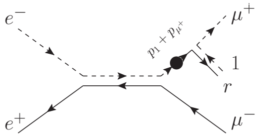
The helicity-flow method

Malin Sjödahl

In collaboration with Andrew Lifson and Christian Reuschle

Lund University



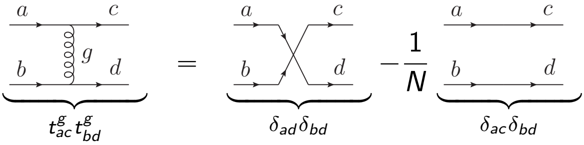
December 3rd 2019

Outline

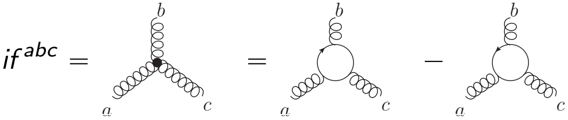
Motivation – the analogy with color

In QCD we translate color structures to flows of color

- SU(N) Fierz identity: remove adjoint indices ($T_R = 1$)



- Remove gluon vertices similarly



In the end every amplitude is a linear combination of products of δs

Can we do something similar for spacetime?

- At the algebra level, the Lorentz group consists of two copies of su(2) $so(3,1) \cong su(2) \oplus su(2)$
- The Dirac spinor structure transforms under the direct sum representation $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$, in the chiral/Weyl basis

$$u(p) \to \begin{pmatrix} e^{i\theta\frac{\bar{\sigma}}{2}+\eta\frac{\bar{\sigma}}{2}} & 0 \\ 0 & e^{i\theta\frac{\bar{\sigma}}{2}-\eta\frac{\bar{\sigma}}{2}} \end{pmatrix} u(p)$$

i.e. actually two copies of SL(2,$\mathbb{C}$), generated by the complexified su(2) algebra, projected onto by $P_{\pm} = \frac{1}{2}(1 \pm \gamma^5)$, $\quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

- For $m = 0$ $\quad u(p) = \begin{pmatrix} u_-(p) \\ u_+(p) \end{pmatrix} = \begin{pmatrix} \tilde{\lambda}_p^{\dot{\alpha}} \\ \lambda_{p,\alpha} \end{pmatrix}, \quad \bar{u}(p) = \begin{pmatrix} \tilde{\lambda}_{p,\dot{\alpha}} & \lambda_p^{\alpha} \end{pmatrix},$

$$v(p) = \begin{pmatrix} v_+(p) \\ v_-(p) \end{pmatrix} = \begin{pmatrix} \tilde{\lambda}_p^{\dot{\alpha}} \\ \lambda_{p,\alpha} \end{pmatrix}, \quad \bar{v}(p) = \begin{pmatrix} \tilde{\lambda}_{p,\dot{\alpha}} & \lambda_p^{\alpha} \end{pmatrix}$$

Can we do something similar for spacetime?

- Lorentz inner products formed using the only SL(2,$\mathbb{C}$) invariant object $\epsilon^{\alpha\beta}$, $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12}$

$$\underbrace{\epsilon^{\alpha\beta}\lambda_{i,\beta}}_{\equiv\lambda_i^{\alpha}}\lambda_{j,\alpha} = \lambda_i^{\alpha}\lambda_{j,\alpha} = \langle ij\rangle,\ \underbrace{\epsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}_i^{\dot{\beta}}}_{\equiv\tilde{\lambda}_{i,\dot{\alpha}}}\tilde{\lambda}_j^{\dot{\alpha}} = \tilde{\lambda}_{i,\dot{\alpha}}\tilde{\lambda}_j^{\dot{\alpha}} = [ij]\ ,\ \langle ij\rangle, [ij] \sim \sqrt{s_{ij}}$$

