Factorization and Resummation for Massive Quark Effects in Exclusive Drell-Yan

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based on arXiv:1703.09702

in collaboration with Piotr Pietrulewicz, Anne Spiering and Frank J. Tackmann

Particle Physics Seminar, Wien

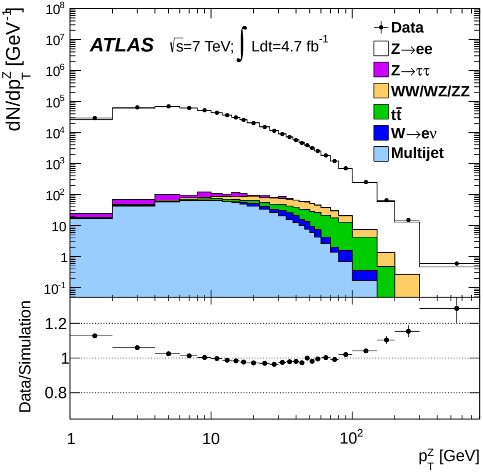
09 May 2017

Outline

Outline

Motivation

- p_T spectrum of Z-boson measured
  with high precision
- NNLL′ analyses available
- no systematic theoretical description of
  b-quark mass effects yet
  can be relevant in m_W measurements
- discrepancies between MC and
  experiment in low p_T region



[ATLAS Collaboration (2014)]

goal: systematic treatment of quark mass effects at NNLL′ accuracy
for transverse momentum ($\mathrm{SCET_{II}}$) and beam thrust ($\mathrm{SCET_{I}}$) spectrum

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- p_T spectrum of Z-boson measured with high precision
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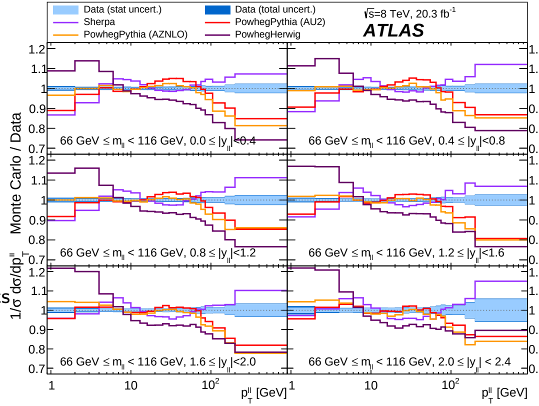

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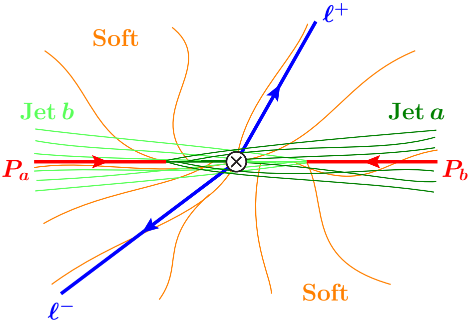
goal: systematic treatment of quark mass effects at NNLL′ accuracy for transverse momentum ($SCET_{II}$) and beam thrust ($SCET_{I}$) spectrum

Transverse Momentum and Beam Thrust

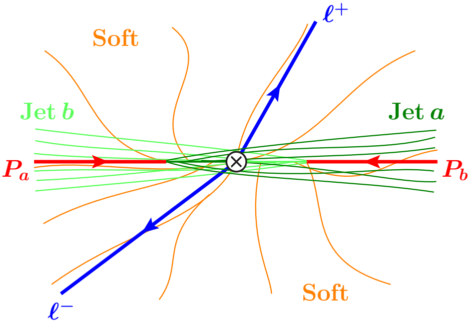


from: I.W.Stewart,F.J.Tackmann,W.J.Waalewijn, *Phys. Rev.* **D81** (2010) 094035

- Drell-Yan + 0 jets
- jet veto to restrict hadronic final state
- requires resummation of logarithms $\mu_{\text{jet}} \ll Q$

- $p_T = |\sum_i \vec{p}_{T,i}| = |\vec{p}_{T,\ell^+} + \vec{p}_{T,\ell^-}|$
- 0-jet limit for $p_T \ll Q$
- 2 scales (SCET_{II}): hard $\sim Q$, collinear/soft $\sim p_T$
- rapidity logarithms

Transverse Momentum and Beam Thrust



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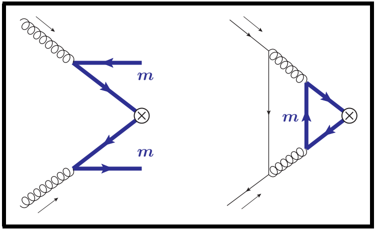
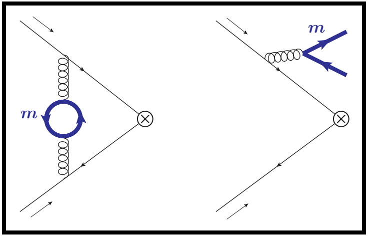
- Drell-Yan + 0 jets
- jet veto to restrict hadronic final state
- requires resummation of logarithms $\mu_{\text{jet}} \ll Q$

- $\mathcal{T} = \sum_i \min\{n_a \cdot p_i, n_b \cdot p_i\} \qquad n^\mu_{a,b} = (1, \pm\hat{z})$
- 0-jet limit for $\mathcal{T} \ll Q$
- closely related to 0-jettines τ_0: $\qquad \mathcal{T} = \tau_0 + \mathcal{O}\big(\frac{p_T^2}{Q^2}\big)$
- 3 scales (SCET_I): hard $\sim Q$, collinear $\sim \sqrt{Q\mathcal{T}}$, soft $\sim \mathcal{T}$
- no rapidity logarithms (in the massless case)

Massive Quarks in Drell-Yan

primary and secondary massive quarks.





- primay: massive quarks go into hard interaction
- secondary: massive quark corrections to light quark induced processes
- both start at $\mathcal{O}\left(\alpha_s^2\right)$, relevant for NNLL′ resummation
- different hierarchies between p_T and m possible:

$m \ll p_T$ $\qquad$ $p_T \sim m$ $\qquad$ $p_T \ll m$

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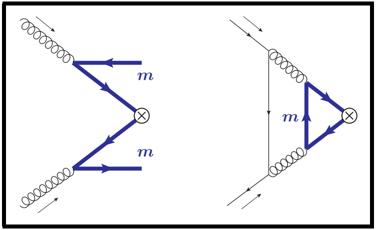
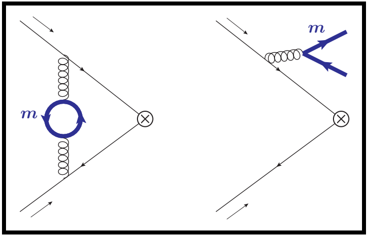




