“TeV scale MSSM Dark Matter and the electroweak Sommerfeld effect”

Wien, 9 June 2017

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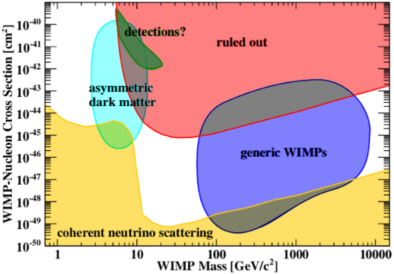
TECHNISCHE UNIVERSITÄT MÜNCHEN

MB, C. Hellmann, P. Ruiz-Femenia, [arXiv:1210.7928, 1411.6924, 1411.6930];
C. Hellmann, P. Ruiz-Femenia [arXiv:1302.0200];
MB, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel, P. Ruiz-Femenia, [arXiv:1601.04718];
MB, A. Bharucha, A. Hryczuk, S. Recksiegel, P. Ruiz-Femenia, [arXiv:1611.00804]

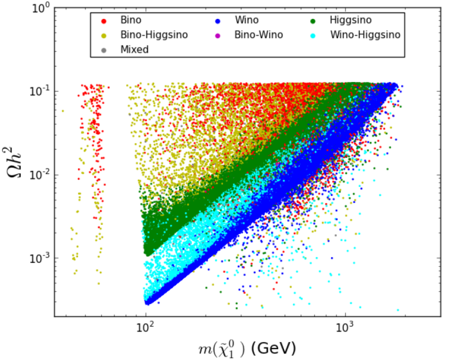
I. Introduction

Introduction



[Image credit: APS/Rafael Lang]

- TeV region natural mass range for a thermal relic DM particle with electroweak charges and correct abundance
- Out of LHC reach



[Cahill-Rowley et al., 1405.6716]

Sommerfeld effect

- Dark matter pair annihilation occurs at small non-relativistic velocities

  - $v \sim 10^{-3}$ in galaxies (up to 10^{-6} in dwarf galaxies)
  - In relic density calculations

$$\langle \sigma v \rangle = \frac{M_\chi}{4\pi T} \int dv \, 4\pi v \, \sigma(v) v \, \mathrm{e}^{-\frac{M_\chi v^2}{4T}}$$

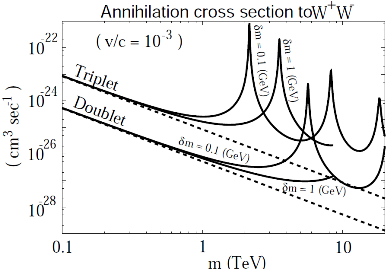
  and freeze-out occurs at $T \approx M/25$, i.e. $v \approx 1/5$.

- For non-singlet EW DM, large quantum corrections due to non-relativistic scattering prior to annihilation for $m_\chi \gg M_{\mathrm{EW}}$ due to electroweak/(Higgs) Yukawa potential
  [Hisano et al., 2004, 2006]

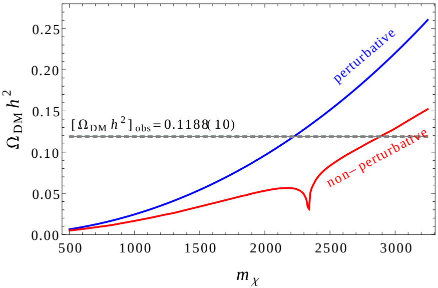
$$V(r) = -\frac{\alpha_2}{r} \, e^{-m_{W,Z} r}$$

- Tree-level calculations not sufficient.

Example: miminal models, in particular triplet (“pure wino”)



[Hisano et al., 2004]



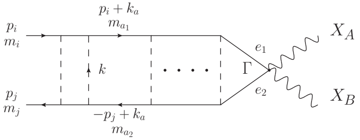
Pure Wino, following [Hisano et al., 2006]

Aim:

- General MSSM with mixed (wino-higgsino-bino) neutralino DM
- Systematic approximations within NREFT (partial-wave separation, potentials, expansion in mass splittings for off-diagonal annihilation)

II. Electroweak Sommerfeld effect and resonance

EW Sommerfeld enhancement



Contribution from one ladder rung from (potential) loop momentum region $k^0 \ll \vec{k} \ll m_\chi$

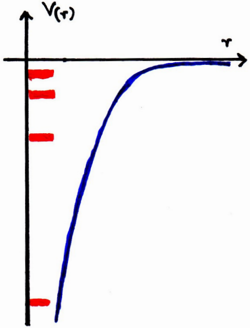
$$I \sim g_2^2 \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{(\vec{k}^{\,2} + m_W^2)(E_{\chi^0\chi^0} - \frac{(\vec{p}+\vec{k})^2}{m_\chi} - 2\delta m_{\chi^+\chi^0})} \sim \max\left(\frac{\pi\alpha_2 m_\chi}{|\vec{p}\,|}, \frac{\pi\alpha_2 m_\chi}{m_W}\right)$$

- Finite range of Yukawa potential cut-off enhancement when $|\vec{p}\,| \ll m_W$. Heavy intermediate states also cut-off the enhancement.
- $O(1)$ effect [$\rightarrow$ summation] for

$$m_\chi \geq \frac{m_W}{\pi\alpha_2} \sim \text{TeV} \qquad \delta m_{\chi^+\chi^0} \lesssim \frac{m_W^2}{m_\chi} \sim \text{GeV}$$

- $E_{\chi^0\chi^0} \sim \vec{p}^{\,2}/m_\chi \sim T$ (temperature of the Universe) for relic density computation.
- Coulomb potential in the $\chi^+\chi^-$ sector

Resonance effect for the Yukawa potential



Range $r \sim 1/m_W$ cuts off Rydberg states $[r_{\rm Ryd} \sim n^2/(M_\chi \alpha_2)]$

Finite number of levels

$$n^2 \lesssim \frac{m_\chi \alpha_2}{m_W}$$

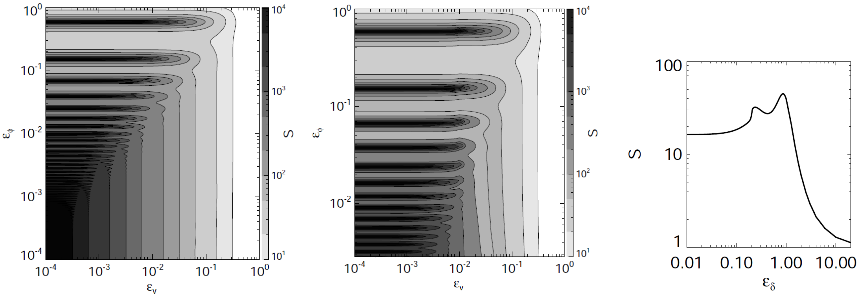
Increasing M_χ adds levels from above. Zero-energy bound states for certain m_χ. Then

$$S \propto \frac{1}{E - E_{\rm bind}} \sim \frac{1}{v^2}$$

stronger than $1/v$ Coulomb enhancement.

