

Top Production Cross Section -

Matching of Relativistic and Non-Relativistic Regimes

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in collaboration with

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Outline

Motivation

Relativistic Regime

Non-Relativistic Regime

Matching

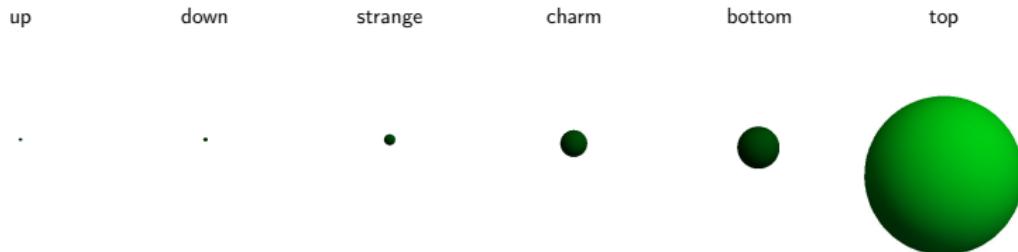
Outlook

Motivation

top quark properties: $m_t^{pole} \approx 170 \text{ GeV}$, $\Gamma_t \approx 2 \text{ GeV}$

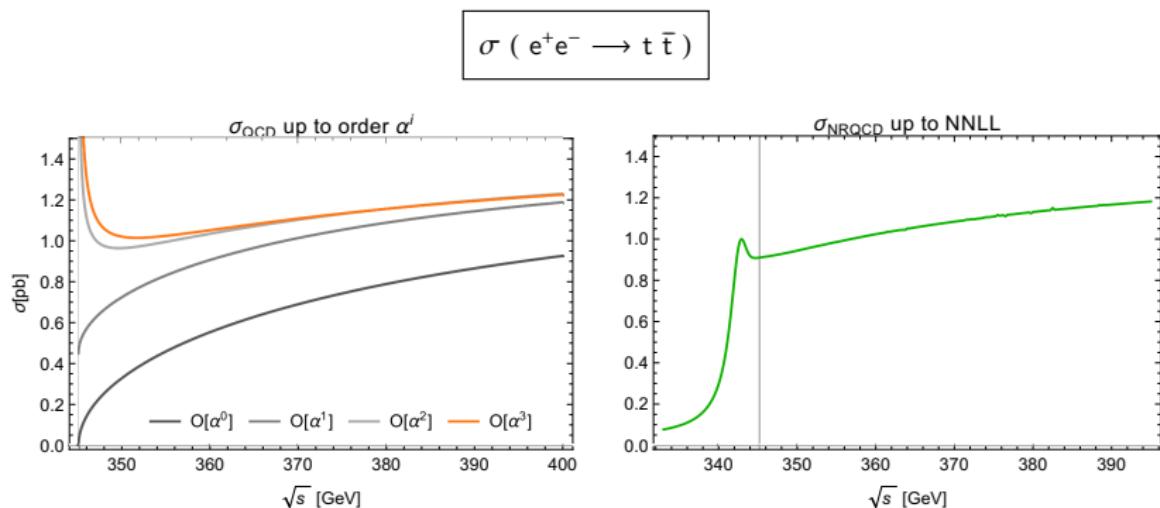
precise measurement of top quark properties for:

- stability of the SM vacuum
- electroweak precision tests
- new physics searches
- ...



Motivation

Top production cross section in the pole mass scheme:



Relativistic Regime

Relativistic Regime

$$\sigma_{t\bar{t}} = \int d\Pi \left| \begin{array}{c} \text{Diagram 1: } e^- \text{ and } e^+ \text{ annihilation into } t \text{ and } \bar{t} \text{ via a virtual photon } \gamma \text{ exchange, with momentum } q. \\ + \\ \text{Diagram 2: } e^- \text{ and } e^+ \text{ annihilation into } t \text{ and } \bar{t} \text{ via a virtual photon } \gamma \text{ exchange, with a gluon loop at the vertex.} \\ + \\ \text{Diagram 3: } e^- \text{ and } e^+ \text{ annihilation into } t \text{ and } \bar{t} \text{ via a virtual photon } \gamma \text{ exchange, with a triangle loop at the vertex.} \\ + \dots \end{array} \right|^2$$

Relativistic Regime

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$$= \text{Im} \left[\begin{array}{c} \text{Diagram 2: } e^- \text{ and } e^+ \text{ annihilation into } t \text{ and } \bar{t} \text{ via a loop diagram with a virtual photon exchange, with momentum } q. \\ + \text{ (sum of other loop diagrams)} \\ + \dots \end{array} \right]$$

Relativistic Regime

$$\sigma_{t\bar{t}} = \int d\Pi \left| \begin{array}{c} \text{Feynman diagram for } t\bar{t} \text{ production via annihilation of } e^+e^- \text{ into } \gamma + t\bar{t} \\ + \text{ higher order corrections} \end{array} \right|^2$$

The first term shows an annihilation process where an electron-positron pair (e^-e^+) annihilates into a virtual photon (γ) and a $t\bar{t}$ pair. The virtual photon then decays into two real photons ($\gamma\gamma$). Subsequent terms represent higher-order corrections involving multiple loops and gluon exchanges.

$$= \text{Im} \left[\begin{array}{c} \text{Feynman diagram for } t\bar{t} \text{ production via annihilation of } e^+e^- \text{ into } t\bar{t} \\ + \text{ higher order corrections} \end{array} \right]$$

The second term shows the same process but with a different loop structure, representing the imaginary part of the scattering amplitude.

$$= \frac{(4\pi\alpha)^2}{q^2} Q_t^2 \text{ Im} \left[\begin{array}{c} \text{Feynman diagram for } t\bar{t} \text{ production via annihilation of } e^+e^- \text{ into } t\bar{t} \\ + \text{ higher order corrections} \end{array} \right]$$

The final term is the result of the dimensional regularization and renormalization process, showing the contribution of the loop diagrams to the cross-section.

