Top mass effects in gluon fusion processes

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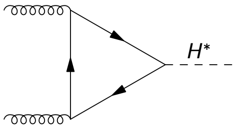
FOR FUNDAMENTAL PHYSICS

Outline

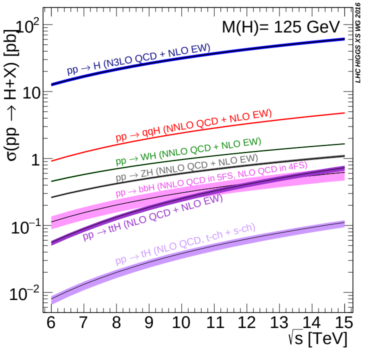
- Gluon fusion processes
- Validity of approximations
- Padé approximants
- Expansion around the top threshold
- Results
- Conclusions & outlook

Gluon fusion processes

Single Higgs production

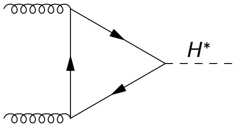


- Dominant Higgs production process

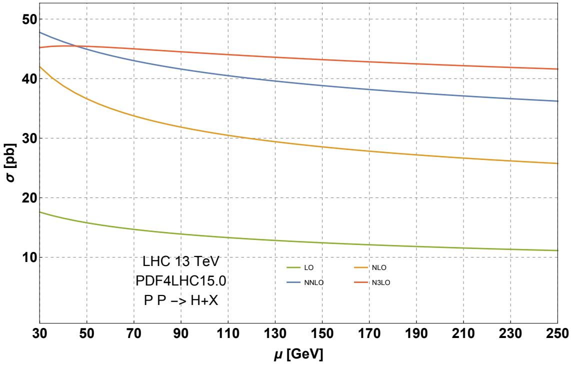


Gluon fusion processes

Single Higgs production



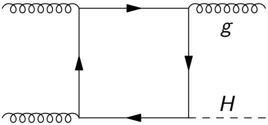
- Dominant Higgs production process
- Convergence is very slow



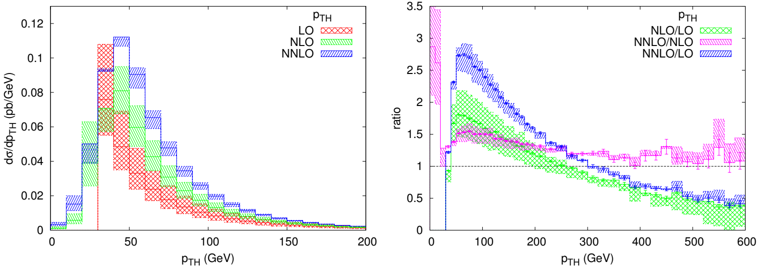
[Mistlberger 2018]

Gluon fusion processes

Higgs plus jet production



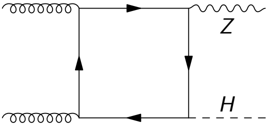
- Transverse momentum dependence
- Convergence is very slow



[Chen, Gehrmann, Glover, Jaquier 2014]

Gluon fusion processes

Higgs plus Z production



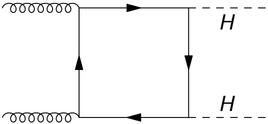
- Formally NNLO, but large
- Convergence is very slow

$\sqrt{s_H}$/GeV	$\sigma^{(\mathrm{exact})}_{\mathrm{H,LO}}$	$\sigma^{(\mathrm{exp-0})}_{\mathrm{H,NLO}}$	$\sigma^{(\mathrm{re-scale})}_{\mathrm{H,NLO}}$	$\sigma^{(\mathrm{diff})}_{\mathrm{H,NLO}}$	$\sigma^{[2/2]}_{\mathrm{H,NLO}}$	NLO scale variation
7	11.2	23.9	24.9	26.1	26.5	$^{+21\%}_{-21\%}$
8	16.0	35.2	35.4	37.2	38.8	$^{+20\%}_{-20\%}$
13	52.4	129	113	121	140	$^{+14\%}_{-17\%}$
14	61.8	155	133	142	168	$^{+13\%}_{-16\%}$

[Hasselhuhn, Luthe, Steinhauser 2016]

Gluon fusion processes

Higgs pair production



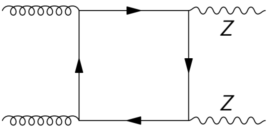
- Measurement of trilinear Higgs coupling
- Convergence is very slow

