Top Production Cross Section -

Matching of Relativistic and Non-Relativistic Regimes

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in collaboration with

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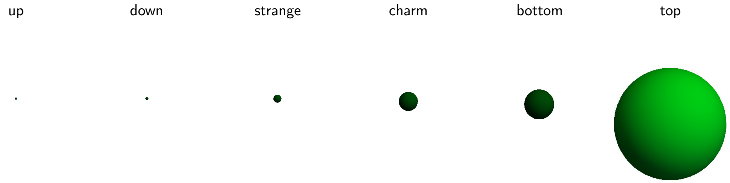
Motivation

top quark properties: $m_t^{pole} \approx 170\,\text{GeV}$, $\Gamma_t \approx 2\,\text{GeV}$

precise measurement of top quark properties for:

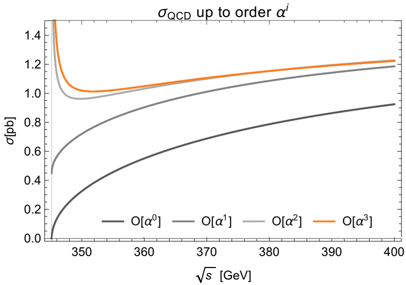
- stability of the SM vacuum
- electroweak precision tests
- new physics searches
- ...

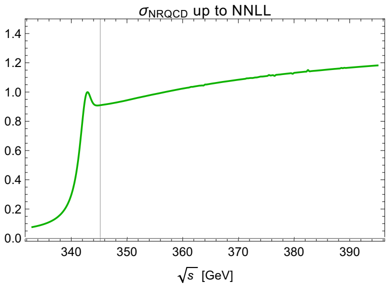


Motivation

Top production cross section in the pole mass scheme:

$$\sigma\ (\ e^+e^- \longrightarrow t\ \bar{t}\ )$$

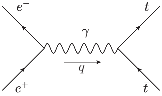




Relativistic Regime

Relativistic Regime

$$\sigma_{t\bar{t}} = \int d\Pi \Bigg|$$



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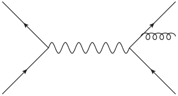

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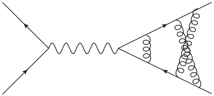

$$+ \dots \Bigg|^2$$

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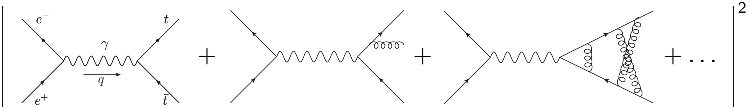
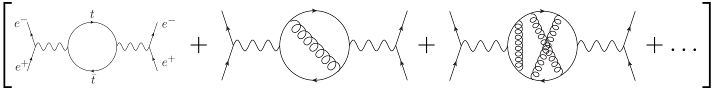


$= \mathrm{Im}$



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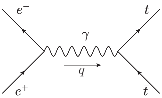

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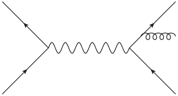
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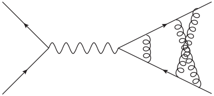
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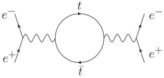

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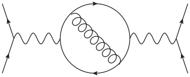
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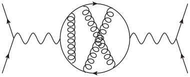
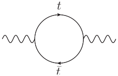

$$+ \ldots \Bigg]$$

$$= \frac{(4\pi\alpha)^2}{q^2} Q_t^2 \ \mathrm{Im}\Bigg[$$


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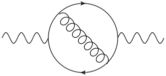
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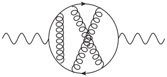
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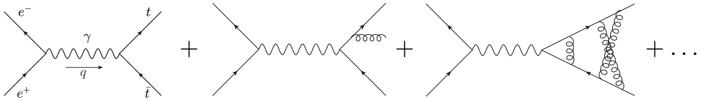
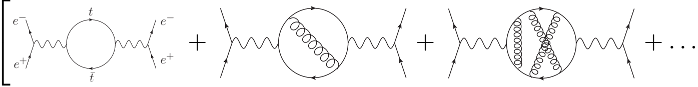


$$= \mathrm{Im}\left[\quad \right]$$



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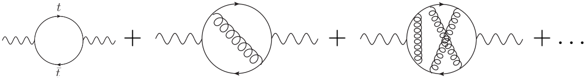

$$= A(q^2)\ \mathrm{Im}\left[-i \int d^4x \ e^{iqx} \langle 0|\mathrm{T}\, j_\mu(x)\, j^\mu(0)|0\rangle \right] , \quad j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$$

