Simulating the earliest stages of heavy-ion collisions

Teilchenphysikseminar Universität Wien
November 12, 2019

Andreas Ipp

based on

Institute for Theoretical Physics, TU Wien, Austria
Outline

- Simulation of early stages of heavy-ion collisions
  - color-glass-condensate (CGC) framework
  - colored particle-in-cell (CPIC)
  - beyond boost-invariance
- Numerical results
  - energy density at different rapidities
  - comparison to RHIC data

![Diagram showing initial state, CGC, "Glasma", QGP, hadronization, and hadronic gas]
QCD phase diagram

Illustration: Swagato Mukherjee, Brookhaven National Laboratory.
Relativistic Heavy Ion Collider (RHIC)

Brookhaven National Laboratory (USA)
Large Hadron Collider (LHC)

CERN
Heavy-ion collisions

- Heavy ion nuclei (gold, lead) ($d \sim 14$ fm)
- formation time of QGP: $t \sim 1$ fm/c $\approx 3$ ys
- QGP phase: RHIC 15 ys, LHC 25 ys
- good agreement with hydrodynamic simulations
Stages of a heavy-ion collision

1 fm/c ≈ 3.3 \cdot 10^{-24} s ≈ 3.3 ys

**Initial state:** Lorentz-contracted nuclei (color glass condensate)

**Collision event**

**Glasma** (τ ≈ 0 - 1 fm/c): quasi-classical fields (classical field equations)

**QGP** (τ ≈ 1 - 10 fm/c): quarks and gluons (relativistic viscous hydrodynamics) (almost) isotropic and in thermal equilibrium

**Hadronization** (τ ≈ 10 fm/c): confinement transition → hadron formation

**Hadronic gas** (τ ≈ 10 - 15 fm/c): hadrons (kinetic transport theory)

**Freeze-out** (τ ≈ 15 fm/c): interactions stop

**scope of this project**
Pancake thickness

French crêpe

LHC (ALICE) @ CERN: Pb+Pb with ~5.5 TeV per nucleon pair ($\gamma \approx 2700$)

American pancake

RHIC @ BNL: Au+Au with ~200 GeV ($\gamma \approx 100$)

RHIC beam energy scan: ~7.7 – 62.4 GeV ($\gamma \approx 4 - 30$)
Nuclei at ultrarelativistic speeds can be described by **classical effective theory** in the color glass condensate (CGC) framework.


Large gluon occupation numbers $\rightarrow$ coherent, classical gluon field

Split degrees of freedom into ...

- Hard partons = classical color charges
- Soft gluons = classical gauge field

- Static field configuration due to time dilation.
- **Collision of two such classical fields creates the **Glasma**.**


Figure from L. McLerran: Proceedings of ISMD08, p.3-18 (2008)
Boost-invariant CGC collision

- color glass condensate (CGC): hard and soft degrees of freedom, weak coupling
- infinitely thin color currents
- boost-invariant solution
- solve Yang-Mills equations numerically in 2+1 D

\[ D_\mu F^{\mu\nu}(\tau, x_T) = 0 \]
Finite nucleus thickness

- extended color currents
- boost-invariance lost
- solve full 3+1 D Yang-Mills equation with currents

\[ D_\mu F^{\mu \nu}(t, z, x_T) = J_1^\nu + J_2^\nu \]
\[ D_\mu J^{\mu}(t, z, x_T) = 0 \]

\[ D_\mu \equiv \partial_\mu + ig [A_\mu, \cdot] \]

→ use Colored particle-in-cell (CPIC) in laboratory frame
Colored particle-in-cell (CPIC)


Nucleus model: 2D McLerran-Venugopalan (MV) model
[McLerran, Venugopalan: PRDD49 (1994) 3352-3355]

\[
\langle \hat{\rho}^a(x_T) \hat{\rho}^b(x'_T) \rangle = g^2 \mu^2 \delta^{(2)}(x_T - x'_T) \delta^{ab}
\]

\[\mu \approx 0.5 \text{ GeV} \quad \text{(Au, RHIC)}\]

Infrared regulation
\[m \approx 200 \text{ MeV}\]

Gaussian profile with thickness \(\sigma\).
\[\sigma \approx R/\gamma\]

(no random longitudinal structure)
Implementation


Continuum equations of motion

\[ D^a_{\mu} F^{b,\mu\nu} = j^{a,\nu} \]