	σ_H [fb]	$K^{(\mathrm{N})\mathrm{NLO}}$	X^{NNLO} [%]
LO	22.7	—	—
LO+NLO$\|_{\rho^0}$	36.4	1.60	—
LO+NLO$\|_{\rho^0}$+NNLO$\|_{\rho^0}$	39.7	1.75	0
LO+NLO$\|_{\rho^0}$+NNLO$\|_{\rho^1}$	38.7	1.70	-2.5
LO+NLO$\|_{\rho^0}$+NNLO$\|_{\rho^2}$	40.5	1.78	$+2.0$

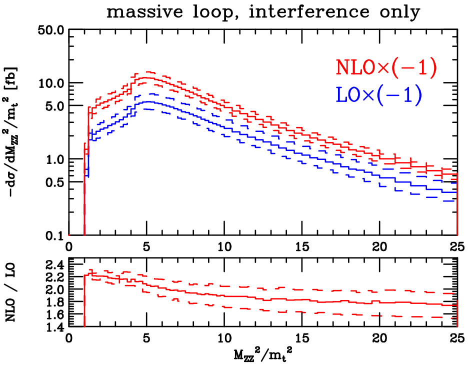
[Grigo, Hoff, Steinhauser 2015]

Gluon fusion processes

Z pair production

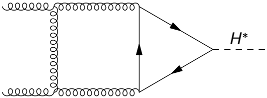


- Higgs width from interference with $gg \to H^* \to ZZ$ [Caola, Melnikov 2013]
- Convergence is very slow

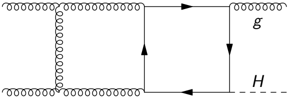


[Campbell, Czakon, Ellis, Kirchner 2016]

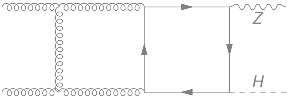
Gluon fusion processes at NLO

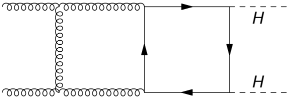


[Spira, Djouadi, Graudenz, Zerwas 1995; . . .]

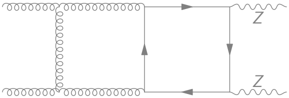


[Jones, Kerner, Luisoni 2018]



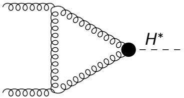


[Borowka et al. 2016]

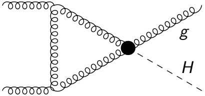


Gluon fusion processes at NLO

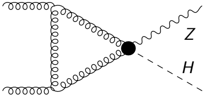
Large mass expansion (LME) $m_t \gg \hat{s}$



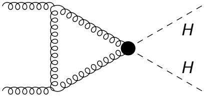
[Dawson 1991; Djouadi, Spira, Zerwas 1991]



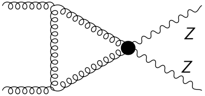
[Harlander, Neumann, Ozeren, Wiesemann 2012; . . .]



[Hasselhuhn, Luthe, Steinhauser 2016]



[Grigo, Hoff, Steinhauser 2015; Degrassi, Giardino, Gröber 2016]

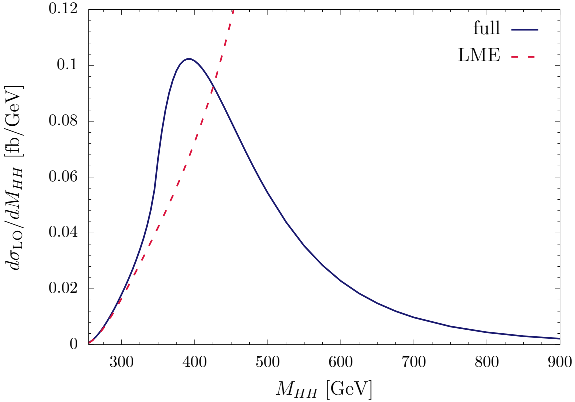


[Melnikov, Dowling 2015; Campbell, Ellis, Czakon, Kirchner 2016; Caola et al. 2016]

