RESUMMATION FOR BOOSTED TOP-QUARK PAIR PRODUCTION

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WHY TOP?



Top is special because of large mass $m_t \sim 173~{
m GeV}$

- plays special role in many BSM models
- decays before hadronizing and losing spin information: "free quark" [Bigi, Dokshitzer, V.A. Khoze, Kuhn, Zerwas '86]
- $\alpha_s(m_t) \ll 1$: can use perturbation theory

Why top now?

1) Tevatron: Top is relatively unstudied ($\sigma_{t\bar{t}X} \sim 7 \text{pb}$)

- discovered 20 years ago at Tevatron but limited statistics on many measurements
- 1 pair per day produced

2) LHC: Top is everywhere ($\sigma_{t\bar{t}X} \sim 250$ pb at LHC8, ~ 800 pb at LHC13)

- top sample at LHC has long surpassed that at Tevatron
- 1 pair per second at LHC13

The LHC is a top factory

The assembly line of top properties



BOOSTED TOP PRODUCTION

Tevatron $\sqrt{s} \approx 2 \ TeV$ LHC: $\sqrt{s} = 7 \ TeV$ $1/\sigma_{\rm tf}^{\rm td} d\sigma_{\rm tf}^{\rm td} dm_{\rm tf}^{\rm tf} [1/{\rm TeV}]$ 100data NLO (MCFM) $\sqrt{s} = 1.96 \, \text{TeV}$ NLO + NNLL $d\sigma/dM_{t\bar{t}}$ [fb/GeV] 10 ATLAS $\int L dt = 2.05 \text{ fb}^{-1}$ 10-3 **NNLO/Data NLO/Data** 0.1 0.5 CDF data 1.5 NLO + NNLL 0.01 200 400 600 800 1000 1200 0.5 $M_{t\bar{t}}$ [GeV] 2000 300 1000 m, [GeV]

more and more LHC results in "boosted regime" M_{tt̄}, p^t_T ≫ m_t
not just "corner of phase space": important for new physics searches

QCD RESUMMATION FOR BOOSTED TOPS

My talk is about precision QCD calculations for boosted top production $(m_t \ll M_{t\bar{t}} \text{ or } m_t \ll p_T)$

Such calculations necessarily involve a combination of

- finesse (effective field theory to deal with multiple scales) [Broggio, Ferroglia, Marzani, BP, Scott, Yang, Wang: arXiv:1205.3662, arXiv:1207.4798, arXiv:1306.1537, arXiv:1310.3836, arXiv:1409.5294, arXiv:1601.07020 (PRL)]
- brute force (NNLO+NNLL QCD calculations for precision)
 [Czakon, Ferroglia, Mitov, BP, Scott, Yang, Wang arXiv:1803.07623]
- and combination with EW for complete description in SM [above plus Pagani, Papanastasiou, Tsinikos arXiv:1901.0828...]

- top pair production: the basics
- factorization in soft and boosted soft limits
- resummation
- comparison with data

FACTORIZATION FOR INCLUSIVE PRODUCTION

Factorization for $h_1h_2 \rightarrow t\bar{t}X$:

$$d\sigma_{h_1,h_2}^{t\bar{t}X} = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^{h_1}(x_1,\mu_{\mathsf{F}}) f_j^{h_2}(x_2,\mu_{\mathsf{F}}) d\hat{\sigma}_{ij}\left(\hat{s},m_t,\ldots,\alpha_{\mathsf{s}}(\mu_{\mathsf{R}}),\mu_{\mathsf{F}},\mu_{\mathsf{R}}\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_t}\right)$$

$$s = (p_{h_1} + p_{h_2})^2$$
, $\hat{s} = x_1 x_2 s$

Strategy:

- take PDFs from data (PDF set collaborations)
- calculate partonic cross sections $d\hat{\sigma}_{ii}$ in QCD (Feynman diagrams)
- simplest scheme is fixed-order perturbation theory

$$d\hat{\sigma}_{ij} = \alpha_s^2 d\hat{\sigma}_{ij}^{(0)} + \alpha_s^3 d\hat{\sigma}_{ij}^{(1)} + \dots$$

