Precision Flavour Physics and Lattice QCD: A path to discovering new physics

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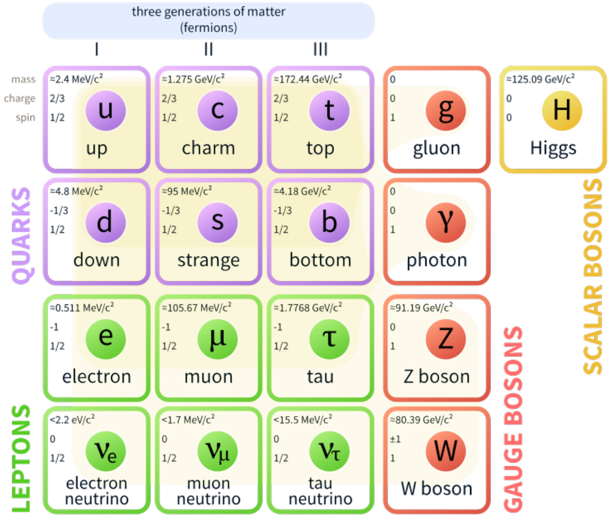
Standard Model of Elementary Particles



Why 3 generations?

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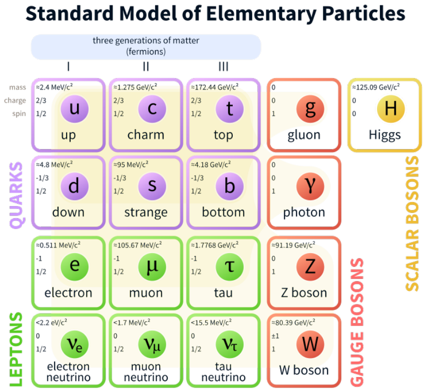
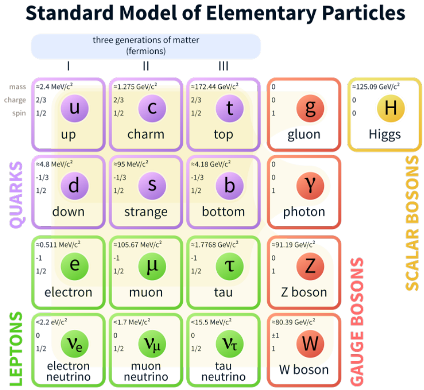


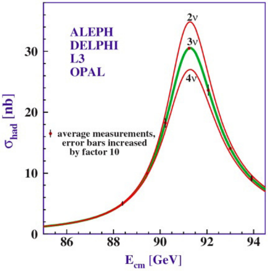
- Who ordered that?

I.I.Rabi, 1936

Discovery of the muon



- Z_0 width



$$N_\nu = 2.9840 \pm 0.0082$$

PDG 2016

There are many reasons to believe that the Standard Model is incomplete:

- Why are the charges of the proton and electron equal and opposite:

$$\frac{Q_p + Q_e}{e} < 1 \times 10^{-21}\,.$$

  - Unification of forces?
  - Cancellation of anomalies?

The SM is incomplete

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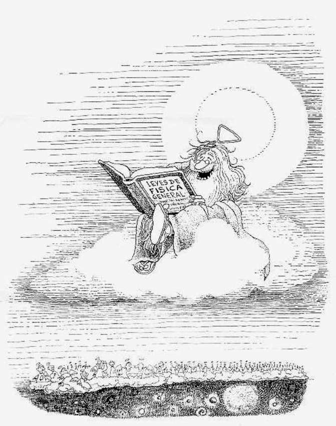
- Why are the charges of the proton and electron equal and opposite:

$$\frac{Q_p + Q_e}{e} < 1 \times 10^{-21} .$$

  - Unification of forces?
  - Cancellation of anomalies?