Relativistic Regime

✓ Vacuum Polarization known up to α^3 [Hoang, Mateu, Zebarjad '08]

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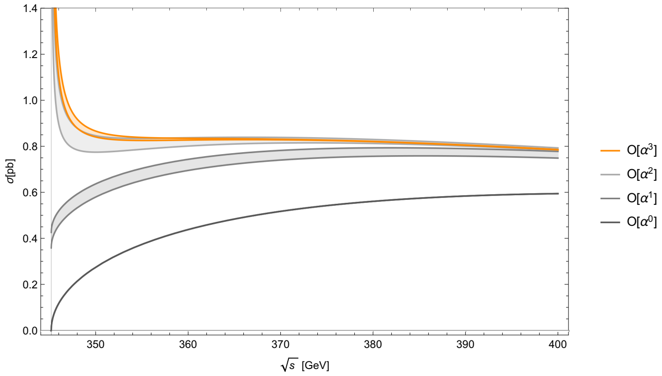
σ_{QCD} up to O[α^{i}]



Non-Relativistic Regime

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- scales: m, $|\vec{p}| \sim mv$, $E \sim mv^2$ $\quad$ $(m_t \sim 170\,\text{GeV}$, $v \sim \alpha \sim 0.1)$

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- problems:

1. large logarithms $\quad \log\left(\frac{E^2}{m^2}\right)$, $\log\left(\frac{\vec{p}^2}{m^2}\right)$

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– problems:

1. large logarithms $\log\left(\frac{E^2}{m^2}\right)$, $\log\left(\frac{\vec{p}^2}{m^2}\right)$
2. Coulomb singularity contributions of order $\frac{\alpha^2}{v^3}, \frac{\alpha^3}{v^4}, \frac{\alpha^4}{v^5}, \ldots$

Non-Relativistic Regime

Fields in vNRQCD and pNRQCD:

field	(E, m)
potential quark	(mv^2, mv)
soft gluon	(mv, mv)
ultrasoft gluon	(mv^2, mv^2)

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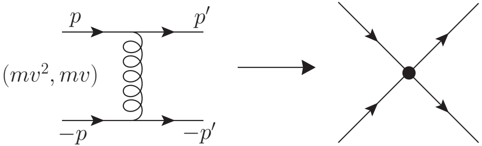
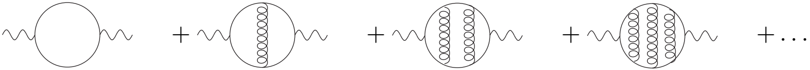
Example:



$\sim \frac{\mathcal{V}_c}{(\vec{p}-\vec{p}\,')^2}$

Coulomb Singularity: LO



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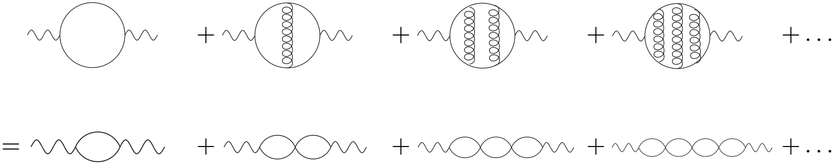

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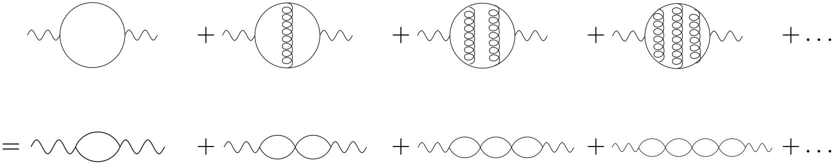

v $\qquad$ α_s $\qquad$ α_s^2/v $\qquad$ α_s^3/v^2

$$\alpha_s \sim v!$$

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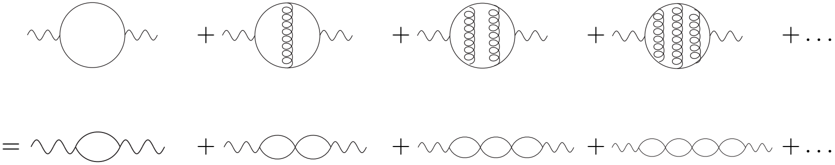

