Hadronic effects and effective field theories at the precision frontier

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Overview

(No) signals beyond the Standard Model

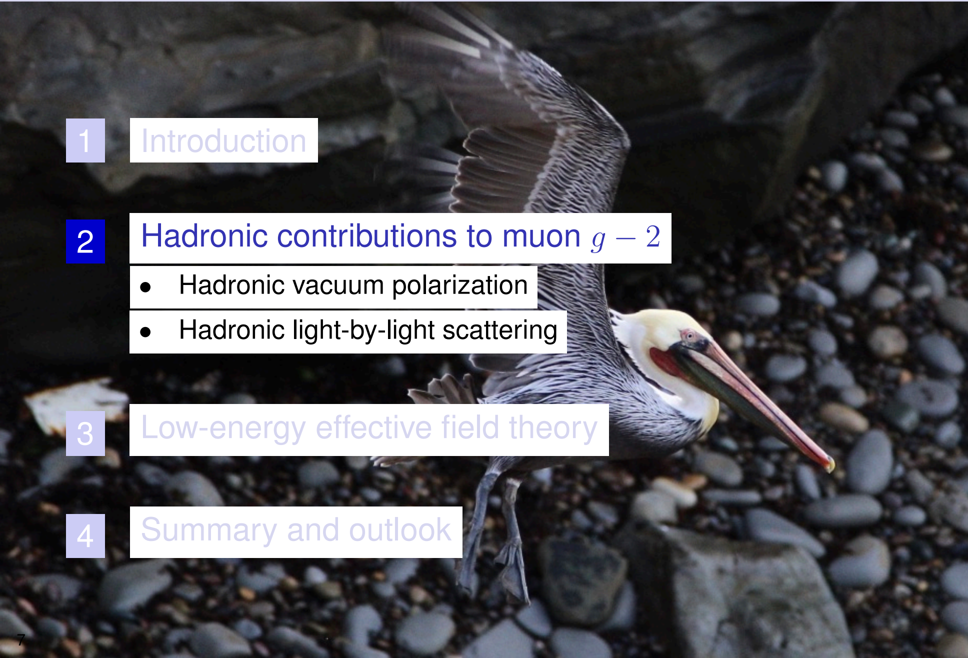
- clear signals of **new physics**, i.e. not explained by the Standard Model (SM):
  - neutrino oscillations and masses
  - baryon asymmetry
  - dark matter
- we expect that there must be **yet unknown particles**
- **direct searches** at high-energy colliders
- **indirect searches** with (low-energy) precision measurements

(No) signals beyond the SM

- so far **no signal of new physics** in direct searches at the Large Hadron Collider (LHC) at CERN
- indirect searches: only a **few discrepancies** around $2 \ldots 4\sigma$, e.g. **muon anomalous magnetic moment**
- could be a combination of statistical fluctuations and systematic effects, or a hint of something new

Searching for new physics

- increasing precision in indirect searches, both at colliders and low energies
- calls for theoretical progress:
  - control hadronic uncertainties at low energies
    $\Rightarrow$ **non-perturbative methods**
  - build generic model-independent framework to combine all constraints
    $\Rightarrow$ **effective field theories**



1 Introduction

2 Hadronic contributions to muon $g-2$

- Hadronic vacuum polarization
- Hadronic light-by-light scattering

3 Low-energy effective field theory

4 Summary and outlook

Magnetic moment

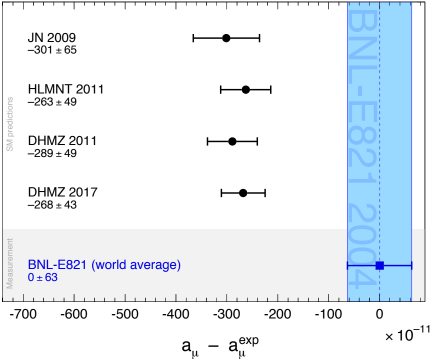
- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio

- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- today: precision test of the SM

$(g-2)_\mu$: comparison of theory and experiment

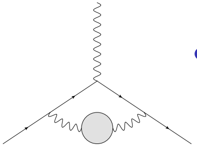


→ PDG 2018

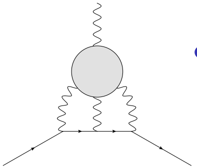
Fermilab $g-2$ experiment aims at improvement by factor 4 (0.14 ppm): first results expected soon