Resummation for differential cross sections

Many differential observables involve two hard scales

 $Q \gg M \gg \Lambda_{
m QCD}$

- if $\alpha_s \ln \frac{Q}{M} \sim 1$, must "resum" logarithmic corrections to get a sensible answer for $d\hat{\sigma}$
- "resummation" = re-organization of large logarithms in perturbative expansion. Defining $L = \ln \left(\frac{Q}{M}\right)$:



will argue such resummation is important for boosted top production

$t\bar{t}$ LO partonic cross section



• LO squared matrix elements (color and spin summed/averaged)

$$< |M_{q\bar{q}}|^{2} > = \frac{4\alpha_{s}(\mu)^{2}}{9} \left[\frac{t_{1}^{2}}{\hat{s}^{2}} + \frac{u_{1}^{2}}{\hat{s}^{2}} + \frac{m_{t}^{2}}{\hat{s}} \right]$$

$$< |M_{q\bar{q}}|^{2} > = \alpha_{s}(\mu)^{2} \left[\left(\frac{\hat{s}^{2}}{6u_{1}t_{1}} - \frac{3}{8} \right) \left(\frac{t_{1}^{2}}{\hat{s}^{2}} + \frac{u_{1}^{2}}{\hat{s}^{2}} + \frac{4m_{t}^{2}}{\hat{s}} - \frac{4m_{t}^{4}}{t_{1}u_{1}} \right) \right]$$

- to be combined with phase space measure and flux factor to get $d\hat{\sigma}$
- gg channel dominates at LHC due to PDFs

NNLO CALCULATIONS

$$d\hat{\sigma}_{t\bar{t}+X}^{\mathrm{NNLO}} = d\hat{\sigma}^{\mathrm{VV}} + d\hat{\sigma}^{\mathrm{RV}} + d\hat{\sigma}^{\mathrm{RR}}$$

- the total $pp \rightarrow t\bar{t}X$ cross section is now known to NNLO [Czakon, Fiedler, Mitov '13]
- the FB asymmetry in $p\bar{p} \rightarrow t\bar{t}X$ cross section is now known to NNLO [Czakon, Fiedler, Mitov '14]
- arbitrary differential cross sections now known at NNLO [Czakon, Heymes, Mitov '15, '16]

NNLO is now the baseline for QCD in top-pair production

TOTAL CS AT NNLO



Impressive agreement between NNLO and ATLAS/CMS for total CS

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Scale choices for distributions

The difficulty for fixed-order calculations: multiplescale process with complicated kinematics!



top quark mass p⊤ of top p⊤ of anti-top rapidity of top rapidity of anti-top Invariant mass M_{tt}

. . .

Which (combination) should be used for the renormalization/factorization scales?

NNLO WITH DYNAMIC SCALES

Czakon, Heymes, Mitov: 1606.03350

Determine optimal "scale scheme" by minimizing higher order corrections

$$\begin{split} & \mu_0 \sim m_t \;, \\ & \mu_0 \sim m_T = \sqrt{m_t^2 + p_T^2} \;, \\ & \mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \;, \\ & \mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \;, \\ & \mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \;, \\ & \mu_0 \sim E_T = \sqrt{\sqrt{m_t^2 + p_{T,t}^2} \sqrt{m_t^2 + p_{T,\bar{t}}^2} \;, \\ & \mu_0 \sim H_{T,\text{int}} = \sqrt{(m_t/2)^2 + p_{T,t}^2} + \sqrt{(m_t/2)^2 + p_{T,\bar{t}}^2} \;, \\ & \mu_0 \sim m_{t\bar{t}} \;, \end{split}$$

$mu \sim H_T$ scale for M_{tt} distribution!

NNLO WITH DYNAMIC SCALES



Vastly different behaviors with different scales, even at NNLO and especially for p_T , $M_{tt} >> m_t$ (boosted regime)

LARGE CORRECTIONS IN BOOSTED PRODUCTION



QCD MADE SIMPLER

Interplay of soft and collinear emissions is characteristic for highenergy processes. In both limits interactions simplify:

Collinear limit, where multiple particles move in a similar directions

• Soft limit, in which particles with small energy and momentum are emitted. Eikonal interactions.



