Double parton scattering: basics and recent theory developments

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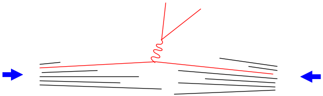
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Hadron-hadron collisions

- standard description based on factorisation formulae

  cross sect = parton distributions × parton-level cross sect



- net transverse momentum p_T of hard-scattering products:
  - p_T integrated cross sect $\rightsquigarrow$ collinear factorisation
  - $p_T \ll$ hard scale of interaction $\rightsquigarrow$ TMD factorisation
- particles resulting from interactions between spectator partons unobserved

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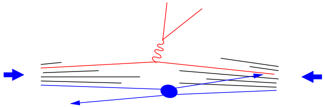
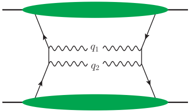


- net transverse momentum p_T of hard-scattering products:
  - p_T integrated cross sect ⇝ collinear factorisation
  - $p_T \ll$ hard scale of interaction ⇝ TMD factorisation
- particles resulting from interactions between spectator partons unobserved
- spectator interactions can be soft ⇝ underlying event
  or hard ⇝ multiparton interactions
- here: double parton scattering with factorisation formula

  cross sect = double parton distributions × parton-level cross sections

Single vs. double parton scattering (SPS vs. DPS)

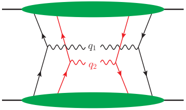
- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$



double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

- for transv. momenta $\sim \Lambda \ll Q$:

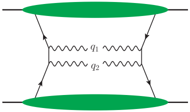
$$\frac{d\sigma_{\text{SPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \gg \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double parton scattering (SPS vs. DPS)

- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



single scattering:

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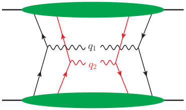


double scattering:

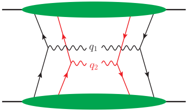
both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

- for small parton mom. fractions x
  double scattering enhanced by parton luminosity
- depending on process: enhancement or suppression
  from parton type (quarks vs. gluons), coupling constants, etc.

  example: $\sigma(qq \to qq + W^-W^-) \propto \alpha_s^2$

  vs. $\sigma(d\bar{u} \to W^-) \times \sigma(d\bar{u} \to W^-) \propto \alpha_s^0$

DPS cross section: collinear factorisation



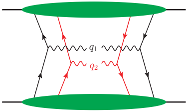
$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

C = combinatorial factor
$\hat{\sigma}_i$ = parton-level cross sections
$F(x_1, x_2, \boldsymbol{y})$ = double parton distribution (DPD)
$\boldsymbol{y}$ = transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s
  get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F

Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2012

DPS cross section: TMD factorisation



- for measured transv. momenta

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C}\,\hat{\sigma}_1\,\hat{\sigma}_2$$

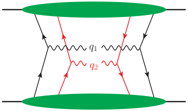
$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2}\,\frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\; e^{-i(\boldsymbol{z}_1\boldsymbol{q}_1+\boldsymbol{z}_2\boldsymbol{q}_2)} \int d^2\boldsymbol{y}\, F(x_i,\boldsymbol{z}_i,\boldsymbol{y})\, F(\bar{x}_i,\boldsymbol{z}_i,\boldsymbol{y})$$

- $F(x_i,\boldsymbol{z}_i,\boldsymbol{y})$ = double-parton TMDs
  $\boldsymbol{z}_i$ = Fourier conjugate to parton transverse mom. $\boldsymbol{k}_i$
- operator definition as for TMDs: schematically have

$$F(x_i,\boldsymbol{z}_i,\boldsymbol{y}) = \underset{z_i^- \to\, x_i p^+}{\mathcal{FT}} \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\;\bar{q}\big(y-\tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y+\tfrac{1}{2}z_1\big)|p\rangle$$

  - to be completed by renormalisation, Wilson lines, soft factors
  - essential for studying factorisation, scale and rapidity dependence

DPS cross section: TMD factorisation



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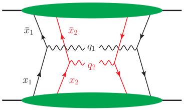
  - to be completed by renormalisation, Wilson lines, soft factors
  - essential for studying factorisation, scale and rapidity dependence
  - analogous def for collinear distributions $F(x_i, \boldsymbol{y})$
    $\Rightarrow$ not a twist-four operator but product of two twist-two operators

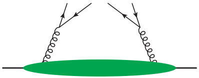
Double parton scattering: ultraviolet problem