  Note: antisymmetric $\langle ij\rangle = -\langle ji\rangle$, $[ij] = -[ji]$

- The Dirac Lagrangian $\bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi$ gives after requiring local gauge invariance couplings $\sim A_\mu \bar{u}(p_1)\gamma^\mu u(p_2)$, i.e., the photon couples to

$$\bar{u}(p_1)\gamma^\mu u(p_2) = \underbrace{\begin{pmatrix}\tilde{\lambda}_{1,\dot{\alpha}} & \lambda_1^{\alpha}\end{pmatrix}}_{\bar{u}(p_1)} \underbrace{\begin{pmatrix} 0 & \sqrt{2}\tau^{\mu,\dot{\alpha}\beta} \\ \sqrt{2}\bar{\tau}^{\mu}_{\alpha\dot{\beta}} & 0 \end{pmatrix}}_{\gamma^\mu} \underbrace{\begin{pmatrix}\tilde{\lambda}_2^{\dot{\beta}} \\ \lambda_{2,\beta}\end{pmatrix}}_{u(p_2)}$$

  where $\sqrt{2}\tau^\mu = (1, \vec{\sigma})$, $\sqrt{2}\bar{\tau}^\mu = (1, -\vec{\sigma})$, $\mathrm{Tr}(\tau^\mu\bar{\tau}^\nu) = g^{\mu\nu}$

- giving vertices $\sim \tilde{\lambda}_{1,\dot{\alpha}}\tau^{\mu,\dot{\alpha}\beta}\lambda_{2,\beta}$ and $\lambda_1^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tilde{\lambda}_2^{\dot{\beta}}$

Can we do something similar for spacetime?

- Lorentz four-vectors transform under a direct product representation $\sim (\frac{1}{2}, \frac{1}{2})$ and are mapped to

$$p^{\dot{\alpha}\beta} \equiv p_\mu \tau^{\mu,\dot{\alpha}\beta} = \frac{1}{\sqrt{2}} p_\mu \sigma^{\mu,\dot{\alpha}\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix},$$

$$\bar{p}_{\alpha\dot{\beta}} \equiv p_\mu \bar{\tau}^\mu_{\alpha\dot{\beta}} = \frac{1}{\sqrt{2}} p_\mu \bar{\sigma}^\mu_{\alpha\dot{\beta}} = \frac{1}{\sqrt{2}} \begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \\ -p_1 - ip_2 & p_0 + p_3 \end{pmatrix},$$

  It can be proved that transforming (only) the spinor indices in $\tau^{\mu,\dot{\alpha}\beta}$ or $\bar{\tau}^\mu_{\alpha\dot{\beta}}$, using the direct product transformation gives the Lorentz four-vector transformation. It can also be can be read off from the Lagrangian that this must be the case.
- For lightlike momenta $p^2 = 0$

$$p^2 = 0 \Leftrightarrow \det[p^{\dot{\alpha}\beta}] = 0 \underbrace{\longrightarrow}_{\text{Dirac}} \not{p} \equiv \sqrt{2} p^{\dot{\alpha}\beta} = \tilde{\lambda}_p^{\dot{\alpha}} \lambda_p^\beta$$

Can we do something similar for spacetime?

- Similarly $\bar{\not p} = \sqrt{2} p_\mu \bar{\tau}^\mu_{\alpha\dot\beta} \overset{p^2=0}{=} \lambda_{p,\alpha}\tilde{\lambda}_{p,\dot\beta}$
- Multiplying this with $\tau^{\nu,\dot\beta\alpha}$, summing over indices, and using $\mathrm{Tr}(\tau^\mu\bar\tau^\nu) = g^{\mu\nu}$ we get

$$\underbrace{\sqrt{2} p_\mu \bar{\tau}^\mu_{\alpha\dot\beta}}_{\lambda_{p,\alpha}\tilde{\lambda}_{p,\dot\beta}} \tau^{\nu,\dot\beta\alpha} = \sqrt{2} p_\mu g^{\mu\nu} = \sqrt{2} p^\nu \implies p^\nu \overset{p^2=0}{=} \frac{1}{\sqrt{2}} \tilde{\lambda}_{p,\dot\beta} \tau^{\nu,\dot\beta\alpha} \lambda_{p,\alpha}$$

