Simulating the earliest stages of heavy-ion collisions

Teilchenphysikseminar Universität Wien
November 12, 2019

Andreas Ipp

based on
D. Gelfand, AI, D. Müller, Phys. Rev. D94 (2016) no.1, 014020
AI, D. Müller, Phys. Lett. B (2017) 771
AI, D. Müller, Eur.Phys.J. C78 (2018) no.11, 884

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INSTITUTE FOR THEORETICAL PHYSICS

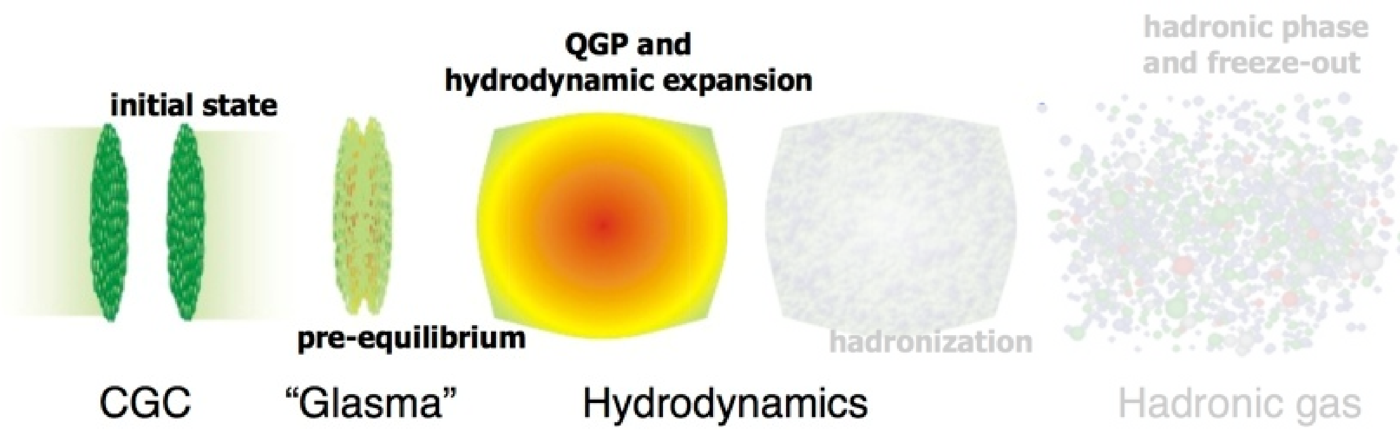
$\int dk\Pi$

FWF Der Wissenschaftsfonds.

TU WIEN

Outline

- Simulation of early stages of heavy-ion collisions
  - color-glass-condensate (CGC) framework
  - colored particle-in-cell (CPIC)
  - beyond boost-invariance
- Numerical results
  - energy density at different rapidities
  - comparison to RHIC data



Berndt Müller, arXiv:1309.7616

QCD phase diagram

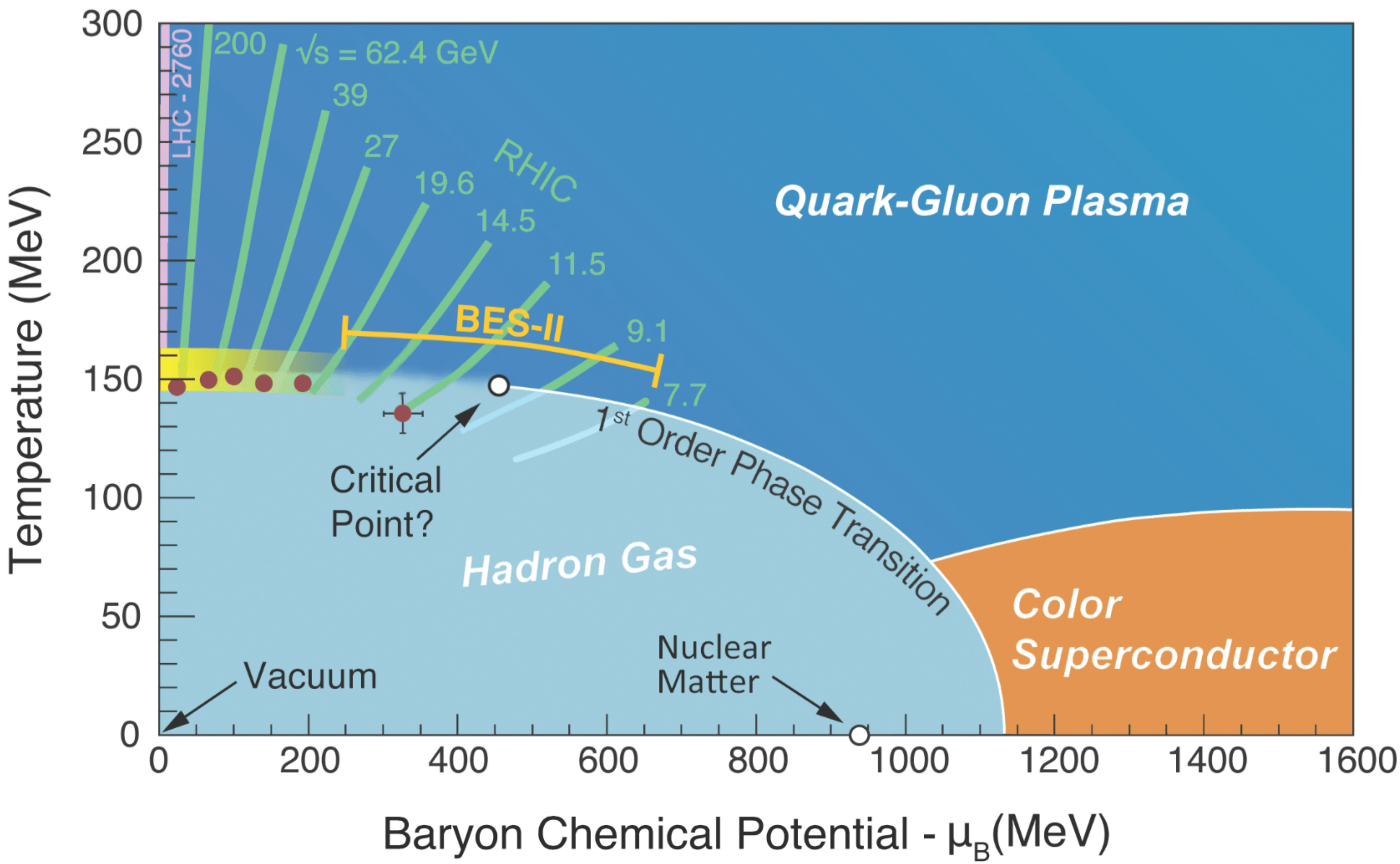
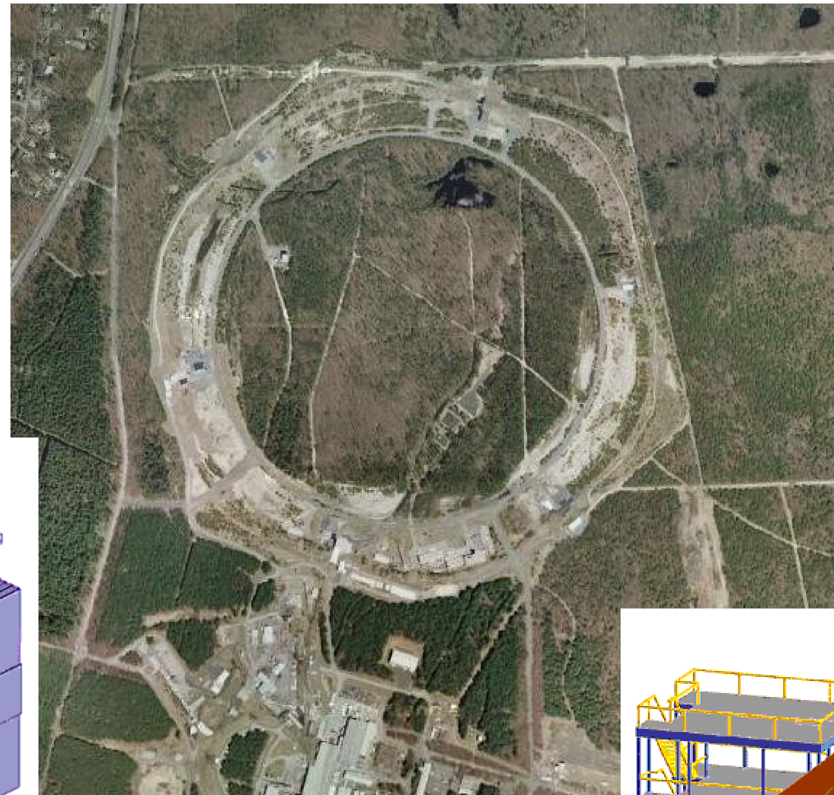


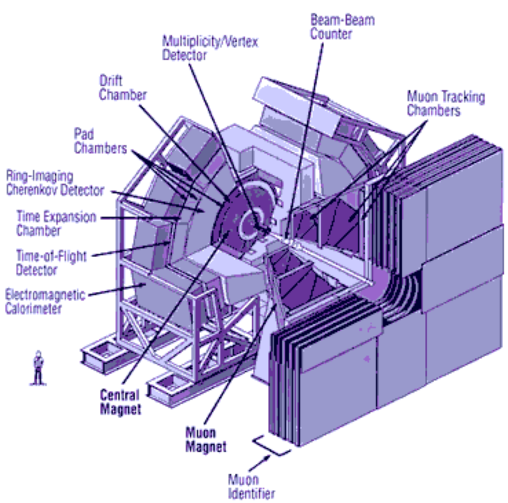
Illustration: Swagato Mukherjee, Brookhaven National Laboratory.

Relativistic Heavy Ion Collider (RHIC)

Brookhaven National Laboratory (USA)



PHENIX



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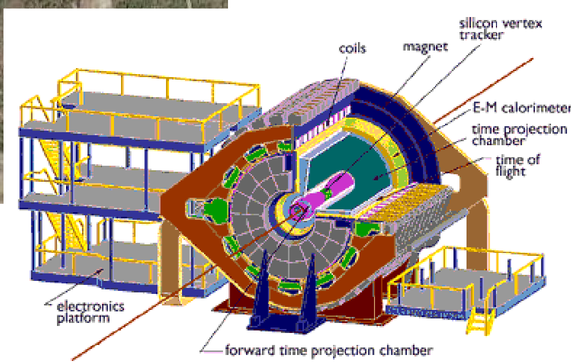
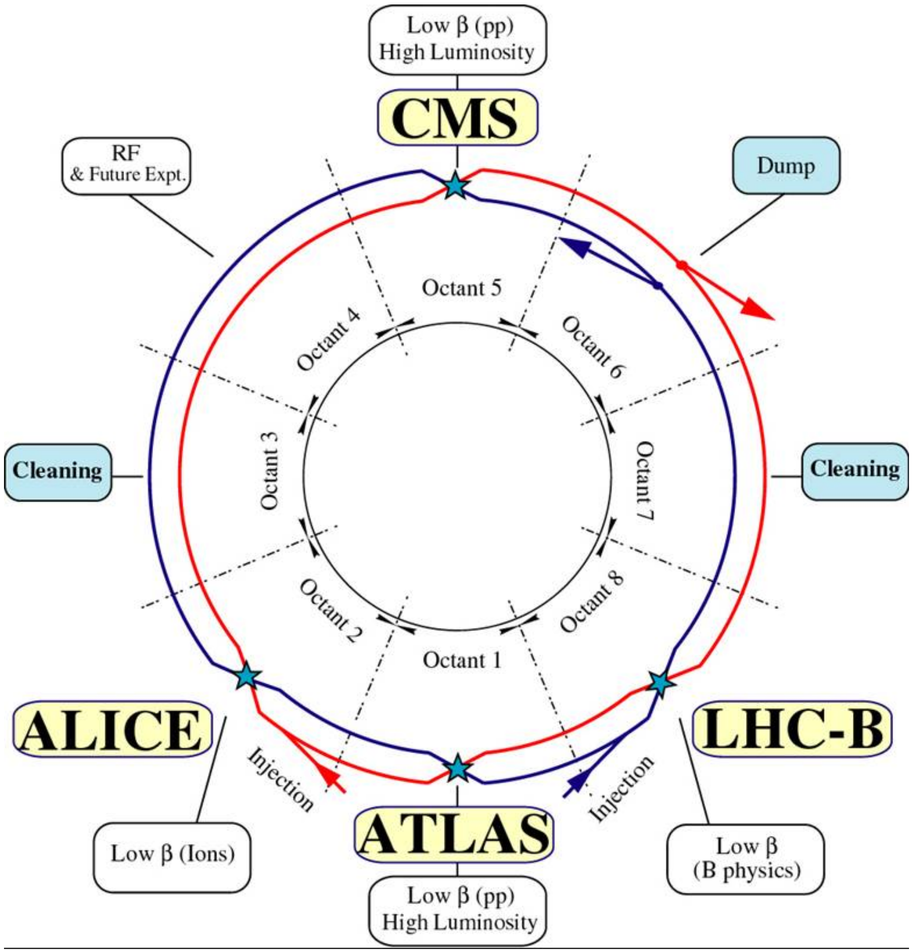
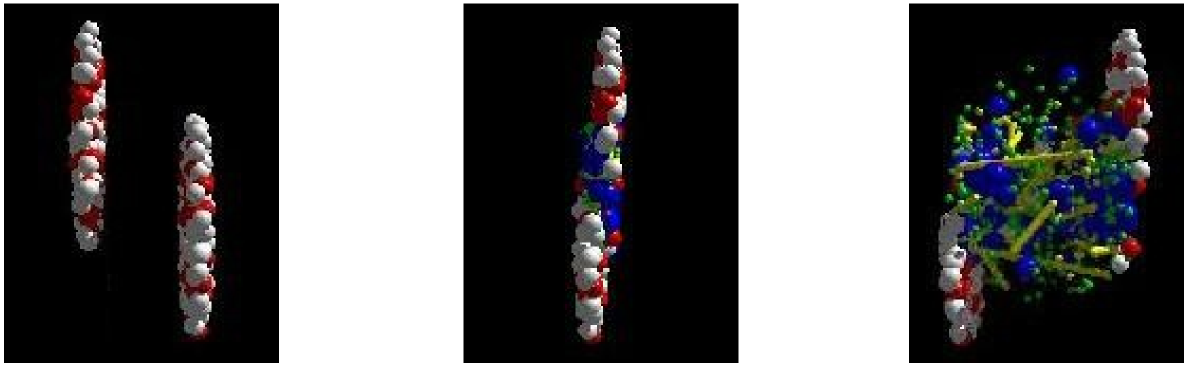


Large Hadron Collider (LHC)

CERN



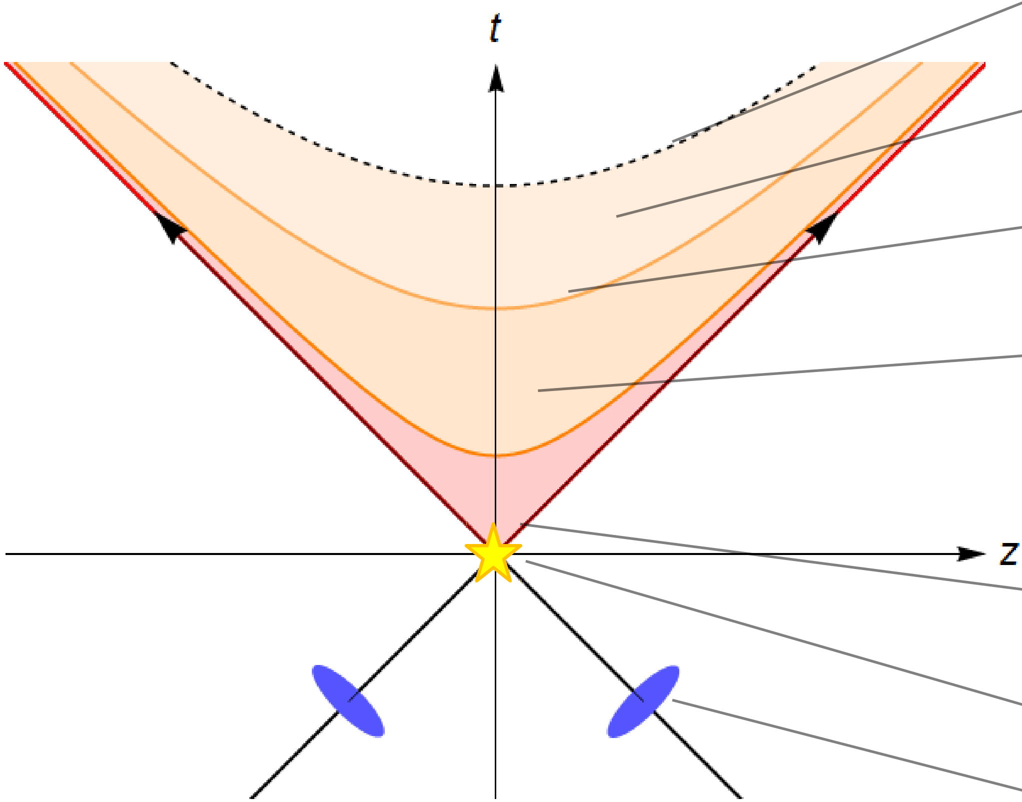
Heavy-ion collisions



(Simulation by UrQMD group, Frankfurt)

- Heavy ion nuclei (gold, lead) (*d*~14 fm)
- formation time of QGP: *t*~1 fm/c ≈ 3 ys
- QGP phase: RHIC 15 ys, LHC 25 ys
- good agreement with hydrodynamic simulations

Stages of a heavy-ion collision



$$1\,\mathrm{fm}/c \approx 3.3 \cdot 10^{-24}\,\mathrm{s} \approx 3.3\,\mathrm{ys}$$

Freeze-out (τ ≈ 15 fm/*c*):
interactions stop

Hadronic gas (τ ≈ 10 - 15 fm/*c*)**:** hadrons
(kinetic transport theory)

Hadronization (τ ≈ 10 fm/*c*)**:**
confinement transition → hadron formation

QGP (τ ≈ 1 - 10 fm/*c*)**:** quarks and gluons
(relativistic viscous hydrodynamics)
(almost) isotropic and in thermal equilibrium

Glasma (τ ≈ 0 - 1 fm/*c*)**:** quasi-classical fields
(classical field equations)

Collision event

Initial state: Lorentz-contracted nuclei
(color glass condensate)

scope of this project

Pancake thickness

French crêpe

© Image: David Monniaux

LHC (ALICE) @ CERN: Pb+Pb
with **~5.5 TeV** per nucleon pair
($\gamma \approx 2700$)

American pancake

RHIC @ BNL: Au+Au
with **~200 GeV** ($\gamma \approx 100$)

RHIC beam energy scan:
~7.7 – 62.4 GeV ($\gamma \approx 4$ - 30)

Color glass condensate

Nuclei at ultrarelativistic speeds can be described by **classical effective theory** in the color glass condensate (CGC) framework.
[Gelis, Iancu, Jalilian-Marian, Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010]

Large gluon occupation numbers → coherent, classical gluon field

Split degrees of freedom into ...