Validity of the large mass expansion

Higgs pair production at LO

Fails near the top threshold $M_{HH} \to 2m_t \approx 350\,\text{GeV}$

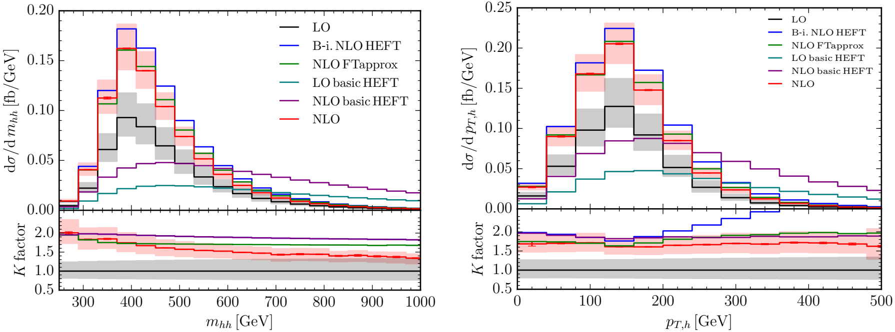


Validity of the large mass expansion

Higgs pair production at NLO

Use a rescaling to improve the prediction from the large-m_t limit

$$d\sigma_{\text{NLO}}^{\text{rescaled LME}}/dX = \frac{d\sigma_{\text{NLO}}^{\text{LME}}/dX}{d\sigma_{\text{LO}}^{\text{LME}}/dX}\, d\sigma_{\text{LO}}^{\text{exact}}/dX\,.$$

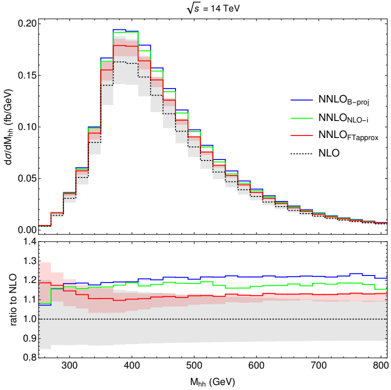


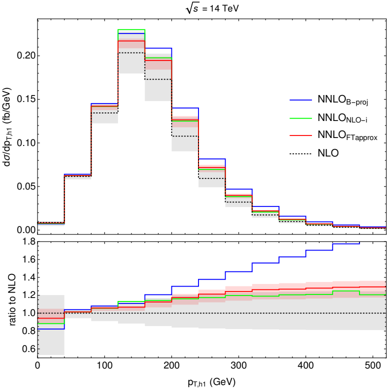
[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke 2016]

FTapprox

Top mass dependence in real radiation

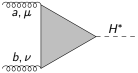
Improvements from including the full top mass dependence in the real part (1 loop) at NLO [Maltoni, Vryonidou, Zaro 2014] and the double real part (1 loop) at NNLO [Grazzini et al. 2018]





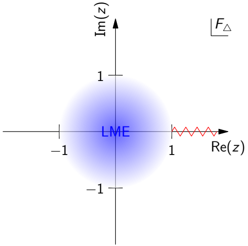
Padé Approximation

[Broadhurst, Fleischer, Tarasov 1993; Fleischer, Tarasov 1994; . . .]



$$= \frac{y_t \hat{s}}{\sqrt{2} m_t} \frac{\alpha_s}{2\pi} \delta_{ab} T_F A_1^{\mu\nu} F_\triangle(z)$$

$$z = \frac{\hat{s}}{4 m_t^2}$$



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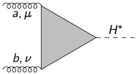

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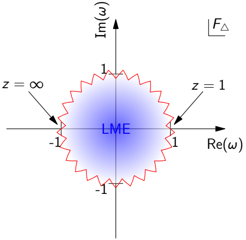
Conformal mapping: $z = \frac{4\omega}{(1+\omega)^2}$



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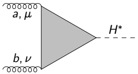
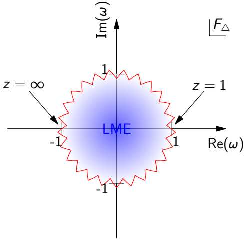


$$= \frac{y_t \hat{s}}{\sqrt{2} m_t} \frac{\alpha_s}{2\pi} \delta_{ab} T_F A_1^{\mu\nu} F_\triangle(z)$$

$$z = \frac{\hat{s}}{4 m_t^2}$$



Conformal mapping: $z = \frac{4\omega}{(1+\omega)^2}$

Padé approximation: $[n/m](\omega) = \frac{\sum_{i=0}^{n} a_i \omega^i}{1+\sum_{i=1}^{m} b_i \omega^i}$

