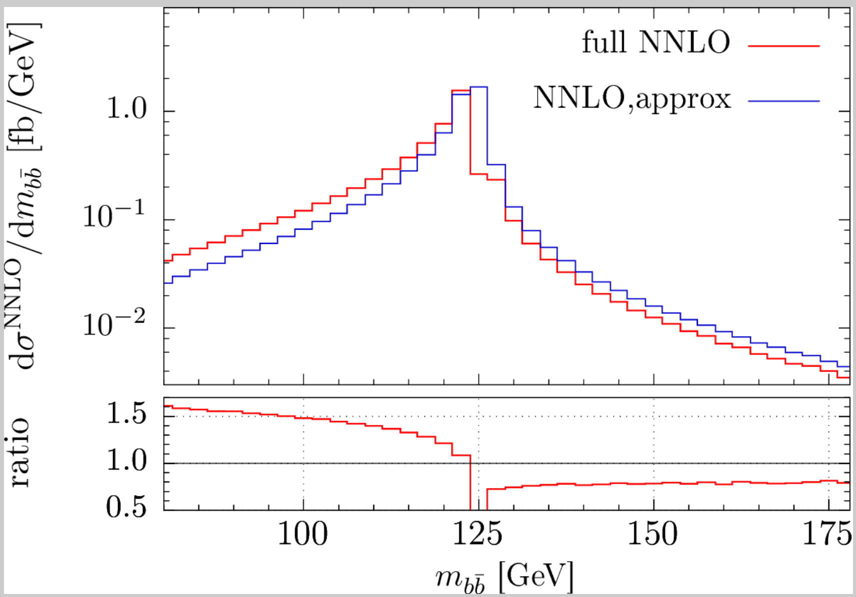
Confirm results of [Ferrera, Somogyi, Tramantano]:

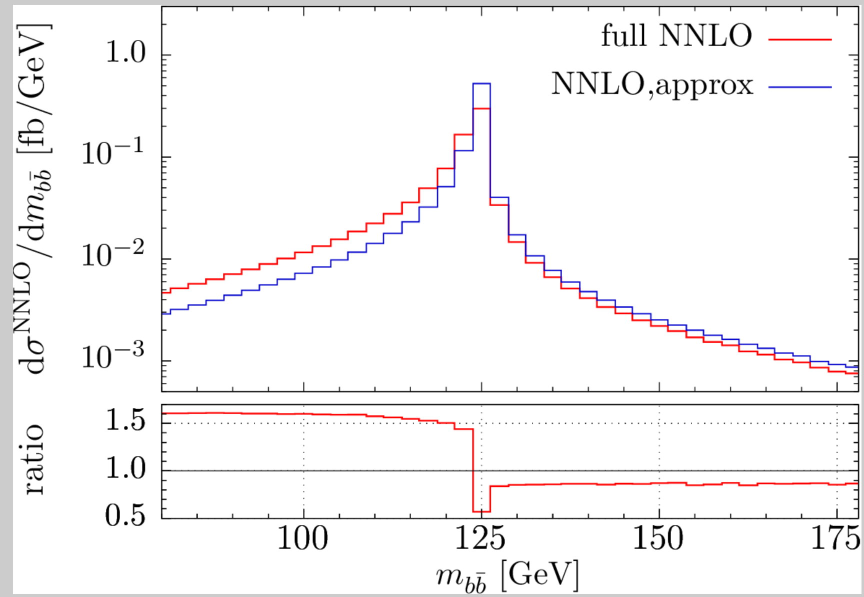
- Large (~60%) at low invariant mass.
- Sharp decrease at Higgs mass.
- ~ 15% depletion at high inv. mass.
- **Expected** as full NNLO includes corrections to decay – reduce inv. mass.



Invariant mass distribution (II)

Comparing results without (L) and with (R) cut on $p_{T,W}$





Minor differences:

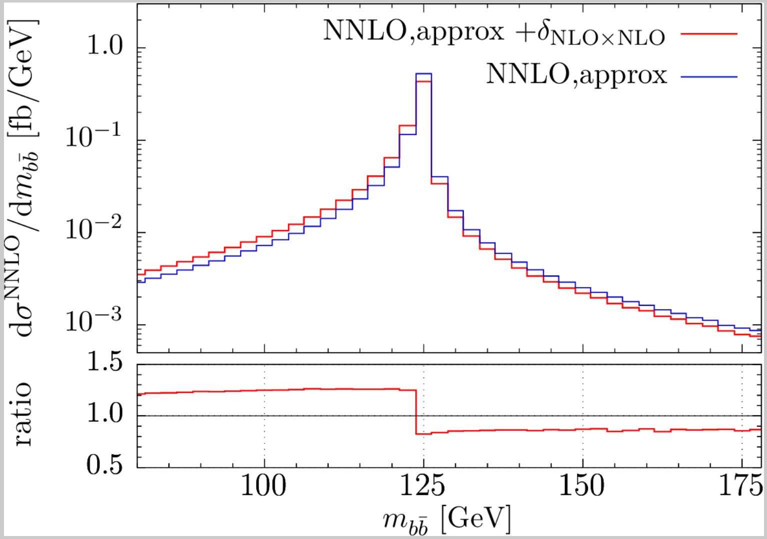
- Full NNLO corrections shift peak without cut.
- Ratio shows sharper decrease with cut than without.