Relativistic Regime

$$\sigma_{t\bar{t}} = \int d\Pi \left| \begin{array}{c} \text{Feynman diagram for } t\bar{t} \text{ production via annihilation of an electron-positron pair } e^+e^- \text{ into a virtual photon } \gamma \text{ and a quark-antiquark pair } t\bar{t}, \text{ followed by the decay of the virtual photon into a real photon } \gamma \text{ and a virtual fermion-antifermion pair } q\bar{q}. \\ + \text{ higher-order terms} \end{array} \right|^2$$

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$$= A(q^2) \text{Im} \left[-i \int d^4x e^{iqx} \langle 0 | T j_\mu(x) j^\mu(0) | 0 \rangle \right], \quad j^\mu(x) = \bar{\psi}(x) \gamma^\mu \psi(x)$$

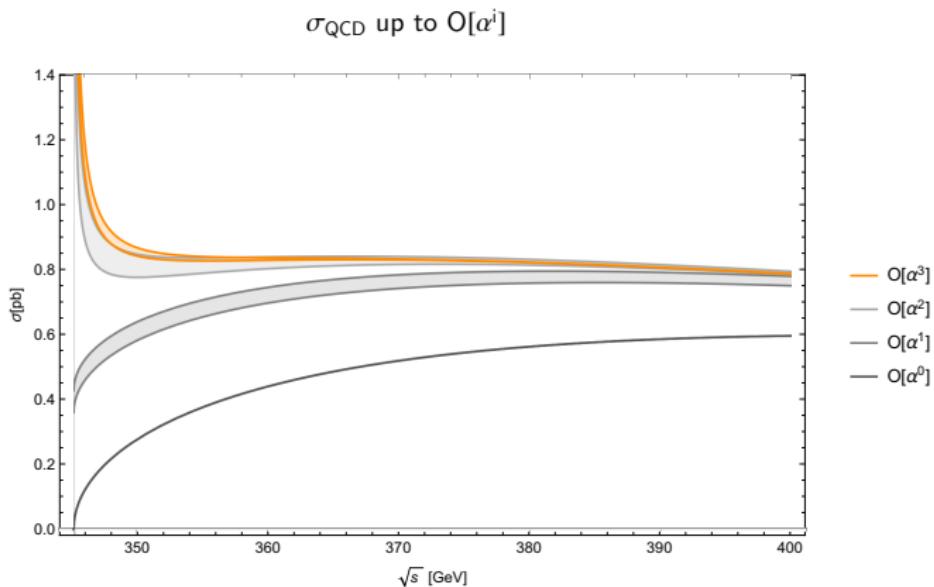
Relativistic Regime

- ✓ Vacuum Polarization known up to α^3 [Hoang, Mateu, Zebarjad '08]

Relativistic Regime

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[Hoang, Mateu, Zebarjad '08]



Non-Relativistic Regime

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- scales: $m, |\vec{p}| \sim mv, E \sim mv^2$ ($m_t \sim 170 \text{ GeV}, v \sim \alpha \sim 0.1$)

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- problems:

1. large logarithms

$$\log\left(\frac{E^2}{m^2}\right), \log\left(\frac{\vec{p}^2}{m^2}\right)$$

Non-Relativistic Regime

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1. large logarithms $\log\left(\frac{E^2}{m^2}\right), \log\left(\frac{\vec{p}^2}{m^2}\right)$

2. Coulomb singularity contributions of order $\frac{\alpha^2}{v^3}, \frac{\alpha^3}{v^4}, \frac{\alpha^4}{v^5}, \dots$

Non-Relativistic Regime

Fields in vNRQCD and pNRQCD:

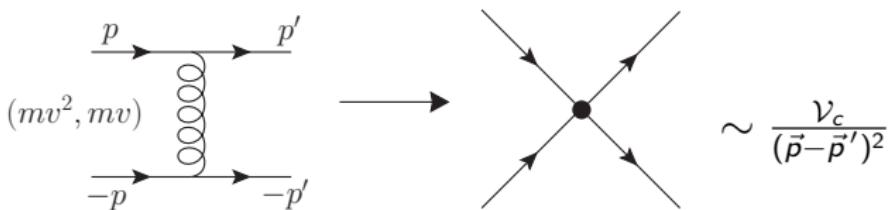
field	(E, m)
potential quark	(mv^2, mv)
soft gluon	(mv, mv)
ultrasoft gluon	(mv^2, mv^2)

Non-Relativistic Regime

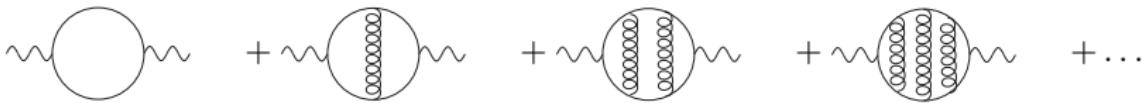
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Example:



Coulomb Singularity: LO



Coulomb Singularity: LO

$$\text{Diagram: } \text{Wavy line} + \text{Wavy line with vertical loop} + \text{Wavy line with two vertical loops} + \text{Wavy line with three vertical loops} + \dots$$

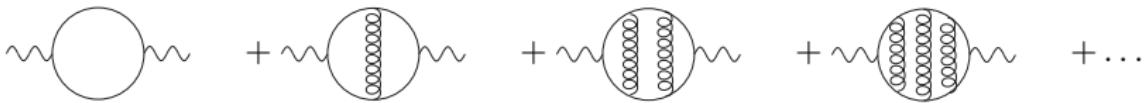
$$= \text{Wavy line with one loop} + \text{Wavy line with two loops} + \text{Wavy line with three loops} + \text{Wavy line with four loops} + \dots$$

Coulomb Singularity: LO

$$\begin{aligned} & \text{Diagram showing a series of loop corrections to a vertex, starting with a single loop and increasing in complexity.} \\ = & \frac{\nu}{\alpha_s} + \frac{\alpha_s^2/\nu}{\alpha_s} + \frac{\alpha_s^4/\nu^2}{\alpha_s} + \frac{\alpha_s^6/\nu^3}{\alpha_s} + \dots \end{aligned}$$

$$\alpha_s \sim \nu!$$

Coulomb Singularity: LO



$$= \frac{v}{v} + \frac{\alpha_s}{\alpha_s} + \frac{\alpha_s^2/v}{\alpha_s^2/v} + \frac{\alpha_s^3/v^2}{\alpha_s^3/v^2} + \dots$$

$$\begin{aligned} & \sim \int \frac{d^3 p}{(2\pi)^3} \left(\frac{-1}{E - \vec{p}^2/m + i\varepsilon} \right) \\ & + \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \left(\frac{-1}{E - \vec{p}_1^2/m + i\varepsilon} \right) \left(\frac{4\pi\alpha_s \cdot C_F}{(\vec{p}_1 - \vec{p}_2)^2} \right) \left(\frac{-1}{E - \vec{p}_2^2/m + i\varepsilon} \right) \\ & + \dots \end{aligned}$$

$$\alpha_s \sim v!$$

Coulomb Singularity: LO

Green function of the non-relativistic Schrödinger equation:

Free Hamiltonian: $H_0 = \vec{p}^2/m$

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$$(H_0 - E) G_0(\vec{p}, \vec{p}') = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}')$$

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$$G_0(\vec{x}, \vec{x}', E) = \int \frac{d^3 p}{(2\pi)^3} \frac{-1}{E - \vec{p}^2/m} e^{i\vec{p}(\vec{x} - \vec{x}')}}$$

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$$G_0(0, 0, E) \sim$$



Coulomb Singularity: LO

Full Hamiltonian: $H = H_0 + V$

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$$(H_0 - E) G(\vec{p}, \vec{p}') + \int \frac{d^3 k}{(2\pi)^3} V(\vec{p} - \vec{k}) G(\vec{k}, \vec{p}) = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}')$$

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$$G(0, 0, E) \sim \text{wavy loop} + \text{two wavy loops} + \text{three wavy loops} + \dots$$

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$$G(0, 0, E) = \lim_{r \rightarrow 0} \frac{m^2}{4\pi} \left[iv + \frac{1}{mr} - \alpha_s C_F \left(\log(-2i mvr) - 1 + 2\gamma_E + \psi \left(1 - i \frac{C_F \alpha_s}{2v} \right) \right) \right]$$

Coulomb Singularity

$$\left(\text{wavy line} + \text{wavy loop} + \text{wavy double loop} + \text{wavy triple loop} + \dots \right) c_1(\nu)$$

$$\nu \qquad \alpha_s(1, L) \qquad \alpha_s^2/\nu \qquad \alpha_s^3/\nu^2 \qquad \text{LO}$$

$$L = \log(\nu)$$

Coulomb Singularity

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ν	$\alpha_s(1, L)$	α_s^2/ν	α_s^3/ν^2	LO
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$\alpha_s \nu$	$\alpha_s^2(1, L, L^2)$	$\alpha_s^3/\nu(1, L)$	NLO
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ν^3	$\alpha_s \nu^2(1, L)$	$\alpha_s^2 \nu(1, L)$	$\alpha_s^3(1, L, L^2, L^3)$	NNLO
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$$L = \log(\nu)$$

→ NNLO Schrödinger equation solved numerically [Hoang, Teubner '99]

Large Logarithms

→ resum logs with vNRQCD

$$m \gg |\vec{p}| \gg E \gg \Lambda_{QCD}$$

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correlated scales

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renormalization scales

$$\mu_u s, \mu_s$$

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renormalization scales

$$\mu_{us}, \mu_s$$

logs

$$\log\left(\frac{E^2}{\mu_{us}^2}\right), \log\left(\frac{\vec{p}^2}{\mu_s^2}\right)$$

Large Logarithms

→ resum logs with vNRQCD

$$m \gg |\vec{p}| \gg E \gg \Lambda_{QCD}$$

correlated scales

$$E = \frac{\vec{p}^2}{m}$$

renormalization scales

$$\mu_{us}, \mu_s$$

logs

$$\log\left(\frac{E^2}{\mu_{us}^2}\right), \log\left(\frac{\vec{p}^2}{\mu_s^2}\right)$$

subtraction velocity ν

$$\mu_{us} = m\nu^2, \mu_s = m\nu$$

Large Logarithms

Expansion of currents:

$$j^i = \bar{\psi}(x) \gamma^i \psi(x) = \sum_{\vec{p}} \left(c_1(\nu) \mathcal{O}_{\vec{p},1}^i + c_2(\nu) \mathcal{O}_{\vec{p},2}^i \right),$$

$$\begin{aligned}\mathcal{O}_{\vec{p},1}^i &= \psi_{\vec{p}}^\dagger \sigma^i(i\sigma_2) \chi_{-\vec{p}}^* \\ \mathcal{O}_{\vec{p},2}^i &= \frac{1}{m^2} \psi_{\vec{p}}^\dagger \vec{p}^2 \sigma^i(i\sigma_2) \chi_{-\vec{p}}^*\end{aligned}$$

Large Logarithms

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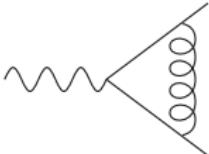
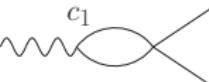
Cross section:

$$\begin{aligned} \sigma_{t\bar{t}} &= A(q^2) \operatorname{Im} \left[-i \int d^4x e^{iqx} \langle 0 | T j_\mu(x) j^\mu(0) | 0 \rangle \right] \\ &= A(q^2) \operatorname{Im} [c_1(\nu) \mathcal{A}_1(\nu, \nu, m) + 2 c_1(\nu) c_2(\nu) \mathcal{A}_2(\nu, m, \nu)] \end{aligned}$$

$$\begin{aligned} \mathcal{A}_1 &= i \sum_{\vec{p}, \vec{p}'} \int d^4x e^{iqx} \langle 0 | T \mathcal{O}_{\vec{p},1}(x) \mathcal{O}_{\vec{p}',1}^\dagger(0) | 0 \rangle \\ \mathcal{A}_2 &= \frac{i}{2} \sum_{\vec{p}, \vec{p}'} \int d^4x e^{iqx} \langle 0 | T \mathcal{O}_{\vec{p},1} \mathcal{O}_{\vec{p}',2}^\dagger + \mathcal{O}_{\vec{p},2} \mathcal{O}_{\vec{p}',1}^\dagger | 0 \rangle \end{aligned}$$