- Hard partons = classical color charges
- Soft gluons = classical gauge field

generates..

- Static field configuration due to time dilation.
- Collision of two such classical fields creates the **Glasma**.
  [Gelis, Int.J.Mod.Phys. A28 (2013) 1330001]

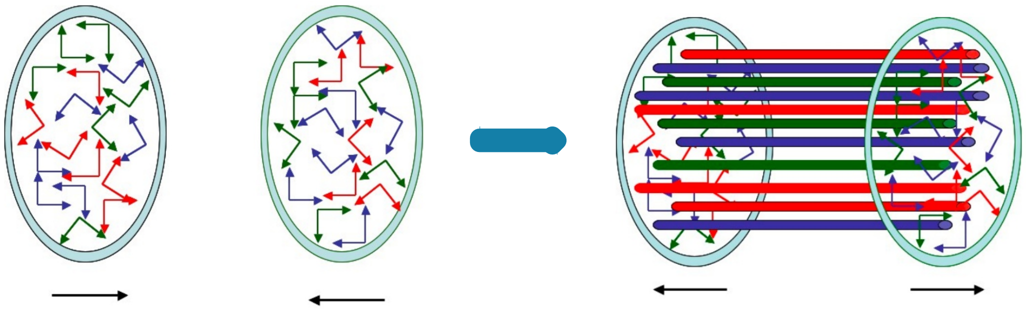
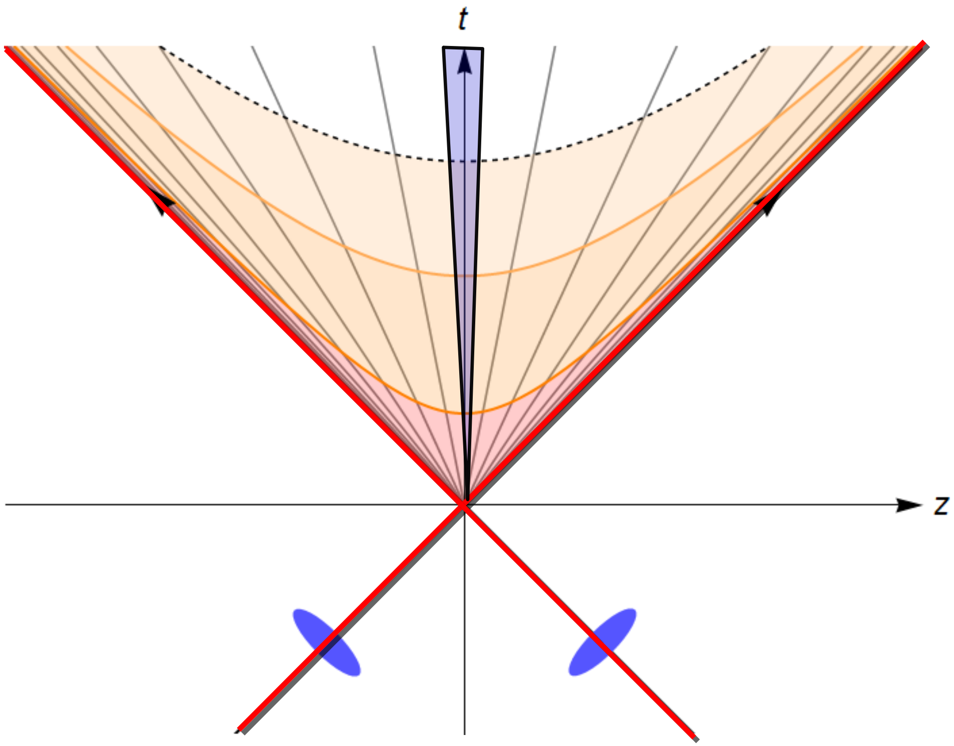


Figure from L. McLerran: Proceedings of ISMD08, p.3-18 (2008)

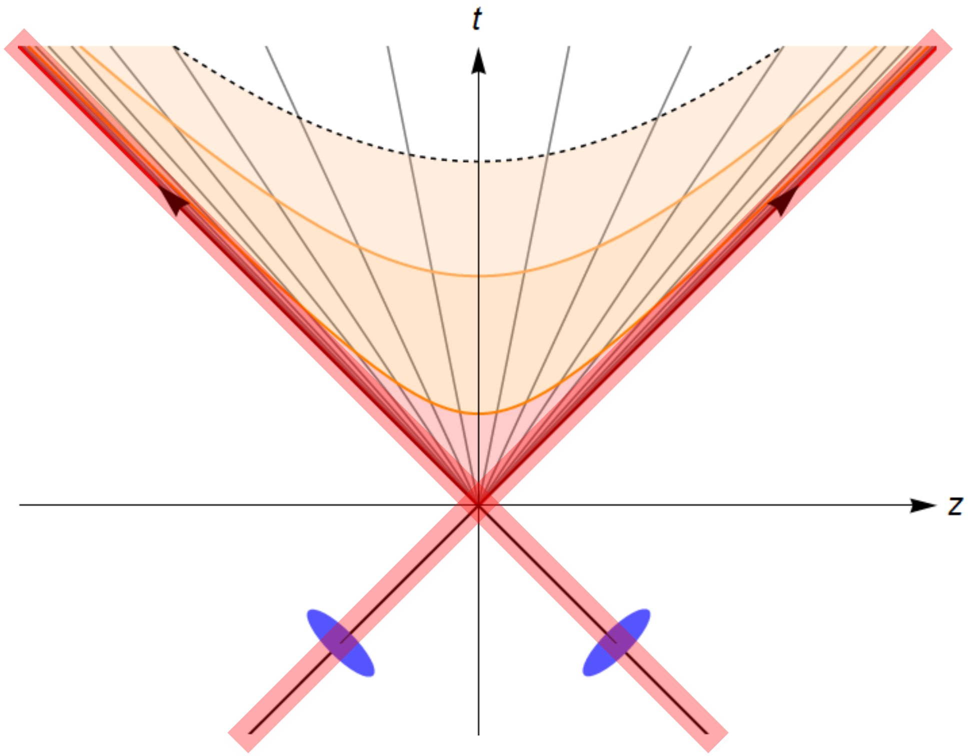
Boost-invariant CGC collision



- color glass condensate (CGC): hard and soft degrees of freedom, weak coupling
- infinitely thin color currents
- boost-invariant solution
- solve Yang-Mills equations numerically in 2+1 D

$$D_\mu F^{\mu\nu}(\tau, x_T) = 0$$

Finite nucleus thickness



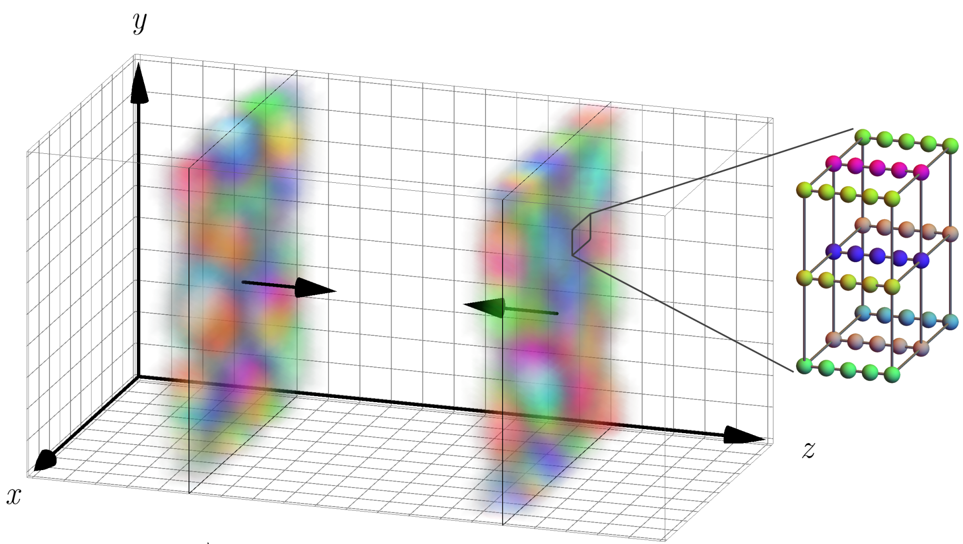
- extended color currents
- boost-invariance lost
- solve full 3+1 D Yang-Mills equation with currents

$$D_\mu F^{\mu\nu}(t,z,x_T) = J_1^\nu + J_2^\nu$$
$$D_\mu J^\mu(t,z,x_T) = 0$$

$$D_\mu \equiv \partial_\mu + ig[A_\mu, \cdot]$$

→ use Colored particle-in-cell (CPIC) in laboratory frame

Colored particle-in-cell (CPIC)



CPIC: non-Abelian generalization of the particle-in-cell method from plasma physics.

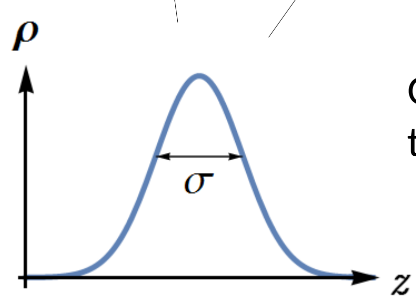
[A. Dumitru, Y. Nara, M. Strickland: PRD75:025016 (2007)]

Nucleus model: 2D McLerran-Venugopalan (MV) model .

[McLerran, Venugopalan: PRDD49 (1994) 3352-3355]

$$\langle \hat{\rho}^a(\boldsymbol{x}_T)\hat{\rho}^b(\boldsymbol{x}'_T)\rangle = g^2\mu^2\delta^{(2)}(\boldsymbol{x}_T-\boldsymbol{x}'_T)\delta^{ab}$$

$\mu \approx 0.5\,\mathrm{GeV}$ (Au, RHIC)



Gaussian profile with thickness σ.

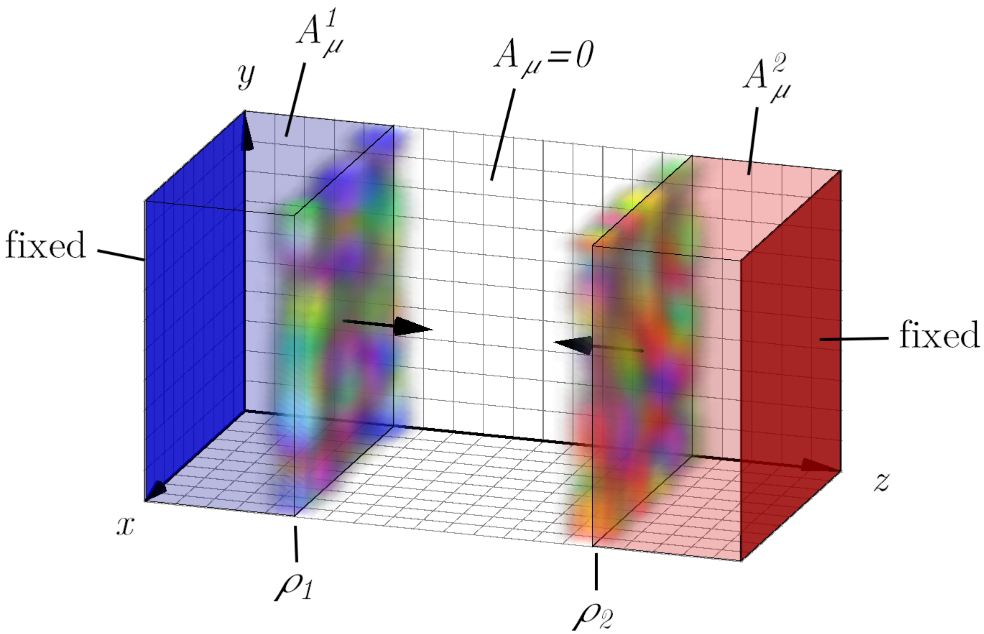
$$\sigma \approx R/\gamma$$

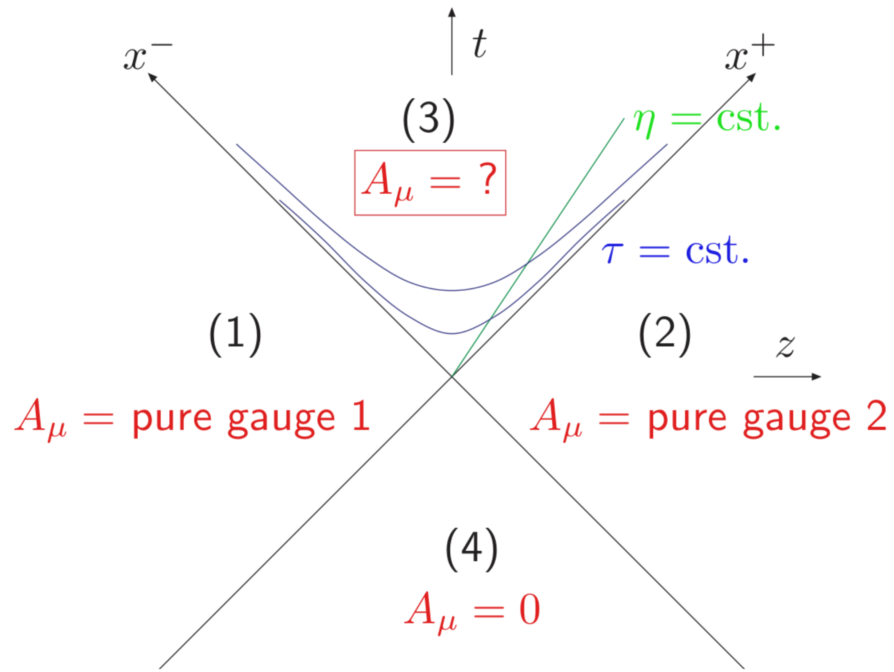
(no random longitudinal structure)

Infrared regulation

$$m \approx 200\,\mathrm{MeV}$$

Implementation





Lappi, Phys.Lett. B643 (2006) 11-16

Continuum equations of motion

$$D^{ab}_\mu F^{b,\mu\nu} = j^{a,\nu}$$

Lattice equations of motion

$$\dot{E}^a_i(x) = \frac{2}{ga^3} \sum_{j\neq i} \mathrm{Im}\left[\mathrm{tr}\left(U_{ij}(x) + U_{i-j}(x)\right)\right] - j^a_i(x)$$

$$U_{ij}(x) = U_i(x)U_j(x+i)U_{-i}(x+i+j)U_{-j}(x+j)$$

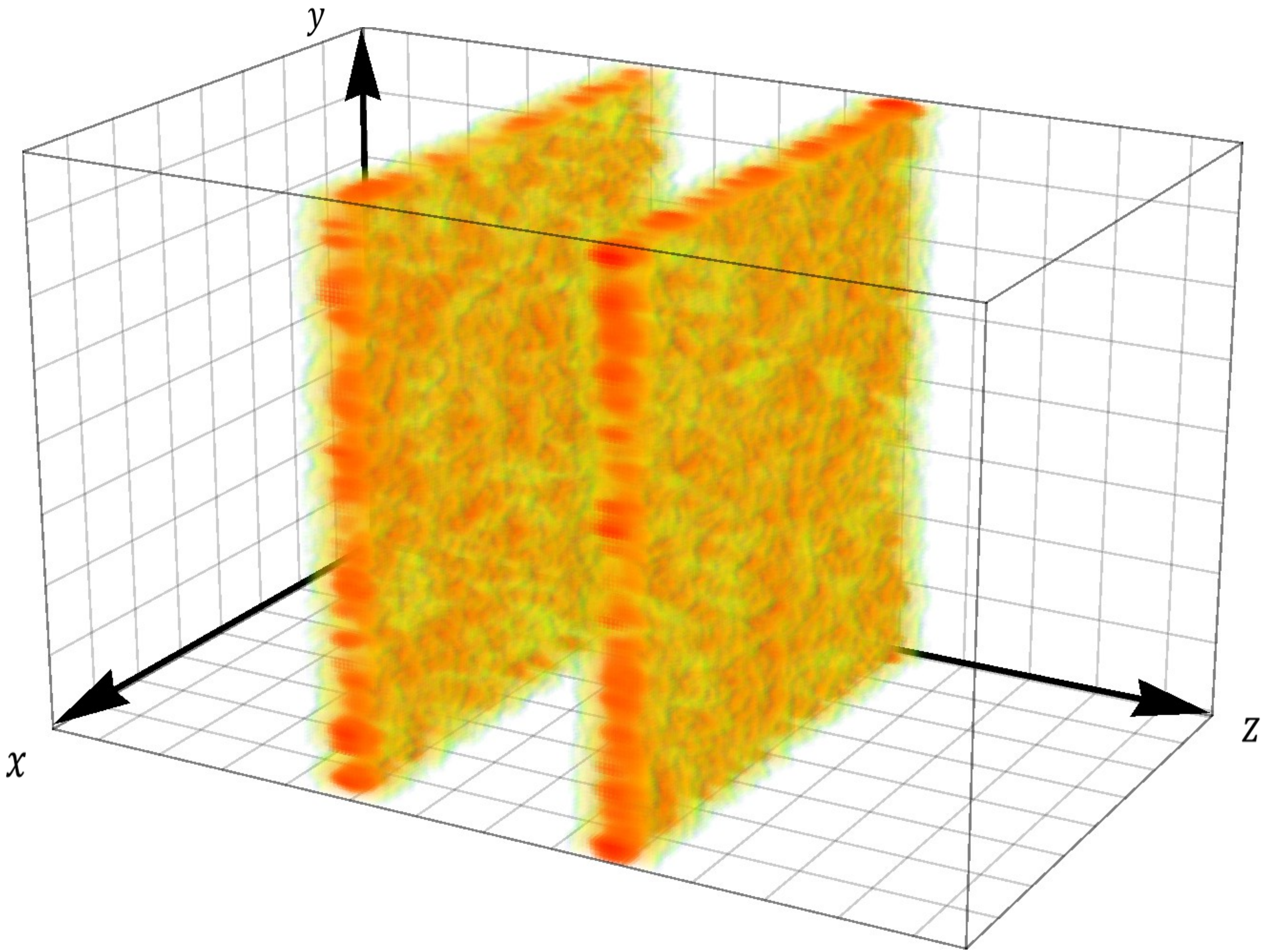
$$\dot{U}_i(x) = -igaE^a_i(x)t^aU_i(x)$$