$\alpha_s \sim v!$

$$\sim \int \frac{d^3p}{(2\pi)^3} \left(\frac{-1}{E - \vec{p}^{\,2}/m + i\varepsilon} \right)$$

$$+ \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \left(\frac{-1}{E - \vec{p}_1^{\,2}/m + i\varepsilon} \right) \left(\frac{4\pi\alpha_s \cdot C_F}{(\vec{p}_1 - \vec{p}_2)^2} \right) \left(\frac{-1}{E - \vec{p}_2^{\,2}/m + i\varepsilon} \right)$$

$$+ \ \ldots$$

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Green function of the non-relativistic Schrödinger equation:

<u>Free Hamiltonian:</u> $H_0 = \vec{p}^{\,2}/m$

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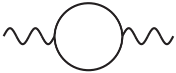
$G_0(0, 0, E) \sim$ 

Coulomb Singularity: LO

Full Hamiltonian: $H = H_0 + V$

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$G\,(0, 0, E) \sim$  $+$  $+$  $+ \dots$

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$G(0,0,E) \sim$  $+$  $+$  $+ \dots$

$$G(0,0,E) = \lim_{r \to 0} \frac{m^2}{4\pi} \left[iv + \frac{1}{mr} - \alpha_s C_F \left(\log(-2i\, mvr) - 1 + 2\gamma_E + \psi\left(1 - i\frac{C_F \alpha_s}{2v}\right)\right)\right]$$

Coulomb Singularity

( +  +  +  $+ \dots \Big)\, c_1(\nu)$

v	$\alpha_s\ (1, L)$	α_s^2/v	α_s^3/v^2	LO

$L = \log(v)$

Coulomb Singularity

$\Big($  $+ \dots \Big)\, c_1(\nu)$

v	$\alpha_s\ (1, L)$	α_s^2/v	α_s^3/v^2	LO
	$\alpha_s v$	$\alpha_s^2\ (1, L, L^2)$	$\alpha_s^3/v\ (1, L)$	NLO
v^3	$\alpha_s v^2\ (1, L)$	$\alpha_s^2 v\ (1, L)$	$\alpha_s^3\ (1, L, L^2, L^3)$	NNLO

$L = \log(v)$

$\longrightarrow$ NNLO Schrödinger equation solved numerically [Hoang, Teubner '99]

Large Logarithms

$\longrightarrow$ resum logs with vNRQCD

$$m \gg |\vec{p}\,| \gg E \gg \Lambda_{QCD}$$

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renormalization scales $\qquad \mu_{us}\,,\ \mu_s$

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correlated scales	$E = \frac{\vec{p}^{\,2}}{m}$
renormalization scales	$\mu_{us}\,,\ \mu_s$
logs	$\log\left(\frac{E^2}{\mu_{us}^2}\right),\ \log\left(\frac{\vec{p}^{\,2}}{\mu_s^2}\right)$

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renormalization scales	$\mu_{us},\ \mu_s$
logs	$\log\left(\frac{E^2}{\mu_{us}^2}\right),\ \log\left(\frac{\vec{p}^{\,2}}{\mu_s^2}\right)$
subtraction velocity ν	$\mu_{us} = m\nu^2,\ \mu_s = m\nu$

Large Logarithms

Expansion of currents:

$$j^i = \bar{\psi}(x)\,\gamma^i\,\psi(x) = \sum_{\vec{p}} \Big(c_1(\nu)\,\mathcal{O}^i_{\vec{p},1} + c_2(\nu)\,\mathcal{O}^i_{\vec{p},2} \Big)\,,$$

$$\mathcal{O}^i_{\vec{p},1} = \psi^\dagger_{\vec{p}}\;\sigma^i(i\sigma_2)\;\chi^*_{-\vec{p}}$$
$$\mathcal{O}^i_{\vec{p},2} = \frac{1}{m^2}\psi^\dagger_{\vec{p}}\;\vec{p}^{\,2}\sigma^i(i\sigma_2)\;\chi^*_{-\vec{p}}$$

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Cross section:

$$\sigma_{t\bar{t}} = A(q^2)\;\mathrm{Im}\left[-i\int d^4x\;e^{iqx}\,\langle 0|\mathrm{T}\,j_\mu(x)\,j^\mu(0)|0\rangle\right]$$

