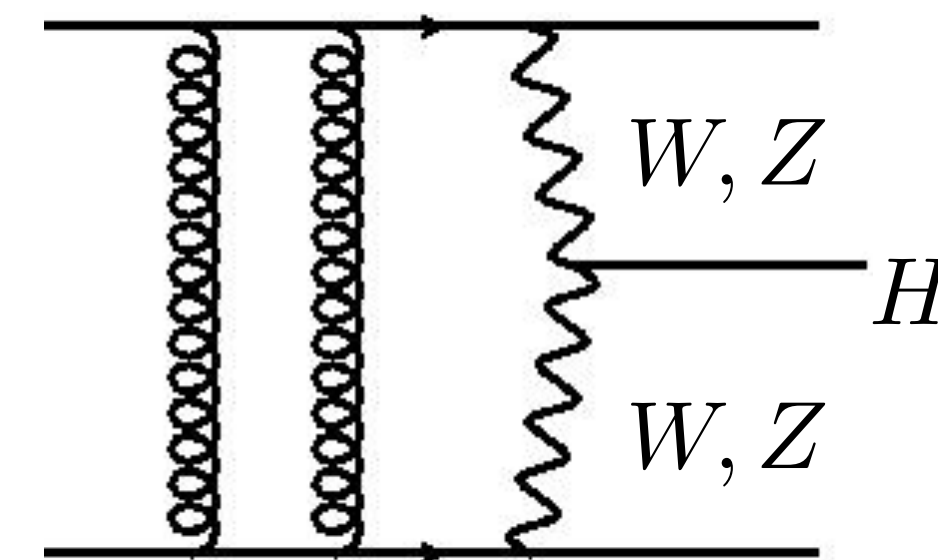


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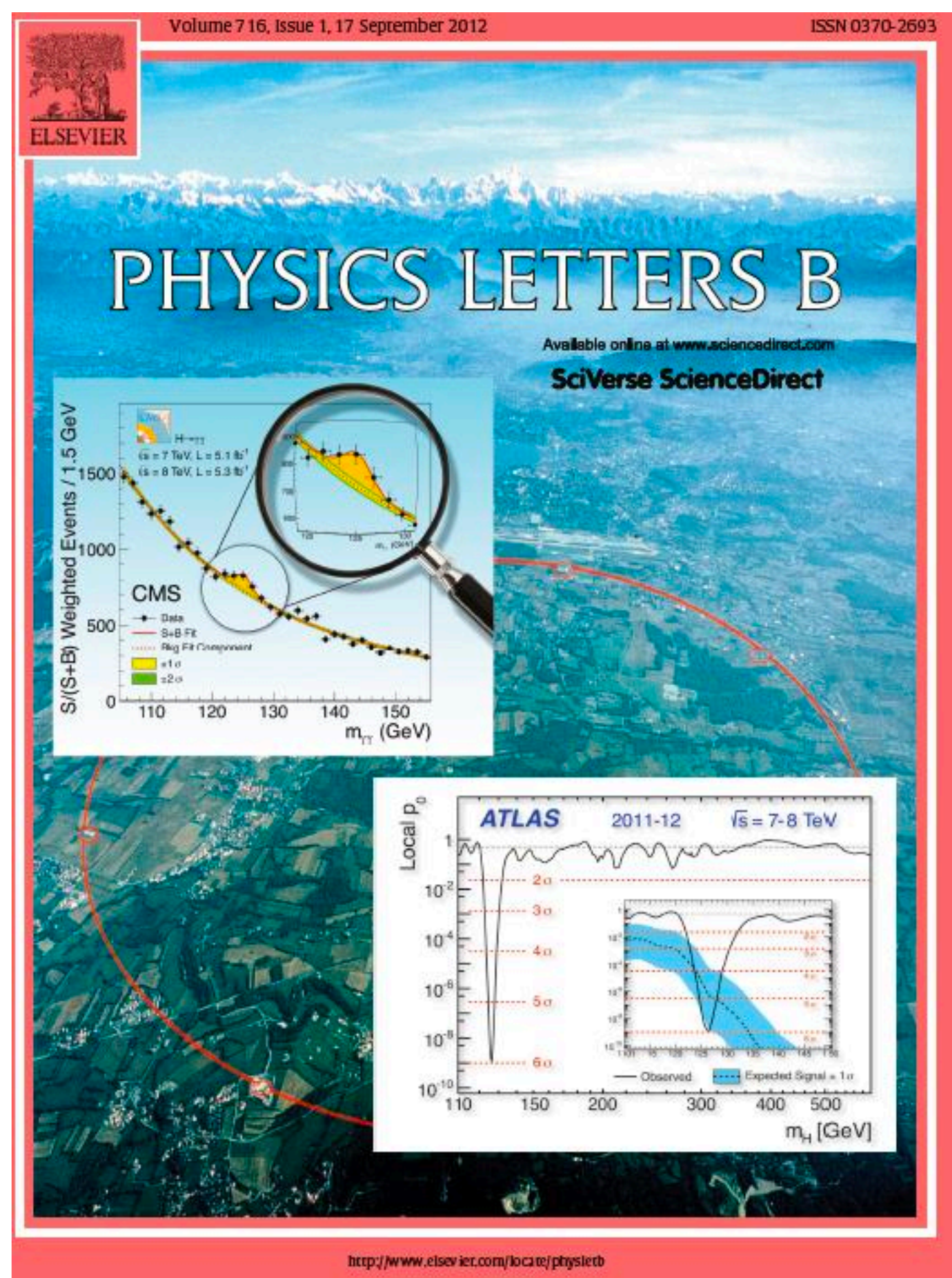
HIGGS BOSON PRODUCTION IN WEAK BOSON FUSION: NON-FACTORIZABLE QCD CORRECTIONS

In collaboration with A. Penin and T. Liu



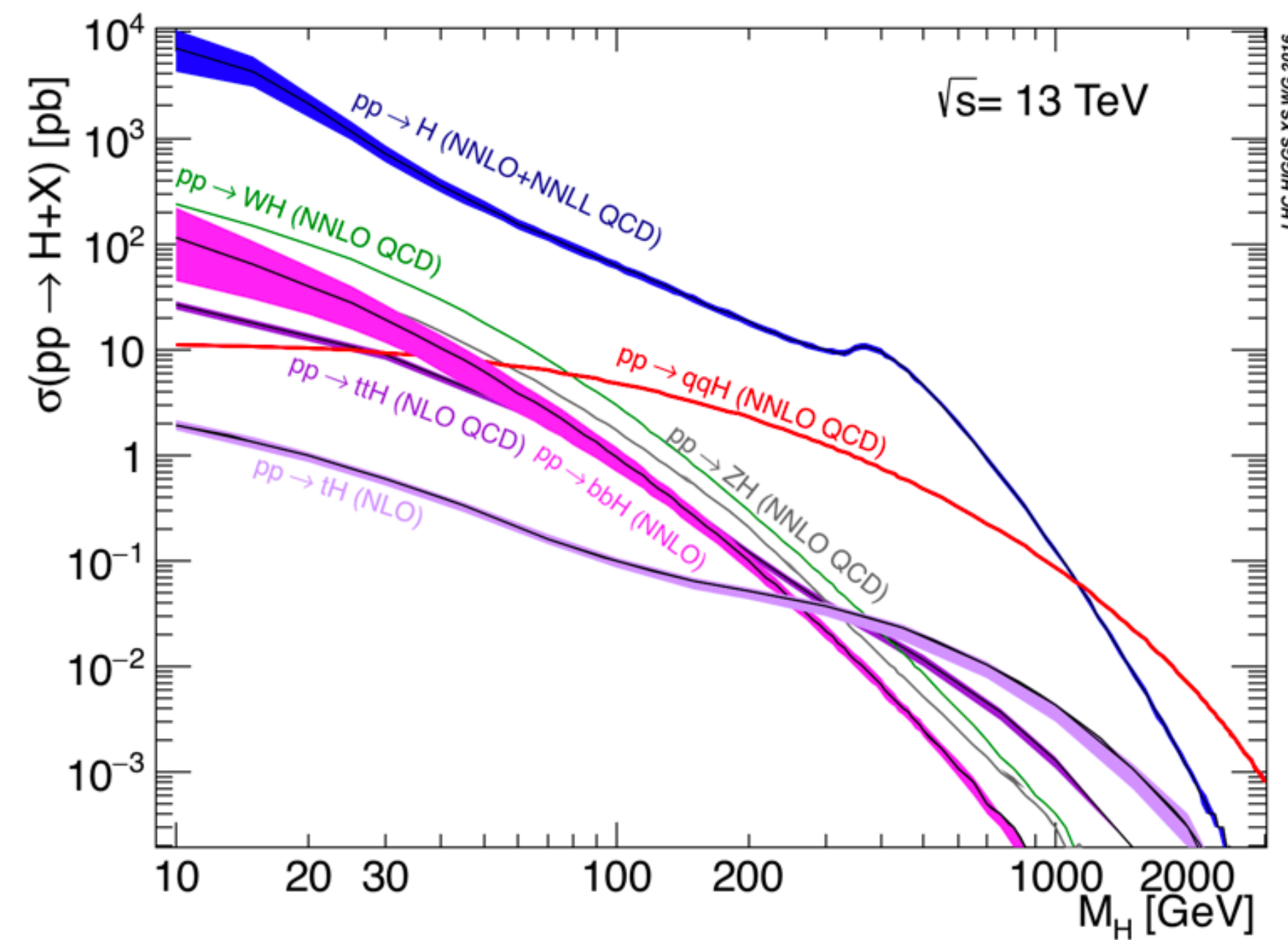
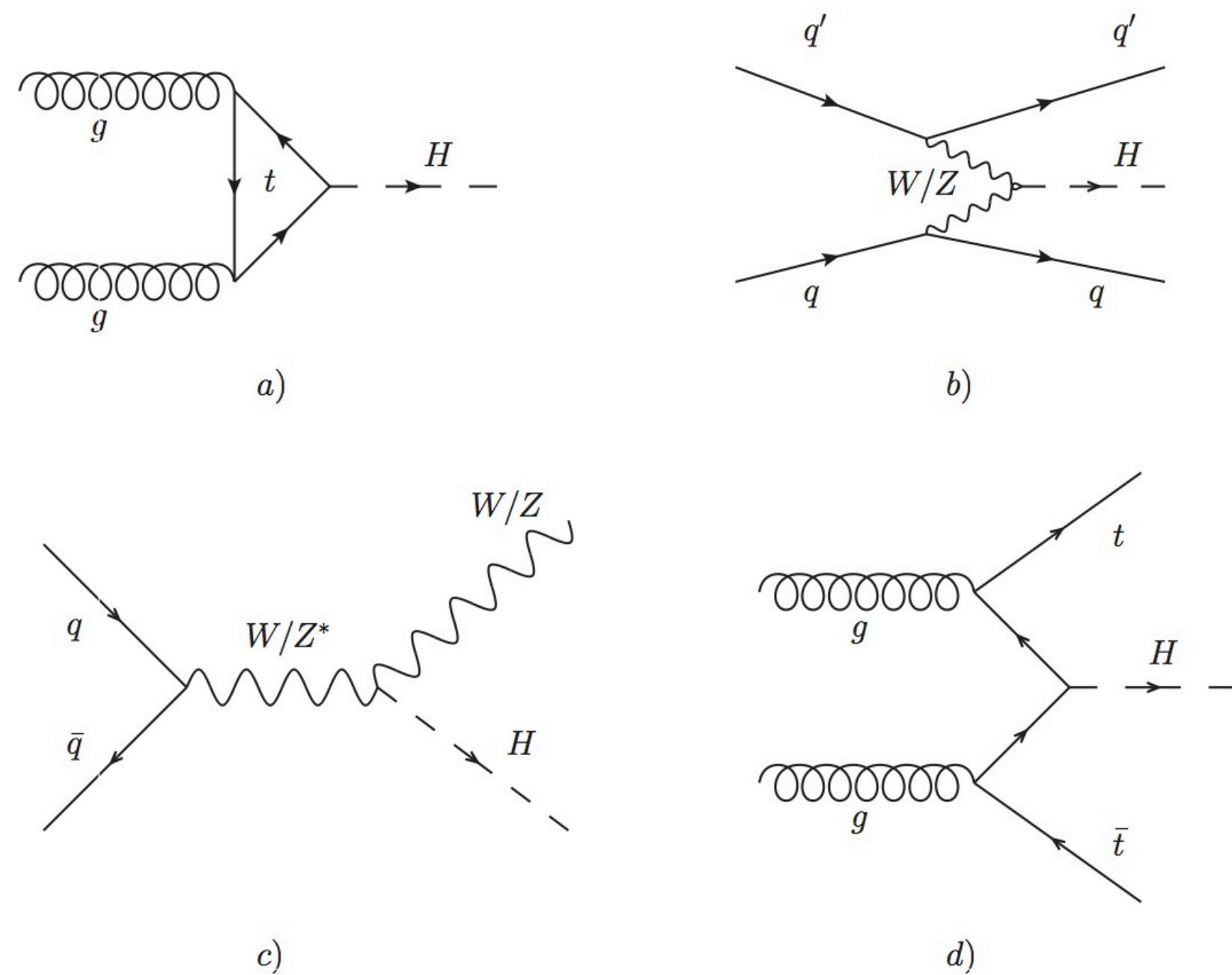
The Higgs discovery

Higgs boson discovery structurally completed the Standard Model and launched an era of the detailed exploration of this particle.



Higgs boson production

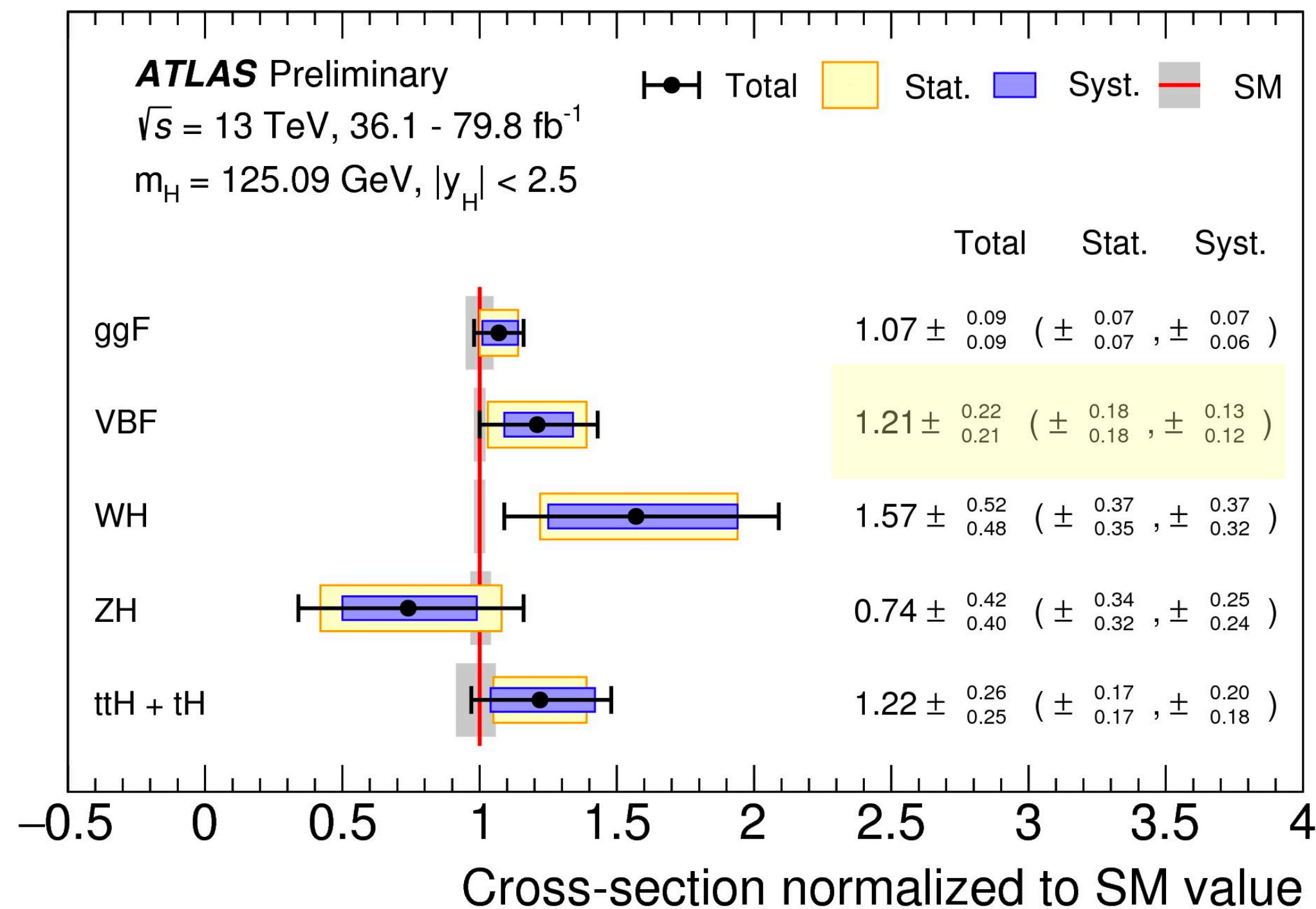
Higgs boson can be produced in a variety of ways in the SM. For the 125 GeV Higgs, the weak boson fusion cross section is next-to-largest. Higgs bosons also decay in a variety of ways offering many opportunities to detect them.



