Nested soft-collinear subtractions for infrared singularities at NNLO

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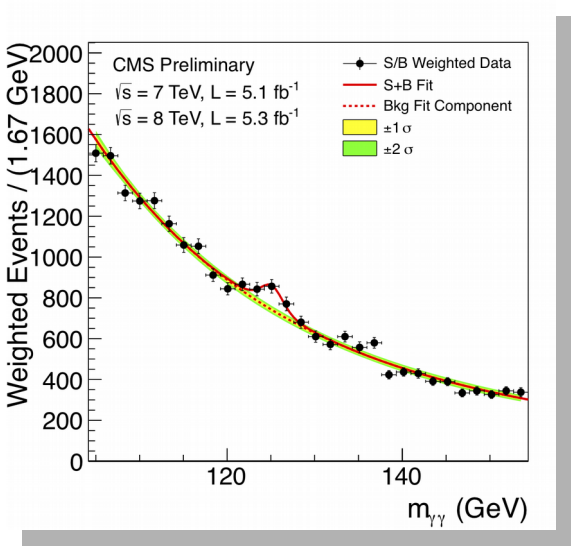
F. Caola, K. Melnikov, R.R. [hep-ph/1702.01352, hep-ph/1902.02081, hep-ph/1905.xxxxx]

F. Caola, M. Delto, H. Frellesvig, K. Melnikov [hep-ph/1807.05835]

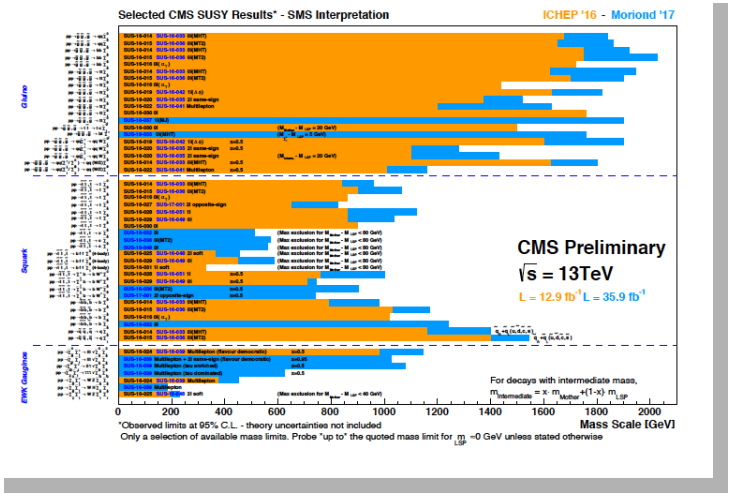
M. Delto, K. Melnikov [hep-ph/1901.05213]

Precision physics at the LHC

Discovery of Higgs boson...



+ absence of enduring evidence for new physics...



➡ **Precision physics programme** at LHC

- Extensive studies of Higgs boson: fully understand the nature of EWSB.
- Search for BSM physics through subtle deviations from SM background.
- Determine fundamental parameters of nature.

How Precise is Precise?

- Suppose we have BSM physics at scale $\Lambda_{NP} \sim 1 - \text{ few TeV}$
- Difficult to produce directly at LHC, but have indirect impact.
- Simple scaling argument: effect is $\frac{Q^2}{\Lambda_{\rm NP}^2} \sim \left(\frac{100 \text{ GeV}}{1 \text{ TeV}}\right)^2 \sim \text{few}\%$

- Achievable experimentally!

> *"Except for rare decays, the overall uncertainties will be dominated by the theoretical systematics, with a precision close to percent level."*
>
> - Report on *Physics Potential of the HL-LHC,* submitted to CERN Council

- Achievable theoretically!
  - Nonperturbative effects enter at ~1% level.
- Requires advances in all aspects of collider physics:
  - Parton distribution functions
  - Fixed order calculations
  - Resummations
  - Parton showers
  - ....

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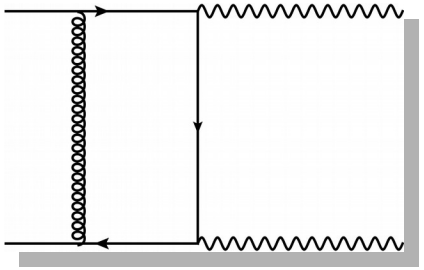
Multiloop amplitudes

Treating IR singularities

Infrared singularities

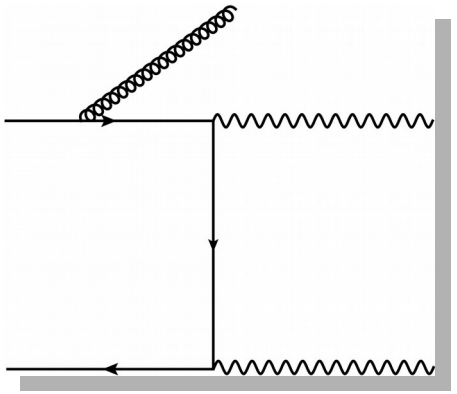
Higher order corrections contain infrared singularities from soft and/or collinear radiation.

- **Virtual corrections**
  - Explicit IR singularities from loop integration – poles in $1/\epsilon$.



- **Real corrections**
  - IR singularities after integration over full phase space of radiated parton.
  - **BUT:** then lose kinematic information

    (needed for distributions, kinematic cuts,...)



IR singularities at NLO and NNLO

Subtraction scheme:

Extract singularities without integrating over full phase space of radiated parton:

- Singularities manifest as poles in $1/\epsilon$ cancel against poles in virtual correction (KLN theorem).

- Solved at NLO (Catani-Seymour, Frixione-Kunszt-Signer,...).
  - Fully local.
  - Explicit, analytic cancellation of poles and expressions for finite counterterms.
  - Applicable to any process at the LHC.
  - Essential precursor to *"NLO revolution"* & automation of NLO calculations.
- Highly non-trivial at NNLO: multiple soft/collinear limits which may overlap – can approach a limit in different ways.
- Two approaches: slicing and subtraction.

Handling IR singularities at NNLO

SLICING

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 F_J d\phi_d\right]_{\text{s.c.}} + \int_\delta^1 |\mathcal{M}_J|^2 F_J d\phi_4 + \mathcal{O}(\delta)$$

Divergent

Born-like;

Treat in soft-collinear approximation

NLO+jet

- Exploits vast experience in NLO calculations.
- Non-local – potential issues of numerical stability.
- NLO+jet term: cutoff as large as possible.
- Power corrections: cutoff as small as possible.
- Ongoing work to better control power corrections.