Resonant enhancement at certain values of m_χ starting in TeV range.

Two-state toy model [Slatyer, 0910.5713]



$\epsilon_\delta = 0$ $\qquad$ $\epsilon_\delta = 0.01$

$$V(r) = \begin{pmatrix} 0 & -\frac{\alpha_2}{r}\, e^{-m_W r} \\ -\frac{\alpha_2}{r}\, e^{-m_W r} & 2\delta m \end{pmatrix}$$

$$\epsilon_v = \frac{|\vec{p}|}{m_\chi \alpha_2}, \quad \epsilon_\phi = \frac{m_W}{m_\chi \alpha_2}, \quad \epsilon_\delta = \sqrt{\frac{2\delta m}{m_\chi \alpha_2^2}}$$

III. Non-relativistic MSSM – matching and Schrödinger equation

Sommerfeld enhancement in the general MSSM

MSSM with $M_\chi \gg M_Z$: degeneracies are natural (electroweak multiplets) $\rightarrow$ coannihilation
In general:

$$14\ \chi_i^0\chi_j^0,\ \chi_i^+\chi_i^-\ \text{charge-0 states,}$$
$$8\ \chi_i^0\chi_j^+\ \text{charge +1 [+ conjugates],}$$
$$3\ \chi_i^+\chi_j^+\ \text{charge +2 states [+ conjugates].}$$

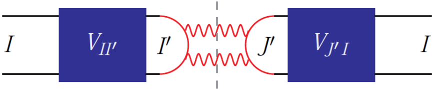
depending on M_1 (bino), M_2 (wino), μ (Higgsino).

Example: Dominantly Wino
$[M_2 < |\mu| \ll |M_1|$ with $m_W \ll |\mu| - M_2]$

$$\delta m_{\tilde{\chi}_1^+} \equiv m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} \approx \frac{12 m_W^4 M_2\, c_{2\beta}^2}{(\mu^2 - M_2^2)^2} \text{ “+” } \underbrace{\frac{1-c_w}{2}\,\alpha_2 m_W}_{\text{radiative, } \approx\ 158\ \text{MeV}}$$

In the charge-0 sector two highly degenerate states $\chi_1^0\chi_1^0$, $\chi_1^+\chi_1^-$,
followed by four states $\chi_1^0\chi_{2,3}^0$, $\chi_1^\pm\chi_2^\mp$, then the four two-Higgsino-like states $\chi_{2,3}^0\chi_{2,3}^0$, $\chi_2^+\chi_2^-$
and finally four states with bino-like states $\chi_{1,2,3}^0\chi_4^0$, $\chi_4^0\chi_4^0$.
Similarly in the charged co-annihilation sectors.

Scatter into one another through Yukawa interaction. Each annihilates into a multitude of SM final states.



$$\sigma_I(v) = \sum_i \Gamma_i(^{2S+1}L_J)_{I'J'} \, \langle [\chi\chi]_I | \mathcal{O}(^{2S+1}L_J)_{I'J'} | [\chi\chi]_I \rangle \overset{\text{Born}}{=} a_I + b_I v^2$$

- I Compute the potentials from Z, W, Higgs exchange (14 x 14 matrix etc.)
- II Compute the tree-level coefficients of *off-diagonal* partial wave forward-amplitudes
- III Solve Schrödinger equation for operator matrix elements (wave-functions + derivatives at origin) for given external velocity and partial wave L.
- IV Tabulate $\sigma_I(v)$ for every two-particle state I and compute the thermally averaged + co-annihilation summed effective annihilation cross section $\langle\sigma_{\text{eff}} v\rangle$)(T) for $x = m_\chi/T \sim 10\ldots 10^8$.
- V Solve Boltzmann equation for relic density
- VI Compute cross sections for exclusive two-particle final states + fragmentation into stable cosmic ray particles.

Previous work [Hisano et al. (2004, 2006); Cirelli et al. (2007, 2008, 2009), Hryczuk et al. (2010, 2014)]: pure-Wino and/or -Higgsino LSP limit; no off-diagonals away from pure-W/H limits; no partial-wave separation.