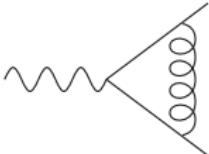
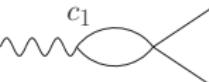
Large Logarithms

$$c_1(\nu) \mathcal{A}_1(\nu, \nu, m)$$

QCD	vNRQCD
	
$\log\left(\frac{E^2}{m^2}\right), \log\left(\frac{\vec{p}^2}{m^2}\right)$	$\log\left(\frac{E^2}{\mu_{us}^2}\right), \log\left(\frac{\vec{p}^2}{\mu_s^2}\right), \frac{1}{\varepsilon^n}$

Large Logarithms

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- $\boxed{\nu = 1}$ $\mu_{us} = m, \mu_s = m \rightarrow$ determine c_1

Large Logarithms

$$c_1(\nu) \mathcal{A}_1(\nu, \nu, m)$$

- logs in \mathcal{A}_1 : $\log\left(\frac{E^2}{\mu_{us}^2}\right) \sim \log\left(\frac{\nu^4}{\nu^4}\right),$
 $\log\left(\frac{\bar{p}^2}{\mu_s^2}\right) \sim \log\left(\frac{\nu^2}{\nu^2}\right)$

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 $\log\left(\frac{\bar{p}^2}{\mu_s^2}\right) \sim \log\left(\frac{\nu^2}{\nu^2}\right)$
- $\nu = 1 \rightarrow \nu = \nu$ use RGE running for $c_1(\nu)$
- $\boxed{\nu = \nu}$ evaluate $c_1(\nu) \mathcal{A}_1(\nu, \nu, m)$

Large Logarithms

$$c_1(\nu) \mathcal{A}_1(\nu, v, m)$$

- logs in \mathcal{A}_1 : $\log\left(\frac{E^2}{\mu_{us}^2}\right) \sim \log\left(\frac{v^4}{\nu^4}\right)$,
 $\log\left(\frac{\bar{p}^2}{\mu_s^2}\right) \sim \log\left(\frac{v^2}{\nu^2}\right)$
- $v = 1 \rightarrow v = v$ use RGE running for $c_1(\nu)$
- $\boxed{\nu = v}$ evaluate $c_1(\nu) \mathcal{A}_1(\nu, v, m)$
- matching at $h \cdot m \longrightarrow \log(h)$
evaluate at $\nu = v \cdot f \longrightarrow \log(f)$

Large Logarithms

$$\left(\text{wavy loop} + \text{wavy loop with one wavy line} + \text{wavy loop with two wavy lines} + \text{wavy loop with three wavy lines} + \dots \right) c_1(\nu)$$

$$\nu \qquad \alpha_s(1, L) \qquad \alpha_s^2/\nu \qquad \alpha_s^3/\nu^2 \qquad \text{LO}$$

$$\alpha_s \nu \qquad \alpha_s^2(1, L, L^2) \qquad \alpha_s^3/\nu(1, L) \qquad \text{NLO}$$

$$\nu^3 \qquad \alpha_s \nu^2(1, L) \qquad \alpha_s^2 \nu(1, L) \qquad \alpha_s^3(1, L, L^2, L^3) \qquad \text{NNLO}$$

$$L = \log(\nu)$$

Large Logarithms

$$\left(\text{wavy loop} + \text{wavy loop} + \dots \right) c_1(\nu)$$

ν

$\alpha_s(1, L)$

α_s^2/ν

α_s^3/ν^2

LL

$\alpha_s \nu$

α_s^2

α_s^3/ν

NLL

ν^3

$\alpha_s \nu^2$

$\alpha_s^2 \nu$

α_s^3

NNLL

$$L = \log(\nu)$$

Non-Relativistic Regime

$$\mu_m = h m, \mu_s = h m(\nu f), \mu_{us} = h m(\nu f)^2$$

Non-Relativistic Regime

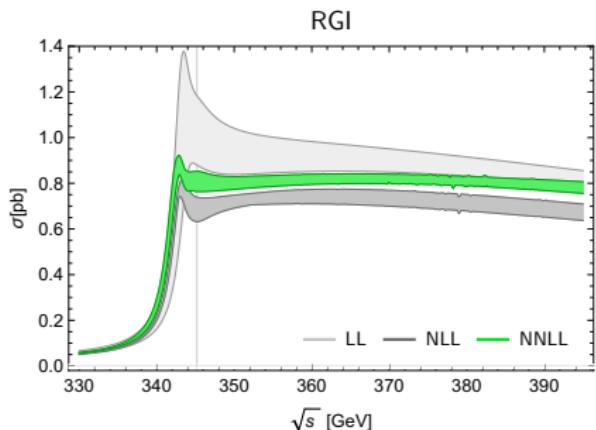
$$\mu_m = h m, \mu_s = h m(\nu f), \mu_{us} = h m(\nu f)^2$$

Renormalization Group Improved (RGI) :

[Hoang, Manohar, Stewart, Teubner '01]

[Hoang, Stahlhofen '14]

- $\nu \sim v$
- f and h variation



Non-Relativistic Regime

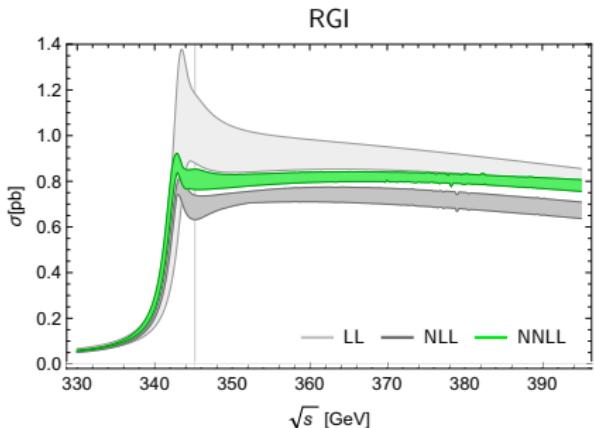
$$\mu_m = h m, \mu_s = h m(\nu f), \mu_{us} = h m(\nu f)^2$$

Renormalization Group Improved (RGI) :

[Hoang, Manohar, Stewart, Teubner '01]