At the same time the cross sections are enhanced in these regions.

EFFECTIVE DIAGRAMMATIC FACTORIZATION: SOFT-COLLINEAR EFFECTIVE THEORY (SCET)

The simple structure of soft and collinear emissions forms the basis of the classic factorization proofs, which were obtained by analyzing Feynman diagrams.

Collins, Soper, Sterman, ...

Advantages of the the SCET approach:

- Simpler to exploit gauge invariance on the Lagrangian level
- Operator definitions for the soft and collinear contributions
- Resummation with renormalization group
- Can include power corrections



All that for \$50?!

Introduction to Soft-Collinear Effective Theory (Lecture Notes in Physics) 2015th Edition

by Thomas Becher (Author), Alessandro Broggio (Author), Andrea Ferroglia (Author)

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Factorization in the soft limit $(M \equiv M_{t\bar{t}})$

Soft limit:
$$z \equiv rac{M_{t\bar{t}}^2}{\hat{s}} o 1$$
 $\hat{s}, \hat{t}_1, m_t^2 \gg \hat{s}(1-z)^2$

Partonic cross section factorizes [Kidonakis, Sterman '97]

$$d\hat{\sigma}_{ij} = \operatorname{Tr}\left[\mathbf{H}_{ij}^{m}(M, m_{t}, \cos\theta, \mu_{f}) \mathbf{S}_{ij}^{m}(\sqrt{\hat{s}}(1-z), M, m_{t}, \cos\theta, \mu_{f})\right] + \mathcal{O}(1-z)$$

H^m_{ij} are color-space matrices related to virtual corrections
 S^m_{ij} are color-space matrices related to real emission in soft limit Soft corrections involve δ(1 - z) or

$$\alpha_s^n \left[\frac{\ln^m (1-z)}{1-z} \right]_+; \quad m = 0, \cdots, 2n-1$$

HARD-SOFT FACTORIZATION (QUARK CHANNEL)



(b)

$$T_{I} \sim C_{I}(M, m_{t}, ...) \times \bar{\chi}_{n_{2}}^{a_{2}} \chi_{n_{1}}^{a_{1}} \bar{\chi}_{n_{3}}^{a_{3}} \chi_{n_{4}}^{a_{4}} \times c_{I}^{\{a\}} \times O_{S,I}$$

$$(c_1^{q\bar{q}})_{\{a\}} = \delta_{a_1 a_2} \delta_{a_3 a_4}, \quad (c_2^{q\bar{q}})_{\{a\}} = t_{a_2 a_1}^c t_{a_3 a_4}^c,$$

$$d\hat{\sigma} \sim \sum_{I,J} C_I S_{IJ} C_J^* \equiv \text{Tr}[\boldsymbol{H} \boldsymbol{S}]$$

Boosted soft limit: $M \gg m_t \gg M(1-z)$

$$d\hat{\sigma}_{ij} = \operatorname{Tr}\left[\mathsf{H}_{ij}^{m}(M, m_{t}, \cos\theta, \mu_{f}) \mathbf{S}_{ij}^{m}(\sqrt{\hat{s}}(1-z), M, m_{t}, \cos\theta, \mu_{f})\right] + \mathcal{O}(1-z)$$

Can no longer use formula derived in soft limit, because both \mathbf{H}_{ij}^m and $\tilde{\mathbf{s}}_{ij}^m$ contain large logs of form $\ln(m_t/M)$ in the boosted soft limit

However, these small-mass logs are of collinear origin: can understand and factorize them

