ALL-ORDERS CALCULATIONS FOR PDFs DETERMINATION

University of Vienna,

Friday 13th April 2018
OUTLINE

- Introduction
- PDFs with large-x resummation
- PDFs with small-x resummation: evidence for BFKL dynamics in inclusive HERA data
- Double resummation
- Conclusion and Outlook
A WORD ON PDFs

- Parton distribution functions describe the non-perturbative structure of the colliding protons
- Collinear factorisation implies their universality (up to power corrections)

\[
\sigma(x, Q) = \sigma_0 C \left( \frac{x}{x_1 x_2}, \alpha_s(\mu) \right) \otimes f_1(x_1, \mu) \otimes f_2(x_2, \mu)
\]

- Coefficient functions (NLO, NNLO, N^3LO)
- Parton evolution (NLO, NNLO)
- Electro-weak corrections
- Quark mass effects
- Target-mass corrections
- ...
HIGHER-ORDER CORRECTIONS

- Higher-order QCD corrections correspond to emission of extra partons or virtual corrections

- these corrections are enhanced in particular regions of phase-space

\[ \alpha_s^k \left[ \frac{\log^{2k-1}(1-z)}{1-z} \right] , \ z \to 1 \]

\[ \alpha_s^k \log^{k-1} \frac{z}{z} , \ z \to 0 \]

WE WILL MOST CONVENIENTLY WORK IN MELLIN SPACE

SOFT-GLUON RESUMMATION: \( z \to 1 \Leftrightarrow \text{LOGS OF N} \)

BFKL RESUMMATION: \( z \to 0 \Leftrightarrow \text{POLES IN N (TYPICALLY AT N=0)} \)
DATASET OF A GLOBAL FIT

- Standard PDFs fits rely on NLO and NNLO calculations of coefficient functions and evolution
- current datasets span several order of magnitude in $Q^2$ and $x$

QUESTIONS THAT COME TO MIND

- Do we trust FO everywhere?
- Do we see evidence of all-order effects in the data?
- Is it ok to use standard PDFs with resummed calculation?
THRESHOLD (LARGE-X) RESUMMATION
PRODUCTION AT THRESHOLD

- absolute threshold: the initial-state energy is just enough to produce the final state with invariant mass $Q$

$$x = \frac{Q^2}{s} \to 1$$

- emissions forced to be soft, leading to log-enhanced contributions order-by-order in perturbation theory

\[
\begin{align*}
C(z, \alpha_s) &\sim \sigma_0 \sum_{n=1}^{2n-1} \sum_{k=-1}^{n} \alpha_s^n \left[ \frac{\ln^k(1 - z)}{1 - z} \right]
\end{align*}
\]
**WHY BOTHER WITH THRESHOLD AT THE LHC?**

- Gluon PDF shows a steep increase at low $x$
  \[ \hat{S} = x_1 x_2 s \]
- region of partonic threshold is enhanced in the convolution

- more precise argument in Mellin space
- a saddle-point approximation indicates the region that gives the bulk of the contribution to the inverse Mellin integral
- this region turns out to be fairly narrow around the (real) saddle-point
THRESHOLD RESUMMATION

- **momentum space:** distributional terms for $z \to 1$

- **moment space:** terms that do not vanish at large $N$

\[
C_{\text{res}}(N, \alpha_s) = \bar{g}_0(\alpha_s, \mu_F^2) \exp \bar{S}(\alpha_s, N),
\]
\[
\bar{S}(\alpha_s, N) = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left( \int_{\mu_F^2}^{m_H^2} \frac{(1-z)^2}{z} d\mu^2 \frac{2A(\alpha_s(\mu^2)) + D(\alpha_s([1 - z]^2 m_H^2))}{\mu^2} \right),
\]
\[
\bar{g}_0(\alpha_s, \mu_F^2) = 1 + \sum_{k=1}^{\infty} \bar{g}_{0,k}(\mu_F^2) \alpha_s^k, \quad \text{Anastasiou et al. (2014)}
\]
\[
A(\alpha_s) = \sum_{k=1}^{\infty} A_k \alpha_s^k, \quad D(\alpha_s) = \sum_{k=1}^{\infty} D_k \alpha_s^k, \quad \text{Catani et al. (2002); Moch, Vogt (2005); Laenen, Magnea (2005) [...]}
\]

- constants can go in the exponent of in front of it
- state of the art $N^3$LL (but the 4-loop cusp)
- next-to-eikonal can be important (e.g. $(1-z)^2/z$)  