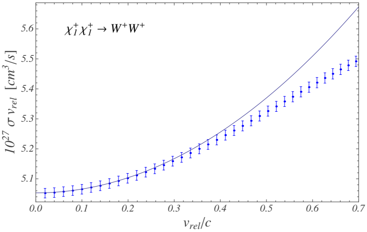
Short-distance matching

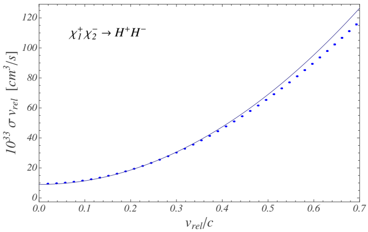
$\mathcal{O}^{\chi\chi\to\chi\chi}({}^1P_1)$	$\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)\xi_{e_1}$
$\mathcal{O}^{\chi\chi\to\chi\chi}({}^3P_0)$	$\frac{1}{3}\,\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\cdot\boldsymbol{\sigma}\right)\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\cdot\boldsymbol{\sigma}\right)\xi_{e_1}$
$\mathcal{O}^{\chi\chi\to\chi\chi}({}^3P_1)$	$\frac{1}{2}\,\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\times\boldsymbol{\sigma}\right)\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\times\boldsymbol{\sigma}\right)\xi_{e_1}$
$\mathcal{O}^{\chi\chi\to\chi\chi}({}^3P_2)$	$\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}^{(i}\boldsymbol{\sigma}^{j)}\right)\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}^{(i}\boldsymbol{\sigma}^{j)}\right)\xi_{e_1}$
$\mathcal{P}^{\chi\chi\to\chi\chi}({}^1S_0)$	$\frac{1}{2}\left[\xi^\dagger_{e_4}\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi_{e_1}+\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\xi_{e_1}\right]$
$\mathcal{P}^{\chi\chi\to\chi\chi}({}^3S_1)$	$\frac{1}{2}\left[\xi^\dagger_{e_4}\boldsymbol{\sigma}\,\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi_{e_1}+\xi^\dagger_{e_4}\boldsymbol{\sigma}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\,\xi_{e_1}\right]$
$\mathcal{Q}_1^{\chi\chi\to\chi\chi}({}^1S_0)$	$(\delta m\, M)\,\xi^\dagger_{e_4}\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\xi_{e_1}$
$\mathcal{Q}_1^{\chi\chi\to\chi\chi}({}^3S_1)$	$(\delta m\, M)\,\xi^\dagger_{e_4}\boldsymbol{\sigma}\,\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\,\xi_{e_1}$
$\mathcal{Q}_2^{\chi\chi\to\chi\chi}({}^1S_0)$	$(\delta\overline{m}\, M)\,\xi^\dagger_{e_4}\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\xi_{e_1}$
$\mathcal{Q}_2^{\chi\chi\to\chi\chi}({}^3S_1)$	$(\delta\overline{m}\, M)\,\xi^\dagger_{e_4}\boldsymbol{\sigma}\,\xi^c_{e_3}\cdot\xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\,\xi_{e_1}$

$$m_1+m_2+\frac{\vec{p}^{\,2}}{\mu}+\ldots$$
$$=m_3+m_4+\frac{\vec{p}^{\prime\,2}}{\mu}+\ldots$$

Mass splitting must be formally smaller than $\mathcal{O}(m_\chi v^2)$ for consistent NR expansion.

- Off-diagonal annihilation matching coefficients cannot be obtained from existing codes (DARKSUSY, micrOmega, ...)
- Analytic computation of all annihilation channels.
  $8 \times (14 \times 14 + 2 \times 8 \times 8 + 2 \times 3 \times 3) = 2736$ matching coefficients,
  83456 exclusive channels (Feynman gauge, MSSM)





- $\mathcal{O}(v^2)$ accuracy sufficient for relic density computations (and, of course, annihlation in present Universe)

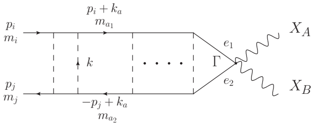
NRMSSM matrix element calculation

$$\mathcal{L}^{\rm NRMSSM} = \sum_i \chi_i^\dagger \left(i\partial_t - (m_i - m_{\rm LSP}) + \frac{\vec{\partial}^{\,2}}{2m_{\rm LSP}} \right) \chi_i$$

$$- \sum_{\chi\chi\to\chi\chi} \int d^3\vec{r}\, V^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(r)\, \chi_{e_4}^\dagger(t,\vec{x})\chi_{e_3}^\dagger(t,\vec{x}+\vec{r})\chi_{e_1}(t,\vec{x})\chi_{e_2}(t,\vec{x}+\vec{r}) + \ldots$$

$$V^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(r) = \left[A\,\mathbf{1}\otimes\mathbf{1} + B(\vec{\sigma}\otimes\mathbf{1} + \mathbf{1}\otimes\vec{\sigma})^2\right]_{e_1e_2e_4e_3} \frac{e^{-m_\phi r}}{r},$$

For $v \ll 1, m_\phi \ll m_\chi$ leading contributions from ladder diagrams. Summation equivalent to solving a multi-channel (matrix) Schrödinger equation.



For the S-wave operators define the ratio to the tree-level matrix element in the $[\chi\chi]_I$ state

$$S_I = \sum_{J,K} \frac{\psi_E(0)_{I\to J}}{\psi_E(0)_{I,\rm free}} \times \frac{\Gamma_{JK}}{\Gamma_{II}} \times \frac{\psi_E^*(0)_{K\to I}}{\psi_E^*(0)_{I,\rm free}}$$

Scattering wave function from numerical solution of a matrix Schrödinger equation.

- Sommerfeld-corrected cross section

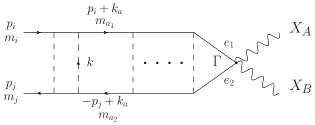
$$\sigma_{ij}|\vec{v}_i - \vec{v}_j| = \sum_{{}^1S_0,\,{}^3S_1} S_I({}^{2s+1}L_J)\,\Gamma_{II}({}^{2s+1}L_J) + \vec{p}_i^{\,2}\Big[\sum_{{}^1P_1,\,{}^3P_J} S_I({}^{2s+1}L_J)\,\Gamma_{II}({}^{2s+1}L_J)$$

$$+ \sum_{{}^1S_0,\,{}^3S_1} S_I^{p^2}({}^{2s+1}L_J)\,\Gamma_{II}^{p^2}({}^{2s+1}L_J)\Big]$$

S_I must be computed for each (relative) velocity.

- $\Gamma_{IJ}^{p^2}({}^{2s+1}L_J)$ can be related to $\Gamma_{IJ}({}^{2s+1}L_J)$ $(L = S)$ by equation of motion.

- Heavy two-particle channels have a small effect, and the dominant contribution comes from the last loop. Include analytically in an effective annihilation matrix for the lower-mass channels to reduce the CPU time.



