\[ \alpha_s \text{ from non-strange hadronic } \tau \text{ decays} \]

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\[ \delta \alpha_s (M_Z) \simeq \left( \frac{\alpha_s (M_Z)}{\alpha_s (m_\tau)} \right)^2 \delta \alpha (m_\tau) \]
QCD in $\tau$ decay

\[ w_T(s; s_0) = \left(1 + 2 \frac{s}{s_0}\right) \left(1 - \frac{s}{s_0}\right)^2 \]

doubly pinched

\[ w_L(s; s_0) = 2 \left(\frac{s}{s_0}\right) \left(1 - \frac{s}{s_0}\right)^2 \]

doubly pinched

\[ s_0 = m_{\tau}^2 \]

\[ \rho_{V,A} = \frac{1}{\pi} \text{Im} \Pi_{V,A} \]

\[ \frac{\Gamma(\tau \to \nu_\tau H_{ud}(\gamma))}{\Gamma[\tau \to \nu_\tau e\bar{\nu}_e(\gamma)]} = 12\pi^2 |V_{ud}|^2 S_{EW} \int_0^{s_0} \frac{ds}{s_0} \left[ w_T(s; s_0) \rho_{V+A}^{(1+0)}(s) - w_L(s; s_0) \rho_{A}^{(0)}(s) \right] \]

\[ \alpha_s \text{ from non-strange hadronic } \tau \text{ decays – p.3/32} \]

-Create pseudo-data:

Boito et al. '11
Davier et al. '14

Since 2014 Aleph correlations underestimated.

\( t < 2014 \) Aleph correlations underestimated.
Theoretical Foundations (I)

Shankar ’77; Braaten-Narison-Pich ’92

- \[ \Pi(q^2) \]

“Cauchy’s Theorem” \((z = q^2 ; \rho(t) = \frac{1}{\pi} \text{Im}\Pi ; w_n = \text{polynomial}) : \]

\[
\int_{s_0}^{s_0} \, dt \, w_n(t) \, \rho(t) = \frac{1}{2i\pi} \oint_{|z|=s_0} dz \, w_n(z) \, \Pi(z)
\]

\[
= \frac{1}{2i\pi} \oint_{|z|=s_0} dz \, w_n(z) \left[ \Pi_{\text{OPE}}(z) + \Pi(z) - \Pi_{\text{OPE}}(z) \right]
\]

\[ \mathcal{O}(\alpha_s^4) \quad \Pi_{\text{DV}}(z) \]

\[ \leftrightarrow \]

\[ \Pi_{\text{DV}} \rightarrow 0 \iff \Pi_{\text{OPE}} \rightarrow \Pi. \]

(Cata-Golterman-S.P. ’05)

However,

- \[ \Pi_{\text{OPE}} \text{ expected asymptotic : } \Pi_{\text{DV}}(z) \rightarrow 0, \ z \rightarrow \infty. \]

- \[ \text{OPE no good on the Minkowski axis (spect. fnct. shows oscillations)} \]

\[ \implies \text{use polynomial } w_n(s_0) = 0 \quad (\equiv \text{"pinching"}) \]
Main Theoretical Message:
(Maltman-Yavin ’08, Boito et al. ’11)

★ No free lunch: with pinching one has a price to pay:

It is not possible to simultaneously suppress DVs and condensates.

★ “Seesaw” mechanism at work:
• Need a better control of systematic error

\[ \Rightarrow \text{need quantitative knowledge of DVs.} \]

• \[ \Pi_{DV}(s) \to 0, \text{ as } s \to \infty. \text{ Then:} \]

\[ -\frac{1}{2\pi i} \oint_{|z|=s_0} dz \ w(z) \ \Pi_{DV}(z) = -\int_{s_0}^{\infty} ds \ w(s) \ \frac{1}{\pi} \text{Im}\Pi_{DV}(s) \]

(Cata-Golterman-S.P. ’05)

extrapolation!
A phenomenological **educated guess**: (See Regge/Asymptotic Theory theoretical discussion later on...)