Laenen et al. (2015, 2016); Larkosi, Neill, Stewart (2015)
PDFs AT LARGE X

Observable: \[ \sigma = \sigma_0 C(\alpha_s(\mu)) \otimes f(\mu) \left[ \otimes f(\mu) \right] \]

Evolution: \[ \mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu) \]

- coefficient functions contain large-x logs
- PDF evolution doesn’t (in MSbar)
  \[ P_{gg}(x) \sim \frac{A(\alpha_s)}{(1 - x)^+} \]
- performing a resummed fit is relatively straightforward
- data set is restricted: no jets
- (\*)global vs non-global

\begin{tabular}{|l|l|l|}
  \hline
  Process & observable & resummation available \\
  \hline
  DIS & \(d\sigma/dx/dQ^2\) (NC, CC, charm, ...) & YES \\
  DY \(Z/\gamma\) & \(d\sigma/dM^2/dY\) & YES \\
  DY \(W\) & differential in the lepton kinematics & NO \\
  \(t\bar{t}\) & total \(\sigma\) & YES \\
  jets & inclusive \(d\sigma/dp_t/dY\) & YES/NO \\
  \hline
\end{tabular}

\textbf{it should be easy to compute}

\textbf{different calculations exist at NLL(\*) but no public implementation}

de Florian, Vogelsang (2007, 2013); Kidonakis, Owens (2000); Liu, Moch, Ringer (2017)

DIS, DY available from TROLL \textit{(TROLL Resums Only Large-x Logarithms)}
\url{www.ge.infn.it/~bonvini/troll}

\(t\bar{t}\) available from \textit{top++}
\url{www.alexandermitov.com/software}
EFFECTS ON THEORY PREDICTIONS

- K-factors reduced when NNLO is included: resummation is perturbative
PDFs FIT WITH THRESHOLD RESUMMATION

- as expected: visible effects at NLO+NLL are very much reduced at NNLO+NNLL
- \( \chi^2 \) slightly worse because of DY fixed-target experiments
- this remains a puzzle

<table>
<thead>
<tr>
<th>Experiment</th>
<th>NNPDF3.0 DIS+DY+top</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NLO</td>
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<td>NMC</td>
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<tr>
<td>NuTeV</td>
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<td>HERA-I</td>
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<td>ZEUS HERA-II</td>
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<td>DY E605</td>
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<tr>
<td>CDF Z rap</td>
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<tr>
<td>D0 Z rap</td>
<td>0.57</td>
</tr>
<tr>
<td>ATLAS Z 2010</td>
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<tr>
<td>ATLAS high-mass DY</td>
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</tr>
<tr>
<td>CMS 2D DY 2011</td>
<td>1.22</td>
</tr>
<tr>
<td>LHCb Z rapidity</td>
<td>0.83</td>
</tr>
<tr>
<td>ATLAS CMS top prod</td>
<td>1.23</td>
</tr>
<tr>
<td>Total</td>
<td>1.233</td>
</tr>
</tbody>
</table>

Table 5. Same as table 4 for the DIS+DY+top fits.
PARTONS WITH THRESHOLD RESUMMATION

› comparison to global fit: larger uncertainties because of reduced dataset

› only “proof-of-concept” studies
effects on SM Higgs negligible

more pronounced for high-mass states, still within PDF errors

large-x PDFs not (yet) competitive because of missing jet data
HIGH-ENERGY (SMALL-X) RESUMMATION
LHC KINEMATICS

- PDFs are largely unconstrained at low $x$
- LHC does probe this region
- Is DGLAP enough to describe this region?
- Do we need to worry about small $x$? and saturation?

DGLAP: $Q^2$ evolution for $N$ moments of the parton density

$$\frac{d}{d \ln(Q^2/\mu^2)} G(N, Q^2) = \gamma(N, \alpha_s) G(N, Q^2)$$

BFKL: small-$x$ evolution for $M$ moments of the parton density

$$\frac{d}{d \ln(1/x)} G(x, M) = \chi(M, \alpha_s) G(x, M)$$

Mellin moments:

$$\ln^k \frac{Q^2}{\mu^2} \leftrightarrow \frac{1}{M^{k+1}}$$

$$\ln^k \frac{1}{x} \leftrightarrow \frac{1}{N^{k+1}}$$

logs $\leftrightarrow$ poles
DGLAP EVOLUTION AT SMALL-X

- DGLAP evolution in the singlet sector
  \[ Q^2 \frac{d}{dQ^2} \left( \begin{array}{c} f_g \\ f_q \end{array} \right) = \Gamma(N, \alpha_s(Q^2)) \left( \begin{array}{c} f_g \\ f_q \end{array} \right), \quad \Gamma(N, \alpha_s) \equiv \left( \begin{array}{cc} \gamma_{gg} & \gamma_{gq} \\ \gamma_{qg} & \gamma_{qq} \end{array} \right) \]

