

Charged lepton decays from soft flavour violation

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$\int dk \prod$ Doktoratskolleg
Particles and Interactions



Overview

1 THE MODEL

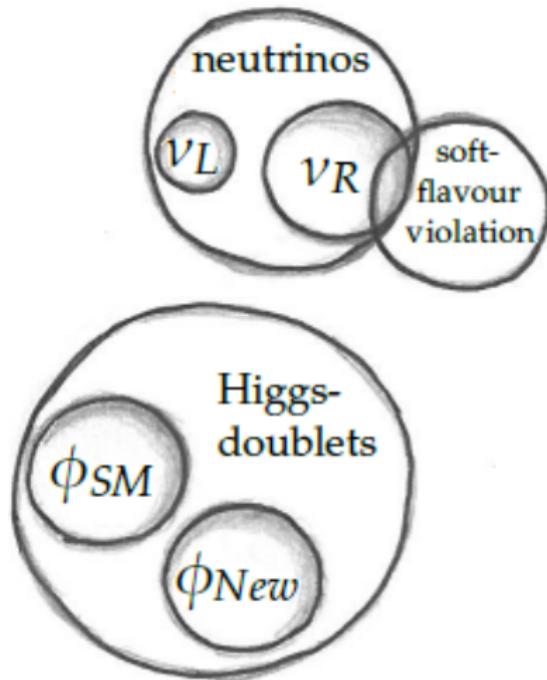
- Neutrinos - right handed neutrinos
- Multi Higgs doublets
- Soft flavour violation
- Properties and advantages

2 CHARGED LEPTON DECAYS

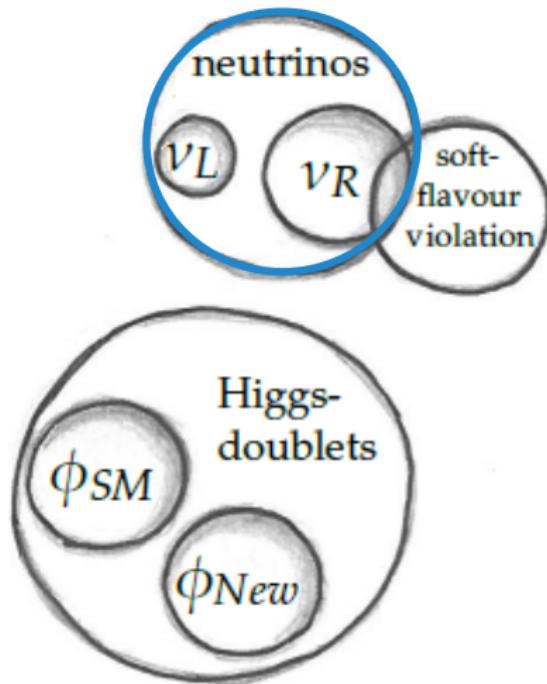
- Free parameters
- Radiative corrections
- Magnetic dipole moments
- Constrains from 4ℓ -decay
- Results

3 SUMMARY

The model



The model



Why we care about neutrinos

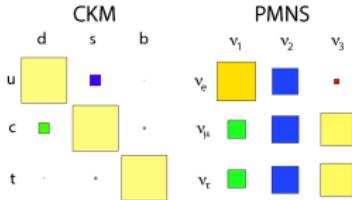
experimentally unsolved: anomalies...

properties:

- just weak interacting
- no observed right handed partner

theoretical unsolved: (all about mass)

- different mixing matrices than quarks
- normal or inverted mass hierarchy
- hierarchy problem: very light mass
- origin of mass: Dirac, Majorana



Why we care about neutrinos

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- origin of mass: Dirac, Majorana

$$\begin{aligned}\mathcal{L}_M &= ? \quad \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Majorana}} \\ &\sim \bar{\nu}_R M_D \nu_L + \bar{\nu}_\alpha M_M \nu_\beta + h.c.\end{aligned}$$

properties:

- just weak interacting
- no observed right handed partner

Desperately seeking sterile

The three known types of neutrino might be "balanced out" by a bashful fourth type