Contributions of different corrections

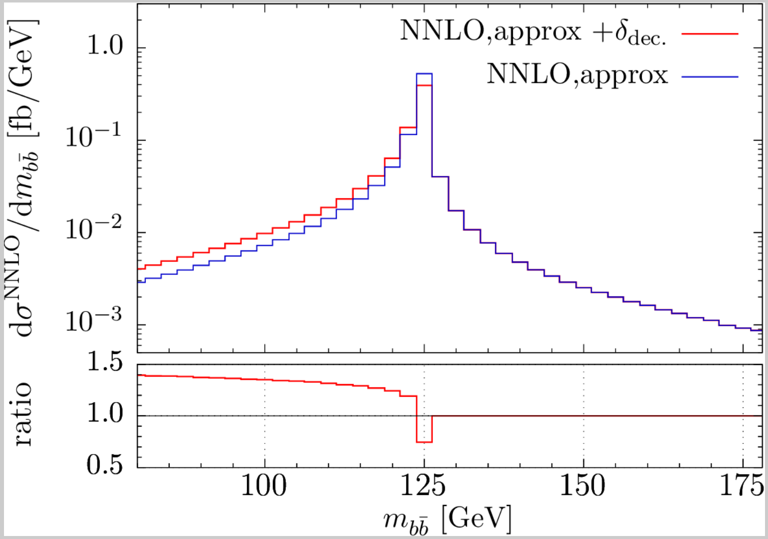
Split into “NLO x NLO” and “NNLO decay”:

$$\delta_{\text{dec.}} = \text{Br}(H \to b\bar{b})\, \text{d}\sigma^{(0)} \left(\text{d}\gamma^{(2)} - \text{d}\gamma^{(1)}\right)$$
$$\delta_{\text{NLO}\times\text{NLO}} = \text{Br}(H \to b\bar{b})\, \text{d}\sigma^{(1)} \left(\text{d}\gamma^{(1)} - \text{d}\gamma^{(0)}\right)$$

Note: $\delta_{\text{dec.}} + \delta_{\text{NLO}\times\text{NLO}} + \text{d}\sigma_{\text{NNLO,approx.}} = 1.$



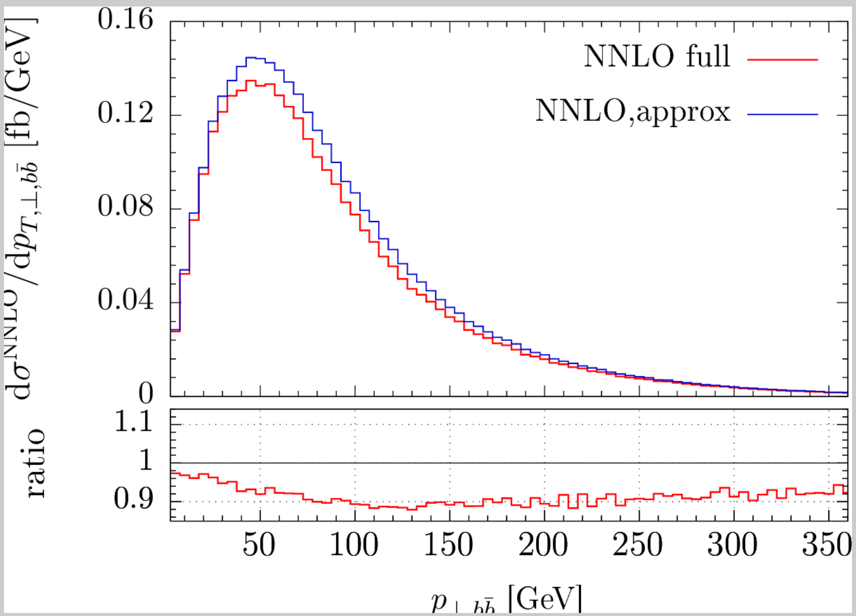
- Below m_H:
  - ~ 20% NLO x NLO
  - ~ 40% NNLO decay
- Above m_H: NLO x NLO only:
  - Expected as NNLO decay reduces invariant mass.



