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Numerical techniques for resummation in Soft-Collinear Effective Theory

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based on work with Guido Bell, Bahman Dehnadi, Tobias Mohrmann (Siegen) and Jim Talbert (DESY)

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Outline

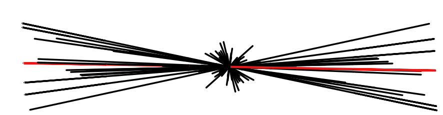
Large (global) logarithms

- At collider experiments we compare theoretical predictions for observables ω to measurement

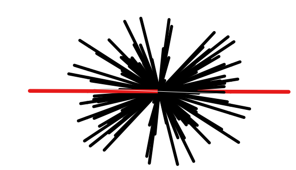
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} \delta(\omega - \omega(\{k_i\}))$$

- Examples:
  - Event shapes, e.g. *Thrust*
  - p_T spectra
  - Energy distributions
  - …

$$\tau = \max_{n} \frac{\sum_i |\vec{p}_i| - |\vec{p}_i \cdot n|}{\sum_i |\vec{p}_i|}$$



$\tau \simeq 0$



$\tau \simeq 0.5$

- These observable can suffer from *Sudakov logarithms* spoiling perturbation theory

$$\alpha_s^n \ln^{2n} \tau$$

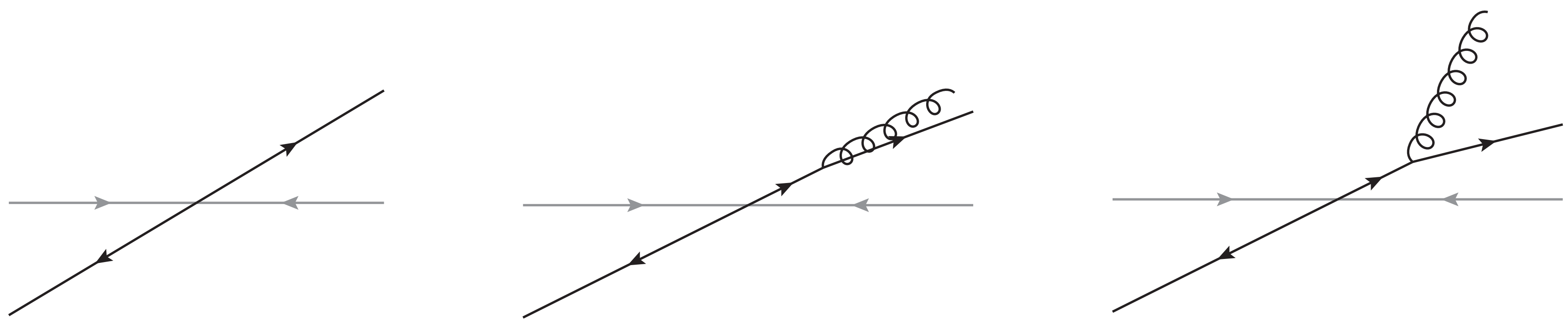
(Thrust)

$$\alpha_s^n \ln^{2n} \left(\frac{m_Z^2}{p_T^2}\right)$$

(Z @ small p_T)

Resummation

- Source: Soft and collinear divergences cancel (KLN) …

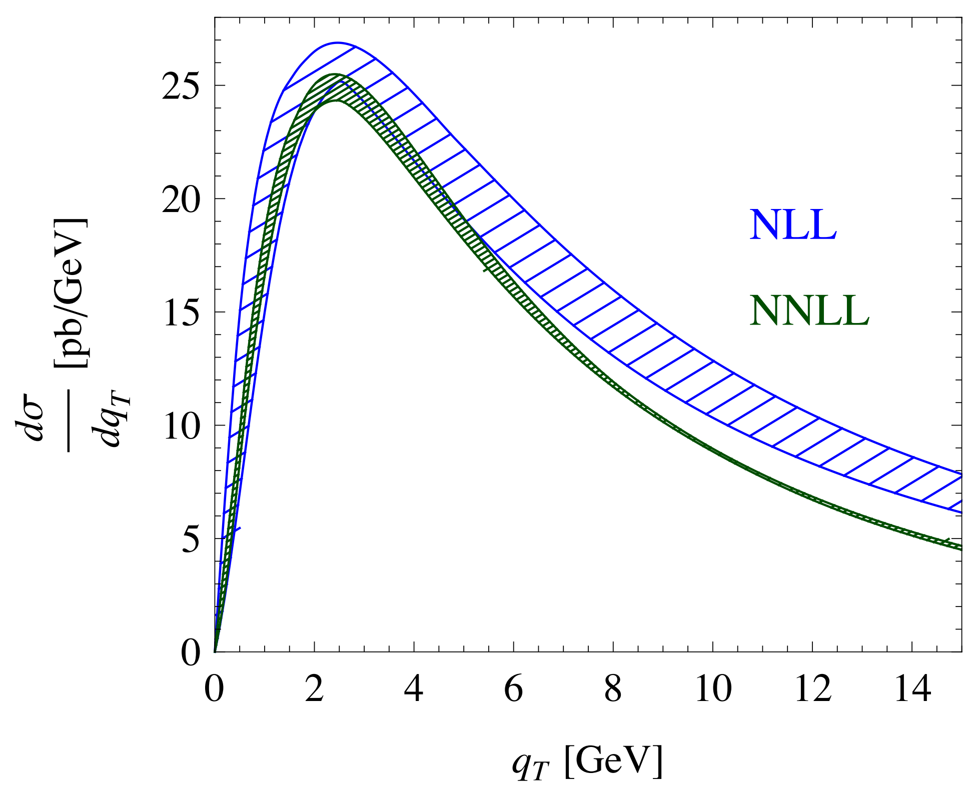


$e^+e^- \to q\bar{q}$: leading $\tau \approx 0$ $\tau > 0$

- … but phase space constraints differ between real and virtual emissions

- Resummation: *Systematically* construct healthy all-order expression, which expands to the pathological perturbative series

- Pure QCD methods rely on coherent branching algorithms, effective theory methods on RGE flow



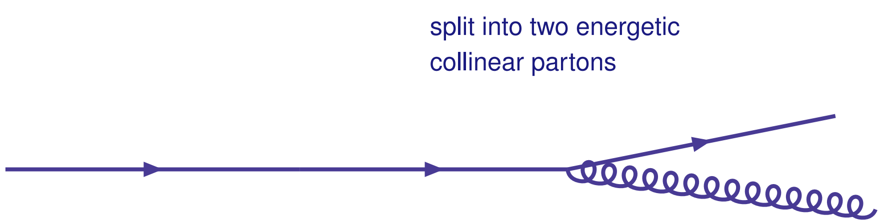
Automation in QCD: CAESAR/ARES:
[Banfi, Salam, Zanderighi, '04], [Banfi, McAslan, Monni, Zanderighi, '14]