ELECTRON NEUTRINO	MUON NEUTRINO	TAU NEUTRINO	STERILE NEUTRINO
ν_e	ν_μ	ν_τ	ν_s
MASS	< 1 electronvolt	> 1 electronvolt	Gravity
FORCES THEY RESPOND TO	Weak force Gravity	All three "left handed"	"Right handed"
DIRECTION OF SPIN			



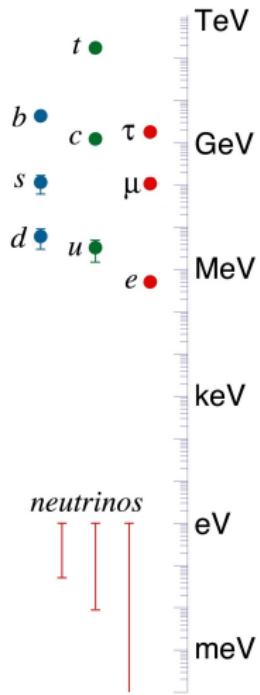
Heavy right handed neutrinos ν_R

Hierarchy problem:

Neutrino mass is small $m_\nu < 0.1 \text{ eV}$ (exp. limits)

Masses are normally $m_e \simeq 0.5 \text{ MeV}$ to $m_t \simeq 173 \text{ GeV}$

\Rightarrow *small Yukawa masses seem to be unnatural*



Heavy right handed neutrinos ν_R

**Soltion: Majorana neutrinos
with heavy righthanded partners
in Seesaw mechanism**

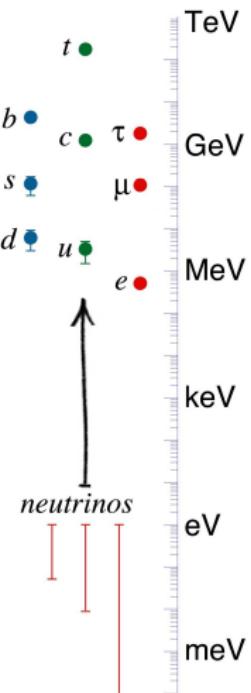
$$\mathcal{L}_{maj} = -\frac{1}{2} \overline{(\nu_R)^c} M_R^* \nu_R + \text{H.c.}, \quad \text{scale } m_R \gtrsim \text{TeV}$$

flavour basis: scale $m_D \sim m_e$

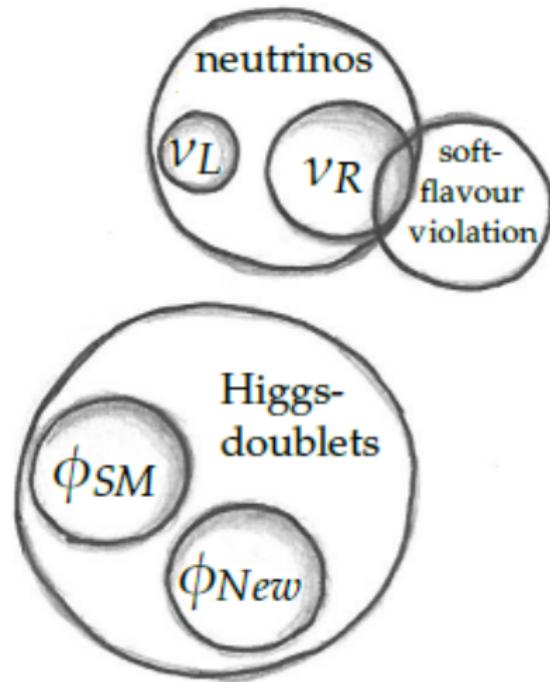
$$\mathcal{L}_Y^{mass} = -\frac{1}{2} \left(\overline{(\nu_L)^c}, \overline{\nu_R} \right) \underbrace{\begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}}_{M_{maj}} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + \text{H.c.}$$

mass basis: diagonalise $M_{maj} \rightarrow \text{diag}(\hat{m}_\nu, \hat{m}_R)$