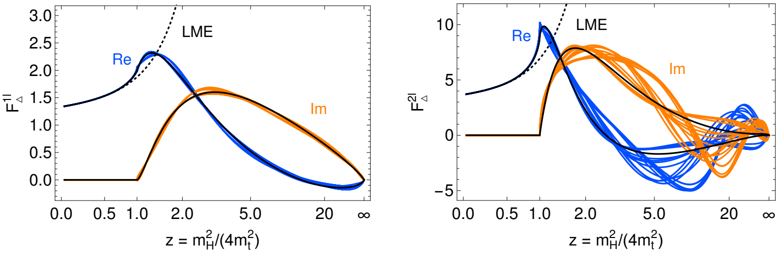
Ansatz:

$$F_\triangle(z) = \frac{[n/m](\omega(z))}{1 + a_R z}\,, \quad a_R \in [0.1, 10]$$

Fix a_i, b_i from LME

from $F_\triangle \xrightarrow{z\to\infty} 0$

Padé Approximation for $gg \to H^*$



20 Padé approximants [1/3], [2/2], [3/1] from LME up to $\frac{1}{m_t^8}$.
Exclude approximants with poles for
$\text{Re}(z) \in [0, 8], \text{Im}(z) \in [-1, 1]$.
At NLO some instabilities emerge above the top threshold.

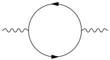
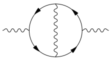
Threshold expansion

$$F_{\triangle}^{1l} \overset{z\to 1}{\asymp} 2\pi(1-z)^{3/2} + \frac{13\pi}{3}(1-z)^{5/2} + \mathcal{O}\left((1-z)^{7/2}\right),$$

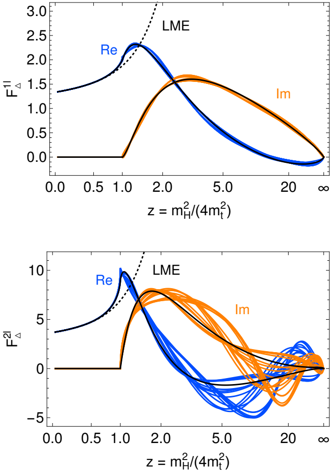
$$F_{\triangle}^{2l} \overset{z\to 1}{\asymp} C_F\pi^2(1-z)\ln(1-z) + \frac{\pi}{12}\left[3\pi^2(C_F - C_A) - 40C_F + 4C_A\right](1-z)^{3/2}$$
$$+C_F\frac{2\pi^2}{3}(1-z)^2\ln(1-z) + \mathcal{O}\left((1-z)^{5/2}\right),$$

$\ln(1-z)$ incompatible with Padé approximation
$\hookrightarrow$ extended ansatz

$$F_{\triangle} = \frac{[n/m](\omega(z))}{1+a_R z} + s(z)$$

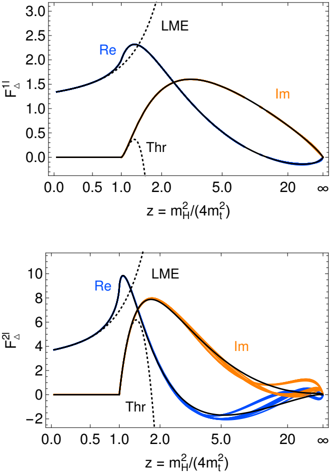
- $\ln(1-z)$ absorbed into subtraction function s
- s analytic for $z \to 0$, constructed from  , 

Padé Approximation with threshold expansion



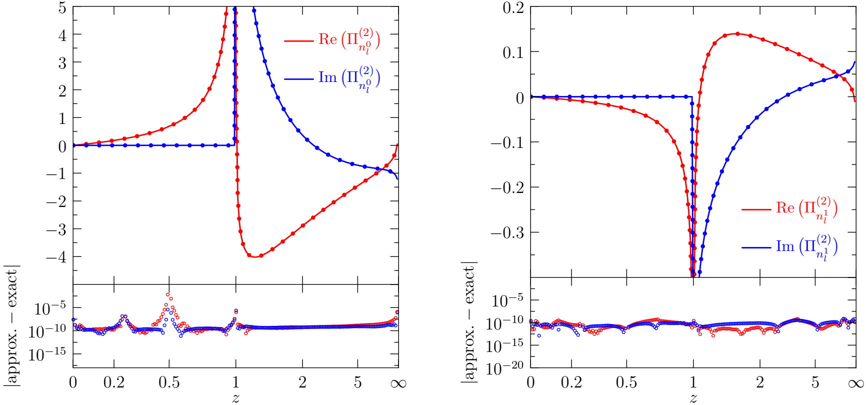
threshold

excl. $(1-z)^i \log^0$



Padé approximation for vacuum polarisation

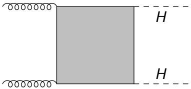
Compare numerical result for vacuum polarization at 3 loops to Padé approximants with $n + m = 60$