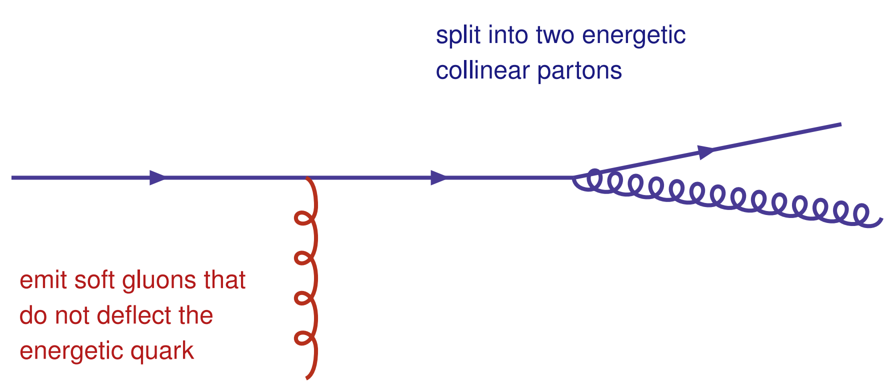
1109.6027(Becher/Neubert/Wilhelm)

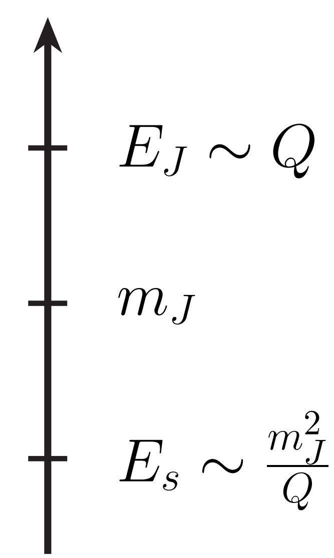
Introducing SCET: intuition

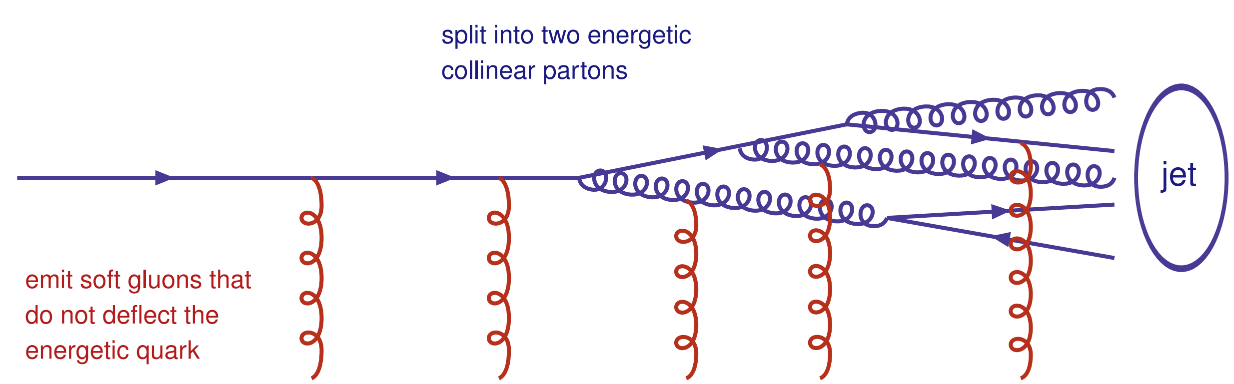
- Soft-Collinear Effective Theory is an effective theory whose degrees of freedom are soft and collinear partons - the modes giving rise to Sudakov logarithms

[Bauer, Fleming, Pirjol, Stewart, ’00]
[Beneke, Chapovsky, Diehl, Feldmann, 02]







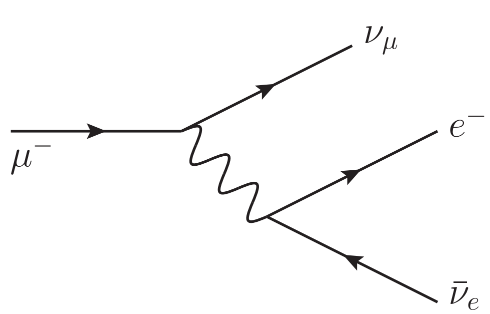


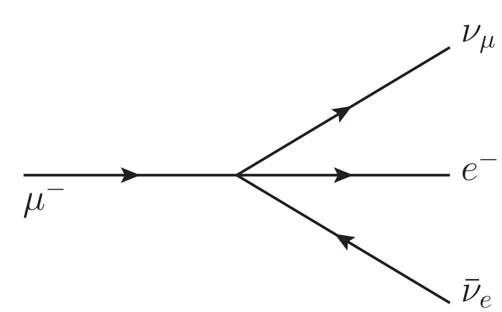
$\Rightarrow$ jet of collinear particles $m_J^2 \ll E_J^2$

soft large-angle radiation $E_s \ll E_J$

More intuition: Recall Fermi

- Used to simplify low energy description, yields low-energy-expanded SM result
- Weak bosons integrated out
- Top-down construction: Integrating out / Matching





$$\mathcal{L}_{EW} \ni \frac{e}{\sin\theta_W}\left(W_+^\mu J_\mu^- + W_-^\mu J_+^\mu\right)$$

$$\mathcal{L}_{eff} \ni \frac{c}{\Lambda^2} J_\mu^+ J_-^\mu + \ldots$$

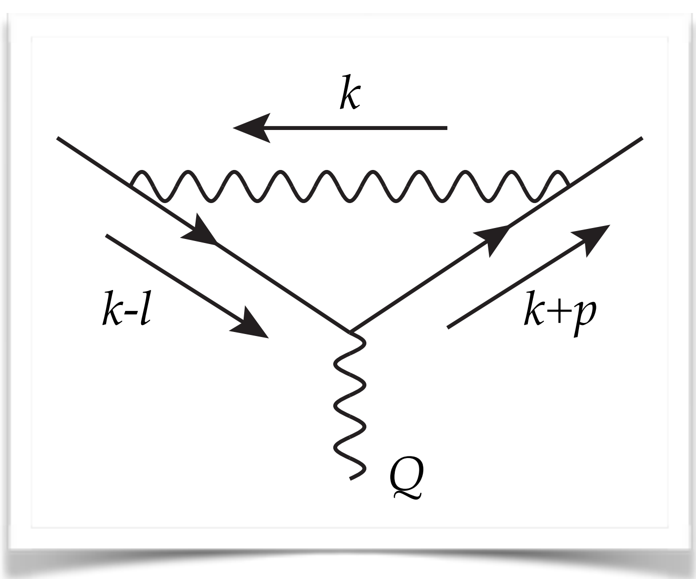
$$\frac{1}{p^2 - M_W^2} \approx -\frac{1}{M_W^2} + \mathcal{O}\left(\frac{p^2}{M_W^2}\right)$$

- Reproduce expanded result from High energy matching + low energy dynamics

Towards SCET: Method of regions

- Method to derive the leading behaviour of a loop integral in some limit:
  - Identify *regions* (typical momentum scalings) that contribute to singularities
  - Expand the *integrand* in these regions (will require regularisation)
  - Add individual results to reproduce leading terms of the full result