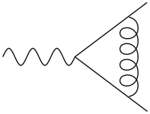
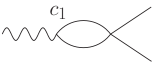
$$= A(q^2)\;\mathrm{Im}\left[c_1(\nu)\;\mathcal{A}_1(\nu,v,m) + 2\,c_1(\nu)c_2(\nu)\;\mathcal{A}_2(\nu,m,v)\right]$$

$$\mathcal{A}_1 = i\sum_{\vec{p},\vec{p}\,'}\int d^4x\;e^{iqx}\,\langle 0|\mathrm{T}\,\boldsymbol{\mathcal{O}}_{\vec{p},1}(x)\,\boldsymbol{\mathcal{O}}^\dagger_{\vec{p}\,',1}(0)|0\rangle$$

$$\mathcal{A}_2 = \frac{i}{2}\sum_{\vec{p},\vec{p}\,'}\int d^4x\;e^{iqx}\,\langle 0|\mathrm{T}\,\boldsymbol{\mathcal{O}}_{\vec{p},1}\,\boldsymbol{\mathcal{O}}^\dagger_{\vec{p}\,',2} + \boldsymbol{\mathcal{O}}_{\vec{p},2}\,\boldsymbol{\mathcal{O}}^\dagger_{\vec{p}\,',1}|0\rangle$$

Large Logarithms

$$\boxed{c_1(\nu)\, \mathcal{A}_1(\nu, v, m)}$$

QCD	vNRQCD
	
$\log\left(\frac{E^2}{m^2}\right)$, $\log\left(\frac{\vec{p}^{\,2}}{m^2}\right)$	$\log\left(\frac{E^2}{\mu_{us}^2}\right)$, $\log\left(\frac{\vec{p}^{\,2}}{\mu_s^2}\right)$, $\frac{1}{\varepsilon^n}$

Large Logarithms

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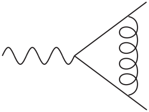
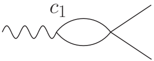
QCD	vNRQCD
	
$\log\left(\frac{E^2}{m^2}\right),\ \log\left(\frac{\vec{p}^2}{m^2}\right)$	$\log\left(\frac{E^2}{\mu_{us}^2}\right),\ \log\left(\frac{\vec{p}^2}{\mu_s^2}\right)\ ,\ \frac{1}{\varepsilon^n}$

- $\boxed{\nu = 1}$ $\mu_{us} = m,\ \mu_s = m \rightarrow$ determine c_1

Large Logarithms

$$\boxed{c_1(\nu)\,\mathcal{A}_1(\nu, v, m)}$$

– logs in $\mathcal{A}_1$: $\log\left(\frac{E^2}{\mu_{us}^2}\right) \sim \log\left(\frac{v^4}{\nu^4}\right)$,

$$\log\left(\frac{\vec{p}^{\,2}}{\mu_s^2}\right) \sim \log\left(\frac{v^2}{\nu^2}\right)$$

Large Logarithms

$$\boxed{c_1(\nu)\,\mathcal{A}_1(\nu, v, m)}$$

- logs in $\mathcal{A}_1$: $\log\left(\frac{E^2}{\mu_{us}^2}\right) \sim \log\left(\frac{v^4}{\nu^4}\right)$,
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- $\nu = 1 \to \nu = v$ use RGE running for $c_1(\nu)$
- $\boxed{\nu = v}$ evaluate $c_1(\nu)\,\mathcal{A}_1(\nu, v, m)$

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- $\nu = 1 \to \nu = v$ use RGE running for $c_1(\nu)$

- $\boxed{\nu = v}$ evaluate $c_1(\nu)\,\mathcal{A}_1(\nu, v, m)$

- matching at $h \cdot m \quad \longrightarrow \log(h)$
  evaluate at $\nu = v \cdot f \longrightarrow \log(f)$

Large Logarithms

( +  +  +  $+ \dots \Big) c_1(\nu)$

v	$\alpha_s\ (1, L)$	α_s^2/v	α_s^3/v^2	LO
	$\alpha_s v$	$\alpha_s^2\ (1, L, L^2)$	$\alpha_s^3/v\ (1, L)$	NLO
v^3	$\alpha_s v^2\ (1, L)$	$\alpha_s^2 v\ (1, L)$	$\alpha_s^3\ (1, L, L^2, L^3)$	NNLO

$L = \log(v)$

Large Logarithms

v	$\alpha_s\ (1, L)$	α_s^2/v	α_s^3/v^2	LL
	$\alpha_s v$	α_s^2	α_s^3/v	NLL
v^3	$\alpha_s v^2$	$\alpha_s^2 v$	α_s^3	NNLL