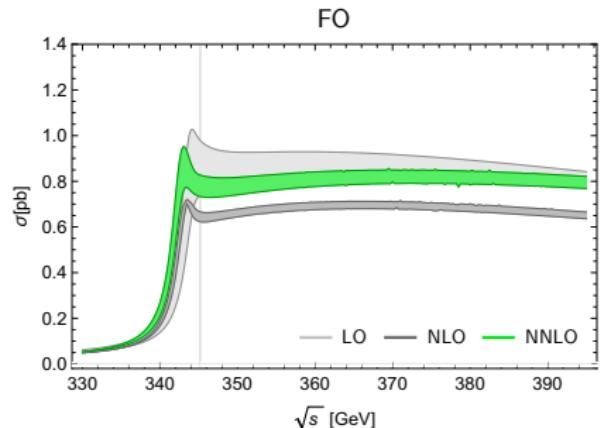
[Hoang, Stahlhofen '14]

- $\nu \sim v$
- f and h variation



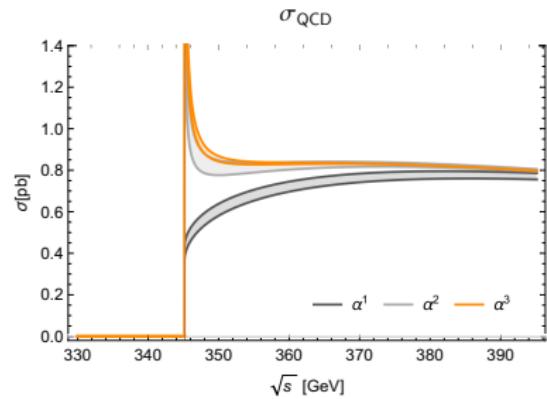
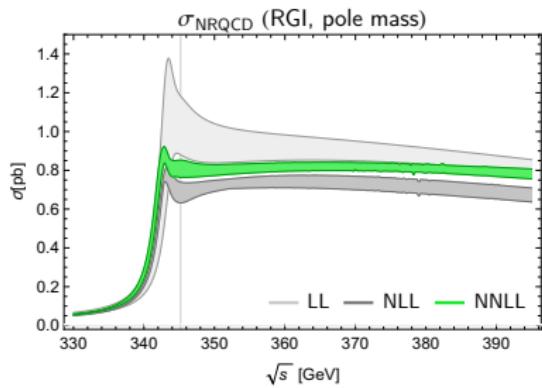
Fixed Order (FO) :

- $\nu \sim 1, h \sim \sqrt{v}$
- h variation



Matching

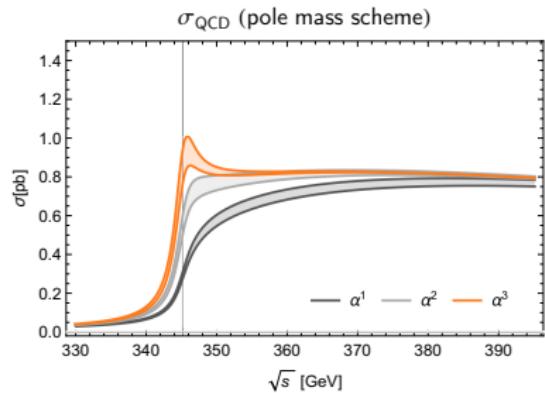
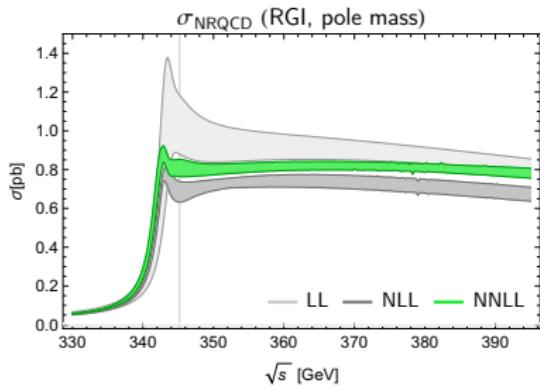
Matching



⇒ include leading order finite width effects:

$$v = \sqrt{\frac{q-2m}{m}} \rightarrow \sqrt{\frac{q-2m+i\Gamma_t}{m}}$$

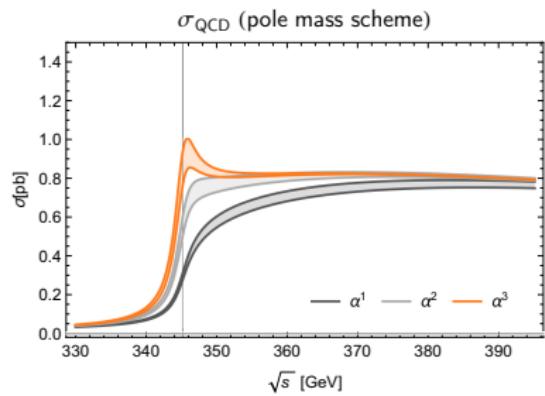
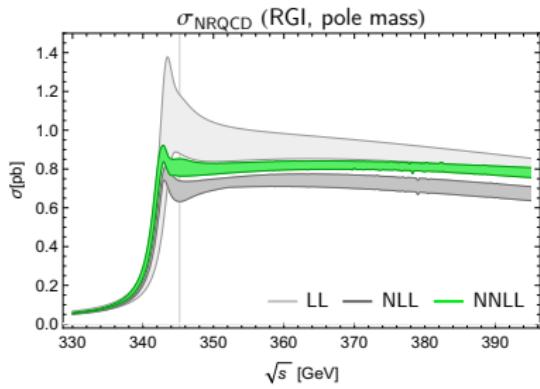
Matching



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Matching



$$q^2 > 400 \text{ GeV}$$

$$q^2 \sim (2m)^2$$

$$\sigma_{matched} = \sigma_{QCD}$$

$$\sigma_{matched} = \sigma_{QCD} + (\sigma_{NRQCD} - \sigma_{expanded})$$

Matching

QCD	α^0	α^1	α^2	α^3	α^4
v	α	$\frac{\alpha^2}{v}$	$\frac{\alpha^3}{v^2}$	$\frac{\alpha^4}{v^3}$	
	αv	α^2	$\frac{\alpha^3}{v}$	$\frac{\alpha^4}{v^2}$	
v^3	αv^2	$\alpha^2 v$	α^3	$\frac{\alpha^4}{v}$	
v^4	αv^3	$\alpha^2 v^2$	$\alpha^3 v$	α^4	
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

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Matching

NRQCD						
LO	v	α	$\frac{\alpha^2}{v}$	$\frac{\alpha^3}{v^2}$	$\frac{\alpha^4}{v^3}$...
NLO		αv	α^2	$\frac{\alpha^3}{v}$	$\frac{\alpha^4}{v^2}$...
NNLO	v^3	αv^2	$\alpha^2 v$	α^3	$\frac{\alpha^4}{v}$...
N^3LO	v^4	αv^3	$\alpha^2 v^2$	$\alpha^3 v$	α^4	...