$(g-2)_\mu$: SM uncertainty

dominant uncertainty are **hadronic effects**, i.e.
quantum corrections due to the strong nuclear force



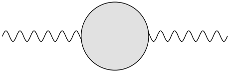
- hadronic vacuum polarization (HVP)



- hadronic light-by-light scattering (HLbL)

Hadronic vacuum polarization (HVP)

Photon HVP function:



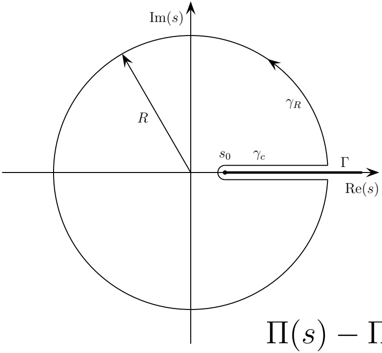
$$= i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

Unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma(e^+e^- \to \mathrm{hadrons})$$

Dispersion relation

Causality implies **analyticity**:



Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s' - s} ds'$$

Deform integration path:

$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s' - s - i\epsilon)s'} ds'$$

HVP contribution to $(g-2)_\mu$

$$a_\mu^{\text{HVP}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\text{thr}}}^{\infty} ds\, \frac{\hat{K}(s)}{s}\, \sigma(e^+e^- \to \text{hadrons})$$

- basic principles: unitarity and analyticity
- direct **relation to data**: total hadronic cross section $\sigma(e^+e^- \to \text{hadrons})$
- dedicated e^+e^- program (BaBar, Belle, BESIII, CMD3, KLOE, SND)

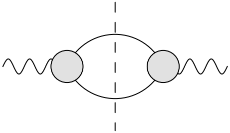
Two-pion contribution to HVP

- $\pi\pi$ contribution amounts to more than 70% of HVP contribution
- responsible for a similar fraction of HVP uncertainty

Unitarity and analyticity

Implications of unitarity (two-pion intermediate states):

1. $\pi\pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi\pi$ scattering
3. $\pi\pi$ scattering—$\pi\pi$ scattering



: $\quad \sigma(e^+e^- \to \pi^+\pi^-) \propto |F_\pi^V(s)|^2$

analyticity $\Rightarrow$ dispersion relation for HVP contribution

Unitarity and analyticity

Implications of unitarity (two-pion intermediate states):

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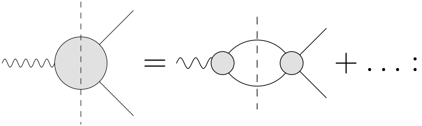

$$F_\pi^V(s) = |F_\pi^V(s)|e^{i\delta_1^1(s)+\ldots}$$

analyticity $\Rightarrow$ dispersion relation for pion VFF

Unitarity and analyticity

Implications of unitarity (two-pion intermediate states):

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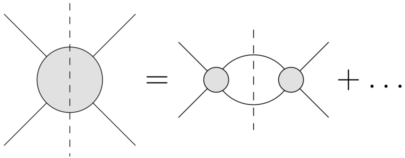
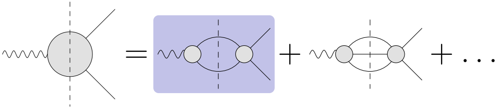


analyticity, crossing, PW expansion $\Rightarrow$ Roy equations

Dispersive representation of pion VFF

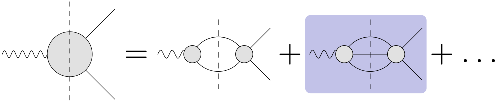


$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

- Omnès function with elastic $\pi\pi$-scattering P-wave phase shift $\delta_1^1(s)$ as input:

$$\Omega_1^1(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^\infty ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}$$

Dispersive representation of pion VFF



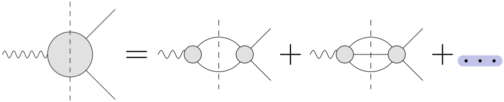
$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

- isospin-breaking 3π intermediate state: negligible apart from ω resonance (ρ–ω interference effect)

$$G_\omega(s) = 1 + \frac{s}{\pi}\int_{9M_\pi^2}^{\infty} ds' \frac{\text{Im}g_\omega(s')}{s'(s'-s)} \left(\frac{1-\frac{9M_\pi^2}{s'}}{1-\frac{9M_\pi^2}{M_\omega^2}}\right)^4,$$

$$g_\omega(s) = 1 + \epsilon_\omega \frac{s}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s}$$