Solving the Schrödinger equation [Slatyer (2009), MB, Hellmann, Ruiz-Femenia (2014)]

$$\left(\left[-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}-m_{\rm LSP}E\right]\delta^{ab}+m_{\rm LSP}V^{ab}(r)\right)[u_l(r)]_{bi}=0$$

Solve for regular solutions

$$[u_l(r_0)]_{ai}=\frac{1}{2l+1}\,\hat{r}_0^{l+1}\,\delta_{ai}\,,\qquad [u_l'(r_0)]_{ai}=\frac{l+1}{2l+1}\,\hat{r}_0^{l}\,\delta_{ai}\,,$$

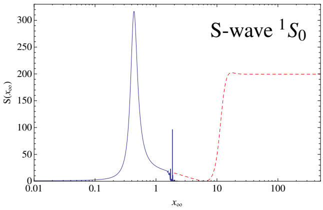
$$U_{aj}(r_\infty)=e^{ik_a r_\infty}\,\left([u_l'(r_\infty)]_{aj}-ik_a[u_l(r_\infty)]_{aj}\right)\,,$$

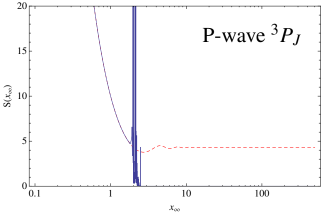
Relation to Sommerfeld factor

$$S_i[^{2S+1}L_J]=\left(\frac{(2L-1)!!}{k_i^L}\right)^2\,\frac{[T^\dagger]_{ie'}\,\Gamma_{ee'}(^{2S+1}L_J)\,T_{ei}}{\Gamma_{ii}(^{2S+1}L_J)|_{\rm LO}}$$

$$T=U^{-1}$$

Matrix inversion practically impossible due to mixing with kinematically closed channels with $M_{\chi\chi}-[2m_{\rm LSP}+E]>M_{\rm EW}^2/m_{\rm LSP}\approx$ few GeV.





Solution is a modification of the modification [Ershov (2011)] of the variable phase method (originally developed for nuclear physics problems)

$$[u_l(x)]_{ai} = f_a(x)\alpha_{ai}(x) - g_a(x)\beta_{ai}(x) \qquad \text{with} \qquad f_a(x)\alpha'_{ai}(x) - g_a(x)\beta'_{ai}(x) = 0$$

$$f_a(x) = \sqrt{\frac{\pi x}{2}}\, J_{l+\frac{1}{2}}(\hat{k}_a x) \qquad g_a(x) = -\sqrt{\frac{\pi x}{2}}\left[Y_{l+\frac{1}{2}}(\hat{k}_a x) - iJ_{l+\frac{1}{2}}(\hat{k}_a x)\right]$$

$$N'_{ab} = \delta_{ab} + \left(\frac{g'_a}{g_a} + \frac{g'_b}{g_b}\right) N_{ab} - N_{ac}\,\frac{\hat{V}_{cd}}{E}\,N_{db}\,,$$

$$\tilde{\alpha}^{-1\,\prime}_{ia} = \tilde{\alpha}^{-1}_{ib} Z_{ba} \qquad \text{with} \qquad Z_{ab} \equiv -\frac{g'_a}{g_a}\,\delta_{ab} + \frac{\hat{V}_{ac}}{E}\,N_{cb}\,.$$

$$T_{ia}(x_\infty) \overset{x_\infty \to \infty}{=} e^{-i\hat{k}_a x_\infty}\,\tilde{\alpha}^{-1}_{ai}(x_\infty)$$

IV. Relic abundance in benchmark models

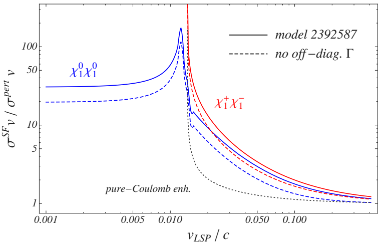
Wino-like χ_1^0 DM

- pMSSM benchmark model "2392587" [Cahill-Rowley et al. (2013)]
- (Approximate) $SU(2)_L$ triplet states χ_1^0, $\chi_1^\pm$. $m_{\chi_1^0} = 1650.664\,\text{GeV}$, $|Z_{N\,21}|^2 = 0.999$
  Mass-splitting due to radiative corrections: $\delta m_{\text{rad}} = m_{\chi^+} - m_{\chi^0} \approx 0.155\,\text{GeV}$
- (co-)annihilation sectors

neutral	$\chi_1^0\,\chi_1^0,\ \chi_1^+\,\chi_1^-$
single charged	$\chi_1^0\,\chi_1^+\ (\chi_1^0\,\chi_1^-)$
double charged	$\chi_1^+\,\chi_1^+\ (\chi_1^-\,\chi_1^-)$

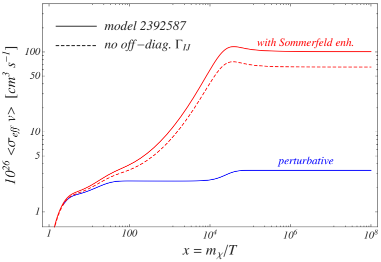
$\rightarrow$ coupled system in neutral $^1S_0$ reactions:
matrix-valued potential V_{IJ}

- Approximate treatment of all other states (CPU time)



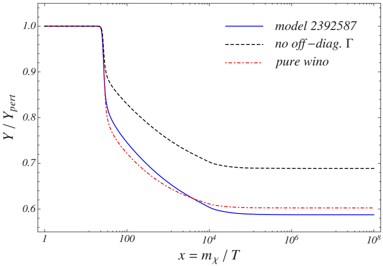
- $\chi_1^0\chi_1^0$:
  v_{LSP}-independent enh. well below,
  resonance region at $\chi_1^+\chi_1^-$ thresh.
- $\chi_1^+\chi_1^-$: Coulomb enh. above threshold
- no off-diagonal Γ:
  $\lesssim 30\%$ reduction in $\sigma^{\text{SF}}v/\sigma^{\text{pert}}v$

Thermally averaged cross section summed over all initial and final states and relic abundance $\Omega_{\chi_1^0} h^2 = \rho^0_{\chi_1^0}/\rho_{crit}\, h^2 = m_\chi s_0 Y_0/\rho_{crit}\, h^2$



$$\langle \sigma_{\text{eff}}\, v\rangle = \sum_{i,j} \langle \sigma_{ij}\, v_{ij}\rangle\, n_i^{\text{eq}} n_j^{\text{eq}}/n_{\text{eq}}^2$$