- Note: A lightlike four-vector same spinor structure as vertex $\sim$ pseudo vertex
- Need polarization vectors for external photons

$$\varepsilon^\mu_+(p,r) = \frac{\tilde{\lambda}_{p,\dot\alpha}\tau^{\mu,\dot\alpha\beta}\lambda_{r,\beta}}{\langle rp\rangle} \quad , \quad \varepsilon^\mu_-(p,r) = \frac{\lambda^\alpha_p \bar\tau^\mu_{\alpha\dot\beta}\tilde{\lambda}^{\dot\beta}_r}{[pr]}$$

- Note: Also same spinor structure as vertex $\sim$ pseudo vertex

Building the flow picture

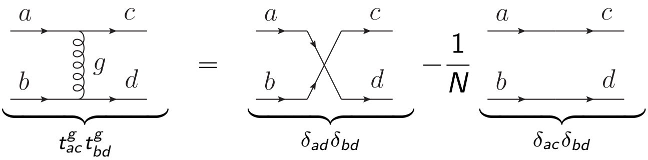
Let's compare to QCD color

- Color single su(N)
- Quarks in fundamental rep.
- Gluons in adjoint rep $\rightarrow$ combination of fundamental rep. indices
- $t^a_{ij} t^a_{kl} \rightarrow \delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}$ SU(3) generators

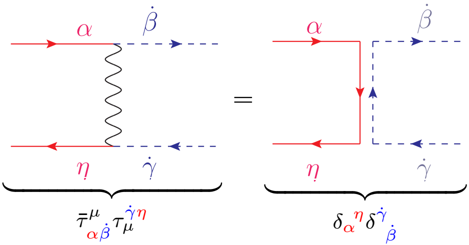
- Lorentz structure $su(2)$, $su(2)$
- Spinors in different irreps. $\lambda, \tilde{\lambda}$
- Four-vectors in direct product rep $\rightarrow$ combination of spinor reps
- $\tau^{\mu,\dot{\alpha}\beta}\ \bar{\tau}_{\mu,\gamma\dot{\delta}} \rightarrow \delta^{\dot{\alpha}}_{\dot{\delta}}\delta^{\beta}_{\gamma}$ not exactly su(2) generators...

Creating a helicity flow picture

- Recall the QCD Fierz identity $(T_R = 1)$



- Spinor Fierz in flow form is (always read indices along arrow):

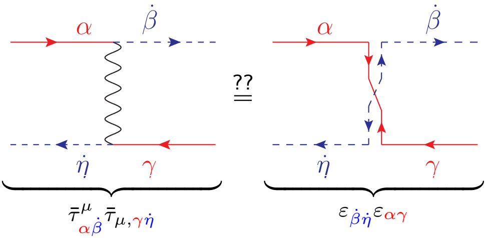


- No $1/N$-suppressed term even better than QCD!

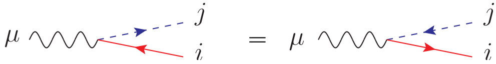
Photon exchange

- Above we had a "flow", coming from photon exchange, applicable for $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau_{\mu}^{\dot{\gamma}\eta}$, but photon exchange may also give two τ or two $\bar{\tau}$
- $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\bar{\tau}_{\mu,\gamma\dot{\eta}} = \varepsilon_{\dot{\beta}\dot{\eta}}\varepsilon_{\alpha\gamma}$ does **not** create a flow!
- Pictorially, problem seen by arrows pointing towards or away from each other

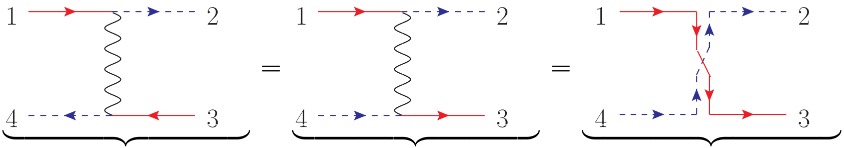
$$\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\bar{\tau}_{\mu,\gamma\dot{\eta}} =$$