[Ebert, Moult, Stewart, Rothen, Tackmann, Vita, Zhu '17-'18];

[Boughezal, Isgro, Liu, Petriello, '17-'18]

- qT

  [Catani, Grazzini '07]
- N-jettiness

  [Gaunt *et al* '15; Boughezal *et al* '15]

Handling IR singularities at NNLO

SUBTRACTION

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int \left(|\mathcal{M}_J|^2 F_J - S\right) d\phi_4 + \int S d\phi_d$$

Divergent

Finite;
integrate in 4-dim.

Counterterm;
Explicit singularities

- Antenna [Gehrmann-de Ridder, Gehrmann, Glover '05, …]
- STRIPPER [Czakon '10, '11]
- Projection-to-Born [Cacciari *et al* '15]
- CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca '05, …]

} "ESTABLISHED"

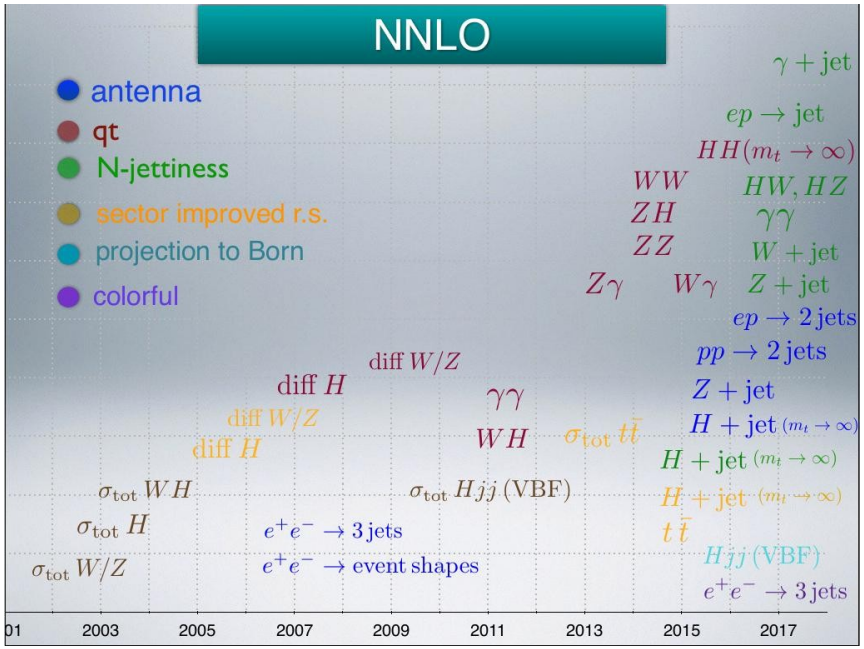
- **Nested soft-collinear [Caola, Melnikov, R.R. '17]**
- Geometric [Herzog '18]
- Local analytic sector [Magnea *et al* '18]

} "2ND GEN."

The NNLO Revolution

Great progress in subtraction & slicing methods ➡ *"The NNLO Revolution":*

All $2 \to 2$ process and a few $2 \to 3$ process (with special kinematics) known at NNLO.



Slide from Gudrun Heinrich, LHCP2017

Problem solved, but solutions **not satisfactory** (esp. compared to situation at NLO).

Current subtraction schemes:

- Are often complicated – difficult to implement.
- May obscure the physical origin of singularities in intermediate steps.
- May not be flexible.
- Usually require large computational times and fast scaling:
  - ~100 CPU hrs for *V* (differential)
  - ~100k CPU hrs for *V+j* (differential).
  - $2 \to 3$ processes, e.g. *H+2j*?

Improving NNLO subtractions

Goal: Replicate success of NLO subtraction methods (FKS/CS).

A "*better*" subtraction scheme should:

- Be fully local
  - *Subtractions point-by-point in phase space.*
  - *Clear physical origins of singularities.*
  - *Avoid large numerical cancellations in intermediate steps.*
- Have analytic expressions for the counterterms
  - *Poles cancel explicitly -- full control over singular structures.*
  - *Improved numerical efficiency.*
- Have a minimal structure displaying a clear origin of physical singularities
  - *Easier for others to implement.*
- Be applicable to all production processes at the LHC.
- Be flexible
  - *Allow freedom in phase-space parametrization/mapping.*

Nested soft-collinear subtraction

[Caola, Melnikov, R.R. '17]

- Extension of FKS subtraction to NNLO.
- Independent subtraction of soft and collinear divergences (color coherence).
- Use of sectors (as in STRIPPER) to separate overlapping *collinear* singularities.

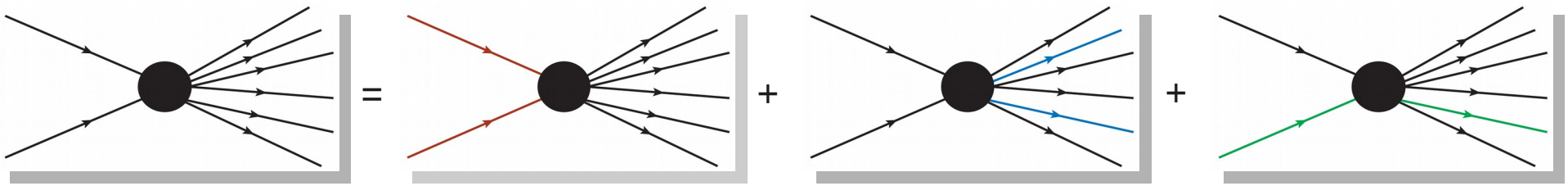
[Czakon '10, '11]

  - Natural splitting by rapidity.

- ✓ Fully local.
- ✓ Fully analytic.
  - Nontrivial integrals in [Caola, Delto, Frellesvig, Melnikov '18; Delto, Melnikov '19]
- ✓ Clear physical origin of singularities (soft & collinear).
- ✓ Not tied to phase space parametrization (currently using STRIPPER parametrization of angular phase space).
- ✓ Highly modular – identify simpler building blocks for subtractions for arbitrary processes.
- ✓ Recombination of sectors leading to simplifications in integrated subtraction terms.

Building Towards Generality

Modularity: split $pp \to n$ parton process into:



Initial state color only (color singlet production)

Final state color only (color singlet decay)

One color particle in initial and final states (deep inelastic scattering)

1. Consider color singlet production, color singlet decay, deep inelastic scattering in turn.
2. Compare against analytic results ➡ ***complete control*** on each block.
3. Combine into general result for arbitrary production process.