- the gluon splitting functions start at LLx
  \[ \begin{align*} 
  \gamma_{gg} & \sim c_1 \frac{\alpha_s}{N} + c_2 \left( \frac{\alpha_s}{N} \right)^2 + \ldots \\
  \gamma_{gq} & \sim \frac{C_F}{C_A} \gamma_{gg} 
  \end{align*} \]

- while the quarks are NLLx
  \[ \begin{align*} 
  \gamma_{qq} & \sim \alpha_s d_0 + d_1 \alpha_s \left( \frac{\alpha_s}{N} \right) + c_2 \alpha_s \left( \frac{\alpha_s}{N} \right)^2 + \ldots \\
  \gamma^{(PS)}_{qq} & \sim \frac{C_F}{C_A} \left( \gamma_{gg} - \alpha_s d_0 \right) 
  \end{align*} \]
**FIXED-ORDER CONSIDERATIONS**

- Note that some of the coefficients can be zero because of accidental cancellations: most notably $c_2$ and $c_3$ in MS-like schemes

\[ \gamma_{gg} \sim c_1 \left( \frac{\alpha_s}{N} \right) + c_2 \left( \frac{\alpha_s}{N} \right)^2 + c_3 \left( \frac{\alpha_s}{N} \right)^3 + c_4 \left( \frac{\alpha_s}{N} \right)^4 + \mathcal{O}(\alpha_s^5) \]

- NNLO is less stable than NLO (subleading logs survive)
- $N^3$LO (calculations underway) is likely to exhibit stronger instabilities
DGLAP–BFLK DUALITY

- (N)LLx behaviour can be determined from the (N)LO BFKL kernel
  \[ G(N, M) = \frac{G_0(N)}{M - \gamma(\alpha_s, N)} \]
  \[ \chi(\gamma(N, \alpha_s), \alpha_s) = N \]
  \[ \gamma(\chi(M, \alpha_s), \alpha_s) = M \]

- however: naive implementation of BFKL leads to results not supported by HERA data (too strong, too soon)

\[ \sigma \equiv \sqrt{\ln \frac{z_0}{x} \ln \frac{t}{t_0}} , \quad \rho \equiv \sqrt{\ln \frac{z_0}{x} / \ln \frac{t}{t_0}} \]

models that naively implement BFKL are disfavoured by HERA data

Ball, Forte (1994)
RESUMMATION OF DGLAP EVOLUTION

- Problem studied by different groups in late ‘90s /early ‘00s: Altarelli, Ball, Forte; Ciafaloni, Colferai, Salam, Stasto; Thorne, White


- recent progress in SCET Rothstein, Stewart (2016)

- we mostly follow the approach by ABF

- key ingredients:
  - duality between DGLAP and BFKL kernels
  - stable solution of the running coupling BFKL equation (important subleading effects)
  - match to standard DGLAP at large \( N (x) \)
COEFFICIENT FUNCTIONS AT SMALL X

- the high-energy behaviour of coefficient function is obtained using $k_t$-factorisation \cite{Catani1991, Collins1991}

- derivation in terms of ladder expansion allowed for its generalisation to differential distributions \cite{Caola2010, Forte2016, Muselli2017}

- for most processes of interest (DIS, DY) resummation starts at NLLx
RESUMMATION OF COEFFICIENT FUNCTIONS

- naive (i.e. fixed-log counting) resummation has same issues as evolution
- running coupling corrections are crucial
- elegant but complex treatment in Mellin space Ball (2008)
- our approach in a nutshell: resummation in momentum space

High-energy ($k_T$) factorization:

$$
\sigma \propto \int \frac{dz}{z} \int d^2k \hspace{0.5em} \hat{\sigma}_g \left( \frac{x}{z}, \frac{Q^2}{k^2}, \alpha_s(Q^2) \right) \mathcal{F}_g(z, k) = \begin{cases} 
\mathcal{F}_g(x, k) & \text{: unintegrated PDF} \\
\hat{\sigma}_g \left( z, \frac{Q^2}{k^2}, \alpha_s \right) & \text{: off-shell xs} 
\end{cases}
$$