[Maier, Marquard 2017]

[Chetyrkin, Kühn, Steinhauser 1995; Hoang, Mateu, Zebarjad 2008; Kiyo, Maier, Maierhöfer, Marquard 2009]

Padé approximation for $2 \to 2$ processes



$$= y_t^2 \frac{\alpha_s}{2\pi} \delta_{ab} T_F \, z \left[A_1^{\mu\nu} F_1 + A_2^{\mu\nu} F_2 \right]$$

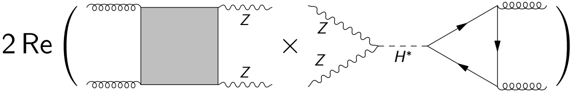
Three ratios: $z = \frac{\hat{s}}{4m_t^2}$, $r_H = \frac{m_H^2}{\hat{s}}$, $r_{p_T} = \frac{p_T^2}{\hat{s}}$

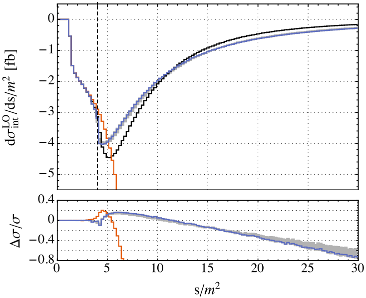
Given a phase space point (z, r_H, r_{p_T}):

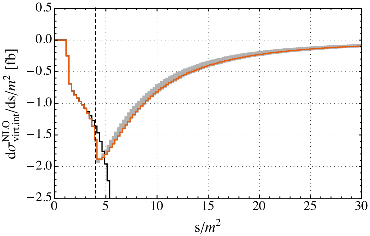
1. Compute threshold and large mass expansion for given r_H, r_{p_T}
2. Construct 100 approximants in z
3. Evaluate approximants for given z (physical m_t)

Padé Approximation for $gg(\to H^*) \to ZZ$

[Campbell, Ellis, Czakon, Kirchner 2016]







- exact
- LME up to $\frac{1}{m_t^{12}}$
- [3/3]
- [n/m] with $n, m \in \{2, 3\}$

- LME up to $\frac{1}{m_t^{12}}$
- [3/3]
- [n/m] with $n, m \in \{2, 3\}$

Threshold expansion

- Near threshold tops can only be on-shell when they are non-relativistic: $p_t^0 - m_t \sim m_t(1-z)$, $\vec{p} \sim m_t\sqrt{1-z}$
  $\Rightarrow$ Hierarchy: $m_t(1-z) \ll m_t\sqrt{1-z} \ll m_t$
- External gluons set directions n, $\bar{n}$ for collinear modes

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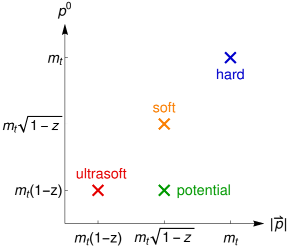
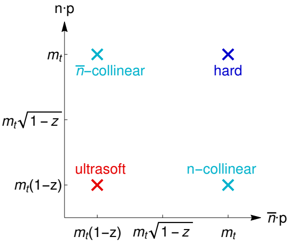


Potential Non-Relativistic QCD

[Pineda, Soto ’97; Beneke et al. ’98; Brambilla et al. ’99]



Soft-Collinear Effective Theory

[Bauer et al. ’00-’01; Beneke et al. ’02]

Threshold expansion

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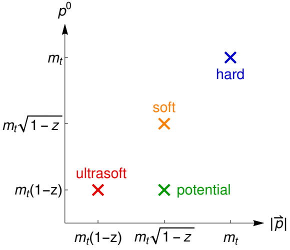
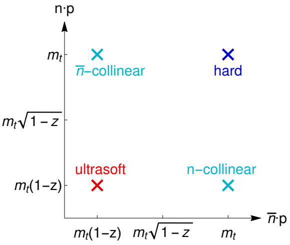




Potential Non-Relativistic QCD

[Pineda, Soto '97; Beneke et al. '98; Brambilla et al. '99]

Soft-Collinear Effective Theory

[Bauer et al. '00-'01; Beneke et al. '02]

→ Unstable-Particle Effective Theory ←

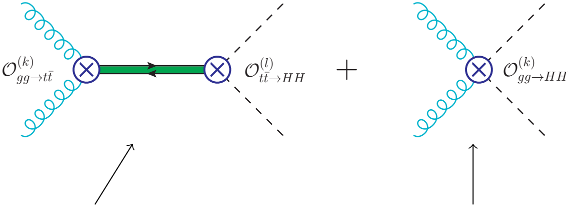
[Beneke, Chapovsky, Signer, Zanderighi '03,'04]