Dispersive representation of pion VFF



$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\rm in}^N(s)$$

- heavier intermediate states: 4π (mainly $\pi^0\omega$), $\bar{K}K$, ...
- described in terms of a conformal polynomial with cut starting at $\pi^0\omega$ threshold

$$G_{\rm in}^N(s) = 1 + \sum_{k=1}^{N} c_k\left(z^k(s) - z^k(0)\right)$$

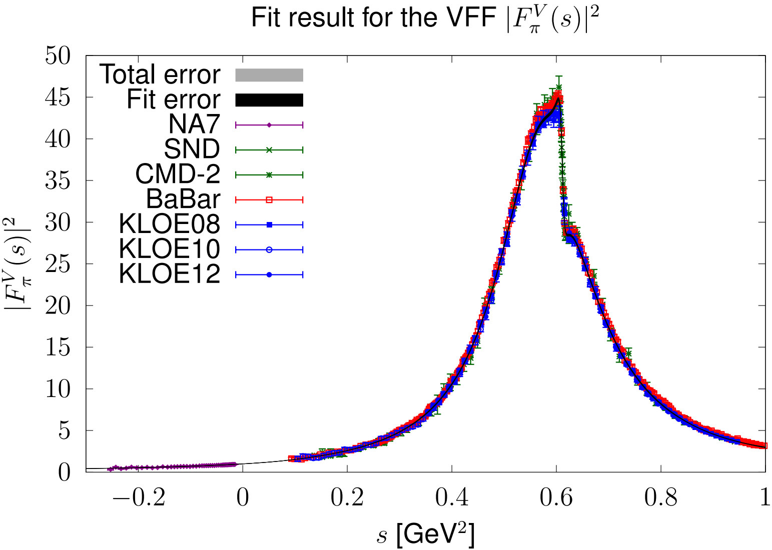
- correct P-wave threshold behavior imposed

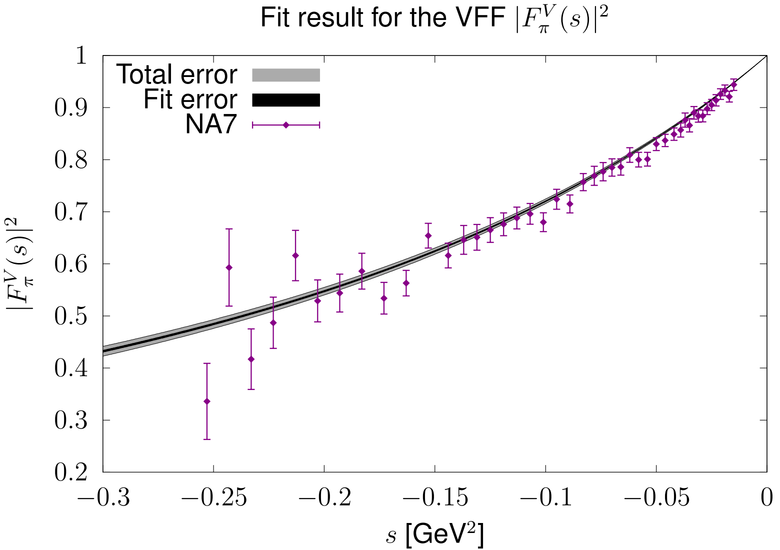
VFF fit to the following data

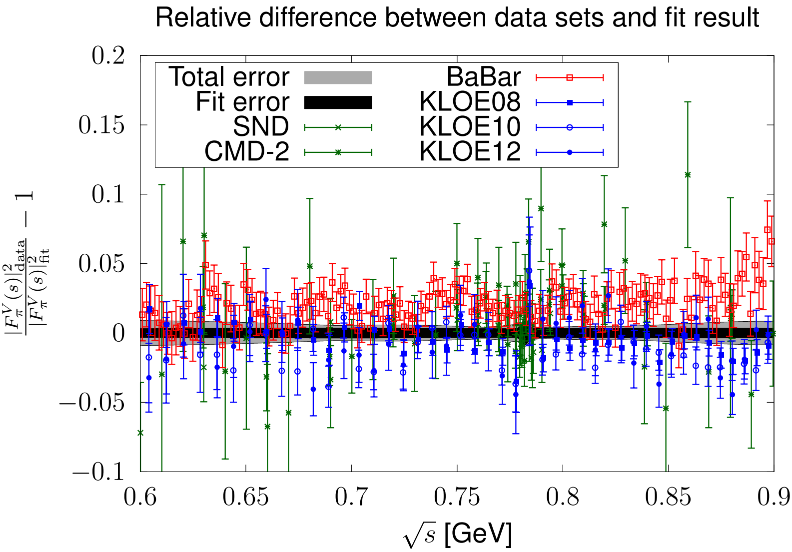
- time-like cross section data from high-statistics e^+e^- experiments SND, CMD-2, BaBar, KLOE
- space-like VFF data from NA7
- Eidelman–Łukaszuk bound on inelastic phase:

  → Eidelman, Łukaszuk, 2004
- iterative fit routine including full experimental covariance matrices and avoiding D'Agostini bias

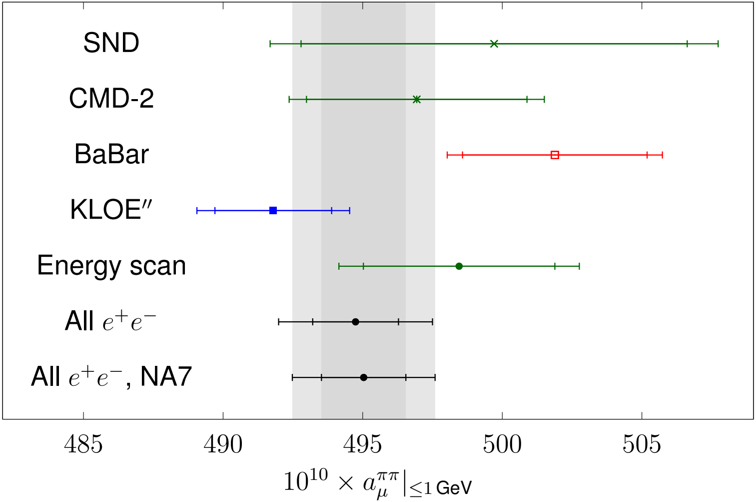
  → D'Agostini, 1994; Ball et al. (NNPDF) 2010


Fit result for the VFF $|F_\pi^V(s)|^2$
Total error
Fit error
NA7
SND
CMD-2
BaBar
KLOE08
KLOE10
KLOE12
$|F_\pi^V(s)|^2$
s [GeV^2]
50
45
40
35
30
25
20
15
10
5
0
−0.2
0
0.2
0.4
0.6
0.8
1


Fit result for the VFF $|F_\pi^V(s)|^2$
Total error
Fit error
NA7
$|F_\pi^V(s)|^2$
s [GeV^2]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
−0.3
−0.25
−0.2
−0.15
−0.1
−0.05
0


Relative difference between data sets and fit result
Total error
Fit error
SND
CMD-2
BaBar
KLOE08
KLOE10
KLOE12
0.2
0.15
0.1
0.05
0
−0.05
−0.1
0.6
0.65
0.7
0.75
0.8
0.85
0.9
$\frac{|F_\pi^V(s)|^2_{\mathrm{data}}}{|F_\pi^V(s)|^2_{\mathrm{fit}}} - 1$
$\sqrt{s}$ [GeV]

Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV



Contribution to $(g-2)_\mu$

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006

- low-energy $\pi\pi$ contribution:

$$a_\mu^{\text{HVP},\pi\pi}|_{\leq 0.63\,\text{GeV}} = 132.8(0.4)(1.0) \times 10^{-10}$$

- $\pi\pi$ contribution up to 1 GeV:

$$a_\mu^{\text{HVP},\pi\pi}|_{\leq 1\,\text{GeV}} = 495.0(1.5)(2.1) \times 10^{-10}$$

Determination of the pion charge radius

$$F_\pi^V(s) = 1 + \frac{1}{6}\langle r_\pi^2\rangle s + \mathcal{O}(s^2)$$

DR for F_π^V implies sum rule for charge radius:

$$\langle r_\pi^2\rangle = \frac{6}{\pi}\int_{4M_\pi^2}^\infty ds \frac{\mathrm{Im}F_\pi^V(s)}{s^2} = 0.429(4)\,\text{fm}^2$$

together with $\langle r_\pi^2\rangle = 0.432(4)$ → Ananthanarayan et al., 2017

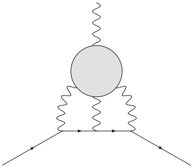
triggered a revision of the PDG value:

PDG 2018: $\langle r_\pi^2\rangle = 0.452(11)\,\text{fm}^2$

PDG 2019: $\langle r_\pi^2\rangle = 0.434(5)\,\text{fm}^2$

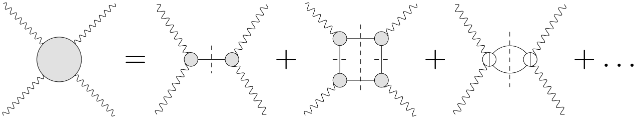
(model-dependent $eN \to e\pi N$ now excluded)

Hadronic light-by-light scattering



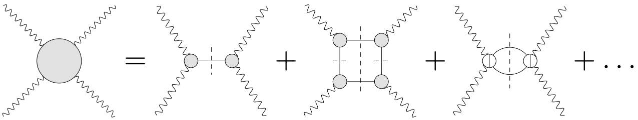
- previously based only on hadronic models
- first lattice-QCD results with $\sim 60\%$ uncertainty
- our work: **dispersive framework**, replacing hadronic models step by step

Hadronic light-by-light scattering



- gauge-invariant **Lorentz decomposition** of HLbL tensor, solution of kinematic constraints
- precise evaluation of **pion box** and **two-pion rescattering** contributions

Hadronic light-by-light scattering

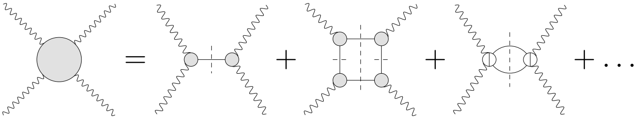


$$a_\mu^{\text{HLbL},\pi\text{-box}} = -15.9(2) \times 10^{-11}$$
$$a_\mu^{\text{HLbL},\pi\pi}\Big|_{J=0}^{\pi\text{-pole LHC}} = -8(1) \times 10^{-11}$$

→ Colangelo, Hoferichter, Procura, Stoffer

JHEP **09** (2014) 091, JHEP **09** (2015) 074, PRL **118** (2017) 232001, JHEP **04** (2017) 161

Hadronic light-by-light scattering

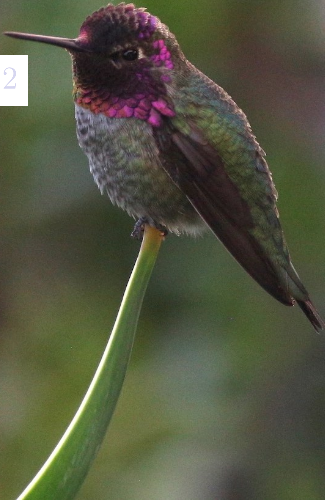


recent work and in progress:

- extension to D-waves
- asymptotic constraints
- beyond two-pion contributions

→ Hoferichter, Stoffer, JHEP **07** (2019) 073

→ Colangelo, Hagelstein, Hoferichter, Laub, Stoffer
arXiv:1910.13432 [hep-ph], arXiv:1910.11881 [hep-ph]



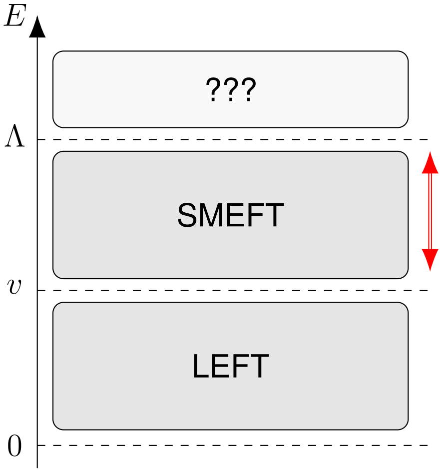
Beyond the SM

- baryon asymmetry requires sources of CP**-violation** beyond the SM
- neutrino masses and oscillations imply **lepton-flavor (LF) violation** at a very high scale
- additional new physics violating LF and CP may appear at energies above the electroweak scale
- its low-energy quantum effects can be described by **effective field theory**, containing only SM particles (SMEFT)
- experimental constraints from precision observables

Effective field theories (EFTs)

- based on a **small set of assumptions**
- generic and systematic **quantum field theories**, can be used "stand-alone" for fits to experiments or in connection with a broad range of specific models
- work only with the **relevant particles** at a particular energy $\Rightarrow$ simplify calculations
- EFT parameters depend on energy scale $\Rightarrow$ **running & mixing**
- connect **different energy regimes** (renormalization group, avoid large logarithms)