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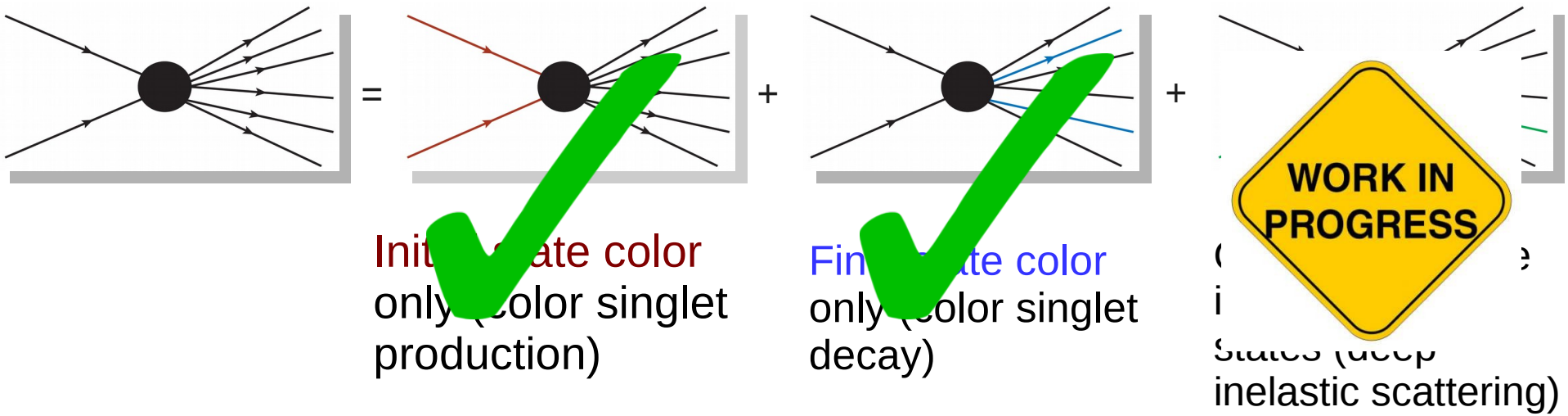

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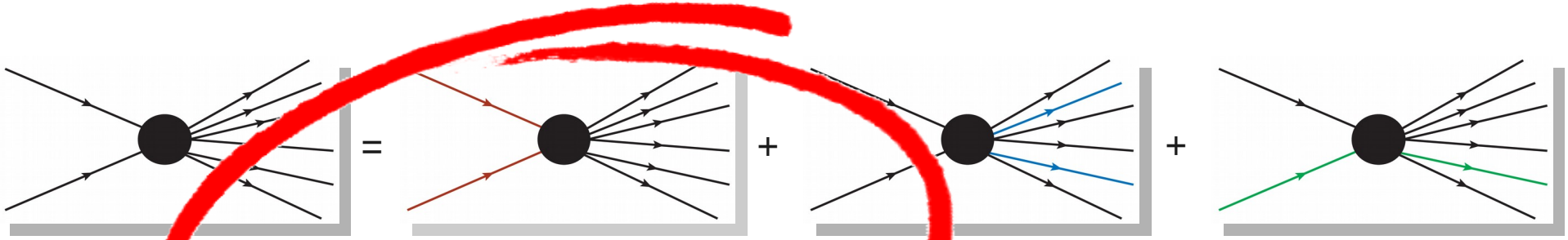

Initial state color only (color singlet production)

Final state color only (color singlet decay)

One color particle in initial and final states (deep inelastic scattering)

This talk.

FKS subtraction at NLO: Notation

Consider real corrections to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4) :$$

$$\boxed{\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}g_4] F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4)\rangle.}$$

$$F_{LM}(1,2,4) = \mathrm{dLips}_V \; |\mathcal{M}(1,2,4,V)|^2 \; \mathcal{F}_{\mathrm{kin}}(1,2,4,V) \qquad [\mathrm{d}g_4] = \frac{\mathrm{d}^{d-1}p_4}{(2\pi)^d 2E_4}\theta(E_{\max} - E_4)$$

Lorentz-inv. Phase space for V **(incl. delta-fn)**

Matrix-element sq.

IR-safe observable

Integration in partonic CoM frame

Define soft and collinear operators:

$$\boxed{S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A}$$

$$\rho_{ij} = 1 - \cos\theta_{ij}$$

FKS subtraction at NLO: Subtraction

Remove singular limits and add back as subtraction terms:

$$\begin{aligned}\langle F_{LM}(1,2,4)\rangle =& \langle (I - C_{41} - C_{42})(I - S_4) F_{LM}(1,2,4)\rangle + \\ & \langle S_4 F_{LM}(1,2,4)\rangle + \\ & \langle (C_{41} + C_{42})(I - S_4) F_{LM}(1,2,4)\rangle\end{aligned}$$

- First term: finite, can be integrated numerically in 4-dimensions.
- Second term: soft subtraction term – gluon decouples completely (need upper bound: $E_{\max}$).
- Third term: collinear and soft+collinear subtraction terms – gluon decouples partially or completely.
- Singularities made explicit by integrating subtraction terms over phase space of unresolved gluon.

FKS subtraction at NLO: finite result

- Combining with virtual corrections and pdf renormalization → cancel poles.
- Take $\epsilon \to 0$ limit to get finite remainder – NLO correction:

$$2s \cdot \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \left\langle F_{LV}^{\mathrm{fin}}(1,2) + \frac{\alpha_s(\mu)}{2\pi}\left[\frac{2}{3}\pi^2 C_F - 2\gamma_q \log\left(\frac{\mu^2}{s}\right)\right] F_{LM}(1,2)\right\rangle$$
$$- \frac{\alpha_s(\mu)}{2\pi}\int_0^1 \mathrm{d}z \left[\hat{P}^{(0)}_{qq,R}(z)\ln\left(\frac{\mu^2}{s}\right) + \mathcal{P}'_{qq}(z)\right]\left\langle \frac{F_{LM}(z\cdot 1,2)}{z} + \frac{F_{LM}(1,z\cdot 2)}{z}\right\rangle$$
$$+ \left\langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4)\right\rangle.$$

$$\hat{O}_{\mathrm{NLO}} = (I - C_{41} - C_{42})(I - S_4)$$

$$\hat{P}^{(0)}_{qq,R} = C_F\left(2D_0(z) - (1+z)\right)$$ (AP splitting function without delta function)

$$\mathcal{P}'_{qq}(z) = -C_F\left[-4D_1(z) - (1-z) + 2(1+z)\log(1-z)\right]$$

$$\gamma_q = 3/2 C_F$$

FKS subtraction at NLO: finite result

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$$+ \left\langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4)\right\rangle.$$

Sum of:

- **LO-like terms,** with or without convolutions with splitting functions.
- Real emission term, with singular configurations removed by iterated subtraction.
- Finite remainder of virtual corrections.

Real-real subtractions at NNLO

Aim to replicate NLO results as much as possible at NNLO.