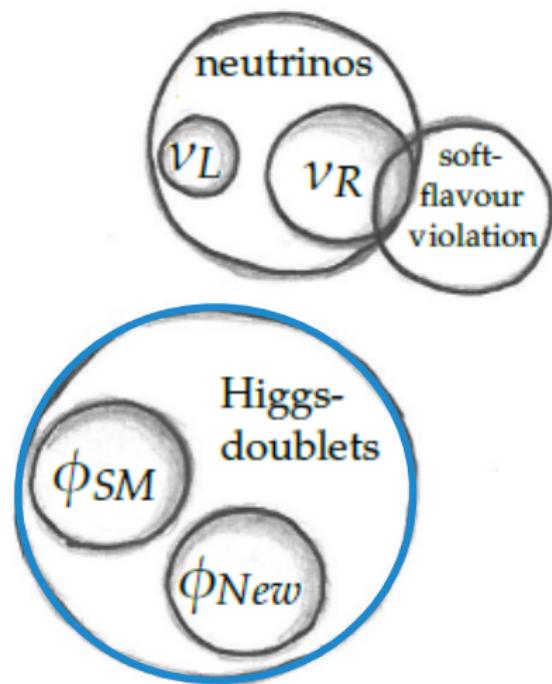
$$\text{scale } m_\nu = -m_D^2/m_R \quad \rightarrow \quad m_\nu \text{ small}$$



The modell



The modell



Multi Higgs doublets Φ_k

**Include: (e.g. $n_H = 2$) Higgs doublets
and lepton flavour breaking**

$$\phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix}, \quad \varphi_k^0 = \frac{\nu_k}{\sqrt{2}} + \varphi_k^{0*}$$

$$\mathcal{L}_Y = - \sum_{k=1}^{n_H} \sum_{\ell, \ell' = e, \mu, \tau} \left[(\varphi_k^-, \varphi_k^{0*}) \Gamma_{k\ell\ell'} \bar{\ell}_R + (\varphi_k^0, -\varphi_k^+) \Delta_{k\ell\ell'} \bar{\nu}_{\ell R} \right] \begin{pmatrix} \nu_{\ell' L} \\ \ell_L' \end{pmatrix} + \text{H.c.}$$

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- NC & mass:

$$\mathcal{L}_Y(\varphi^0) = - \sum_{k\ell\ell'} \left\{ \ell'_L (\varphi_k^{0,*} \Gamma_{k\ell\ell'} + (M_\ell)_{\ell\ell'}) \bar{\ell}_R + \nu_{\ell' L} (\varphi_k^{0'} \Delta_{k\ell\ell'} + (M_D)_{\ell\ell'}) \bar{\nu}_{\ell R} \right\}$$

- CC :

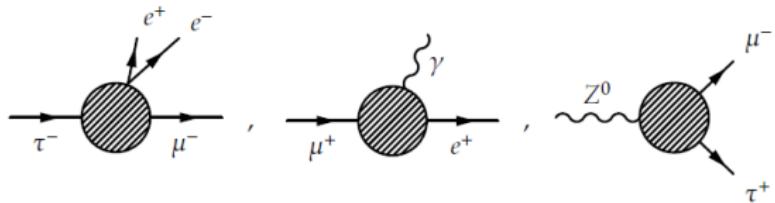
$$\mathcal{L}_Y(\varphi^\pm) = - \sum_{k\ell\ell'} \left\{ \nu_{\ell' L} (\varphi_k^- \Gamma_{k\ell\ell'}) \bar{\ell}_R - \ell'_L (\varphi_k^+ \Delta_{k\ell\ell'}) \bar{\nu}_{\ell R} + \text{H.c.} \right\}$$

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e.g.



interesting effect:
observable processes!