Factorized hard function in $m_t^2/\hat{s} \ll 1$ limit

Three pieces

$$|\mathcal{M}(\epsilon, M, m_t, t_1)\rangle = Z_{[q]}(\epsilon, m_t, \mu) |\mathcal{M}(\epsilon, M, t_1)\rangle \quad [\mathsf{Mitov}/\mathsf{Moch}]$$
$$\lim_{\epsilon \to 0} \mathsf{Z}_m^{-1}(\epsilon, M, m_t, t_1, \mu) |\mathcal{M}(\epsilon, M, m_t, t_1)\rangle = |\mathcal{M}_{\mathsf{ren}}(M, m_t, t_1, \mu)\rangle \quad [\mathsf{Neubert}/\mathsf{Becher}]$$
$$\lim_{\epsilon \to 0} \mathsf{Z}^{-1}(\epsilon, M, t_1, \mu) |\mathcal{M}(\epsilon, M, t_1)\rangle = |\mathcal{M}_{\mathsf{ren}}(M, t_1, \mu)\rangle \quad [\mathsf{Catani; Neubert}/\mathsf{Becher}]$$

Combine to get a relation between Z factors and finite matching function C_D [Ferroglia, BP, Yang '12]

$$Z_{[q]}(\epsilon, m_t, \mu) \mathbf{Z}_m^{-1}(\epsilon, M, m_t, t_1, \mu) = C_D(m_t, \mu) \mathbf{Z}^{-1}(\epsilon, M, t_1, \mu).$$

It follows that

$$\mathsf{H}^m_{ij}(M,m_t,t_1,\mu) = \mathcal{C}^2_D(m_t,\mu)\mathsf{H}_{ij}(M,t_1,\mu) + \mathcal{O}\left(rac{m_t^2}{\hat{s}}
ight) \,.$$

Factorized soft function in $m_t^2 \ll \hat{s}$ limit

Factorization of soft-function relies on analysis of phase-space integrals using method of regions [Ferroglia, BP, Marzani, Yang '13]

$$\begin{split} \mathbf{S}_{ij}^{m}(\omega,\hat{s},\hat{t}_{1},m_{t},\mu) &= \int d\omega_{s} \, d\omega_{d} \, d\omega_{d}' \, \delta(\omega-\omega_{s}-\omega_{d}-\omega_{d}') \\ &\times \mathbf{S}_{ij}\left(\omega_{s},\frac{\omega_{s}}{\sqrt{\hat{s}}},-\frac{t_{1}}{\hat{s}},\mu\right) \, S_{D}\left(\omega_{d},\frac{\omega_{d}m_{t}}{\hat{s}},\mu\right) \, S_{D}\left(\omega_{d}',\frac{\omega_{d}'m_{t}}{\hat{s}},\mu\right) \\ &+ \mathcal{O}(m_{t}\omega/\hat{s}) + \mathcal{O}(m_{t}^{2}/\hat{s}) \end{split}$$

- ${f S}_{ij}$ related to wide-angle soft emission to massless gg, qar q o ar Q Q
- S_D related to soft emission collinear to top or anti-top (soft part of heavy-quark fragmentation function)

 $d\hat{\sigma}_{ij} \sim \mathrm{Tr}[\mathbf{H}_{ij}(M,\mu)\mathbf{S}_{ij}(M(1-z),\mu)] \otimes C_D^2(m_t,\mu)S_D^2(m_t(1-z),\mu) + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$

All component functions known at NNLO

- C_D and S_D from fragmentation function for generic z [Melnikov, Mitov '04] (along with calculation of [Becher, Neubert '05])
- H_{ij} from virtual corrections to massless $gg, q\bar{q} \rightarrow \bar{Q}Q$ scattering [Glover et. al '00-'01] after IR renormalization procedure [Broggio, Ferroglia, BP, Zhang '14]
- S_{ij} from double real emission corrections to massless $gg, q\bar{q} \rightarrow \bar{q}'q'$ in soft limit [Ferroglia, BP, Yang '12]

$$d\hat{\sigma}_{ij} = \mathrm{Tr}\left[\mathbf{H}^m_{ij}(M, m_t, \cos{ heta}, \mu_f) \mathbf{S}^m_{ij}(\sqrt{\hat{s}}(1-z), M, m_t, \cos{ heta}, \mu_f)
ight] + \mathcal{O}(1-z)$$