Consider real-real correction to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4) + g(p_5):$$

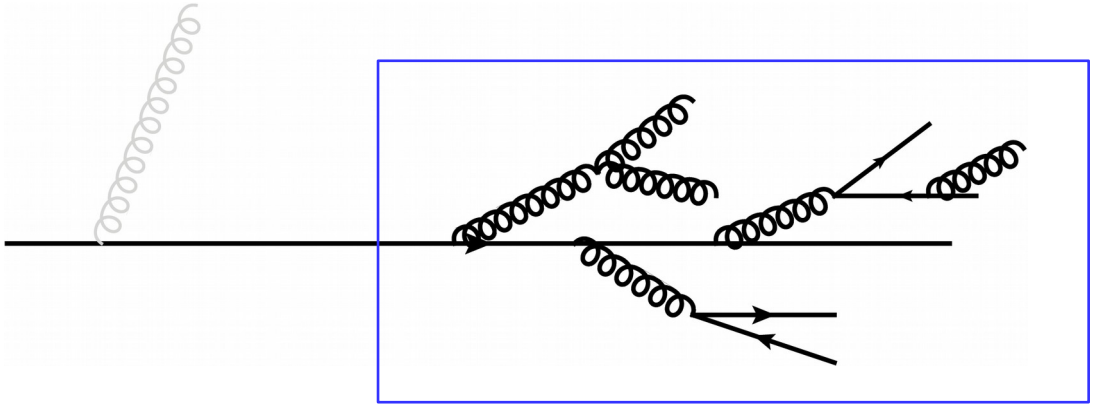
$$\mathrm{d}\sigma^{\mathrm{RR}} = \frac{1}{2s}\int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5)$$

IR singularities from

- g_4 and/or g_5 → soft.
- g_4 or g_5 → collinear to initial state partons.
- g_4 or g_5 → collinear to each other.
- g_4 and g_5 collinear to same initial state parton (triple collinear limit).

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Used in resummation & parton showers; not manifest in subtractions.
- Soft gluon cannot resolve details of collinear splittings; only sensitive to total color charge.



➡ No overlap between soft and collinear limits -- can be treated independently:

- Regularize soft singularities first, then collinear singularities.
- Energies and angles decouple.

Treatment of real-real singularities

- **Step 1:** Limit operators.
  - Recall $\boxed{S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A.}$ $\quad (\rho_{ij} = 1 - \cos\theta_{ij})$
  - NNLO-like:

$$\mathbb{S} A = \lim_{E_4, E_5 \to 0} A, \text{ at fixed } E_5/E_4,$$

$$\mathbb{C}_i A = \lim_{\rho_{4i}, \rho_{5i} \to 0} A, \text{ with non vanishing } \rho_{4i}/\rho_{5i}, \rho_{45}/\rho_{4i}, \rho_{45}/\rho_{5i}.$$

- **Step 2:** Order gluon energies $E_4 > E_5$.

$$2\,\mathrm{s} \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5]\theta(E_4 - E_5) F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4,5) \rangle.$$

  - Gluon energies bounded by $E_{\max}$.
  - Energies defined in CoM frame.
  - Soft singularities: either double soft or g_5 soft.

Soft singularities

- **Step 3:** Regulate the *soft* singularities:

$$\big\langle F_{LM}(1,2,4,5)\big\rangle = \big\langle \mathbb{S} F_{LM}(1,2,4,5)\big\rangle + \big\langle S_5(I-\mathbb{S})F_{LM}(1,2,4,5)\big\rangle + \big\langle (I-S_5)(I-\mathbb{S})F_{LM}(1,2,4,5)\big\rangle.$$

- First term: both g_4 and g_5 soft.
- Second term: g_5 soft, soft singularities in g_4 are regulated.
- Third term: regulated against all soft singularities,
- All three terms contain **(potentially overlapping)** collinear singularities.

Phase-space partitioning

- **Step 4:** Introduce **phase-space partitions**

$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

with

$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0 \quad \Rightarrow \quad w^{14,15} \text{ contains } C_{41},\ C_{51},\ C_{45}$$

$$C_{41} w^{24,25} = C_{51} w^{24,25} = 0 \quad \quad w^{24,25} \text{ contains } C_{42},\ C_{52},\ C_{45}$$

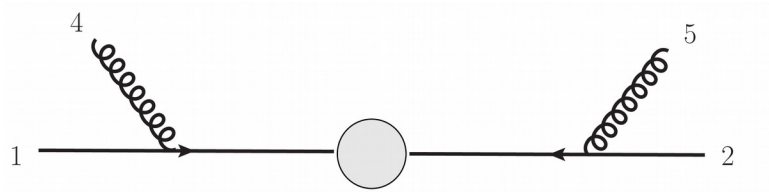
Triple collinear partition

and

$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \quad \Rightarrow \quad w^{14,25} \text{ contains } C_{41},\ C_{52}$$

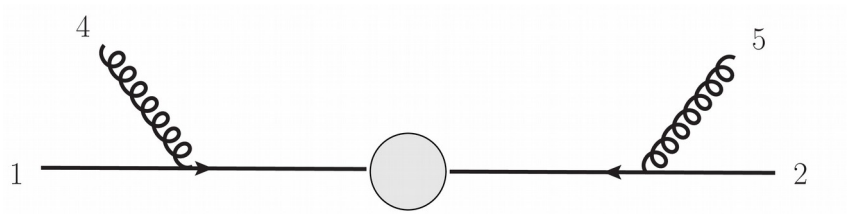
$$C_{41} w^{15,24} = C_{52} w^{15,24} = C_{45} w^{15,24} = 0 \quad \quad w^{15,24} \text{ contains } C_{42},\ C_{51}$$

Double collinear partition



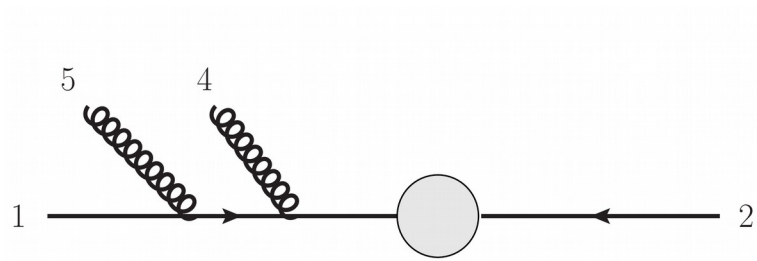
Phase-space partitioning

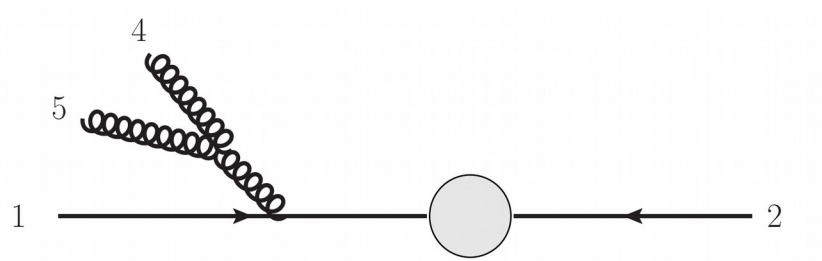
- Double collinear partition – large rapidity difference.