Lattice equations of motion

\[ \dot{E}^a_i(x) = \frac{2}{g a^3} \sum_{j \neq i} \text{Im} \left[ \text{tr} \left( U_{ij}(x) + U_{i-j}(x) \right) \right] - j^a_i(x) \]

\[ U_{ij}(x) = U_i(x) U_j(x + i) U_{-i}(x + i + j) U_{-j}(x + j) \]

\[ \dot{U}_i(x) = -iga E_i^a(x) t^a U_i(x) \]

Parallel transporters (gauge links):
\[ U_i(x) = \exp (iga A_i^a(x)t^a) \]
Results

Austrian Kaiserschmarrn („Emperor's mess“)

© Image: Aleksi Pihkanen
3D energy density
3D energy density
3D energy density
3D energy density
3D energy density
3D energy density
Comparison to boost-invariant results

Energy density component $\text{tr } E_L^2(x_T)$ in the transverse plane at $t = 0$.

Au-Au collision in the McLerran-Venugopalan (MV) model for SU(2)

256×128² cells, $a_s = 0.028$ fm
Shown: 64×64 cells
Correlation analytic ↔ numerical

(too thin – numerically unstable)

$256 \times 128^2$ cells, $a_s = 0.028$ fm, $a_t = a_s / 2$
Observables

Main observable: energy-momentum tensor $T^{\mu\nu}(x)$

- Build $T^{\mu\nu}(x)$ from electric and magnetic fields $E_i^a(x), B_i^a(x)$
- Average over configurations and integrate over transverse plane

$$
\langle T^{\mu\nu} \rangle = \begin{pmatrix}
    \langle \varepsilon \rangle & 0 & 0 & \langle S_L \rangle \\
    0 & \langle p_T \rangle & 0 & 0 \\
    0 & 0 & \langle p_T \rangle & 0 \\
    \langle S_L \rangle & 0 & 0 & \langle p_L \rangle \\
\end{pmatrix}
$$

$$
\langle \varepsilon \rangle = \frac{1}{2} \langle E_T^2 + B_T^2 + E_L^2 + B_L^2 \rangle \\
\langle p_T \rangle = \frac{1}{2} \langle E_L^2 + B_L^2 \rangle \\
\langle p_L \rangle = \frac{1}{2} \langle E_T^2 + B_T^2 - E_L^2 - B_L^2 \rangle \\
\langle S_L \rangle = \left\langle \left( \vec{E}^a \times \vec{B}^a \right)_L \right\rangle
$$

- Diagonalize, obtain local rest-frame energy density

$$
\langle \varepsilon_{\text{loc}} \rangle = \frac{1}{2} \left( \langle \varepsilon \rangle - \langle p_L \rangle + \sqrt{\left( \langle \varepsilon \rangle + \langle p_L \rangle \right)^2 - 4 \langle S_L \rangle^2} \right)
$$
Pressure anisotropy

Longitudinal pressure $p_L(z)$ and transverse pressure $p_T(z)$

$\rightarrow$ Pronounced pressure anisotropy

\[
\begin{align*}
\langle p_T \rangle &= \frac{1}{2} \langle E_L^2 + B_L^2 \rangle \\
\langle p_L \rangle &= \frac{1}{2} \langle E_T^2 + B_T^2 - E_L^2 - B_L^2 \rangle
\end{align*}
\]

$t_0 = -1$ fm/c (before collision)

$t_1 = +2$ fm/c (after collision)

$t_2 = +5$ fm/c (late times)
Pressure anisotropy at midrapidity

observe very slow isotropization

longitudinal pressure $p_L(z)$
transverse pressure $p_T(z)$
Rapidity profiles

Plot (space-time) rapidity profile of local rest-frame energy density

Compare to measured **rapidity profile of particle multiplicity (RHIC)** and **Landau model** prediction

- Simulation data in interval $\eta_s \in (-1,1)$ at $\tau = 1 \text{ fm/c}$
- Fit to Gaussian profile (dashed)
- Dependency on thickness (or rather $\sqrt{s}$ )
- Strong dependency on IR regulator, but $m=0.2 \text{ GeV}$ gives realistic shape
- However: no hydrodynamic expansion included
- Limited rapidity interval

\[ \sqrt{s_{NN}} = 200 \text{ GeV} \]

\[ \eta_s \in (-1,1) \text{ at } \tau = 1 \text{ fm/c} \]

\[ m = 0.2 \text{ GeV} \]

\[ m = 0.4 \text{ GeV} \]

\[ m = 0.8 \text{ GeV} \]