- nature of dark matter and dark energy;
- naturalness and mass hierarchies;
- strong CP-problem;
- origin of neutrino masses;
- gravity, $\cdots$



1. General introduction
2. Brief introduction to flavour physics
3. Brief introduction to lattice Quantum Chromodynamics (QCD)
4. Novel directions in lattice flavour physics
   - $K \to \pi\pi$ decays
5. Two *tensions*
   - Lepton flavour violation?
   - $(g-2)_\mu$
6. Summary, prospect and conclusions

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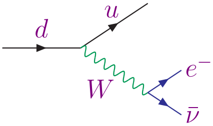
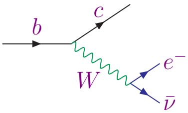
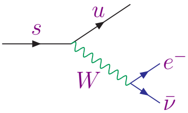
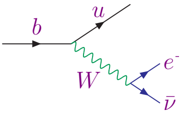
- At the level of quarks we understand nuclear β decay in terms of the fundamental process:



- With the 3 generations of quarks and leptons in the standard model this is generalized to other *charged current* processes, e.g.:



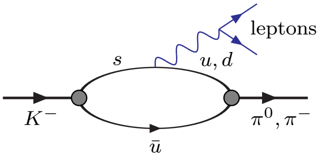




- Weak interaction eigenstates $\neq$ mass eigenstates.

Two Experimental Numbers:

$$B(K^- \to \pi^0 e^- \bar{\nu}_e) \simeq 5\% \ (K_{e3} \text{ Decay}) \quad \text{and} \quad B(K^- \to \pi^- e^+ e^-) = (3.00 \pm 0.09) \times 10^{-7}\,.$$



- Measurements like this show that $s \to u$ (charged-current) transitions are not very rare, but that *Flavour Changing Neutral Current* (FCNC) transitions, such as $s \to d$ are.
  - Since FCNC processes are *rare* in the SM, they provide an excellent laboratory for searches for new physics.
- The existence of decays such as $K^- \to \pi^0 e^- \bar{\nu}_e$ implies that we need to have a mechanism for transitions between quarks of different generations.
- The picture which has emerged is the Cabibbo-Kobayashi-Maskawa (CKM) theory of quark mixing.

Weak interaction eigenstates $\neq$ mass eigenstates:

$$U_W = \begin{pmatrix} u_W \\ c_W \\ t_W \end{pmatrix} = U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix} = U_u\, U \quad \text{and} \quad D_W = \begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix} = U_d D$$

where U_u and U_d are unitary matrices.

- For neutral currents:

$$\bar{U}_W \cdots U_W = \bar{U} \cdots U \quad \text{and} \quad \bar{D}_W \cdots D_W = \bar{D} \cdots D$$

  and no FCNC are induced. The $\cdots$ represent Dirac Matrices, but the identity in flavour.

- For charged currents:

$$J_W^{\mu\,+} = \frac{1}{\sqrt{2}}\,\bar{U}_W \gamma_L^\mu D_W = \frac{1}{\sqrt{2}}\,\bar{U}_L \gamma^\mu (U_u^\dagger U_d) D_L \equiv \frac{1}{\sqrt{2}}\,\bar{U} \gamma_L^\mu V_{\text{CKM}} D$$

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- The charged-current interactions are of the form

$$J_\mu^+ = (\bar{u}, \bar{c}, \bar{t}\,)_L \gamma_\mu V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

- 2016 Particle Data Group summary for the magnitudes of the entries:

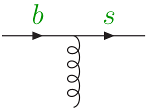
$$\begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}$$

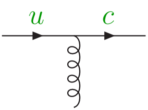
$\Rightarrow$ we can write (Wolfenstein parametrisation)

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)\,.$$

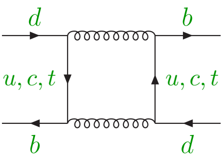
- A, ρ and η are real numbers that a priori were intended to be of order unity.

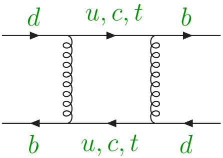
We have seen that in the SM, unitarity implies that there are no FCNC reactions at tree level, i.e. there are no vertices of the type:



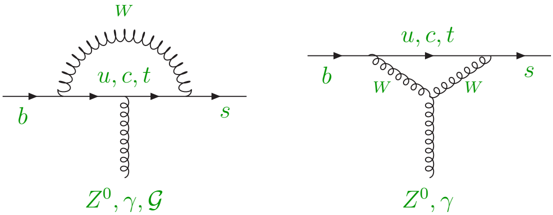


Quantum loops, however, can generate FCNC reactions, through *box* diagrams or *penguin* diagrams.
Example relevant for $\bar{B}^0 - B^0$ mixing:





Examples of penguin diagrams relevant for $b \to s$ transitions:



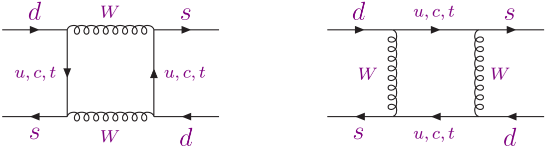
We will discuss several of the physical processes induced by these loop-effects. The Glashow-Illiopoulos-Maiani (GIM) mechanism $\Rightarrow$ FCNC effects vanish for degenerate quarks ($m_u = m_c = m_t$). For example unitarity implies

$$V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0$$

$\Rightarrow$ each of the above penguin vertices vanish.

- Consider the neutral-kaon system:
  - Strong interaction eigenstates: $|K_0\rangle = |\bar{s}d\rangle$ and $|\bar{K}_0\rangle = |s\bar{d}\rangle$.
  - CP-eigenstates: $|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K_0\rangle \pm |\bar{K}_0\rangle)$.
  - Mass eigenstates: $|K_S\rangle \propto (|K_1\rangle + \varepsilon|K_2\rangle)$ and $|K_L\rangle \propto (|K_2\rangle + \varepsilon|K_1\rangle)$.

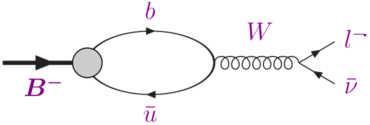
$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6)\times 10^{-12}\,\text{MeV} \ll \Lambda_{\text{QCD}}.$$



- It is frequently said that Flavour Physics can probe scales which are unreachable in colliders.
  - Here, if we could reproduce the experimental value of Δm_K in the SM to 10% accuracy and if we imagine an effective new-physics $\Delta S = 2$ contribution $\frac{1}{\Lambda^2}(\bar{s}\cdots d)(\bar{s}\cdots d)$ then $\Lambda \gtrsim (10^3 - 10^4)$ TeV.

- We (RBC-UKQCD collaboration) are well on our way to an *ab initio* calculation of Δm_K.

- The difficulty in making predictions for weak decays of hadrons is in controlling the non-perturbative strong interaction effects.
- As a particularly simple example consider the leptonic decays of pseudoscalar mesons in general and of the B-meson in particular.



- Non-perturbative QCD effects are contained in the matrix element

$$\langle 0|\,\bar{b}\gamma^\mu(1-\gamma^5)u\,|B(p)\rangle\ .$$

- Lorentz Inv. + Parity $\Rightarrow\ \langle 0|\,\bar{b}\gamma^\mu u\,|B(p)\rangle = 0.$
- Similarly $\quad \langle 0|\,\bar{b}\gamma^\mu\gamma^5 u\,|B(p)\rangle = if_B p^\mu\ .$

All QCD effects are contained in a single constant, f_B, the B-meson's *(leptonic) decay constant.* ($f_\pi \simeq 132$ MeV)

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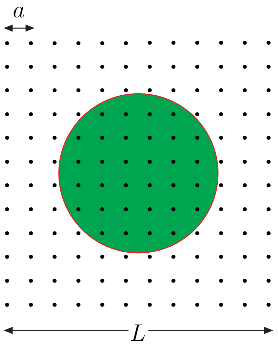
2. Introduction to Lattice QCD

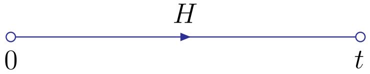


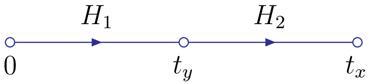
- Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0|\, O(x_1,x_2,\cdots,x_n)\, |0\rangle \;=\; \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\bar\psi]\, e^{-S}\, O(x_1,x_2,\cdots,x_n)\,,$$

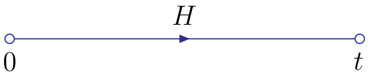
where $O(x_1,x_2,\cdots,x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function.

- The physics which can be studied depends on the choice of the multilocal operator O.





- The functional integral is performed by discretising Euclidean space-time and using Monte-Carlo Integration.