Higgs boson properties are known but not quite

Higgs boson properties are in good agreement with the SM expectations. Nevertheless, there is still room for O(10-20) percent deviations even in the largest cross sections and the best known couplings.

Weak boson fusion cross sections are measured to about 20 percent; uncertainties are still dominated by statistics (note that the data set is still incomplete).



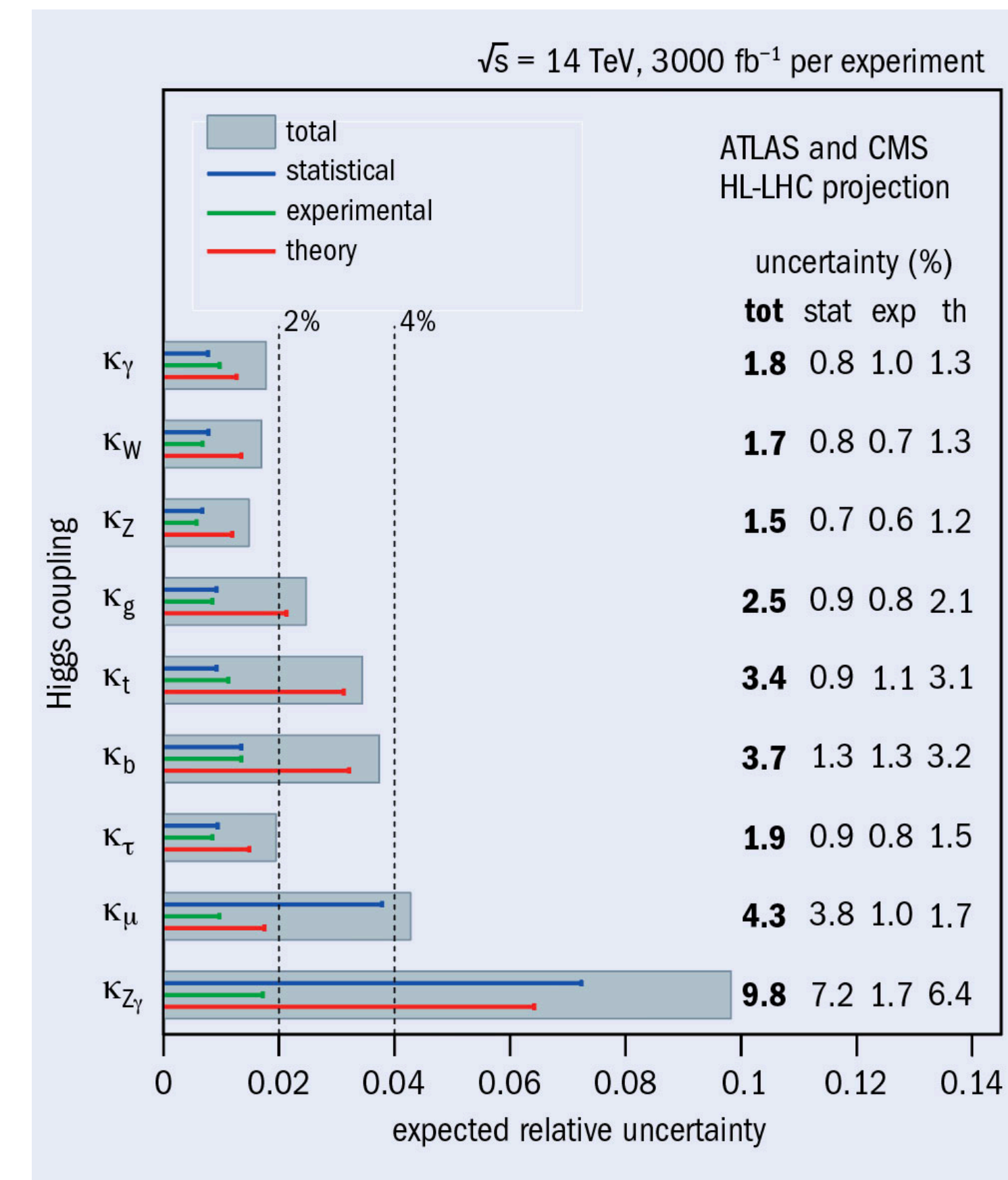
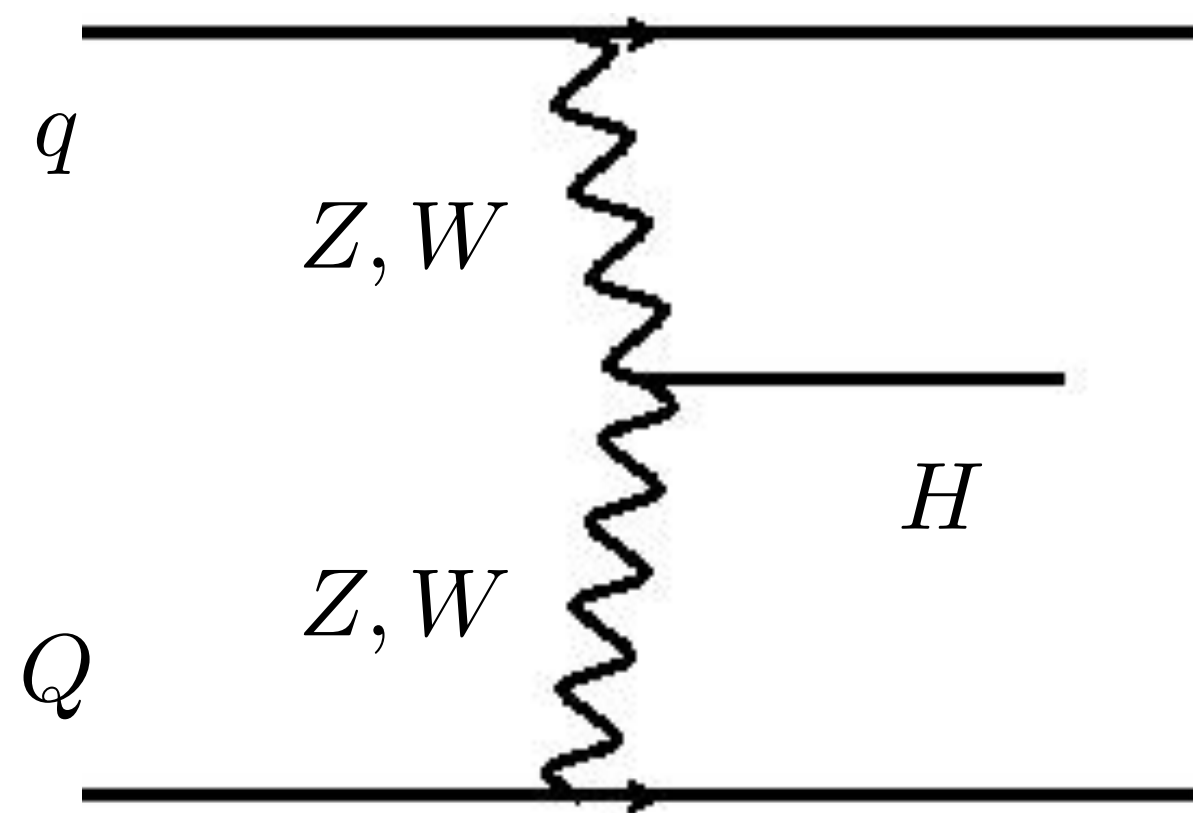
Production process	Best fit value	Uncertainty	
		stat.	syst.
ggH	1.22	$+0.14$ -0.12 (+0.11) (-0.11)	$+0.08$ -0.08 (+0.07) (-0.07)
VBF	0.73	$+0.30$ -0.27 (+0.29) (-0.27)	$+0.24$ -0.23 (+0.24) (-0.23)
WH	2.18	$+0.58$ -0.55 (+0.53) (-0.51)	$+0.46$ -0.45 (+0.43) (-0.42)
ZH	0.87	$+0.44$ -0.42 (+0.43) (-0.41)	$+0.39$ -0.38 (+0.38) (-0.37)
ttH	1.18	$+0.30$ -0.27 (+0.28) (-0.25)	$+0.26$ -0.21 (+0.23) (-0.20)

CMS, 13 TeV, 36 fb⁻¹

Weak boson fusion

Higgs boson production in weak boson fusion (WBF) is interesting because

- a) this process has a relatively large cross section;
- b) the cross section depends on HWW and HZZ couplings that are fixed by Higgs quantum numbers and the SM renormalizability requirement;
- c) the process is sensitive to HVV anomalous couplings;
- d) it provides useful signatures to study CP properties of a Higgs boson;
- e) different Higgs boson decays can be studied.

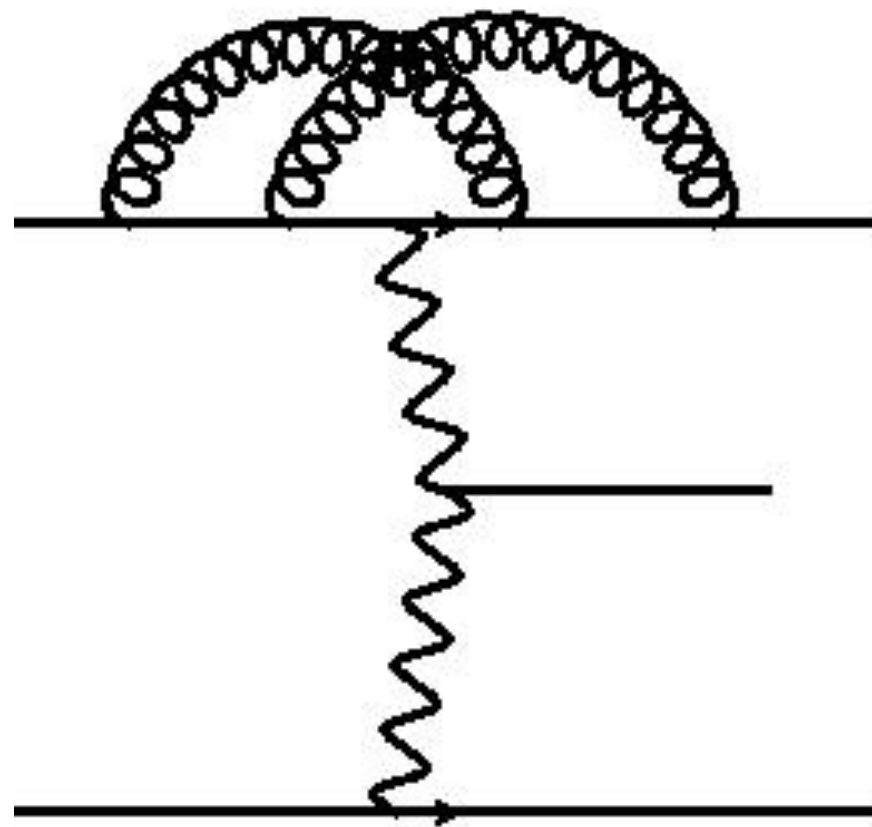


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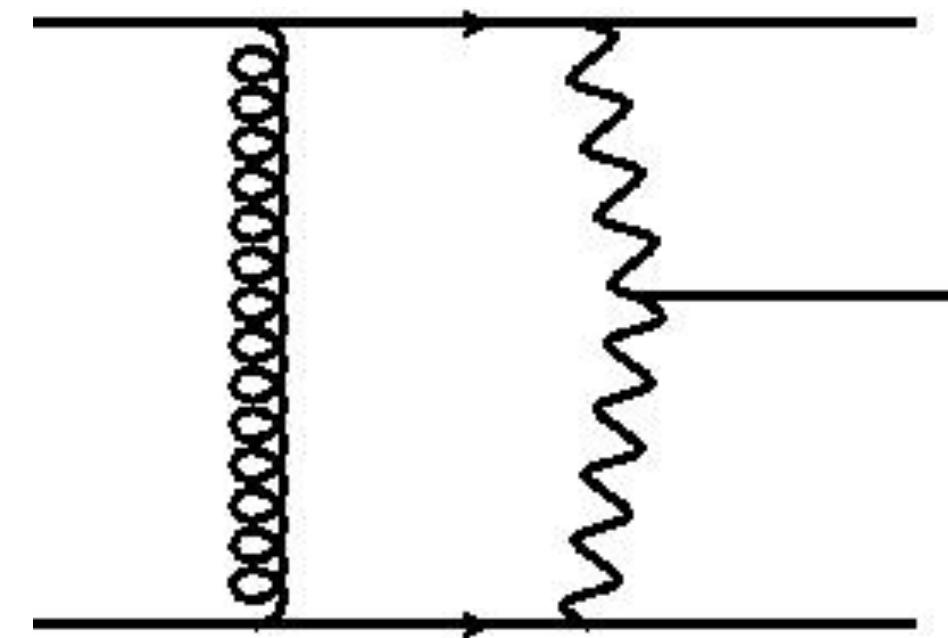
We will focus on the QCD corrections to Higgs boson production in weak boson fusion; these corrections involve two distinct types of contributions:

- a) corrections to the same fermion line (factorizable);
- b) interactions between two fermion lines (non-factorizable).

There are other contributions to WBF that do not fit into this template: the interference of the two fermion lines (e.g. $uu \rightarrow H u u$) or interference of WBF and other contributions ($gg \rightarrow H$ with two forward jets or $V^* H$ production followed by the decay $V^* \rightarrow 2$ jets). These contributions have been computed at leading and sometimes at next-to-leading order in the strong coupling constant and were found to be small (permille).