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Matching

	α^0	α^1	α^2	α^3		
LO	v	α	$\frac{\alpha^2}{v}$	$\frac{\alpha^3}{v^2}$	$\frac{\alpha^4}{v^3}$...
NLO		αv	α^2	$\frac{\alpha^3}{v}$	$\frac{\alpha^4}{v^2}$...
NNLO	v^3	αv^2	$\alpha^2 v$	α^3	$\frac{\alpha^4}{v}$...
	v^4	αv^3	$\alpha^2 v^2$	$\alpha^3 v$	α^4	...
	\vdots	\vdots	\vdots	\vdots	\vdots	

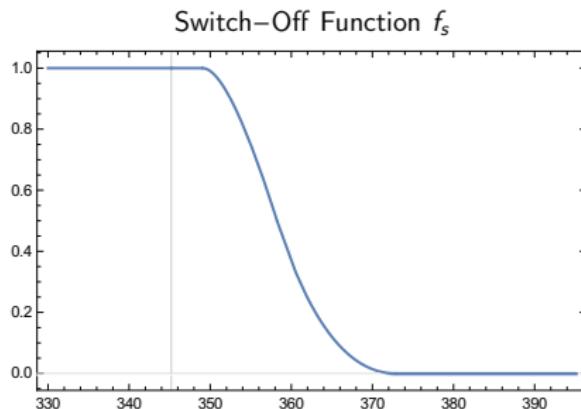
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Matching



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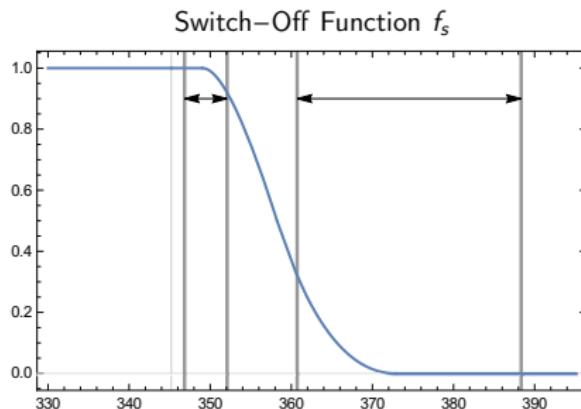
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$$(2m)^2 < q^2 < 400 \text{ GeV}$$

$$\sigma_{\text{matched}} = \sigma_{QCD} + (\sigma_{NRQCD} - \sigma_{\text{expanded}}) \cdot f_s$$

Matching



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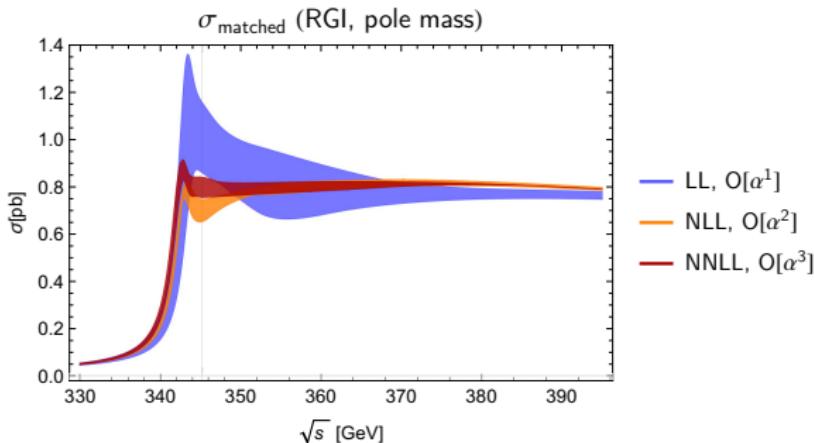
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Matching

	σ_{NRQCD}	σ_{QCD}
matching:	LL	$\longleftrightarrow \alpha^1$
	NLL	$\longleftrightarrow \alpha^2$
	NNLL	$\longleftrightarrow \alpha^3$

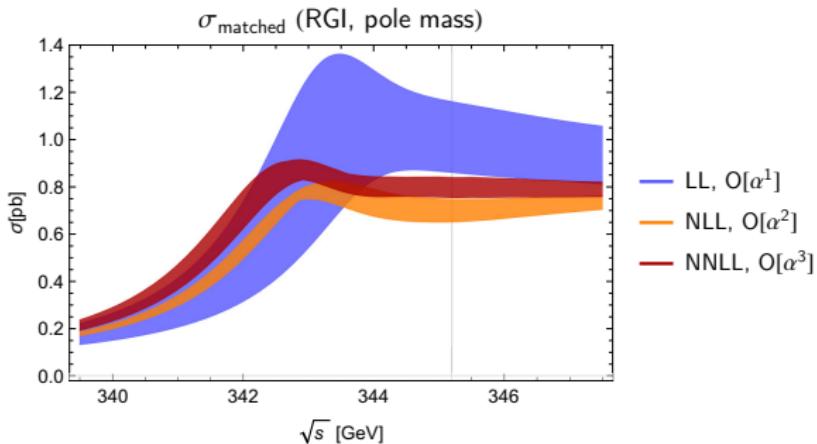
results in the pole mass scheme:



Matching

	σ_{NRQCD}	σ_{QCD}
matching:	LL	$\longleftrightarrow \alpha^1$
	NLL	$\longleftrightarrow \alpha^2$
	NNLL	$\longleftrightarrow \alpha^3$

results in the pole mass scheme:



Matching

1S mass ✓ renormalon free

[Hoang, Ligeti, Manohar '98]

$$\begin{aligned} m_{1S} &= m_{pole} + (C_F \alpha_s(\mu) m_{pole}) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log \left(\frac{\mu}{C_F \alpha_s(\mu) m_{pole}} \right) \\ &= m_{pole} - \frac{2}{9} \alpha_s^2 m_{pole} + \dots \end{aligned}$$

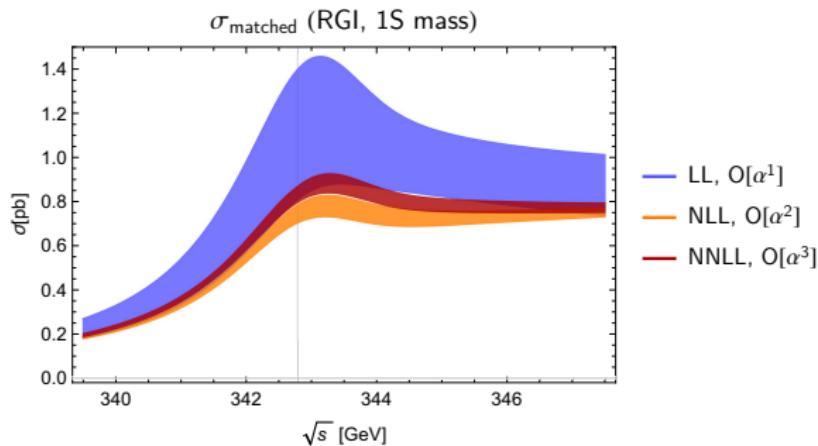
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Pole mass:

	α^0	α^1	α^2	α^3
L0	v			...
NLO				...
NNLO				...
	:	:	:	:

Matching

$$m_{1S} = m_{pole} + (C_F \alpha_s(\mu) m_{pole}) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log \left(\frac{\mu}{C_F \alpha_s(\mu) m_{pole}} \right)$$
$$= m_{pole} - \frac{2}{9} \alpha_s^2 m_{pole} + \dots$$

Pole mass:

	α^0	α^1	α^2	α^3
LO	v			...
NLO				...
NNLO				...
⋮	⋮	⋮	⋮	⋮

1S mass:

	α^0	α^1	α^2	α^3
LO	v		$\frac{\alpha^2}{v}$...
NLO				$\frac{\alpha^3}{v}$
NNLO			$\alpha^2 v$...
⋮	⋮	⋮	⋮	⋮

Matching

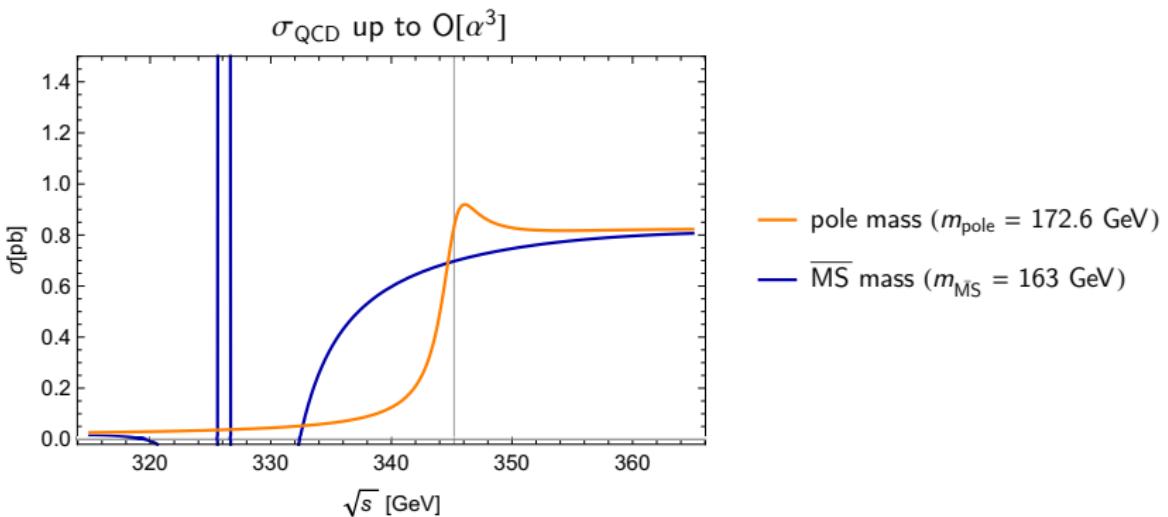
\overline{MS} mass :

$$\begin{aligned} m_{pole} &= \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h) \alpha_s(\overline{m})^n \\ &= \overline{m} + \overline{m} \alpha_s a_1 + \dots \quad (\overline{m} = \overline{m}^{(nl+1)} (\overline{m}^{(nl+1)})) \end{aligned}$$

Matching

\overline{MS} mass :

$$\begin{aligned} m_{pole} &= \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h) \alpha_s(\overline{m})^n \\ &= \overline{m} + \overline{m} \alpha_s a_1 + \dots \quad (\overline{m} = \overline{m}^{(nl+1)}(\overline{m}^{(nl+1)})) \end{aligned}$$



Matching

Pole mass:

	α^0	α^1	α^2	α^3
LO	v			...
NLO				...
NNLO				...
	:	:	:	:

\overline{MS} mass:

	α^0	α^1	α^2	α^3
			$\frac{\alpha^3}{v^5}$...
			$\frac{\alpha^2}{v^3}$...
		$\frac{\alpha}{v}$	$\frac{\alpha^3}{v^3}$...
LO	v		$\frac{\alpha^2}{v}$...
NLO		α	v	...
NNLO				...
	:	:	:	:

Matching

MSR mass :

[Hoang, Jain, Scimemi, Stewart '08]