- involves two one-scale functions, so large logs appear for any choice of μ_f
- standard EFT solution of deriving and solving RG equations resums logarithms in hard function

$$\boldsymbol{H}(\boldsymbol{M},\mu_f) = \boldsymbol{U}_H(\mu_f,\mu_h)\boldsymbol{H}(\boldsymbol{M},\mu_h \sim \boldsymbol{M})$$

 for technical reasons, resummation for soft function done in Mellin space (≈ Laplace space in soft limit)

Mellin transforms

Under Mellin transforms

$$\tilde{f}(N) = \mathcal{M}[f](N) = \int_0^1 dx \ x^{N-1}f(x); \ \mathcal{M}^{-1}[\tilde{f}](x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \ x^{-N}f(N)$$
$$d\tilde{\sigma}(N) = \sum_{ij} \widetilde{\mathcal{L}}_{ij}(N, \mu_f)_{ij} d\tilde{\sigma}_{ij}(N, M, m_t, \cos\theta, \mu_f)$$

• Factorization in Mellin space soft limit: $M, m_t \gg M/N$:

$$d\widetilde{\hat{\sigma}}_{ij} = \operatorname{Tr}\left[\mathbf{H}_{ij}^{m}(\boldsymbol{M}, \boldsymbol{m}_{t}, \cos\theta, \mu_{f})\widetilde{\mathbf{s}}_{ij}^{m}\left(\ln\frac{M^{2}}{\bar{N}^{2}\mu_{f}^{2}}, \boldsymbol{M}, \boldsymbol{m}_{t}, \cos\theta, \mu_{f}\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

• Mellin-space soft function has simple RG equation

$$\frac{d}{d\ln\mu}\widetilde{\mathbf{s}}_{ij}^{m} = -\left[\Gamma_{\mathrm{cusp}}\ln\frac{M^{2}}{\bar{N}^{2}\mu^{2}} - \gamma_{ij}^{m,h\dagger}\right]\widetilde{\mathbf{s}}_{ij}^{m} - \widetilde{\mathbf{s}}_{ij}^{m}\left[\Gamma_{\mathrm{cusp}}\ln\frac{M^{2}}{\bar{N}^{2}\mu^{2}} - \gamma_{ij}^{m,h}\right]$$

 can solve with standard RG techniques to get resummed partonic cross section in Mellin space

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Resummed cross section in soft limit

$$\begin{aligned} d\tilde{\sigma}_{ij}(\mu_f) &= \exp\left[\frac{4\pi}{\alpha_s(\mu_h)}g_1^m(\lambda,\lambda_f) + g_2^m(\lambda,\lambda_f) + \frac{\alpha_s(\mu_h)}{4\pi}g_3^m(\lambda,\lambda_f) + \dots\right] \\ &\times \operatorname{Tr}\left[\widetilde{\mathbf{u}}_{ij}^m(\mu_h,\mu_s)\mathbf{H}_{ij}^m(M,m_t,\cos\theta,\mu_h)\widetilde{\mathbf{u}}_{ij}^{m\dagger}(\mu_h,\mu_s)\widetilde{\mathbf{s}}_{ij}^m\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},M,m_t,\cos\theta,\mu_s\right)\right] \end{aligned}$$

$$\lambda \equiv \beta_0 \frac{\alpha_s(\mu_h)}{2\pi} \ln(\mu_h/\mu_s), \qquad \lambda_f \equiv \beta_0 \frac{\alpha_s(\mu_h)}{2\pi} \ln(\mu_h/\mu_f)$$
$$\widetilde{\mathbf{u}}_{ij}^m(\mu_h,\mu_s) \equiv \mathcal{P} \exp\left\{\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} \frac{d\alpha}{\beta(\alpha)} \gamma_{ij}^{m,h}(M,m_t,\cos\theta,\alpha)\right\}$$