[RHIC data: Bearden et al., PRL 94 (2005) 162301]
Rapidity profiles

Plot (space-time) rapidity profile of local rest-frame energy density

Compare to measured rapidity profile of particle multiplicity (RHIC) and Landau model prediction

- Simulation data in interval $\eta_s \in (-1,1)$ at $\tau = 1 \text{ fm}/c$
- Fit to Gaussian profile (dashed)
- Dependency on thickness (or rather $\sqrt{s}$)
- Strong dependency on IR regulator, but $m = 0.2 \text{ GeV}$ gives realistic shape
- However: no hydrodynamic expansion included
- Limited rapidity interval

$\sqrt{s_{NN}} = 130 \text{ GeV}$

$\sqrt{s_{NN}} = 130 \text{ GeV}$

\[ \eta, \tau \]

\[ y \]

\[ \varepsilon_{\text{loc}}(\eta, \eta_s) / \varepsilon_{\text{loc}}(0, 0) \]

\[ (0) \cdot \frac{\varepsilon_{\text{loc}}(\eta, \eta_s)}{N \cdot \varepsilon_{\text{loc}}(0, 0)} \]

\( (a) \) $m = 0.2 \text{ GeV}$
\( (b) \) $m = 0.4 \text{ GeV}$
\( (c) \) $m = 0.8 \text{ GeV}$
Rapidity profiles

Limited rapidity interval due to ..

- longitudinal simulation box length / simulation time
- “interference” from fields of the nuclei

Blue: $p_L(x)$
Red: $p_T(x)$
Rapidity profiles

Combining rapidity profiles for increasingly boosted collisions

preliminary
Compare local velocity of glasma to free streaming condition $v = z/t$

Lines (almost) on top of each other.

Black solid: measured $v_z$
Red dashed: free streaming $v_{fs} = z/t$
Transverse pressure distribution

- Transverse pressure \( p_T(x) \) generated by longitudinal fields

\[
\langle p_T \rangle = \frac{1}{2} \langle E_L^2 + B_L^2 \rangle
\]

- **Boost-invariant case**: initial conditions at \( \tau = 0 \) for longitudinal \( E \) and \( B \) fields, i.e. constant \( p_T \) along the boundary of the forward light cone

[3+1 Yang-Mills](Casalderrey-Solana et al., PRL (2013) 181601)

Holographic model

\[
P_T/\rho^4
\]
Non-Abelian version of Poynting theorem

\[ \frac{d\varepsilon}{dt} + \frac{1}{V} \int \partial_i S_i \, d^3x + \frac{1}{V} \int E_i^a J_i^a \, d^3x = 0 \]

Energy production caused by longitudinal chromoelectric fields
Chromo-magnetic suppression?

Ratio of longitudinal magnetic over longitudinal electric contributions

“Thin” nuclei ($\gamma \sim 200 \sim 1000$)

“Thick” nuclei ($\gamma \sim 20 \sim 40$)

$tr(B_L^2) / tr(E_L^2)$

$t$ [fm/c]

$\sigma = 0.008$ fm

$\sigma = 0.016$ fm

$\sigma = 0.032$ fm

$\sigma = 0.16$ fm

$\sigma = 0.24$ fm

$\sigma = 0.32$ fm

Small ratio for thick nuclei
Chromo-magnetic suppression?

Ratio of longitudinal magnetic over longitudinal electric contributions

\[ \frac{\text{tr}(B_L^2)}{\text{tr}(E_L^2)} \text{ vs. } t \text{ [fm/c]} \]

256³ cells, \( a_s = 0.02 \text{ fm} \)

\[ \sigma = 0.08 \iff \gamma \approx 45 \]

Strong dependence on IR regulator \( m \)

\( \rightarrow \) spurious effect of our choice of initial conditions?
Longitudinal structure

Current implementation

Longitudinal randomness

boosted

“at rest”

“at rest”

boosted

Longitudinal randomness…

• leads to higher energy density in the glasma
  [Fukushima, PRD 77 (2008) 074005]

• could provide boost-invariance breaking perturbations, which can drive system towards isotropization
  [Epelbaum, Gelis, PRL 111 (2013) 232301]
Wilson line expectation value $\langle \text{tr}(V) \rangle$ of a single nucleus is sensitive to longitudinal structure.