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$



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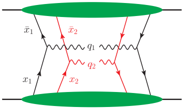

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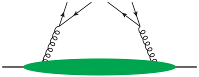
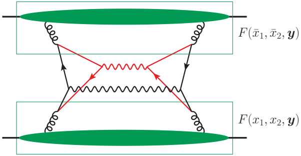
  $$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

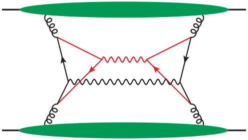
  gives UV divergent cross section $\propto \int d^2\boldsymbol{y}/\boldsymbol{y}^4$
  in fact, formula not valid for $|\boldsymbol{y}| \sim 1/Q$



- problem also for two-parton TMDs
  UV divergences logarithmic instead of quadratic

. . . and more problems





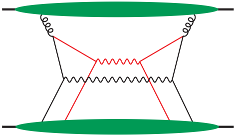
- double counting problem between double scattering with splitting (1v1) and single scattering at loop level

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- also have graphs with splitting in one proton only: "2v1"

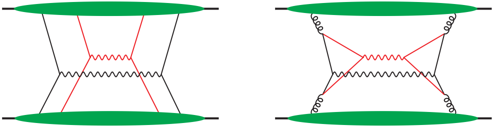
$$\sim \int d^2\boldsymbol{y}/\boldsymbol{y}^2 \; \times F_{\text{int}}(x_1, x_2, \boldsymbol{y})$$

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015



A consistent solution

MD, J. Gaunt, K. Schönwald 2017

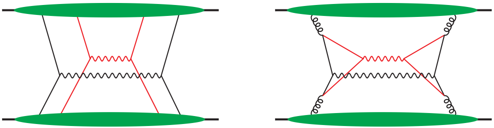


- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions

  analogous regulator for transverse-momentum dependent DPDs
- keep definition of DPDs as operator matrix elements
  cutoff in $\boldsymbol{y}$ does not break symmetries that haven't already been broken

A consistent solution

MD, J. Gaunt, K. Schönwald 2017



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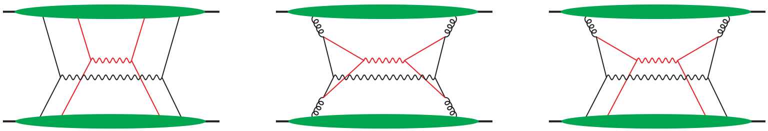
  analogous regulator for transverse-momentum dependent DPDs

- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order

A consistent solution

MD, J. Gaunt, K. Schönwald 2017



- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
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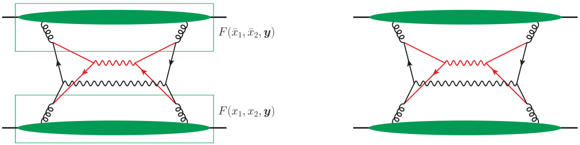
  analogous regulator for transverse-momentum dependent DPDs
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (1v1 + 2v1)}} + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order
  - can also include twist 2 $\times$ twist 4 contribution
    and double counting subtraction for 2v1 term

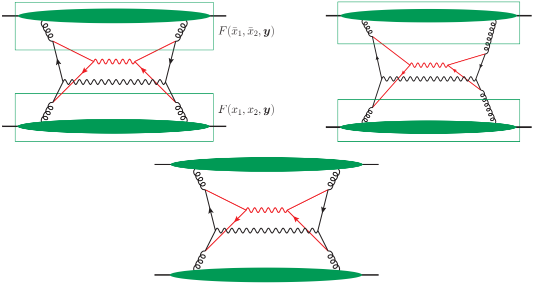
Subtraction formalism at work



$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \sim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \; \sigma \approx \sigma_{\text{SPS}}$ dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub}
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \; \sigma \approx \sigma_{\text{DPS}}$ with regulator fct. $\Phi(\nu y) \approx 1$
- same argument for 2v1 term and $\sigma_{\text{tw2}\times\text{tw4}}$ (were neglected above)
- subtraction formalism works order by order in perturb. theory
  J. Collins, Foundations of Perturbative QCD, Chapt. 10

Double counting in TMD factorisation for DPS



- left and right box can independently be collinear or hard:
  $\rightsquigarrow$ DPS, DPS/SPS interference and SPS
- get nested double counting subtractions