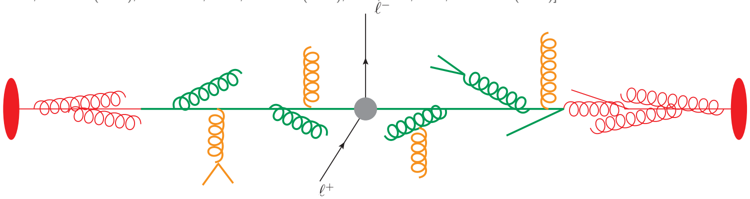
- primay: massive quarks go into hard interaction
- secondary: massive quark corrections to light quark induced processes
- both start at $\mathcal{O}\left(\alpha_s^2\right)$, relevant for NNLL′ resummation
- different hierarchies between $\mathcal{T}$ and m possible:

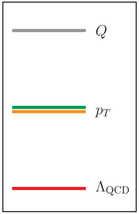
$m \ll \mathcal{T}$ $\quad$ $\mathcal{T} \sim m$ $\quad$ $\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$ $\quad$ $\sqrt{Q\mathcal{T}} \sim m$ $\quad$ $\sqrt{Q\mathcal{T}} \ll m$

Outline

Massless Factorization for p_T-spectrum in Drell-Yan

[Collins, Soper, Sterman (1985)]
[Becher, Neubert (2011); Echevarria, Idilbi, Scimemi (2012); Chiu, Jain, Neill, Rothstein (2012)]

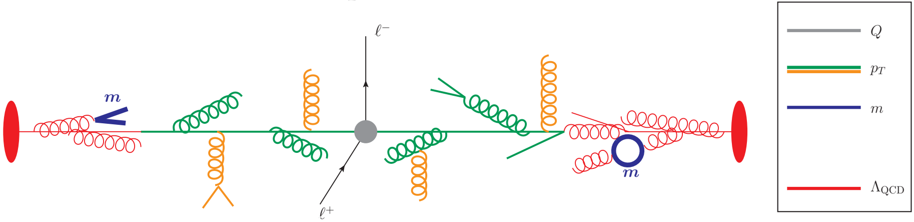




$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \sum_{i\in\{q\}} H_i^{(n_f)}(Q) \times \Big[\sum_{j\in\{q,g\}} \mathcal{I}_{ij}^{(n_f)}(p_T, x) \otimes f_j^{(n_f)}(x) \Big]^2 \otimes_\perp S^{(n_f)}(p_T) + \mathcal{O}\Big(\frac{p_T}{Q}, \frac{\Lambda_{\rm QCD}^2}{p_T^2}\Big)$$

- H: hard function, scale Q
- $\mathcal{I}$: beam function, scale p_T
- S: soft function, scale p_T
- f: PDF, scale $\Lambda_{\rm QCD}$

Factorization for $m \ll p_T$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \sum_{i\in\{q,Q\}} H_i^{(5)}(Q) \times \Big[\sum_{j\in\{q,Q,g\}} \mathcal{I}_{ij}^{(5)}(p_T, x) \otimes f_j^{(5)}(x) \Big]^2 \otimes_\perp S^{(5)}(p_T)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{p_T^2}\right)$

- matching: $f_j^{(5)}(x) = \sum_k \mathcal{M}_{jk}(x, m) \otimes f_k^{(4)}(x)$
- secondary $\mathcal{M}_{qq}$ and primary $\mathcal{M}_{Qg}$ heavy quarks

  [M. Buza, Y. Matinounine, J. Smith, W. van. Neerven (1998)]
- hard, beam and soft functions with 5 massless flavors

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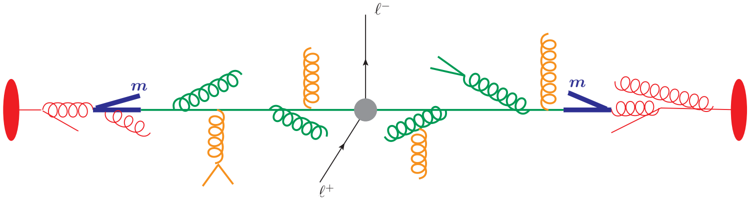

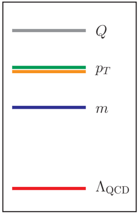

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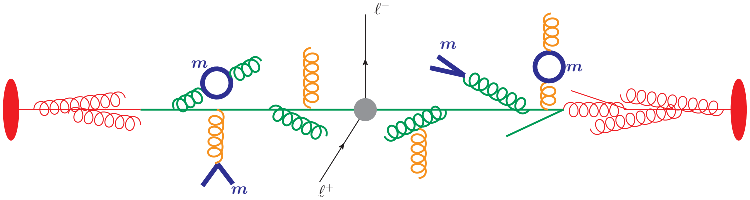
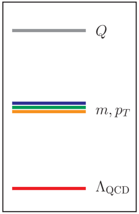
Factorization for $p_T \sim m$



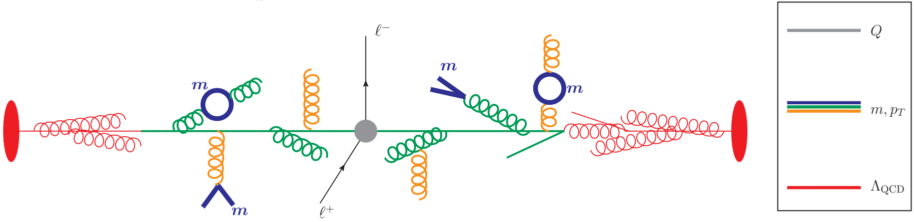


$$\frac{\mathrm{d}\sigma_{\rm sec}}{\mathrm{d}p_T} = \sum_{i\in\{q\}} H_i^{(5)}(Q) \times \Big[\sum_{j\in\{q,g\}} \mathcal{I}_{ij}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \Big]^2 \otimes_\perp S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\Big(\frac{m^2}{Q^2}\Big)$

- secondary massive quarks in beam function $\mathcal{I}_{qq}$ and soft function S

Factorization for $p_T \sim m$



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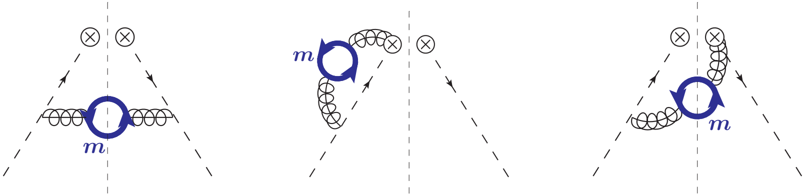

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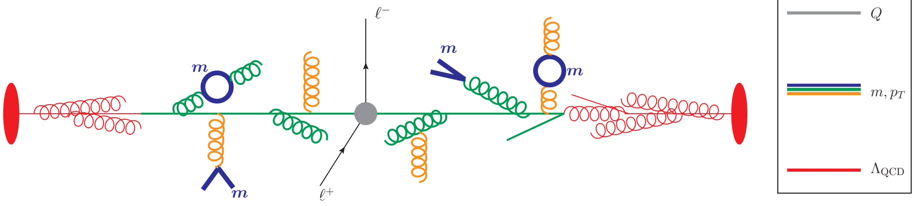