$L = \log(v)$

Non-Relativistic Regime

$$\mu_m = h\, m,\ \mu_s = h\, m(\nu\, f),\ \mu_{us} = h\, m(\nu\, f)^2$$

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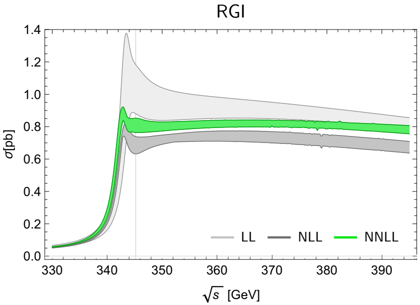
Renormalization Group Improved (RGI) :

[Hoang, Manohar, Stewart, Teubner '01]

[Hoang, Stahlhofen '14]

- $\nu \sim v$
- f and h variation



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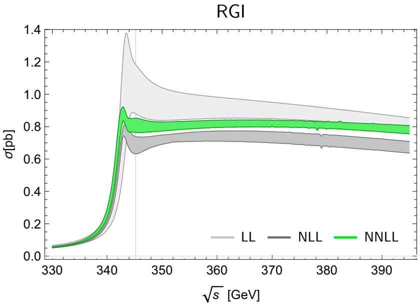
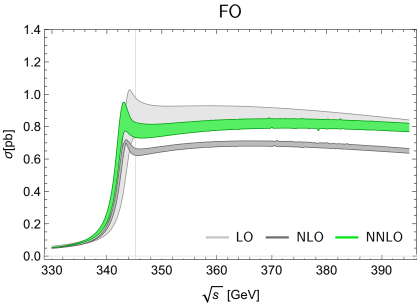
- $\nu \sim v$
- f and h variation

Fixed Order (FO) :

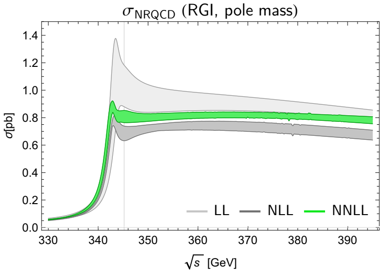
- $\nu \sim 1,\ h \sim \sqrt{v}$
- h variation

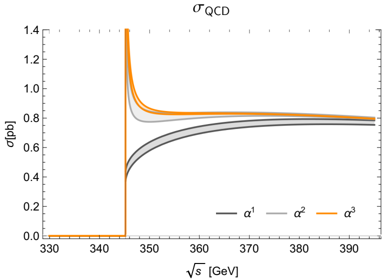




Matching

Matching

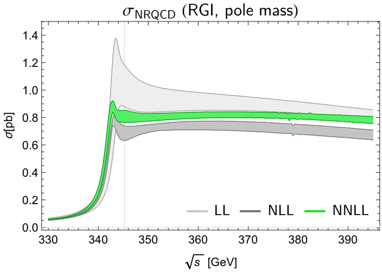


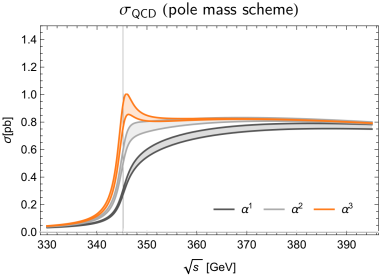


$\Rightarrow$ include leading order finite width effects:

$$v = \sqrt{\frac{q-2m}{m}} \rightarrow \sqrt{\frac{q-2m+i\,\Gamma_t}{m}}$$

Matching





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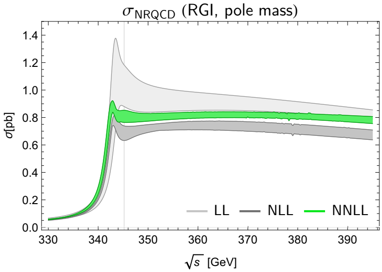

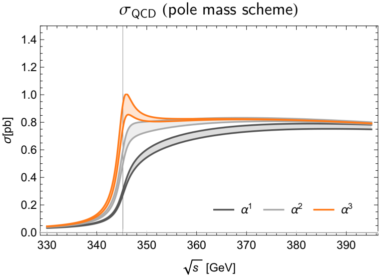

$q^2 > 400\,\text{GeV}$ $\qquad \sigma_{matched} = \sigma_{QCD}$

$q^2 \sim (2m)^2$ $\qquad \sigma_{matched} = \sigma_{QCD} + (\sigma_{NRQCD} - \sigma_{expanded})$