Photon exchange: The arrow flip

- Can fix with charge conjugation of a current
  - $\lambda_i^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tilde{\lambda}_j^{\dot{\beta}} = \tilde{\lambda}_{j,\dot{\alpha}}\tau^{\mu,\dot{\alpha}\beta}\lambda_{i,\beta}$
- Or in pictures, an arrow flip:
  - 

- Can replace $\tau \leftrightarrow \bar{\tau}$ if also replace the spinors
- Considering the complete diagram we have:



$(\lambda_1^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tilde{\lambda}_2^{\dot{\beta}})(\lambda_3^{\gamma}\bar{\tau}_{\mu,\gamma\dot{\eta}}\tilde{\lambda}_4^{\dot{\eta}})$ $\quad$ $(\lambda_1^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tilde{\lambda}_2^{\dot{\beta}})(\lambda_{4,\dot{\eta}}\tau_{\mu}^{\dot{\eta}\gamma}\tilde{\lambda}_{3,\gamma})$ $\quad$ $\lambda_1^{\alpha}\lambda_{3,\alpha}\tilde{\lambda}_{4,\dot{\beta}}\tilde{\lambda}_2^{\dot{\beta}} \equiv \langle 13\rangle[42]$

Creating a helicity flow picture

- Here we have used
  - $\lambda_i^{\alpha}\lambda_{j,\alpha} = \langle ij \rangle =$ 

  - $\tilde{\lambda}_{i,\dot{\beta}}\tilde{\lambda}_j^{\dot{\beta}} = [ij] =$ 

- and before we had
  - $\delta_{\alpha}^{\ \beta} =$ 

  - $\delta^{\dot{\beta}}_{\ \dot{\alpha}} =$ 

  analogous to QCD $\delta_{ij} =$ 

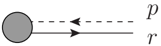
  (color delta function)
- In general we let
  - $\lambda_{j,\alpha} =$ 

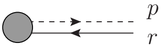
    , $\lambda_i^{\alpha} =$ 

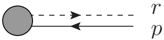
  - $\tilde{\lambda}_{i,\dot{\alpha}} =$ 

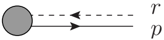
    , $\tilde{\lambda}_j^{\dot{\alpha}} =$ 

Creating a helicity flow picture: external photons

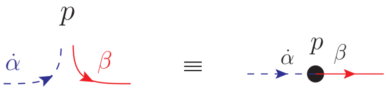
- We also need external photons
  - $\varepsilon_+^\mu(p,r) = \frac{\tilde{\lambda}_{p,\dot{\alpha}}\tau^{\mu,\dot{\alpha}\beta}\lambda_{r,\beta}}{\langle rp\rangle}$, $\varepsilon_-^\mu(p,r) = \frac{\lambda_p^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tilde{\lambda}_r^{\dot{\beta}}}{[pr]}$
- External photons are just $f\bar{f}\gamma$ vertices with a denominator
- So we can Fierz (with possible arrow swap) any external photon
- $\varepsilon_+^\mu(p,r) \to \frac{1}{\langle ri\rangle}$ 

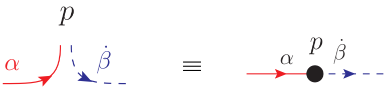
, or $\varepsilon_+^\mu(p,r) \to \frac{1}{\langle ri\rangle}$ 

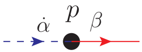
- $\varepsilon_-^\mu(p,r) \to -\frac{1}{[ri]}$ 

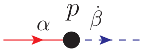
, or $\varepsilon_-^\mu(p,r) \to -\frac{1}{[ri]}$ 