~ NLO x NLO ➡ simple

- Triple collinear partition – large/small rapidity difference.





Overlapping singularities remain – need one last step to separate these.

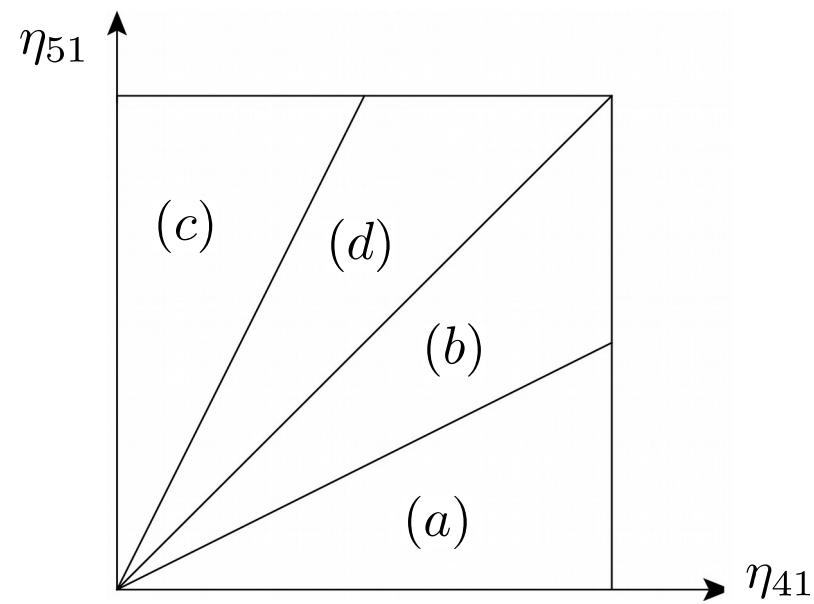
Sector Decomposition

- **Step 5: Sector decomposition:**

- Define angular ordering to separate singularities.

$$\eta_{ij} = \rho_{ij}/2$$

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right)$$
$$+ \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right)$$
$$\equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$



- Thus the limits are

$$\left.\begin{array}{l}\theta^{(a)} : C_{51} \\ \theta^{(c)} : C_{41}\end{array}\right\} \text{Large rapidity difference}$$

$$\left.\begin{array}{l}\theta^{(b)} : C_{45} \\ \theta^{(d)} : C_{45}\end{array}\right\} \text{Small rapidity difference}$$

- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall <u>energy ordering.</u>
- Angular phase space parametrization [Czakon '10].

Removing collinear singularities

Then we can write soft-regulated term as

$$\left\langle (I - S_5)(I - \mathbb{S})F_{LM}(1,2,4,5)\right\rangle = \left\langle F_{LM}^{s_r c_s}(1,2,4,5)\right\rangle + \left\langle F_{LM}^{s_r c_t}(1,2,4,5)\right\rangle + \left\langle F_{LM}^{s_r c_r}(1,2,4,5)\right\rangle,$$

$\left\langle F_{LM}^{s_r c_r}(1,2,4,5)\right\rangle$

- All singularities removed through nested subtractions – evaluated in 4-dimensions.
- Only term involving fully-resolved real-real matrix element.

$\left\langle F_{LM}^{s_r c_{s,t}}(1,2,4,5)\right\rangle$

- Contain (soft-regulated) single and triple collinear singularities.
- Matrix elements of lower multiplicity.
- Partitioning factors and sectors: one collinear singularity in each term.

Treating singular limits

We have four singular subtraction terms:

$$\langle \mathbb{S} F_{LM}(1,2,4,5)\rangle \quad \langle S_5(I-\mathbb{S})F_{LM}(1,2,4,5)\rangle \quad \langle F_{LM}^{s_r c_s}(1,2,4,5)\rangle \quad \langle F_{LM}^{s_r c_t}(1,2,4,5)\rangle$$

We know how to treat them:

- Gluon(s) decouple partially or completely.
- Decouple completely:
  - Integrate over gluonic angles and energy.
- Decouple partially:
  - Integrate over gluonic angles.
  - Integral(s) over energy → integrals over splitting function in *z*.
- Analytic results for nontrivial integrals from double-soft and triple-collinear limits calculated in [Caola, Delto, Frellesvig, Melnikov '18; Delto, Melnikov '19].
- Significant analytic simplifications on <u>recombining sectors</u> after integration.

Treating singular limits

After integration: subtraction terms written as lower multiplicity terms:

- LO-like:

$$\langle F_{LM}(z\cdot 1, \bar{z}\cdot 2)\rangle\,,\ \langle F_{LM}(z\cdot 1, 2)\rangle\,,\ \langle F_{LM}(1, z\cdot 2)\rangle\,,\ \langle F_{LM}(1, 2)\rangle$$

(no final state partons).

- NLO-real-like (regulated by iterative subtraction):

$$\langle \mathcal{O}_{NLO} F_{LM}(z\cdot 1, 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1, z\cdot 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1, 2, 4)\rangle$$

(maximum one final state parton).

convoluted with splitting functions with explicit singularities.

- Pole cancellation within each structure.

Finite remainders

- Relatively compact expressions for finite remainders for each lower-multiplicity structure.
- Familiar structures appear, e.g.

$$\mathrm{d}\sigma_{z1,2,4} = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 \mathrm{d}z \Bigg\{ \hat{P}^{(0)}_{qq,R}(z) \left\langle \log \frac{\rho_{41}}{4} \mathcal{O}_{\mathrm{NLO}} \left[\tilde{w}^{41,51}_{5||1} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \right] \right\rangle$$
$$+ \left[\mathcal{P}'_{qq}(z) - \hat{P}^{(0)}_{qq,R}(z) \log\left(\frac{\mu^2}{s}\right) \right] \mathcal{O}_{\mathrm{NLO}} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \Bigg\}$$

$$\mathrm{d}\sigma_{z1,\bar{z}2} = \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \int_0^1 \mathrm{d}z\mathrm{d}\bar{z} \left[\mathcal{P}'_{qq}(z) - \log\left(\frac{\mu^2}{s}\right) \hat{P}^{(0)}_{qq,R}(z) \right]$$
$$\times \left[\mathcal{P}'_{qq}(\bar{z}) - \log\left(\frac{\mu^2}{s}\right)) \hat{P}^{(0)}_{qq,R}(z) \right] \frac{F_{LM}(z\cdot 1, \bar{z}\cdot 2)}{z\bar{z}}$$

- Same functions that appeared at NLO (as expected...)