Defining

$$
\mathcal{F}_g(N, k) = U \left( N, \frac{k^2}{\mu^2} \right) f_g(N, \mu^2)
$$

we get

$$
C_g(N, \alpha_s) = \int d^2k \hspace{0.5em} \hat{\sigma}_g \left( N, \frac{Q^2}{k^2}, \alpha_s \right) U \left( N, \frac{k^2}{\mu^2} \right)
$$

At LLx accuracy, $U$ has a simple form, in terms of small-$x$ resummed anom dim $\gamma$

$$
U \left( N, \frac{k^2}{\mu^2} \right) \approx k^2 \frac{d}{dk^2} \exp \int_{\mu^2}^{k^2} \frac{dv^2}{v^2} \gamma(N, \alpha_s(v^2))
$$

- until recent: very little phenomenology because a comprehensive code was missing
public code that computes resummed splitting functions and perturbative coefficient functions

HELL-x: pheno tool with pre-tabulated results, interfaced with evolution code APFEL

in current HELL 2.0 version

- DIS (both NC and CC)
- heavy-quark matching conditions

HELL 3.0 will appear soon (Higgs, DY)

https://www.ge.infn.it/~bonvini/hell/
Results from Hell: Splitting Functions

- resummation matched up to NNLO
- uncertainty bands obtained by varying subleading corrections
- quark splitting functions under less control (they start at NLL)
RESULTS FROM HELL: DIS COEFFICIENT FUNCTIONS

- parton level results
- large theoretical uncertainty (they start at NLL)

we can already see $N^3LO$ instabilities
A FIT WITH SMALL-\(X\) RESUMMATION: THE DATASET

- exploit NNPDF state-of-art technology to perform fits with small-\(x\) resummation

- for DIS with have a consistent implementation of small-\(x\) resummation (both evolution and coefficient functions)

- similar dataset as standard NNLO analysis (NNPDF 3.1)

- lower the initial scale of the fit to \(Q_0=1.64\) GeV to include an extra bin of the HERA data \((Q^2=2.7\) GeV\(^2\))

- what about hadronic data?

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(N_{\text{dat}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMC</td>
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<tr>
<td>NuTeV dimuon</td>
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<tr>
<td>HERA I+II incl. NC</td>
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<tr>
<td>HERA I+II incl. CC</td>
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<td>HERA (\sigma_{\text{NC}})</td>
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<tr>
<td>HERA (F_2^b)</td>
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</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3231</strong></td>
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</table>
THE ISSUE WITH HADRONIC DATA

- resummation for coefficient functions in pp collisions is known but not yet implemented in HELL
- resummation only included in the evolution
- to avoid biases we cut away hadronic low-x data (mostly LHCb DY)
- we discard points for which (based on LO kinematics)

\[ \alpha_s(Q^2) \ln \frac{1}{x} \geq H_{\text{cut}} \]

- the smaller \( H_{\text{cut}} \), the tighter the cut
- we find \( H_{\text{cut}} = 0.6 \) to be a good compromise
- we keep \( \sim 70\% \) of hadronic data

\[ \ln \frac{1}{x} \geq \beta_0 H_{\text{cut}} \ln \frac{Q^2}{\Lambda^2} \]
### FIT RESULTS

<table>
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<tr>
<th></th>
<th>$\chi^2/N_{\text{dat}}$</th>
<th>$\Delta\chi^2$</th>
<th>$\chi^2/N_{\text{dat}}$</th>
<th>$\Delta\chi^2$</th>
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<td>HERA $F_2^b$</td>
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<td>1.49</td>
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<td>CMS total</td>
<td>0.97</td>
<td>0.92</td>
<td>−13</td>
<td>0.86</td>
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<td>CMS Drell-Yan 2D 2011</td>
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<td>−</td>
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<td>CMS Z $p_T$ TeV ($p_T, y_{ll}$)</td>
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<td>1.16</td>
<td>1.19</td>
<td>−</td>
<td>1.08</td>
</tr>
</tbody>
</table>

- the quality of NLO+NLLx and NLO fits is comparable
- it’s expected because the two theories are rather similar
- situation changes dramatically at NNLO
- NNLO+NLLx provides the best fit
- the bulk of the improvement comes from HERA data
PARTON DENSITIES WITH SMALL-\(x\) RESUMMATION

- resulting PDFs show interesting features
- agreement at large \(x\) but they’re much steeper at low \(x\)
IMPACT OF THEORETICAL UNCERTAINTIES