- Sketched example: Sudakov form factor



$$n^\mu = (1, 0, 0, 1)$$
$$\bar{n}^\mu = (1, 0, 0, -1)$$

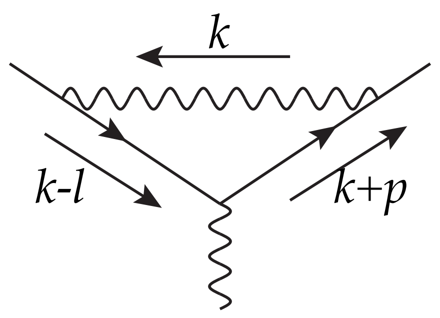
$$l^2 \sim p^2 \sim \lambda^2 Q^2, \quad \lambda << 1$$

More intuition: Method of regions II

- Notation:

$$n^\mu = (1,0,0,1)$$
$$\bar{n}^\mu = (1,0,0,-1)$$

$$p^\mu = \frac{\bar{n}^\mu}{2} n\cdot p + \frac{n^\mu}{2}\bar{n}\cdot p + p^{\perp,\mu} = (p^+, p^-, p^\perp)$$



$$l \sim Q(\lambda^2, 1, \lambda) \qquad p \sim Q(1, \lambda^2, \lambda)$$

- Contributing regions:

hard	$k \sim Q(1,1,1)$
collinear	$k \sim Q(1,\lambda^2,\lambda)$
	$k \sim Q(\lambda^2,1,\lambda)$
soft	$k \sim Q(\lambda^2,\lambda^2,\lambda^2)$

- Expansion, e.g. in hard region:

$$(k-l)^2 = k^2 + l^2 - 2kl$$
$$\to k^2 - 2k^+ l^-$$

- Individual region integrals need regularisation, but the sum is finite

Soft-Collinear Effective Theory

- Construct effective theory for QCD to reproduce method of regions

- Each region finds a place:
  (a) high energy region (hard region) goes into matching
  (b) low energy regions get a dynamical field each

- Full result from high energy matching and low energy dynamics

- Alternative view: Expanded QCD diagrams match onto diagrams in SCET

- *Crucial*: Low energy fields can be decoupled in SCET Lagrangian for QCD:

$$\mathcal{L} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

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The current operator

- For applications, we also need the matched current
- Naively, we would expect

$$\bar{\psi}(0)\gamma^\mu\psi(0) \to C_V\,\bar{\zeta}_{\bar{n}}(0)\gamma_\perp^\mu\zeta_n(0)$$

- We actually find

$$\bar{\psi}(0)\gamma^\mu\psi(0) \to \int dsdt\, C_V(s,t)\,\left(\bar{\zeta}_{\bar{n}}W_{\bar{n}}^\dagger S_{\bar{n}}^\dagger\right)(sn)\gamma_\perp^\mu\left(S_n W_n \zeta_n\right)(t\bar{n})$$

with *Wilson lines* for soft and collinear fields:

$$W_c = Pexp\left(ig\int_{-\infty}^0 ds\bar{n}\cdot A_c(x+s\bar{n})\right) \qquad S_n(s) = Pexp\left(ig\int_{-\infty}^0 ds\, n\cdot A_s(x+s\bar{n})\right)$$

- Both non-locality and Wilson line appearance collect operators at same power
- See e.g. derivative tower: there are large momentum components, so $\frac{(n\cdot\partial)^2}{Q^2}\sim 1$

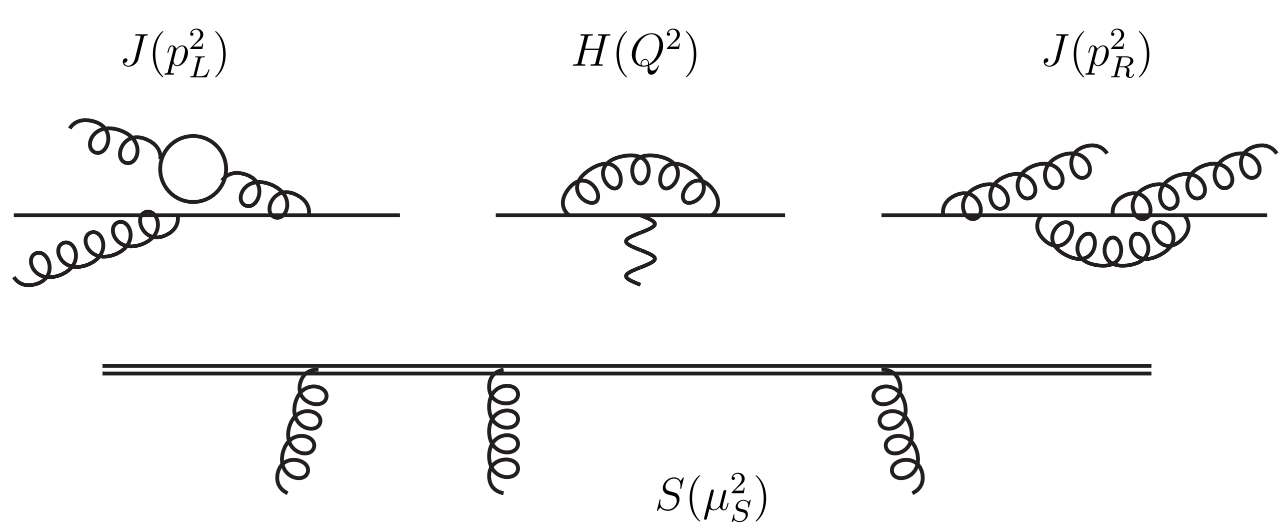
SCET: Factorisation

- Use this operator in a matrix element and it decomposes (if the observable factorises as well):

$$|C_V|^2 \sum_X |\langle 0|\mathcal{O}_{n\bar{n}}|X\rangle|^2$$
$$= |C_V|^2 \; \langle 0| \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right] \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right]^\dagger |0\rangle \; \langle 0| \left[W^0_n \zeta^0_n\right] \left[W^0_n \zeta^0_n\right]^\dagger |0\rangle \; \langle 0| \left[S^\dagger_{\bar{n}} S_n\right] \left[S^\dagger_{\bar{n}} S_n\right]^\dagger |0\rangle$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dp_L^2 \int dp_R^2 \;\; J(p_L^2,\mu) \;\; J(p_R^2,\mu) \;\; S(\tau Q - \frac{p_L^2 + p_R^2}{Q}, \mu)$$

(for dijet thrust)



SCET: resummation

- H, J^i, and S contain logarithms involving their respective natural scales:

$$\ln \frac{\mu^2}{Q^2}, \;\; \ln \frac{\mu^2}{\lambda^2 Q^2}, \;\; \ln \frac{\mu^2}{\lambda^4 Q^2},$$

- They must be evaluated at a common scale, but there is no common scale that avoids large logs. Any choice can at best remove one source of large logarithms:

$$\mu_h \sim Q \quad \mu_J \sim Q\lambda \quad \mu_s \sim Q\lambda^2$$

- So we solve the RGEs and evolve our functions from their natural scales instead:

$$\frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln\left(\frac{Q^2}{\mu^2}\right) + 4\gamma_H(\alpha_s)\right] H(Q^2,\mu)$$

Motivation for Automation

- We need to compute anomalous dimensions and finite terms to some perturbative order required to achieve any given desired logarithmic accuracy, like 2-loop for NNLL

- So far we proceed observable by observable individually:

  - Thrust
    [Becher, Schwartz, '08]
  - C-Parameter
    [Hoang, Kolodrubetz, Mateu, Stewart, '14]
  - Angularities
    [Bell, Hornig, Lee, Talbert, WIP]
  - …
  - Threshold Drell-Yan
    [Becher, Neubert, Xu, '07]
  - W/Z/H @ large p_T
    [Becher, Bell, Lorentzen, Marti, '13,'14]
  - Jet veto
    [Becher et al. '13, Stewart et al., '13]
  - …

- Can this be made more systematic? It's possible in full QCD…

Automation in QCD: CAESAR/ARES: [Banfi, Salam, Zanderighi, '04], [Banfi, McAslan, Monni, Zanderighi, '14]

Universal dijet soft functions

- The generic form of the dijet soft function we get from the factorisation:

$$S(\tau,\mu) = \frac{1}{N_c}\sum_X \mathcal{M}(\tau,k_i)\ \mathrm{Tr}\ |\langle 0|S^\dagger_{\bar{n}}(0)S_n(0)|X\rangle|^2 \qquad S_n(x) = Pexp(ig_s\int_{-\infty}^{0} n\cdot A_s(x+sn)ds)$$

- The **matrix element** is *independent of the observable* and is the source of divergences

- The **measurement function** (*M*) is *observable dependent* and harmless, e.g.

$$\mathcal{M}_{thrust}(\tau,\{k_i\}) = \exp\left(-\tau\textstyle\sum_i \min(k_i^+,k_i^-)\right)$$ (in Laplace space)

- **Idea**: isolate singularities at each order and calculate the associated coefficient numerically:

$$\bar{S}(\tau) \sim 1+\alpha_s\{\frac{c_2}{\epsilon^2}+\frac{c_1}{\epsilon^1}+c_0\}+\mathcal{O}(\alpha_s^2)$$

Universal soft functions: NLO

- The virtual corrections are scaleless in dim reg, so the NLO soft function is:

$$S^{(1)}(\tau,\mu) = \frac{\mu^{2\varepsilon}}{(2\pi)^{d-1}} \int \delta(k^2)\ \theta(k^0)\ \frac{16\pi\alpha_s C_F}{k_+ k_-}\ \mathcal{M}(\tau,k)\ d^d k$$

- To disentangle the soft and collinear divergences we substitute:

$$k_- \to \frac{k_T}{\sqrt{y}} \qquad k_+ \to k_T\sqrt{y}$$

- We also must specify the measurement function *M*. We assume its form:

$$\mathcal{M}^{(1)}(\tau,k) = \exp\left(-\tau\, k_T\, y^{\frac{n}{2}}\ f(y,\vartheta)\right)$$

Measurement functions: NLO examples

$$\mathcal{M}^{(1)}(\tau, k) = \exp\left(-\tau\, k_T\, y^{\frac{n}{2}}\; f\,(y, \vartheta)\right)$$

Observable	n	$f(y,\vartheta)$
Thrust	1	1
Angularities	$1-A$	1
Recoil-free broadening	0	$1/2$
C-Parameter	1	$1/(1+y)$
Threshold Drell-Yan	-1	$1+y$
W @ large p_T	-1	$1+y-2\sqrt{y}\,\cos\theta$
e^+e^- transverse thrust	1	$\frac{1}{s\sqrt{y}}\left(\sqrt{\left(c\cos\theta+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)\frac{s}{2}\right)^2+1-\cos^2\theta}-\left\lvert c\cos\theta+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)\frac{s}{2}\right\rvert\right)$

- For transverse thrust, $s=\sin\theta_B$, $c=\cos\theta_B$, with $\theta_B = \angle$beam axis, thrust axis

Universal soft functions: NLO master formula

- The k_T integration can now be performed analytically

- The NLO Master soft function then reads:

$$S^{(1)}(\tau,\mu) \sim \Gamma(-2\epsilon) \int_0^\pi \mathrm{d}\vartheta \int_0^1 \mathrm{d}y \; y^{-1+n\epsilon} f(y,\vartheta)^{2\epsilon}$$

- To extract the singularity, subtract and leave a plus-distribution behind:

$$\int_0^1 \mathrm{d}y\, y^{-1+n\epsilon}\, g(y) = \int_0^1 \mathrm{d}y\, y^{-1+n\epsilon} \left[g\left(y\right) - g\left(0\right) + g\left(0\right)\right]$$

$y^{-1+n\epsilon}\, g(y)$ → divergent

$g(y) - g(0)$ →
- finite/O(y)
- expand in ϵ
- integrate numerically

$g(0)$ → $\sim \dfrac{1}{n\epsilon}$
- singularity isolated

Assumptions and classification: NLO

- *Assume*: Exponential function, motivated by Laplace space

$$\exp(-\tau\omega(\{k_i\})) = \int_0^\infty d\omega \exp(-\tau\omega)\,\delta(\omega - \omega(\{k_i\}))$$

- *Assume*: ω is *linear* in mass dimension

$$\mathcal{M} = \exp(-\tau k_T \hat{f}(y, \vartheta))$$

- *Classify*: How does *f* behave as *y* vanishes?

$$\mathcal{M} = \exp(-\tau k_T y^{\frac{n}{2}} f(y, \vartheta))$$

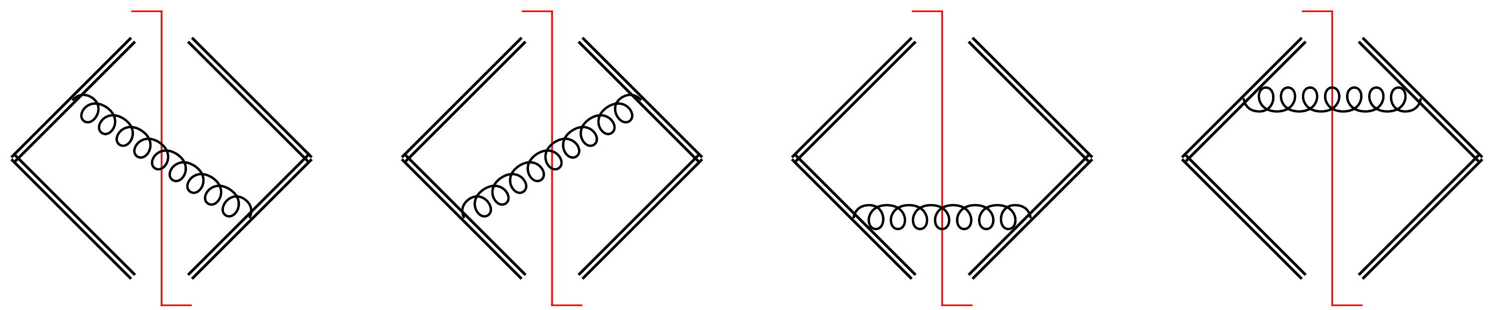
- Combined with Infrared and collinear safety (IRC) this is enough to ensure the behaviour of the observable is under control in the critical limits:

Soft	$(k_T \to 0) \Rightarrow$	vanishes, fixed by mass dimension
Collinear	$(y \to 0) \Rightarrow$	*f* finite