Matching

QCD	α^0	α^1	α^2	α^3	α^4
	v	α	$\frac{\alpha^2}{v}$	$\frac{\alpha^3}{v^2}$	$\frac{\alpha^4}{v^3}$
		$\alpha\ v$	α^2	$\frac{\alpha^3}{v}$	$\frac{\alpha^4}{v^2}$
	v^3	$\alpha\ v^2$	$\alpha^2\ v$	α^3	$\frac{\alpha^4}{v}$
	v^4	$\alpha\ v^3$	$\alpha^2\ v^2$	$\alpha^3\ v$	α^4
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

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Matching

NRQCD						
LO	v	α	$\frac{\alpha^2}{v}$	$\frac{\alpha^3}{v^2}$	$\frac{\alpha^4}{v^3}$	...
NLO		$\alpha\ v$	α^2	$\frac{\alpha^3}{v}$	$\frac{\alpha^4}{v^2}$	...
NNLO	v^3	$\alpha\ v^2$	$\alpha^2\ v$	α^3	$\frac{\alpha^4}{v}$	...
N^3LO	v^4	$\alpha\ v^3$	$\alpha^2\ v^2$	$\alpha^3\ v$	α^4	...

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Matching

	α^0	α^1	α^2	α^3		
LO	v	α	$\frac{\alpha^2}{v}$	$\frac{\alpha^3}{v^2}$	$\frac{\alpha^4}{v^3}$	$\dots$
NLO		$\alpha\ v$	α^2	$\frac{\alpha^3}{v}$	$\frac{\alpha^4}{v^2}$	$\dots$
NNLO	v^3	$\alpha\ v^2$	$\alpha^2\ v$	α^3	$\frac{\alpha^4}{v}$	$\dots$
	v^4	$\alpha\ v^3$	$\alpha^2\ v^2$	$\alpha^3\ v$	α^4	$\dots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

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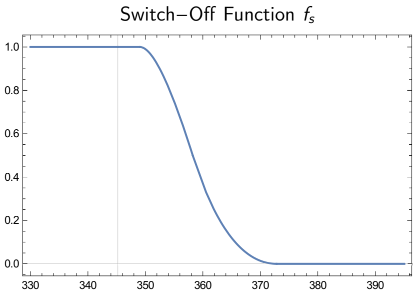
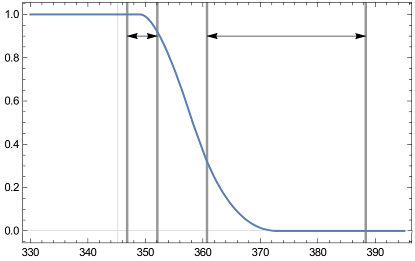
Matching



$q^2 > 400$ GeV	$\sigma_{matched} = \sigma_{QCD}$
$q^2 \sim (2m)^2$	$\sigma_{matched} = \sigma_{QCD} + (\sigma_{NRQCD} - \sigma_{expanded})$
$(2m)^2 < q^2 < 400$ GeV	$\sigma_{matched} = \sigma_{QCD} + (\sigma_{NRQCD} - \sigma_{expanded}) \cdot f_s$

Matching



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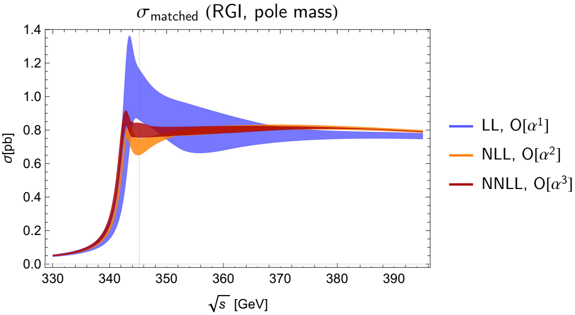
Matching

matching:

σ_{NRQCD}		σ_{QCD}
LL	$\longleftrightarrow$	α^1
NLL	$\longleftrightarrow$	α^2
NNLL	$\longleftrightarrow$	α^3

results in the pole mass scheme:



Matching

	σ_{NRQCD}		σ_{QCD}
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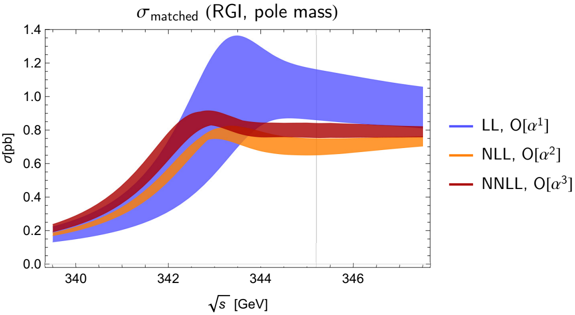