- we have seen that quark splitting functions and coefficient functions suffer from large theoretical uncertainties
- the inclusion of theory errors in PDF fit is currently an active area of research
- we can use a setting that varies from the standard one beyond NNLLx
- a DIS-only study shows that the fit quality is unchanged
- qualitative behaviour on solid grounds, however quantitative results do change

**Figure 4.4.** Comparison between the gluon (left) and the total quark singlet (right plots) from the NNLO and NNLO+NLLx resummation. The e

This shows the need for NNLLx resummation (at least in the quark sector)
PERTURBATIVE STABILITY

- NNLO and NNLO+NLLx differ quite dramatically
- one could question the reliability of the resummed procedure
- what gives us confidence we’re not talking rubbish?
- resummation cures perturbative instability of NNLO
the improved description of DIS structure functions is clearly visible

\[ F_L(x, Q^2) \]

this is particularly true for \( F_L \) where resummation effects starts at its LO
BFKL: THE GHOST OF CHRISTMAS PAST

- How does the fit-quality change if we include data at smaller and smaller $x$?
- similar strategy as for hadronic data

\[ \alpha_s(Q^2) \ln \frac{1}{x} \geq D_{\text{cut}} \]
BFKL: THE GHOST OF CHRISTMAS PRESENT

- to investigate LHC phenomenology we need resummed coefficient functions
- we can have a look at parton luminosities: $q\bar{q}$ doesn’t change much but the change in $gg$ is striking!
- consistent phenomenology for cosmic ray neutrinos (CC-DIS)
- unique “lab” for low-x physics
**BFKL: THE GHOST OF CHRISTMAS YET-TO-COME**

- **small-x physics** will be crucial at future circular colliders
- **$e$ (60 GeV) - $p$ (7 TeV or 50 TeV) collisions**
- to gauge the impact: fits including (resummed) pseudo-data

<table>
<thead>
<tr>
<th></th>
<th>$N_{\text{dat}}$</th>
<th>$\chi^2/N_{\text{dat}}$</th>
<th>$\Delta \chi^2$</th>
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<tr>
<td>HERA I+II incl. NC</td>
<td>922</td>
<td>1.22</td>
<td>1.07</td>
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<tr>
<td>LHeC incl. NC</td>
<td>148</td>
<td>1.71</td>
<td>1.22</td>
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<tr>
<td>FCC-eh incl. NC</td>
<td>98</td>
<td>2.72</td>
<td>1.34</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1168</strong></td>
<td><strong>1.407</strong></td>
<td><strong>1.110</strong></td>
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</table>
DOUBLE RESUMMATION: HIGGS PRODUCTION
**DOUBLE RESUMMATION: PARTONIC COEFFICIENT FUNCTIONS**

\[ C_{ij}(x, \alpha_s) = C_{ij}^{\text{fo}}(x, \alpha_s) + \Delta C_{ij}^{\text{lx}}(x, \alpha_s) + \Delta C_{ij}^{\text{rsx}}(x, \alpha_s) \]

- how to merge together the two resummation developed so far?
- look at singularity structure in Mellin space

\[ \left( \frac{\alpha_s}{N - 1} \right)^n \]

- double-resummed result respects singularity structure order-by-order

\[ \alpha_s^n \ln^{2n} N \]

\[ \text{Re } N \]

\[ \alpha_s^n \psi^{2n}(N) \]

Ball, Bonvini, Forte, SM, Ridolfi (2013)
Ideally we would like to use double-resummed PDFs.

We have to make a choice: small-\(x\) resummation strongly affects the NNLO gluon PDF, while threshold is a small correction.

Use small-\(x\) resummed PDFs for double resummation.

Bonvini and SM (2018)
DOUBLE RESUMMATION: RESULTS

- faster convergence of perturbative expansion
- reliable theoretical uncertainties using scale variations and subleading logs)
- large effect at 100 TeV driven by small-x resummation of the gluon

Bonvini and SM (2018)
CONCLUSIONS & OUTLOOK

- Better determinations of PDFs require both data and theory
- Resummation offers a complementary direction
- Large-\(x\) resummed fits performed with restrict data set
- Small-\(x\) resummed fit shows evidence of BFKL dynamics in HERA inclusive data
- LHC application: double-resummed Higgs cross-section
- Towards truly global resummed fits:
  - DY at small \(x\) is the next item on the agenda
  - Then jets, both at large- and small-\(x\)
THANK YOU!

if we have seen further it is only by standing on the shoulders of giants