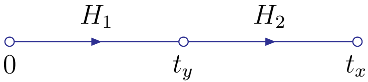


$$
\begin{aligned}
C_2(t) &= \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, \phi(\vec{x},t)\, \phi^\dagger(\vec{0},0)\, |0\rangle \\
&= \sum_n \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, \phi(\vec{x},t)\, |n\rangle\, \langle n|\phi^\dagger(\vec{0},0)\, |0\rangle \\
&= \int d^3x e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, \phi(\vec{x},t)\, |H\rangle\, \langle H|\phi^\dagger(\vec{0},0)\, |0\rangle + \cdots
\end{aligned}
$$

$$
= \frac{1}{2E} e^{-iEt} \left|\langle 0|\phi(\vec{0},0)|H(p)\rangle\right|^2 + \cdots \Rightarrow \frac{1}{2E} e^{-Et} \left|\langle 0|\phi(\vec{0},0)|H(p)\rangle\right|^2 + \cdots \quad \text{(Euclidean)}
$$

where $E = \sqrt{m_H^2 + \vec{p}^{\,2}}$ and we have taken H to be the lightest state created by $\phi^\dagger$.
The $\cdots$ represent contributions from heavier states.

- By fitting $C(t)$ to the form above, both the energy (or, if $\vec{p}=0$, the mass) and the modulus of the matrix element $\left|\langle 0|J(\vec{0},0)|H(p)\rangle\right|$ can be evaluated.
- Example: if $\phi = \bar{b}\,\gamma^\mu\gamma^5 u$ then the decay constant of the B-meson can be evaluated, $\left|\langle 0|\,\bar{b}\,\gamma^\mu\gamma^5 u\,|B^+(p)\rangle\right| = f_B\, p^\mu$.



$$C_3(t_x,t_y) \;=\; \int d^3x\,d^3y\; e^{i\vec p\cdot\vec x}\; e^{i\vec q\cdot\vec y}\; \langle 0|\,\phi_2(\vec x,t_x)\,O(\vec y,t_y)\,\phi_1^\dagger(\vec 0,0)\,|0\rangle\;,$$

$$\simeq \frac{e^{-E_1 t_y}}{2E_1}\;\frac{e^{-E_2(t_x-t_y)}}{2E_2}\;\langle 0|\phi_2(0)|H_2(\vec p)\rangle\langle H_2(\vec p)|O(0)|H_1(\vec p+\vec q)\rangle\langle H_1(\vec p+\vec q)|\phi_1^\dagger(0)|0\rangle\;,$$

for sufficiently large times t_y and $t_x - t_y$ and $E_1^2 = m_1^2 + (\vec p+\vec q)^2$ and $E_2^2 = m_1^2 + \vec p^{\,2}$.

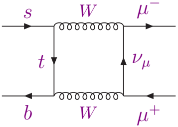
- Thus from 2- and 3-point functions we obtain transition matrix elements of the form $|\langle H_2|O|H_1\rangle|$.
- Important examples include $\langle \bar K^0|(\bar s\gamma_L^\mu d)\,(\bar s\gamma_{\mu\,L} d)|K^0\rangle$ and $\langle \pi^0|(\bar s\gamma^\mu u)|K^+\rangle$.

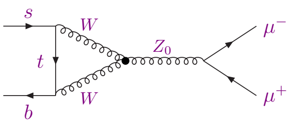
- In Lattice QCD, while it is natural to think in terms of the lattice spacing a, the input parameter is $\beta = 6/g^2(a)$.
- Imagine performing a simulation with $N_f = 2+1$ with $m_{ud} = m_u = m_d$ around their "physical" values.
- At each β, take two dimensionless quantities, e.g. m_π/m_Ω and m_K/m_Ω, and find the bare quark masses m_{ud} and m_s which give the corresponding physical values.These are then defined to be the physical (bare) quark masses at that β.
- Now consider a dimensionful quantity, e.g. m_Ω. The value of the lattice spacing is defined by

$$a^{-1} = \frac{1.672\,\mathrm{GeV}}{m_\Omega(\beta, m_{ud}, m_s)}$$

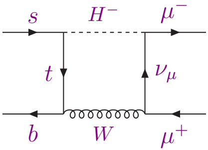
  where $m_\Omega(\beta, m_{ud}, m_s)$ is the computed value in lattice units.
- Other physical quantities computed at the physical bare-quark masses will now differ from their physical values by artefacts of $O(a^2)$.
- Repeating this procedure at different β defines a scaling trajectory. Other choices for the 3 physical quantities used to define the trajectory are clearly possible.
- If the simulations are performed with m_c and/or $m_u \neq m_d$ then the procedure has to be extended accordingly.