Creating a helicity flow for QED: fermion propagators

- So far: vertices, internal and external photons, external fermions
- Missing QED piece: Fermion propagators, containing $\not{p}$
- We split $\not{p}_{4\times4} \equiv p_\mu \gamma^\mu$ split into two terms
  - $\not{p} = \sqrt{2} p^\mu \tau_\mu^{\dot{\alpha}\beta} \overset{p^2=0}{=} \tilde{\lambda}_p^{\dot{\alpha}} \lambda_p^\beta =$ 

  - $\bar{\not{p}} = \sqrt{2} p_\mu \bar{\tau}^\mu_{\alpha\dot{\beta}} \overset{p^2=0}{=} \lambda_{p,\alpha} \tilde{\lambda}_{p,\dot{\beta}} =$ 

- For massless tree-level propagators we have $p^\mu = \sum p_i^\mu$, $p_i^2 = 0$
- Convenient shorthand:
  - $\not{p} =$ 

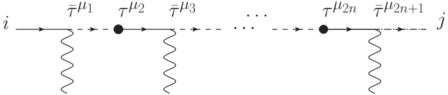
$= \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^\beta$ for $p_i^2 = 0$
  - $\bar{\not{p}} =$ 

$= \sum_i \lambda_{i,\alpha} \tilde{\lambda}_{i,\dot{\beta}}$ for $p_i^2 = 0$

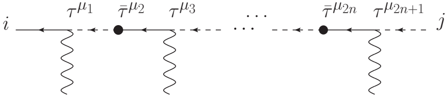
Add extra fermion lines

- What if we now exchange more than 2 pairs of fermions?
- Can we still use the flow picture?
  - Yes (at least at tree level)
  - Conjugation of a current holds for full fermion line
  - $\lambda_i \bar{\tau}^{\mu_1} \tau^{\mu_2} \ldots \bar{\tau}^{\mu_{2n+1}} \tilde{\lambda}_j = \tilde{\lambda}_j \tau^{\mu_{2n+1}} \bar{\tau}^{\mu_{2n}} \ldots \tau^{\mu_1} \lambda_i$
- Pictorially:

$\lambda_i \bar{\tau}^{\mu_1} \tau^{\mu_2} \ldots \bar{\tau}^{\mu_{2n+1}} \tilde{\lambda}_j =$



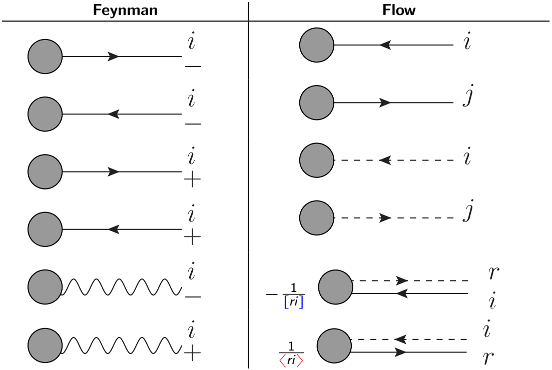
$=$



$= \tilde{\lambda}_j \tau^{\mu_{2n+1}} \bar{\tau}^{\mu_{2n}} \ldots \tau^{\mu_1} \lambda_i$

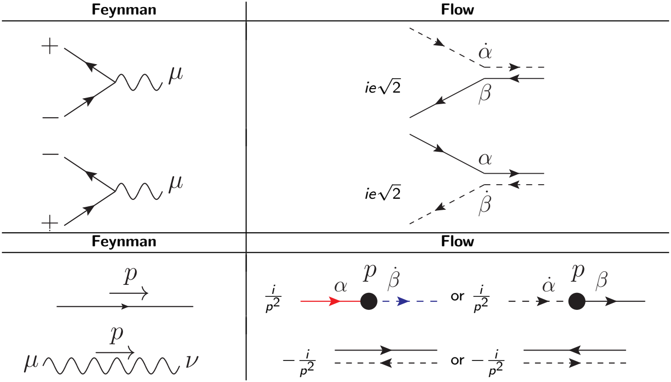
- i.e. arrow swap (and Fierz) works for any fermion line!