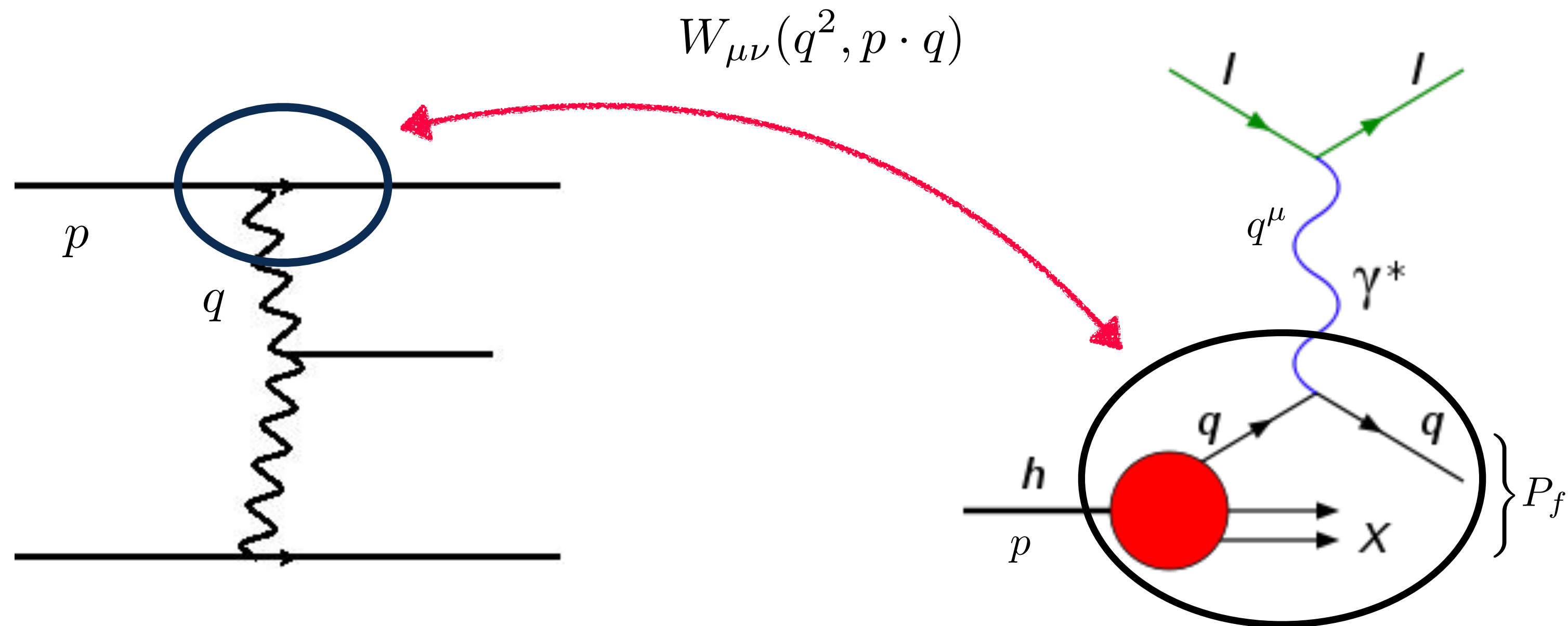


DIS-like, factorizable effects



Non-factorizable corrections: cross-talk between two fermion lines

The DIS-like contribution is called so for a good reason: it can be directly mapped onto the DIS structure functions.



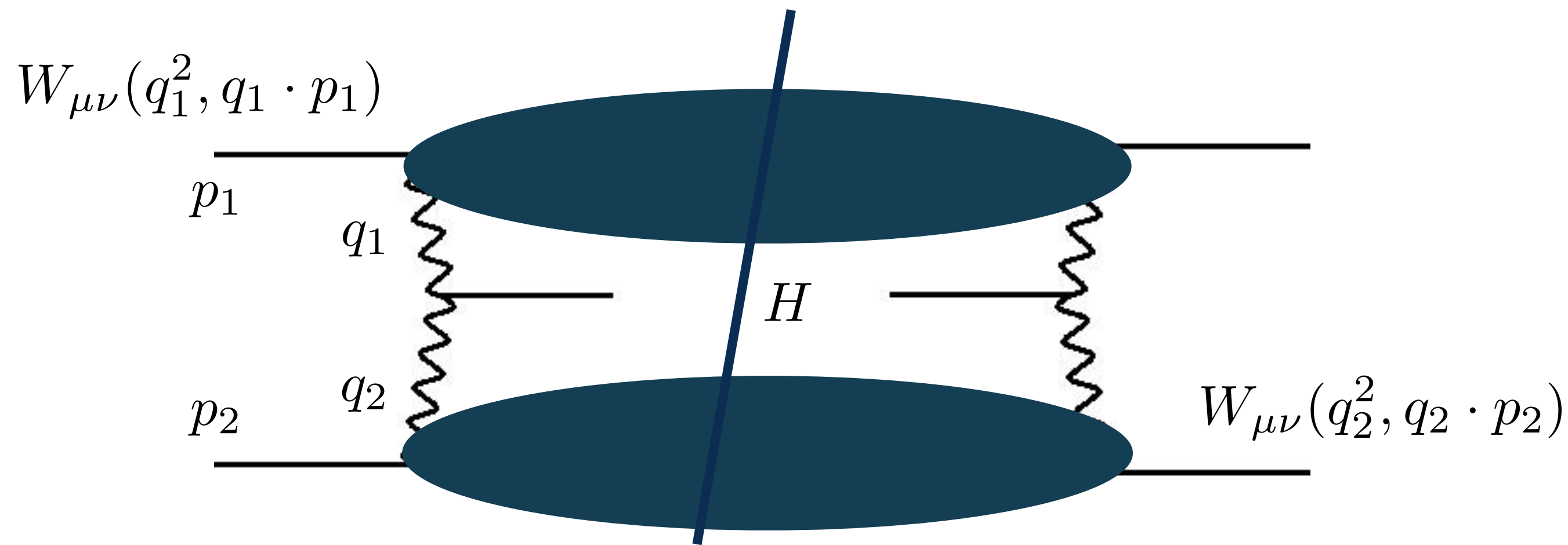
$$W_{\mu\nu} = \frac{1}{2} \sum_{\lambda} \sum_{X_f} \int [dP_f] (2\pi)^d \delta^{(d)}(p - P_f - q) \langle p, \lambda | J^\mu | f \rangle \langle f | J^\nu | p, \lambda \rangle$$

$$W^{\mu\nu} = W_1(q^2, p \cdot q) \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2(q^2, p \cdot q) \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

In case of WBF, where exchanges of electroweak gauge bosons contribute, also structure functions of the axial-vector current are needed. However, all DIS coefficient functions are currently known through N3LO and can be used to describe the WBF.

van Neerven, Zijlstra; Moch, Vermaseren, Vogt

$$W^{\mu\nu} = W_1(q^2, p \cdot q) \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2(q^2, p \cdot q) \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) - i \epsilon^{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{2p \cdot q} W_3(q^2, p \cdot q)$$



$$d\sigma_{\text{VBF}} \sim \frac{(2\pi)^4 \delta^{(4)}(q_1 + q_2 - p_H) d^4 q_1 d^4 q_2}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)} W^{\mu\nu}(q_1^2, q_1 \cdot p_1) W^{\mu'\nu'}(q_2^2, q_2 \cdot p_2) \mathcal{M}_{VV \rightarrow H}^{\mu\mu', \nu\nu'} \frac{d^3 p_H}{2E_H (2\pi)^3}$$

If the DIS structure functions are known up to a certain order in the perturbative expansion in QCD, the WBF cross section can be immediately computed through the same perturbative order as well. In practice, this has been done through N3LO QCD, since this is the perturbative order through which DIS structure functions are known.

Boltoni, Maltoni, Moch, Zaro; Karlberg, Dreyer

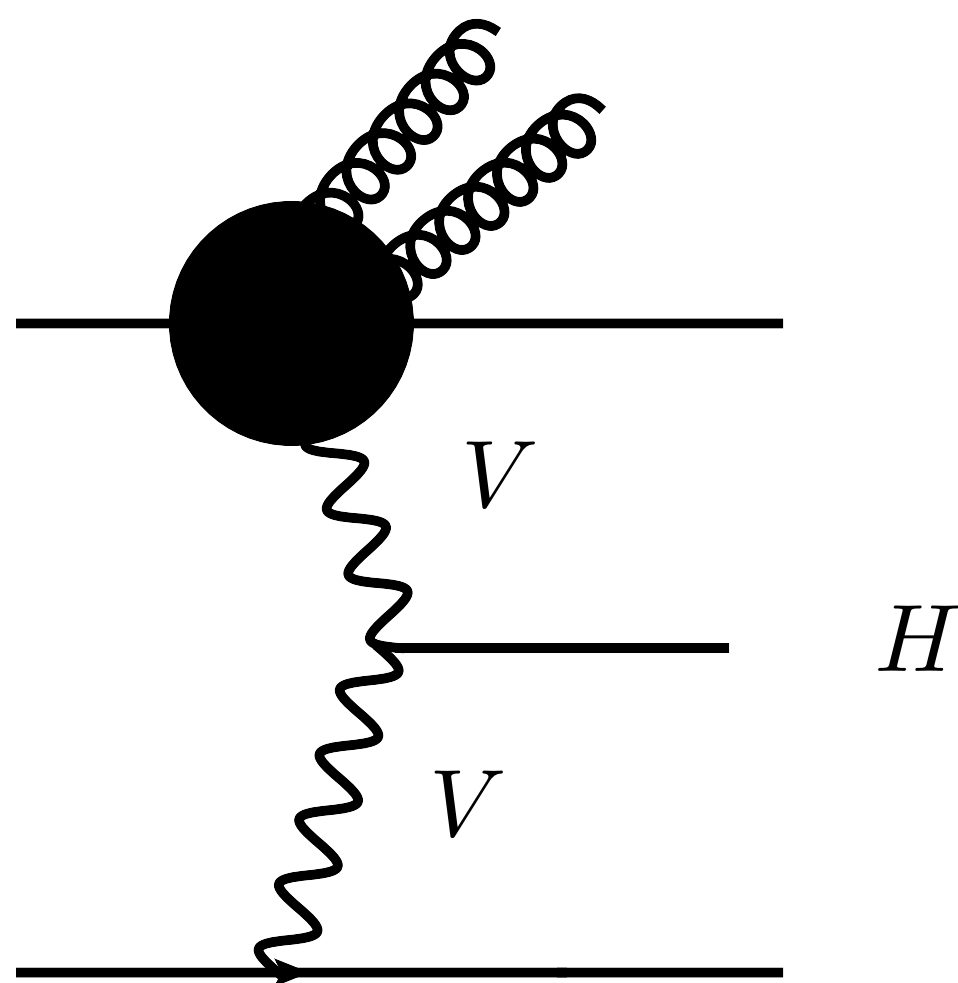
Factorizable QCD corrections to weak boson fusion

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Within the structure functions approach, quite moderate QCD corrections to WBF were found ($\sim 3\%$ at NLO, $\sim 1\%$ at NNLO, $\sim 0.1\%$ at N3LO. However it is unclear to what extent these results are relevant for physics of weak boson fusion.

Indeed, the structure function approach involves integration over partons in the final state and does not allow us to impose constraints on QCD radiation. This is not ideal since WBF cuts are quite severe (the WBF cross section after cuts is only about 20 percent of the cross section without the WBF cuts) and involve forward tagging jets.

For this reason, it is important to perform a fully differential computation (even within the DIS approximation!) that accounts for WBF cuts on the tagging jets.



Typical WBF cuts

$$\begin{aligned} p_{\perp}^{j_{1,2}} &> 25 \text{ GeV}, \quad |y_{j_{1,2}}| < 4.5, \\ \Delta y_{j_1, j_2} &= 4.5, \quad m_{j_1, j_2} > 600 \text{ GeV}, \\ y_{j_1} y_{j_2} &< 0, \quad \Delta R > 0.4 \end{aligned}$$

A [fully differential](#) NNLO QCD computation within the DIS-approximation has been performed using two different methods: projection-to-Born and antenna subtraction.

It was observed that the QCD corrections to the total cross section [with VBF cuts](#) are larger, [by almost a factor of 3](#), than the QCD corrections computed in the structure function approximation.

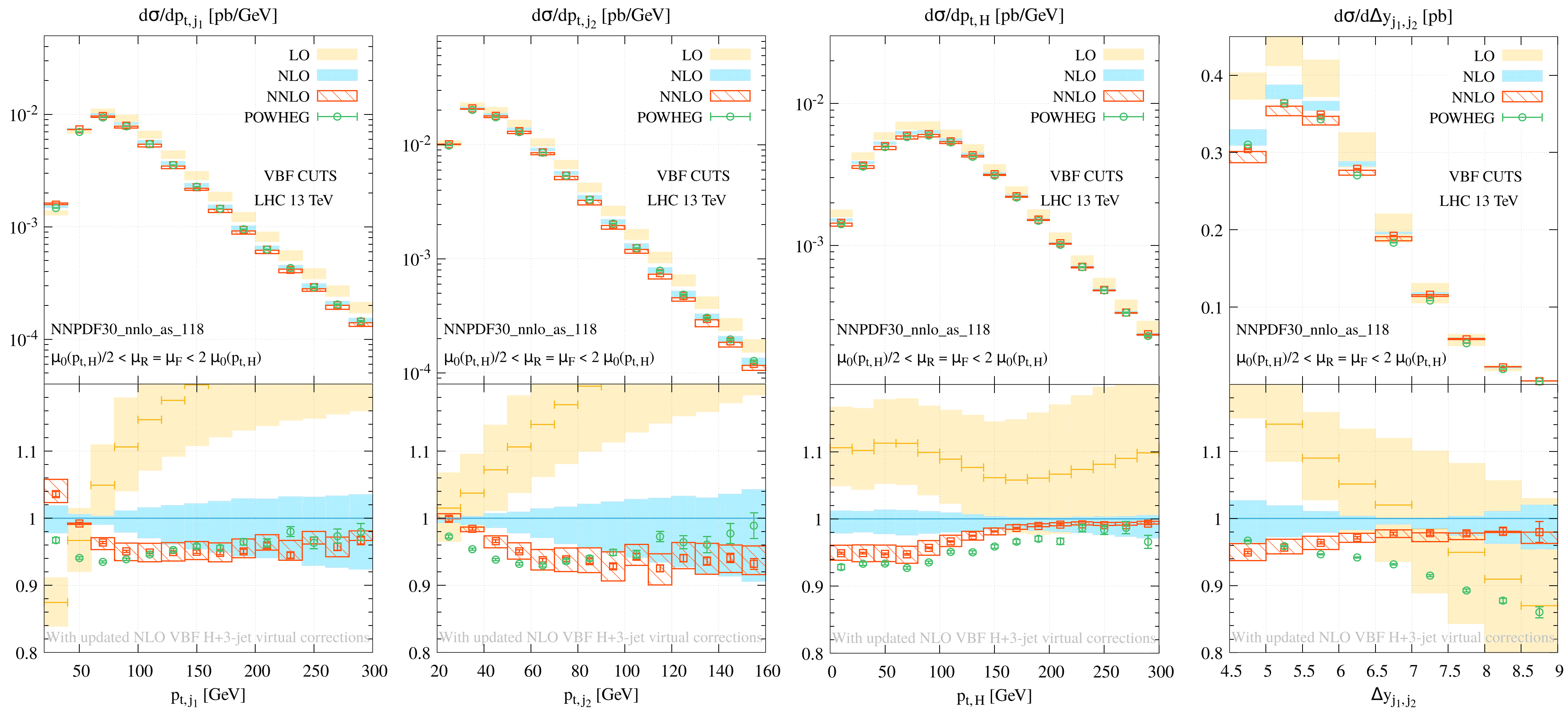
	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.844^{+0.008}_{-0.008}$

Cacciari, Dreyer, Karlberg, Salam, Zanderighi

Cruz-Martinez, Glover, Gehrmann, Huss

NNLO QCD kinematic distributions

Relatively large (4-8 percent) NNLO QCD corrections appear in various kinematic distributions. These corrections are **kinematic-dependent**; they are not flat K-factors that can be obtained by **re-weighting leading order distributions by corrections to the inclusive cross section**. POWHEG catches many of the NNLO QCD corrections to kinematic distributions quite decently.



Towards N3LO QCD predictions for WBF

As we already mentioned, thanks to the connection between WBF and DIS, it is possible to estimate N3LO corrections to WBF. Indeed, since N3LO structure functions in QCD are known, one can integrate them to obtain N3LO corrections to inclusive WBF cross section.

Karlberg, Dreyer

Moreover, although we do not (quite) know how to subtract real-emission singularities at N3LO QCD, the projection-to-Born method allows us to construct the fully differential N3LO cross section once the NNLO QCD corrections to single-jet production in DIS or WBF are known. Such calculation has been done and this implies that N3LO fully differential prediction for WBF is within reach.