Finite remainders

- New functions are relatively simple...
- Extension of NLO calculation to NNLO:
  - LO and NLO results convoluted with known functions.
  - Nested subtraction for real-real contribution.

$$
\begin{aligned}
&\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{F_{LM}(z\cdot 1,2)}(\mu^2 = s) = \\
&\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \int_0^1 \mathrm{d}z \Bigg\{ C_F^2 \Bigg[8\tilde{\mathcal{D}}_3(z) + 4\tilde{\mathcal{D}}_1(z)(1+\ln 2) + 4\tilde{\mathcal{D}}_0(z)\left[\frac{\pi^2}{3}\ln 2 + 4\zeta_3\right] \\
&+ \frac{5z-7}{2} + \frac{5-11z}{2}\ln z + (1-3z)\ln 2 \ln z + \ln(1-z)\left[\frac{3}{2}z - (5+11z)\ln z\right] \\
&+ 2(1-3z)\mathrm{Li}_2(1-z) \\
&+ (1-z)\Bigg[\frac{4}{3}\pi^2 + \frac{7}{2}\ln^2 2 - 2\ln^2(1-z) + \ln 2\left[4\ln(1-z) - 6\right] + \ln^2 z \\
&+ \mathrm{Li}_2(1-z)\Bigg] + (1+z)\Bigg[-\frac{\pi^2}{3}\ln z - \frac{7}{4}\ln^2 2 \ln z - 2\ln 2 \ln(1-z)\ln z \\
&+ 4\ln^2(1-z)\ln z - \frac{\ln^3 z}{3} + \left[4\ln(1-z) - 2\ln 2\right]\mathrm{Li}_2(1-z)\Bigg] \\
&+ \left[\frac{1+z^2}{1-z}\right]\ln(1-z)\left[3\mathrm{Li}_2(1-z) - 2\ln^2 z\right] - \frac{5-3z^2}{1-z}\mathrm{Li}_3(1-z) \\
&+ \frac{\ln z}{(1-z)}\left[12\ln(1-z) - \frac{3-5z^2}{2}\ln^2(1-z) - \frac{7+z^2}{2}\ln 2 \ln z\right]\Bigg] \\
&+ C_A C_F \Bigg[-\frac{22}{3}\tilde{\mathcal{D}}_2(z) + \left(\frac{134}{9} - \frac{2}{3}\pi^2\right)\tilde{\mathcal{D}}_1(z) + \Bigg[-\frac{802}{27} + \frac{11}{18}\pi^2 \\
&+ (2\pi^2 - 1)\frac{\ln 2}{3} + 11\ln^2 2 + 16\zeta_3\Bigg]\tilde{\mathcal{D}}_0(z) + \frac{37-28z}{9} + \frac{1-4z}{3}\ln 2 \\
&- \left(\frac{61}{9} + \frac{161}{18}z\right)\ln(1-z) + (1+z)\ln(1-z)\left[\frac{\pi^2}{3} - \frac{22}{3}\ln 2\right] \\
&- (1-z)\left[\frac{\pi^2}{6} + \mathrm{Li}_2(1-z)\right] - \frac{2+11z^2}{3(1-z)}\ln 2 \ln z - \frac{1+z^2}{1-z}\mathrm{Li}_2(1-z)\times \\
&\times \left[2\ln 2 + 3\ln(1-z)\right]\Bigg] + R_+^{(c)}\mathcal{D}_0(z) + R^{(c)}(z)\Bigg\}\left\langle \frac{F_{LM}(z\cdot 1,2)}{z}\right\rangle.
\end{aligned}
$$

Other partonic channels

Straightforward extension to other partonic channels.

E.g. $g(p_1)g(p_2) \to V + g(p_4)g(p_5)$

$$\Rightarrow \mathrm{d}\sigma_{z1,2,4} = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 \mathrm{d}z \Bigg\{ \hat{P}^{(0)}_{gg,R}(z) \left\langle \log \frac{\rho_{41}}{4} \mathcal{O}_{\mathrm{NLO}} \left[\tilde{w}^{41,51}_{5||1} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \right] \right\rangle$$
$$+ \left[\mathcal{P}'_{gg}(z) - \hat{P}^{(0)}_{gg,R}(z) \log\left(\frac{\mu^2}{s}\right) \right] \mathcal{O}_{\mathrm{NLO}} \frac{F_{\mathrm{LM}}(z\cdot 1,2,4)}{z} \Bigg\}$$

and similar changes elsewhere, e.g. $C_F \to C_A; \;\; \gamma_q = 3/2 C_F \to \gamma_g = \beta_0.$

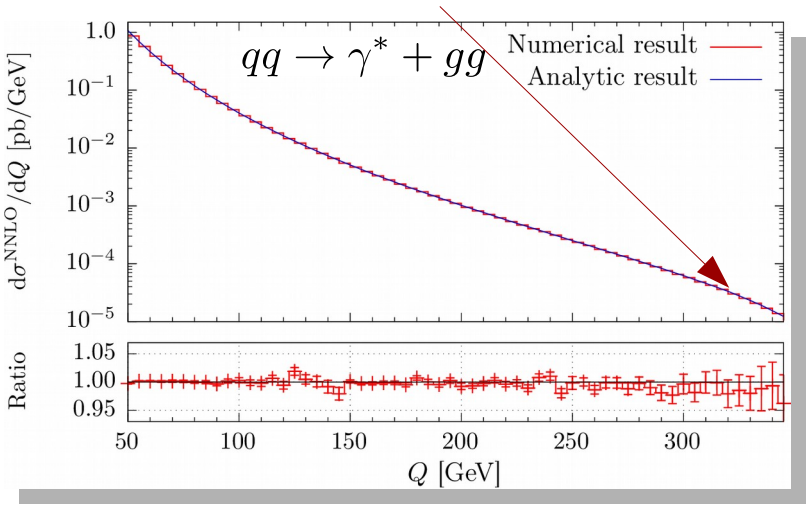
Minor modifications for certain channels, e.g.

- No energy ordering,
- No partitioning or sector decomposition of phase space.

Validation of Results

- Exhaustively tested against analytic results for
  - ✔ Drell-Yan production [Hamberg, Matsuura, van Neerven '89]
  - ✔ Higgs production [Anastasiou, Melnikov '04]

- Good control in extreme kinematic regions.



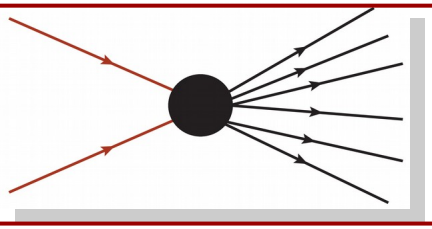
[Caola, Melnikov, R.R. '17]

- < per mille agreement for all NNLO *contributions*, including numerically tiny ones.