The QED flow rules: external particles



Everything already Fierzed, in terms of spinors

The QED flow rules: vertices and propagators

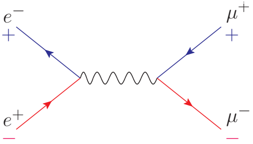


Everything already Fierzed, in terms of spinors

QED examples (massless)

Simplest QED example, all particles outgoing

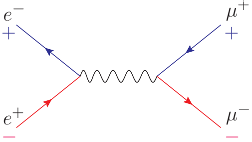
- Regular spinor-helicity ≡ easy



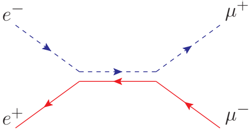
$$= \frac{2ie^2}{s_{e^+e^-}} (\tilde{\lambda}_{e^-,\dot{\alpha}} \tau_{\mu}^{\dot{\alpha}\beta} \lambda_{e^+,\beta})(\lambda_{\mu^-}^{\alpha} \bar{\tau}^{\mu}_{\alpha\dot{\beta}} \tilde{\lambda}^{\dot{\beta}}_{\mu^+})$$

$$= \frac{2ie^2}{s_{e^+e^-}} \tilde{\lambda}_{e^-,\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}}_{\mu^+} \lambda^{\beta}_{\mu^-} \lambda_{e^+,\beta} \equiv \frac{2ie^2}{s_{e^+e^-}} [e^- \mu^+] \langle \mu^- e^+ \rangle$$

- Helicity flow ≡ super easy and intuitive

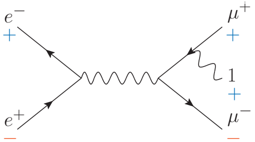


$$= \frac{2ie^2}{s_{e^+e^-}}$$



Next simplest QED example

- Regular spinor-helicity $\equiv$ easy



$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{\mu+1}} \left(\tilde\lambda_{e-,\dot\alpha}\tau_\mu^{\dot\alpha\beta}\lambda_{e+,\beta}\right)\left(\lambda^\alpha_{\mu-}\bar\tau^\mu_{\alpha\dot\beta}\underbrace{(\not p_1+\not p_{\mu+})^{\dot\beta\eta}}_{\not p_i=\tilde\lambda_i^{\dot\beta}\lambda_i^\eta}\underbrace{\not\epsilon^+_{\eta\dot\gamma}(1,r)}_{\frac{\lambda_{r,\eta}\tilde\lambda_{1,\dot\gamma}}{\langle r1\rangle}}\tilde\lambda^{\dot\gamma}_{\mu+}\right)$$

$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{\mu+1}\langle r1\rangle} \left(\tilde\lambda_{e-,\dot\alpha}\tau_\mu^{\dot\alpha\beta}\lambda_{e+,\beta}\right)\tilde\lambda_{1,\dot\gamma}\tilde\lambda^{\dot\gamma}_{\mu+}$$

$$\times\left(\lambda^\alpha_{\mu-}\bar\tau^\mu_{\alpha\dot\beta}\tilde\lambda_1^{\dot\beta}\lambda_1^\eta\lambda_{r,\eta}+\lambda^\alpha_{\mu-}\bar\tau^\mu_{\alpha\dot\beta}\tilde\lambda_{\mu+}^{\dot\beta}\lambda_{\mu+}^\eta\lambda_{r,\eta}\right)$$

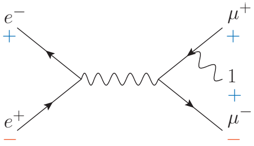
$$\sim\left(\tilde\lambda_{e-,\dot\alpha}\tilde\lambda_1^{\dot\alpha}\lambda_1^\eta\lambda_{r,\eta}+\tilde\lambda_{e-,\dot\alpha}\tilde\lambda_{\mu+}^{\dot\alpha}\lambda_{\mu+}^\eta\lambda_{r,\eta}\right)\tilde\lambda_{1,\dot\gamma}\tilde\lambda^{\dot\gamma}_{\mu+}\lambda^\beta_{\mu-}\lambda_{e+,\beta}$$