Threshold expansion

Master formula

Integrating out the hard scale gives the following structure

$$i\mathcal{A}_{gg\to F} \overset{z\to 1}{=} \sum_{k,l} C^{(k)}_{gg\to t\bar{t}} C^{(l)}_{t\bar{t}\to F} \int d^4x \left\langle F \middle| T\left[i\mathcal{O}^{(l)}_{t\bar{t}\to F}(x) i\mathcal{O}^{(k)}_{gg\to t\bar{t}}(0)\right] \middle| gg \right\rangle_{\text{EFT}}$$
$$+ C_{gg\to F} \langle F \,|\, i\mathcal{O}_{gg\to F}(0) \,|\, gg\rangle_{\text{EFT}} \,.$$



Resonant contribution:
Propagation of non-relativistic top pair. Contains non-analytic terms $\sqrt{1-z}$ and $\ln(1-z)$.

Non-resonant contribution:
Contribution from hard top loop (local operator in EFT). Analytical in $(1-z)$.

Threshold expansion

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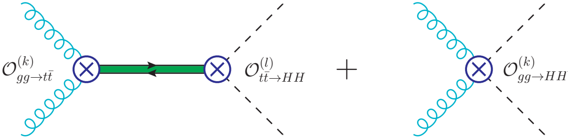
Integrating out the hard scale gives the following structure

$$i\mathcal{A}_{gg\to F} \overset{z\to 1}{=} \sum_{k,l} C^{(k)}_{gg\to t\bar{t}}\, C^{(l)}_{t\bar{t}\to F} \int d^4x \left\langle F \,\middle|\, T\left[i\mathcal{O}^{(l)}_{t\bar{t}\to F}(x)\, i\mathcal{O}^{(k)}_{gg\to t\bar{t}}(0)\right] \,\middle|\, gg \right\rangle_{\mathrm{EFT}}$$
$$+ C_{gg\to F}\, \langle F \,|\, i\mathcal{O}_{gg\to F}(0) \,|\, gg\rangle_{\mathrm{EFT}}\,.$$

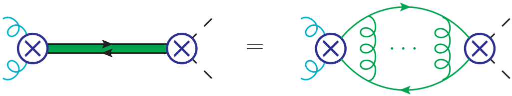


The experience with the triangle form factor (single Higgs production) shows that knowledge of non-analytic terms is sufficient for a good reconstruction. Therefore, we only consider the resonant part.

Threshold expansion

Resonant matrix element

- Ladder exchanges of potential gluons yield Coulomb singularities $(\alpha_s/\sqrt{1-z})^k$



- Can be resummed into a non-relativistic Green function

$$\frac{\mathcal{A}_{\text{'resonant'}}}{\mathcal{A}_0(z=1)} \sim \sqrt{1-z}^{\,2l+1} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\sqrt{1-z}}\right)^k \times \begin{cases} 1 & \text{nrLO,} \\ \alpha_s,\, \sqrt{1-z} & \text{nrNLO,} \\ \alpha_s^2,\, \alpha_s\sqrt{1-z},\, (1-z) & \text{nrNNLO,} \end{cases}$$

Threshold expansion

Resonant matrix element

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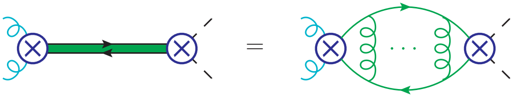

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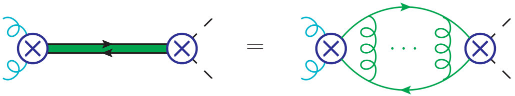
- Collinear nrNLO corrections are scaleless.

Threshold expansion

Resonant matrix element

- Ladder exchanges of potential gluons yield Coulomb singularities $(\alpha_s/\sqrt{1-z})^k$



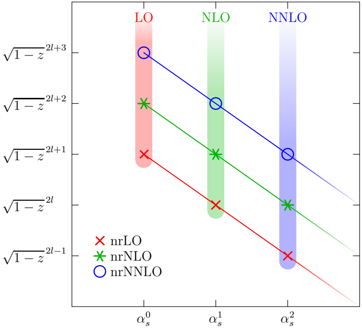
- Can be resummed into a non-relativistic Green function

$$\frac{\mathcal{A}_{\text{'resonant'}}}{\mathcal{A}_0(z=1)} \sim \sqrt{1-z}^{\,2l+1} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\sqrt{1-z}}\right)^k \times \begin{cases} 1 & \text{nrLO,} \\ \alpha_s, \sqrt{1-z} & \text{nrNLO,} \\ \alpha_s^2, \alpha_s\sqrt{1-z}, (1-z) & \text{nrNNLO,} \end{cases}$$