Matching

1S mass ✓ renormalon free

[Hoang, Ligeti, Manohar '98]

$$m_{1S} = m_{pole} + \left(C_F\,\alpha_s(\mu) m_{pole}\right) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log\left(\frac{\mu}{C_F\,\alpha_s(\mu) m_{pole}}\right)$$

$$= m_{pole} - \frac{2}{9}\alpha_s^2 m_{pole} + \ldots$$

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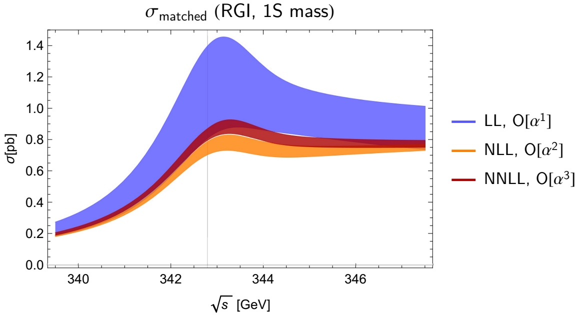

Matching

$$m_{1S} = m_{pole} + (C_F\,\alpha_s(\mu) m_{pole}) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log\left(\frac{\mu}{C_F\,\alpha_s(\mu) m_{pole}}\right)$$
$$= m_{pole} - \frac{2}{9}\alpha_s^2 m_{pole} + \dots$$

Pole mass:

	α^0	α^1	α^2	α^3	
LO	v				...
NLO					...
NNLO					...
	⋮	⋮	⋮	⋮	

Matching

$$m_{1S} = m_{pole} + \left(C_F\, \alpha_s(\mu) m_{pole}\right) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log\left(\frac{\mu}{C_F\, \alpha_s(\mu) m_{pole}}\right)$$

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Pole mass:

	α^0	α^1	α^2	α^3	
LO	v				...
NLO					...
NNLO					...
	⋮	⋮	⋮	⋮	

1S mass:

	α^0	α^1	α^2	α^3	
LO	v		$\frac{\alpha^2}{v}$		...
NLO				$\frac{\alpha^3}{v}$	...
NNLO			α^2 v		...
	⋮	⋮	⋮	⋮	

Matching

$\overline{MS}$ mass :

$$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h)\, \alpha_s(\overline{m})^n$$

$$= \overline{m} + \overline{m}\, \alpha_s\, a_1 + \dots \qquad (\ \overline{m} = \overline{m}^{(nl+1)}(\overline{m}^{(nl+1)})\ )$$

Matching

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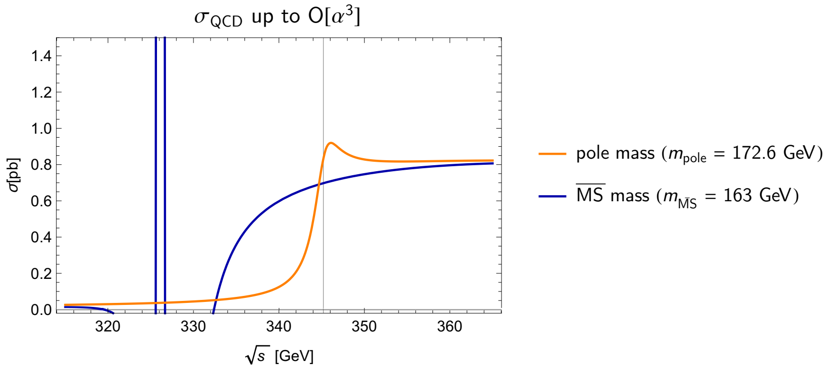


Matching

Pole mass:

	α^0	α^1	α^2	α^3	
LO	v				...
NLO					...
NNLO					...
	⋮	⋮	⋮	⋮	

$\overline{MS}$ mass:

	α^0	α^1	α^2	α^3	
				$\frac{\alpha^3}{v^5}$	...
			$\frac{\alpha^2}{v^3}$		...
		$\frac{\alpha}{v}$		$\frac{\alpha^3}{v^3}$	...
LO	v		$\frac{\alpha^2}{v}$		...
NLO		α v			...
NNLO					...
	⋮	⋮	⋮	⋮	

Matching

MSR mass :