M Buffing, MD, T Kasemets 2017

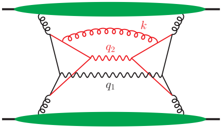
DGLAP evolution

- define DPDs as matrix elements of renormalised twist-two operators:

  $F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p|\mathcal{O}_1(\mathbf{0}; \mu_1)\, \mathcal{O}_2(\boldsymbol{y}; \mu_2)|p\rangle \qquad f(x;\mu) \sim \langle p|\mathcal{O}(\mathbf{0};\mu)|p\rangle$

  $\Rightarrow$ separate DGLAP evolution for partons 1 and 2:

$$\frac{\partial}{\partial \log \mu_i^2} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F \qquad \text{for } i = 1, 2$$



- DGLAP logarithm from strongly ordered region $|\boldsymbol{q}_1| \ll |\boldsymbol{k}| \sim |\boldsymbol{q}_2| \ll Q_2$ repeats itself at higher orders (ladder graphs)
- resummed by DPD evolution in σ_{DPS} if take $\nu \sim \mu_1 \sim Q_1$, $\mu_2 \sim Q_2$ and appropriate initial conditions ($\rightarrow$ next slide)
- can enhance DPS region over SPS region $|\boldsymbol{q}_1| \sim |\boldsymbol{q}_2| \sim Q_{1,2}$ which dominates by power counting

A model study

- take DPD model with $F = F_{\text{spl}} + F_{\text{int}}$

$$F_{\text{spl}}(x_1, x_2, \boldsymbol{y}; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*)\, e^{-y^2 \Lambda^2} \quad \text{with} \quad y^* = \frac{y}{\sqrt{1 + y^2/y_{\text{max}}^2}}$$

inspired by b^* of Collins, Soper, Sterman

$$F_{\text{int}}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0)\, \Lambda^2 e^{-y^2 \Lambda^2} / \pi$$

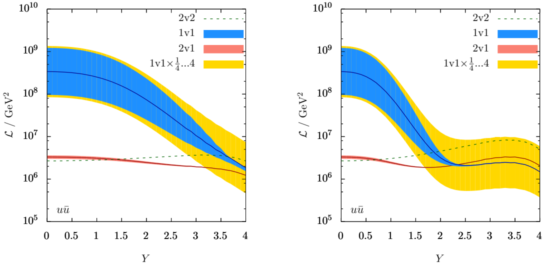
description simplified, actual model slightly refined

- $F_{\text{perturb.}}(y)$ ensures correct perturbative behaviour at small y
  DGLAP logarithms built up between splitting scale $\sim 1/y^*$ and $\sim Q$
- in SPS subtraction term take instead
  $F_{\text{spl}}(x_1, x_2, \boldsymbol{y}; Q, Q) = F_{\text{perturb.}}(y)$
  hard scattering at fixed order, no resummation here
- following plots: show double parton luminosity
  $\mathcal{L} = \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  with separate contributions from 1v1, 2v1, 2v2

DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity Y of q_1 with q_2 central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14\,\mathrm{TeV}$
- blue band: vary ν from $0.5\,M_W \ldots 2\,M_W$
  yellow band: naive scale variation for $\sigma_{\text{1v1}} \propto \nu^2$ from $\int_{b_0^2/\nu^2} dy^2\, \left(1/y^2\right)^2$

$u\bar{u}$



- if ν variation large then need $-\,\sigma_{\text{sub (1v1)}} + \sigma_{\text{SPS}}$
  $\rightsquigarrow$ use 1v1 as estimate for importance of SPS at high orders

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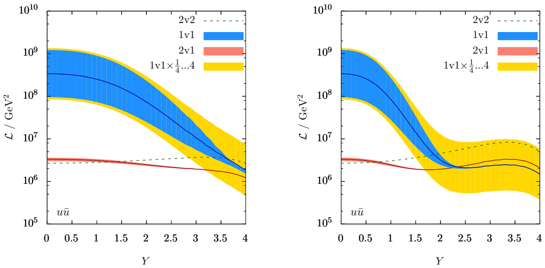

- large rapidity separation $\rightsquigarrow$ x_1 and x_2 asymmetric
  $\rightsquigarrow$ region $y \gg 1/\nu$ in 1v1 enhanced by DPD evolution
  $\rightsquigarrow$ evolved F_{spl} less steep than fixed-order $1/y^2$