-For \( s \) large enough:

\[
\frac{1}{\pi} \text{Im} \Pi_{DV}(s) \simeq e^{-\delta} e^{-\gamma s} \sin(\alpha + \beta s)
\]

(asympt. exp. \( \Leftrightarrow \) recall renormalons \( \sim e^{-\gamma/\alpha s} \))

independently for \( V \) and \( A \) (i.e. 8 DV parameters in total).

- Assuming no DVs \( \equiv \) assuming \( e^{-\delta} = 0 \) \( \) (unlike data, which is not flat).
\textbf{Pert. Theory: CIPT vs. FOPT}

Caprini-Fischer ’09; Menke ’09; Descotes-Malaescu ’09; Cvetic ’10,...

\begin{itemize}
  \item Partial integration: \( \oint dz \; w_n(z) \; \Pi(z) = \oint dz \; \tilde{w}_n(z) \left( -z \frac{d}{dz} \Pi(z) \right) \)
  \item \( D(z) = \frac{1}{4\pi^2} \sum_{n,k} c_{n,k} \; \alpha_s(\mu)^n \; \log^k \left( -\frac{z}{\mu^2} \right) \)
  \item Since \( z = s_0 \; e^{i\phi} \), two (extreme) choices:
  \begin{itemize}
    \item \textbf{CIPT}: \( \mu^2 = -z \). Good \textit{if} Adler function series stops at finite order.
      \begin{itemize}
        \item LeDiberder-Pich ’92, Pivovarov ’92
      \end{itemize}
    \item \textbf{FOPT}: \( \mu^2 = s_0 \). Good \textit{if} Adler function series grows factorially (renormalons).
      \begin{itemize}
        \item Beneke-Jamin ’08
      \end{itemize}
  \end{itemize}
\end{itemize}

For many years thought to be the dominant source of error...
A Change of Strategy (I)

Old Strategy: (LeDiberder-Pich ’92)

- Use 5 pinched weights

\[ w_{kl}(y) = (1 - y)^2 (1 + 2y)(1 - y)^k y^l \], \( y = s/s_0 \), \( s_0 = m_\tau^2 \) (only)

with \((k, l) = \{(0, 0), (1, 0), (1, 1), (1, 2), (1, 3)\}\).

- Fit to extract 4 param. : \(\alpha_s\) and \(C_D=4,6,8\).

- Assume (unknown) OPE condensates \(C_D=10,12,14,16 = 0\). (~ OPE convergent)

- Assume (unknown) Duality Violations =0.

- May use \(V\) and \(A\), but assume \(V + A\) more reliable.

(Davier et al. ’14)

\[
\langle \frac{\alpha_s}{\pi} GG \rangle = (-0.5 \pm 0.3) \times 10^{-2} \text{ GeV}^4 , \quad \chi^2 = 0.43, \ p = 51\% \quad V ,
\]

\[
(-3.4 \pm 0.4) \times 10^{-2} \text{ GeV}^4 , \quad \chi^2 = 3.4, \ p = 7\% \quad A ,
\]

\[
(-2.0 \pm 0.3) \times 10^{-2} \text{ GeV}^4 , \quad \chi^2 = 1.1, \ p = 29\% \quad V + A .
\]

- Check Weinberg sum rules.
A Change of Strategy (II)

New Strategy (Boito et al. ’11 and ’12):

• Do not use $w(y)$ with a term linear in $y$. (Beneke et al. ’13)

• Do not assume any condensate is zero. (Let the data speak.)

• Do not assume that Duality Violations are zero. (Let the data speak.)

For $s \geq s_{\text{min}}$ (Regge/asymp. series assumption; see later on):

$$\rho_{DV}^{V,A}(s) = e^{-\delta_{V,A}-\gamma_{V,A}s} \sin (\alpha_{V,A} + \beta_{V,A}s)$$

c.f. old strategy model assumption: $e^{-\delta_{V,A}} = 0$.