Correct Answer

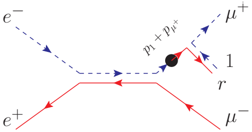
$$\frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{\mu+1}\langle r1\rangle}\left([e^-1]\langle 1r\rangle+[e^-\mu^+]\langle\mu^+r\rangle\right)[1\mu^+]\langle\mu^-e^+\rangle$$

Next simplest QED Example

- Helicity flow $\equiv$ super easy and intuitive



$$= \frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{\mu^+1}\langle r1\rangle}$$



- Immediately read off inner products

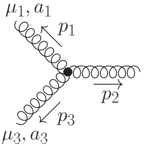
Correct Answer

$$\frac{-i2\sqrt{2}e^3}{s_{e^+e^-}s_{\mu^+1}\langle r1\rangle}\Big([e^-1]\langle 1r\rangle + [e^-\mu^+]\langle \mu^+ r\rangle\Big)[1\mu^+]\langle \mu^- e^+\rangle$$

QCD helicity flow (massless)

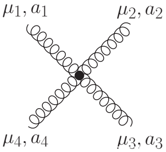
Extending to QCD: What's different?

- Color is added – can be stripped away so no problem
- Non-abelian vertices:
  - 3-gluon:



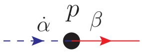
$$= -\frac{g_s f^{abc}}{\sqrt{2}} g^{\mu_1\mu_2}(p_1 - p_2)^{\mu_3} + \circlearrowleft$$

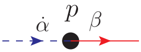
  - 4-gluon:

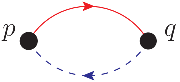


$$= i g_s^2 \sum_{Z(2,3,4)} f^{a_1 a_2 b} f^{b a_4 a_3} \left(g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_3} g^{\mu_2\mu_4} \right)$$

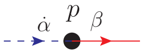
Momentum: The last piece of the flow puzzle

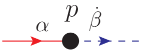
- Recall $p^\mu = \frac{1}{\sqrt{2}} \lambda_p^{\alpha} \bar{\tau}^{\mu}_{\alpha\dot{\beta}} \tilde{\lambda}_p^{\dot{\beta}} = \frac{1}{\sqrt{2}} \tilde{\lambda}_{p,\dot{\alpha}} \tau^{\mu\dot{\alpha}\beta} \lambda_{p,\beta}$
  - $\Rightarrow$ we can see p^μ as a pseudo-vertex!
  - $\Rightarrow$ we can use it as a helicity flow!
- What does p^μ get contracted with?
  - $\tau_\mu \rightarrow \not{p}/\sqrt{2} = \frac{1}{\sqrt{2}}$ 

, $\bar{\tau}_\mu \rightarrow \bar{\not{p}}/\sqrt{2} = \frac{1}{\sqrt{2}}$ 

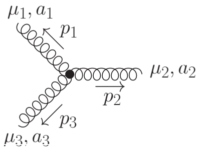
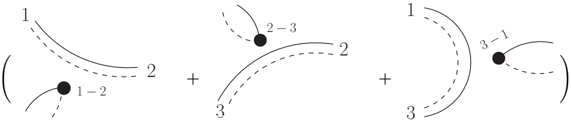
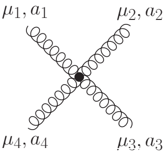
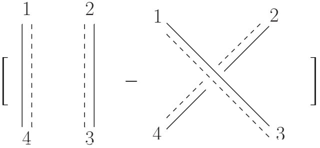
  - $k_\mu \rightarrow p \cdot k = \frac{\mathrm{Tr}(\not{p}\bar{\not{k}})}{2} = \frac{1}{2}$ 