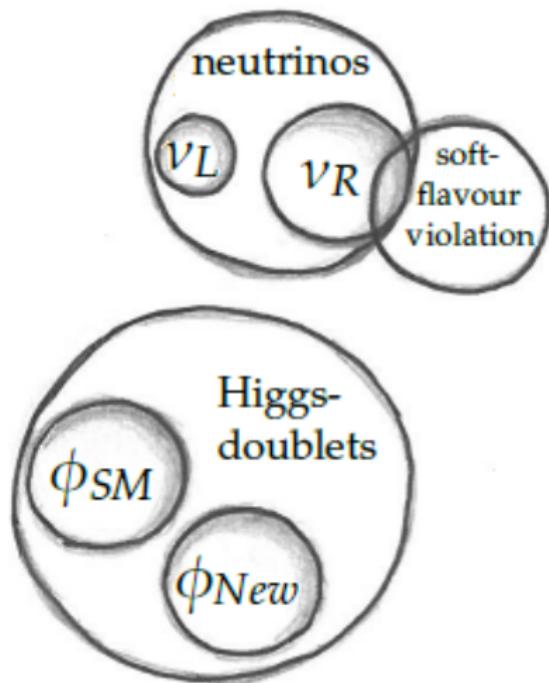
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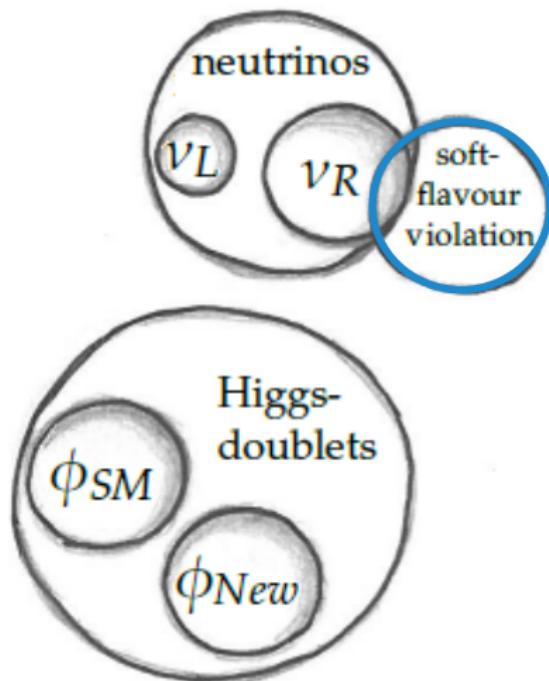
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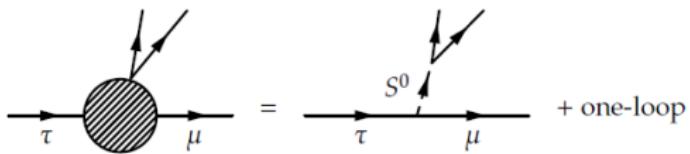
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Soft flavour violation \mathcal{L}_α

Problem:

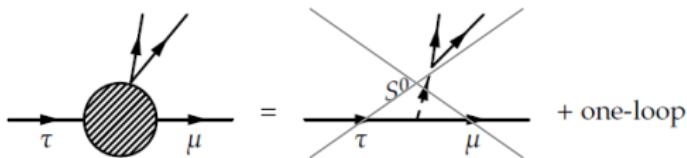
strong experimental bounds on FCNIs at tree level



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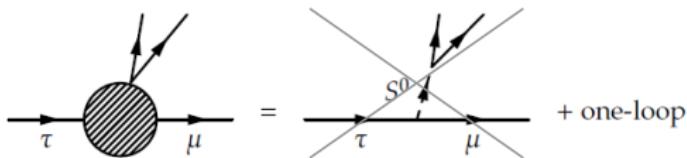
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Soft flavour violation L_α

Problem:

strong experimental bounds on FCNIs at tree level



Solution: SOFT flavour violation $L_\alpha \{ \alpha = e, \mu, \tau \}$

L_α conservation:

in Yukawa interactions

\Rightarrow diag. $\Gamma(\ell)$, $\Delta(\nu)$

L_α explicit soft breaking:

in Majorana term \Rightarrow non-diag. \mathbf{M}_R

Soft flavour violation L_α

- Flavour violation comes solely from neutrinos.
 - FCNIs appear at one loop in charged Lepton decays.
- e.g.
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$$M_\ell \sim \sum_k v_k^* Y_{\ell k}, \quad M_D \sim \sum_k v_k Y_{\nu k}$$

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\Rightarrow diag. $\Gamma(\ell)$, $\Delta(\nu)$ \Rightarrow **diag.** $\mathbf{M}_\ell, \mathbf{M}_D = \text{diag}(m_e, m_\mu, m_\tau)$

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Short analysis of FCNLs

Evtl. experimentally testable processes:

$$\mathcal{A}(Z \rightarrow \tau^+ \mu^-) \propto 1/m_R^2,$$

$$\mathcal{A}(\tau^- \rightarrow \mu^- \gamma) \propto 1/m_R^2,$$

$$\mathcal{A}(\tau^- \rightarrow \mu^- e^+ e^-) \propto \begin{cases} 1/m_R^2 & n_H = 1 \\ \text{const.} & n_H > 1 \end{cases}$$

Processes including the sub-process $\ell^- \rightarrow \ell'^- S^{0*}$, ($S^{0*} \rightarrow e^+ e^-$) have ($n_H \geq 2$) non- m_R -suppressed contributions from graphs with charged-scalar exchange S^\pm (plot) in their Amplitudes \mathcal{A} , [Grimus, Lavoura, 02].