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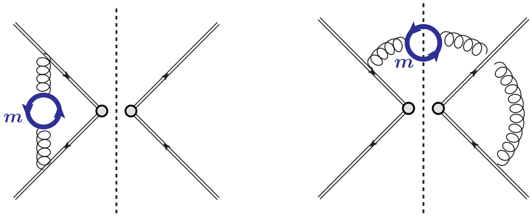

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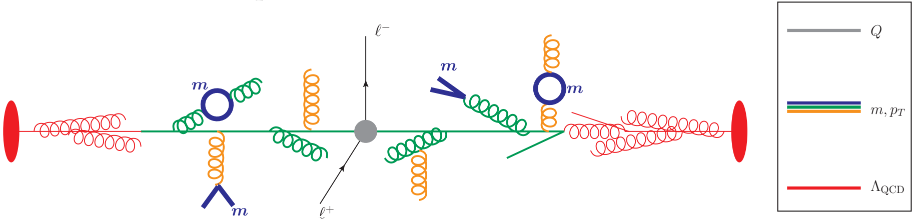

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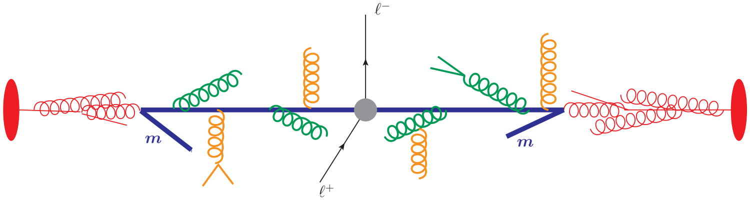

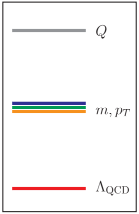

$$\frac{d\sigma_{\text{prim}}}{dp_T} = H_Q^{(5)}(Q) \times \Big[\sum_{j\in\{q,g\}} \mathcal{I}_{Qj}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \Big]^2 \otimes_\perp S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$

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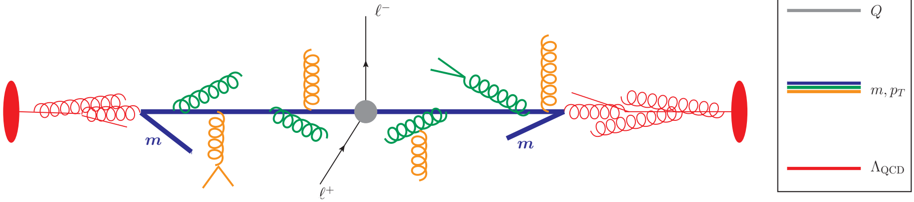
  [A. Balyaev, P. Nadolsky, C.-P. Yuan (2005); S. Berge, P. Nadolsky, F. Olness (2005)]

Factorization for $p_T \sim m$



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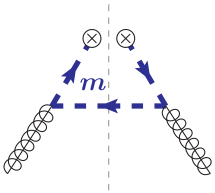
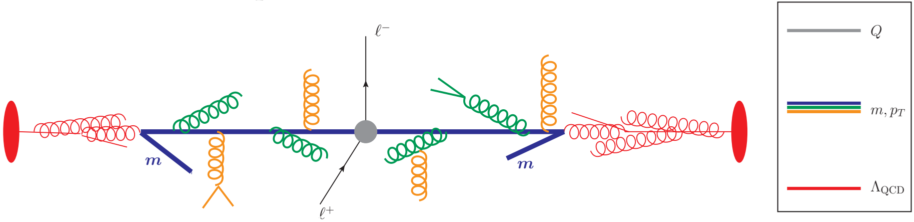


Factorization for $p_T \sim m$



$$\frac{\mathrm{d}\sigma_{\rm prim}}{\mathrm{d}p_T} = H_Q^{(5)}(Q) \times \Big[\sum_{j\in\{q,g\}} \mathcal{I}_{Qj}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \Big]^2 \otimes_\perp S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\Big(\frac{m^2}{Q^2}\Big)$

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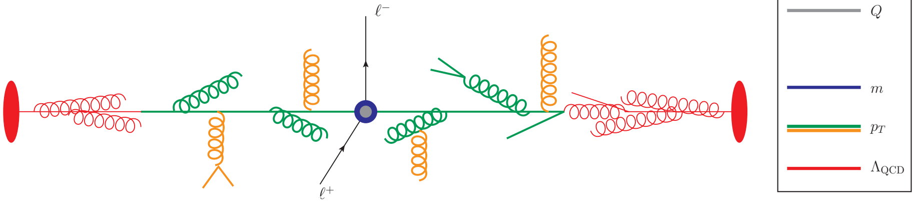
- PDF with 4 and hard function with 5 massless flavors

Factorization for $p_T \ll m$



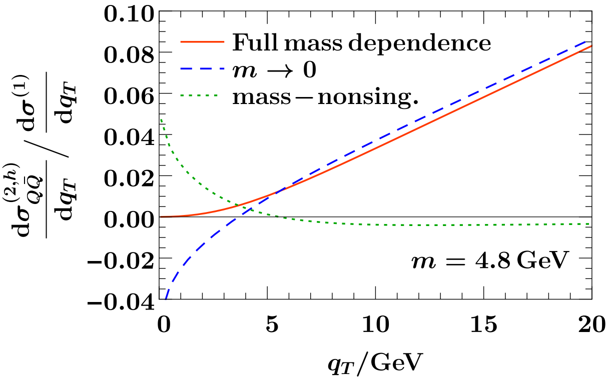
$$\frac{d\sigma}{dp_T} = \sum_{i\in\{q\}} H_i^{(5)}(Q) \times H_m(m) \times \Big[\sum_{j\in\{q,g\}} \mathcal{I}_{ij}^{(4)}(p_T, x) \otimes f_j^{(4)}(x) \Big]^2 \otimes_\perp S^{(4)}(p_T)$$

Power Corrections: $\mathcal{O}\Big(\frac{p_T^2}{m^2}, \frac{m^2}{Q^2}\Big)$

- mass mode matching function H_m from integrating out the heavy flavor from the current
  [S. Gritschacher, A. Hoang, I. Jemos , V. Mateu, P. Pietrulewicz (2014)]

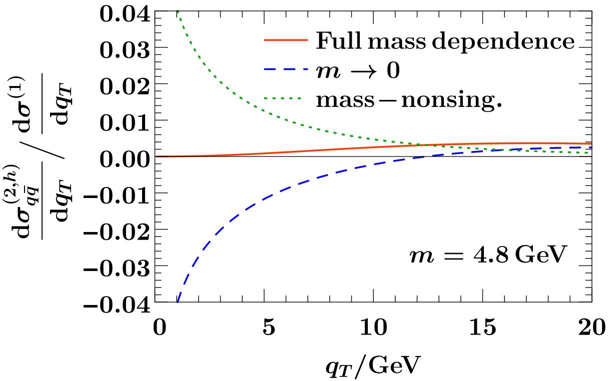
- beam function, soft function, PDF with 4 massless flavors,
  hard function with 5 massless flavors