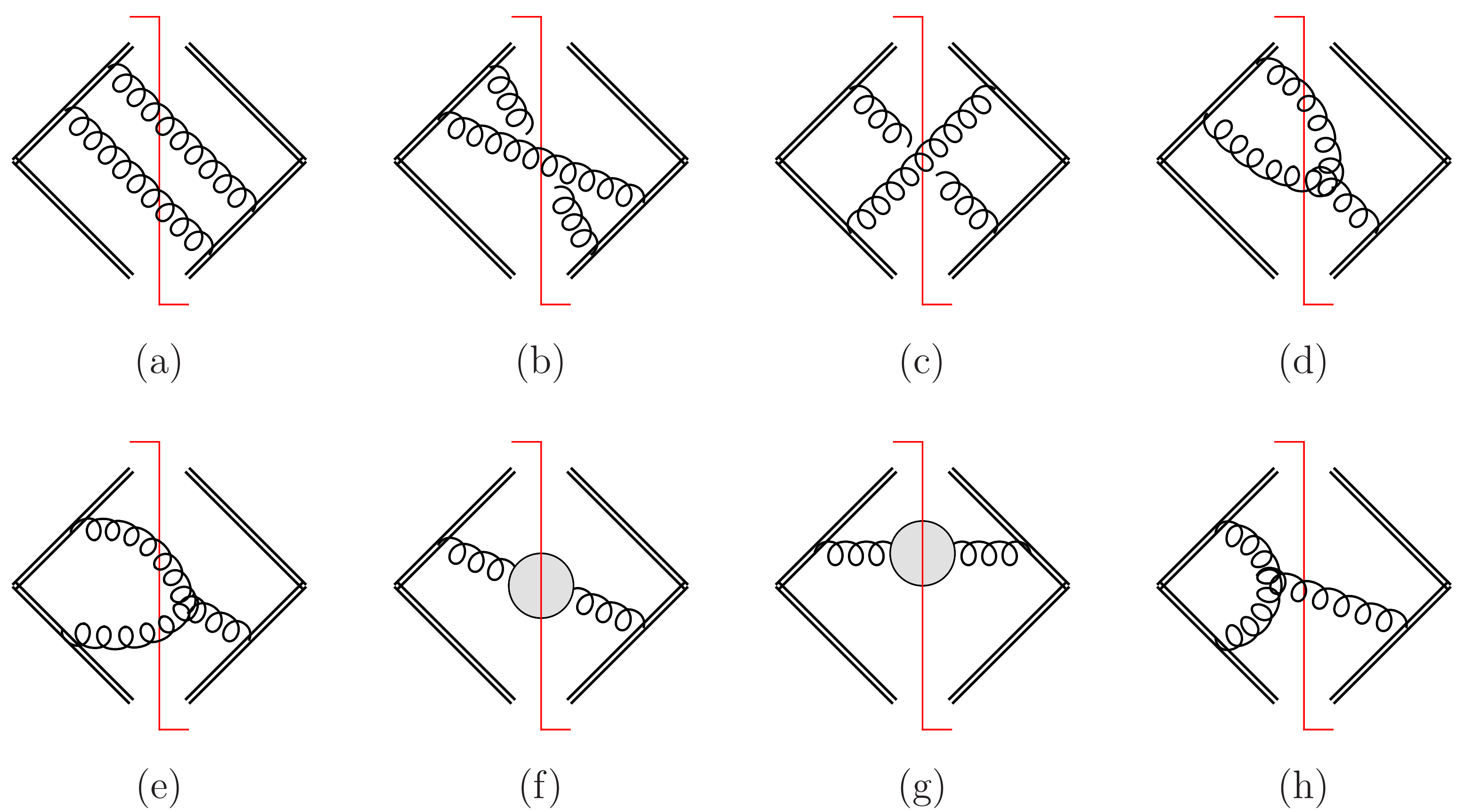
- *Also Assume*: *f* non-vanishing over almost all of phase space

Automation: NLO vs. NNLO

NLO:



NNLO:



Automation: NLO vs. NNLO

- Consider the double real emission:

$$S^2_{RR}(\tau) = \frac{\mu^{4\epsilon}}{(2\pi)^{2d-2}} \int d^dk\ \delta(k^2)\ \theta(k^0) \int d^dl\ \delta(l^2)\ \theta(l^0)\ |\mathcal{A}(k,l)|^2\ \mathcal{M}(\tau,k,l)$$

- The matrix elements are no longer nice and easy, see e.g., the $C_F T_F n_f$ color structure:

$$|\mathcal{A}(k,l)|^2 = 128\pi^2\alpha_s^2 C_F T_F n_f \frac{2k\cdot l(k_- + l_-)(k_+ + l_+) - (k_- l_+ - k_+ l_-)}{(k_- + l_-)^2(k_+ + l_+)^2(2k\cdot l)^2}$$

- The singularities are partially overlapping, not as easy to extract, but it's possible
- We then again assume the form of the measurement function:

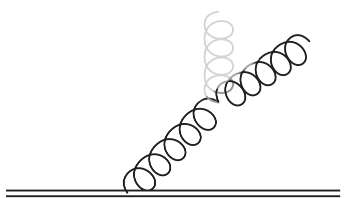
$$\mathcal{M}^{(2)}(\tau,k,l) = \exp\left(-\tau\, p_T\, y^{\frac{n}{2}}\ F\left(y,a,b,\vartheta,\vartheta_k,\vartheta_l\right)\right)$$

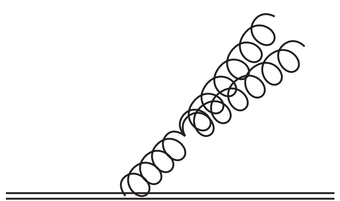
- Why is this enough?

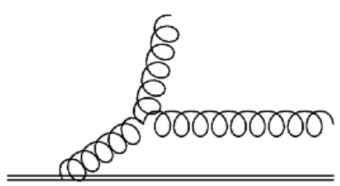
2-loop - Correlated emissions: $C_F C_A$, $C_F T_f n_f$

- Matrix element divergent in four critical limits: Behaviour

 (Global soft) — fixed by mass dimension

 (Individual soft) — fixed by IR safety

 (emissions collinear) — fixed by collinear safety

 ("jet-collinear") — unconstrained ➜ Classify!