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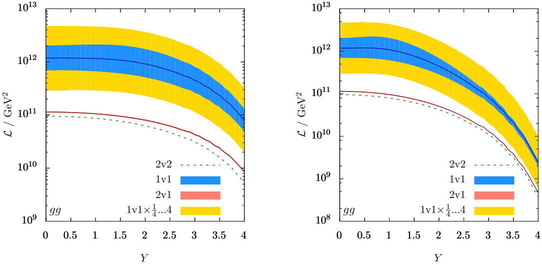
gg



- gluons: prominent evolution effects at all Y

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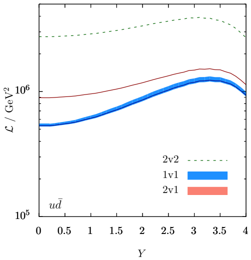
$u\bar{d}$

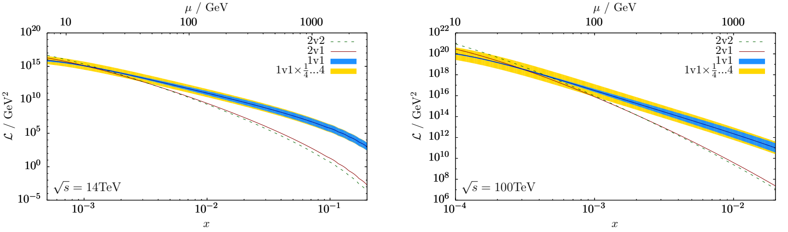


- $u\bar{d}$ induced by splitting at $\mathcal{O}(\alpha_s^2)$, e.g. by $u \to ug \to ud\bar{d}$

DPS parton luminosities for illustration, model parameters not tuned

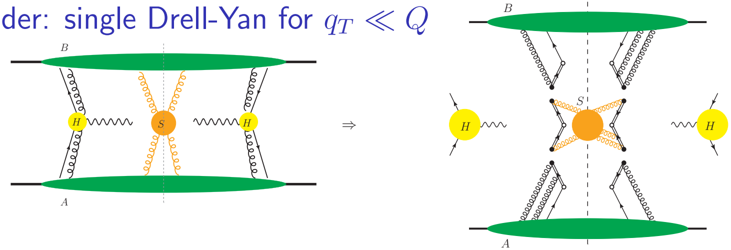
- plot $\mathcal{L}$ vs. $x = x_1 = x_2 = \bar{x}_1 = \bar{x}_2$ at fixed $\sqrt{s}$
  $\mu_{1,2} = Q_{1,2} = x\sqrt{s}$

gg



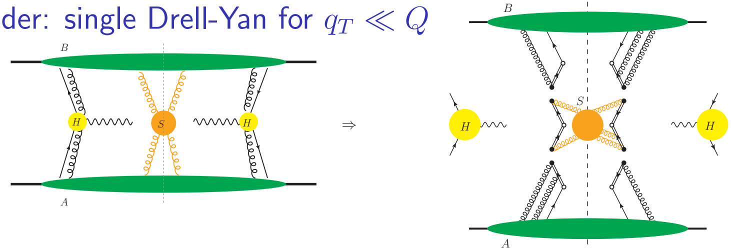
- DPS region enhanced for small x by evolution

Reminder: single Drell-Yan for $q_T \ll Q$



- fast-moving longitudinal gluons coupling to hard scattering
  - include in Wilson lines in parton density
- soft gluon exchange between left- and right-moving partons
  - include in soft factors = vevs of Wilson lines
    needs: eikonal approximation, Ward identities, Glauber cancellation
  - essential for establishing factorisation
  - permits resummation of Sudakov logarithms
    TMD factorisation Collins, Soper, Sterman 1980s; Collins 2011

Reminder: single Drell-Yan for $q_T \ll Q$



- absorb soft factor into parton densities:

$$\sigma = \hat{\sigma}\, BSA = \hat{\sigma}\,(BS)\, S^{-1}(SA) = \hat{\sigma}\, f_B\, f_A$$

  with $f_A = S^{-1/2} f_{A,\text{unsub}}$ and $f_{A,\text{unsub}} = SA$ and same for B

- S requires a rapidity cutoff for the gluons:
  right-moving gluons $\rightsquigarrow f_A$, left-moving ones $\rightsquigarrow f_B$
- separation at central rapidity Y (or equivalent variable)

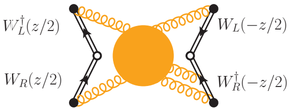
$$\zeta = 2(x p_A^+ e^{-Y})^2 \qquad \bar{\zeta} = 2(\bar{x} p_B^- e^{+Y})^2 \qquad \zeta\bar{\zeta} = Q^4$$