Mass Effects - Primary



- $\mathcal{O}(\alpha_s^2)$ contributions from primary massive quarks at fixed order, normalized to the LO spectrum
- $E_{\mathrm{cm}} = 13\,\mathrm{TeV}$, $m_b = 4.8\,\mathrm{GeV}$, $Q = m_Z$, $Y = 0$, MSTW NLO PDFs

Mass Effects - Secondary



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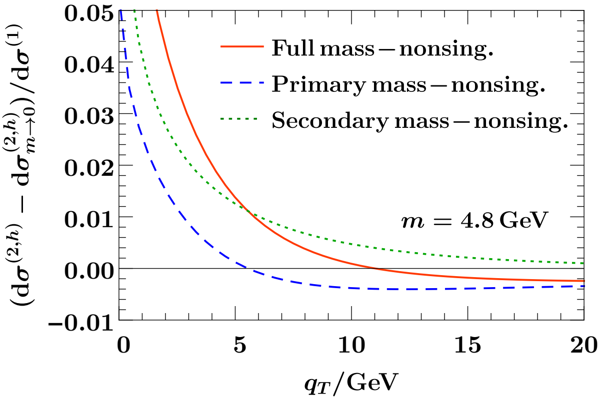
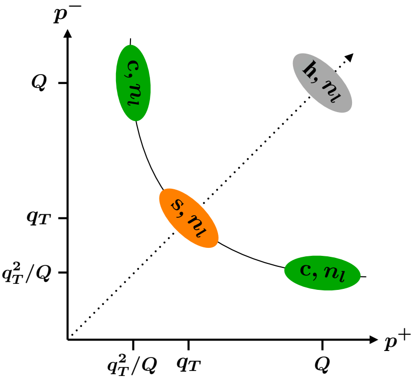
Mass Effects



- $\mathcal{O}(\alpha_s^2)$ quark mass corrections to the Z-boson p_T-spectrum at fixed order, normalized to the LO spectrum
- $E_{\mathrm{cm}} = 13\,\mathrm{TeV}$, $m_b = 4.8\,\mathrm{GeV}$, $Q = m_Z$, $Y = 0$, MSTW NLO PDFs

Outline

Massless Factorization for p_T



(non-pert. coll.-modes (PDF) not shown)

- hard matching coefficient

$$J^\mu_{\text{QCD}} = C \times J^\mu_{\text{SCET}}$$

- measurement function

$$\mathcal{M}(\vec{p}_T) = \delta^{(2)}(\vec{p}_T - \vec{\mathcal{P}}_\perp)$$

- beam function

$$B_{ij}(\vec{p}_T, \frac{\omega}{p^-}) = \langle j|\overline{\chi}\mathcal{M}(\vec{p}_T)\frac{\bar{n}\!\!\!/}{2}[\delta(\omega - \mathcal{P}_n)\chi]|j\rangle$$

$$B_{ij}(\vec{p}_T, x) = \sum_k \mathcal{I}_{ik}(\vec{p}_T, x) \otimes f_{k/j}(x)$$

$$f_{i/j}(\frac{\omega}{p^-}) = \langle j|\overline{\chi}\frac{\bar{n}\!\!\!/}{2}[\delta(\omega - \mathcal{P}_n)\chi]|j\rangle$$

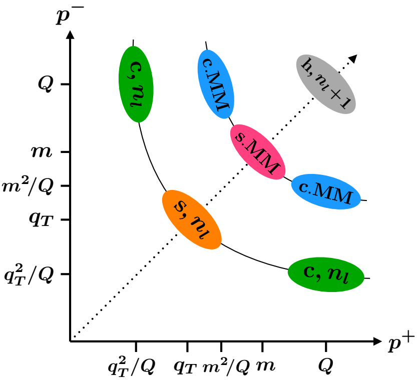
- soft function

$$S(\vec{p}_T) = \frac{1}{N_c}\text{tr}\langle 0|\overline{\text{T}}[S_n^\dagger S_{\bar{n}}]\mathcal{M}(\vec{p}_T)\text{T}[S_{\bar{n}}^\dagger S_n]|0\rangle$$

SCET_{II} - rapidity divergences cancel between soft and beam functions.

$$\frac{\text{d}\sigma}{\text{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T}{Q}\right)$$

$p_T \ll m$



- integrate out massive flavor in SCET current
  ⇒mass mode matching functions

$$\left(J_{\rm SCET}^{(n_l+1)}\right)^\mu = C_c(m) \times C_{\bar{c}}(m) \times C_s(m) \times \left(J_{\rm SCET}^{(n_l)}\right)^\mu$$

[S. Gritschacher, A. Hoang, I. Jemos, V. Mateu, P. Pietrulewicz (2014)]

[A. Hoang, P. Pietrulewicz, D.S. (2016)]

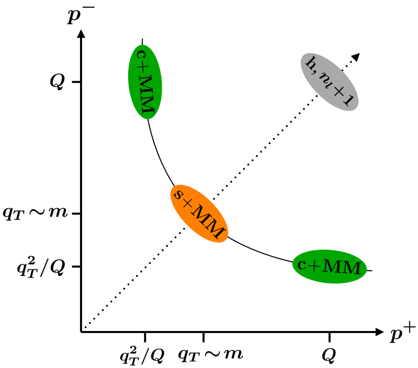
- rapidity divergences cancle between
  these mathing coefficients
  ⇒rapidity RGE to resum associated logs

- hard function with $(n_l + 1)$ massless flavors

- beam and soft function with (n_l) massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_c(m) \times H_{\bar{c}}(m) \times H_s(m) \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_{k/P}^{(n_l)}\right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)$$

$p_T \sim m$



- mass scale coincides with beam/soft scale
  $\Rightarrow$ massive beam and soft functions

- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

$$i \in \{q, Q, g\} \qquad k \in \{q, g\}$$

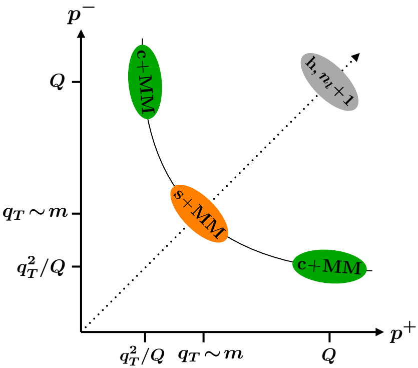
- secondary massive quarks in soft function

- secondary massive quarks change rapidity RGE

- hard function with $(n_l + 1)$ massless flavors

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m)$$

$p_T \sim m$



- mass scale coincides with beam/soft scale
  $\Rightarrow$ massive beam and soft functions
- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