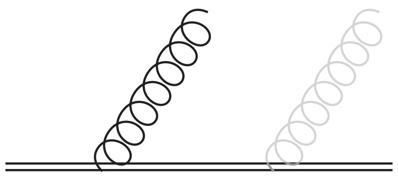
- Only one unconstrained variable
- Variable definition ensures commuting limits

$$\mathcal{M}^{(2)}(\tau, k, l) = \exp\left(-\tau\, p_T\, y^{\frac{n}{2}}\, F\left(y, a, b, \vartheta, \vartheta_k, \vartheta_l\right)\right)$$

2-loop - Uncorrelated emissions: C_F^2

- 4 critical limits: Behaviour

(Global soft) fixed by mass dimension



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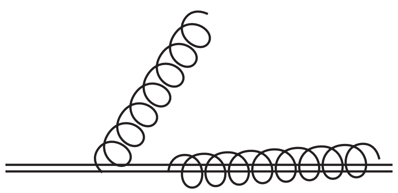
(

(one emission "jet-collinear") unconstrained)^2

- Two "unconstrained" limits
- Worse: *Overlapping zeroes*:

$$\omega(k,l) = k_T\, y_k^{\frac{n}{2}}\, f(y_k) + l_T\, y_l^{\frac{n}{2}}\, f(y_l)$$

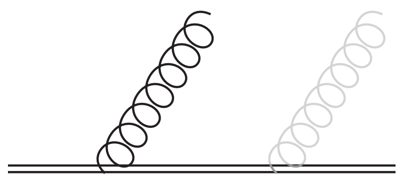
- Solution: adapt parametrisation for k_T, l_T:

$$k_T = q_T \frac{b}{1+b}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n,\; l_T = q_T \frac{1}{1+b}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n$$

2-loop - Uncorrelated emissions: C_F^2

- 4 critical limits: — Behaviour

(Global soft) — fixed by mass dimension



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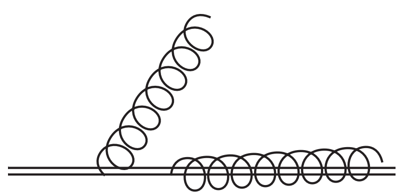
(

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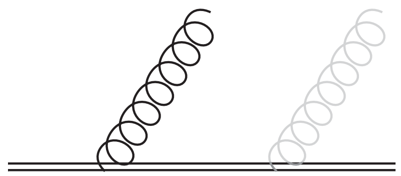
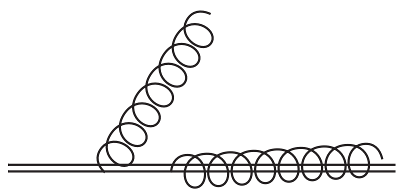
2-loop - Uncorrelated emissions: C_F^2

- 4 critical limits: Behaviour

(Global soft) — fixed by mass dimension



(individual soft) — fixed by IR safety

$\Big($ 

(one emission "jet-collinear") — unconstrained $\Big)^2$

- Suitable parametrisation solves overlapping limit problem

$$\mathcal{M}^{(2)}(\tau;k,l) = \exp\left(-\tau\, q_T\, y_k^{\frac{n}{2}}\, y_l^{\frac{n}{2}}\; G(y_k, y_l, b, \vartheta, \vartheta_k, \vartheta_l)\right)$$

The completed framework

- Semi-Analytic expressions are available for anomalous dimensions
  [1805.12414]

- For finite parts and anomalous dimensions implementations exist using *pySecDec* and a dedicated C++ based program:
  [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke, 1703.09692]

(Soft function Simulation and Evaluation for Real and Virtual Emissions)

- `SoftSERVE` uses the *Cuba* library's *Divonne* integrator, implements numerical improvements, and has multiprecision variable support
  [Hahn, hep-ph/0404043]

- Correlated emission variant online on `HEPForge`: [1812.08690]

SoftSERVE

- First release (0.9) has correlated emissions only
- User required input (C++ syntax):
  (a) correlated: two functions F (roughly: one for each hemisphere)
  (b) uncorrelated: three functions G
  (c) two parameters
  (d) optional: parameter values, integrator settings
- Two integrator settings pre-defined
- Scripts available for renormalisation in Laplace (and later momentum) space, and for dealing with Fourier space observables
- For now: limited to *collinear anomaly* framework (more later)

Results: $\mathrm{SCET_I}$

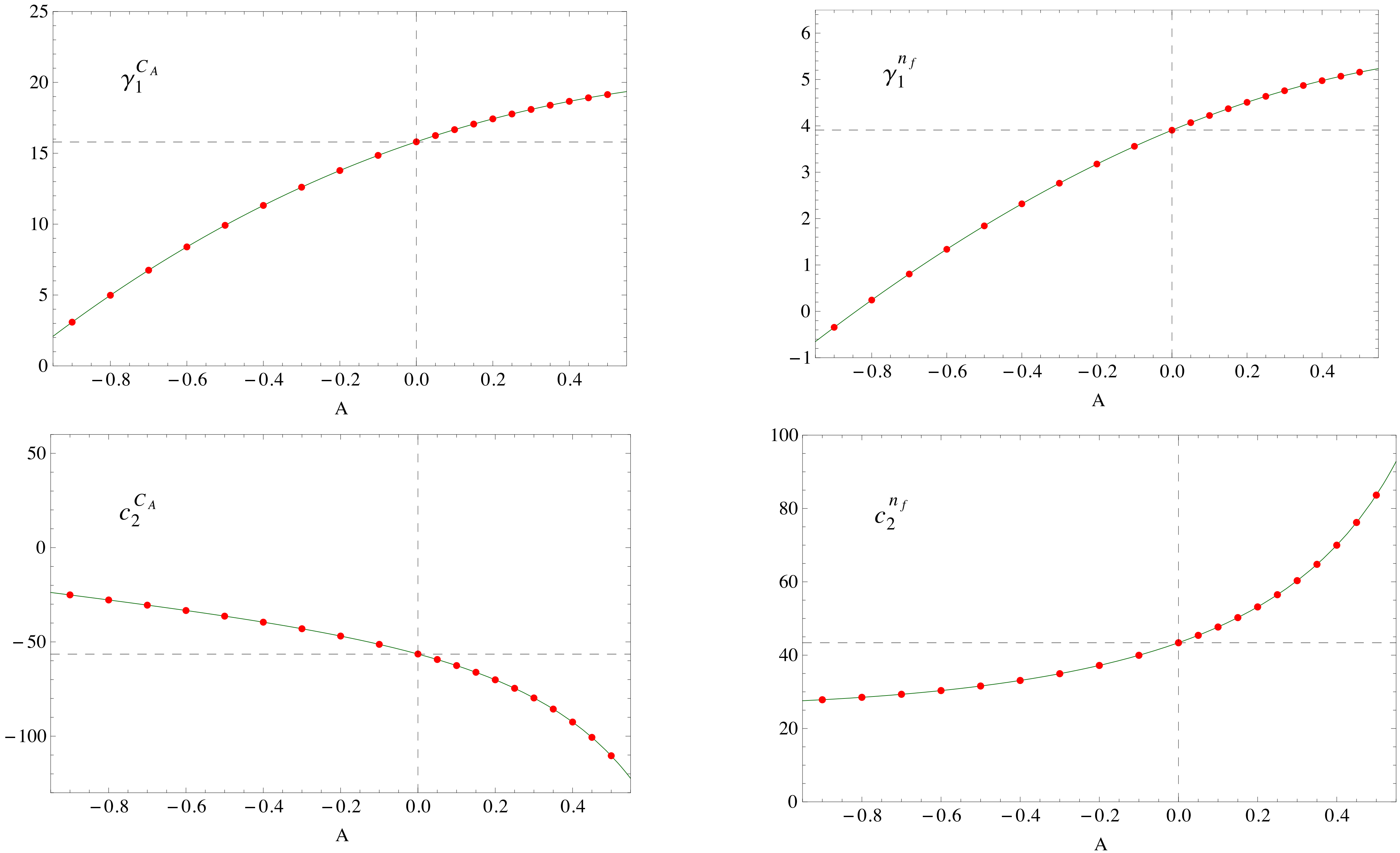
Soft function	$\gamma_1^{C_A}$	$\gamma_1^{n_f}$	$c_2^{C_A}$	$c_2^{n_f}$
Thrust [Kelley et al, '11] [Monni et al, '11]	15.7945 (15.7945)	3.90981 (3.90981)	−56.4992 (−56.4990)	43.3902 (43.3905)
C-Parameter [Hoang et al, '14]	15.7947 (15.7945)	3.90980 (3.90981)	−57.9754 [-58.16 ± 0.26]	43.8179 [43.74 ± 0.06]
Threshold Drell-Yan [Belitsky, '98]	15.7946 (15.7945)	3.90982 (3.90981)	6.81281 (6.81287)	−10.6857 (−10.6857)
W @ large p_T [Becher et al, '12]	15.7947 (15.7945)	3.90981 (3.90981)	−2.65034 (−2.65010)	−25.3073 (−25.3073)
Transverse Thrust [Becher, Garcia, Piclum, '15]	−158.278 [$-148\pm^{20}_{30}$]	19.3955 [$18\pm^{2}_{3}$]	—	—