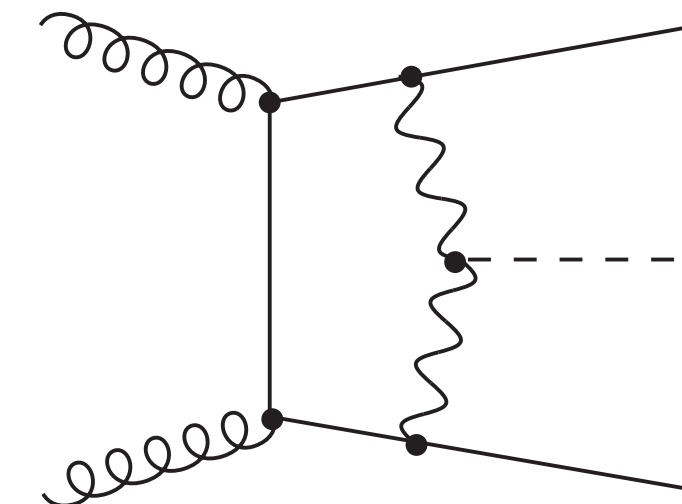
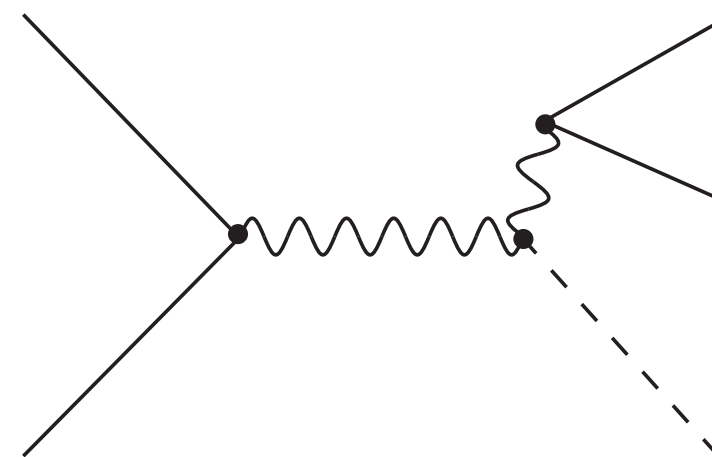
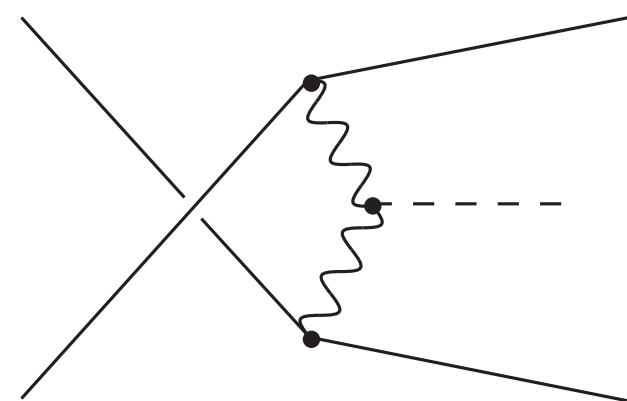
Gehrmann et al.

An opportunity to make such high-order predictions for the WBF process is exciting and perhaps even useful (QCD dynamics of forward jets vs. parton showers). However, it also forces us to think about other effects that are being neglected if we only focus on factorizable, DIS-like contributions.

These effects are:

- 1) non-factorizable gluon exchanges between incoming quark lines;
- 2) t/u channel interferences (identical quarks); they contribute $\mathcal{O}(5\%)$ at the inclusive level and $\mathcal{O}(0.5\%)$ once the WBF cuts are applied;
- 3) HV(jj) final state contribution to WBF final states; they are negligible after WBF cuts;
- 4) contribution of the “single-quark line processes” to WBF; less than a permille after the VBF cuts;

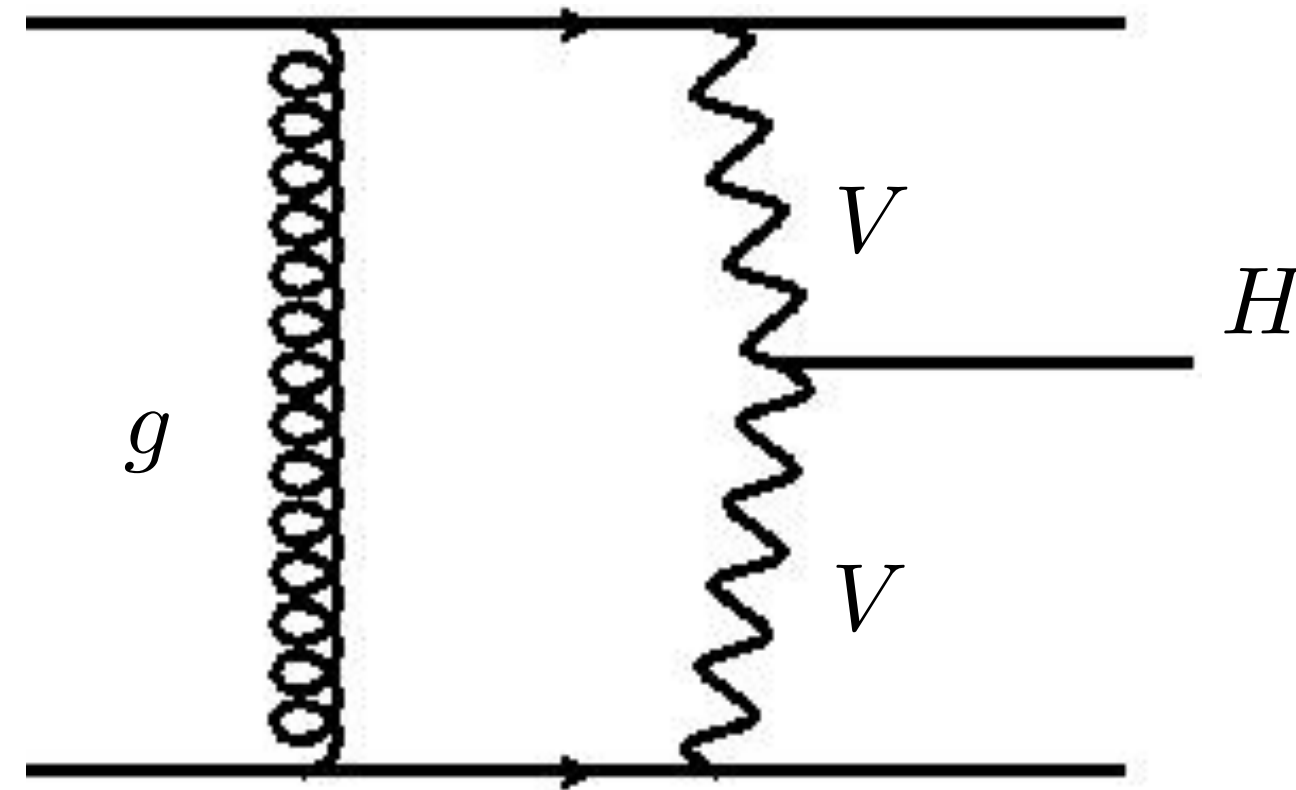
The majority of these effects can be (and has been) studied at lower-orders of perturbation theory and the above estimates are based on that; this cannot be done for non-factorizable gluon exchanges as we explain below. The non-factorizable contribution is, in a way, a very unique unknown.



Non-factorizable contributions

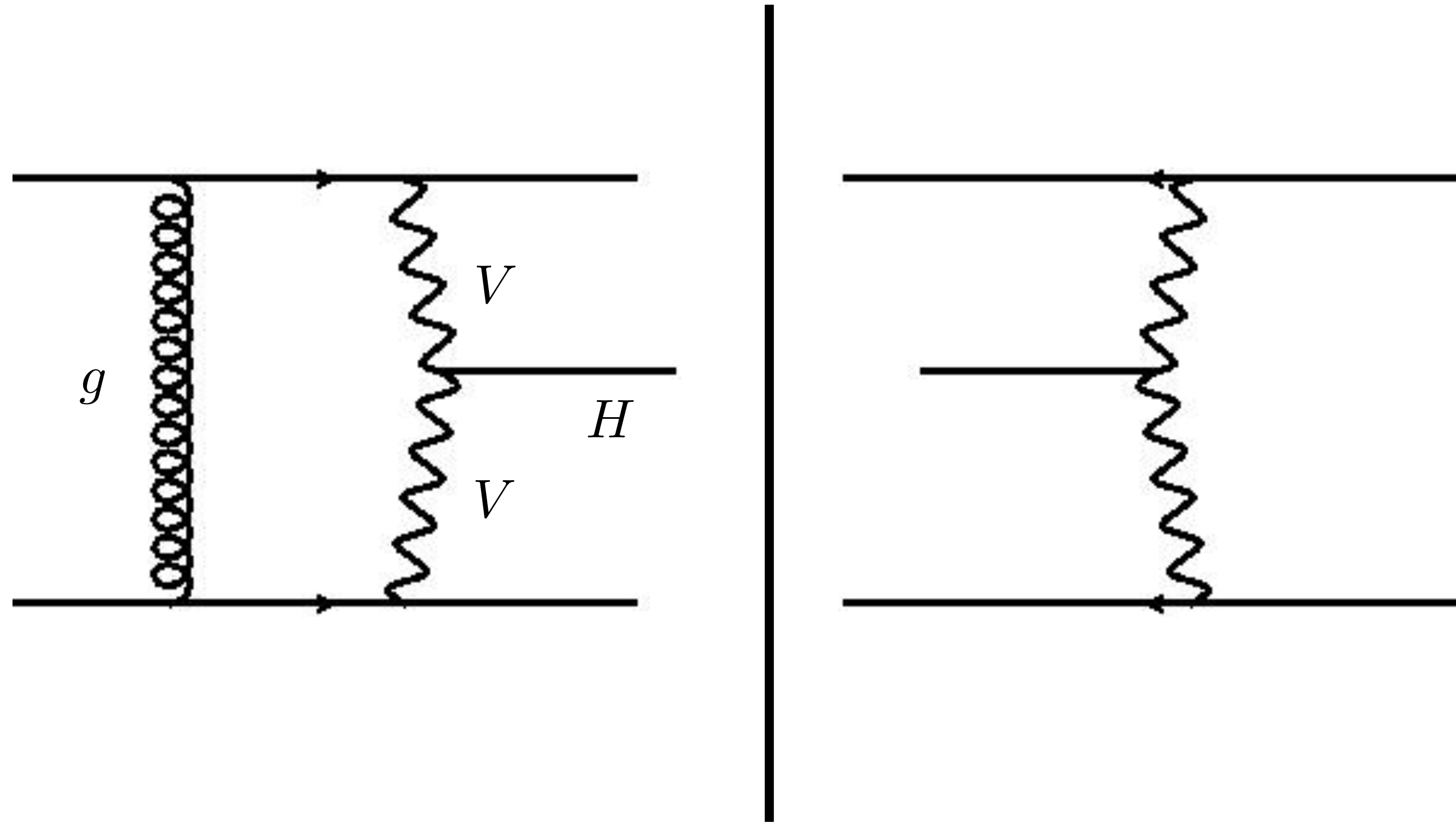
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It may appear that non-factorizable corrections contribute **at next-to-leading order** since the diagram below is definitely not zero.



Non-factorizable contributions

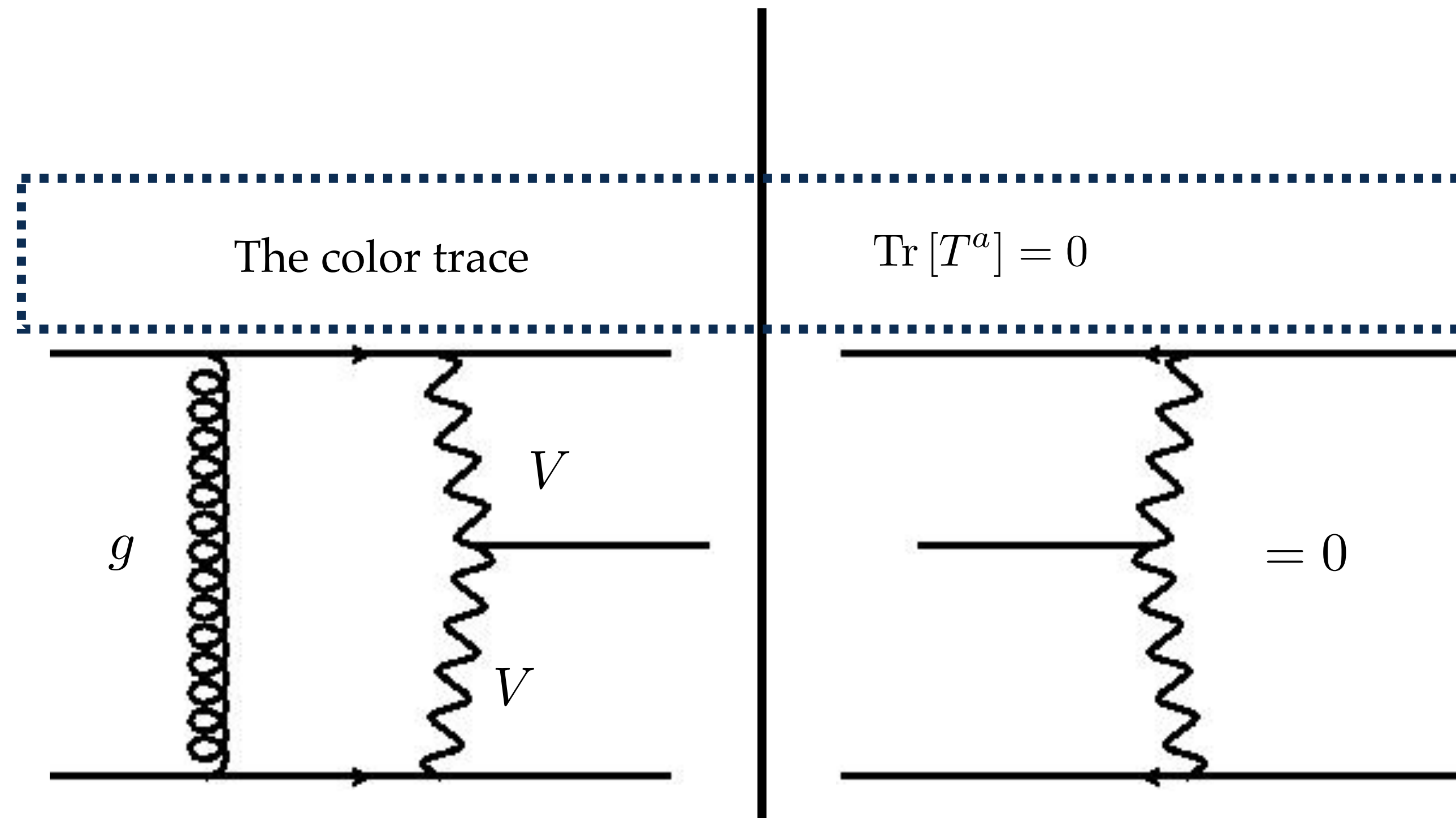
However, this is not the case because of the color conservations...



