Higher-spin generalisations of gravity

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1. Introduction

Quantum description of gravity: unsolved puzzle.

Einstein field equations:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

classical geometry

quantum matter

No guidance by experimental results.

Only theoretical requirements: consistency, correct classical limit, compatibility with other forces

asymptotic safety

loop quantum gravity

string theory

group field theory

$\boxed{\text{Quantum gravity}}$

supergravity

causal dynamical triangulation

higher-spin gravity

causal set theory

matrix models

noncommutative geometry

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causal dynamical triangulation

causal set theory

Lisa

noncommutative geometry

Quantum gravity

Pascal Dario

string theory

Thorsten David

supergravity

Jan

Olaf Sergio Stefan

higher-spin gravity

matrix models

Harold

Gravity nonrenormalisable

Naive approach to perturbative quantum gravity:

Quantise gravity as other field theories perturbatively with background field method.

Problem: coupling G w. negative mass dim., non-renormalisable.

2-loop divergence [Goroff,Sagnotti 1985; van de Ven 1992]

$$\Gamma^{(2)}_{\text{div}} = \frac{1}{\epsilon}\frac{209}{2880}\frac{G}{8\pi^2}\int d^4x\sqrt{g}\,C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\lambda\tau}C_{\lambda\tau}{}^{\mu\nu}$$

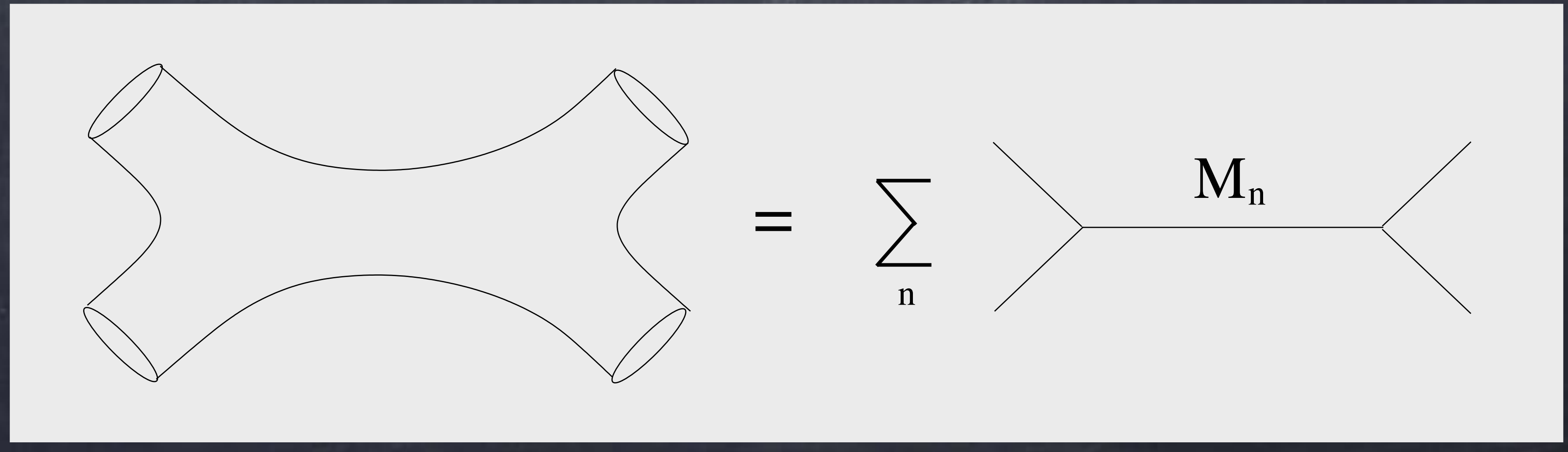
String theory

General idea:

strings as fundamental constituents

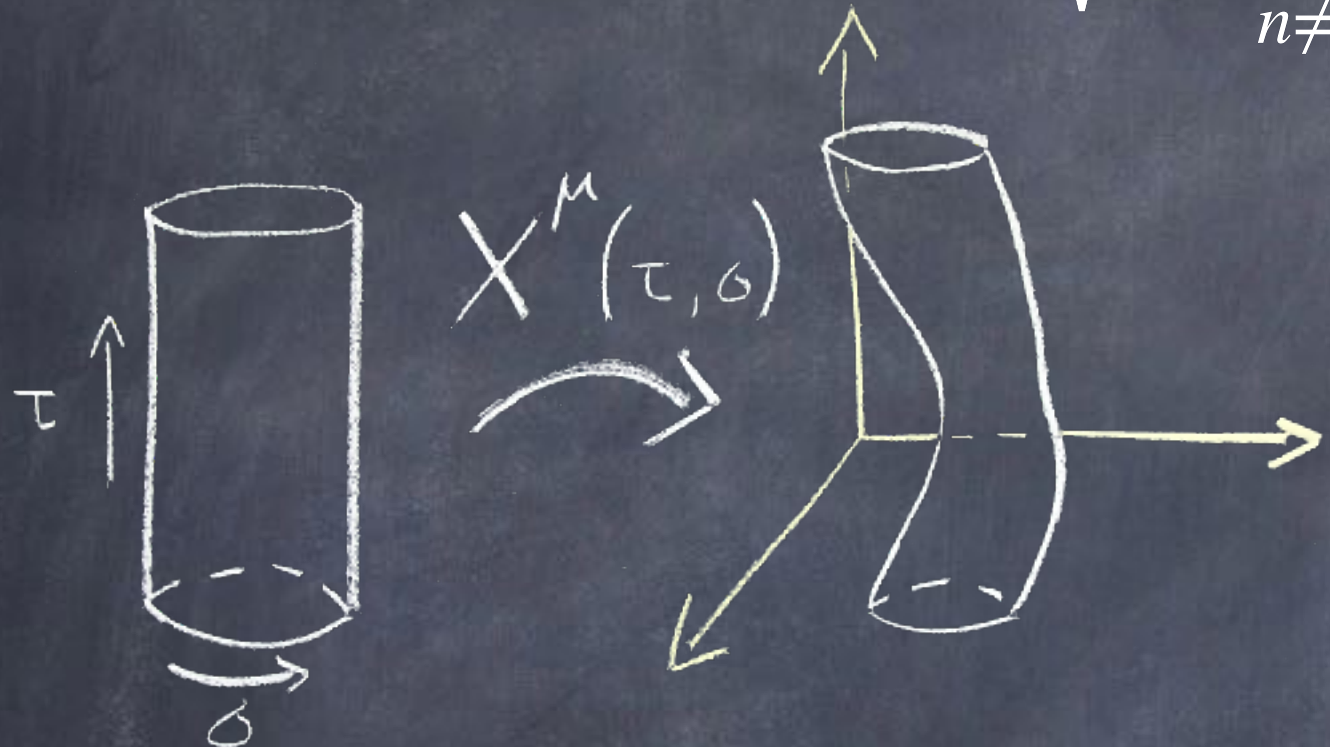
- Different string modes <-> different particles

- Graviton mode
- UV-divergence of quantum gravity cured



$$X^\mu(\tau,\sigma) = x^\mu + \alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\Big(\alpha_n^\mu e^{-in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)}\Big)$$

infinitely many harmonic oscillators



Excited states: $\alpha_{-m_1}^{\mu_1} \ldots \alpha_{-m_s}^{\mu_s} \, \tilde{\alpha}_{-n_1}^{\nu_1} \ldots \tilde{\alpha}_{-n_t}^{\nu_t} |p\rangle$

correspond to higher-spin tensor fields

$\phi^{\mu_1 \ldots \mu_s \nu_1 \ldots \nu_t}$ with mass $M^2 = \dfrac{4}{\alpha'}\Big(\sum_i m_i - 1\Big)$

In a point-particle limit, one considers energies far below the string scale set by α' where the massive modes do not play a role. The low energy effective theory for superstring theory is supergravity.