- Collinear nrNLO corrections are scaleless.
- Virtual ultrasoft gluon exchange between collinear gluons is scaleless. Ultrasoft $\bar{t}tg$ interaction is power-suppressed after decoupling transformation (nrNNNLO). [Beneke et al. 2010]

Threshold expansion

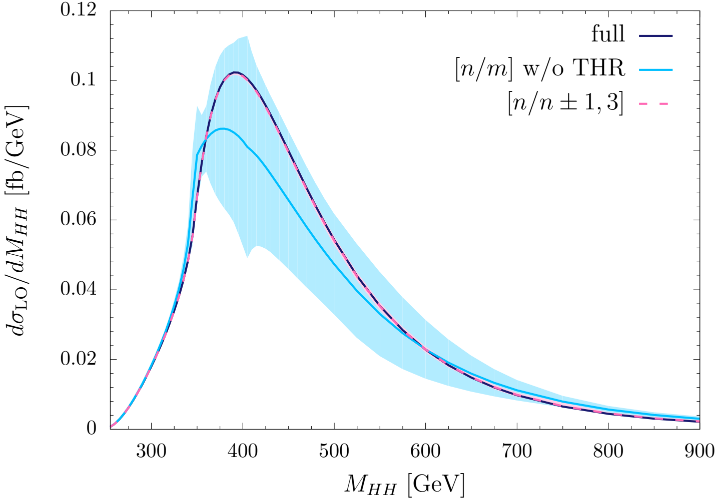
Resonant matrix element: ingredients



- nrLO:
  - $C^{(k)}_{gg\to t\bar{t}}$, $C^{(l)}_{t\bar{t}\to HH}$ at tree level
  - P-wave Green function $G_P(1-z)$ at nrLO [Beneke, Piclum, TR 2013]
- nrNLO:
  - $C^{(k)}_{gg\to t\bar{t}}$, $C^{(l)}_{t\bar{t}\to HH}$ at one loop
  - P-wave Green function $G_P(1-z)$ at nrNLO [Beneke, Piclum, TR 2013]
- $\mathcal{O}(\alpha_s^0, \alpha_s)$ at nrNNLO:
  - $C^{(k)}_{gg\to t\bar{t}}$, $C^{(l)}_{t\bar{t}\to HH}$ for $(1-z)$ power-suppressed operators
  - $\mathcal{O}(\alpha_s^0, \alpha_s)$ of P-wave Green function $G_P(1-z)$ at nrNNLO

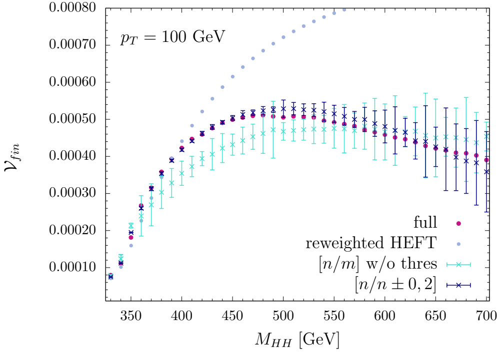
Padé approximation for $gg \to HH$

LO results



Padé approximation for $gg \to HH$

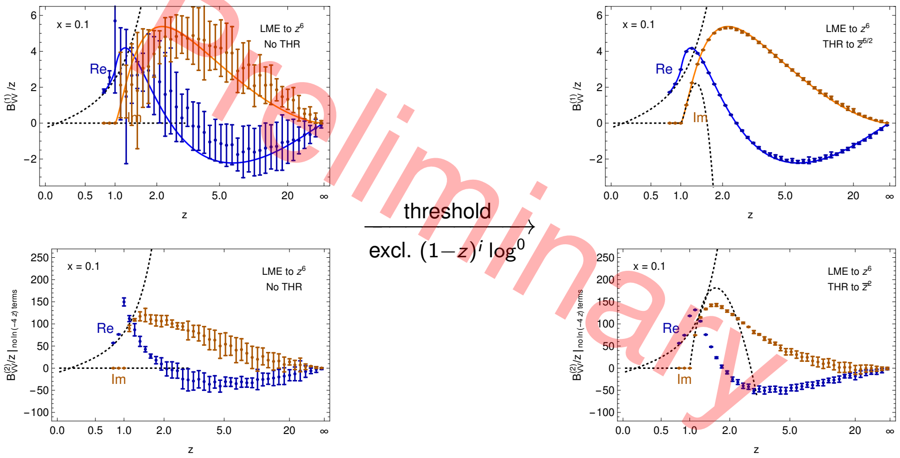
NLO results compared to [Borowka et al. 2016]