Parallel transporters (gauge links): $U_i(x) = \exp\left(igaA^a_i(x)t^a\right)$

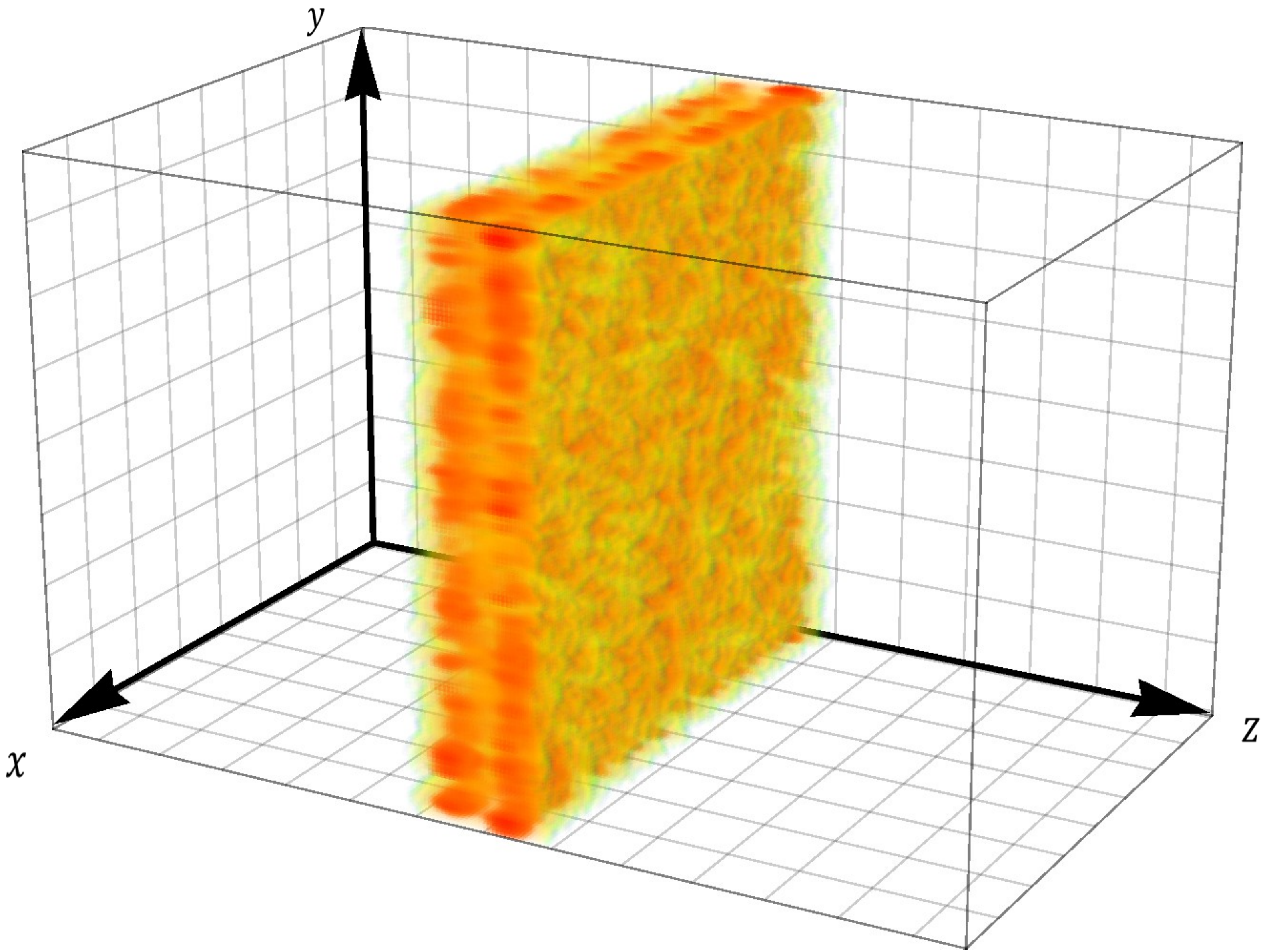
Results

Austrian Kaiserschmarrn („Emperor's mess")

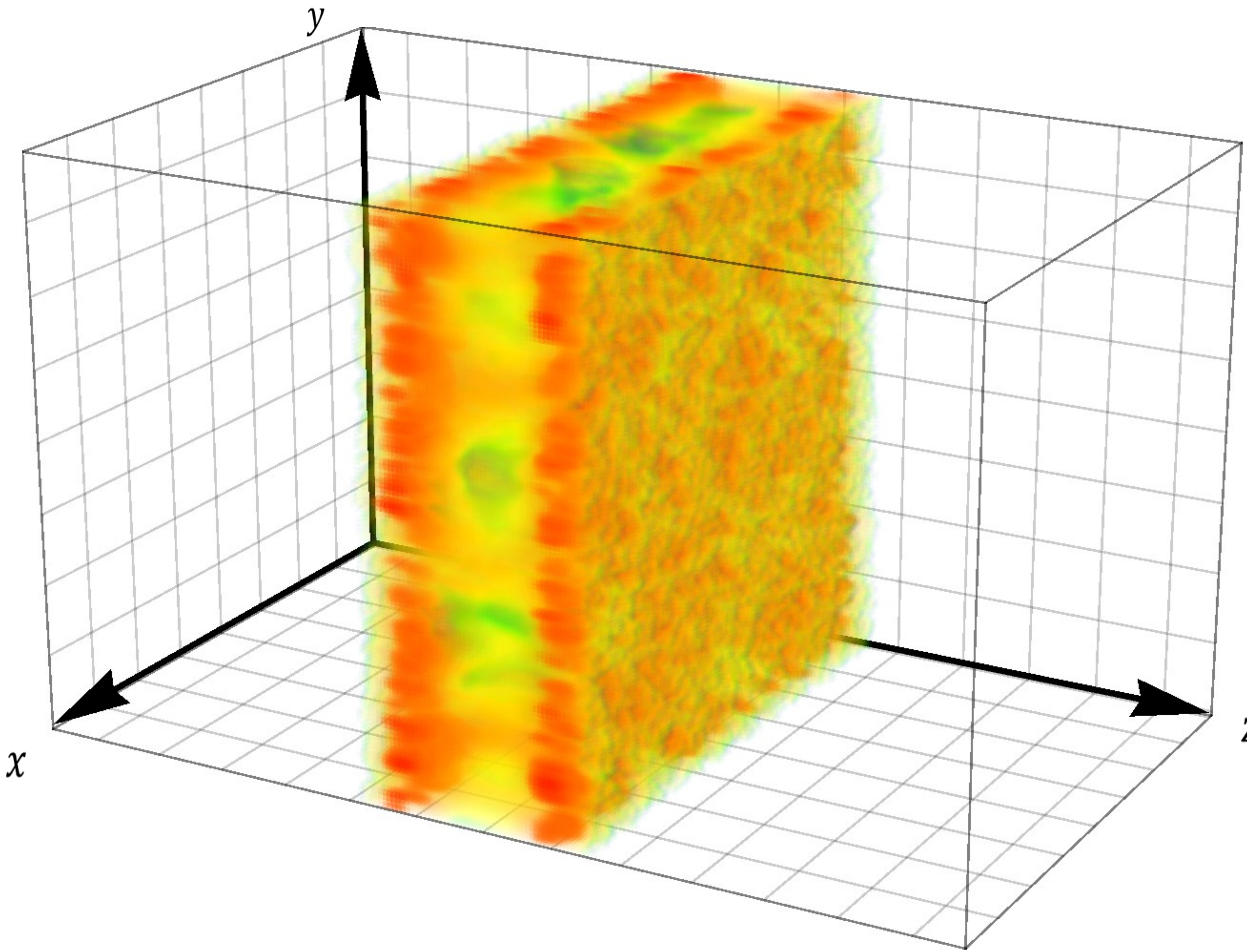
3D energy density



3D energy density



3D energy density



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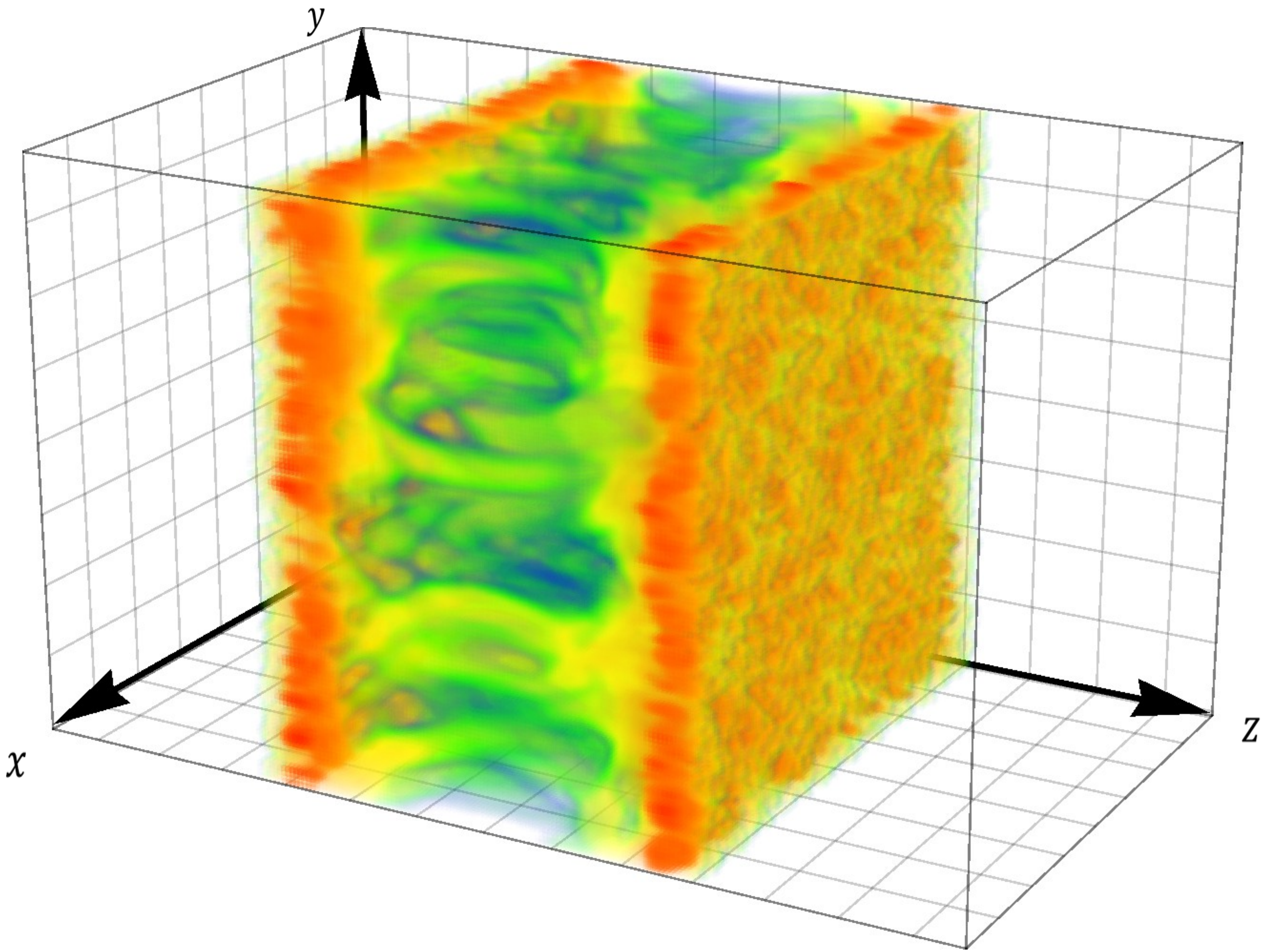
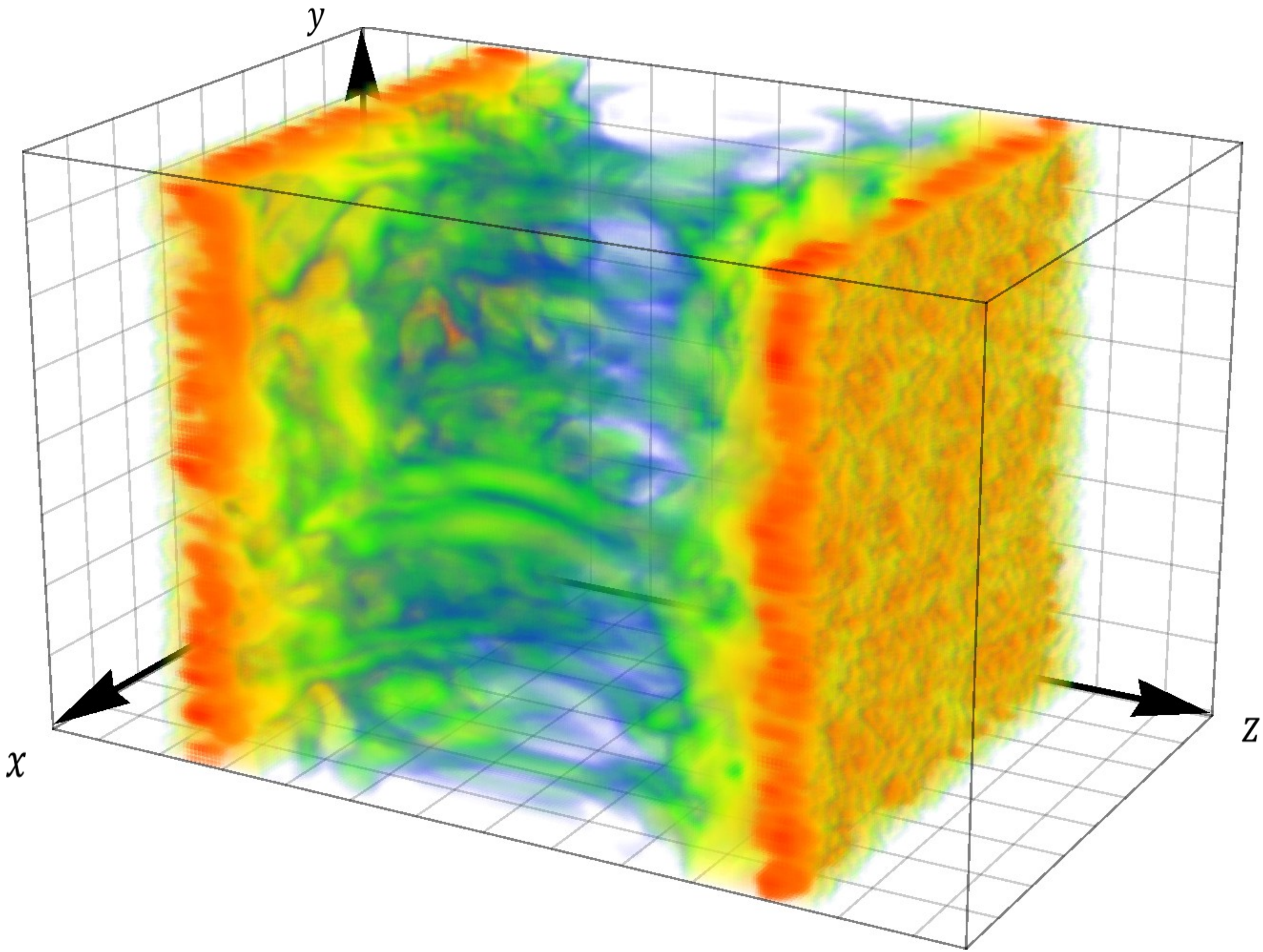
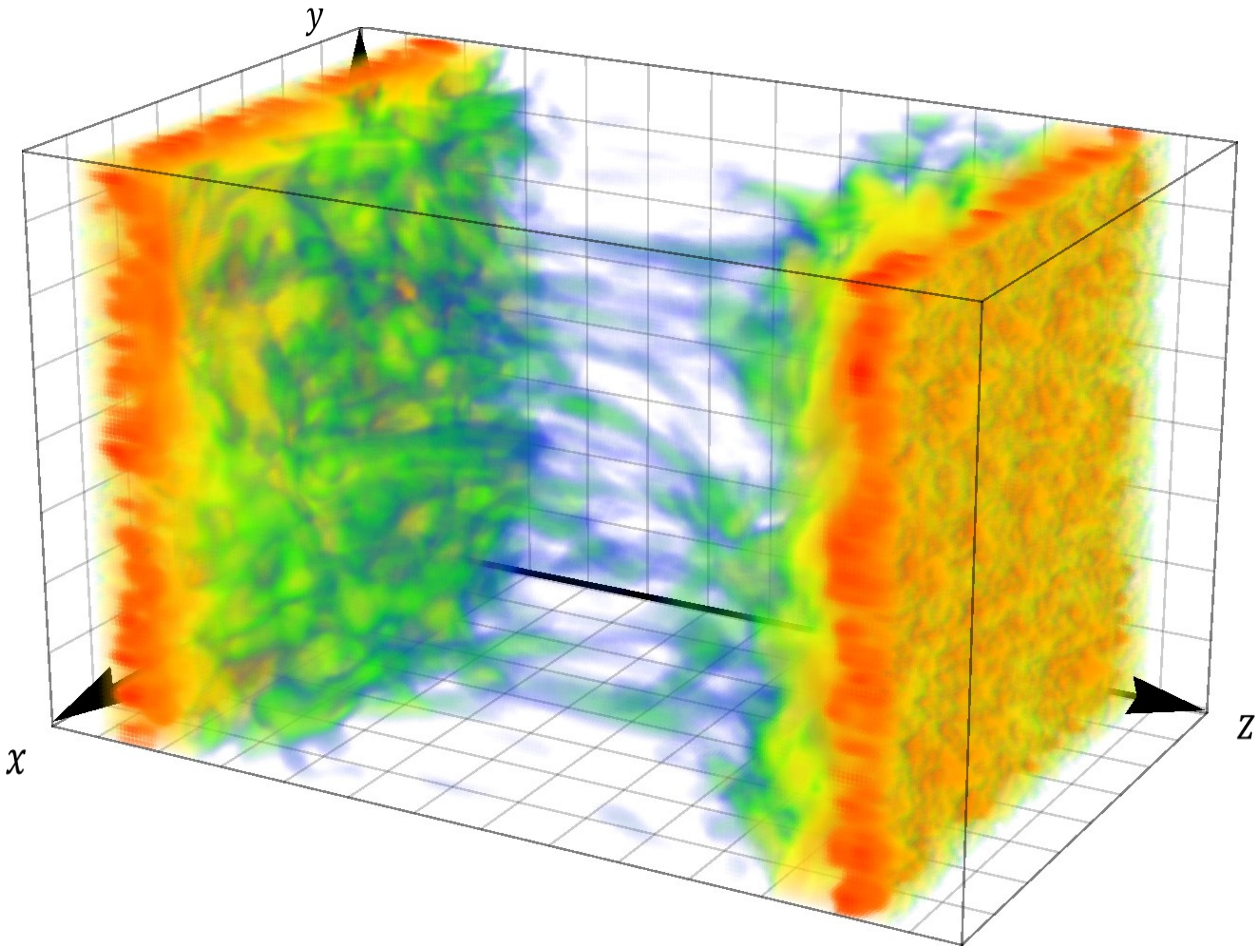