Channel	Color structures	Numerical result (nb)	Analytic result (nb)
$q_i\bar{q}_i \to gg$	–	8.351(1)	8.3516
$q_i\bar{q}_i \to q_j\bar{q}_j$	$C_F T_R n_{\rm up}$, $C_F T_R n_{\rm dn}$	-2.1378(5)	-2.1382
	$C_F(C_A - 2C_F)$	$-4.8048(3) \cdot 10^{-2}$	$-4.8048 \cdot 10^{-2}$
	$C_F T_R$	$5.441(7) \cdot 10^{-2}$	$5.438 \cdot 10^{-2}$
$q_i q_j \to q_i q_j \quad (i \neq -j)$	$C_F T_R$	0.4182(5)	0.4180
	$C_F(C_A - 2C_F)$	$-9.26(1) \cdot 10^{-4}$	$-9.26 \cdot 10^{-4}$
$q_i g + g q_i$	–	-9.002(9)	-8.999
gg	–	1.0772(1)	1.0773

Table 1: Different contributions to the NNLO *coefficient* for on-shell Z production at the 13 TeV LHC with $\mu_R = \mu_F = 2m_Z$. All the color factors are included in the numerical results. The residual Monte-Carlo integration error is shown in brackets. See text for details.

[Caola, Melnikov, R.R. '19]

This building block  is reliable!

Validation of Results

Implies absolute control on physical results.

- Higgs production cross sections: per mille accuracy in ~ 1 CPU hr.

LHC. For this study, we set $\mu_R = \mu_F = m_H$. Running for less than an hour on a single core of a standard laptop, we obtain

$$\sigma_{\rm H}^{\rm LO} = 17.03(0)\ \text{pb}; \qquad \sigma_{\rm H}^{\rm NLO} = 30.25(1)\ \text{pb}; \qquad \sigma_{\rm H}^{\rm NNLO} = 39.96(2)\ \text{pb}. \qquad (5.1)$$

- Drell-Yan production *with symmetric cuts on final state leptons:* 2 per mille accuracy in ~1 CPU hr.

In this case, we use $\mu_R = \mu_F = m_Z$. Running on a single core of a standard laptop for about an hour, we obtain

$$\sigma_{\rm DY}^{\rm LO} = 650.4 \pm 0.1\ \text{pb}; \qquad \sigma_{\rm DY}^{\rm NLO} = 700.2 \pm 0.3\ \text{pb}; \qquad \sigma_{\rm DY}^{\rm NNLO} = 734.8 \pm 1.4\ \text{pb}. \qquad (5.3)$$

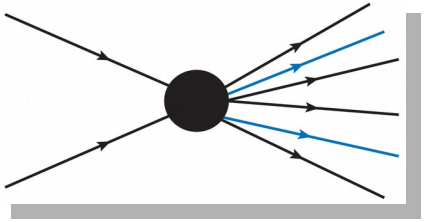
- By comparison

Process (${process_id})	LO runtime (relative uncertainty)	NLO runtime (relative uncertainty)	NNLO runtime (relative uncertainty)	NNLO runtime estimate for 10^{-3} uncertainty
$pp \to H$ (pph21)	0 d 0 h 2 min (1.5×10^{-4})	0 d 0 h 12 min (2.7×10^{-4})	35 d 23 h 23 min (7.2×10^{-4})	19 d
$pp \to e^- e^+$ (ppeex02)	0 d 0 h 48 min (1.0×10^{-4})	0 d 2 h 24 min (2.8×10^{-4})	173 d 20 h 36 min (3.6×10^{-4})	22 d

[Grazzini, Kallweit, Wiesemann, 2018]

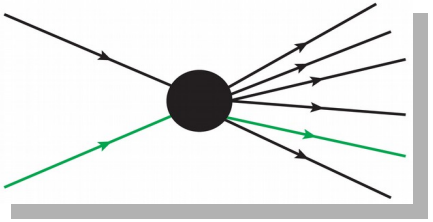
Other Building Blocks

- Color singlet decay is simpler!
  - No initial state parton evolution.
  - ➡ Convolutions become integrals over energies.
  - Tested against analytic results in
    - ✓ $H \to b\bar{b}$ [Baikov, Chetyrkin, Kühn '06]
    - ✓ $H \to gg$ [Schreck, Steinhauser '07]



➡ Similar level of agreement found. [Caola, Melnikov, R.R. 1905.xxxxx]

- Deep inelasic scattering: new challenges.
  - E.g. at LO, partons are **not back-to-back.**
  - ➡ Introduce new angle.
  - Work in progress.



Conclusions

- Demands of the high precision programme require significant advances in collider physics.
- Despite success of IR subtraction schemes, ultimate scheme yet to be developed.
- Proposed **nested soft-collinear scheme:**
  - ✔ Fully local, fully analytic, remarkably straightforward.
- Discussed color singlet production:
  - ✔ Building block for subtraction for arbitrary processes.
- Future directions:
  - ➢ Remaining building blocks: color singlet decay, deep inelastic scattering.
  - ➢ Phenomenological potential.
  - ➢ ....

STAY TUNED!

THANK YOU FOR YOUR ATTENTION!

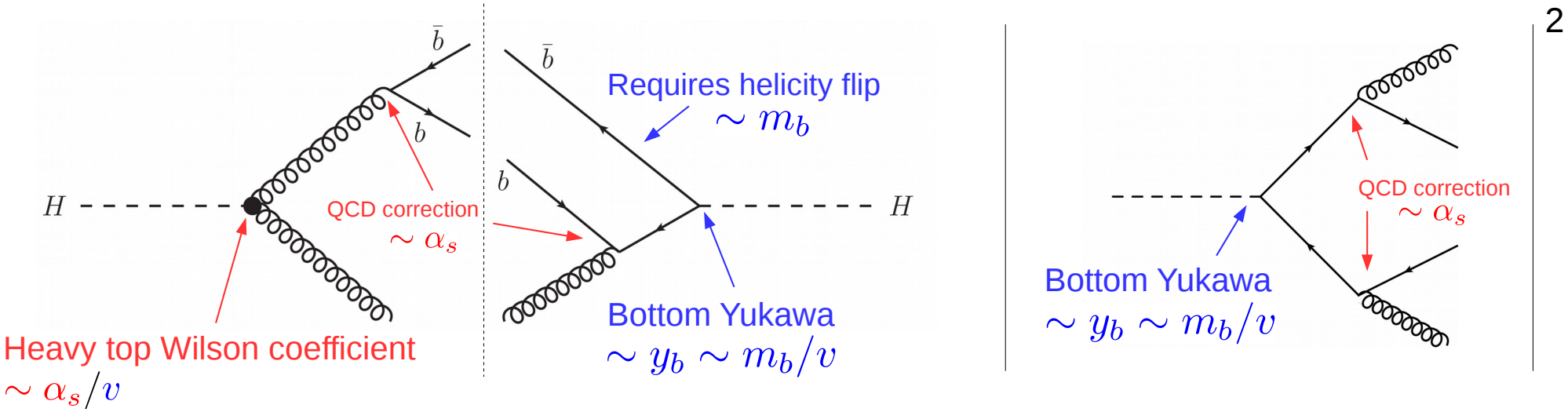
BACKUP SLIDES

Bottom mass effects in $H \to bb$

- In $H \to bb$ decay, want massless b-quarks but non-zero y_b

$$m_b \ll m_H \Rightarrow \mathrm{d}\sigma \sim y_b^2 (A + B\, m_b^2/m_H^2 + \ldots) = A y_b^2$$