- $x \simeq 20$:
  χ_1^0, $\chi_1^\pm$ (chem.) freeze-out off thermal bath
- $x \simeq 10^4$: $\chi_1^\pm$ decouple
  $\left[n_{\chi_1^+}/n_{\chi_1^0} \propto \exp(-\delta m/m_{\chi_1^0}\, x)\right]$



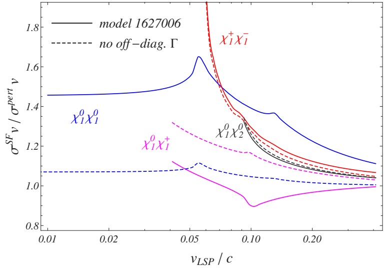
$$Y = \sum_i n_i/s$$

- $\Omega_\chi^{\text{pert}}\, h^2 = 0.109(3)$
- $\Omega_\chi^{\text{SF}}\, h^2 = 0.064(5) \rightarrow$ 40% reduction
- $\sim$ 15% error on $\Omega_\chi^{\text{SF}}\, h^2$ if no off-diag. Γ
- pure-wino: $\Omega_\chi^{\text{SF}}\, h^2 = 0.033$,
  $\Omega_\chi^{\text{pert}}\, h^2 = 0.055$

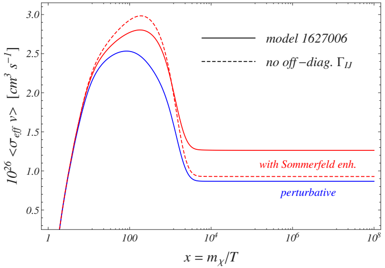
Higgsino-like χ_1^0 DM

- pMSSM benchmark model "1627006" [Cahill-Rowley et al. (2013)]
- Two approximate $SU(2)_L$ doublet states $\chi^0_{1,2}$, $\chi_1^\pm$. $|Z_{N\,31}|^2 + |Z_{N\,41}|^2 = 0.98$
  $m_{\chi_1^0} = 1172.31\,\text{GeV}$
  Tree-level mass-splittings: $\delta m_{\chi_1^+} = 1.8\,\text{GeV}$, $\delta m_{\chi_2^0} = 9.5\,\text{GeV}$
- (co-)annihilation sectors

neutral	$\chi_1^0\,\chi_1^0,\, \chi_2^0\,\chi_1^0,\, \chi_2^0\,\chi_2^0,\; \chi_1^+\,\chi_1^-$
single charged	$\chi_1^0\,\chi_1^+,\, \chi_2^0\,\chi_1^+\; (\chi_1^0\,\chi_1^-,\, \chi_2^0\,\chi_1^+)$
double charged	suppressed by $\mathcal{O}(M_{\text{EW}}/m_\chi)$

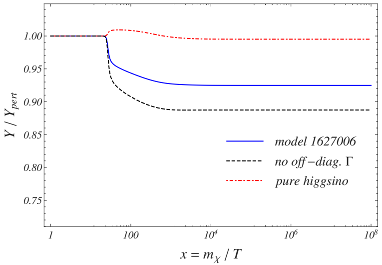


- Smaller effect (larger mass splittngs, smaller couplings)
- Destructive interference for $\chi_1^0\chi_1^\pm$ due to off-diagonal annihilation (solid magenta)



$\langle \sigma_{\rm eff}\, v \rangle$

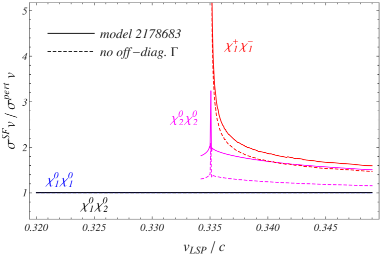
- Note destructive interference from single-charged sector near $x \simeq 20$.

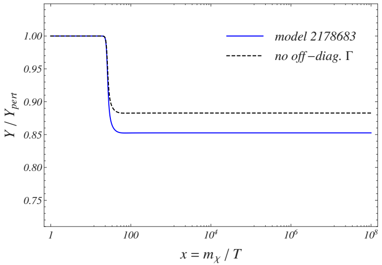


$$Y = \sum_i n_i/s$$

- $\Omega_\chi^{\rm pert}\, h^2 = 0.108$
- $\Omega_\chi^{\rm SF}\, h^2 = 0.100 \rightarrow$ 8% reduction
- pure-Higgsino: almost no effect due to cancellation between enhancement in charge-neutral and suppression in single-charged (co-)annihilation sector.

Bino-like χ_1^0 DM



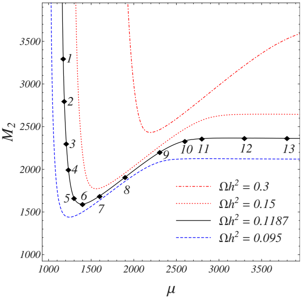


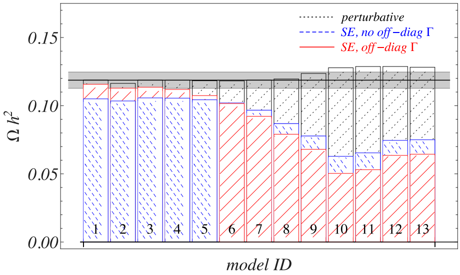
- pMSSM benchmark model "2178683"
  [Cahill-Rowley et al. (2013)]
- Bino-LSP $m_{\chi_1^0} = 488.8\,\text{GeV}$
- Co-annihilating wino sector with $m_{\chi_2^0} = 516.0\,\text{GeV}$, $m_{\chi_1^\pm} = 516.2\,\text{GeV}$.
  Annihilation cross sections 10^3 times larger than bino annihilation.

$$Y = \sum_i n_i/s$$

- $\Omega_\chi^{\text{pert}}\, h^2 = 0.120$
- $\Omega_\chi^{\text{SF}}\, h^2 = 0.102 \to$ 15% reduction

Higgsino-to-wino trajectory





- 13 models with $\Omega^{\text{DarkSUSY}} h^2 = 0.1187$
- trajectory in $\mu - M_2$ plane
  - models $1-6$: higgsino-like χ_1^0: $\Omega^{\text{SF}} h^2 / \Omega^{\text{pert}} h^2 \sim 0.97 - 0.86$
  - models $7-9$: mixed wino-higgsino χ_1^0: $\Omega^{\text{SF}} h^2 / \Omega^{\text{pert}} h^2 \sim (0.78,\ 0.66,\ 0.55)$
  - models $10-13$: wino-like χ_1^0: $\Omega^{\text{SF}} h^2 / \Omega^{\text{pert}} h^2 \sim 0.39 - 0.50$ [note resonance]
- Off-diagonal terms strongly enhance the variation of the SE along the trajectory.