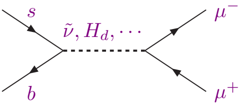
- Within the Standard Model there are box and penguin diagrams leading to the decay $B_S \to \mu^+\mu^-$:





- We shall see that the SM branching ratio is tiny and BSM there are many other potential contributions e.g.:





- This decay therefore constrains models of new physics and their parameter space.

- Standard Model prediction for $B(B_s \to \mu^+\mu^-) \propto f_{B_s}^2 \times \frac{m_\mu^2}{m_{B_s}^2}$:

$$B(B_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}\,.$$

C.Bobeth et al, arXiv:1311.0903

- Particle Data Group results for the branching ratios over the past decade:

Year	PDG $B(B_s \to \mu^+)$
2006	$< 1.5 \times 10^{-7}$
2008,2010	$< 4.7 \times 10^{-8}$
2012	$< 6.4 \times 10^{-9}$
2014	$(3.1 \pm 0.7) \times 10^{-9}$
2016,2017	$(2.4^{+0.9}_{-0.7}) \times 10^{-9}$

- 2017 update from LHCb $\quad B(B_s \to \mu^+\mu^-) = \left(3.0 \pm 0.6^{+0.3}_{-0.2}\right) \times 10^{-9}\,.$

- Most of the compilations in this talk are taken from the current results of the FLAG collaboration: "Review of lattice results concerning low energy particle physics,"
  S. Aoki, Y. Aoki, D.Bečirević, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, P. Dimopoulos, S. Dürr, H. Fukaya, M. Golterman, S. Gottlieb, S. Hashimoto, U.M. Heller, R. Horsley, A. Jüttner, T. Kaneko, L. Lellouch, H. Leutwyler, C.-J. Lin, V. Lubicz, E. Lunghi, R. Mawhinney, T. Onogi, C. Pena, C. Sachrajda, S. Sharpe, S. Simula, R. Sommer, A. Vladikas, U. Wenger, H. Wittig. EPJC **77** (2017) 112; arXiv:1607.00299 (383 pages!)

- This third edition is an extension and continuation of the work of the Flavianet Lattice Averaging Group:
  G. Colangelo, S. Dürr, A. Juttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. T. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig arXiv:1011.4408

- Motivation - to present to the wider community an *average* of lattice results for important quantities obtained after a critical expert review.
- Danger - It is important that original papers (particularly those which pioneer new techniques) get recognised and cited appropriately by the community.
- The closing date for arXiv:1607.00299 was Nov 30th 2015.

FLAG summary in light-quark physics

Quantity	■	$N_f=2+1+1$	■	$N_f=2+1$	■	$N_f=2$
m_s(MeV)	2	93.9(1.1)	5	92.0(2.1)	2	101(3)
m_{ud}(MeV)	1	3.70(17)	5	3.373(80)	1	3.6(2)
m_s/m_{ud}	2	27.30(34)	4	27.43(31)	1	27.3(9)
m_d(MeV)	1	5.03(26)	Flag(4)	4.68(14)(7)	1	4.8(23)
m_u(MeV)	1	2.36(24)	Flag(4)	2.16(9)(7)	1	2.40(23)
m_u/m_d	1	0.470(56)	Flag(4)	0.46(2)(2)	1	0.50(4)
m_c/m_s	3	11.70(6)	2	11.82	1	11.74
$f_+^{K\pi}(0)$	1	0.9704(24)(22)	2	0.9667(27)	1	0.9560(57)(62)
f_{K^+}/f_{π^+}	3	1.193(3)	4	1.192(5)	1	1.205(6)(17)
f_K(MeV)	3	155.6(4)	3	155.9(9)	1	157.5(2.4)
f_π(MeV)			3	130.2(1.4)		
$\Sigma^{\frac{1}{3}}$(MeV)	1	280(8)(15)	4	274(3)	4	266(10)
F_π/F	1	1.076(2)(2)	5	1.064(7)	4	1.073(15)
$\bar{\ell}_3$	1	3.70(7)(26)	5	2.81(64)	3	3.41(82)
$\bar{\ell}_4$	1	4.67(3)(10)	5	4.10(45)	2	4.51(26)
$\hat{B}_K$	1	0.717(18)(16)	4	0.7625(97)	1	0.727(22)(12)