3D energy density

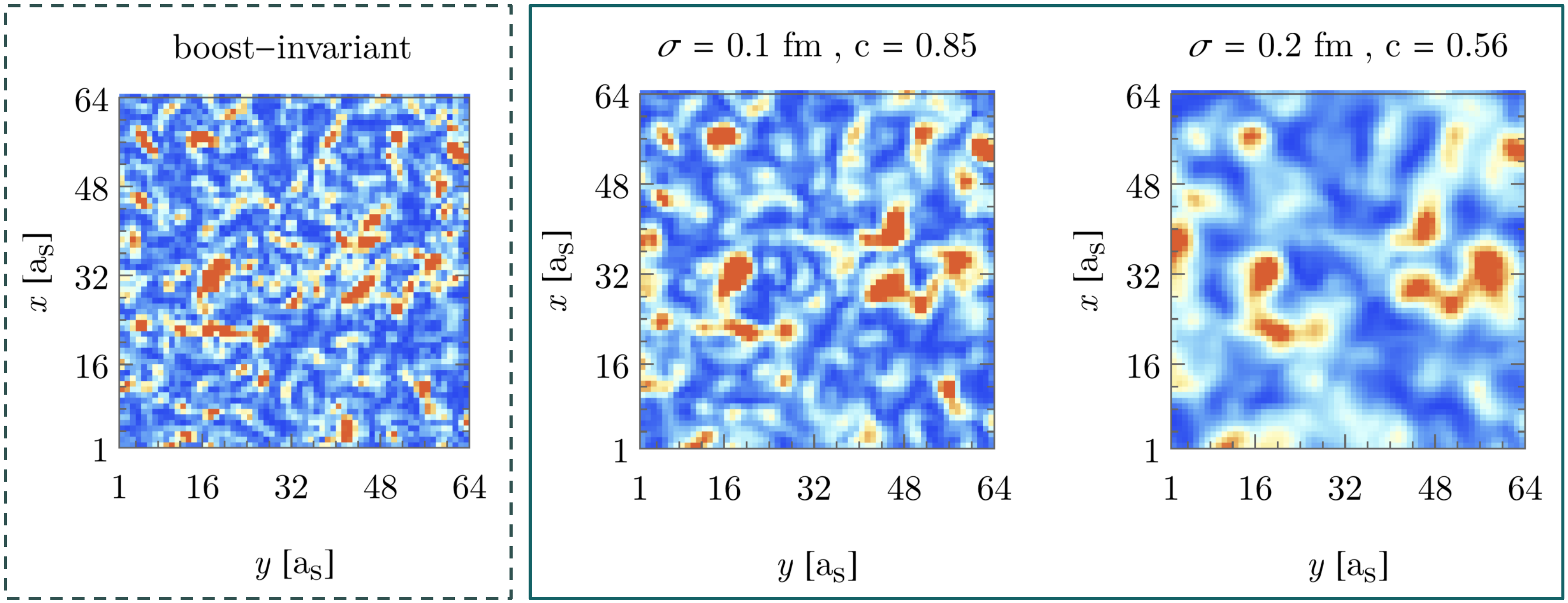


3D energy density



Comparison to boost-invariant results

Energy density component tr $E_L^2(x_T)$ in the transverse plane at $t = 0$.



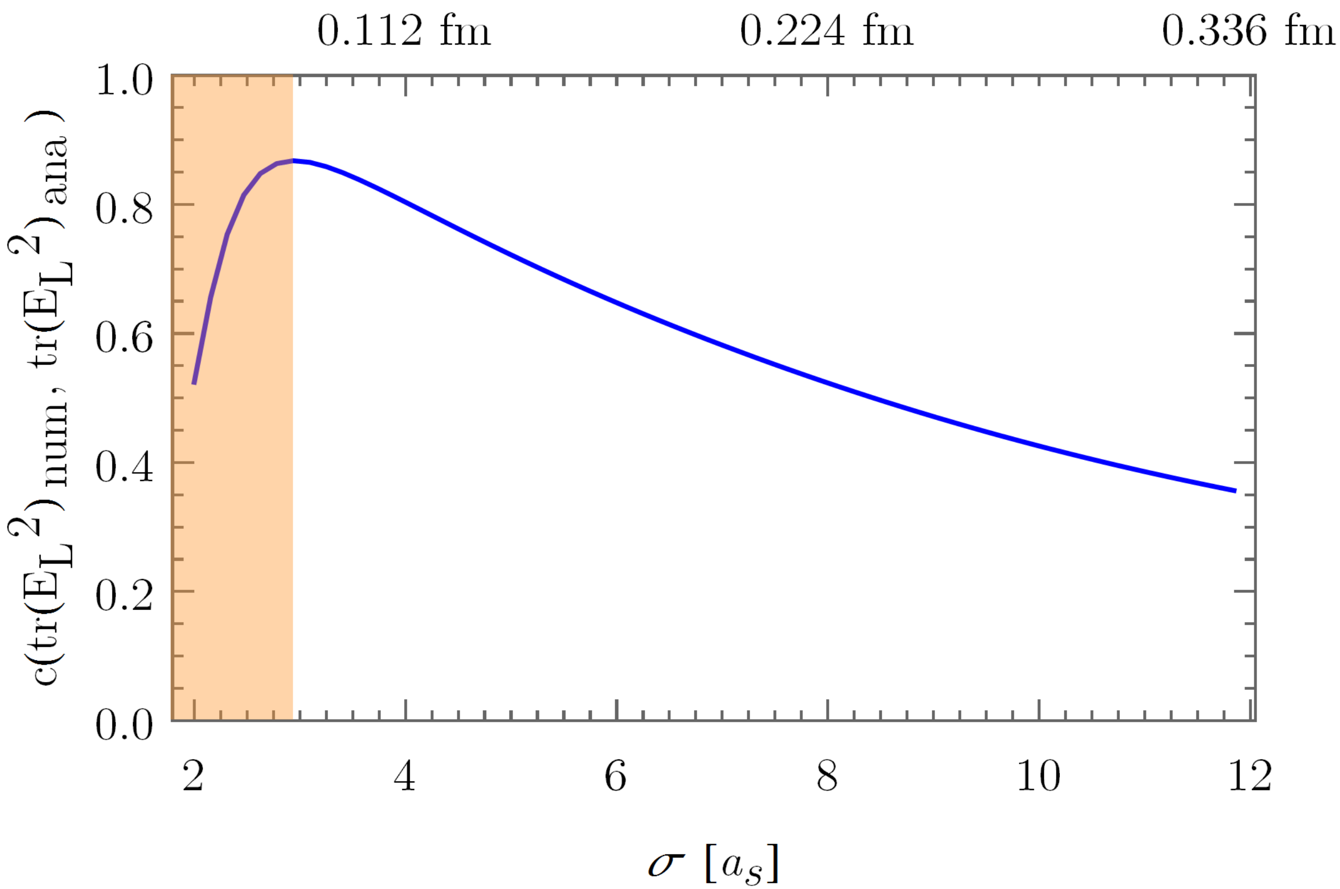
analytic result
(infinitely thin)

simulation result
(thin) (thicker)

Au-Au collision in the McLerran-Venugopalan (MV) model for SU(2)

256×128^2 cells, $a_s = 0.028$ fm
Shown: 64×64 cells

Correlation analytic ↔ numerical



[thickness]

(too thin – numerically unstable)

256×128^2 cells, $a_s = 0.028$ fm, $a_t = a_s / 2$

Observables

Main observable: energy-momentum tensor $T^{\mu\nu}(x)$

- Build $T^{\mu\nu}(x)$ from electric and magnetic fields $E_i^a(x), B_i^a(x)$
- Average over configurations and integrate over transverse plane

$$
\langle T^{\mu\nu} \rangle = \begin{pmatrix} \langle \varepsilon \rangle & 0 & 0 & \langle S_L \rangle \\ 0 & \langle p_T \rangle & 0 & 0 \\ 0 & 0 & \langle p_T \rangle & 0 \\ \langle S_L \rangle & 0 & 0 & \langle p_L \rangle \end{pmatrix}
$$

$$
\begin{aligned}
\langle \varepsilon \rangle &= \frac{1}{2} \left\langle E_T^2 + B_T^2 + E_L^2 + B_L^2 \right\rangle \\
\langle p_T \rangle &= \frac{1}{2} \left\langle E_L^2 + B_L^2 \right\rangle \\
\langle p_L \rangle &= \frac{1}{2} \left\langle E_T^2 + B_T^2 - E_L^2 - B_L^2 \right\rangle \\
\langle S_L \rangle &= \left\langle \left(\vec{E}^a \times \vec{B}^a \right)_L \right\rangle
\end{aligned}
$$

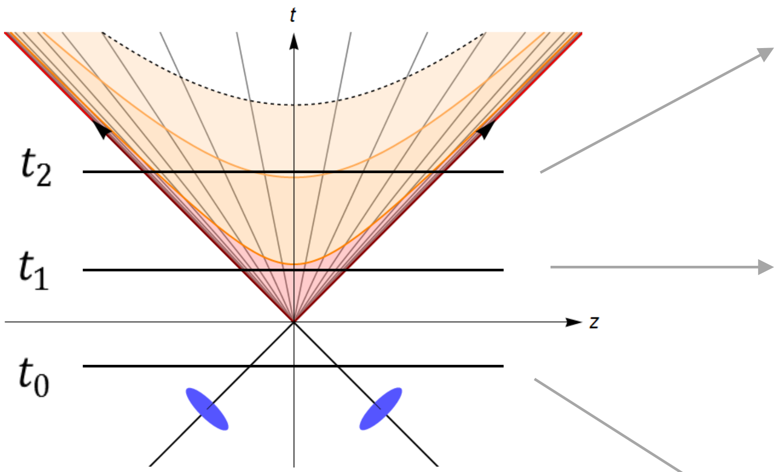
- Diagonalize, obtain local rest-frame energy density

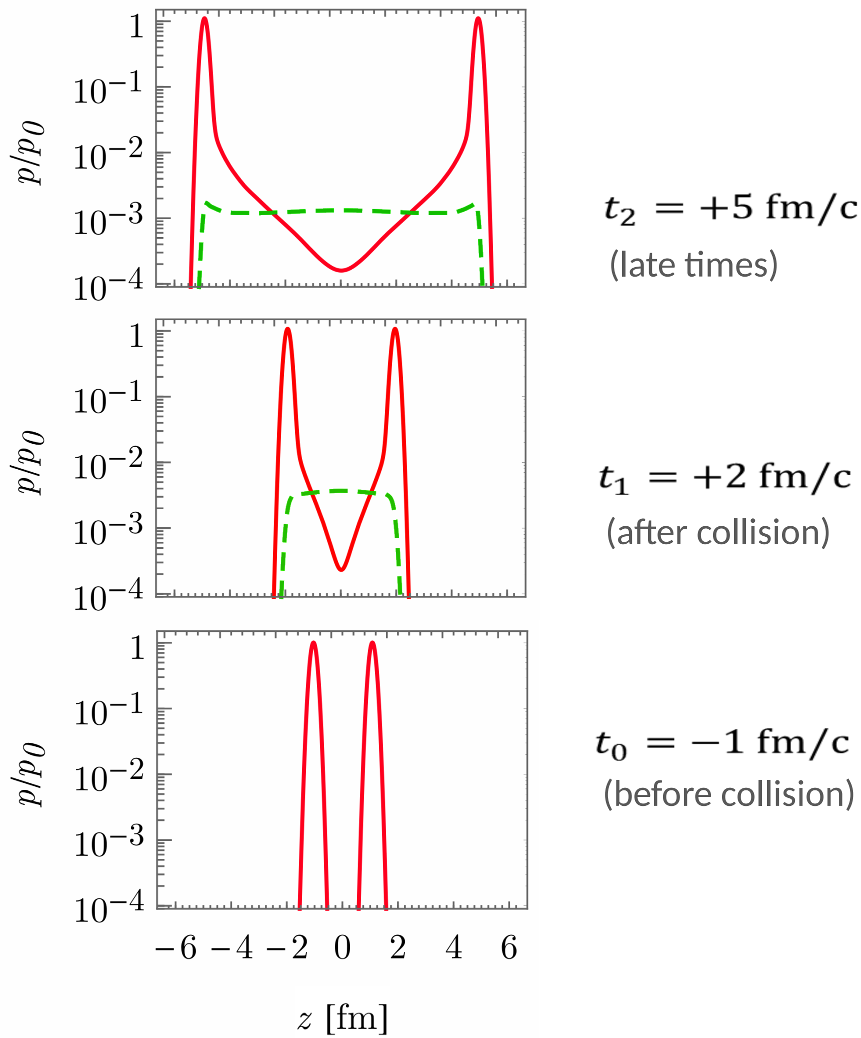
$$
\langle \varepsilon_{\mathrm{loc}} \rangle = \frac{1}{2} \left(\langle \varepsilon \rangle - \langle p_L \rangle + \sqrt{\left(\langle \varepsilon \rangle + \langle p_L \rangle \right)^2 - 4 \langle S_L \rangle^2} \right)
$$