V. Relic abundance of the dominantly wino-like neutralino

General investigation [MB, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel, P. Ruiz-Femenia, 1601.04718 and 1611.00804]

Relic density

- Identify MSSM parameter space where SE is the dominant radiative correction (not necessarily $\mathcal{O}(1)$).
- Include exact 1-loop on-shell mass splittings.
- Analyse thermal corrections.
- Identify regions of observed relic density.

Indirect detection

- Separate exclusive final states.
- Strong constraints on pure wino from indirect detection [Cohen et al. (2013); Fan, Reece (2013); Hryczuk et al. (2014)]
  In MSSM dependence on sfermion masses, Higgsino-, bino-fraction. Allowed parameter space?
- Direct detection constraints.

Also a CPU problem. Require 10min/parameter point (relic density).

Thermal effects [1601.04718]

Relevant temperature range $T_f \simeq m_\chi/20 \simeq 50\ldots200\,\text{GeV}$ to $T_s \simeq m_W^2/m_\chi \simeq 1\ldots4\,\text{GeV}$ includes or is close to the electroweak phase transition.

Theoretical question: real-virtual IR cancellation in Boltzmann equation (multiplied by different phase-space distributions)

Thermal correction to the short-distance cross section

- negligibly small [MB, Dighera, Hryczuk, 1409.3049, 1607.03910]

$$\mathcal{O}(T^4/m_\chi^4) \quad (\chi^0\chi^0), \qquad\qquad \mathcal{O}(T^2/m_\chi^2) \quad (\chi^\pm\chi^0, \chi^\pm\chi^\mp)$$

Compute OPE of forward scattering amplitude in the soft thermal background.

Thermal corrections to the Sommerfeld factor

$$v(T) = v\sqrt{1-\frac{T^2}{T_c^2}}, \qquad T_c \approx 165\,\text{GeV}$$

Thermal effects (II)

$$v(T) = v\sqrt{1-\frac{T^2}{T_c^2}}, \qquad T_c \approx 165\,\text{GeV}$$

- Electroweak potential become long-ranged, $m_{W,Z} \to 0$.
  Thermal self-energy
  $$[\Pi_{00}]_{\text{thermal}} = -\frac{g_2^2 T^2}{9}$$
- Chargino-neutralino mass difference for a dominantly wino LSP goes to 0. Thermal mass difference
  $$[\delta m_{\chi^+}]_{\text{thermal},\gamma} = \frac{\pi\alpha_{\text{em}}}{3}\,\frac{T^2}{m_\chi}$$

Nevertheless small, essentially because $\vec{p} \sim \sqrt{m_\chi T} \gg T$ always.
Numerical implementation (very CPU expensive) and analytical estimates: thermal effect on relic density below 1%.

Capture into WIMPonium bound-state effects

Capture followed by bound-state annihilation vs. direct annihilation of the scattering state attracted a lot of interest recently

[von Harling, Petraki, 1407.7874; An, Wise, Zhang, 1604.01776; Asadi, Baumgart, Fitzpatrick, Krupczak, Slatyer, 1610.07617; Petraki, Postma, de Vries, 1611.01394; Cirelli, Panci, Petraki, Sala, Taoso, 1612.07295; Mitridate, Redi, Sminrov, Strumia, 1702.01141]

Pure wino model

$$V^{(2)}_{\text{even } L+S}(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2\,\frac{e^{-M_W r}}{r} \\ -\sqrt{2}\,\alpha_2\,\frac{e^{-M_W r}}{r} & -\frac{\alpha}{r} - \alpha_2\,c_W^2\,\frac{e^{-M_Z r}}{r} \end{pmatrix}$$

$$V^{(2)}_{\text{odd } L+S}(r) = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\alpha}{r} - \alpha_2\,c_W^2\,\frac{e^{-M_Z r}}{r} \end{pmatrix}$$

Because $\chi_0\chi_0$ consists of two identical fermions, bound states of $\chi_0\chi_0$ must have $L+S$ = even. Even $L+S$ potential stronger than odd $L+S$.

Bound state for $m_\chi \gtrsim 2.6$ TeV, for odd L+S only for $m_\chi \gtrsim 5.6$ TeV.

Capture into bound-state effects (II)

Because $\chi_0\chi_0$ consists of two identical fermions, bound states of $\chi_0\chi_0$ must have $L+S$ = even.
Even $L+S$ potential stronger than odd $L+S$.
Bound state for $m_\chi \gtrsim 2.6$ TeV, for odd L+S only for $m_\chi \gtrsim 5.6$ TeV.

- Present Universe: The $^1S_0$ bound state is not accessible via E1 ($\Delta L = 1,\ \Delta S = 0$) transition for $\chi_0\chi_0$ annihilation.

$$^3P_J[\chi_0\chi_0 \text{ cont.}] \to {}^3S_1[\chi^+\chi^- \text{ bound}] + \gamma$$

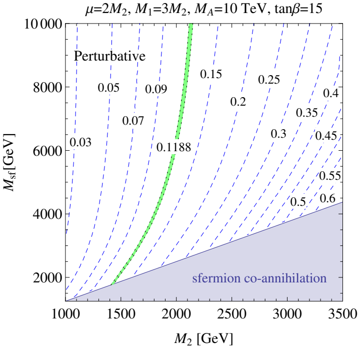
  only for $m_\chi \gtrsim 5.6$ TeV. Capture rate suppressed, because bound state has weaker potential.
  Bound state formation not relevant for present Universe annihilation [Asadi et al., 1610.07617]