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Quantity	■	$N_f=2+1+1$	■	$N_f=2+1$	■	$N_f=2$
f_D(MeV)	2	212.15(1.45)	2	209.2(3.3)	1	208(7)
f_{D_s}(MeV)	2	248.83(1.27)	3	249.8(2.3)	1	250(7)
f_{D_s}/f_D	2	1.716(32)	2	1.187(12)	1	1.20(2)
$f_+^{D\pi}(0)$			1	0.666(29)		
$f_+^{DK}(0)$			1	0.747(19)		
f_B(MeV)	1	186(4)	4	192.0(4.0)	3	188(7)
f_{B_s}(MeV)	1	224(5)	4	228.4(3.7)	3	227(7)
f_{B_s}/f_B	1	1.205(7)	4	1.201(17)	3	1.206(23)
$f_{B_d}\sqrt{\hat{B}_{B_d}}$(MeV)			2	219(14)	1	216(10)
$f_{B_s}\sqrt{\hat{B}_{B_s}}$(MeV)			2	270(16)	1	262(10)
$\hat{B}_{B_d}$			2	1.26(9)	1	1.30(6)
$\hat{B}_{B_s}$			2	1.32(6)	1	1.32(5)
ξ			2	1.239(46)	1	1.225(31)
$\hat{B}_{B_s}/\hat{B}_{B_d}$			2	1.039(63)	1	1.007(21)

$$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1182(12) \quad \text{from 5 papers}$$
$$\Lambda^{(5)}_{\overline{\rm MS}} = 211(14)\,\text{MeV} \quad \text{from 5 papers}$$

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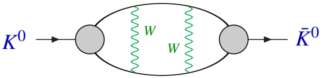
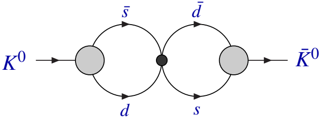
- "Standard" lattice calculations in flavour physics are of matrix elements of local operators between single hadron states $\langle h_2(p_2)|O(0)|h_1(p_1)\rangle$ (or $\langle 0|O(0)|h(p)\rangle$).
- For example, in the evaluation of ε_K, we need to calculate (schematically)



(gluons and quark loops not shown.)

- The process is short-distance dominated and so we can approximate the above by a perturbatively calculable (Wilson) coefficient C times



where the black dot represents the insertion of the local operator $(\bar{s}\gamma_\mu(1-\gamma^5)d)\,(\bar{s}\gamma_\mu(1-\gamma^5)d)$.

  - In the standard model only this single operator contributes.
  - In generic BSM theories there are 5 possible $\Delta S=2$ operators contributing.
- RBC-UKQCD collaboration developing methods for the evaluation of long-distance effects (Δm_K, $\varepsilon_K, A(K\to\pi\ell^+\ell^-), A(K^+\to\pi\nu\bar{\nu})$).

The RBC & UKQCD collaborations

BNL and RBRC

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Taku Izubuchi
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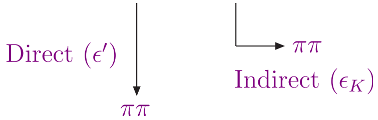
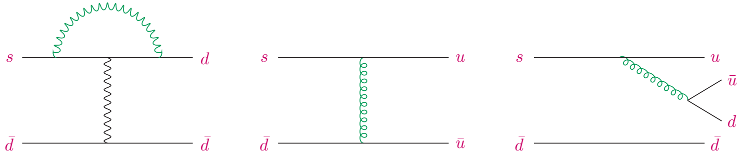
- $K \to \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.
- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.
- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule (Re A_0/Re $A_2 \simeq 22.5$) and an understanding of the experimental value of ε'/ε, the parameter which was the first experimental evidence of direct CP-violation.
- The evaluation of $K \to \pi\pi$ matrix elements requires an extension of the standard computations of $\langle 0\,|\,O(0)\,|\,h\rangle$ and $\langle h_2\,|\,O(0)\,|\,h_1\rangle$ matrix elements with a single hadron in the initial and/or final state.