Non-factorizable contributions

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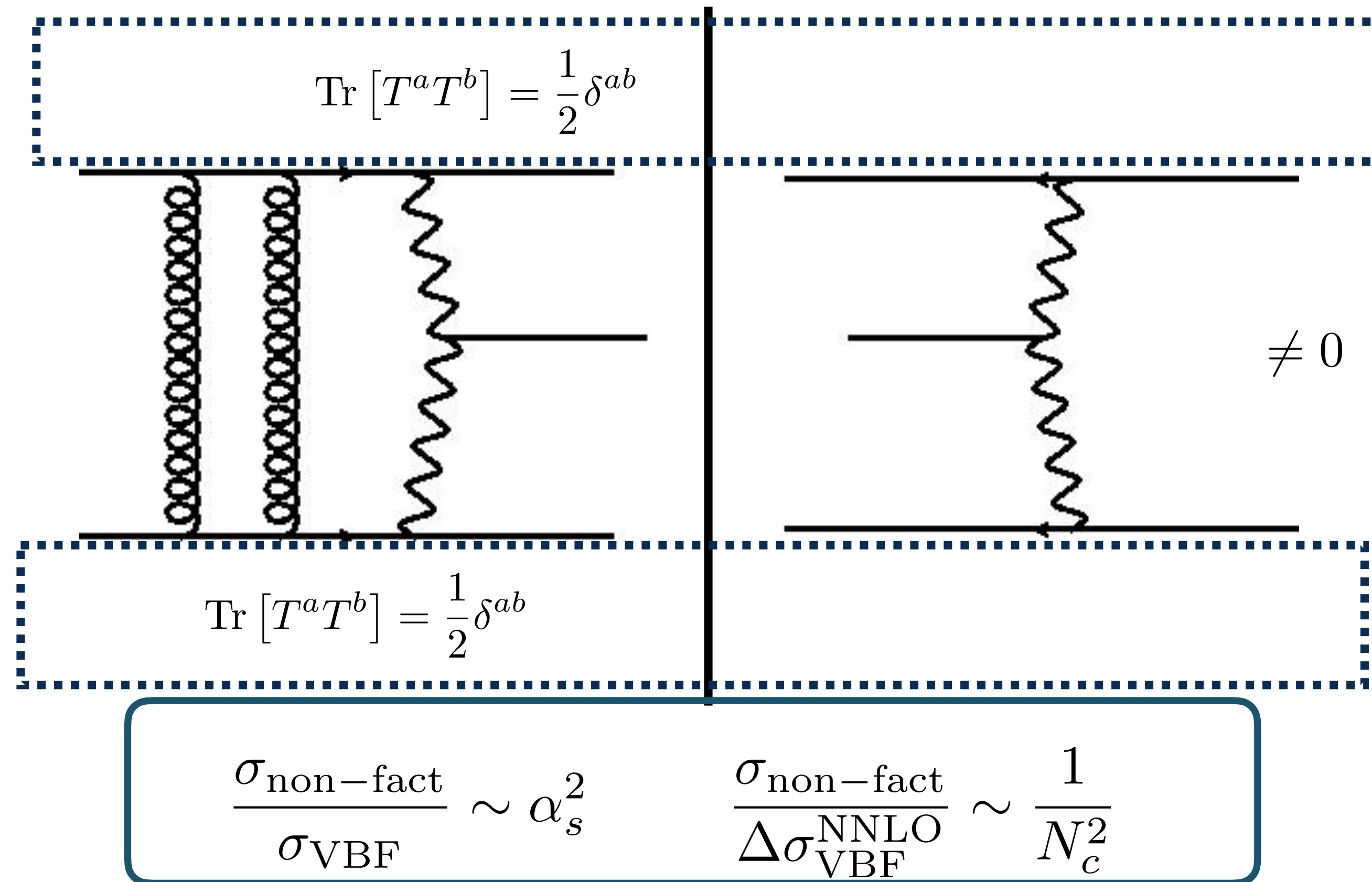
However, this is not the case because of the color conservations... Although we illustrate this for virtual contributions, it is clear that this is the feature of real-emission diagrams as well.



At two loops, the situation changes. If the two gluons exchanged between the quark lines are in the color-singlet state, **the non-factorizable contribution is non-vanishing**. However, it is **color-suppressed** relative to factorizable contributions.

$$\text{fact}_{\text{color}} = C_F^2 N_c^2 = \frac{(N_c^2 - 1)^2}{4}$$

$$\text{non/fact}_{\text{color}} = \frac{1}{4} \delta^{ab} \delta_{ab} = \frac{(N_c^2 - 1)}{4}$$

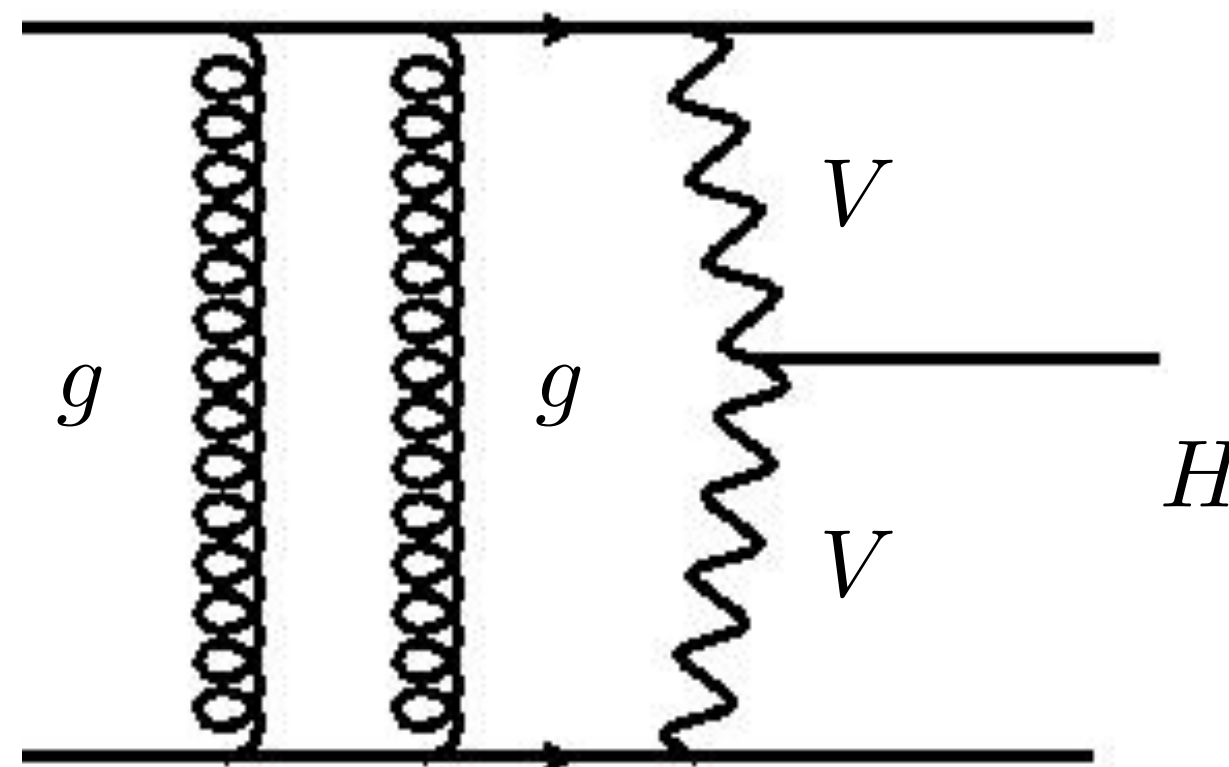


These estimates suggest that non-factorizable contributions are about 1%. However, it was also argued that these contributions are further suppressed because they are **kinematically** disfavoured. As we will see, this feature is quite subtle.

It is important to scrutinize these claims and to estimate the non-factorizable contributions in a convincing manner (we really do not have much experience with them).

Since such corrections are a part of a NNLO computation, understanding them at a fully-differential level requires us to compute the double-virtual, real-virtual and double-real contributions. It is clear that the double-real and the real-virtual contributions can be managed since they are at most one-loop computations supplemented with NNLO subtraction schemes.

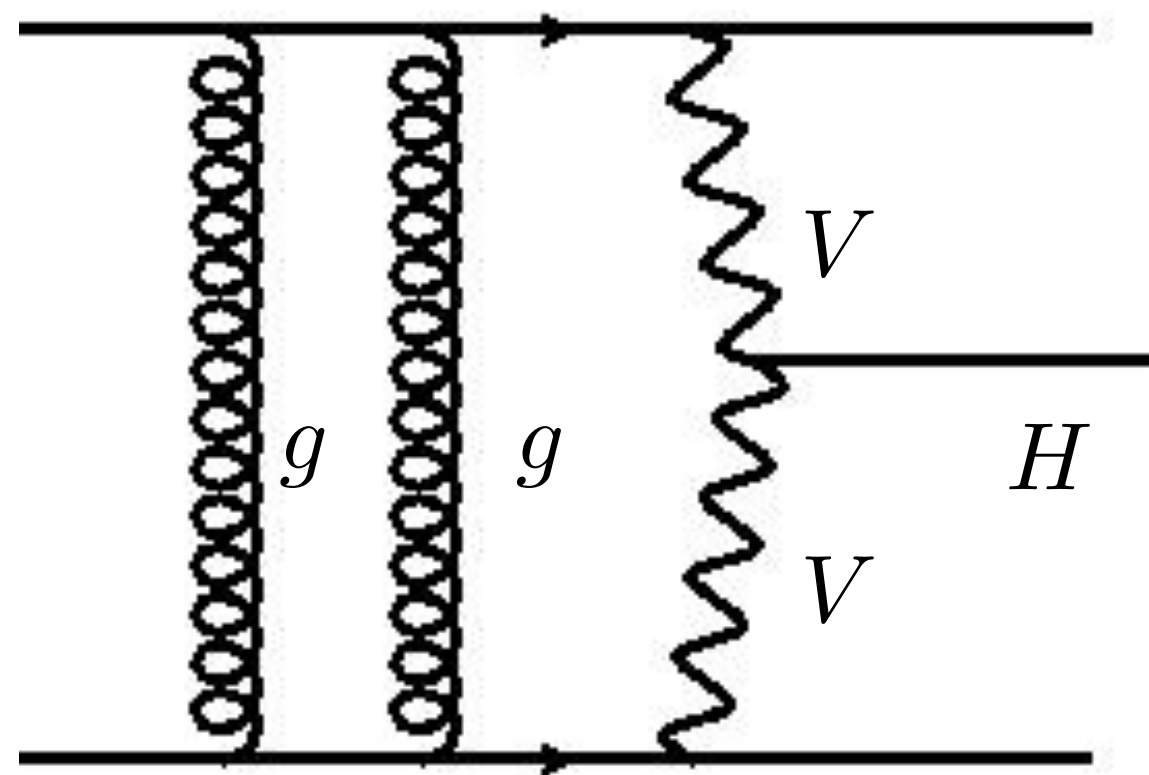
The **double-virtual non-factorizable** corrections are a problem since they require a two-loop five-point function with internal (the Higgs) and external (the weak bosons) masses. **It is impossible to compute such complicated diagrams / amplitudes with the existing technology.**



However, the weak boson fusion kinematics is particular.

Indeed, we have two jets flying in the opposite directions with relatively small transverse momenta and very high energies. We also have rapidity gap between the jets and between the jets and the Higgs. In other words, jets are energetic and forward and all (jet and Higgs) transverse momenta are comparable and (fairly) small.

Is it possible to construct an expansion of the virtual amplitudes taking into account the smallness of transverse momenta relative to energies of projectiles?



$$p_{\perp}^{j_1, j_2} > 25 \text{ GeV}, \quad |y_{j_1, j_2}| < 4.5$$

$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600 \text{ GeV}$$

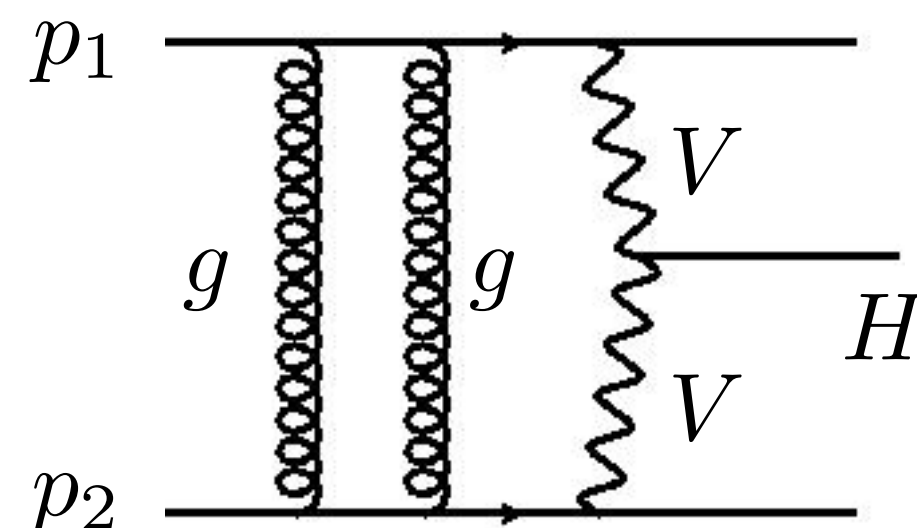
The answer to this question is affirmative. In fact, the leading term in the required expansion is known as the high-energy scattering (Regge) limit.

We will only need the abelian version of the Regge limit that was studied in the early days of physics at electron-positron colliders by many famous people.

Sudakov, Lipatov, Gribov, Cheng, Wu, Chang, Ma

The bottom line: virtual gluons are “soft”. The leading high-energy asymptotic is obtained by employing the following approximations:

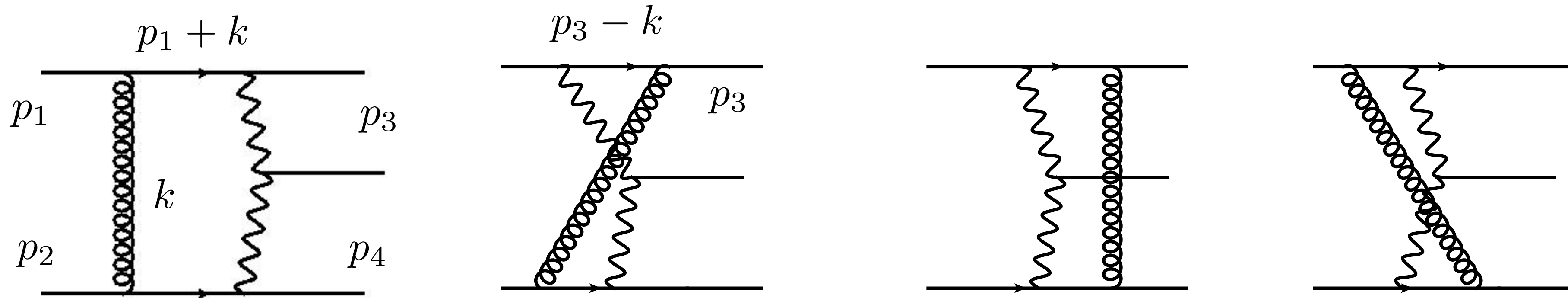
- 1) use eikonal propagators for quark lines: $\frac{1}{2pk + i0} \quad k = \alpha p_1 + \beta p_2 + k_\perp$
- 2) use eikonal couplings of quarks to photons: $-2ie p_\mu$
- 3) neglect longitudinal momenta components in photon propagators: $\frac{1}{k^2} \rightarrow -\frac{1}{k_\perp^2}$



$$p_\perp^{j_1, j_2} > 25 \text{ GeV}, \quad |y_{j_1, j_2}| < 4.5$$

$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600 \text{ GeV}$$

Since we only need the interference of the two-loop amplitude with the leading order amplitude, **the sum over colors makes the contribution abelian, i.e. QED-like. Abelianization**, together with the eikonal approximation, makes the calculation quite simple. **Corrections receive an additional factor π** .