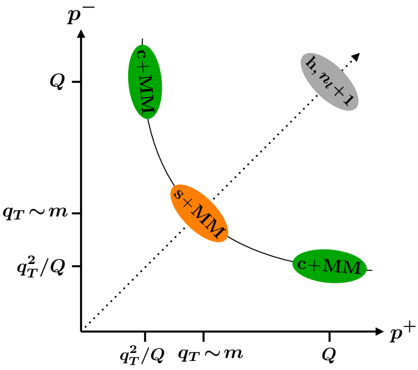
new

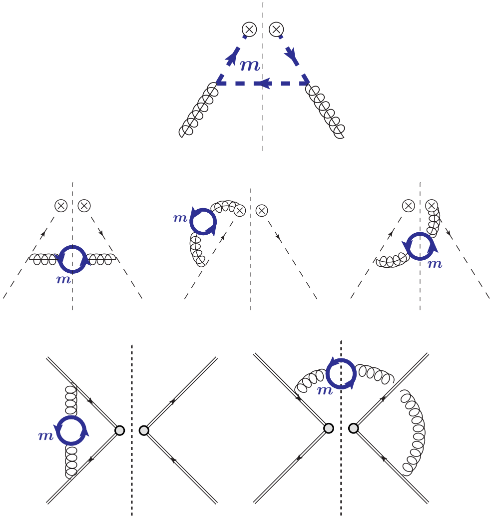
$$i \in \{q, Q, g\} \qquad k \in \{q, g\}$$

- secondary massive quarks in soft function new
- secondary massive quarks change rapidity RGE
- hard function with $(n_l + 1)$ massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)}(m)$$

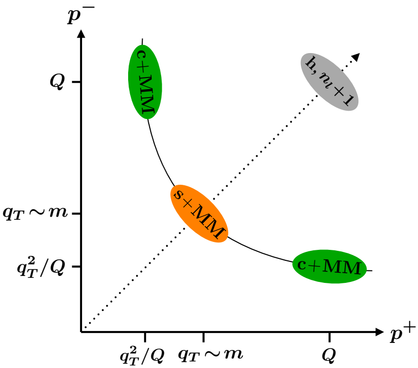
$p_T \sim m$





$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \Big]^2 \otimes S^{(n_l+1)}(m)$$

$p_T \sim m$



- mass scale coincides with beam/soft scale
  $\Rightarrow$ massive beam and soft functions

- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

new

$$i \in \{q, Q, g\} \qquad k \in \{q, g\}$$

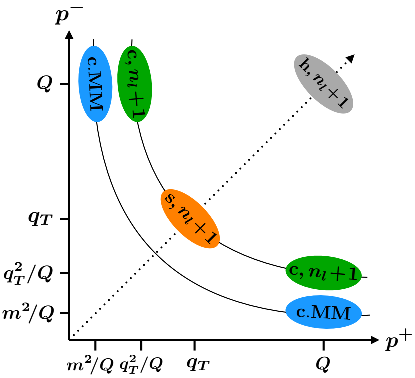
- secondary massive quarks in soft function — new

- secondary massive quarks change rapidity RGE

- hard function with $(n_l + 1)$ massless flavors

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m)$$

$m \ll p_T$



- matching in the PDF evolution

$$f_{i/j}^{(n_l+1)}(m) = \sum_k \mathcal{M}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

$$i \in \{q, Q, g\} \qquad k \in \{q, g\}$$

[M. Buza, Y. Matiounine, J. Smith, W. van Neerven (1998)]

- beam/soft function with $(n_l + 1)$ massless flavors

$$B_{ij}^{(n_l+1)} = \sum_k \mathcal{I}_{ik} \otimes f_{kj}^{(n_l+1)}$$

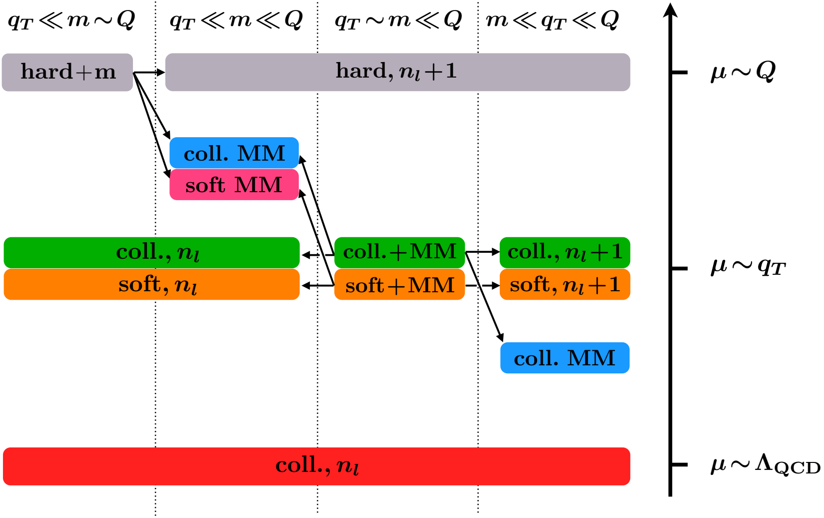
$$i \in \{q, Q, g\} \qquad k \in \{q, Q, g\}$$

- no rapidty divergences in PDFs

- hard function with $(n_l + 1)$ massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{p_T^2}\right)$$

Summary of all Modes



Relations between Hierarchies

- components for the different hierarchies are related.

- beam function matching coefficients:

$$\mathcal{I}_{ik}(m) = \mathcal{I}_{ik}^{(n_l)} \times H_c(m) \times \Big[1 + \mathcal{O}\Big(\frac{p_T^2}{m^2}\Big)\Big]$$

$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \times \Big[1 + \mathcal{O}\Big(\frac{m^2}{p_T^2}\Big)\Big] \qquad j \in \{q, Q, g\}$$

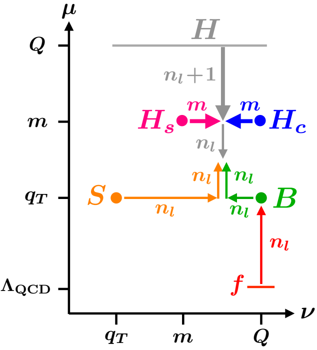
- soft function:

$$S^{(n_l+1)}(m) = S^{(n_l)} \times H_s(m) \times \Big[1 + \mathcal{O}\Big(\frac{p_T^2}{m^2}\Big)\Big]$$

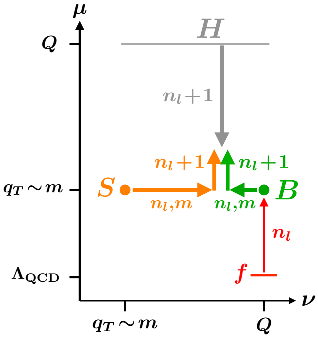
$$S^{(n_l+1)}(m) = S^{(n_l+1)} \times \Big[1 + \mathcal{O}\Big(\frac{m^2}{p_T^2}\Big)\Big]$$

- can be used to systematically include all power corrections.