- Directly CP-violating decays are those in which a CP-even (-odd) state decays into a CP-odd (-even) one: $K_L \;\propto\; K_2 \;+\; \bar{\epsilon} K_1$.



- Consider the following contributions to $K \to \pi\pi$ decays:



$I = 0$, Complex (a) | $I = 0$, Real (b) | $I = 0$ or 2, Real (c)

Direct CP-violation in kaon decays manifests itself as a non-zero relative phase between the $I = 0$ and $I = 2$ amplitudes.

- We also have *strong phases*, δ_0 and δ_2 which are independent of the form of the weak Hamiltonian.

- In 2015 RBC-UKQCD published our first result for ε'/ε computed at physical quark masses and kinematics, albeit still with large relative errors:

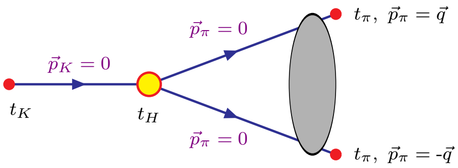
$$\frac{\varepsilon'}{\varepsilon}\bigg|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

to be compared with

$$\frac{\varepsilon'}{\varepsilon}\bigg|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

- This is by far the most complicated project that I have ever been involved with.
- This single result hides much important (and much more precise) information which we have determined along the way.



- $K \to \pi\pi$ correlation function is dominated by lightest state, i.e. the state with two-pions at rest. Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003

For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.

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- These are based on the Poisson summation formula:

$$\frac{1}{L}\sum_{n=-\infty}^{\infty} f(p_n^2) = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\, f(p^2) + \sum_{n\neq 0}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\, f(p^2)e^{inpL}\,,$$

- For single-hadron states the finite-volume corrections decrease exponentially with the volume $\propto e^{-m_\pi L}$. For multi-hadron states, the finite-volume corrections generally fall as powers of the volume.
- For two-hadron states, there is a huge literature following the seminal work by Lüscher and the effects are generally understood.
  - The spectrum of two-pion states in a finite volume is given by the scattering phase-shifts. M. Luscher, Commun. Math. Phys. 105 (1986) 153, Nucl. Phys. B354 (1991) 531.
  - The $K\to\pi\pi$ amplitudes are obtained from the finite-volume matrix elements by the Lellouch-Lüscher factor which contains the derivative of the phase-shift. L.Lellouch & M.Lüscher, hep-lat/:0003023, C.h.Kim, CTS & S.R.Sharpe, hep-lat/0507006 $\cdots$
  - Recently we have also determined the finite-volume corrections for $\Delta m_K = m_{K_L} - m_{K_S}$. N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170
- For three-hadron states, there has been a major effort by Hansen and Sharpe leading to much theoretical clarification.

M.Hansen & S.Sharpe, arXiv:1408.4933, 1409.7012, 1504.04248

- It is necessary to improve the statistics to establish that the results are robust.
  - 2015 PRL - Measurements were performed on 216 configurations.
    We currently (25th May 2017) have 836 additional independent configurations on 304 of which measurements have been made.
    - June 7th 2017 - 889 independent configurations on which measurements have been made on 352.
    - October 9th 2017 - > 1000 independent configurations on which measurements have been made on 841.
  - Each additional independent G-parity configuration took 31.2 hours to generate on 512 nodes BG/Q and a set of measurements on one configuration takes 18.8 hours.
  - The gauge configuration generation has been reduced to 7.6 hours per independent configuration by the use of an *exact one flavour algorithm*.

    Y-C Chen & T-W Chiu, arXiv1403.1683; D.J.Murphy, arXiv:1611.00298
  - We envisage presenting updated results from ≥ 1500 configurations in early 2018, including important systematic improvements.