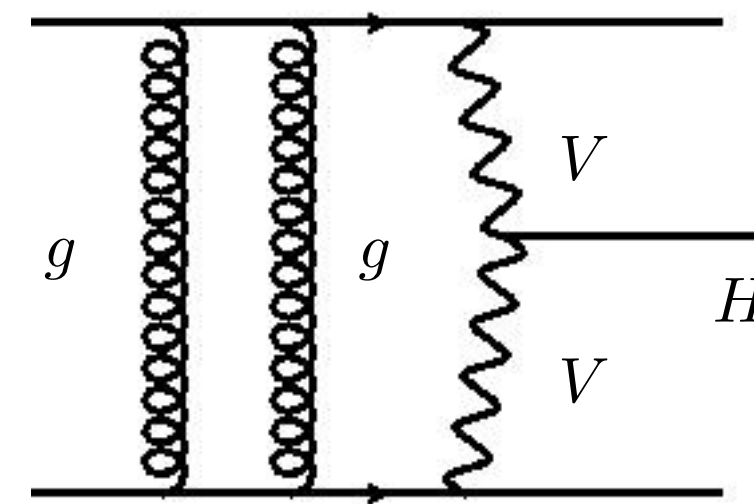
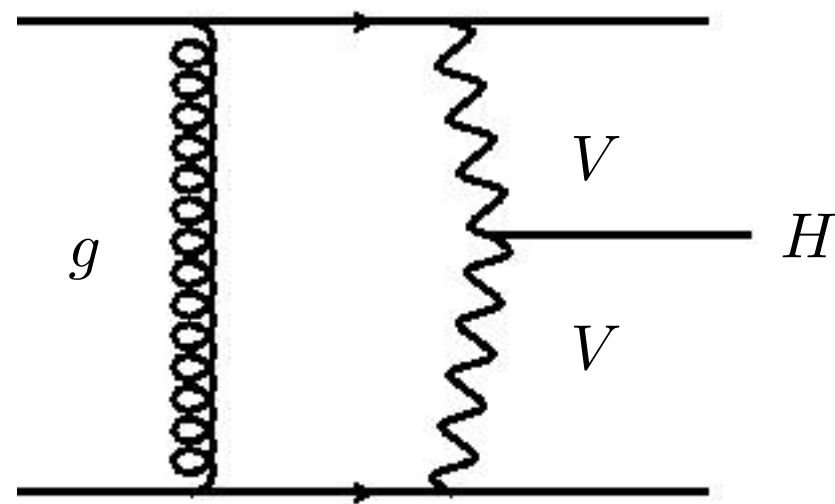
$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{k_{\perp}^2} \frac{1}{(k_{\perp} - q_{3,\perp})^2 + M_V^2} \frac{1}{(k_{\perp} + q_{4,\perp})^2 + M_V^2} \left[\frac{1}{2p_1 k + i0} + \frac{1}{-2p_3 k + i0} \right] \left[\frac{1}{-2p_2 k + i0} + \frac{1}{2p_4 k + i0} \right]$$

$$\lim_{p_3 \rightarrow p_1} \left[\frac{1}{2p_1 k + i0} - \frac{1}{-2p_3 k + i0} \right] = -\frac{2i\pi}{s} \delta(\beta) \quad \lim_{p_4 \rightarrow p_2} \left[\frac{1}{-2p_2 k + i0} + \frac{1}{2p_4 k + i0} \right] = -\frac{2i\pi}{s} \delta(\alpha)$$

$$k = \alpha p_1 + \beta p_2 + k_{\perp}$$

Non-factorizable contributions

A very similar computation can be done for the two-loop amplitude. Upon integrating over longitudinal loop momenta fractions, we obtain the following results for the one- and two-loop eikonal amplitudes. The results are infra-red divergent; we introduce the gluon mass to regulate these divergences. **Note that the one-loop amplitude is pure imaginary.**



$$\left| \frac{p_{\perp}}{\sqrt{s}} \right| \ll 1.$$

$$\mathcal{M}^{(1)} = i\tilde{\alpha}_s \chi^{(1)}(\mathbf{q}_3, \mathbf{q}_4) \mathcal{M}^{(0)}$$

$$\mathcal{M}^{(2)} = -\frac{\tilde{\alpha}_s^2}{2!} \chi^{(2)}(\mathbf{q}_3, \mathbf{q}_4) \mathcal{M}^{(0)}$$

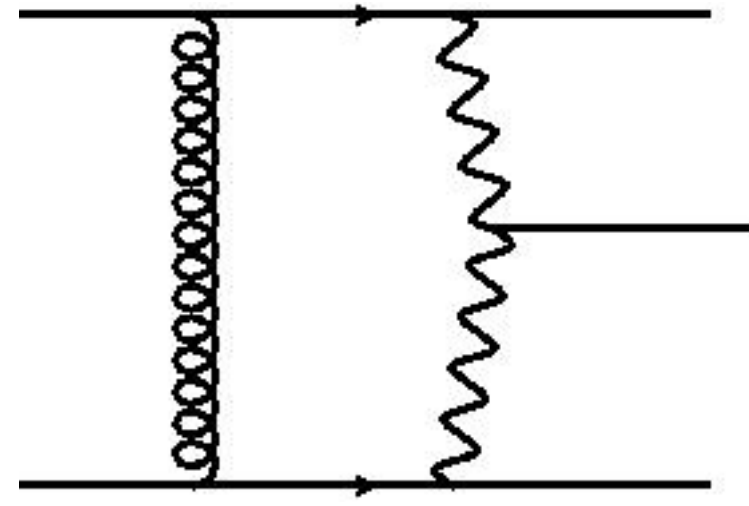
$$\tilde{\alpha}_s = \left[\frac{N_c^2 - 1}{4N_c^2} \right]^{1/2} \alpha_s$$

$$\chi^{(1)}(\mathbf{q}_3, \mathbf{q}_4) = \frac{1}{\pi} \int \frac{d^2 \mathbf{k}}{\mathbf{k}^2 + \lambda^2} \times \frac{\mathbf{q}_3^2 + M_V^2}{(\mathbf{k} - \mathbf{q}_3)^2 + M_V^2} \frac{\mathbf{q}_4^2 + M_V^2}{(\mathbf{k} + \mathbf{q}_4)^2 + M_V^2},$$

$$\chi^{(2)}(\mathbf{q}_3, \mathbf{q}_4) = \frac{1}{\pi^2} \int \left(\prod_{i=1}^2 \frac{d^2 \mathbf{k}_i}{\mathbf{k}_i^2 + \lambda^2} \right) \times \frac{\mathbf{q}_3^2 + M_V^2}{(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{q}_3)^2 + M_V^2} \frac{\mathbf{q}_4^2 + M_V^2}{(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{q}_4)^2 + M_V^2}$$

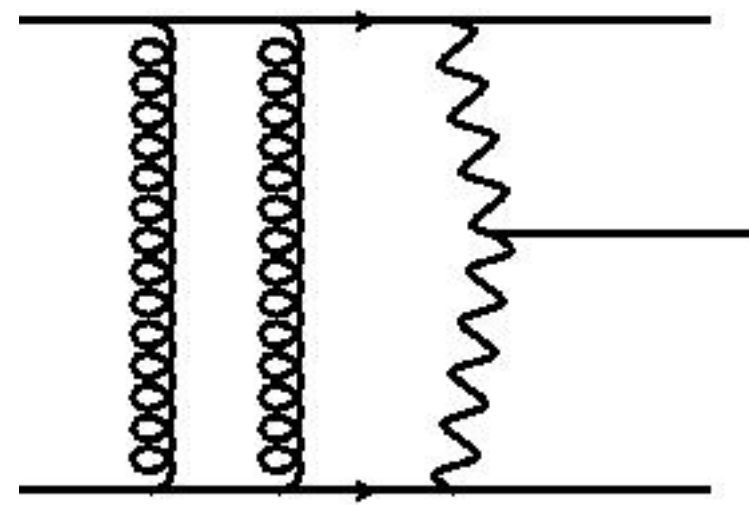
Color projection has been taken; this one-loop contribution is meant to be used in the NNLO contribution only.

Non-factorizable contributions



$$\chi^{(1)} = \frac{1}{\pi} \int \frac{d^2\mathbf{k}}{\mathbf{k}^2 + \lambda^2} \times \frac{\mathbf{q}_3^2 + M_V^2}{(\mathbf{k} - \mathbf{q}_3)^2 + M_V^2} \frac{\mathbf{q}_4^2 + M_V^2}{(\mathbf{k} + \mathbf{q}_3)^2 + M_V^2}$$

$$\chi^{(1)} = -\ln\left(\frac{\lambda^2}{M_V^2}\right) + f^{(1)}(\mathbf{q}_3, \mathbf{q}_4, M_V^2)$$



$$\chi^{(2)} = \frac{1}{\pi^2} \int \prod_{i=1}^2 \frac{d^2\mathbf{k}_i}{\mathbf{k}_i^2 + \lambda^2} \times \frac{\mathbf{q}_3^2 + M_V^2}{(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{q}_3)^2 + M_V^2} \frac{\mathbf{q}_4^2 + M_V^2}{(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{q}_4)^2 + M_V^2}$$

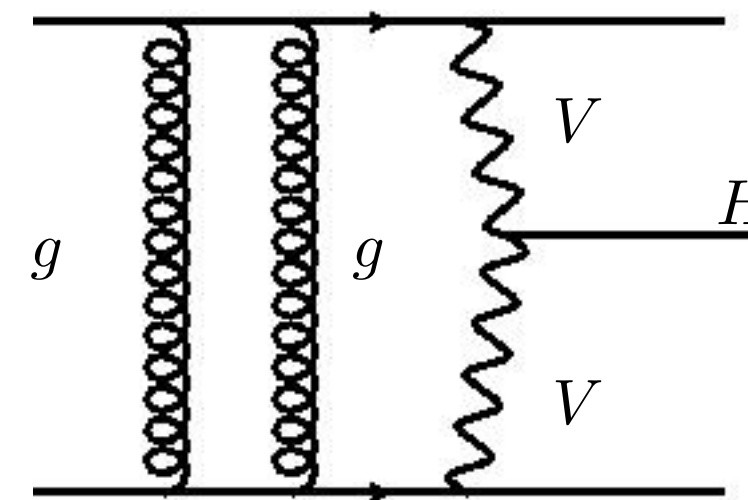
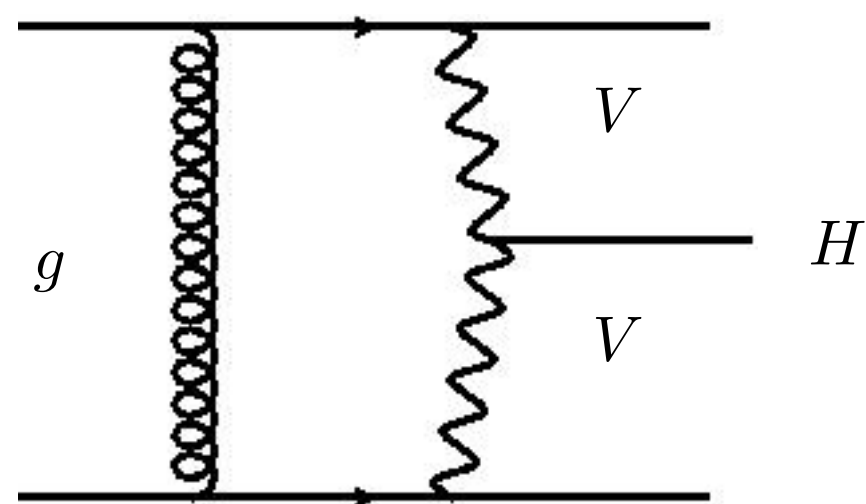
$$\chi^{(2)} = \ln^2\left(\frac{\lambda^2}{M_V^2}\right) - 2 \ln\left(\frac{\lambda^2}{M_V^2}\right) f^{(1)}(\mathbf{q}_3, \mathbf{q}_4, M_V^2) + f^{(2)}(\mathbf{q}_3, \mathbf{q}_4, M_V^2)$$

The infrared-red divergences that appear in non-factorizable contributions **exponentiate into a Coulomb (Glauber) phase and cancel on their own, without any reference to real emission contributions.**

$$\mathcal{M} = \mathcal{M}_0 e^{-i\tilde{\alpha}_s \ln \frac{\lambda^2}{M_V^2}} \left[1 + i\tilde{\alpha}_s f^{(1)} - \frac{\tilde{\alpha}_s^2}{2} f^{(2)} + \dots \right].$$

Non-factorizable contributions

The final result is given by the sum of the one-loop amplitude squared and the interference of the two-loop and tree amplitudes. In this combination, the dependence on the gluon mass (i.e. the infra-red divergence) cancels out. Hence, we obtain a physical result that can be studied independently of real emissions.



$$d\sigma_{\text{nf}}^{\text{NNLO}} = \left(\frac{N_c^2 - 1}{4N_c^2} \right) \alpha_s^2 \chi_{\text{nf}} d\sigma^{\text{LO}}$$

$$\chi_{\text{nf}}(\mathbf{q}_3, \mathbf{q}_4) = [f^{(1)}(\mathbf{q}_3, \mathbf{q}_4)]^2 - f^{(2)}(\mathbf{q}_3, \mathbf{q}_4)$$

$$r_1 = \mathbf{q}_3^2 x + \mathbf{q}_4^2 (1 - x) - \mathbf{q}_H^2 x(1 - x),$$

$$r_2 = \mathbf{q}_H^2 x(1 - x) + M_V^2,$$

$$r_{12} = r_1 + r_2,$$

$$\Delta_i = \mathbf{q}_i^2 + M_V^2.$$

$$f^{(1)} = \int_0^1 dx \frac{\Delta_3 \Delta_4}{r_{12}^2} \left[\ln \left(\frac{r_{12}^2}{r_2 M_V^2} \right) + \frac{r_1 - r_2}{r_2} \right],$$

$$f^{(2)} = \int_0^1 dx \frac{\Delta_3 \Delta_4}{r_{12}^2} \left[\left(\ln \left(\frac{r_{12}^2}{r_2 M_V^2} \right) + \frac{r_1 - r_2}{r_2} \right)^2 - \ln^2 \left(\frac{r_{12}}{r_2} \right) - \frac{2r_{12}}{r_2} \ln \left(\frac{r_{12}}{r_2} \right) - 2 \text{Li}_2 \left(\frac{r_1}{r_{12}} \right) - \left(\frac{r_1 - r_2}{r_2} \right)^2 + \frac{\pi^2}{3} \right],$$

Non-factorizable contributions

It is instructive to consider some limits, where analytic computations simplify.

1) The forward limit: $\mathbf{q}_{3,4} \rightarrow 0$ $\chi_{\text{nf}} = 1 - \frac{\pi^2}{3}$; Negative corrections.

2) The forward Higgs production: $\mathbf{q}_H \rightarrow 0$

$$\chi_{\text{nf}} = \ln^2 \frac{1+x}{x} + 2\text{Li}_2 \left(\frac{1}{1+x} \right) - \frac{\pi^2}{3} + 2 \frac{1+x}{x} \ln \frac{1+x}{x} + \left(\frac{1-x}{x} \right)^2. \quad x = M_V^2 / \mathbf{q}_3^2$$