Pressure anisotropy

Longitudinal pressure $p_L(z)$ and transverse pressure $p_T(z)$

→ Pronounced pressure anisotropy



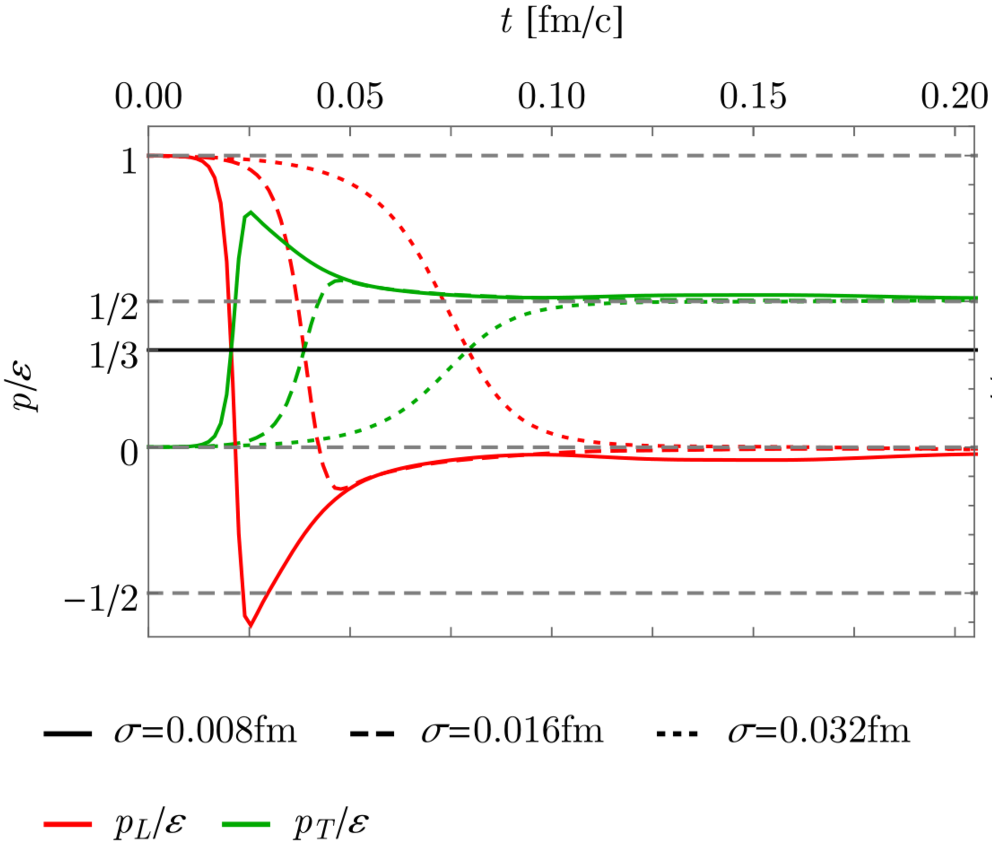


$$\langle p_T \rangle = \frac{1}{2} \langle E_L^2 + B_L^2 \rangle$$

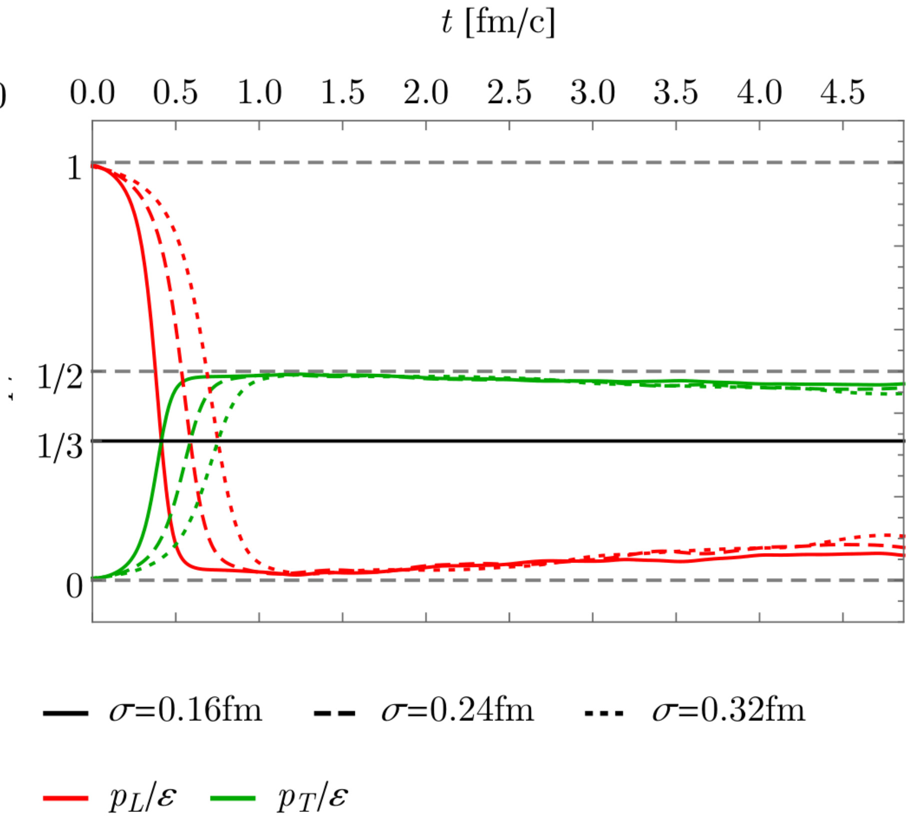
$$\langle p_L \rangle = \frac{1}{2} \langle E_T^2 + B_T^2 - E_L^2 - B_L^2 \rangle$$

Pressure anisotropy at midrapidity

“Thin” nuclei ($\gamma \sim 200 - 1000$)



“Thick” nuclei ($\gamma \sim 20 - 40$)



observe very slow isotropization

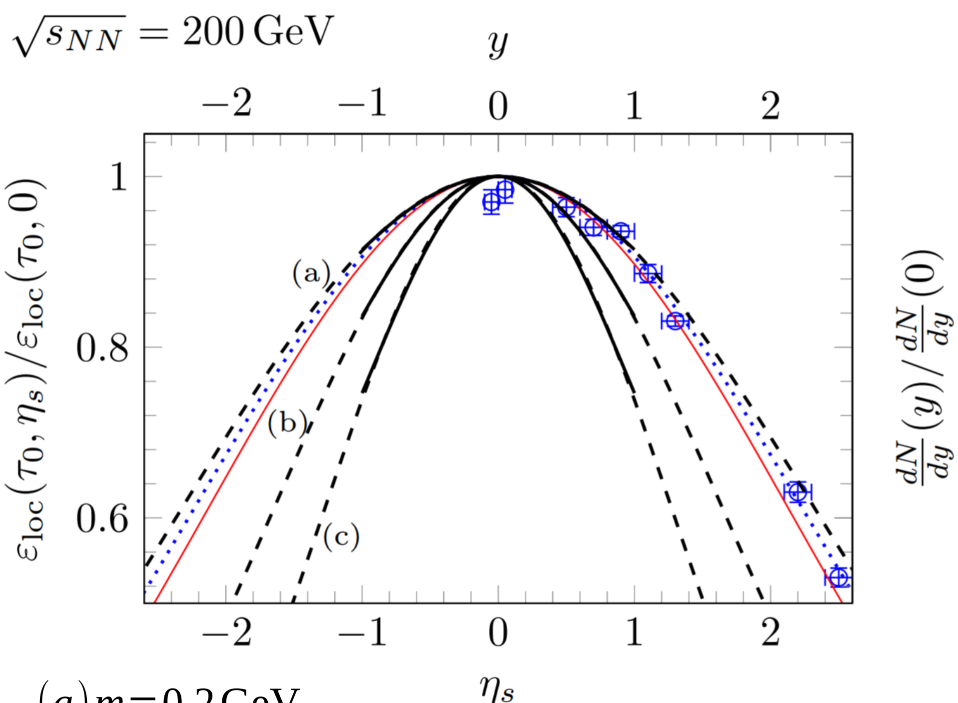
longitudinal pressure $p_L(z)$

transverse pressure $p_T(z)$

Rapidity profiles

Plot (space-time) rapidity profile of local rest-frame energy density

Compare to measured **rapidity profile of particle multiplicity (RHIC)** and **Landau model** prediction



$(a)\, m = 0.2\,\mathrm{GeV}$
$(b)\, m = 0.4\,\mathrm{GeV}$
$(c)\, m = 0.8\,\mathrm{GeV}$

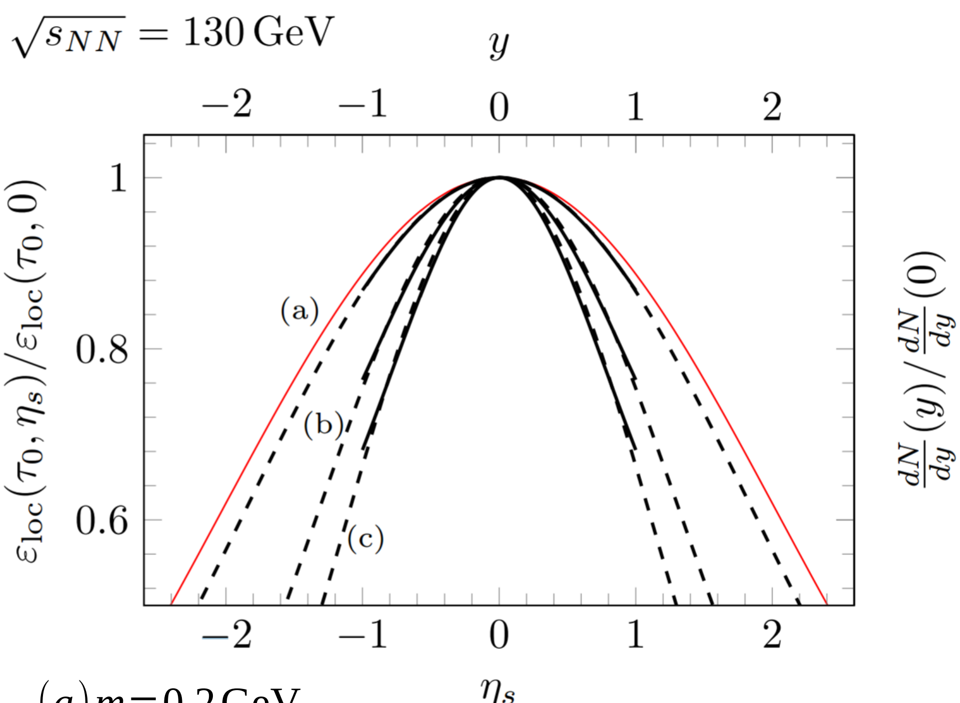
[RHIC data: Bearden et al., PRL 94 (2005) 162301]

- Simulation data in interval $\eta_s \in (-1, 1)$ at $\tau = 1\,\mathrm{fm}/c$
- Fit to Gaussian profile (dashed)
- Dependency on thickness (or rather $\sqrt{s}$)
- Strong dependency on IR regulator, but $m = 0.2\,\mathrm{GeV}$ gives realistic shape
- However: no hydrodynamic expansion included
- Limited rapidity interval

Rapidity profiles

Plot (space-time) rapidity profile of local rest-frame energy density

Compare to measured **rapidity profile of particle multiplicity (RHIC)** and **Landau model** prediction



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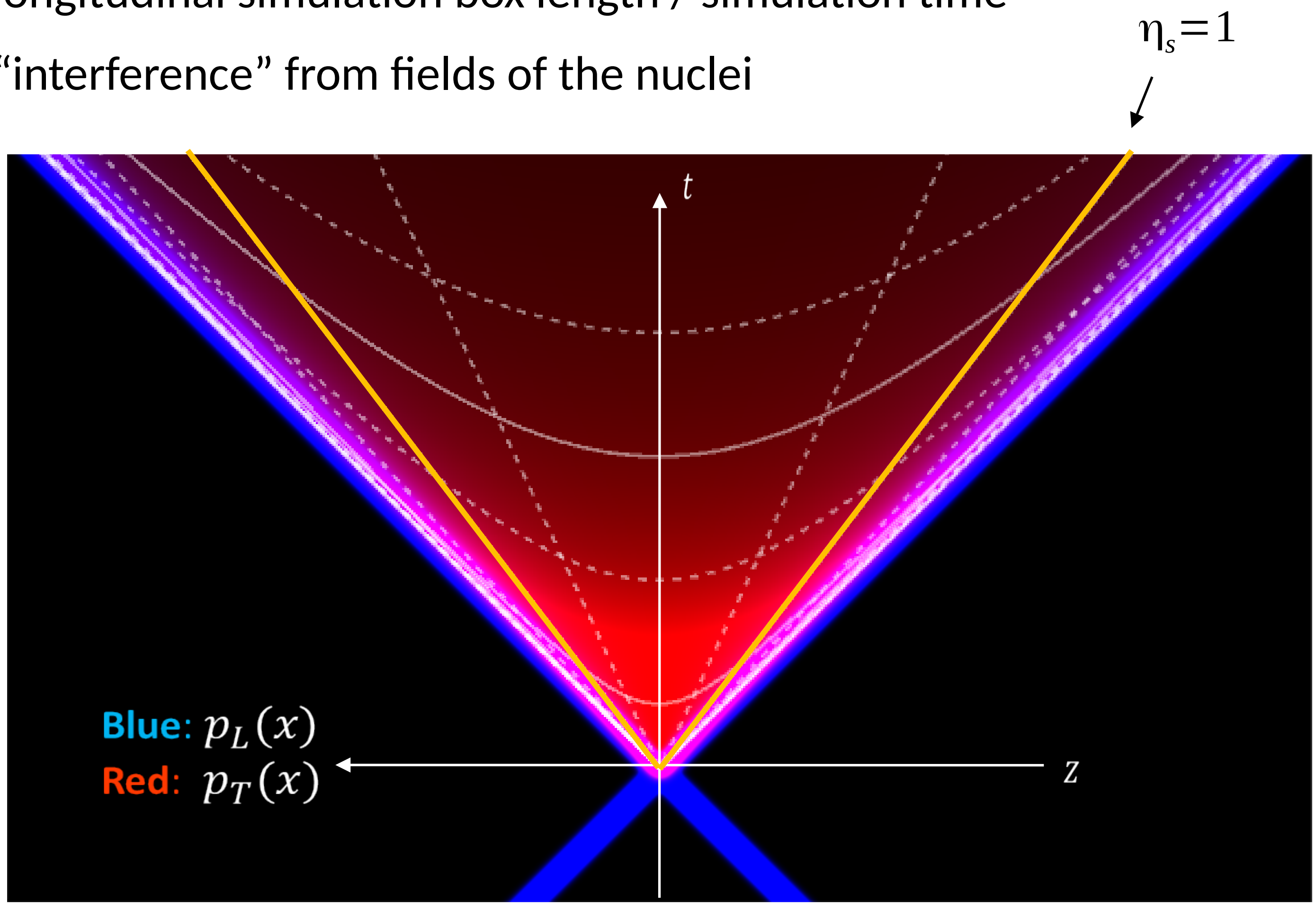
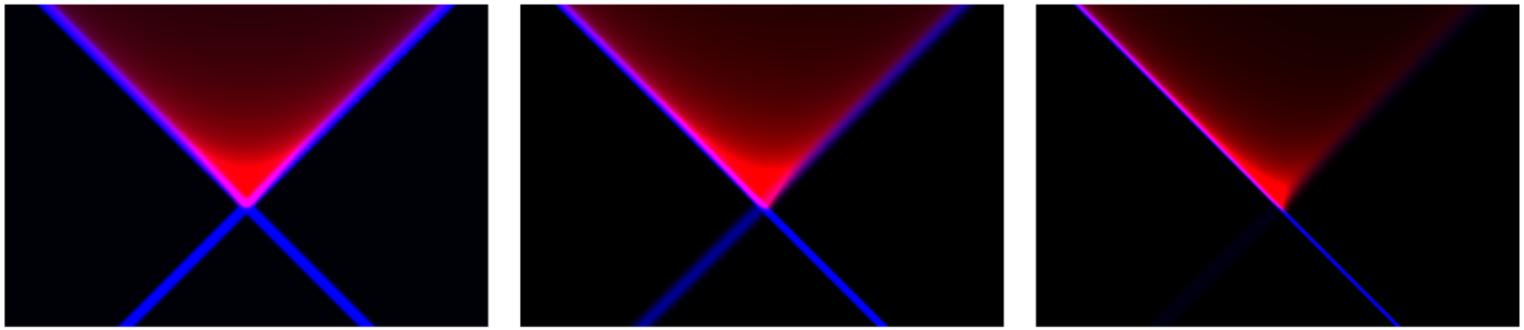
Rapidity profiles

Limited rapidity interval due to ..

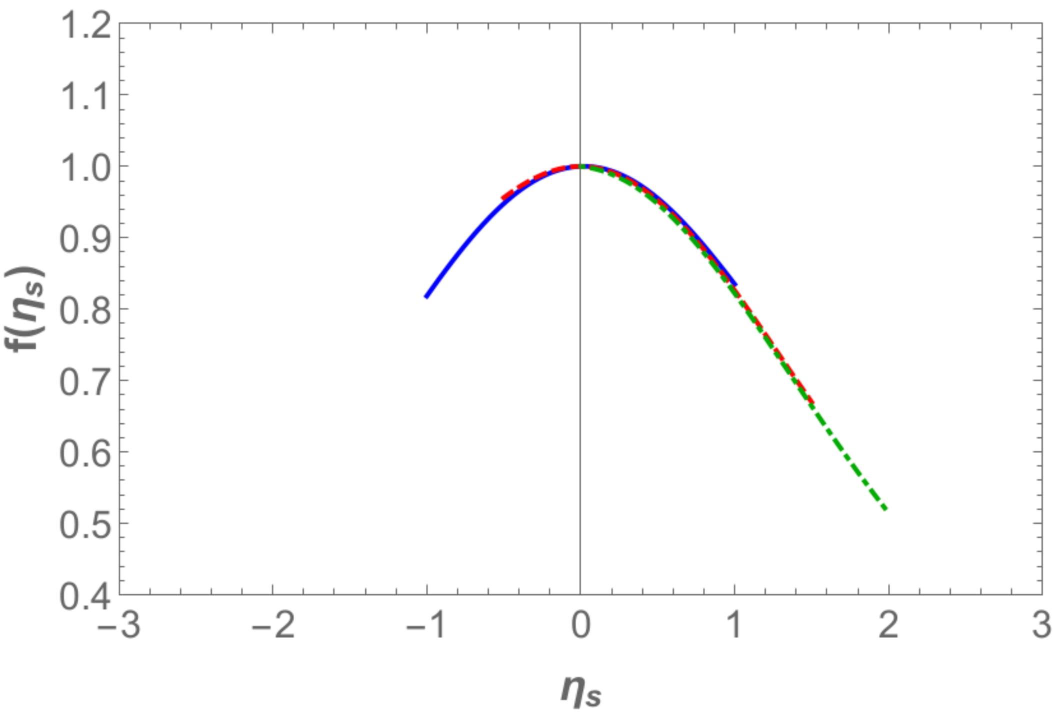
- longitudinal simulation box length / simulation time
- “interference” from fields of the nuclei



Rapidity profiles



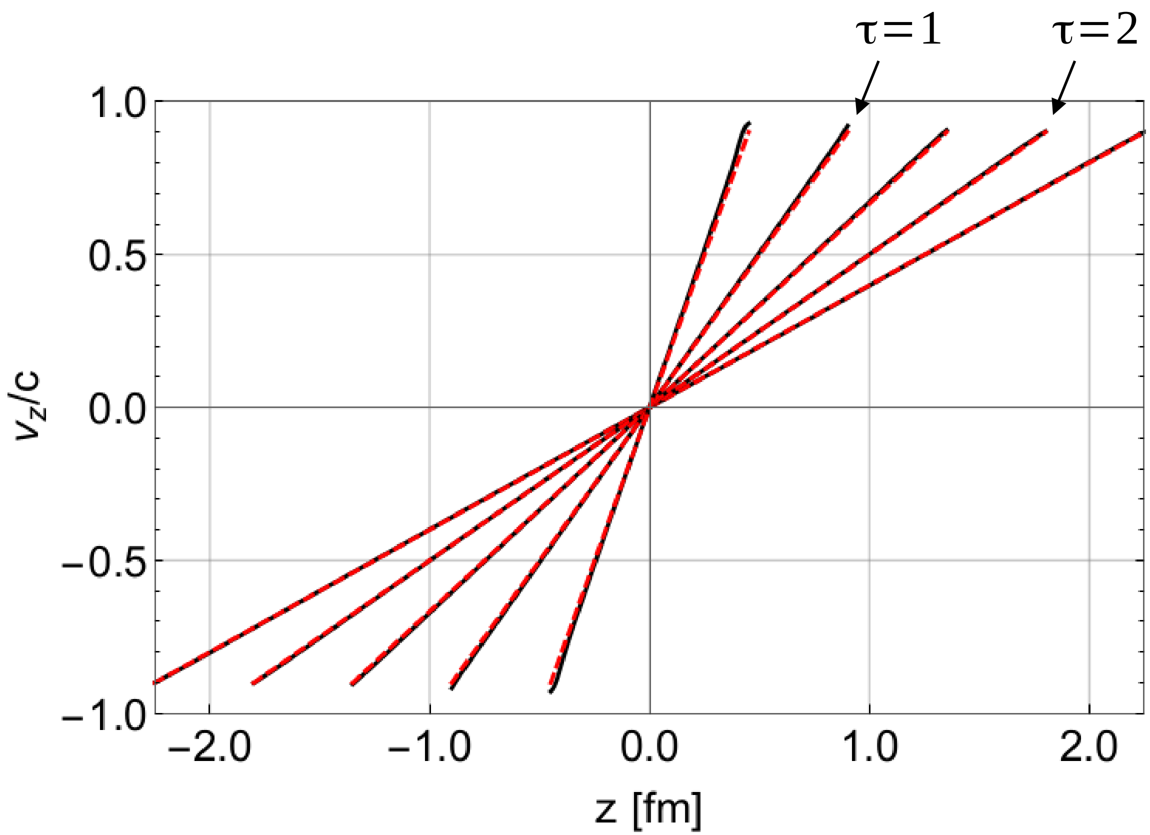
(a) $\eta' = 0$ (b) $\eta' = 0.5$ (c) $\eta' = 1$



Combined rapidity profile for increasingly boosted collisions

preliminary

Free streaming glasma



Compare local velocity of glasma to free streaming condition $v = z/t$

Lines (almost) on top of each other.