EFTs for new physics



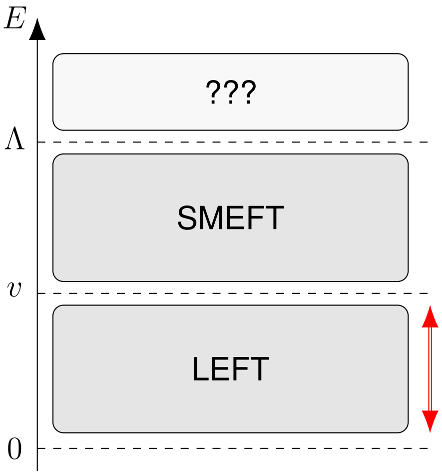
- SMEFT Lagrangian at dimension six known explicitly

  → Buchmüller, Wyler (1986)
  → Grzadkowski et al. (2010)

- one-loop running for SMEFT known

  → Jenkins et al. (2013, 2014)
  → Alonso et al. (2014)

EFTs for new physics



- partial LEFT operator basis and running previously studied in detail
- first complete treatment up to dimension six:

  → Jenkins, Manohar, Stoffer (2018)

EFT for new physics above the weak scale

"**Standard Model EFT**" (SMEFT) assumptions:

→ Buchmüller, Wyler (1986)

- new physics **at high energies** $\Lambda \gg v \approx 246$ GeV
- underlying theory respects the **same symmetry principles** as the SM
- Higgs particle part of electroweak doublet (as in SM)

EFT for new physics above the weak scale

SMEFT Lagrangian:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i C_i^{(5)} \mathcal{O}_i^{(5)} + \sum_i C_i^{(6)} \mathcal{O}_i^{(6)} + \dots$$

- dimension 5: one operator + h.c.
- dimension 6: 3045 operators

EFT below the electroweak scale

"**Low-energy EFT**" (LEFT):

- only **light SM particles** (no Higgs, weak bosons, or top quark)
- basically the old Fermi theory of weak interaction
- complete and systematic treatment up to dimension 6 recently worked out

→ Jenkins, Manohar, Stoffer, JHEP **01** (2018) 084, JHEP **03** (2018) 016

EFT below the electroweak scale

LEFT Lagrangian:

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QED+QCD}} + \sum_i L_i \mathcal{O}_i + \dots$$

Additional effective operators:

- dimension 3: Majorana-neutrino masses ($\Delta L = \pm 2$)
- dimension 5: $\Delta B = \Delta L = 0$ **dipole** operators for $\psi = u, d, e$ and $\Delta L = \pm 2$ neutrino-dipole operators
- dimension 6: CP-even and CP-odd **three-gluon** operators, as well as **four-fermion** operators

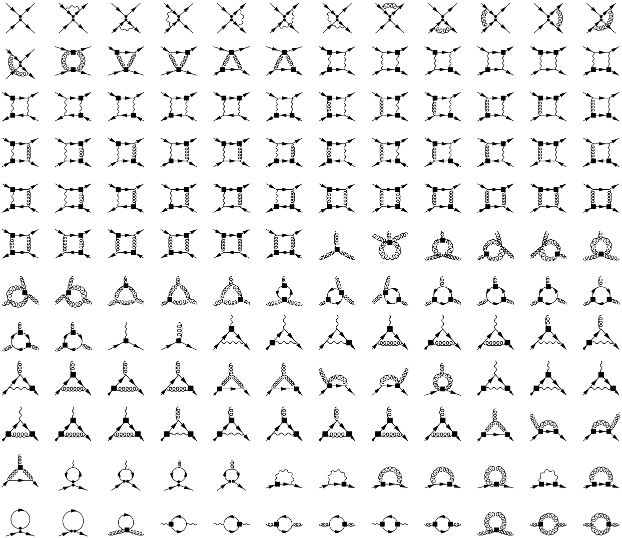
LEFT operators

→ Jenkins, Manohar, Stoffer, JHEP **03** (2018) 016

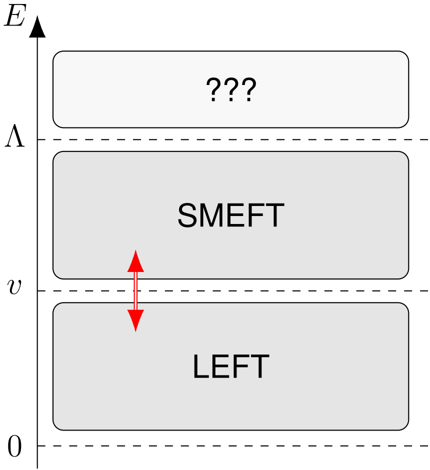
- in total 5963 operators at dimensions three, five, and six: 3099 CP-even and 2864 CP-odd
- basis **free of redundancies** (EOM, Fierz, etc.)
- cross-checked with Hilbert series