[Hoang, Jain, Scimemi, Stewart '08]

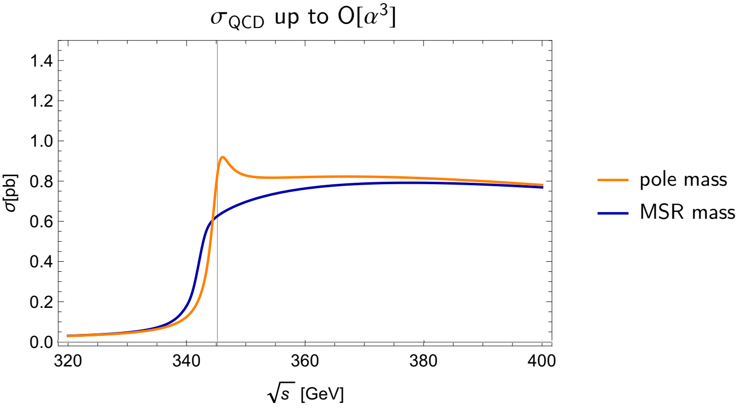
$$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h)\, \alpha_s(\overline{m})^n = \overline{m} + \overline{m}\, \alpha_s\, a_1 + \ldots$$

$$m_{pole} = m_{MSRn}(R) + R \sum_{n=1}^{\infty} a_n(n_l, 0)\, \alpha_s(R)^n = m_{MSRn}(R) + \alpha_s\, R\, a_1 + \ldots$$

$\Rightarrow$ choose $R \sim m\, v$

Matching

MSR mass with $R \sim m\, v$:



Matching

Pole mass:

	α^0	α^1	α^2	α^3	
LO	v				...
NLO					...
NNLO					...
	⋮	⋮	⋮	⋮	

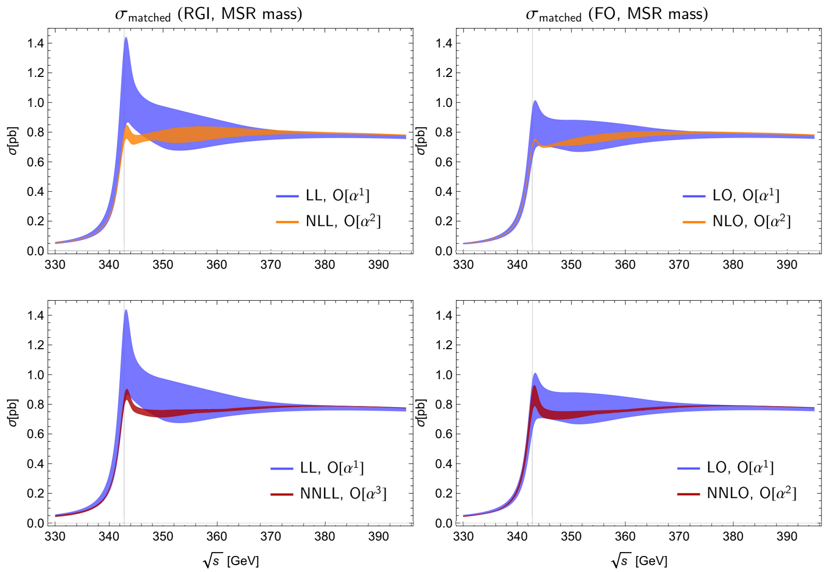
MSR mass:

	α^0	α^1	α^2	α^3	
LO	v	$\frac{\alpha}{v}\frac{R}{m}$	$\frac{\alpha^2}{v^3}\frac{R^2}{m^2}$	$\frac{\alpha^3}{v^5}\frac{R^3}{m^3}$	...
NLO			$\frac{\alpha^2}{v}\frac{R}{m}$	$\frac{\alpha^3}{v^3}\frac{R^2}{m^2}$	...
NNLO		$\alpha\ v\ \frac{R}{m}$	$\frac{\alpha^2}{v}\frac{R^2}{m^2}$	$\frac{\alpha^3}{v^3}\frac{R^3}{m^3}, \frac{\alpha^3}{v}\frac{R}{m}$	...
	⋮	⋮	⋮	⋮	

$$R \sim m\, v \sim m\, \alpha$$

Matching

Results in the MSR mass scheme:



Conclusion

Summary

- matched σ_{NRQCD} and σ_{QCD}
- implemented of the MSR mass scheme

Outlook

- N^3LO corrections for the fixed order cross section
- higher order electroweak effects

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Thank you for your attention!