Black solid: measured v_z

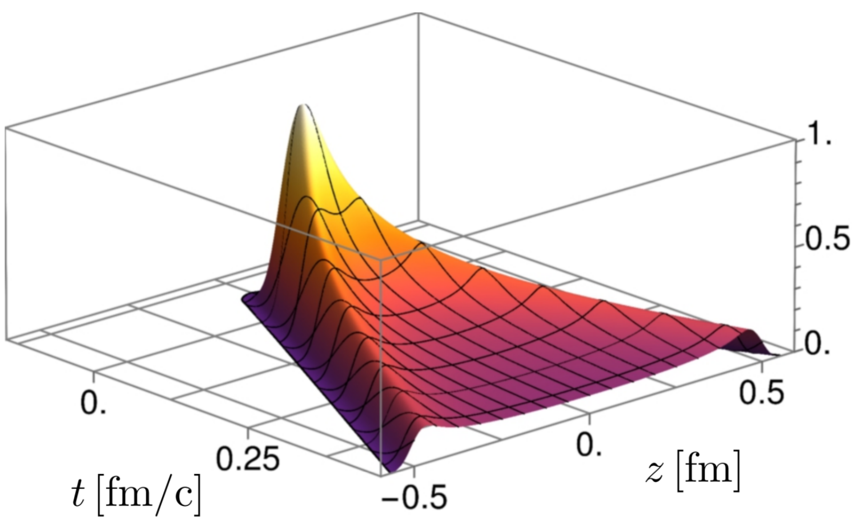
Red dashed: free streaming $v_{fs} = z/t$

Transverse pressure distribution

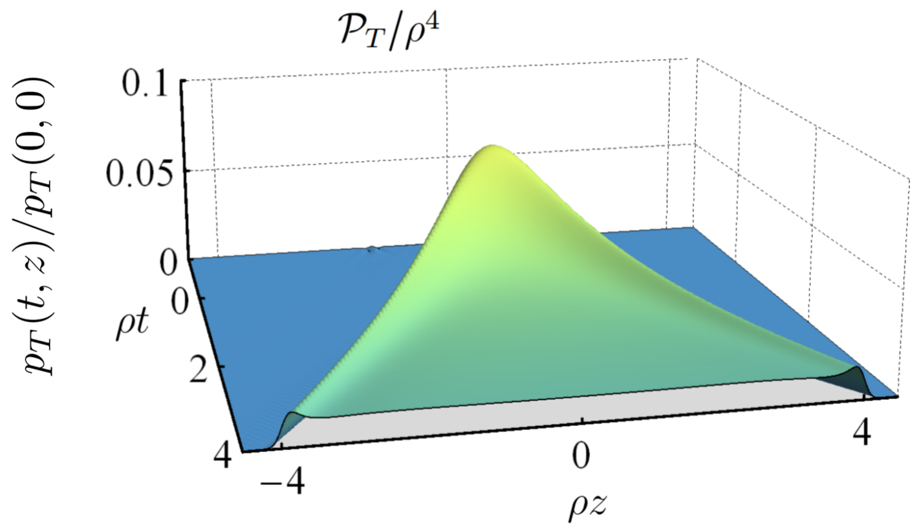
- Transverse pressure $p_T(x)$ generated by longitudinal fields

$$\langle p_T \rangle = \frac{1}{2} \langle E_L^2 + B_L^2 \rangle$$

- **Boost-invariant case**: initial conditions at $\tau = 0$ for longitudinal ***E*** and ***B*** fields, i.e. constant p_T along the boundary of the forward light cone



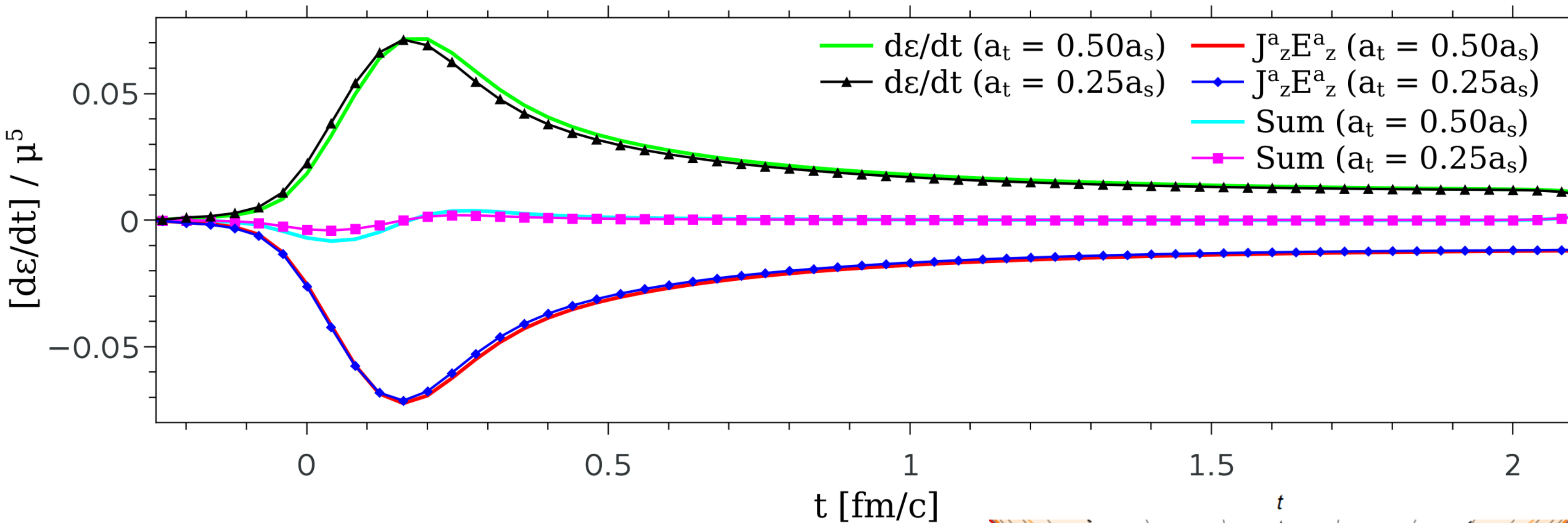
3+1 Yang-Mills



Holographic model

[Casalderrey-Solana et al., PRL (2013) 181601]

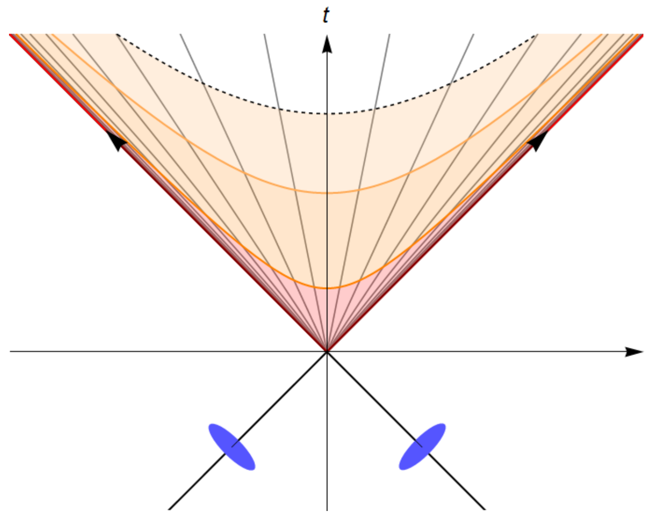
Energy production



Non-Abelian version of Poynting theorem

$$\frac{d\varepsilon}{dt}+\frac{1}{V}\int \partial_i S_i d^3x+\frac{1}{V}\int E_i^a J_i^a d^3x=0$$

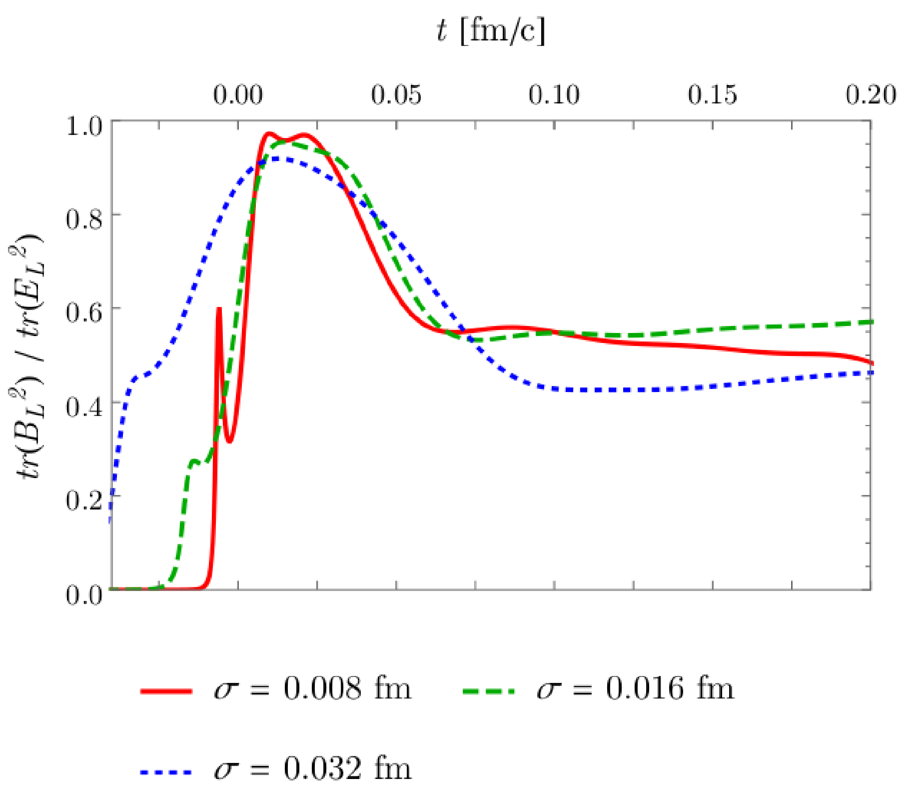
Energy production caused by longitudinal chromoelectric fields



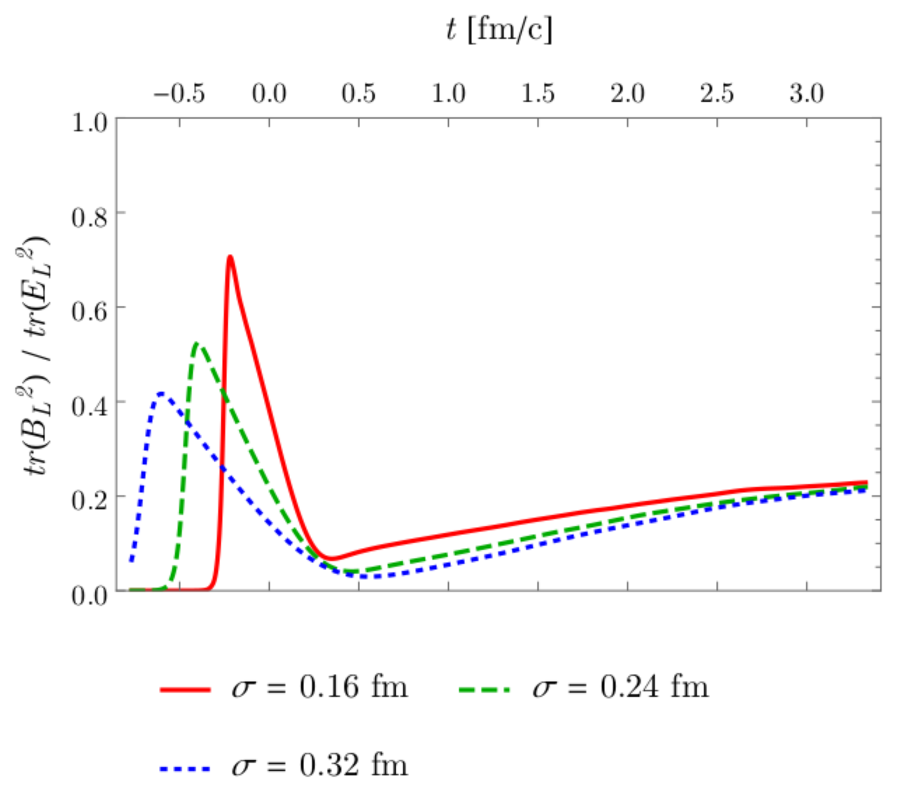
Chromo-magnetic suppression?

Ratio of longitudinal magnetic over longitudinal electric contributions

“Thin” nuclei ($\gamma \sim 200 - 1000$)



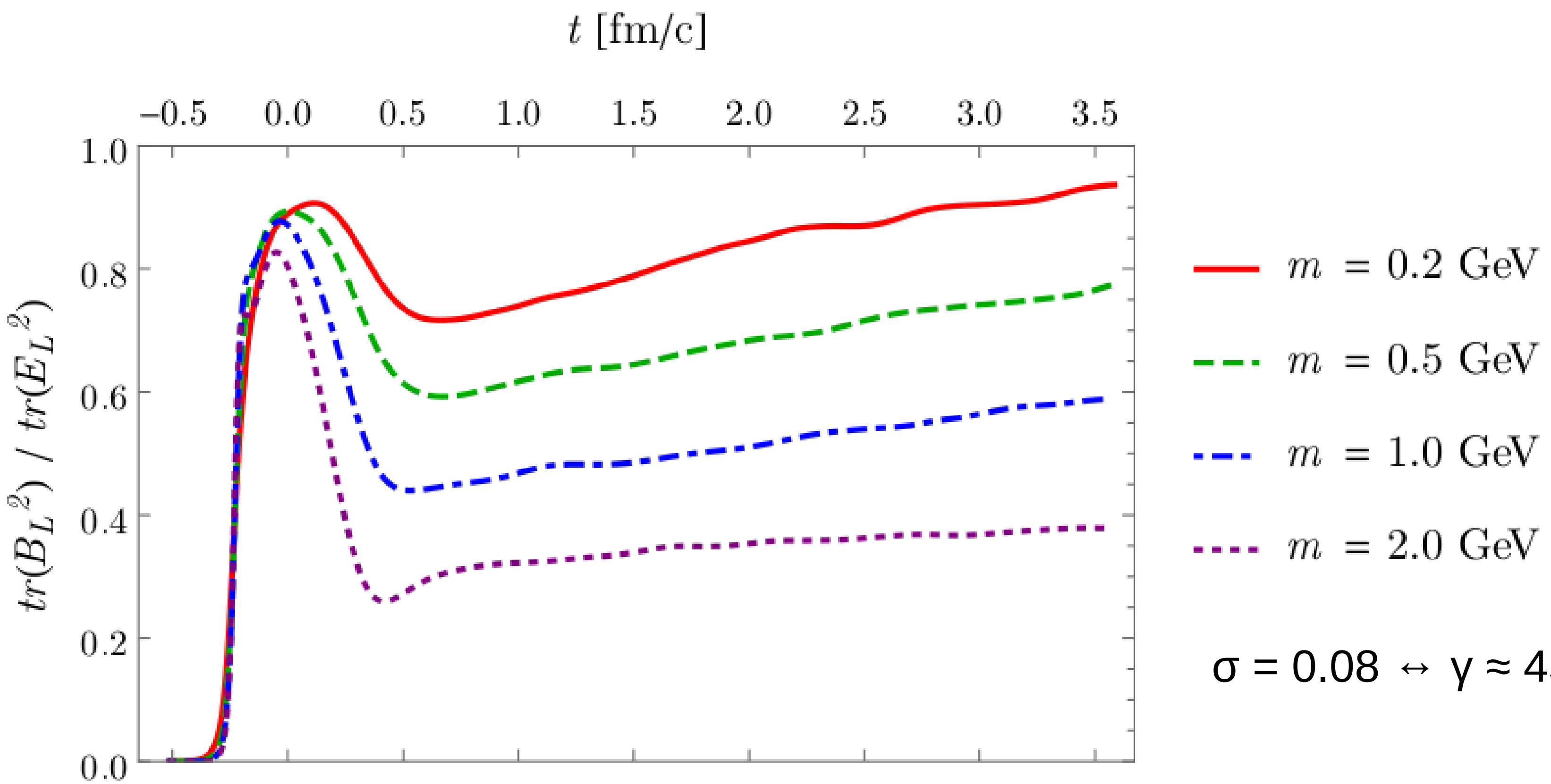
“Thick” nuclei ($\gamma \sim 20 - 40$)



Small ratio for thick nuclei

Chromo-magnetic suppression?