Full set of one-loop diagrams for LEFT running

→ Jenkins, Manohar, Stoffer, JHEP **01** (2018) 084



Matching between the EFTs



- tree-level matching from SMEFT to LEFT

  → Jenkins, Manohar, Stoffer
  JHEP **03** (2018) 016

- complete one-loop matching

  → Dekens, Stoffer
  JHEP **10** (2019) 197

- leads to relations between LEFT operator coefficients

SMEFT in the broken phase

- Higgs in unitary gauge:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \left[1 + c_{H,\text{kin}}\right] h + v_T \end{pmatrix} ,$$

where

$$c_{H,\text{kin}} := \left(C_{H\Box} - \frac{1}{4} C_{HD} \right) v^2 , \quad v_T := \left(1 + \frac{3 C_H v^2}{8\lambda} \right) v$$

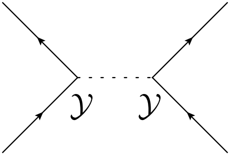
- **modifications from SM** due to dimension-six Higgs operators in SMEFT

SMEFT in the broken phase

- dimension-six modifications of fermion masses and Yukawa couplings $\Rightarrow$ no longer proportional
- modifications of gauge-boson mass terms
- weak charged and neutral currents modified as well, e.g. coupling of $\mathcal{W}^+$ to **right-handed current** $\bar{u}_R \gamma^\mu d_R$
- after rotation to mass eigenstates, modified weak currents lead to **non-unitary effective CKM** quark-mixing matrix

Integrating out weak-scale SM particles

consider Higgs-exchange diagram:



$$[\mathcal{Y}_\psi]_{rs} = \frac{1}{v_T}[M_\psi]_{rs}[1 + c_{H,\mathrm{kin}}] - \frac{v^2}{\sqrt{2}} C^*_{\substack{\psi H \\ sr}}$$

$\mathcal{Y}^2$ has terms of order $(m/v)^2$, mv/Λ^2, v^4/Λ^4

$\Rightarrow$ diagram $\mathcal{Y}^2/m_h^2$ is of same order as dimension-7 or 8 contributions in LEFT or dimension-8 in SMEFT

Integrating out weak-scale SM particles

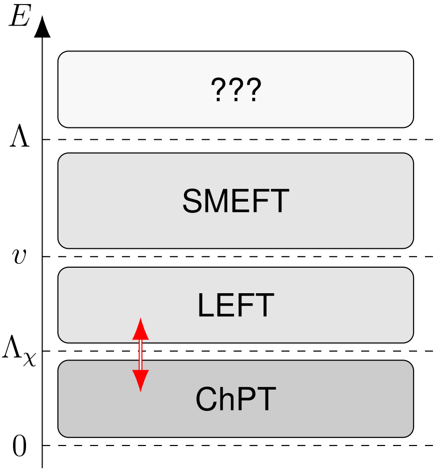
- for SMEFT $\Rightarrow$ LEFT matching: rewrite terms

$$\ldots \frac{1}{\Lambda^n} = \underbrace{\ldots \frac{1}{v^n}}_{\text{LEFT counting}} \times \underbrace{\frac{v^n}{\Lambda^n}}_{\text{SMEFT counting}}$$

- tree-level matching simple: fix Higgs field to vev and compute $\mathcal{W}/\mathcal{Z}$-exchange diagrams
- **one-loop matching**: 754 diagrams including finite parts → Dekens, Stoffer, JHEP **10** (2019) 197
- using background-field gauge → Helset, Paraskevas, Trott (2018)

Matching to Chiral Perturbation Theory (ChPT)

→ Dekens, Jenkins, Manohar, Stoffer, JHEP **01** (2019) 088



- at the hadronic scale, QCD is **non-perturbative**
- **ChPT** can be formulated including LEFT effects
- non-perturbative matching required, e.g. using lattice QCD simulations