• Fit to $\alpha_s, C_{D=6,8}$ and DV parameters with 3 weights:

$$w_0 = 1, w_2 = 1 - y^2 \quad \text{and} \quad w_3 = (1 - y)^2(1 + 2y)$$

Use all data for $s_0 \geq s_{\text{min}}$, ($s_{\text{min}}$ to be determined by the fit as well).

• Use $V$ and $A$. Check spectral functions.

• Check Weinberg sum rules.
We did lots of other fits as well...

Fits:

- V channel, \( w_0 = 1 \).
- V and A channels, \( w_0 = 1 \).
- V channel, \( w_0 = 1 \) and \( w_2 = 1 - y^2 \).
- V and A channels, \( w_0 = 1 \) and \( w_2 = 1 - y^2 \).
- V channel, \( w_0 = 1, w_2 = 1 - y^2 \) and \( w_3 = (1 - y)^2(1 + 2y) \).
- V and A channels, \( w_0 = 1, w_2 = 1 - y^2 \) and \( w_3 = (1 - y)^2(1 + 2y) \).

Consistent results in all cases.
Example: Fit to $w_0 = 1$, V channel (I).

$s_{\text{min}} = 1.55 \text{ GeV}^2$, $\chi^2/d\text{of} = 24.5/16$ ($p = 8\%$) (This is FOPT, CIPT similar)

curves: red = CIPT, blue = FOPT, black = no DV

$w_0 = 1$ spectral integral

ALEPH's V spectrum
Example: Fit to $w_0 = 1$, V channel (II).

$(68\% \text{ and } 95\% \text{ contour plots})$, FOPT.  
Clearly $DV \neq 0$.  

$\alpha_s$ from non-strange hadronic $\tau$ decays – p.14/32
Tests: $w_{11}, w_{12}, w_{13}$

Looking only at $s = m_{\tau}^2$ potentially misleading. (Maltman-Yavin '08).

(Davier et al. '14)

CIPT, V+A:

$w_{11}$

$w_{12}$

$w_{13}$

$\alpha_s$ from non-strange hadronic $\tau$ decays – p.15/32
$D > 8$ condensates vital!

**Tests:** $w_{11}, w_{12}, w_{13}$  

(Boito et al. ’15)

CIPT, V+A:

$w_{11}$,

$s_0 [\text{GeV}^2]$,

$w_{11} \text{ OPE+DV, spectral integrals}$

$w_{12}$,

$s_0 [\text{GeV}^2]$,

$w_{12} \text{ OPE+DV, spectral integrals}$

$w_{13}$,

$s_0 [\text{GeV}^2]$,

$w_{13} \text{ OPE+DV, spectral integrals}$

Not part of our fit!

Used $s_{\text{min}} = 1.55 \text{ GeV}^2$

$w = 1, 1 - y^2, w_\tau$
red=CIPT  blue =FOPT (Boito et al. ’15).

Weinberg sum rule: \( \int_0^\infty ds \left( \rho_V^{(1)}(s) - \rho_A^{(1)}(s) \right) - 2f_\pi^2 = 0 \)
Results

Aleph:

\[
\begin{align*}
\text{(FOPT)} & \quad \alpha_s(m_\tau) = 0.296 \pm 0.010 \rightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014 \\
\text{(CIPT)} & \quad \alpha_s(m_\tau) = 0.310 \pm 0.014 \rightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019
\end{align*}
\]

- N.B. “Old Strategy” produces a shift, i.e.

\[
\alpha_s(m_\tau) \sim +0.03 \text{ higher, (and } \sim \text{ half errors) (Davier et al. '14)}
\]

- Using **Aleph + Opal** data, we get:

\[
\alpha_s(m_Z) = 0.1165 \pm 0.0012 \text{ (FOPT)} \quad \alpha_s(m_Z) = 0.1185 \pm 0.0015 \text{ (CIPT)}
\]

  (Current PDG 2017 world average: \( \alpha_s(m_Z) = 0.1181 \pm 0.0011 \)).