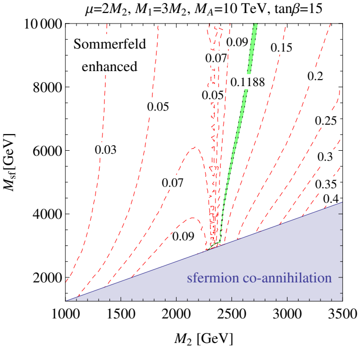
- Early Universe: coexistence of $\chi^+\chi^-$. Also

$$^1P_1[\chi^+\chi^- \text{ cont.}] \to {}^1S_0[\chi_0\chi_0 \text{ bound}] + \gamma$$

  for $m_\chi \gtrsim 2.6$ TeV and now the bound state experiences the stronger force enhancing capture.
  But direct $^1S_0$ annihilation still dominant and bound state formation not relevant for relic density computation (below 1% effect) [Mitridate et al., 1702.01141, Urban]

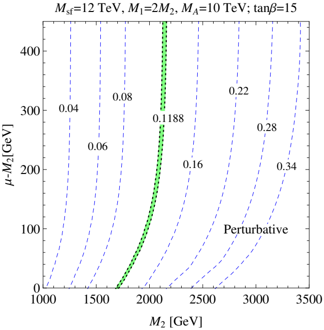
Sfermion mass dependence [close to pure Wino]

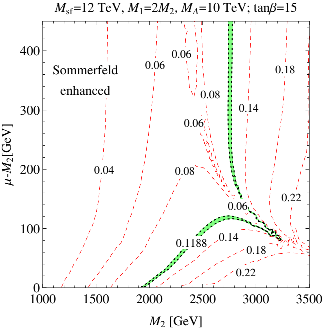




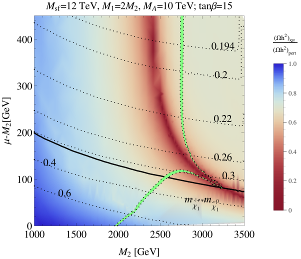
- Lighter sfermion *reduce* the annihilation cross section due to destructive interference between s- and t, u-channel diagrams.
- SE shifts correct relic density to larger masses, above the Sommerfeld resonance.
- SE reduces relic density by $\approx 40\%$.

Mixed Wino-Higgsino

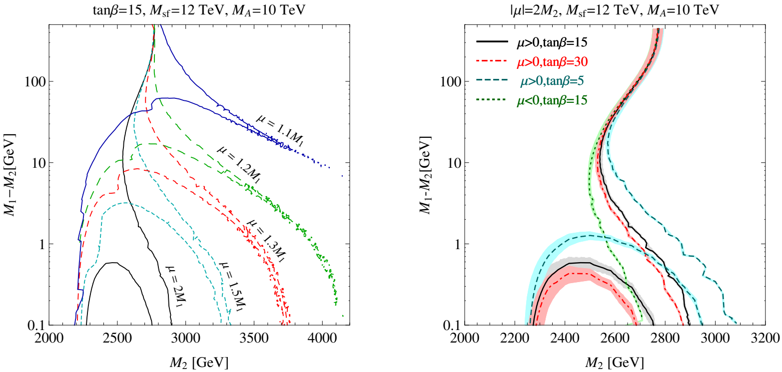




- SE shifts correct relic density to larger masses, above the Sommerfeld resonance.
- Correct relic density line is pulled into the resonance (mass splitting effect)
- Correct relic density for a wide range of wino-like LSP masses 2.0 . . . 3.3 TeV.



Dominantly wino with bino/Higgsino



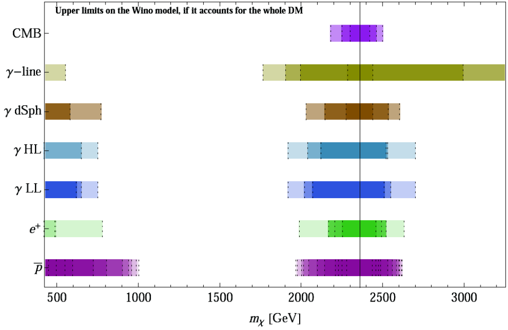
- Bino admixture modifies the cross section and can push the correct relic density into the Sommerfeld resonance region whose location depends also on the Higgsino admixture.
- Strong dependence on other parameters only near the resonance.

Previously studied pure-Wino/Higgsino limits of the MSSM do not nearly capture the full MSSM parameter space where correct relic density is attained for wino-like dark matter.

VI. Indirect and direct search constraints

Indirect detection (cosmic ray) constraints

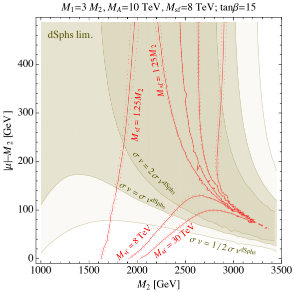
Sommerfeld effect/resonance leads to large annihilation cross sections in the late Universe. Pure wino often said to be excluded by non-observation of cosmic ray signals.



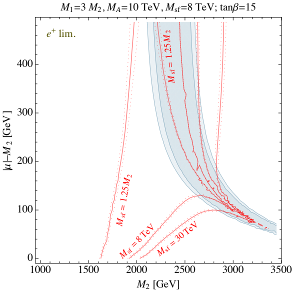
[from Hryczuk et al., 1401.6212]

- Diffuse γ flux from dwarf spheroidal galaxies [FERMI-LAT, MAGIC]; galactic positrons, protons, B/C, Helium [AMS-02 data, DRAGON propagation code]; energy deposition into CMB before and after re-combination [PLANCK].

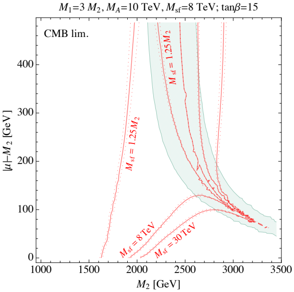
ID constraints on the mixed wino-Higgsino



Diffuse γ from dSphs



Charged cosmic rays

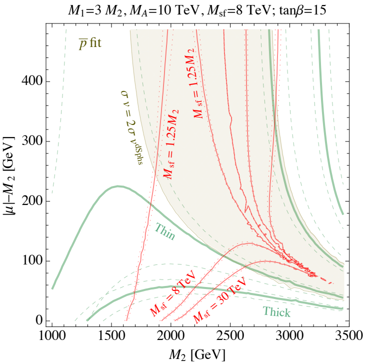


CMB energy injection

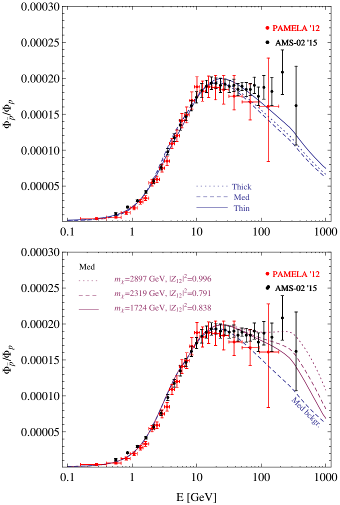
Strongest constraint from diffuse γs from dwarf spheroidal galaxies.