Constraining operator coefficients

- huge number of free parameters, but for particular processes often only a few contribute
- **powerful constraints** most easily derived for operators mediating processes forbidden or suppressed in the SM
- example: **lepton-flavor violation**

Lepton-flavor violation: $\mu \to e\gamma$

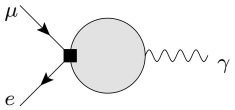
$\to$ Dekens, Jenkins, Manohar, Stoffer, JHEP **01** (2019) 088

- hadronic effects can show up in purely leptonic process
- perform matching to ChPT
- LF violation due to many operators, e.g.

$$\mathcal{O}_{eq}^{S,RR} = (\bar{e}_{Lp} e_{Rr})(\bar{q}_{Ls} q_{Rt})$$
$$\mathcal{O}_{eq}^{V,LL} = (\bar{e}_{Lp} \gamma^\mu e_{Lr})(\bar{q}_{Ls} \gamma_\mu q_{Lt})$$
$$\mathcal{O}_{eq}^{T,RR} = (\bar{e}_{Lp} \sigma^{\mu\nu} e_{Rr})(\bar{q}_{Ls} \sigma_{\mu\nu} q_{Rt})$$

Lepton-flavor violation: $\mu \to e\gamma$

- semileptonic tensor operators contribute to $\mu \to e\gamma$:



- non-perturbative effects **not suppressed** by light quark masses

Lepton-flavor violation: $\mu \to e\gamma$

- matching to ChPT at $\mathcal{O}(p^4)$:

$$\bar{q}_L \sigma^{\mu\nu} t_{\mu\nu} q_R \to \Lambda_1 \langle t_{\mu\nu}(U F_L^{\mu\nu} + F_R^{\mu\nu} U)\rangle + i\Lambda_2 \langle t^{\mu\nu} D_\mu U U^\dagger D_\nu U\rangle$$

- **no external Goldstone** bosons:

$$(\bar{e}_{Lp}\sigma^{\mu\nu} e_{Rr})(\bar{q}_L \sigma_{\mu\nu} q_R) \to -2 Q_q e\, \Lambda_1\, (\bar{e}_{Lp}\sigma^{\mu\nu} e_{Rr}) F^{\mu\nu}\,, \quad q = u, d, s$$

- $\Lambda_{1,2}$: low-energy constants for ChPT with tensor sources. NDA: $\Lambda_1 = c_T \frac{F_\pi}{4\pi}$ with $c_T = \mathcal{O}(1)$
- or use lattice results for Λ_2 and relate Λ_1 to Λ_2 with vector-meson dominance: $c_T \approx -1.0(2)$

Lepton-flavor violation: $\mu \to e\gamma$

$\to$ Dekens, Jenkins, Manohar, Stoffer, JHEP **01** (2019) 088

- limit by MEG collaboration $\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ gives the constraint

$$|c_T| \left(\left| L^T_{e\mu} \right|^2 + \left| L^T_{\mu e} \right|^2 \right)^{1/2} \lesssim 1.65 \times 10^{-5}\,\text{TeV}^{-2}\,,$$

  at the hadronic scale $\approx 2\,$GeV, with $c_T \approx -1.0(2)$

- constraints on SMEFT operators at the weak scale through **running** and **matching** SMEFT $\Rightarrow$ LEFT

Indirect search for new physics

- high precision at experimental frontier must be matched by high-precision theory calculations
- power of low-energy observables with SM contributions often limited by **hadronic effects**, which require non-perturbative methods: e.g. $g-2$
- **strong constraints** from observables with vanishing or suppressed SM contributions: e.g. LF violation

Indirect search for new physics

- still need to evaluate and control **non-perturbative effects**
- they can also lead to interesting enhancements and new constraints: $\mu \to e\gamma$, $\mu \to 3e$

Dispersion relations

- used to describe non-perturbative hadronic effects by providing **model-independent** relations between observables
- example: hadronic contributions (HVP, HLbL) to muon $g-2$

EFTs

- used to describe effects of new physics by providing **model-independent** relations between observables
- one-loop SMEFT-LEFT matching: first step towards a framework at next-to-leading-logarithm accuracy
- many interesting applications: LF violation, CP-violation (EDMs), connection with collider physics
- apply non-perturbative methods (ChPT, dispersion relations) to evaluate **hadronic effects**