\( \alpha_s \) from non-strange hadronic \( \tau \) decays – p.17/32
April 2016

$\alpha_s$ from non-strange hadronic $\tau$ decays – p.18/32
New reanalysis with the Old Strategy ⇒ same conclusions (not a surprise).
(and same $s_0$ dependence problems)
All criticisms have been addressed in Boito et al. Phys. Rev. \textbf{D95} (2017) 034024.

Example: when DVs are included they obtain (FOPT)

\begin{align*}
\alpha_s(m^2) & \text{ vs. } s_0 \\
\text{Number of points fitted} & \\
40 & 35 & 30 & 25 & 20 & 15 & 10
\end{align*}

$\alpha_s$ from non-strange hadronic $\tau$ decays – p.19/32

Question of Principle:
If modeling DVs is useful for determining condensates in $V - A$
(as, e.g., in Glez-Alonso, Pich, Rodriguez-Sanchez '15,'16)
why wouldn’t it be useful for $\alpha_s$ in $V + A$ as well?
Towards a Theory of DVs ? (I)

Boito, Caprini, Golterman, Maltman, SP ‘17

-A theory of DVs requires NP input.

- Let’s start with (exact !) Disp. Rel. of Adler’s function \( q^2 < 0 \), Euclidean:

\[
A(q^2) = -q^2 \int_{0}^{\infty} d\sigma \ e^{\sigma q^2} \ \sigma B(\sigma) , \quad B(\sigma) = \int_{0}^{\infty} dt \ \rho(t) \ e^{-\sigma t}
\]

\[\sigma q^2 = (\sigma > 0, q^2 < 0) = (\sigma < 0, q^2 > 0), \ i.e. \ \text{rotate } \sigma \leftrightarrow q^2 \ \text{analytic continuation.}\]
Towards a Theory of DVs? (II)

\[ \Pi_{DV}(q^2) \sim \]

\[ \alpha_s \text{ from non-strange hadronic } \tau \text{ decays} \]
Towards a Theory of DVs? (III)

- Asymptotic Regge-like spectrum ($\Lambda_{QCD} = 1$, Regge slope) for $N_c = \infty$ in chiral limit:
  - Masses
    \[ M^2(n) = n + b \log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right), \quad n \gg 1. \]
  - Decay constants
    \[ F(n) = 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right), \quad n \gg 1. \]

- When $N_c = 3$ ($d = 2$ QCD, strings, phenomenology):
  Blok et al. '98; Shifman et al. '08; Masjuan et al. '12
  \[ \varphi_{N_c} = - \frac{\Gamma}{M} \sim - \frac{a}{N_c} + ... \]
  we have a branch point in $\sigma$ plane at
  \[ \sigma_0 = 2\pi e^{i\Phi_0} + ... \quad , \quad \Phi_0 = \frac{a}{N_c} + \frac{\pi}{2} + ... \]
  producing ($\text{Re } q^2 > 0, \text{Im } q^2 > 0$)
  \[ \Pi_{DV}(q^2) \sim e^{-2\pi q^2 \frac{a}{N_c}} e^{i2\pi(q^2 - c - b \log q^2)} \left(1 + \mathcal{O}\left(\frac{1}{N_c}; \frac{1}{q^2}; \frac{1}{\log q^2}\alpha_s\right)\right) \]
Towards a Theory of DVs? (IV)

Agreement DVs in tau decay $\Leftrightarrow$ Regge spectrum.

Fits of meson spectrum to radial trajectories:

$\Lambda^2 = 1.35(4) \text{ GeV}^2$, \quad $\frac{\Gamma}{M} = 0.12(8) \simeq \frac{a}{N_c}$

leading to

$\beta_V = \frac{2\pi}{\Lambda^2} = 4.7(2) \text{ GeV}^2$, \quad $\gamma_V = \frac{2\pi}{\Lambda^2} \frac{a}{N_c} = 0.6(4) \text{ GeV}^{-2}$

to be compared with results from fits to $\tau$ decay:

$\beta_V = 4.2(5) \text{ GeV}^2$, \quad $\gamma_V = 0.7(3) \text{ GeV}^{-2}$

coincidence?,... Notice the $2\pi$. 