Ratio of longitudinal magnetic over longitudinal electric contributions



σ = 0.08 ↔ γ ≈ 45

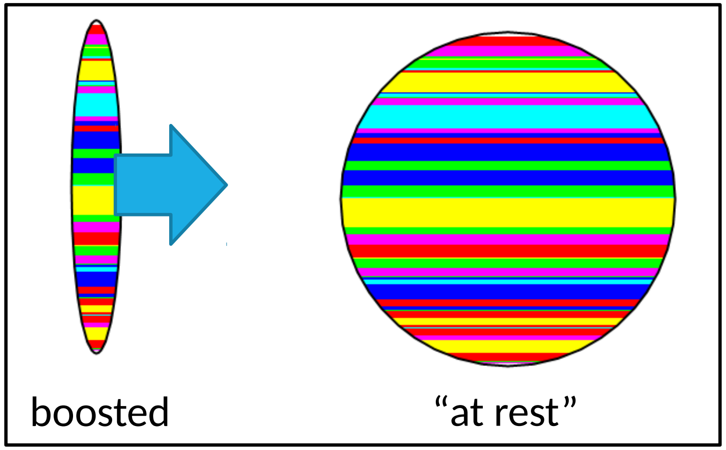
256^3 cells, $a_s = 0.02$ fm

Strong dependence on IR regulator *m*

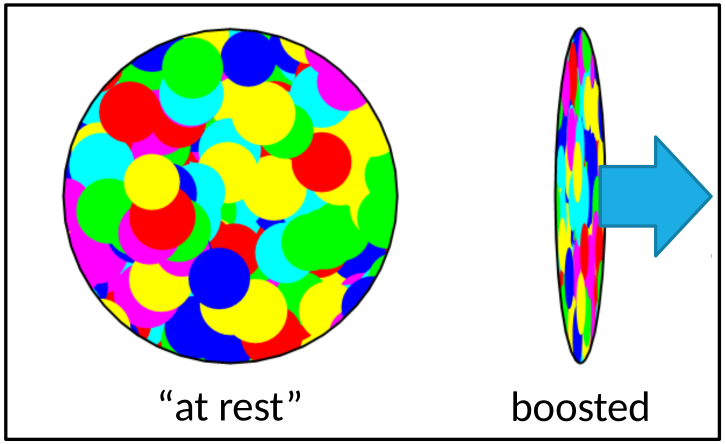
→ spurious effect of our choice of initial conditions?

Longitudinal structure

Current implementation



Longitudinal randomness



Longitudinal randomness…

- leads to higher energy density in the glasma
  [Fukushima, PRD 77 (2008) 074005]
- could provide boost-invariance breaking perturbations, which can drive system towards isotropization
  [Epelbaum, Gelis, PRL 111 (2013) 232301]

Longitudinal structure

Wilson line expectation value $\langle \mathrm{tr}(V) \rangle$ of a single nucleus is sensitive to longitudinal structure.

Embedded 2D MV-model:

$$\langle \hat{\rho}^a(\boldsymbol{x}_T)\hat{\rho}^b(\boldsymbol{x}'_T)\rangle = g^2\mu^2\delta^{(2)}(\boldsymbol{x}_T-\boldsymbol{x}'_T)\delta^{ab}$$

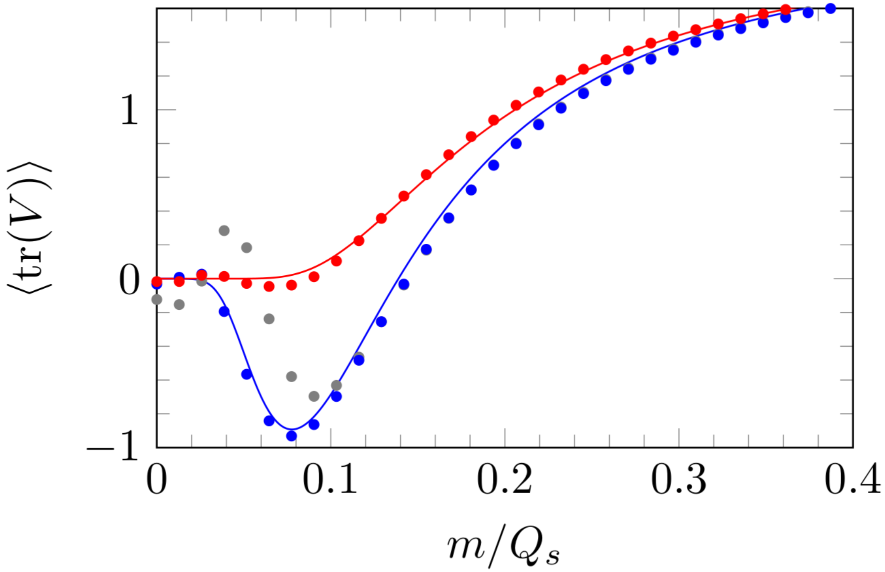
$$\rho(t,z,\boldsymbol{x}_T) = f(z-t)\hat{\rho}(\boldsymbol{x}_T)$$

3D MV-model:
(with random longitudinal structure)

$$\langle \rho^a(x^-,\boldsymbol{x})\rho^b(x^-,\boldsymbol{x}')\rangle$$
$$= g^2\mu^2 f(x^-)\delta(x^- - x'^-)\delta^{(2)}(\boldsymbol{x}_T-\boldsymbol{x}'_T)\delta^{ab}$$

f(z) ... longitudinal profile function $\quad x^- = \dfrac{t-z}{\sqrt{2}}$

Introducing independent "sheets" in longitudinal direction
[Fukushima, PRD 77 (2008) 074005]



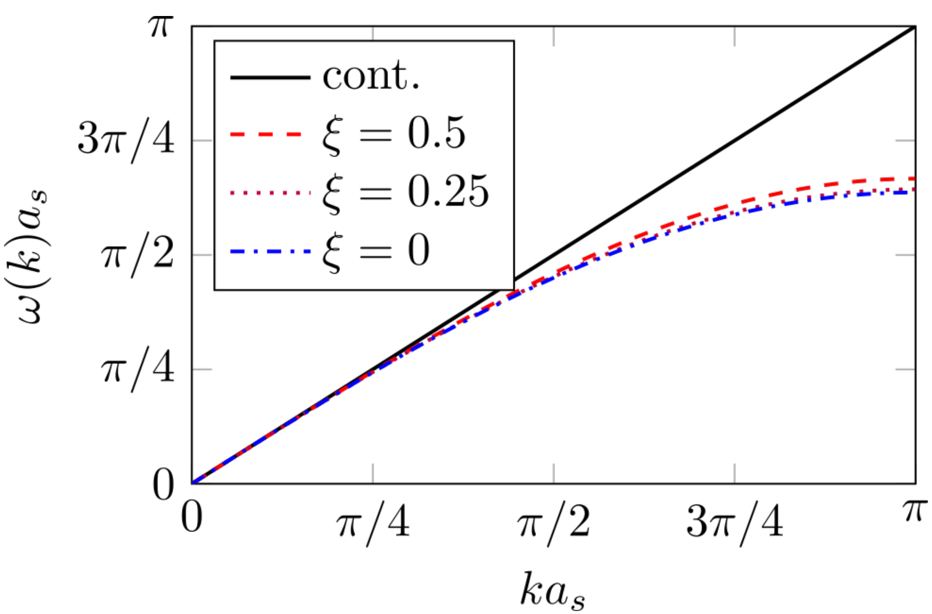
Lines: analytical result
Dots: numerical result

Blue: 2D MV-model
Red: 3D MV-model
Gray: intermediate

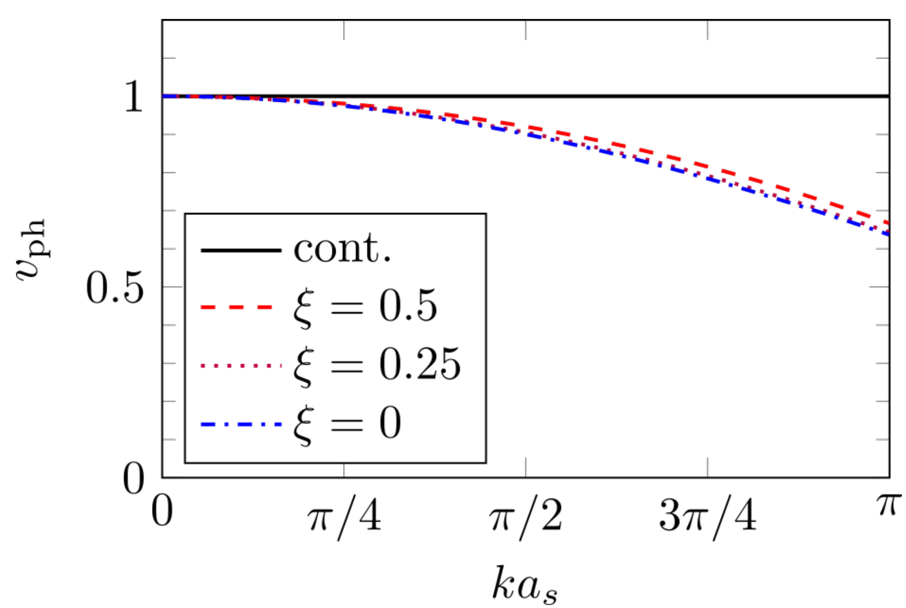
preliminary

Lattice dispersion

Dispersion relation



Phase velocity



Temporal to spatial lattice spacing: $\xi = \frac{a_t}{a_s}$

Courant-Friedrichs-Lewy (CFL) condition: $\xi \leq \frac{1}{\sqrt{d}}$ in *d* dimensions
(for explicit solvers like the Leapfrog algorithm)

Explicit vs. implicit solvers

1D scalar field example

Continuum action: $S[\phi]=\frac{1}{2}\int_x \partial_\mu \phi \, \partial^\mu \phi$

Equations of motion : $\delta S=0 \;\Rightarrow\; \partial_\mu \partial^\mu \phi=\partial_t^2 \phi-\partial_x^2 \phi=0$

Discretized: $$\frac{\phi(x,t+1)-2\phi(x,t)+\phi(x,t-1)}{a_t}=\frac{\phi(x+1,t)-2\phi(x,t)+\phi(x-1,t)}{a_s}$$

Explicit solver: $\phi(x,t+1)=F\left(\phi(\ldots,t),\phi(\ldots,t-1)\right)$

Implicit solver: $\phi(x,t+1)=F\left(\phi(\ldots,t+1),\phi(\ldots,t),\phi(\ldots,t-1)\right)$

Discretized action for explicit solver:

$$S[\phi]=\frac{1}{2}V\sum_x \left\{\left(\frac{\phi(x,t+1)-\phi(x,t)}{a_t}\right)^2+\left(\frac{\phi(x+1,t)-\phi(x,t)}{a_s}\right)^2\right\}$$

Variational integrators

Usual approach

Variation

$$\delta S = 0$$

Discretization
(finite differences, sums, ...)

Variational integrators

Discretization
(finite differences, sums, ...)

Discrete Variation

$$\delta S = 0$$

"Inherited" symmetries

Continuum

$$S[\phi] = \int d^4x \mathcal{L}\left(\phi(x), \partial_\mu \phi(x), \ldots\right)$$

Equations of motion + preserved constraints

$$\partial_\mu \partial^\mu \phi(x) + \ldots = 0$$

$$C\left(\phi(x)\right) = 0$$

Discretized action

$$S[\phi] = V \sum_x \mathcal{L}\left(\phi_x, \bar{\partial}_\mu \phi_x, \ldots\right)$$

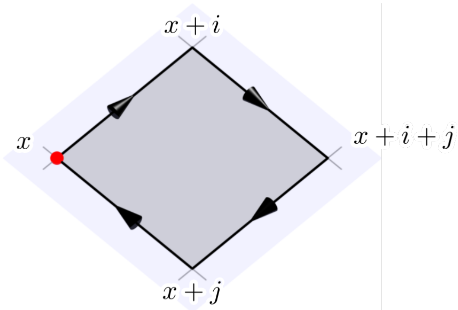
If possible: keep symmetries!

Discretized equations of motion + constraints (?)

$$\bar{\partial}_\mu \bar{\partial}^\mu \phi(x) + \ldots = 0$$

$$\bar{C}\left(\phi_x\right) = 0$$

Lattice gauge theory

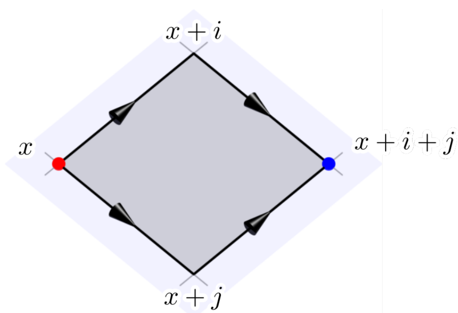


Link variable: $U_{x,\mu} \simeq \exp\left(i g a^{\mu} A_{x,\mu}\right)$

Plaquette: $U_{x,\mu\nu} = U_{x,\mu} U_{x+\mu,\nu} U_{x+\mu+\nu,-\mu} U_{x+\nu,-\mu}$

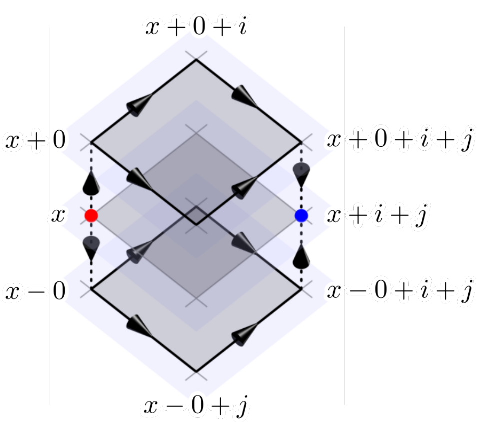
Relation to field strength tensor:

$$\mathrm{tr}\left(2 - U_{x,\mu\nu} - U^{\dagger}_{x,\mu\nu}\right) \simeq \frac{1}{2} \sum_a \left(g a^{\mu} a^{\nu} F^a_{\mu\nu}(x)\right)^2$$

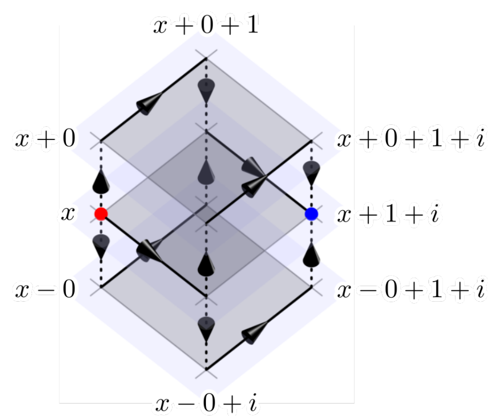


$$C_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\mu,\nu} - U_{x,\nu} U_{x+\nu,\mu}$$

Identity: $C_{x,\mu\nu} C^{\dagger}_{x,\mu\nu} = 2 - U_{x,\mu\nu} - U^{\dagger}_{x,\mu\nu}$

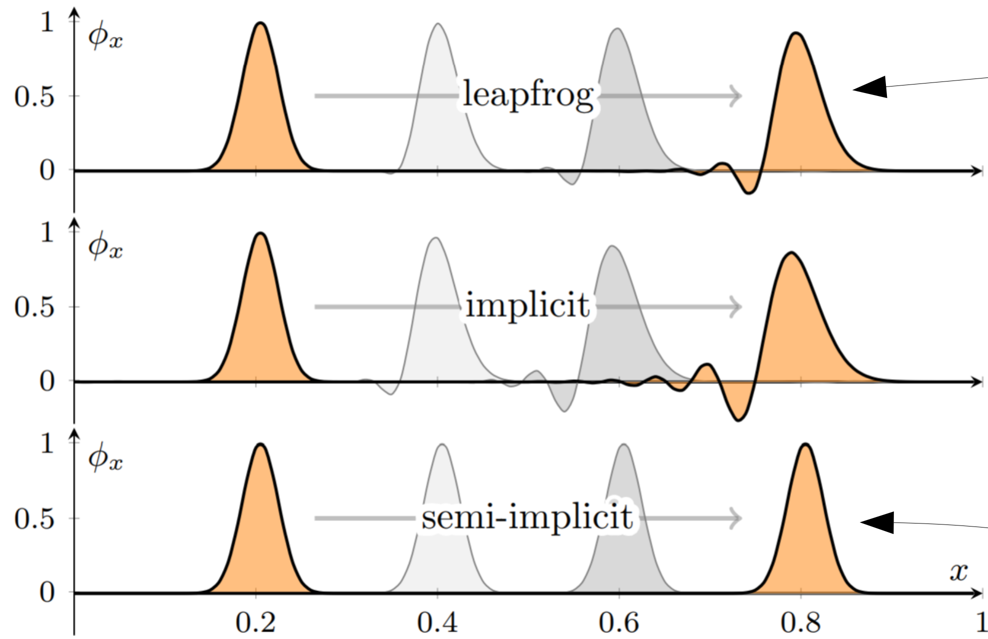


$M_{x,ij}$



$W_{x,1i}$

Dispersion-free propagation



Standard **Wilson action**:

$$S[U] = \frac{V}{g^2}\sum_x \left(\sum_i \frac{1}{(a^0 a^i)^2} \mathrm{tr}\left(2 - U_{x,0i} - U^\dagger_{x,0i}\right) - \frac{1}{2}\sum_{i,j} \frac{1}{(a^i a^j)^2} \mathrm{tr}\left(2 - U_{x,ij} - U^\dagger_{x,ij}\right)\right)$$