[Gross, Mende 1987]

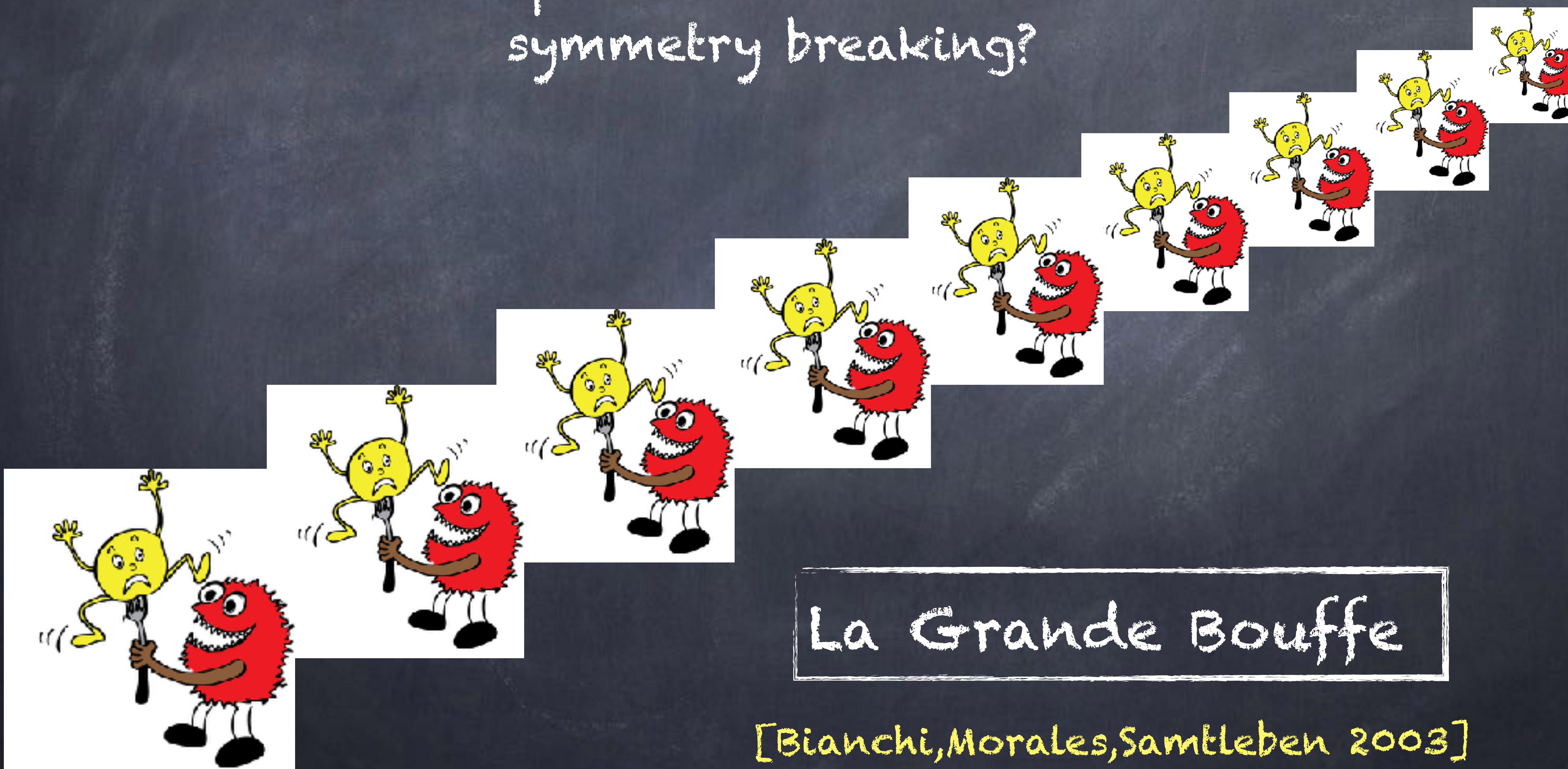
One can now ask what happens in the opposite limit, i.e. at ultra-high energies, or, similarly, a limit where the typical string length becomes comparable to geometric length scales: tensionless limit