- Works at LO & NLO, but not at NNLO – **interference terms.**



Top-loop interference contribution

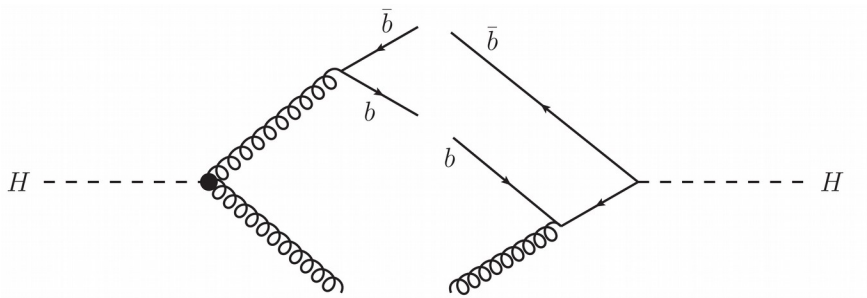
$\sim \alpha_s^2 m_b^2/v^2 \sim \alpha_s^2 y_b^2$

"Regular" contribution squared

$\sim \alpha_s^2 m_b^2/v^2 \sim \alpha_s^2 y_b^2$

Interference contribution has **identical parametric scaling** to other NNLO corrections.

Bottom mass interference



Obvious strategy: factor out one power of m_b and then take $m_b = 0$

BUT:

- Reduced matrix elements have unusual IR behaviour: *subleading power singularities*, e.g. **soft singularities from quarks!**
- $\log(m_b/m_H)$ **don't cancel** between real and virtual interference terms – cannot take massless limit!
- **Cannot** be regulated using flavor-kT algorithm (doesn't regulate soft quark singularity).
- **Cannot** define an inclusive cross section for $H \to bb$ at NNLO with massless b-quarks.
- Calculation in double-log approx: **~ 30%** of NNLO corrections to $H \to bb$ decay.
  - Effect on kinematic distributions?
- Different dependence on bottom Yukawa – **different behavior in BSM models.**

➡ NNLO calculation of $H \to bb$ to massive bottom quarks required.

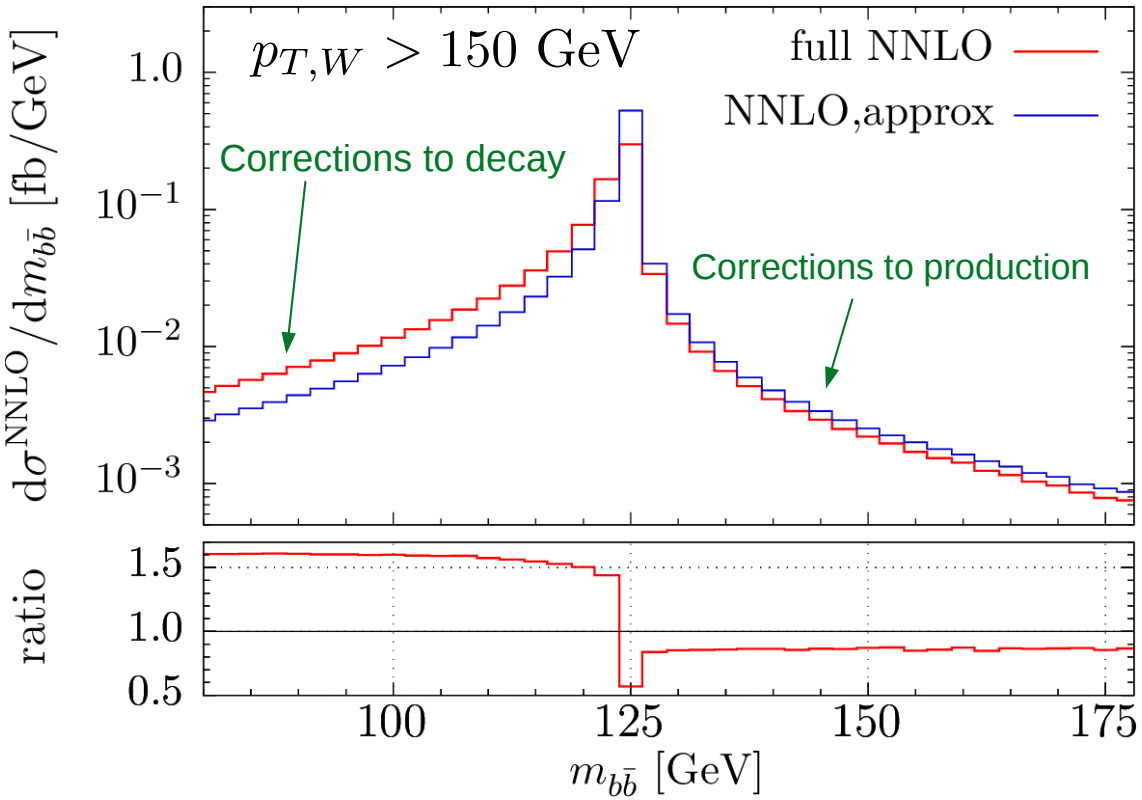
$VH(\to b\bar{b})$ to NNLO in production and decay

[Caola, Luisoni, Melnikov, R.R. '17]

NNLO corrections in production and decay in NWA.

Confirm results of [Ferrera, Somogyi, Tramantano '17]:

- Large (~60%) at low invariant mass.
- Sharp decrease at Higgs mass.
- ~ 15% depletion at high inv. mass.
- **Expected** as full NNLO includes corrections to decay – reduce inv. mass.



Comparison with parton shower

- Can parton showers capture these effects?
  - 🙂 Reasonable high boost $p_{T,W} > 150$ GeV
  - ☹ Low invariant mass requires **hard** gluon.
- `HWJ` generator from `POWHEG-Box` with `MiNLO`; $H \to bb$ through `PYTHIA`.
- `NNLOPS` analysis by [Astill, Bizon, Re, Zanderighi '18]