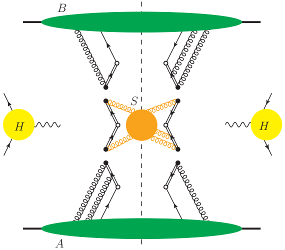
- resum Sudakov logarithms $\log(q_T/Q)$ via evolution equations

$$\frac{d}{d\log\zeta} f_A(\zeta) \quad \text{and} \quad \frac{d}{d\log\bar{\zeta}} f_B(\bar{\zeta})$$

Reminder: single Drell-Yan for $q_T \ll Q$

$$W(z) = \mathrm{P}\exp\left[-igt^a \int_{-\infty}^{0} d\lambda\, vA^a(\lambda v + z)\right]$$





- transverse variables
  - $\boldsymbol{z}$ Fourier conjugate to $\boldsymbol{q}$:
    $$d\sigma/d^2\boldsymbol{q} \propto \int d^2\boldsymbol{z}\, e^{i\boldsymbol{z}\boldsymbol{q}} f_A(x,\boldsymbol{z};\zeta) f_B(\bar{x},\boldsymbol{z};\bar{\zeta})$$
  - soft factor $S = \frac{1}{N_c} \langle 0 | \operatorname{tr} W_L^\dagger(\frac{z}{2})\, W_R(\frac{z}{2})\, W_R^\dagger(-\frac{z}{2})\, W_L(-\frac{z}{2}) | 0 \rangle$
  - collinear factorisation: in $\int d^2\boldsymbol{q}\ (d\sigma/d^2\boldsymbol{q})$ have $\boldsymbol{z} = \boldsymbol{0}$

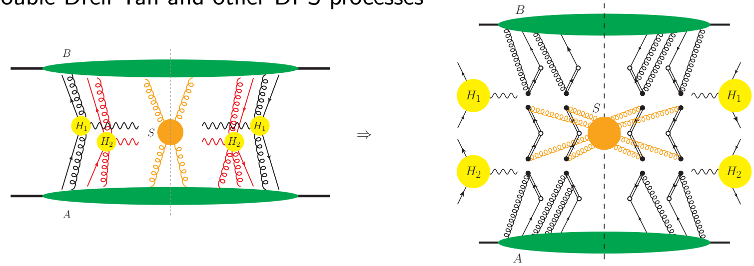
    $\Rightarrow\quad S = 1$

    $\rightsquigarrow$ soft gluon exchanges cancel in sum over all graphs

    $\rightsquigarrow$ no Sudakov logarithms

DPS: factorisation and colour

- generalise previous treatment from single to double Drell-Yan and other DPS processes



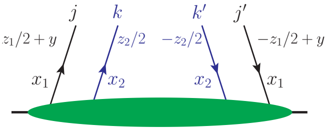
- basic steps can be repeated:
  - collinear gluons ⇝ Wilson lines in DPDs
  - soft gluons ⇝ soft factor
  - Glauber gluons cancel

  MD, D Ostermeier, A Schäfer 2011; MD, J Gaunt, P Plößl, A Schäfer 2015

- absorb soft factors into DPDs M Buffing, T Kasemets, MD 2017

DPS: colour complications

- DPDs have several colour combinations of partons



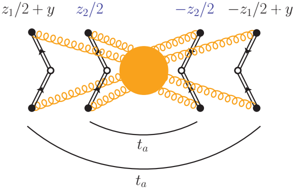
- colour projection operators
- singlet: $P_1^{jj',kk'} = \delta^{jj'}\delta^{kk'}/3$ as in usual PDFs
- octet: $P_8^{jj',kk'} = 2t_a^{jj'}t_a^{kk'}$
- for gluons: $8_A, 8_S, 10, \overline{10}, 27$

- corresponding combinations in soft factor
  - soft factor $\to$ matrix in colour space
  - for colour octet (and other non-singlets): $W_R t^a W_R^\dagger \neq 1$ when at same position

$$\Rightarrow \quad S \neq 1$$

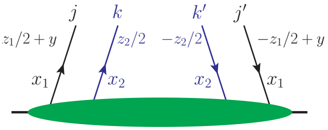
$\rightsquigarrow$ Sudakov factors even in collinear factoris'n