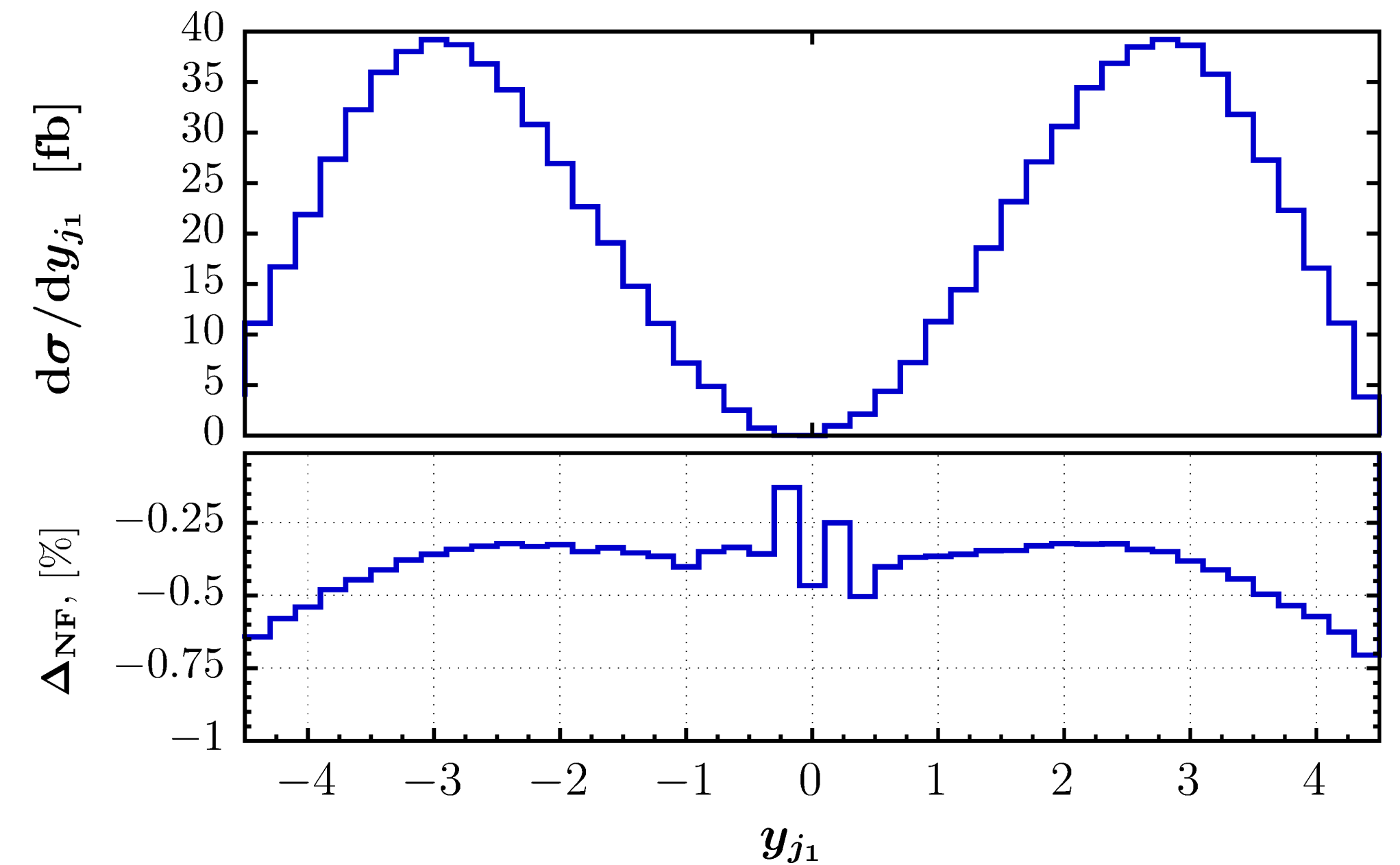
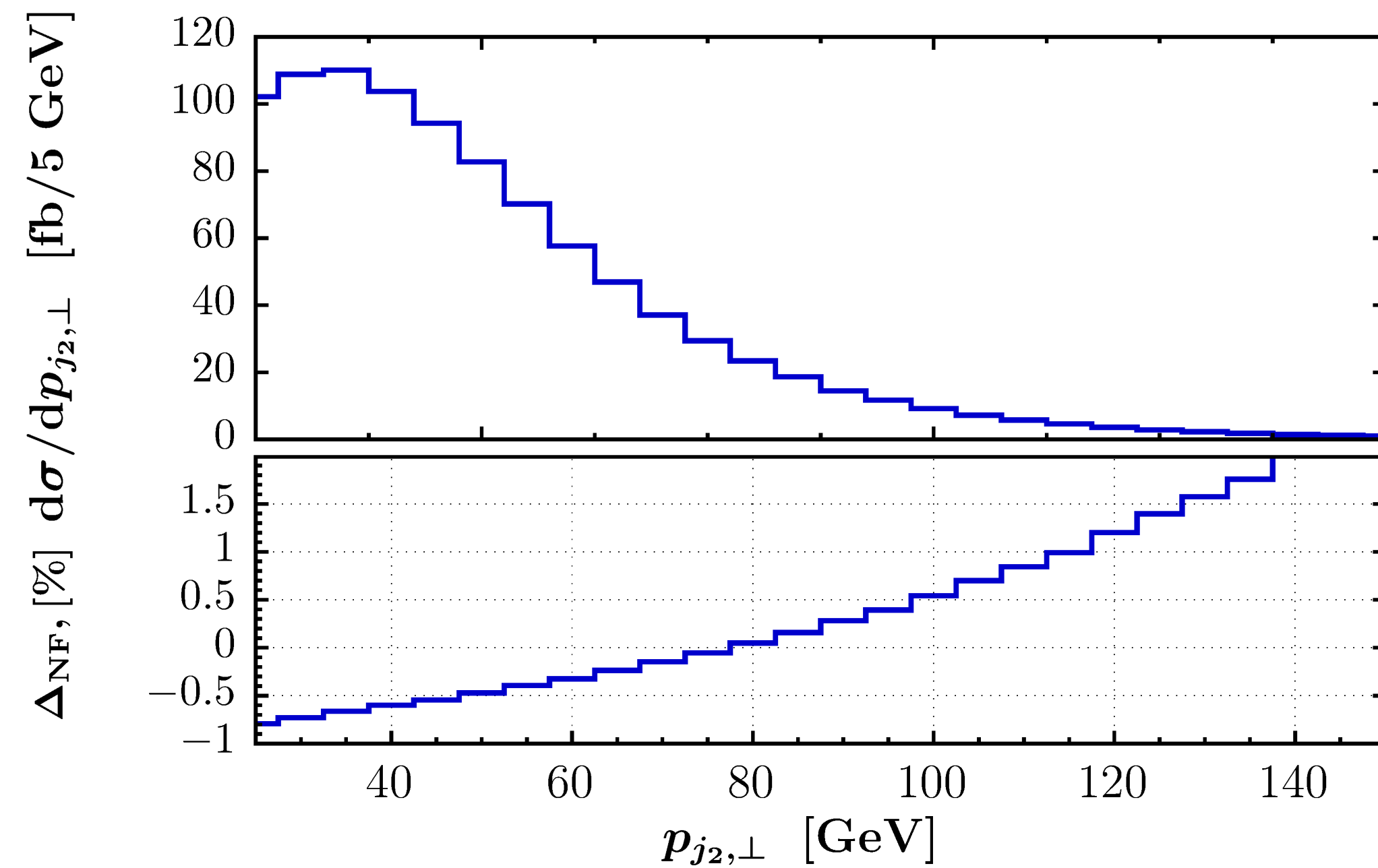
Enhanced positive corrections at small x (large jet transverse momentum).

From these considerations, it can be expected that the non-factorizable corrections significantly change over the phase space. This expectation is confirmed by results of numerical simulation.

It is also useful to emphasize that, although non-factorizable corrections appear at NNLO, there is currently no information about their renormalization scale dependence.

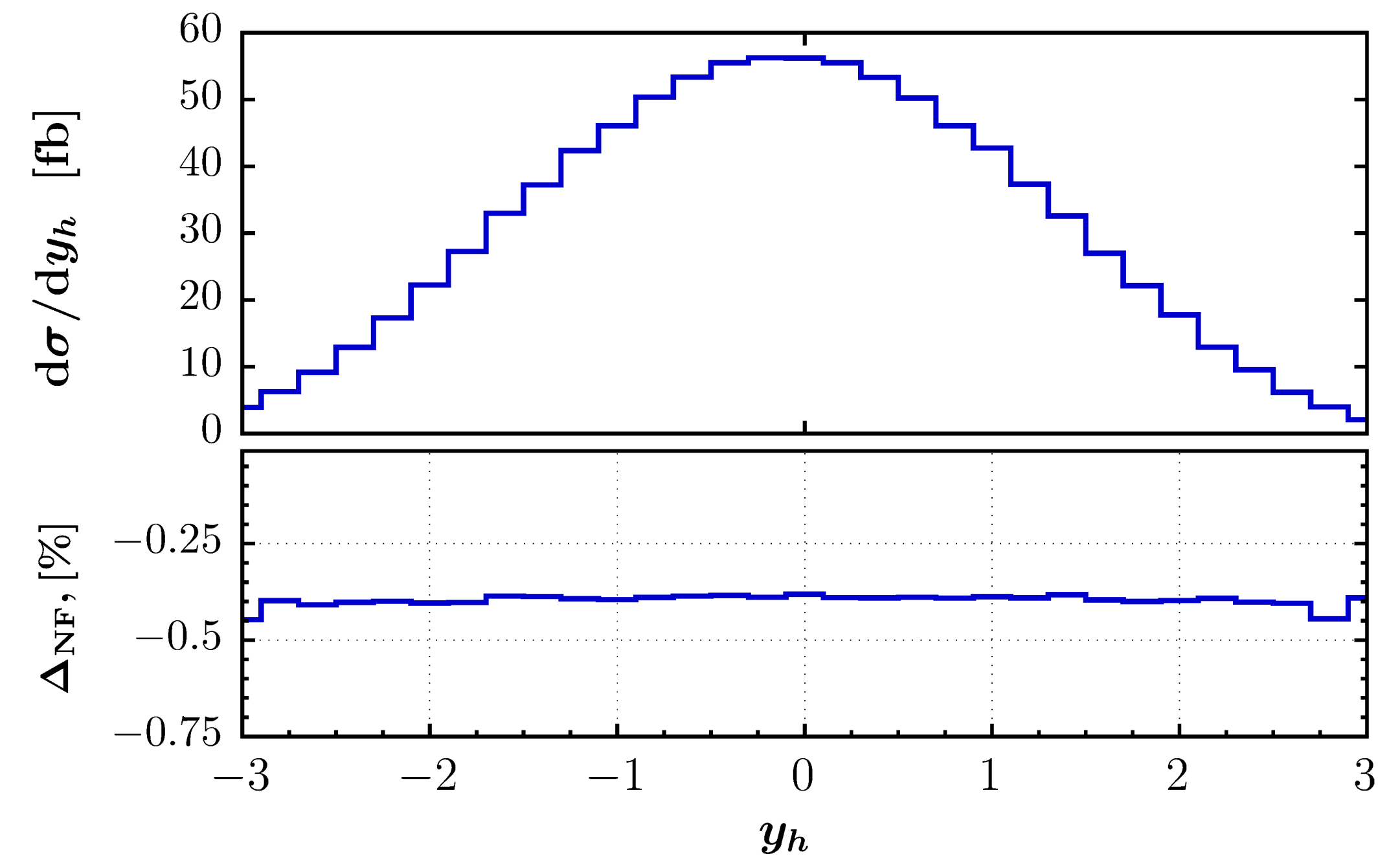
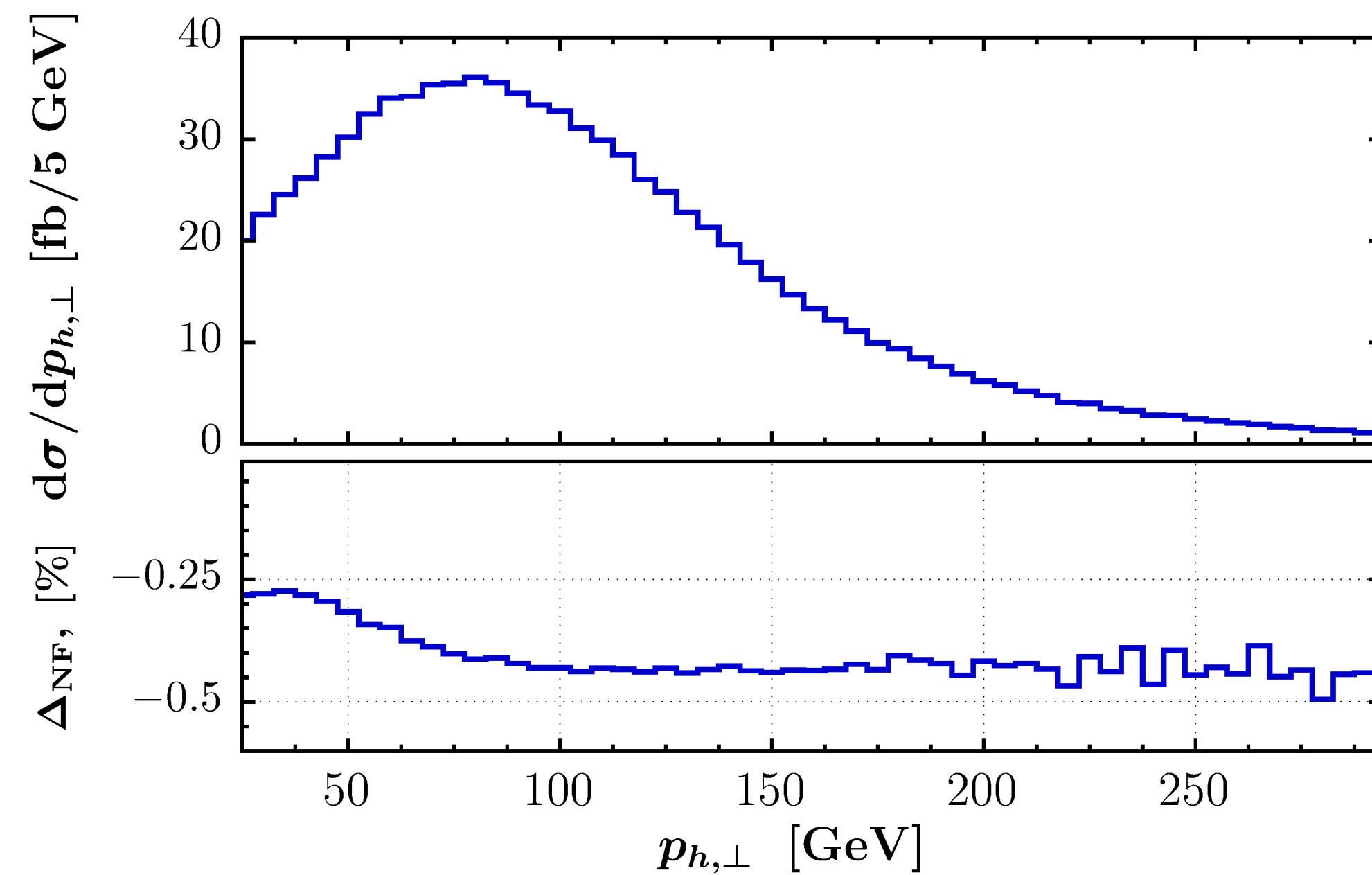
Finally, these effects are not treated by a parton shower, even approximately, since the contribution that we have computed is fully independent of the real emission.

Kinematic distributions: non-factorizable corrections



Percent level non-factorizable effects; corrections to transverse momentum distribution of a second jet change from negative to positive.

Kinematic distributions: non-factorizable corrections



Corrections to the Higgs boson transverse momentum and rapidity distributions are much less volatile... Both are about half a percent. The non-factorizable corrections to the cross section with WBF cuts are quite similar.