- To conclude, we can always write

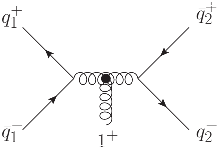
$p^\mu \rightarrow$ 

, or $p^\mu \rightarrow$ 

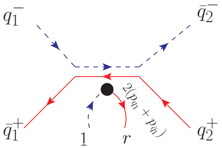
The non-abelian massless QCD flow vertices

Feynman	Flow
	$-\frac{g_s f^{abc}}{2}$ 
	$ig_s^2 \sum_{Z(2,3,4)} f^{a_1 a_2 b} f^{b a_4 a_3}$ 

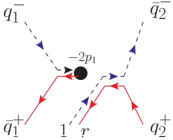
QCD example: $q_1\bar{q}_1 \to q_2\bar{q}_2 g$



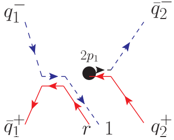
$$= \frac{i g_s^3}{2 s_{q_1\bar{q}_1} s_{q_2\bar{q}_2} \langle r1 \rangle} \Big[$$



$+$



$+$



$\Big]$

$$\Big[\,\Big] \equiv \Big\{ 2[q_1\bar{q}_2]\langle q_2\bar{q}_1\rangle\big([1q_1]\langle q_1 r\rangle + [1\bar{q}_1]\langle 1 r\rangle\big)$$

$$- 2[q_1 1]\langle 1\bar{q}_1\rangle\langle q_2 r\rangle[1\bar{q}_2] + 2[q_1 1]\langle r\bar{q}_1\rangle\langle q_2 1\rangle[1 q_2]\Big\}$$

Conclusion and outlook

Conclusion

- The helicity flow formalism gives a transparent and intuitive way of understanding the Lorentz inner products appearing in amplitudes
  - Spinor helicity formalism: 4×4 matrices $\gamma^\mu \rightarrow$ to 2×2 matrices σ^μ
  - Helicity flow method: 2×2 matrices $\sigma^\mu \rightarrow$ scalars
- Shorter calculation of Feynman diagrams
- In contrast, spinor-hel method:
  - Requires intermediate steps
  - Final result non-transparent/unintuitive
- Massless QED and QCD tree-level done, initial paper coming soon
- Should be useful for any generator using diagrams to avoid dealing with Lorentz algebra

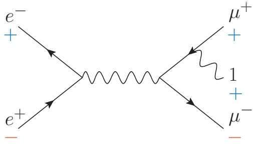
Outlook

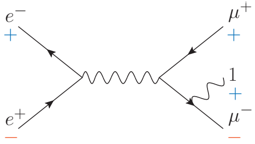
- Add masses complicates calculations, but seems doable...
- Electroweak sector easy?
- Loop calculations
- Applications within generators
- Amplitude-level calculations

Backup Slides

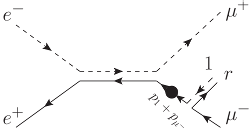
A word about reference momenta

- Reference momentum r represents a gauge choice
- Only require $r^2 = 0, r \cdot p_1 \neq 0$
- Choose r to simplify life the most
  - r can be different for each gauge-invariant sum
- analogy with QCD color-ordering

$\Rightarrow$ 

$+$ 

is gauge invariant

- Inner product is anti-symmetric ($\langle ii \rangle = [ii] = 0$)
- Choosing $r = \mu^- \Rightarrow$ 

$= 0$

What if $k^2 \neq 0$

For momenta with $k^2 = m^2$ we can use a decomposition. Consider an arbitrary light-like four-vector a^μ with $a^2 = 0,\ k \cdot a \neq 0$ and define

$$\alpha = \frac{m^2}{2a \cdot k}, \quad k'^\mu = k^\mu - \alpha a^\mu$$

such that

$$k^\mu = \alpha a^\mu + k'^\mu$$

with

$$k'^2 = k^2 - 2\alpha a \cdot k = m^2 + 2\frac{m^2}{2a \cdot k} a \cdot k = 0$$

So we can treat a massive spinor as a linear combination of two massless spinors