**Embedded 2D MV-model:**

$$\langle \hat{\rho}^a(x_T)\hat{\rho}^b(x'_T) \rangle = g^2 \mu^2 \delta^{(2)}(x_T - x'_T) \delta^{ab}$$

$$\rho(t, z, x_T) = f(z - t) \hat{\rho}(x_T)$$

**3D MV-model:**

(with random longitudinal structure)

$$\langle \rho^a(x^-, x) \rho^b(x^-, x') \rangle = g^2 \mu^2 f(x^-) \delta(x^- - x'^-) \delta^{(2)}(x_T - x'_T) \delta^{ab}$$

$$f(z) \ldots \text{longitudinal profile function} \quad x^- = \frac{t - z}{\sqrt{2}}$$

Introducing independent “sheets” in longitudinal direction

[Fukushima, PRD 77 (2008) 074005]
**Lattice dispersion**

Temporal to spatial lattice spacing: \( \xi = \frac{a_t}{a_s} \)

Courant-Friedrichs-Lewy (CFL) condition: \( \xi \leq \frac{1}{\sqrt{d}} \) in \( d \) dimensions
(for explicit solvers like the Leapfrog algorithm)
Explicit vs. implicit solvers

Continuum action: \[ S[\phi] = \frac{1}{2} \int_x \partial_\mu \phi \partial^\mu \phi \]  

Equations of motion: \[ \delta S = 0 \Rightarrow \partial_\mu \partial^\mu \phi = \partial_t^2 \phi - \partial_x^2 \phi = 0 \]

Discretized: \[
\frac{\phi(x,t+1) - 2\phi(x,t) + \phi(x,t-1)}{a_t} = \frac{\phi(x+1,t) - 2\phi(x,t) + \phi(x-1,t)}{a_s}
\]

Explicit solver: \[ \phi(x,t+1) = F(\phi(...,t),\phi(...,t-1)) \]

Implicit solver: \[ \phi(x,t+1) = F(\phi(...,t+1),\phi(...,t),\phi(...,t-1)) \]

Discretized action for explicit solver:
\[ S[\phi] = \frac{1}{2} V \sum_x \left( \left( \frac{\phi(x,t+1) - \phi(x,t)}{a_t} \right)^2 + \left( \frac{\phi(x+1,t) - \phi(x,t)}{a_s} \right)^2 \right) \]

1D scalar field example
Variational integrators

**Usual approach**

**Variation**

\[ \delta S = 0 \]

**Equations of motion + preserved constraints**

\[ \partial_\mu \partial^\mu \phi(x) + \ldots = 0 \]

\[ C(\phi(x)) = 0 \]

**Discretization**

(finite differences, sums, ...)

**Discretized action**

\[ S[\phi] = V \sum_x \mathcal{L}(\phi_x, \bar{\partial}_\mu \phi_x, \ldots) \]

If possible: keep symmetries!

**Variational integrators**

**Continuum**

\[ S[\phi] = \int d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x), \ldots) \]

**Discretized equations of motion + constraints (?)**

\[ \bar{\partial}_\mu \bar{\partial}^\mu \phi(x) + \ldots = 0 \]

\[ \bar{C}(\phi_x) = 0 \]

**Discrete Variation**

\[ \delta S = 0 \]

“Inherited” symmetries
Lattice gauge theory

Link variable: \( U_{x,\mu} \cong \exp(ig a^{\mu} A_{x,\mu}) \)

Plaquette: \( U_{x,\mu,\nu} = U_{x,\mu} U_{x+\mu,\nu} U_{x+\mu+\nu,-\mu} U_{x+\nu,-\mu} \)

Relation to field strength tensor:
\[
\text{tr}(2 - U_{x,\mu,\nu} - U_{x,\mu,\nu}^\dagger) \approx \frac{1}{2} \sum_a \left( g a^\mu a^\nu F^{a}_{\mu \nu}(x) \right)^2
\]