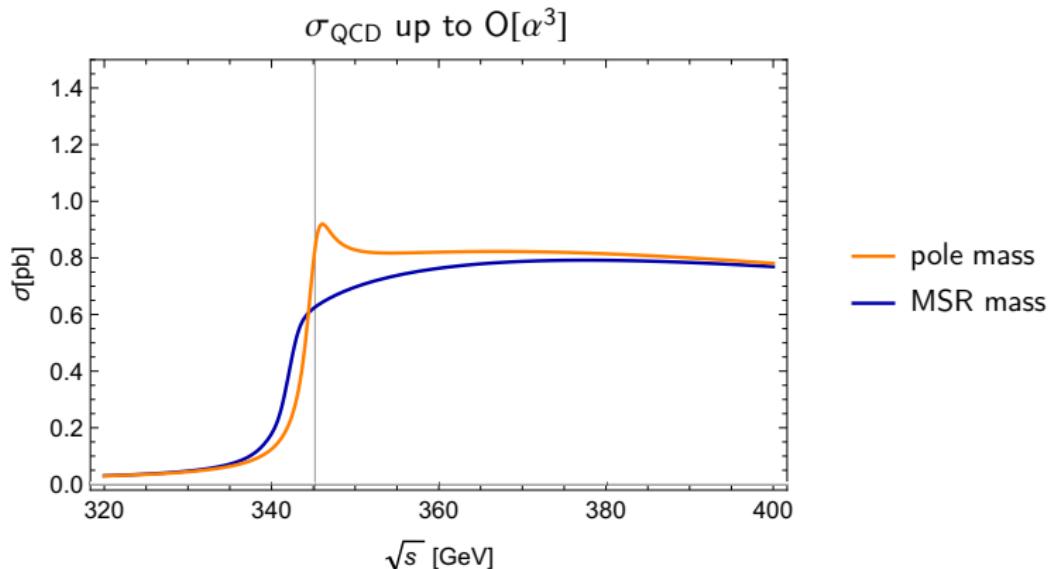
$$m_{pole} = \bar{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h) \alpha_s(\bar{m})^n = \bar{m} + \overline{m} \alpha_s a_1 + \dots$$

$$m_{pole} = m_{MSRn}(R) + R \sum_{n=1}^{\infty} a_n(n_l, 0) \alpha_s(R)^n = m_{MSRn}(R) + \alpha_s R a_1 + \dots$$

\Rightarrow choose $R \sim m v$

Matching

MSR mass with $R \sim m v$:



Matching

Pole mass:

	α^0	α^1	α^2	α^3
LO	v			...
NLO				...
NNLO				...
⋮	⋮	⋮	⋮	⋮

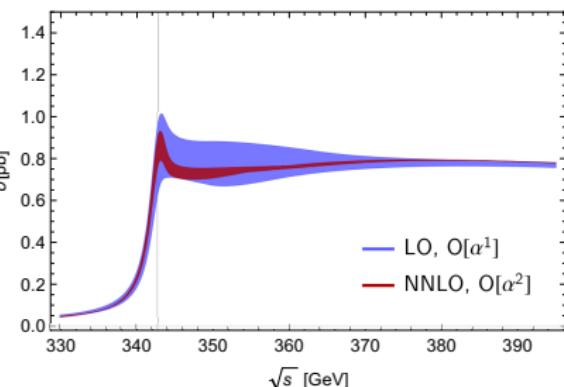
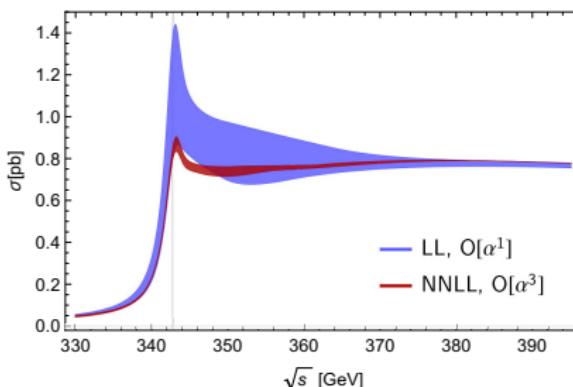
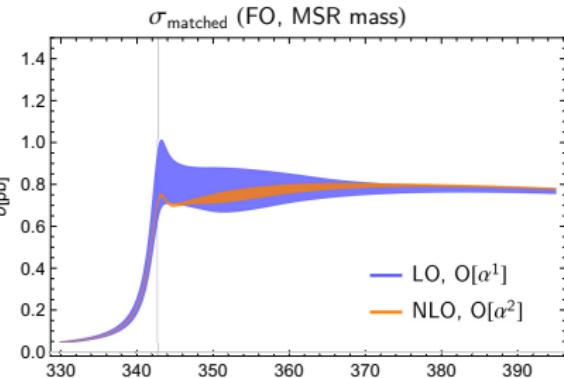
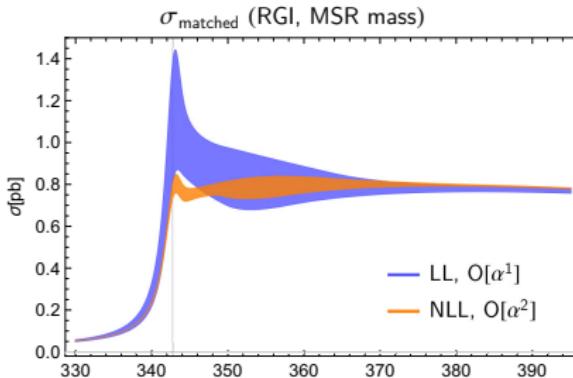
MSR mass:

	α^0	α^1	α^2	α^3	
LO	v	$\frac{\alpha}{v} \frac{R}{m}$	$\frac{\alpha^2}{v^3} \frac{R^2}{m^2}$	$\frac{\alpha^3}{v^5} \frac{R^3}{m^3}$...
NLO			$\frac{\alpha^2}{v} \frac{R}{m}$	$\frac{\alpha^3}{v^3} \frac{R^2}{m^2}$...
NNLO		$\alpha \frac{v}{m} \frac{R}{m}$	$\frac{\alpha^2}{v} \frac{R^2}{m^2}$	$\frac{\alpha^3}{v^3} \frac{R^3}{m^3}, \frac{\alpha^3}{v} \frac{R}{m}$...
⋮	⋮	⋮	⋮	⋮	

$$R \sim m v \sim m \alpha$$

Matching

Results in the MSR mass scheme:



Conclusion

Summary

- matched σ_{NRQCD} and σ_{QCD}
- implemented of the MSR mass scheme

Outlook

- N^3LO corrections for the fixed order cross section
- higher order electroweak effects

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Thank you for your attention!