Many massless higher-spin modes!

tensionless limit

String theory ⇄ Higher-spin gauge theories

spontaneous symmetry breaking?



La Grande Bouffe

[Bianchi,Morales,Samtleben 2003]

One approach: coming bottom-up, try to understand:

- what HS gauge theories are possible
- how can we construct them

Rest of the talk:

2. Higher-spin gauge fields

3. Noether-Fronsdal program

2. Higher-spin gauge fields

Maxwell: spin-1 field A_μ

equations of motion $\Box A_\mu - \partial_\mu \partial^\nu A_\nu = 0$

gauge transformation $\delta A_\mu = \partial_\mu \xi$

Linearised gravity: spin-2 field $h_{\mu\nu}$

equations of motion

$$\Box h_{\mu\nu} - \partial_\mu \partial^\lambda h_{\lambda\nu} - \partial_\nu \partial^\lambda h_{\lambda\mu} + \partial_\mu \partial_\nu h^\lambda{}_\lambda = 0$$

gauge transformation $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Fronsdal fields

Free massless spin s particle [Fronsdal 1978]

$\longrightarrow$ fully symmetric tensor $\phi_{\mu_1\ldots\mu_s}$

- $\phi_{\mu_1\ldots\mu_{s-4}\lambda}{}^{\lambda}{}_{\nu}{}^{\nu} = 0$ (double traceless)
- equations of motion
  $\Box\phi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1|}\partial^{\lambda}\phi_{|\mu_2\ldots\mu_s)\lambda} + \partial_{(\mu_1}\partial_{\mu_2}\phi_{\mu_3\ldots\mu_s)\lambda}{}^{\lambda} = 0$
- gauge transformation $\delta\phi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)}$
  where $\xi_{\mu_1\ldots\mu_{s-3}\lambda}{}^{\lambda} = 0$

Problems with interactions

- no minimal coupling of higher-spin fields to gravity [Aragone,Deser 1971]

- strong no-go theorems on flat space and dimension >3 (scattering arguments): HS gauge fields cannot couple to any field that is minimally coupled to gravity. [Porrati 2008]

Avoid no-go results

A. Consider 3 space-time dimensions

B. Go to asymptotically dS or AdS

Avoid no-go results

A. In 3 dimensions:

- no propagating degrees of freedom

$$R_{\mu\nu\lambda\sigma} = C_{\mu\nu\lambda\sigma} + \frac{2}{d-2}(g_{\mu[\lambda}R_{\sigma]\nu} - g_{\nu[\lambda}R_{\sigma]\mu})$$

$$\equiv 0 \qquad - \frac{2}{(d-1)(d-2)} R\, g_{\mu[\lambda}g_{\sigma]\nu}$$

- pure HS gauge theory understood in terms of Chern-Simons theories [Blencowe 1988]

- rich playground to investigate effects of non-linear HS gauge transformations and of the HS AdS/CFT correspondence

[Campoleoni,S.F.,Pfenninger,Theisen 2010]
[Gaberdiel,Gopakumar 2010][...]