Note: DM density assumed to be the observed one throughout. Implies non-thermal production in some parameter space.

Leaves room for AMS-02 anti-proton excess ...



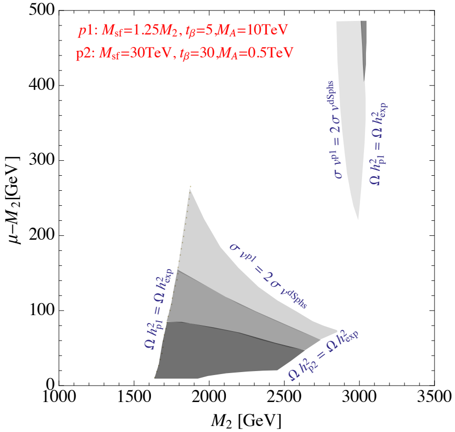
Anti-proton/proton ratio energy spectrum



Mixed wino-Higgsino parameter space

- Compatible with ID constraints
- Correct relic density (for some $M_{\rm sf}$, tan β)

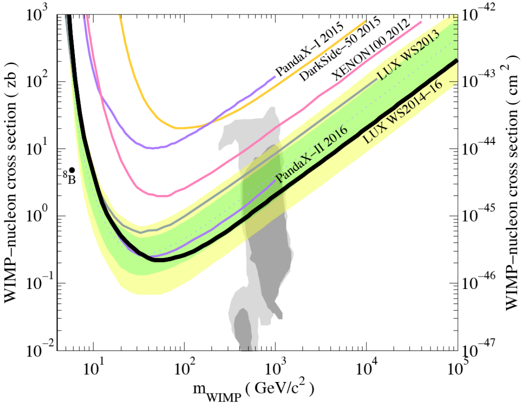
Small region near pure wino and sizable mixed wino-Higgsino region survives. Significant range of viable DM masses.



Direct detection constraints

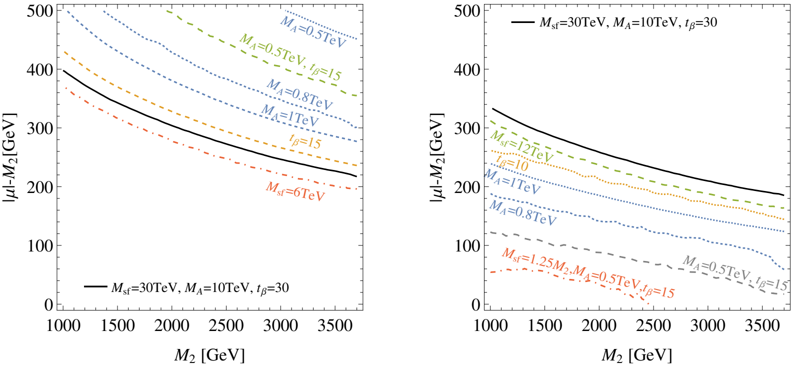
SI scattering cross section loop-suppressed for pure wino.

No constraint from direct DM searches on the parameter space considered here – prior to July 2016



New LUX and PandaX-II limits a factor of four stronger.

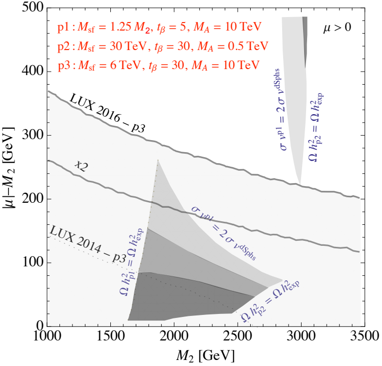
LUX limit in the wino-Higgsino plane



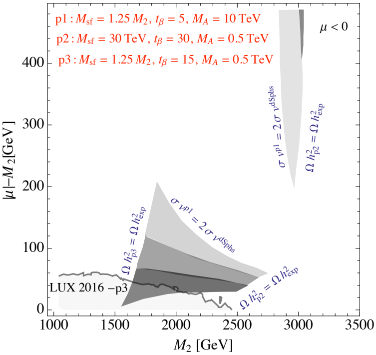
$\mu > 0$ $\qquad\qquad$ $\mu < 0$

Sign of μ, value of $\tan\beta$ matters for DM-nucleon scattering through Higgs t-channel and sfermion s-channel scattering, contrary to annihilation.

The last refuges of mixed wino-Higgsino dark matter



$\mu > 0$



$\mu < 0$

Summary

- Electroweak Sommerfeld enhancement often relevant in still viable dominantly-wino MSSM parameter regions.

- Provide method for χ_1^0 relic abundance calculation including $\sigma^{\mathrm{SF}} v$ for

  - general MSSM (generic χ_1^0 composition)
  - leading order potential interactions in coupled two-particle systems
  - up to $\mathcal{O}(v^2)$ effects in the short-distance annihilation
  - including partial-wave separation and off-diagonal annihilation
  - New method to solve the multi-channel Schrödinger equation without numerical instabilities.
  - Approximate treatment of heavy channels.

- Previously studied pure-wino/Higgsino limits of the MSSM do not nearly capture the full MSSM parameter space where correct relic density is attained for wino-like dark matter.

- Small region near pure-wino and a mixed wino-Higgsino region survives cosmic-ray constraints.

- Recent direct search limits cut into the mixed wino-Higgsino region and practically eliminate it for $\mu > 0$.

- Future γ-ray telescopes (CTA) will rule out entire region or discover as DM signal.