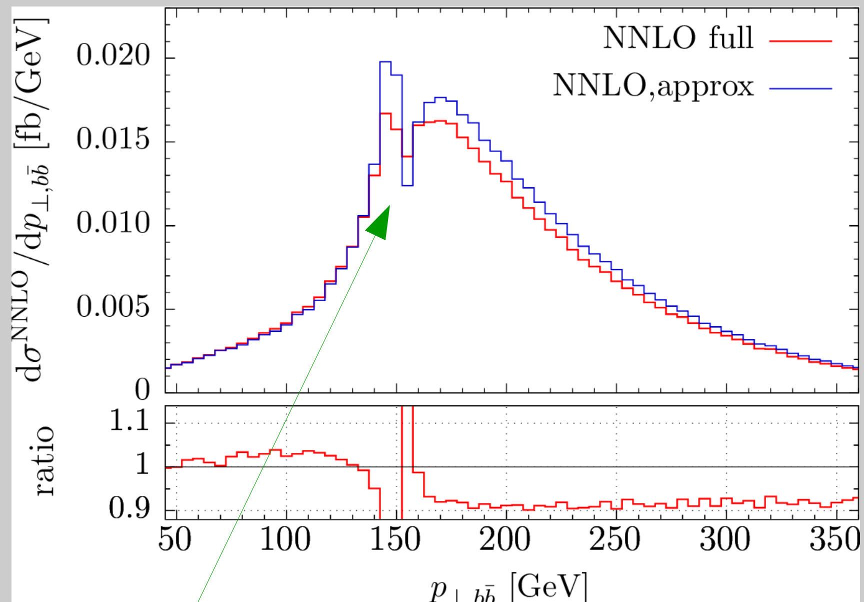
Transverse momentum distribution

Transverse momentum of *b*-jets reconstructing the Higgs:

Without cut on $p_{T,W}$



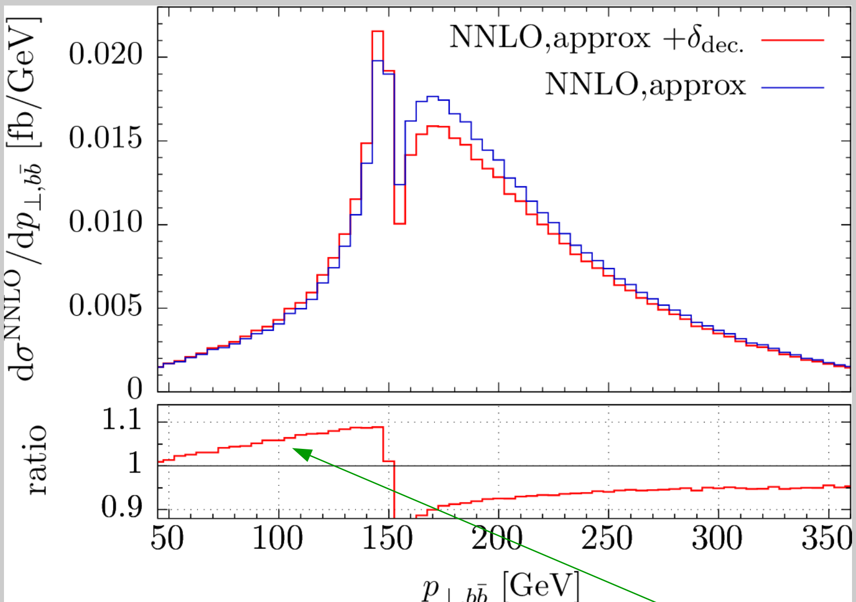
With cut on $p_{T,W}$

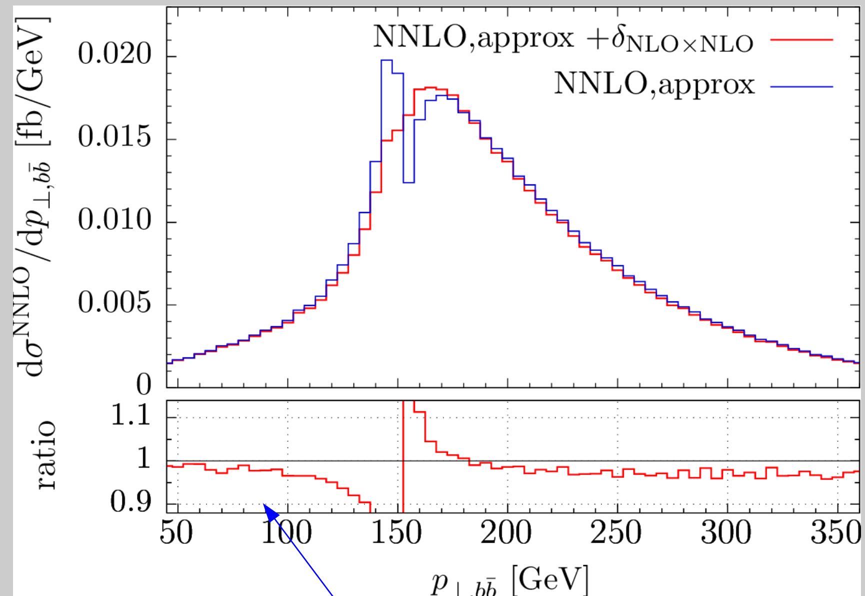


- Cut substantially reshapes distribution.
  - At LO, $p_{T,W} > 150 \text{ GeV} \Rightarrow p_{T,b\bar{b}} > 150 \text{ GeV}.$
- Cut induces Sudakov shoulder below cut 150 GeV.
  - Less prominent when NNLO corrections to decays included.
- Results similar to [Ferrera, Somogyi, Tramontano].

Contributions of different corrections

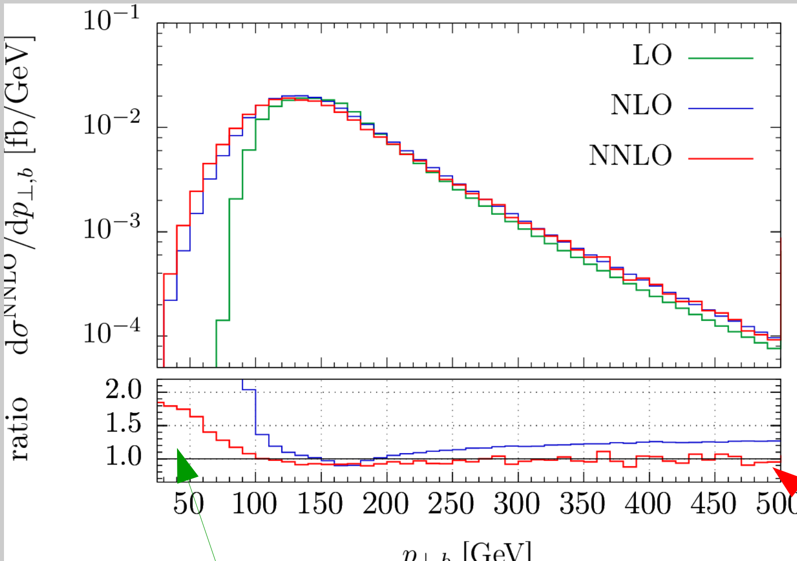
Breakdown into NLO x NLO and NNLO decays:

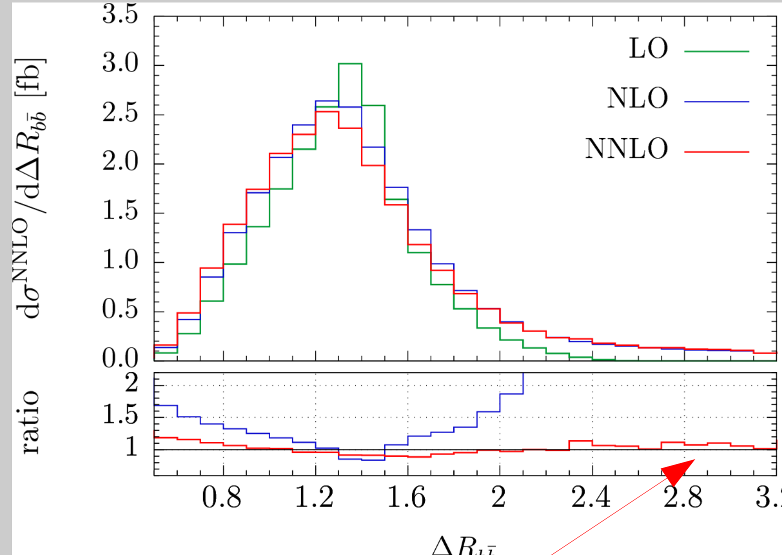




- Low p_T: cancellation between NNLO decay and NLO x NLO.
- Sudakov shoulder smeared by NLO x NLO but not NNLO decay.

Parton showers?





Large corrections in regions not populated at LO, only by real emissions.

Small effects elsewhere.

Can parton showers model these effects?

- $p_{T,W} > 150$ GeV so semi-boosted regime → expect radiation to be soft/collinear and PS to do (reasonably) well.
- Sudakov shoulder at $p_{T,b\bar{b}} = 150$ GeV → expect PS to do well.
- Low invariant mass requires **hard** gluon → PS not ideal.