$$\gamma_1 = \gamma_1^{C_A} C_F C_A + \gamma_1^{N_f} C_F T_F n_f \qquad c_2 = c_2^{C_A} C_F C_A + c_2^{N_f} C_F T_F n_f + \frac{1}{2}(c_1)^2$$

- Derived in few minutes to hours on an 8 core desktop machine
- Deviations from analytic results compatible with 1σ error estimate

Results: Angularities

- Generalisation of thrust
- Obeys non-abelian observation
- New result, used in the NNLL′ resummation in [1808.07867, Bell, Hornig, Lee, Talbert]



Interesting physics I: SCET-1

- Formulae for anomalous dimensions can be derived
  - Simpler than full bare calculation (fewer integrations)
  - Only forward limit contributes
  - Many observables share anomalous dimensions (e.g. Threshold Drell-Yan, Thrust, C-Parameter, W at large p_T,…)
  - Remaining observable dependence via mode choice, parameter n

- For simple cases, formulae for finite parts are possible
  - C-Parameter matching corrections now available analytically

$$c_2 = \left(-\frac{2212}{81} - \frac{67\pi^2}{54} + \frac{13\pi^4}{15} - \frac{770\zeta_3}{9}\right) C_F C_A + \left(\frac{224}{81} + \frac{10\pi^2}{27} + \frac{280\zeta_3}{9}\right) C_F T_F n_f + \frac{\pi^4}{2} C_F^2$$

Interesting physics II: SCET-2

- Recall: For SCET-2 observables, soft and collinear scale match
    - ➡ second regulator required
    - ➡ resummation via *collinear anomaly* or *rapidity renormalisation group*
- RRG assumes regularisation of *connected webs*
- We're using phase space regularisation, ***not*** c-webs
    - ➡ A priori (and unexpectedly) incompatible with RRG

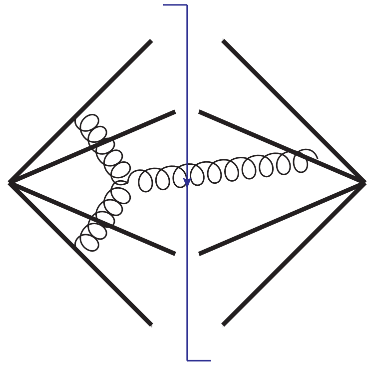
- Collinear anomaly exponent (rapidity anomalous dimension) formulae can be derived and is ***very*** similar to SCET-1 anom. dim.
    - ➡ Can be extrapolated from family of SCET-1 observables (e.g. angularities)
    - ➡ Only difference related to running coupling (1-loop term)
- No such connection obvious for matching corrections

Extension to N jet directions

There are now more jet/beam directions -> more Wilson lines:

$$S(\tau,\mu) = \sum_X \mathcal{M}(\tau,k_i)\,\langle 0|\,(S_{n_1}S_{n_2}S_{n_3}\ldots)^\dagger\,|X\rangle\langle X|S_{n_1}S_{n_2}S_{n_3}\ldots|0\rangle$$

- Tripole and Quadrupole diagrams
  - ✓ Assume non-abelian exponentiation: only one tripole (RV)



- Dipole directions are no longer back-to-back
  - ✓ Use boost-invariant parametrisation
  - ✓ Consequence: transverse temporal direction

$$k = (k\cdot n, k\cdot\bar{n}, k_\perp)$$

$$k = (k\cdot n_a, k\cdot n_b, k_\perp)$$

- more complicated angular integrations
  - ✓ 5 angles instead of 3 at NNLO
  - ✓ Higher poles appear, multiprecision needed
- external geometry must be sampled

N-Jettiness

[Stewart, Tackmann, Waalewijn, ’10]

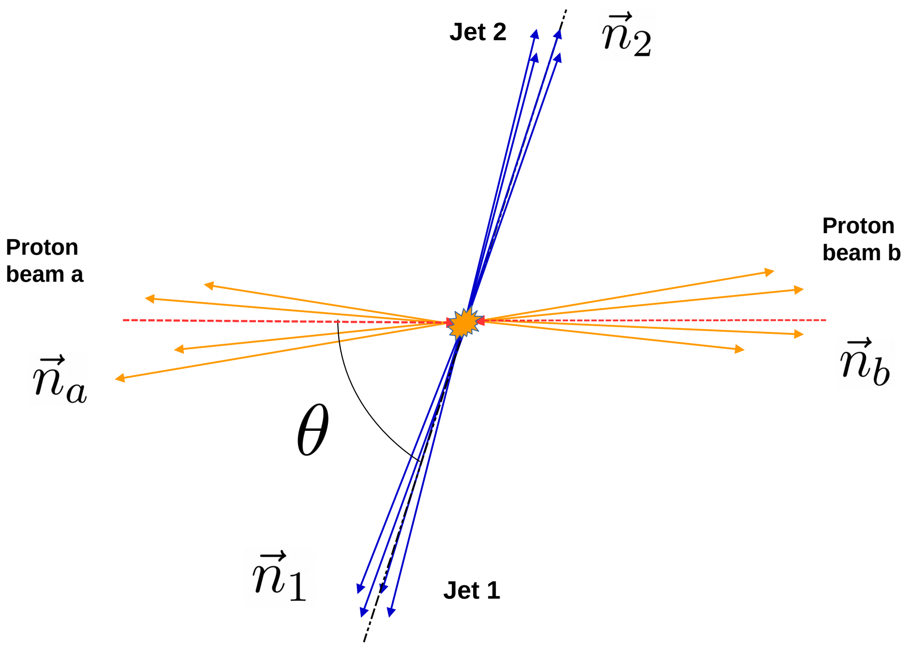
- Recall Thrust: “How much does the event look like 2 jets?”
- N-jettiness: “How much does the event look like (N+2) jets/beams?”

$$\mathcal{T}_N\left(\{k_i\}\right) = \sum_i \min_j \, n_j \cdot k_i \qquad j = \underbrace{a, b}_{beams}, \underbrace{1, ..., N}_{jets}$$