αs from non-strange hadronic $\tau$ decays – p.23/32
Conclusions and Outlook

- DVs are clearly visible in the data.

(DVs are not a question of principle, they exist in practice.)

- Pinching does not allow a simultaneous reduction of DVs and higher-dim condensates

(Unlike what has been assumed so far in the “Old Strategy" Method).

This introduced an unquantified systematic error.
Conclusions and Outlook (II)

- The new strategy using DV’s passes all known tests, experimental and theoretical, performing better than the "Old Strategy".

- New theoretical framework to analyze DVs, based on analytical properties in the Borel-Laplace variable $\sigma$.

So far, analytic result for $II_{DV}$ based on asymptotic Regge espectrum. We think that this result is general, though (at least in the chiral limit).

Generalization to heavy quarks?

- Better data (Babar and Belle ?) would help significantly.

    We are currently working on the analysis using $e^+e^-$ data...
THANK YOU!
Spectral Functions (I)

Sometimes $V + A$ is presented as “evidence” that $D_Vs \simeq 0$ at large $s$:

![Graph showing $\rho_{V+A}(s)$ vs. $s$]

However...
Spectral Functions (II)

A closer look reveals a (partial) fortuitous cancellation for $2 \text{GeV}^2 \lesssim s \lesssim 2.8 \text{GeV}^2$

There is no reason why it should persist at higher $s$:

The $V + A$ oscillation at $s = 3 \text{GeV}^2$ is larger than at $2.5 \text{GeV}^2$. 
Comments on $V + A$

- DV oscillations are still present in $V + A$ (although of a smaller size than in $V$ and $A$).

- Since we have a good representation of $V$ and $A$, we also have it of their sum $V + A$.

- Fit results show that DV’s exponent in $A$ is $\ll$ than in $V$, so the reduction for $s \sim 2 - 3 \, \text{GeV}^2$ is accidental. At still larger $s$, the DVs in $V$ will dominate.

- The expectation of strong cancellation of $D = 6$ terms in OPE in $V + A$ is based on the vacuum saturation ansatz. However, the data shows that this ansatz is a very poor approximation.
Asymptotic vs. Convergent

\[ f(z) = \sum_{n=1}^{\infty} C(n) \ z^n \]

\[ C(n) = \left( \frac{10^n}{n!} \right), \quad \text{Convergent.} \]

\[ C(n) = \left( \frac{n!}{10^n} \right), \quad \text{Divergent.} \]

Poincaré (circa 1893): discussion Geometers vs. Astronomers ⇔ Convergent vs. Divergent
\[ OPE + DV \ (N_c = \infty) \]

-Asymptotic Regge behavior:

\[
M^2(n) = n + b \log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right)
\]

\[
F(n) = 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right)
\]

produces a Dirichlet series

\[
\sigma B(\sigma) = \sum_{n}^{\infty} F(n) \ e^{-\sigma M^2(n)} = 1 + \mathcal{O}\left(\log^{k_1} \sigma\right) + \mathcal{O}\left(\sigma^{k_2} \log^{k_3} \sigma\right) + \mathcal{O}\left(\frac{1}{(\sigma - \hat{\sigma})^{1+\gamma}}\right)
\]

Pert. Theory \hspace{1cm} Power Corrections \hspace{1cm} Duality Violations

and leads to

\[
A(q^2) = 1 + \mathcal{O}\left(\frac{1}{\log^{p_1} (-q^2)}\right) + \mathcal{O}\left(\frac{\log^{p_2} (-q^2)}{(q^2)^{p_3}}\right) + \mathcal{O}\left(e^{\hat{\sigma} q^2 (-q^2) \gamma}\right)
\]

Pert. Theory \hspace{1cm} Power Corrections \hspace{1cm} Duality Violations

\[ \alpha_s \text{ from non-strange hadronic } \tau \text{ decays – p.32/32} \]