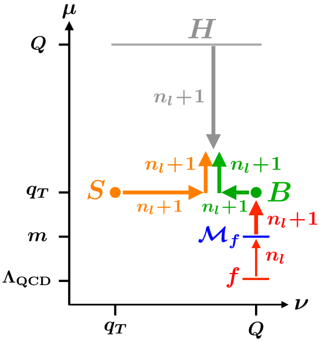
Renormalizaton Group Evolution



$q_T \ll m$



$q_T \sim m$



$m \ll q_T$

$$\gamma_{\nu,B,m}^{(n_l+1)} \neq \gamma_{\nu,B}^{(n_l+1)}$$

$$\gamma_{\nu,S,m}^{(n_l+1)} \neq \gamma_{\nu,S}^{(n_l+1)}$$

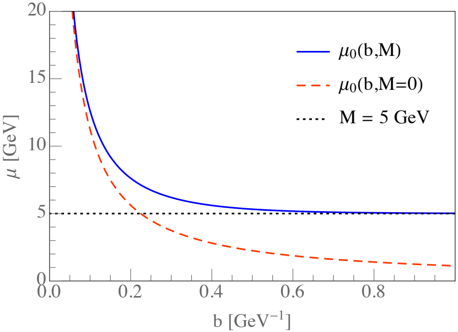
Rapidity Logs from Massive Flavors

- secondary massive quarks lead to mass dependent rapidity anomalous dimension $\tilde{\gamma}_\nu(b,m,\mu)$ (here in b space)

$$\tilde{\gamma}_\nu(b,m,\mu) \overset{b\to 0}{\longrightarrow}= \Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^2 C_F T_F \Big(\frac{16}{3}L_b^2 + \frac{160}{9}L_b + \frac{224}{27}\Big) \qquad L_b = \ln\Big(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\Big)$$

$$\tilde{\gamma}_\nu(b,m,\mu) \overset{b\to \infty}{\longrightarrow}= \Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^2 C_F T_F \Big(-\frac{16}{3}L_m^2 - \frac{160}{9}L_m - \frac{224}{27}\Big) \qquad L_m = \ln\frac{m^2}{\mu^2}$$

- large logarithms in $\tilde{\gamma}_\nu$ can be avoided e.g. by b space setting of μ



$$\mu_0(b,m) \overset{b\to 0}{\longrightarrow} \frac{2e^{-\gamma_E}}{b}$$

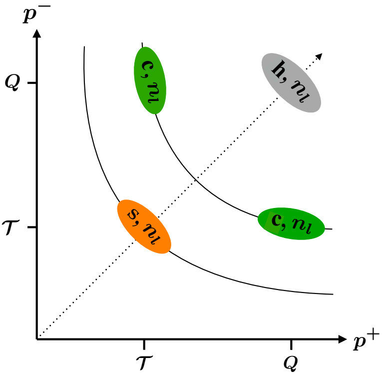
$$\mu_0(b,m) \overset{b\to \infty}{\longrightarrow} m$$

$\Rightarrow$ mass introduces IR cutoff

$\Rightarrow$ no non-pert. regime for $b \to \infty$

Outline

Massless Factorization for $\mathcal{T}$



(non-pert. coll.-modes (PDF) not shown)

SCET$_\mathrm{I}$ - no rapidity divergences.

- hard matching coefficient

$$J^\mu_{\mathrm{QCD}} = C \times J^\mu_{\mathrm{SCET}}$$

- measurement function

$$\mathcal{M}_a(k, \pm) = \delta(k - \hat{p}_a^\pm)$$

- beam function

$$B_{ij}\big(\omega b, \frac{\omega}{p^-}\big) = \langle j|\overline{\chi}\mathcal{M}(b,+)\frac{\bar{\not n}}{2}\big[\delta\big(\omega - \mathcal{P}_n\big)\chi\big]|j\rangle$$

$$B_{ij}(t,x) = \sum_k \mathcal{I}_{ik}(t,x) \otimes f_{k/j}(x)$$

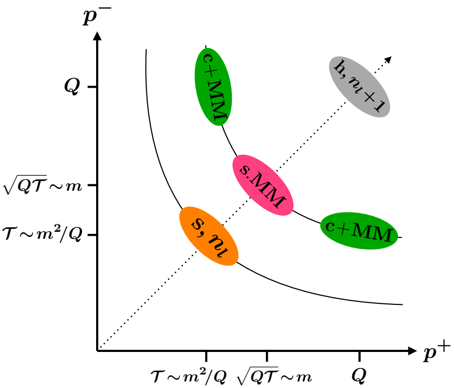
$$f_{i/j}\big(\frac{\omega}{p^-}\big) = \langle j|\overline{\chi}\frac{\bar{\not n}}{2}\big[\delta\big(\omega - \mathcal{P}_n\big)\chi\big]|j\rangle$$

- (u)soft function

$$S(k_a, k_b) = \frac{1}{N_c}\mathrm{tr}\langle 0|\overline{\mathrm{T}}\big[Y_n^\dagger Y_{\bar{n}}\big]\mathcal{M}_a(k_a,+)\mathcal{M}_b(k_b,-)\mathrm{T}\big[Y_{\bar{n}}^\dagger Y_n\big]|0\rangle$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\mathcal{T}}{Q}\right)$$

$\sqrt{Q\mathcal{T}} \sim m$



- prim. and sec. massive quarks in beam function

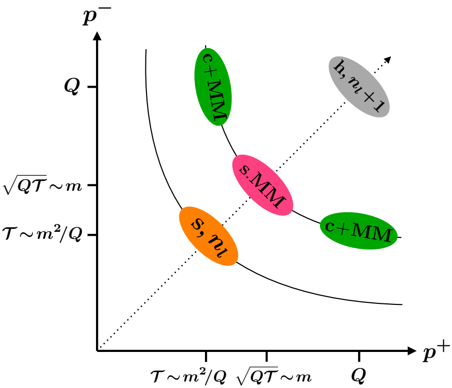
$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

$$i \in \{q, Q, g\} \qquad k \in \{q, g\}$$

- secondary massive quarks introduce rapidity divergences in beam function
- mass scale still above soft scale
  $\Rightarrow$ soft MM integrated out in current $\Rightarrow H_s$
- resummation of rapidity logs between massive beam function and H_s
- hard function with $(n_l + 1)$ massless flavors
- soft function with (n_l) massless flavors

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\mathcal{T}^2}{m^2}\right)$$

$\sqrt{Q\mathcal{T}} \sim m$



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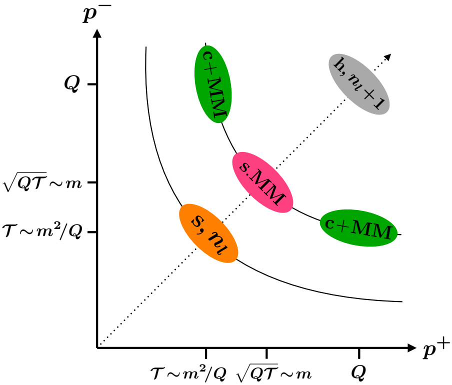

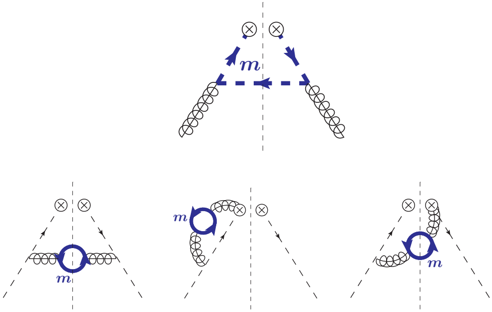

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\mathcal{T}^2}{m^2}\right)$$