• choosing $\mu_h \sim M$, $\mu_s \sim M/N$ resums all logs into exponentials

 inverse Mellin transforming (using Minimal prescription [Catani, Mangano, Nason, Trentadue '96]) gives resummed cross section Can derive and solve RG equations to get joint resummation formula [BP, Scott, Wang, Yang '16]

$$\begin{split} d\widetilde{\sigma}_{ij}(\mu_f) &= \operatorname{Tr}\left[\widetilde{\mathbf{U}}_{ij}(\mu_f, \mu_h, \mu_s) \mathbf{H}_{ij}(M, \cos \theta, \mu_h) \widetilde{\mathbf{U}}_{ij}^{\dagger}(\mu_f, \mu_h, \mu_s) \right. \\ & \left. \times \widetilde{\mathbf{s}}_{ij} \left(\ln \frac{M^2}{\bar{N}^2 \mu_s^2}, M, \cos \theta, \mu_s \right) \right] \times \widetilde{U}_D^2(\mu_f, \mu_{dh}, \mu_{ds}) \, \mathcal{C}_D^2(m_t, \mu_{dh}) \, \widetilde{\mathbf{s}}_D^2\left(\ln \frac{m_t}{\bar{N} \mu_{ds}}, \mu_{ds} \right) \\ & \left. + \mathcal{O}\left(\frac{1}{N}\right) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right) \end{split}$$

- remove large logs in matching functions through choices $\mu_h \sim M$, $\mu_s \sim M/N$, $\mu_{dh} \sim m_t \ \mu_{ds} \sim m_t/N$
- \tilde{U}_{ij} and \tilde{U}_D are RG factors which resum logs
- can estimate perturbative uncertainties by varying all scales independently

LOGARITHMIC ACCURACY: THE FRONTIER

Resummation involves anomalous dimensions $\mathsf{\Gamma},\gamma$ and matching functions

	Γ_{cusp}^{i}	γ_i	$oldsymbol{H},\widetilde{oldsymbol{s}},oldsymbol{c}_{D},\widetilde{oldsymbol{s}}_{D}$
NLL	2-loop	1-loop	0-loop
NNLL	3-loop	2-loop	1-loop
NNLL'	3-loop	2-loop	2-loop

Analytic frontier (differential cross sections):

- soft limit: NNLL [Ahrens, Ferroglia, Neubert, BP, Yang, Kidonakis]
- boosted soft limit: NNLL': [Ferroglia, BP, Yang]

Numerical implementations of resummation (not fixed-order approximations!)

- soft limit: NLL in Mellin space for A_{FB} [Almeida, Sterman, Vogelsang '08]
- soft limit: NNLL in momentum space [Ahrens, Ferroglia, Neubert, BP, Yang '10,'11]
- soft limit to NNLL and boosted soft limit to NNLL' in Mellin space [Ferroglia, BP, Scott, Yang, '16]

We have 3 different calculations

	NNLO	Soft resummation	Soft+small-mass resummation
\checkmark	$\alpha_s^n (n \le 2)$	$\alpha_s^n \ln^m N$	$\alpha_s^n \ln^m N; \alpha_s^n \ln^m (m_t/M_{t\bar{t}})$
X	$\alpha_s^n (n > 2)$	1/N suppressed terms	1/N suppressed terms; mt/Mtt suppressed terms

Need to remove double- and triple-counting when combining them!

NNLO+NNLL'





NNLO+NNLL' FOR THE $M_{t\bar{t}}$ distribution



Czakon, Ferroglia, Heymes, Mitov, BP, Scott, Wang, Yang: 1803.07623

NNLO+NNLL' FOR THE TOP- p_T DISTRIBUTION



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Comparison to data: $M_{t\bar{t}}$ distribution



COMPARISON TO DATA: p_T DISTRIBUTION



Persistent shape difference in the transverse momentum spectrum

The high precision of the theoretical calculations and the experimental measurements allows to see this difference clearly! Summary:

- QCD for boosted top production is an interesting multi-scale problem
- presented formalism for joint resummation of soft and small-mass logs in Mellin space
- brought together fixed-order and resummation (groups) for NNLO+NNLL'
- resummation effects significant at high M_{tt} , less so for p_T distribution