- 1-Jettiness soft function known to NNLO [Boughezal, Liu, Petriello, ’15; Campbell, Ellis, Mondini, Williams, ‘17]
- (N>1)-Jettiness soft function known to NLO [Jouttenus, Stewart, Tackmann, Waalewijn, ’11]

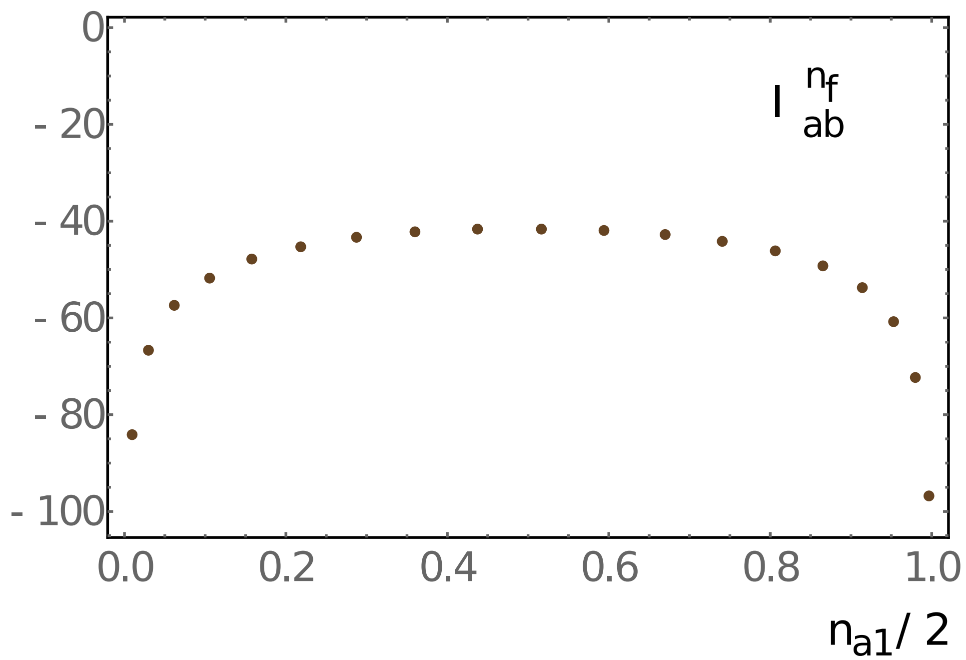
2-Jettiness

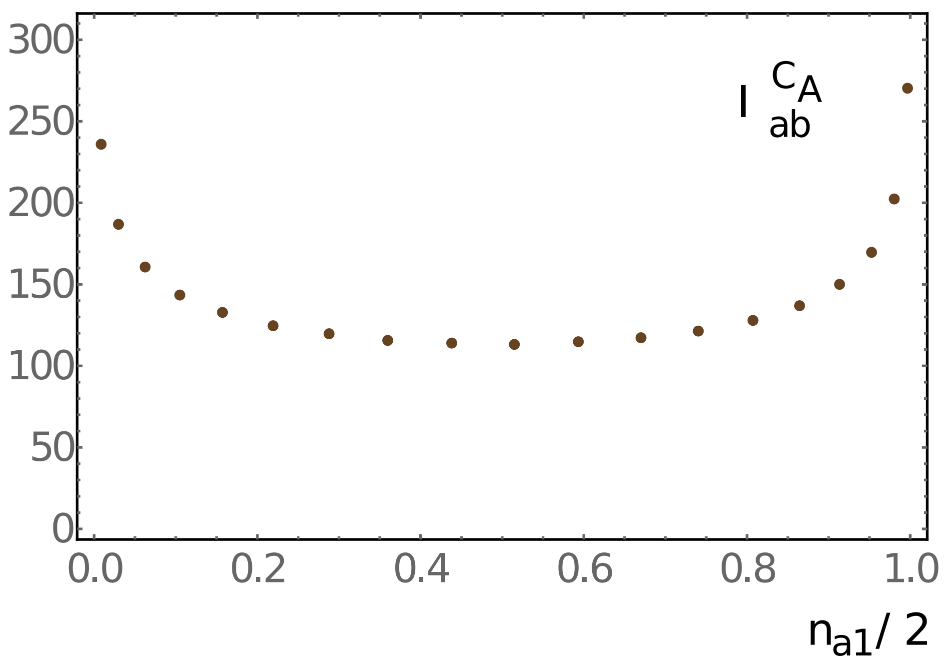
- Kinematics and Sampling



$$n_a \cdot n_b = n_1 \cdot n_2 = 2$$
$$n_a \cdot n_1 = n_b \cdot n_2 = 1 - \cos\theta = n_{a1}$$
$$n_a \cdot n_2 = n_b \cdot n_1 = 1 + \cos\theta$$

- Some preliminary results - dipoles





Conclusion

- SCET provides an efficient, analytic approach to high-order resummations necessary for precision collider physics.

- We have developed a framework to systematically compute generic *NNLO* dijet soft functions for wide ranges of observables at lepton and hadron colliders

- The program(s) based on this framework are being released into the wild

- An extension to N-jet observables seems possible, and we have already re-derived a few known results and are working on new ones

- Work remains to be done there (error bars, performance,…)

That's all folks!

Thank you!

Parametrisation uncorrelated

The parametrisation for the uncorrelated emissions

$$k_+ = q_T \frac{b}{1+b}\sqrt{y_k}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n \qquad k_- = q_T \frac{b}{1+b}\frac{1}{\sqrt{y_k}}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n$$

$$l_+ = q_T \frac{1}{1+b}\sqrt{y_l}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n \qquad l_- = q_T \frac{1}{1+b}\frac{1}{\sqrt{y_l}}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n$$

leads to divergences in b, y_k, y_l, q_T (analytic)

$$y_k = \frac{k_+}{k_-} \qquad b = \sqrt[1+n]{\frac{k_+k_-}{l_+l_-}}\left(\frac{l_+ + l_-}{k_+ + k_-}\right)^n$$

$$y_l = \frac{l_+}{l_-} \qquad q_T = \sqrt{l_+l_-}\left(\frac{k_+ + k_-}{\sqrt{k_+k_-}}\right)^n + \sqrt{k_+k_-}\left(\frac{l_+ + l_-}{\sqrt{l_+l_-}}\right)^n$$

Parametrisation correlated

The parametrisation for the correlated emissions

$$k_+ = p_T \frac{b}{a+b}\sqrt{y} \qquad k_- = p_T \frac{ab}{1+ab}\frac{1}{\sqrt{y}}$$

$$l_+ = p_T \frac{a}{a+b}\sqrt{y} \qquad l_- = p_T \frac{1}{1+ab}\frac{1}{\sqrt{y}}$$

leads to divergences in y, b, p_T (analytic), and an overlapping divergence in $a \to 1$ (with transverse angle)

$$a = \sqrt{\frac{k_- l_+}{l_- k_+}} = \sqrt{\frac{y_l}{y_k}} \qquad b = \sqrt{\frac{k_+ k_-}{l_+ l_-}} = \frac{k_T}{l_T}$$

$$y = \frac{k_+ + l_+}{k_- + l_-} \qquad p_T = \sqrt{(k_+ + l_+)(k_- + l_-)}$$