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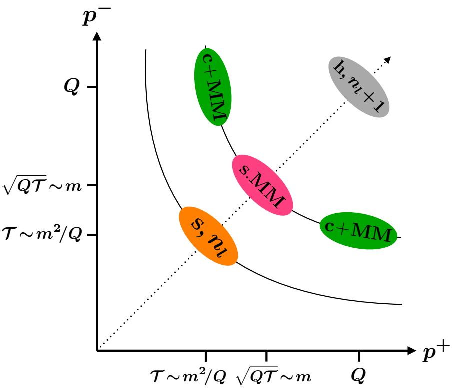

- prim. and sec. massive quarks in beam function

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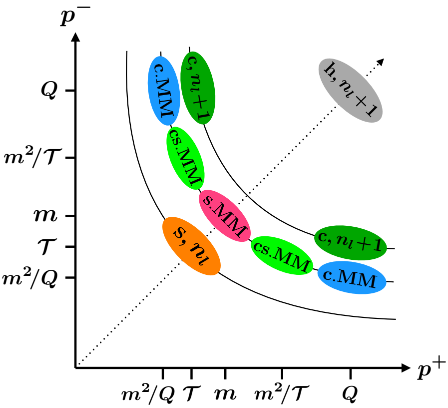
$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$



- collinear-soft function S_c

$$S_c(\ell) = \frac{1}{N_c} \mathrm{tr}\langle 0|\overline{\mathrm{T}}\big[X_n^\dagger V_n\big]\mathcal{M}(\ell,+)\mathrm{T}\big[V_n^\dagger X_n\big]|0\rangle$$

- X_n, V_n : boosted Wilson lines of collinear-soft gluon fields

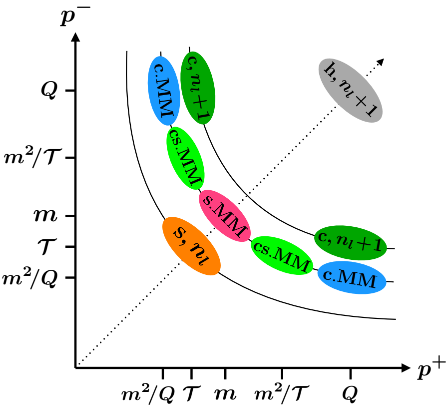
$$A_c \sim \quad (Q, \mathcal{T}, \sqrt{Q\mathcal{T}}) \qquad Q\mathcal{T}$$
$$A_{cs} \sim (\frac{m^2}{\mathcal{T}}, \mathcal{T}, m) \qquad m^2$$
$$A_{us} \sim \quad (\mathcal{T}, \mathcal{T}, \mathcal{T}) \qquad \mathcal{T}^2$$

- vanishes for purely massless contributions

- rapidity divergences for massive quarks

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$



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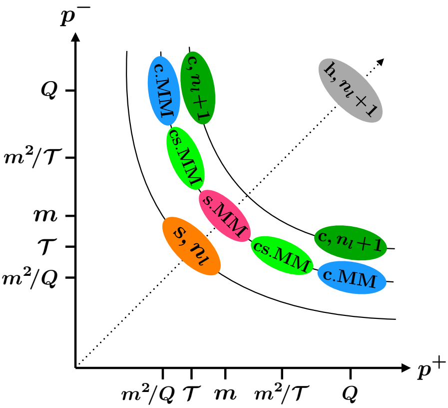
$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$



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new

- X_n, V_n : boosted Wilson lines of collinear-soft gluon fields

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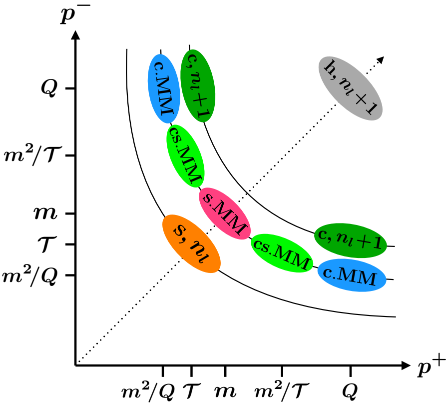
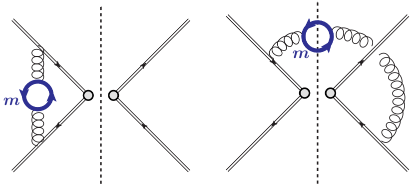
$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$





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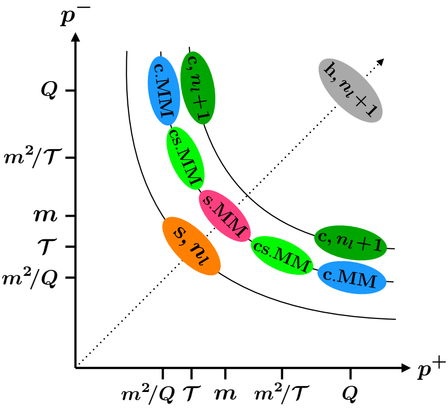
$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$



- collinear soft function S_c

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new

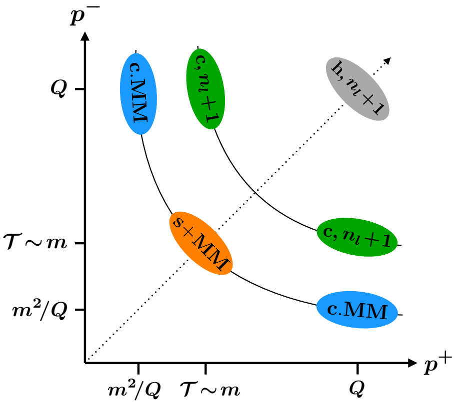
- matching in PDFs: $(n_l+1) \to (n_l)$
- soft mass mode matching function H_s
- resummation of rapidity logs between S_c and H_s
- hard/beam function with (n_l+1) massless flavors
- soft function with (n_l) massless flavors

results with csoft function equivalent to threshold corrections in previous works

[S. Gritschacher, A. Hoang, I. Jemos, V. Mateu, P. Pietrulewicz (2014); A. Hoang, P. Pietrulewicz, D.S. (2016)]

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)}\Big]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

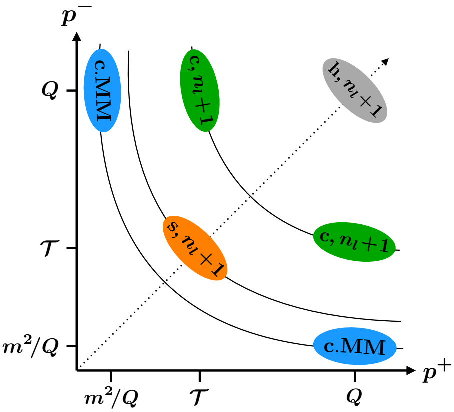
$\mathcal{T} \sim m$



- soft function with secondary massive quarks
  [S. Gritschacher, A. Hoang, I. Jemos, P. Pietrulewicz (2014)]
- matching in PDFs: $(n_l + 1) \to (n_l)$
- hard/beam function with $(n_l + 1)$ massless flavors
- no rapidity logarithms