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- Defining

$$R_H = \frac{\int dq^2 \, \frac{d\Gamma(B\to H\mu^+\mu^-)}{dq^2}}{\int dq^2 \, \frac{d\Gamma(B\to He^+e^-)}{dq^2}}$$

where $H = K, K^*$ and $q^2 = (p_{\ell^+} + p_{\ell^-})^2$, the LHCb collaboration find

$$R_{K^+} = 0.745^{+0.090}_{-0.074} \pm 0.036 \text{ for } 1 < q^2 < 6\,\text{GeV}^2$$ arXiv:1406.6482

$$R_{K^{*0}} = 0.66^{+0.11}_{-0.07} \pm 0.03 \text{ for } 0.045 < q^2 < 1.1\,\text{GeV}^2$$ arXiv:1705.05802

$$R_{K^{*0}} = 0.69^{+0.11}_{-0.07} \pm 0.05 \text{ for } 1.1 < q^2 < 6\,\text{GeV}^2$$ arXiv:1705.05802

- For R_K above and the higher q^2 value of R_{K^*} the SM theoretical prediction is 1 to within 1% or so.
- For R_{K^*} at lower q^2 the theoretical uncertainty is a little larger, e.g. Bordone, Isidori and Pattori find

$$R^{\text{SM}}_{K^{*0}} = 0.906 \pm 0.028 \text{ for } 0.045 < q^2 < 1.1\,\text{GeV}^2$$ arXiv:1705.05802

and advocate raising the lower limit to 0.1 GeV^2 in future analyses (which would decrease the theoretical uncertainty to 0.014).

- For R_K and R_{K^*} hadronic uncertainties do not play a rôle.

- The magnetic moment $\vec{\mu}$ determines the energy shift of a particle in a magnetic field $\vec{B}$: $V = -\vec{\mu}\cdot\vec{B}$.
- If the particle has spin $\vec{S}$, this contributes

$$\vec{\mu} = g\frac{e}{2m}\vec{S}\,.$$

- The anomalous magnetic moment $a = \frac{g-2}{2}$ accounts for the radiative corrections to Dirac's value $g = 2$.
- For the muon the experimental result is

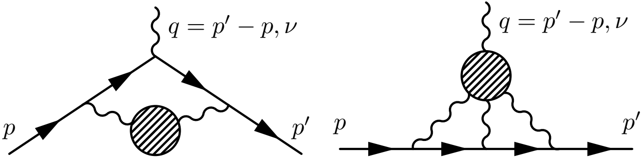
$$a_\mu^{\rm exp} = (11659209.1 \pm 6.3)\times 10^{-10}$$

BNL E821, hep-ex:0602035

Theory status for a_μ – summary

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

We currently observe a $\sim 3\sigma$ tension.



(L) HVP (R) HLbL

- Currently the statistical error for the *pure* lattice result for the HVP is about $\pm 15 \times 10^{-10}$.
  - Reaching the current experimental precision ($\pm 5 \times 10^{-10}$) is expected within the next few years.
  - “We have not run out of knobs to turn yet."
- The evaluation of the HLbL contribution is less advanced.
  - Result from a pioneering calculation on a (5.5 fm)^3 lattice with $a^{-1} = 1.73$ GeV gave $a_\mu^{\rm HLbL} = (5.35 \pm 1.35) \times 10^{-10}$. T.Blum et al, arXiv:1610.04603
  - Large effort being invested by this collaboration to control the systematics, and in particular the finite-volume effects. T.Blum et al, arXiv:1705.01067

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Summary, Prospects and Conclusions

- Precision flavour physics is a particularly powerful tool for exploring the limits of the standard model and for searching for the effects of new physics.
- If, as expected/hoped the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
- The rôle of Lattice QCD is to evaluate the hadronic effects from first principles and with controllable uncertainties.
  - In recent years, for many important quantities the precision of lattice calculations has approached, or even exceeded $O(1\%)$.
  - This implies that for further progress to me made electromagnetic and other isospin-breaking effects have to be included.
    This is currently a major area of research and development.
- Other ongoing developments include the evaluation of $K \to \pi\pi$ matrix elements and the evaluation of long-distance contributions to Δm_K, ε_K and to the rare decays $K^+ \to \pi^+ \nu\bar{\nu}$. RBC-UKQCD Collaboration
- I have not had time here to do justice to the huge effort in b and c physics undertaken by all collaborations.