M Mekhfi 1988; A Manohar, W Waalewijn 2012



DPS: colour complications

- DPDs have several colour combinations of partons



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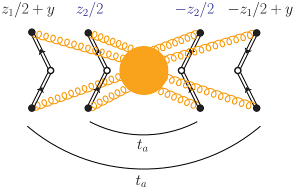
- for gluons: $8_A, 8_S, 10, \overline{10}, 27$

- corresponding combinations in soft factor
  - soft factor $\to$ matrix in colour space
  - in collinear factorisation $(\boldsymbol{z_i = 0})$ soft matrix diagonal in colour space $\rightsquigarrow$ soft gluons do not mix colour channels



TMD factorisation for DPS

- ${}^{RR'}S = \underline{S}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; Y)$ nontrivial matrix in colour space
- rapidity evolution of $\underline{S}$ understood at perturbative two-loop level

A Vladimirov 2016

- assume that general structure valid beyond two loops:

$$\frac{\partial}{\partial Y}\, \underline{S}(Y) = \underline{\widehat{K}}\, \underline{S}(Y) \quad \text{for} \quad Y \gg 1$$

work towards an all-order proof: A Vladimirov 2017

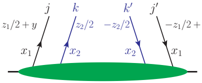
- can then construct matrices $\underline{s}$ and $\underline{K}$ with

$$S(Y_1 + Y_2) = s(Y_1)\, s^{\dagger}(Y_2) \qquad\qquad \frac{\partial}{\partial Y}\, \underline{s}(Y) = \underline{K}\, \underline{s}(Y)$$

for large Y_1, Y_2, Y

- define $\underline{F}_A = \underline{s}^{-1}\, \underline{F}_{A,\text{unsub}}$ as analogue of single TMD $f_A = S^{-1/2}\, f_{A,\text{unsub}}$
- cross section $\sigma \propto \hat{\sigma}_1\, \hat{\sigma}_2 \sum_R {}^{R}F_B\; {}^{R}F_A$

Evolution



- evolution of ${}^{R}F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; \mu_1, \mu_2, \zeta)$

$$\frac{2\partial}{\partial \log \zeta}\, \underline{F} = \underline{K}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; \mu_1, \mu_2)\; \underline{F} \qquad -\frac{\partial}{\log \mu_1}\, \underline{K} = \underline{\mathbb{1}}\; \gamma_K(\mu_1)$$

$$\frac{\partial}{\log \mu_1}\, \underline{F} = \gamma_F(\mu_1, x_1\zeta/x_2)\, \underline{F} \qquad -\frac{2\partial}{\partial \log \zeta}\, \gamma_F = \gamma_K$$

  - γ_F and γ_K same as for single-parton TMDs
    where have Collins-Soper kernel $K(\boldsymbol{z}, \mu)$
  - write $\underline{K} = \underline{\mathbb{1}}\, [K(\boldsymbol{z}_1, \mu_1) + K(\boldsymbol{z}_2, \mu_2)] + \underline{M} \quad \Rightarrow \quad \underline{M}$ indep't of $\mu_{1,2}$

- solution:

$$\underline{F}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \zeta) = e^{-E(\boldsymbol{z}_1; \mu_1, x_1\zeta/x_2) - E(\boldsymbol{z}_2; \mu_2, x_2\zeta/x_1)} \times e^{\underline{M}(\boldsymbol{z}_i, \boldsymbol{y}) \log(\zeta/\zeta_0)}\, \underline{F}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_0, \mu_0, \zeta_0)$$

  - $E(\boldsymbol{z}; \mu, \zeta)$ = Sudakov exponent for single-parton TMD
    contains double logarithm, is colour independent
  - matrix exponential of $\underline{M}$ gives single logarithms

Summary

- double parton scattering important in specific kinematics/for specific processes
- recent years: progress towards a systematic formulation of factorisation in QCD
- solution for UV problem of DPS $\leftrightarrow$ double counting with SPS
  - simple UV regulator for DPS using distance y between partons
  - simple subtraction term to avoid double counting

  naturally includes "2v1" contributions and DGLAP logarithms in DPS

  - at large scales Q find dominant 1v1 contributions in many cases $\rightsquigarrow$ SPS required at high order in α_s before DPS becomes important
  - DPS can dominate for small x_1 and/or x_2, enhanced by evolution
- soft factor and rapidity evolution: matrix structure in colour space can generalise Collins' "square root construction" to two-parton TMDs
  - leading double logarithms universal, same as for single TMDs