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q\mathcal{T}}\right)$$

$m \ll \mathcal{T}$



- soft function with $(n_l + 1)$ massless flavors
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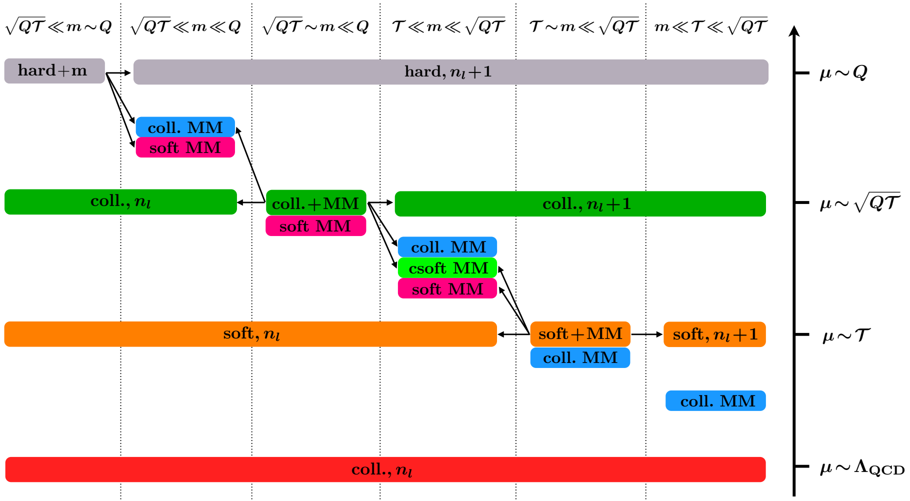
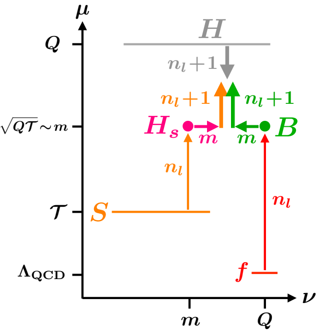
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{\mathcal{T}^2}\right)$$

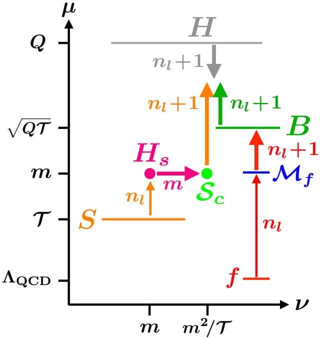
Summary of all Modes



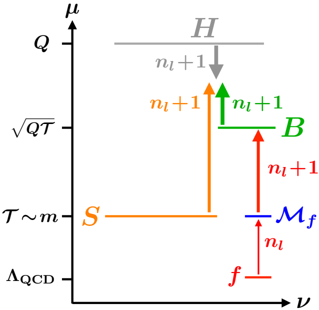
Renormalizaton Group Evolution



$\sqrt{Q\mathcal{T}} \sim m$



$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$



$\mathcal{T} \sim m$

$$\gamma_{B,m}^{(n_l+1)} \neq \gamma_B^{(n_l+1)}$$

$$\gamma_{H_s} + \gamma_{B,m}^{(n_l+1)} + \gamma_S^{(n_l)} = \gamma_B^{(n_l+1)} + \gamma_S^{(n_l+1)}$$

$$\gamma_{\nu,B,m} = \gamma_{\nu,S_c} = \gamma_{\nu,H_c}$$

Outline

1 Introduction

2 Factorization for the p_T spectrum

3 Details on the factorization for the p_T spectrum

4 Details on the factorization for the $\mathcal{T}$ spectrum

5 Outlook: effects on W boson mass measurements

6 Conclusions

W boson mass measurements

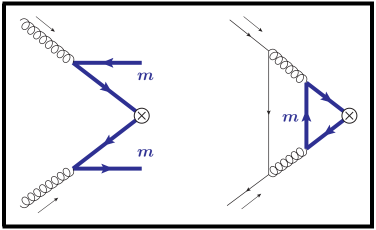
- experimentalists rely on ratio of W and Z boson spectrum

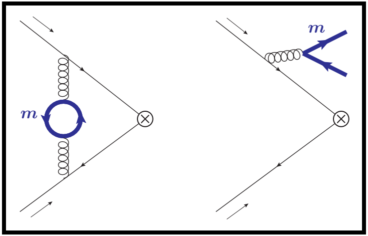
$$\left[\frac{\mathrm{d}\sigma^W}{\mathrm{d}q_T}\right]_{\text{prediction}} = \left[\frac{\mathrm{d}\sigma^W}{\mathrm{d}q_T} \times \left(\frac{\mathrm{d}\sigma^Z}{\mathrm{d}q_T}\right)^{-1}\right]_{\text{theory}} \times \left[\frac{\mathrm{d}\sigma^Z}{\mathrm{d}q_T}\right]_{\text{measured}}$$

- Z boson spectrum measured with very high accuracy
- many things cancel to large extent in the ratio
- every difference between Z and W production can become relevant

Massive Quarks in Z production

primary and secondary massive quarks.





- primay: massive quarks go into hard interaction
- secondary: massive quark corrections to light quark induced processes
- both start at $\mathcal{O}\left(\alpha_s^2\right)$, relevant for NNLL$'$ resummation

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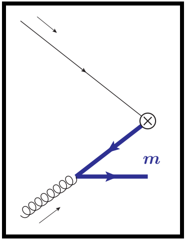

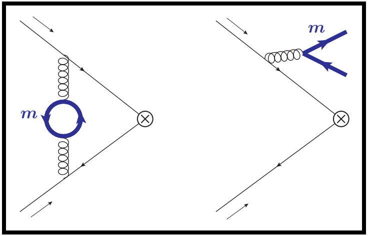

- secondary massive quark effects the same as for Z production
- primary charm quarks already contribute at $\mathcal{O}(\alpha_s)$
- primary bottom quarks CKM suppressed

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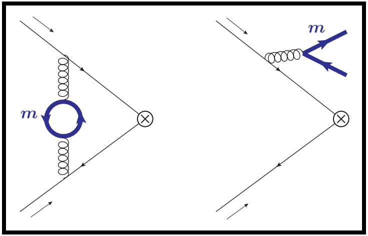

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Outline

Outlook and Conclusions

- included massive quarks into the factorization theorem for Drell-Yan $+$ 0 jets for a SCET_{I} and a SCET_{II} jet veto
- resummation of all mass related logarithms at NNLL$'$ accuracy (one and two loop beam and soft functions with massive quarks)
- different structure of rapidity logarithms due to secondary massive quarks
- relevant for precision measuremnts of W boson mass
- parts of this also relevant for other processes, e.g. $b\bar{b} \to H$

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Thank you for your attention!