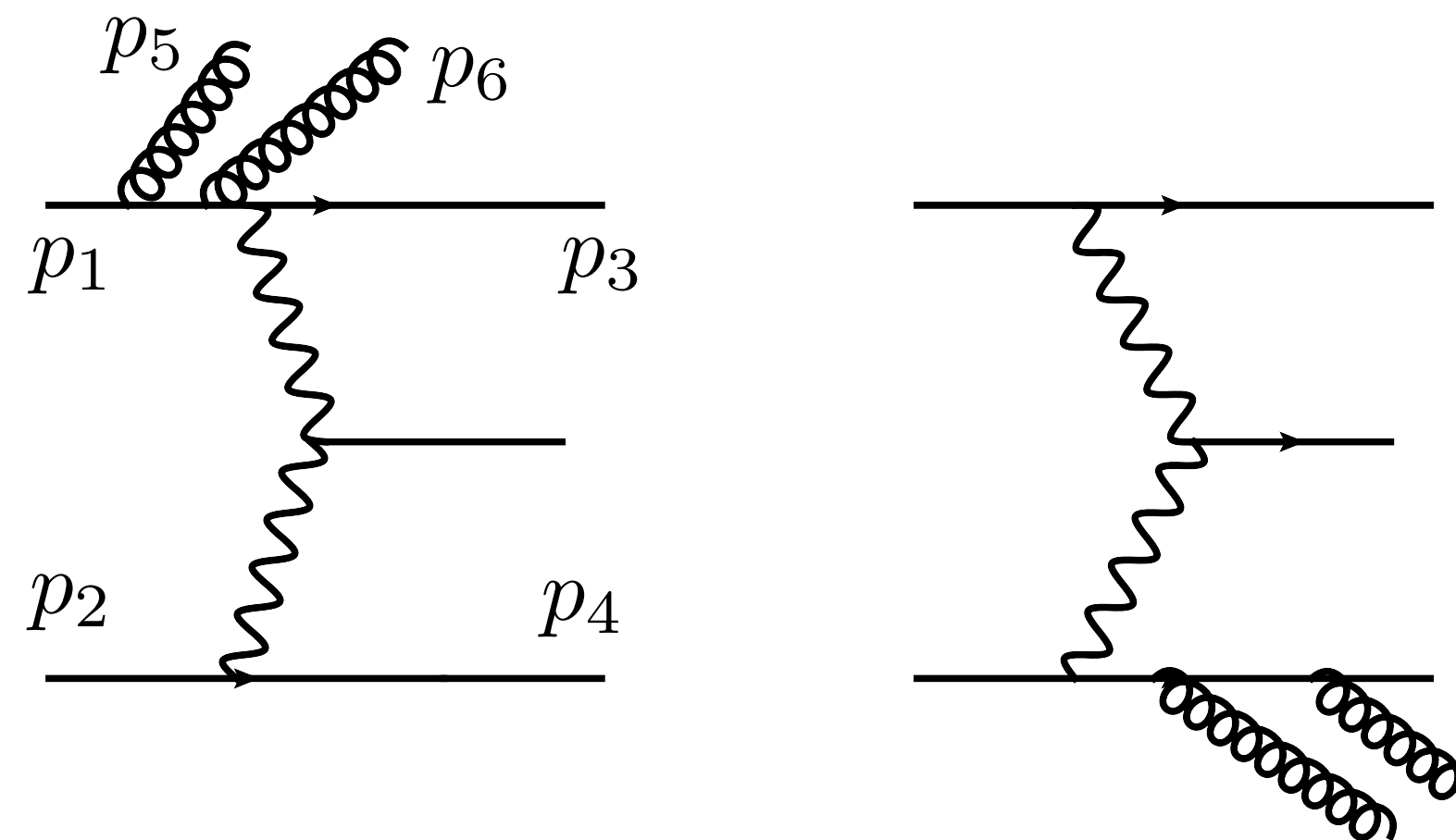
Non-factorizable corrections beyond eikonal approximation

Eikonal approximation (forward scattering) gives us the Coulomb phase; that is why the infra-red singularities cancel on their own. Because of the difference between a gluon and a two weak bosons fusing into the Higgs boson, the result is different from zero.

Other effects, e.g. real emissions, require going beyond the forward scattering limit.

$$\lim_{p_5, p_6 \rightarrow 0} |\mathcal{M}(1_q, 2_Q, 3_q, 4_Q)|_{\text{nf}}^2 = (N_c^2 - 1) \text{Eik}_{\text{nf}}(p_5) \text{Eik}_{\text{nf}}(p_6) \mathcal{A}_0^2(1, 2, 3, 4)$$

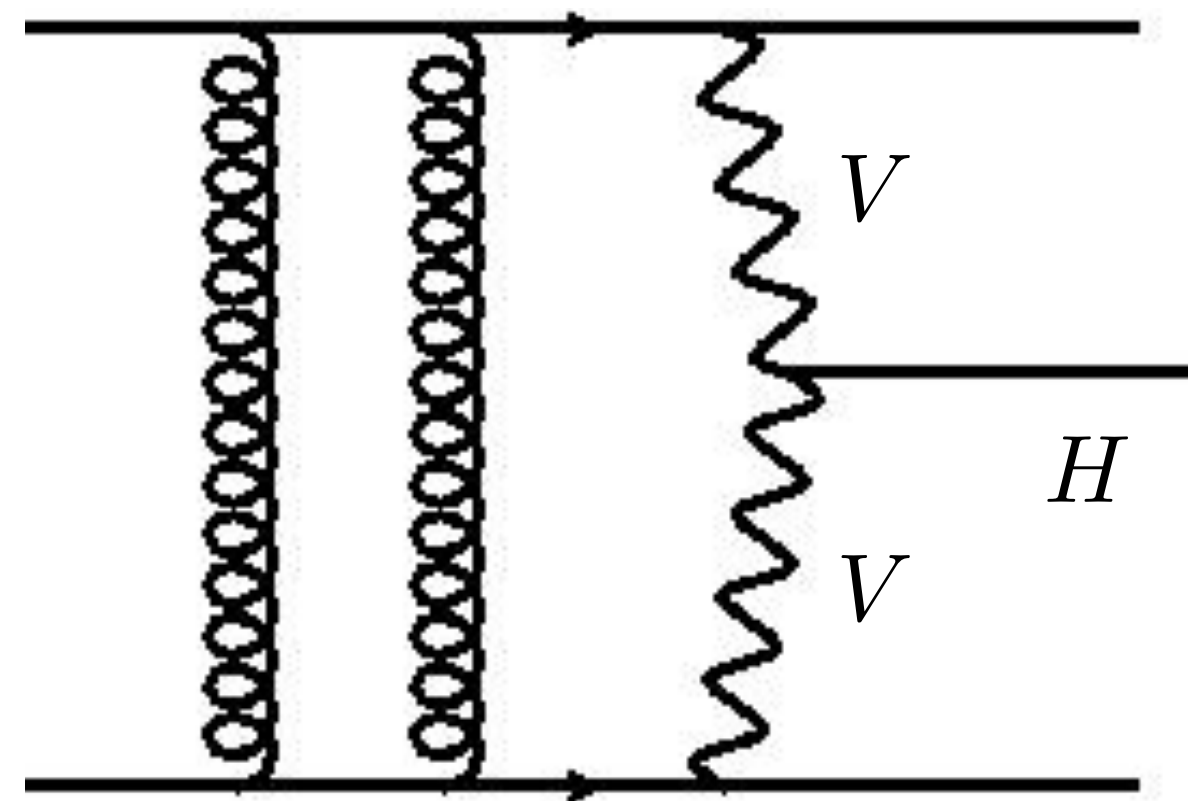
$$\text{Eik}(p) = \sum_{i \in [1, 3]; j \in [2, 4]} \lambda_{ij} \frac{p_i p_j}{(p_i p)(p_j p)} \quad \lambda_{ij} = \begin{cases} 1 & i, j \text{ both incoming or outgoing} \\ -1 & \text{otherwise} \end{cases}$$



Summary

Non-factorizable corrections to Higgs boson production in weak boson fusion

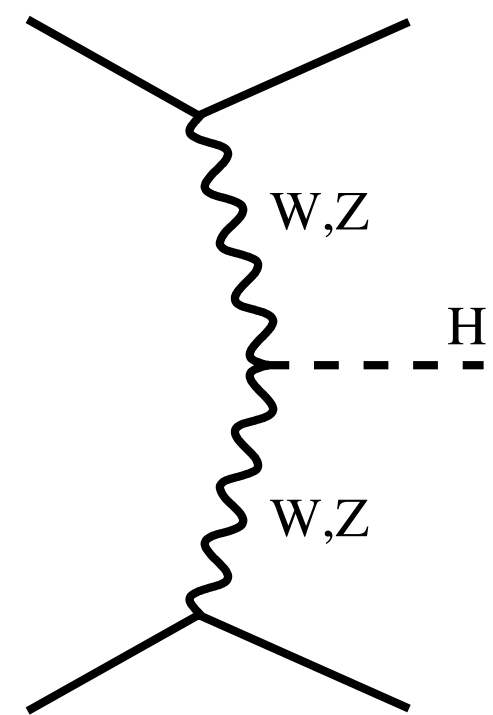
- 1) appear at next-to-next-to-leading order for the first time;
- 2) are color-suppressed but π - enhanced; their origin is related to the Coulomb phase;
- 3) are **not** part of effects simulated by parton showers;
- 4) can reach **a percent level** in kinematic distributions and are strongly kinematic-dependent; change the WBF cross section by -0.5 percent;
- 5) appear to be equally if not more important than factorizable N3LO corrections.



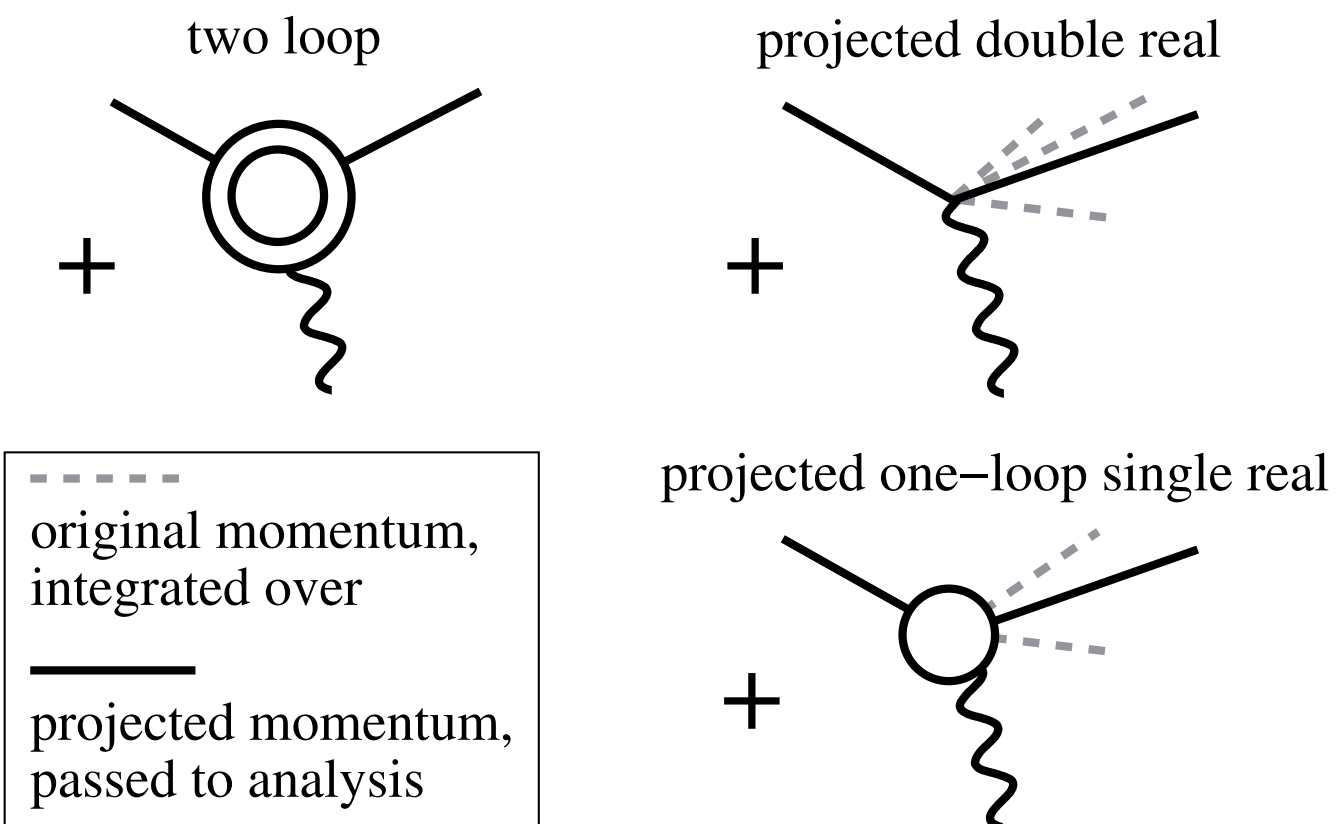
The following picture and formula illustrate the projection-to-Born method. **It is important that this method can be easily extended to one order higher in perturbative QCD provided, of course, that NNLO QCD corrections to VBF+jet production are known.**

$$q_1 : (x_1 P - q_1)^2 = 0 \Rightarrow x_1 = \frac{q_1^2}{2q_1 \cdot P} \Rightarrow p_{\text{in}} = x_1 P, \quad p_{\text{fin}} = q_1 - x_1 P$$

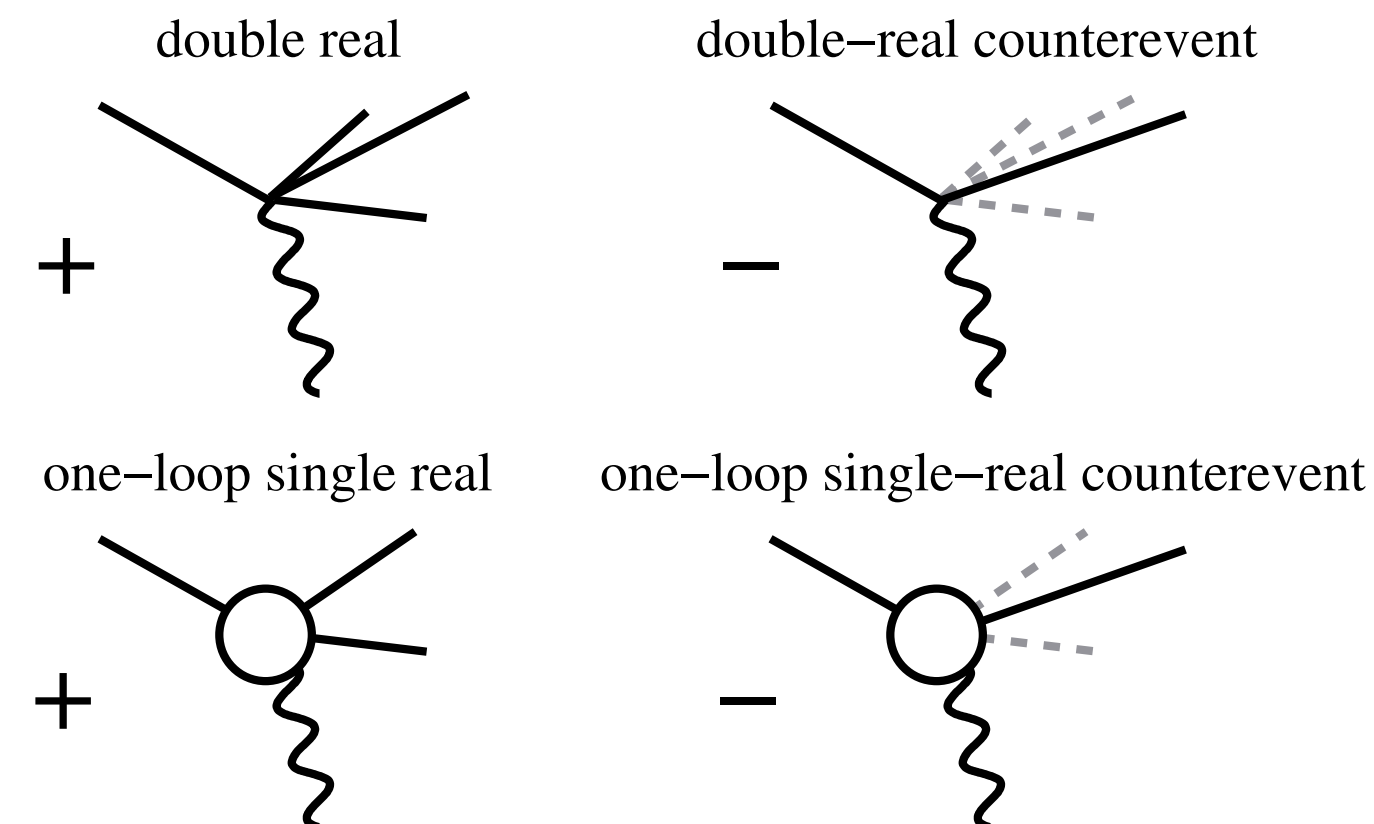
(a) Born VBF process



(b) NNLO "inclusive" part (from structure function method)



(c) NNLO "exclusive" part (from VBF H+3j@NLO)



$$d\sigma_{\text{VBF}}^{\text{NNLO}}|_{q_1} = w(\{p\}) \frac{d\Phi_{\text{VBF}+1j}}{\Phi_{\text{VBF}+1j}} - w(\{p\}) \frac{d\Phi_{\text{B}}}{\Phi_{\text{B}}} + C(q_1) \frac{d\Phi_{\text{B}}}{\Phi_{\text{B}}}$$

$$\int d\sigma_{\text{VBF}}^{\text{NNLO}}|_{q_1} = W(q_1^2, Pq_1) \Leftrightarrow C(q_1)$$

Cacciari, Dreyer, Karlberg, Salam, Zanderighi