Variational integrator: Discretized equations of motion from discretized action

Discretized action for the **semi-implicit scheme**:

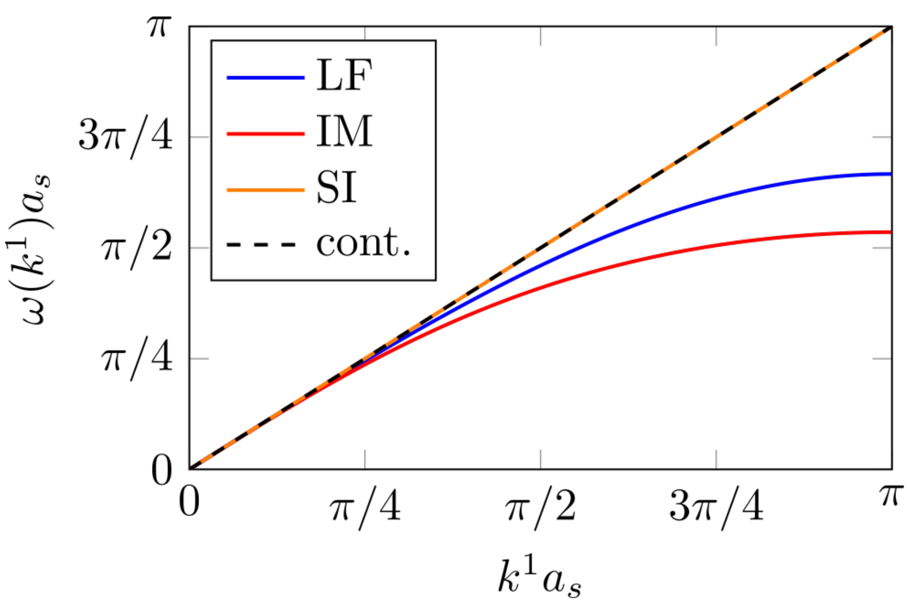
$$S[U] = \frac{V}{g^2}\sum_x \left(\frac{1}{(a^0 a^1)^2} \mathrm{tr}\left(C_{x,01} C^\dagger_{x,01}\right) + \sum_i \frac{1}{(a^0 a^i)^2} \mathrm{tr}\left(C_{x,0i} C^\dagger_{x,0i}\right) \right.$$
$$\left. \underbrace{- \frac{1}{4}\sum_{i,|j|} \frac{1}{(a^i a^j)^2} \mathrm{tr}\left(C_{x,ij} M^\dagger_{x,ij}\right)}_{\text{implicit part}} \underbrace{- \frac{1}{4}\sum_{|j|} \frac{1}{(a^1 a^j)^2} \mathrm{tr}\left(C_{x,1j} W^\dagger_{x,1j} + \mathrm{h.c.}\right)}_{\text{semi-implicit part}} \right)$$

with $C_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\mu,\nu} - U_{x,\nu} U_{x+\nu,\mu}$ etc.

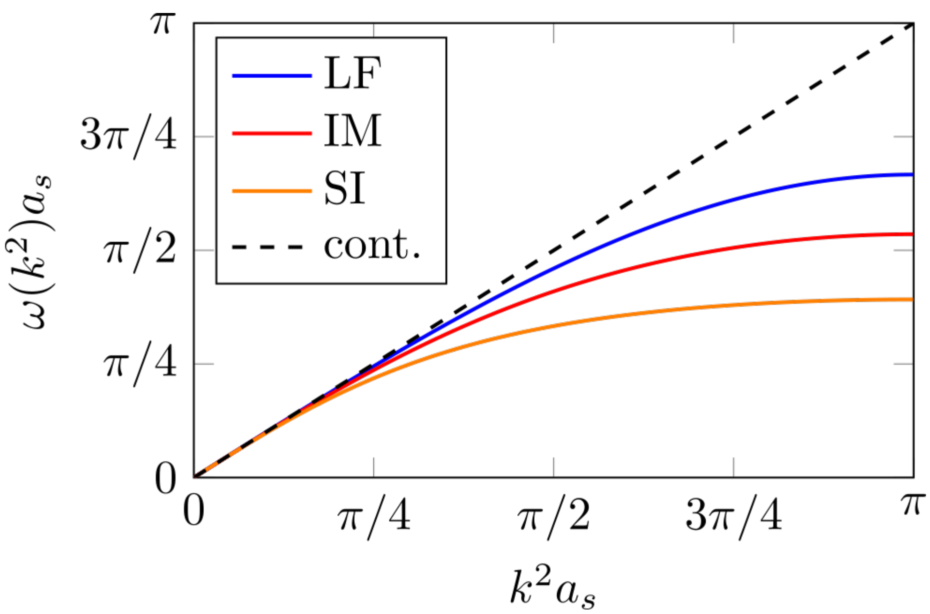
For details see:
AI, D. Müller, Eur.Phys.J. C78 (2018) no.11, 884

Lattice dispersion

Longitudinal direction

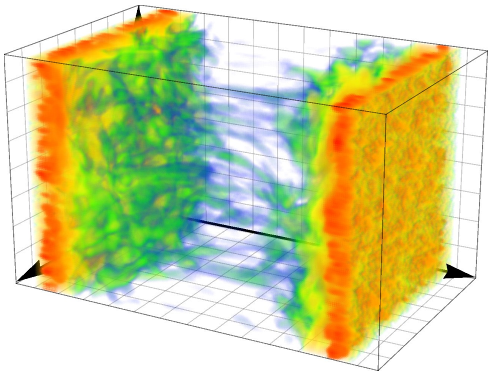


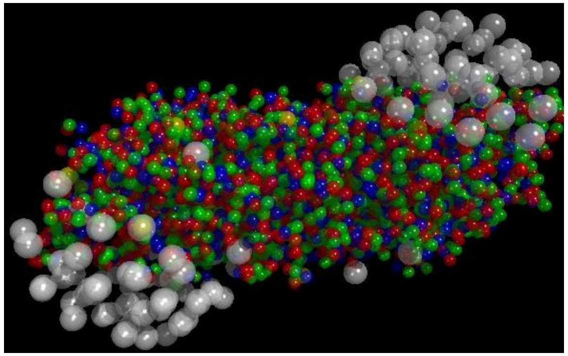
Transverse direction



Lattice dispersion for leapfrog (LF), implicit (IM) and semi-implicit (SI) schemes.

Computational challenges





Simulating small part of nuclei at RHIC energies:

γ-factor: 100
Lattice: 2048 × 192^2 cells
Gauge group: SU(2)
Color sheets: 1
Simulation box: $(6\text{ fm})^3$

→ **25 GB** simulation data
→ **192 core hours** on Vienna Scientific Cluster (VSC-3)

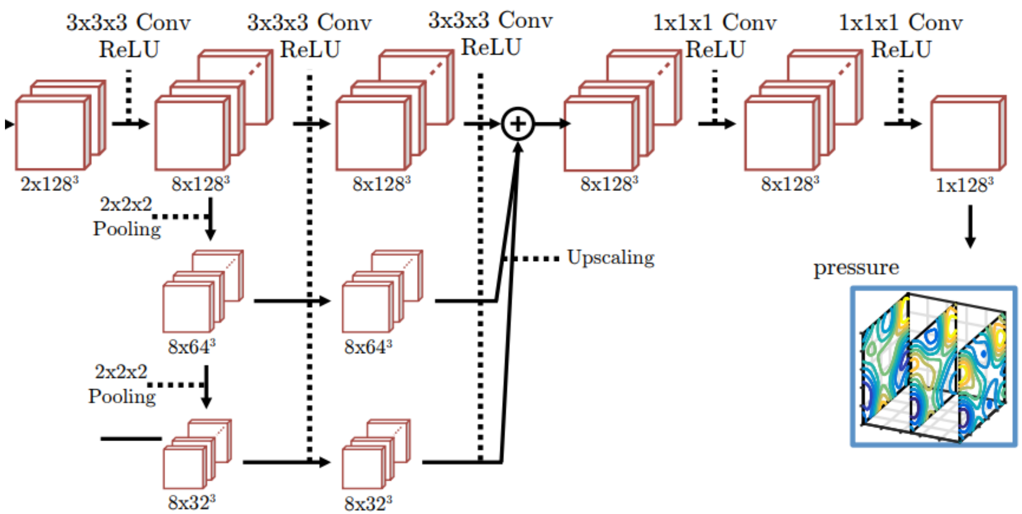
Simulating realistic off-central full size nuclei at LHC energies:

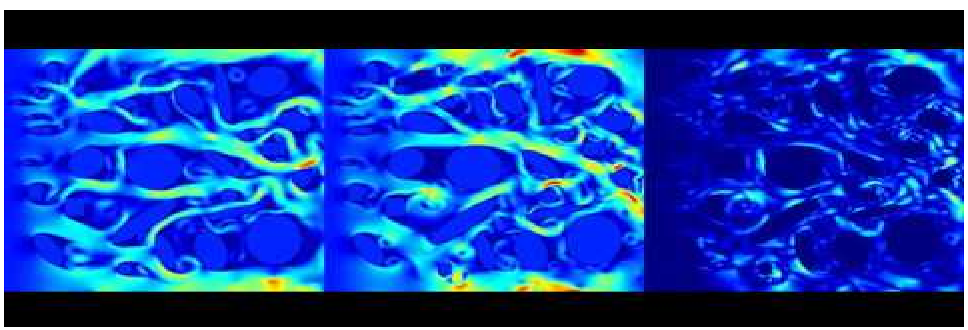
γ-factor: 2500
Lattice: (25×20480) × 1920^2 cells
Gauge group: SU(3)
Color sheets: 100
Simulation box: $(60\text{ fm})^3$

→ **25 PB** simulation data
→ **5 million core years** on VSC-3
(150 years on VSC3; but only 130 TB available)

Machine learning in fluid dynamics

Accelerating Eulerian Fluid Simulation With Convolutional Networks
Tompson et al, arxiv:1607.03597



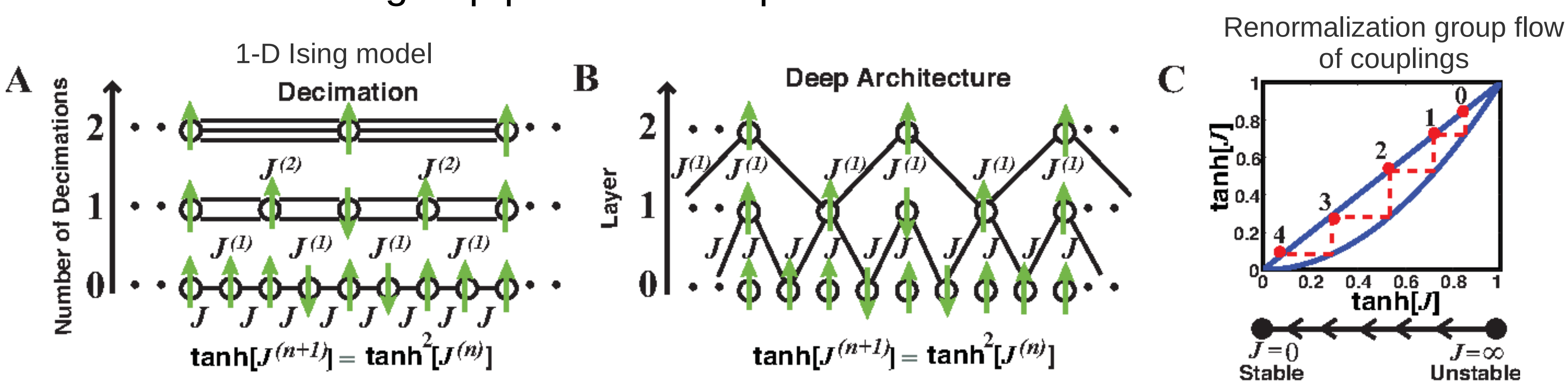


- Compress computation time and memory usage
- Use convolutional autoencoders to compress state size
- Learn dynamics on compressed form
- Can generalize to larger grid sizes

Lat-Net: Compressing Lattice Boltzmann Flow Simulations using Deep Neural Networks
Hennigh, arxiv:1705.09036

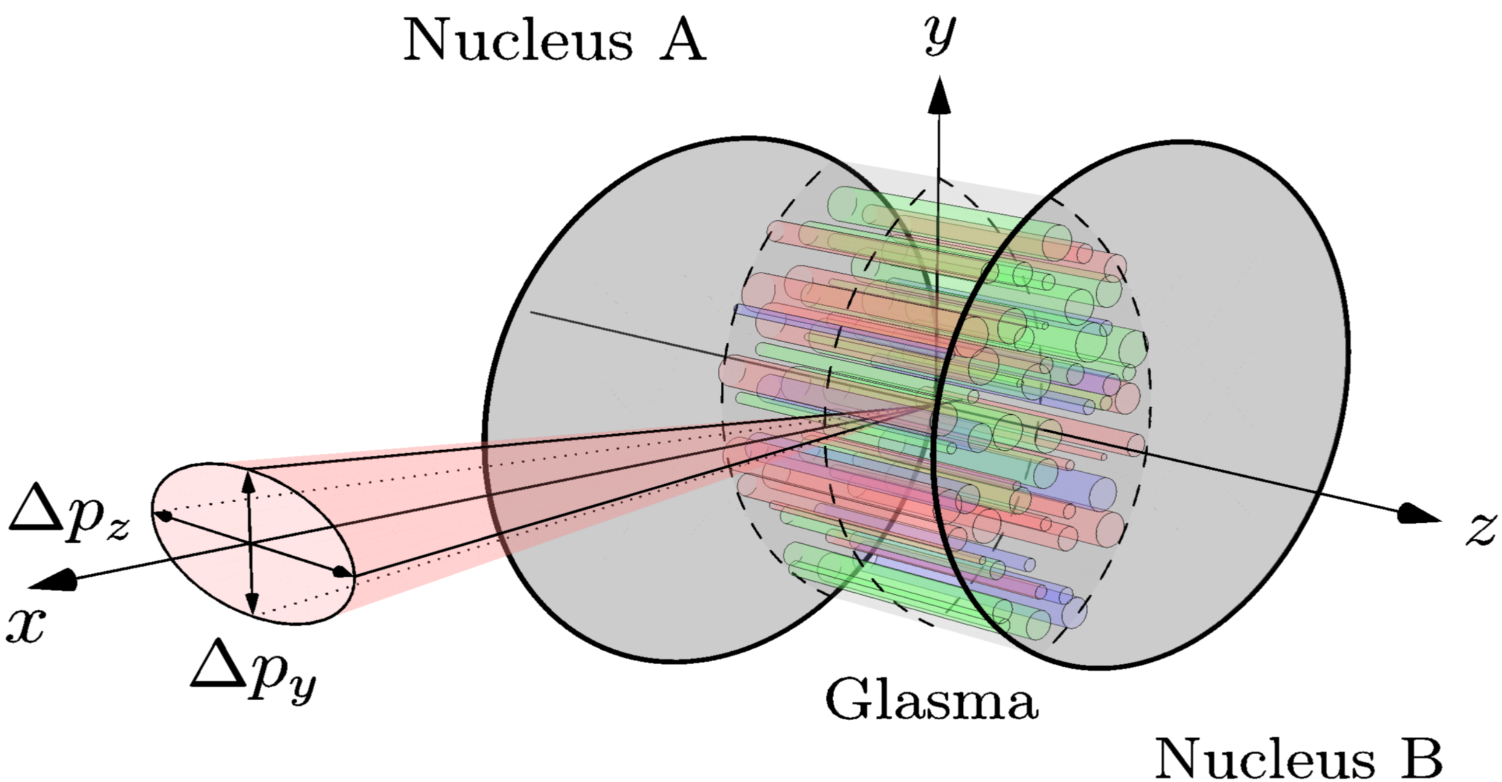
Beyond speeding up simulations...

- Why do neural networks work surprisingly well in some cases, and why and how can they fail in others?
  → we can compare to ground-truth physical result
- Learn about physical system from new viewpoint involving machine learning tools
  → latent space representation can capture relevant degrees of freedom
  → irrelevant degrees of freedom are integrated out into weight parameters
- Renormalization group picture of deep neural networks:



An exact mapping between the Variational Renormalization Group and Deep Learning
Mehta, Schwab, arxiv:1410.3831

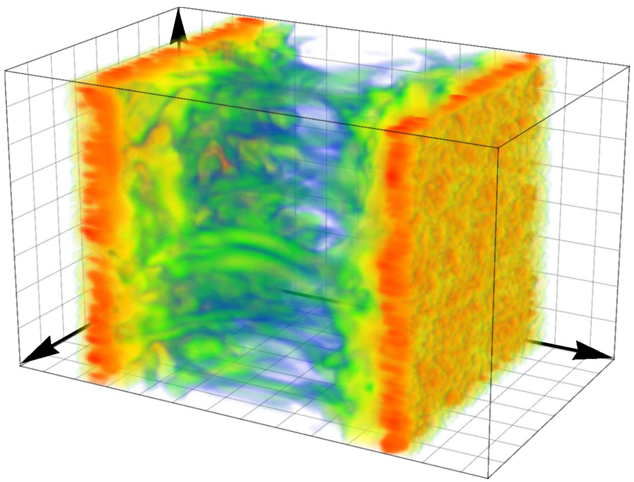
Jet momentum broadening



Work in preparation, together with Daniel Schuh and David Müller

Conclusions & Outlook

- Simulate CGC collisions in 3D+1 using Colored Particle-In-Cell
- Finite thickness breaks boost invariance → Gaussian rapidity profiles
- **Outlook**:
  - Study effect of random longitudinal structure
  - Observables at early times: gluon production, momentum broadening, ...
  - Explore machine learning



D. Gelfand, AI, D. Müller, Phys. Rev. D94 (2016) no.1, 014020
AI, D. Müller, Phys. Lett. B (2017) 771
AI, D. Müller, Eur.Phys.J. C78 (2018) no.11, 884

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www.openpixi.org