M_{HH} [GeV]	p_T [GeV]	$\mathcal{V}_{fin} \times 10^4$			
		HEFT	$[n/m]$	$[n/n \pm 0, 2]$	full
336.85	37.75	0.912	0.996 ± 0.004	0.990 ± 0.001	0.996 ± 0.000
350.04	118.65	1.589	1.933 ± 0.012	1.937 ± 0.010	1.939 ± 0.061
411.36	163.21	4.894	4.326 ± 0.183	4.527 ± 0.069	4.510 ± 0.124
454.69	126.69	6.240	5.300 ± 0.192	5.114 ± 0.051	5.086 ± 0.060
586.96	219.87	7.797	4.935 ± 0.583	5.361 ± 0.281	4.943 ± 0.057
663.51	94.55	8.551	5.104 ± 1.010	4.096 ± 0.401	4.120 ± 0.018

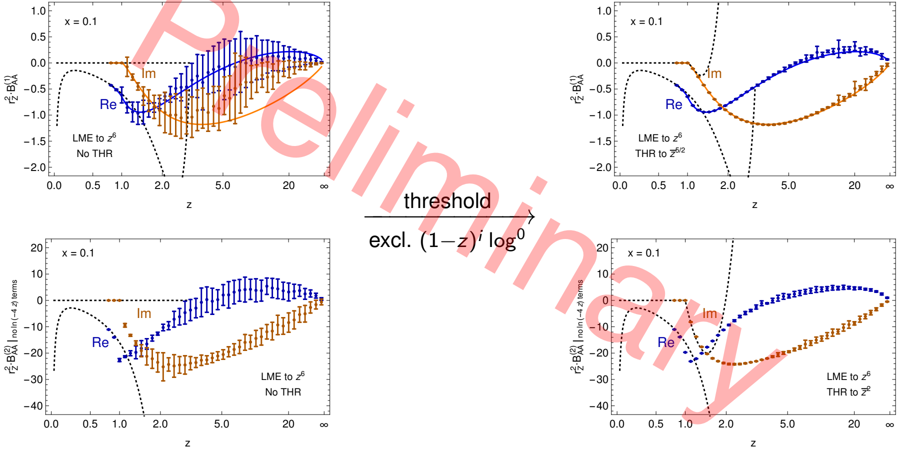
Padé approximation for $gg \to ZZ$

Vector coupling



Padé approximation for $gg \to ZZ$

Axial-vector coupling



Conclusions

- Proof of principle: Padé approximations can capture top mass corrections in gluon fusion processes
- Threshold expansion is necessary to model the peak region
- Systematic improvement through more expansion terms (LME, threshold expansion, small mass expansion)

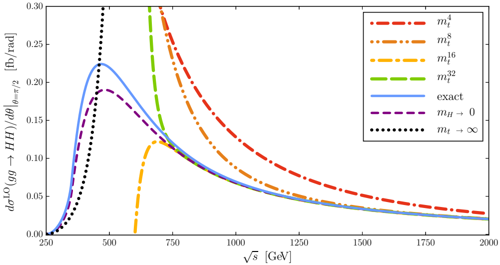
Outlook

- Higher orders in threshold expansion 

Outlook

- Higher orders in threshold expansion 

- Combination with small-mass expansion 



[Davies, Mishima, Steinhauser, Wellmann 2018]

Outlook

- Higher orders in threshold expansion 

- Combination with small-mass expansion 

- Mass effects in three-loop form factors 

Outlook

- Higher orders in threshold expansion 

- Combination with small-mass expansion 

- Mass effects in three-loop form factors 

- Mass effects in $gg(\to H^*) \to ZZ$ interference 

Outlook

- Higher orders in threshold expansion 

- Combination with small-mass expansion 

- Mass effects in three-loop form factors 

- Mass effects in $gg(\to H^*) \to ZZ$ interference 

- Electroweak corrections to Higgs pair production

Backup

Small-p_T expansion

[Bonciani, Degrassi, Giardino, Gröber 2018]

Expand for $p_T^2 + m_H^2 \ll \hat{s}, 4m_t^2$