Avoid no-go results

B. In (A)dS:

- proposal for non-linear HS gauge theory in a first oder formalism exists (Vasiliev theories) which is based on a generalisation of the vielbein formalism [Vasiliev 1990]

$$\varphi_{\mu_1\dots\mu_s} = \bar{e}^{a_1}_{(\mu_1}\dots\bar{e}^{a_{s-1}}_{\mu_{s-1}}\underline{e^{b_1\dots b_{s-1}}_{\mu_s)}}\,\eta_{a_1b_1}\dots\eta_{a_{s-1}b_{s-1}}$$

- no (standard) action known
- difficult to understand issues of non-locality

3. Noether-Fronsdal program

Goal: Construct an action for interacting Fronsdal fields

Method: Add interactions order by order in powers of the fields while keeping gauge invariance

quadratic action   cubic vertices

$$S = S^{(2)} + \epsilon S^{(3)} + \epsilon^2 S^{(4)} + \dots$$

$$\delta\phi_s = \delta^{(0)}\phi_s + \epsilon\delta^{(1)}\phi_s + \epsilon^2\delta^{(2)}\phi_s + \dots$$

transf. of free fields $\partial\xi_{s-1}$

$$S = S^{(2)} + \epsilon S^{(3)} + \epsilon^2 S^{(4)} + \dots$$

$$\delta\phi_s = \delta^{(0)}\phi_s + \epsilon\delta^{(1)}\phi_s + \epsilon^2\delta^{(2)}\phi_s + \dots$$

Gauge invariance:

$$\begin{aligned} 0 = \delta S = \delta^{(0)} S^{(2)} & \\ & + \epsilon\left(\delta^{(0)}S^{(3)} + \delta^{(1)}S^{(2)}\right) \\ & + \epsilon^2\left(\delta^{(0)}S^{(4)} + \delta^{(1)}S^{(3)} + \delta^{(2)}S^{(2)}\right) + \dots \end{aligned}$$

Classify step by step:

$\delta^{(0)}S^{(3)} + \delta^{(1)}S^{(2)} = 0 \implies \delta^{(0)}S^{(3)} \approx 0$ (on shell)

Classify step by step: cubic order

1. $\delta^{(0)}S^{(3)} + \delta^{(1)}S^{(2)} = 0 \implies \delta^{(0)}S^{(3)} \approx 0$

Find all possible cubic vertices. on shell

2. For the most general $S^{(3)}$ find $\delta^{(1)}$.

Next: quartic order

1. $\delta^{(0)}S^{(4)} + \delta^{(1)}S^{(3)} + \delta^{(2)}S^{(2)} = 0$

$$\implies \delta^{(0)}S^{(4)} + \delta^{(1)}S^{(3)} \approx 0$$

a) Find particular solution for $S^{(4)}$

b) Find general solution of $\delta^{(0)}S^{(4)} \approx 0$

2. Find the corresponding $\delta^{(2)}$

Results

In 3 dimensions:

- homogeneous problem solved
- complete classification of cubic vertices [Mkrtchyan; Kessel, Mkrtchyan]
- higher vertices: uniquely fixed by cubic ones [S.F.,Krüger,Mkrtchyan]
- for HS theories on AdS: structures match completely with structures of correlators in corresponding two-dimensional conformal field theory. [S.F.,Krüger,Mkrtchyan]

Results

In higher dimensions:

- cubic vertices understood [Metsaev 1993,...]
- homogeneous problem solved for almost all vertices [S.F.,Krüger,Mkrtchyan (in preparation)]
- higher vertices not fixed by cubic ones (also not expected)

Discussion

The results for the Noether-Fronsdal program in 3 dimensions may allow

- to formulate an interacting theory of Fronsdal fields
- to construct an action for a HS theory coupled to matter (only possible in Vasiliev formulation so far)
- and thus to start to systematically quantise such theories

Summary and Outlook

- there are many approaches to quantum gravity
- HS generalisations of gravity are motivated by an ultra-stringy limit of string (field) theory
- two approaches: vielbein-like (Vasiliev) and metric-like (Fronsdal)
- some progress & many open questions:

locality? action? quantisation? AdS/CFT? HS geometry? Relation to string theory?