Identity: \( C_{x,\mu,\nu} \equiv U_{x,\mu} U_{x+\mu,\nu} - U_{x,\nu} U_{x+\nu,\mu} \)

\[
M_{x,ij} \quad \text{and} \quad W_{x,1i}
\]
Dispersion-free propagation

Standard *Wilson action:*

$$S[U] = \frac{V}{g^2} \sum_x \left( \sum_i \frac{1}{(a^0 a^i)^2} \text{tr} \left( 2 - U_{x,0i} - U_{x,0i}^\dagger \right) - \frac{1}{2} \sum_{i,j} \frac{1}{(a^i a^j)^2} \text{tr} \left( 2 - U_{x,ij} - U_{x,ij}^\dagger \right) \right)$$

Discretized action for the *semi-implicit scheme:*

$$S[U] = \frac{V}{g^2} \sum_x \left( \sum_{i,j} \frac{1}{(a^0 a^i)^2} \text{tr} \left( C_{x,0i} C_{x,0i}^\dagger \right) + \sum_i \frac{1}{(a^0 a^i)^2} \text{tr} \left( C_{x,0i} C_{x,0i}^\dagger \right) \right) - \frac{1}{4} \sum_{i,j} \frac{1}{(a^i a^j)^2} \text{tr} \left( C_{x,ij} M_{x,ij}^\dagger \right) - \frac{1}{4} \sum_{i,j} \frac{1}{(a^i a^j)^2} \text{tr} \left( C_{x,1j} W_{x,1j}^\dagger + \text{h.c.} \right)$$

Implicit part

Semi-implicit part

with $C_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\mu,\nu} - U_{x,\nu} U_{x+\nu,\mu}$ etc.

For details see:
Lattice dispersion for leapfrog (LF), implicit (IM) and semi-implicit (SI) schemes.
Computational challenges

Simulating small part of nuclei at RHIC energies:

\[ \gamma \text{-factor: } 100 \]
\[ \text{Lattice: } 2048 \times 192^2 \text{ cells} \]
\[ \text{Gauge group: SU(2)} \]
\[ \text{Color sheets: } 1 \]
\[ \text{Simulation box: } (6 \text{ fm})^3 \]

\[ \rightarrow 25 \text{ GB simulation data} \]
\[ \rightarrow 192 \text{ core hours on Vienna Scientific Cluster (VSC-3)} \]

Simulating realistic off-central full size nuclei at LHC energies:

\[ \gamma \text{-factor: } 2500 \]
\[ \text{Lattice: } (25 \times 20480) \times 1920^2 \text{ cells} \]
\[ \text{Gauge group: SU(3)} \]
\[ \text{Color sheets: } 100 \]
\[ \text{Simulation box: } (60 \text{ fm})^3 \]

\[ \rightarrow 25 \text{ PB simulation data} \]
\[ \rightarrow 5 \text{ million core years on VSC-3} \]
\[ (150 \text{ years on VSC3; but only 130 TB available}) \]
Machine learning in fluid dynamics

Accelerating Eulerian Fluid Simulation With Convolutional Networks
Tompson et al, arxiv:1607.03597

- Compress computation time and memory usage
- Use convolutional autoencoders to compress state size
- Learn dynamics on compressed form
- Can generalize to larger grid sizes

Lat-Net: Compressing Lattice Boltzmann Flow Simulations using Deep Neural Networks
Hennigh, arxiv:1705.09036
Beyond speeding up simulations...

- Why do neural networks work surprisingly well in some cases, and why and how can they fail in others?
  → we can compare to ground-truth physical result
  - Learn about physical system from new viewpoint involving machine learning tools
    → latent space representation can capture relevant degrees of freedom
    → irrelevant degrees of freedom are integrated out into weight parameters

- Renormalization group picture of deep neural networks:

An exact mapping between the Variational Renormalization Group and Deep Learning
Mehta, Schwab, arxiv:1410.3831
Jet momentum broadening

Work in preparation, together with Daniel Schuh and David Müller
Conclusions & Outlook

• Simulate CGC collisions in 3D+1 using Colored Particle-In-Cell
• Finite thickness breaks boost invariance $\rightarrow$ Gaussian rapidity profiles

• **Outlook:**
  - Study effect of random longitudinal structure
  - Observables at early times: gluon production, momentum broadening, ...
  - Explore machine learning

D. Gelfand, AI, D. Müller, Phys. Rev. D94 (2016) no.1, 014020

**open source:**
www.openpixi.org