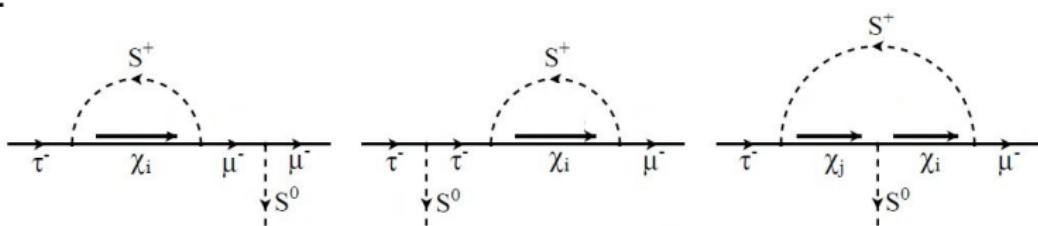


Figure: non-suppressed diagrams for $\tau^- \rightarrow \mu^- S^{0*}$

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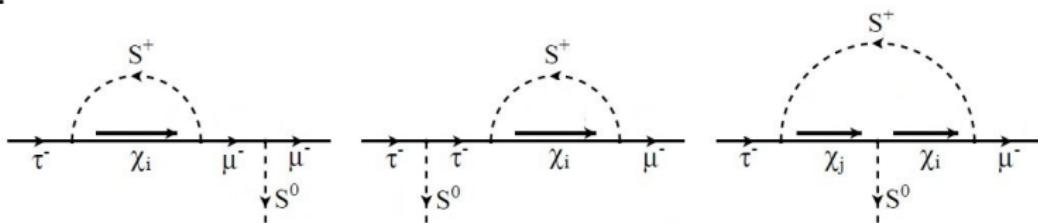


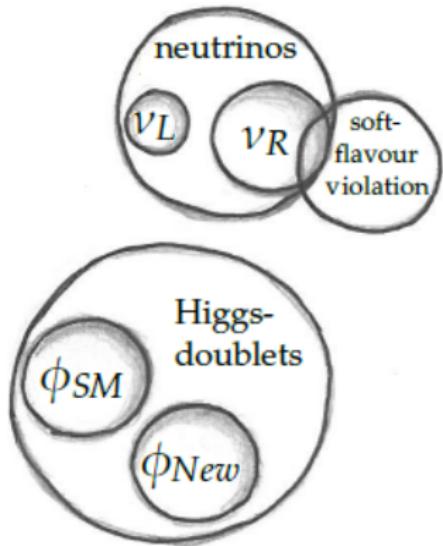
Figure: non-suppressed diagrams for $\tau^- \rightarrow \mu^- S^{0*}$

Properties and advantages

- explain mass hierarchy
- observable processes: charged ℓ -decays

Additional advantages:

- ampl. of FC processes are finite at one-loop
- ampl. are stable under radiative corrections

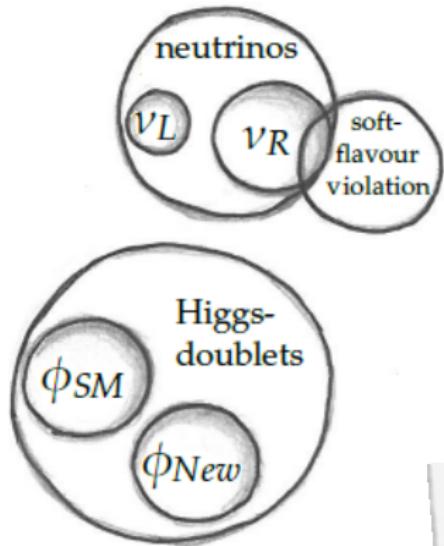


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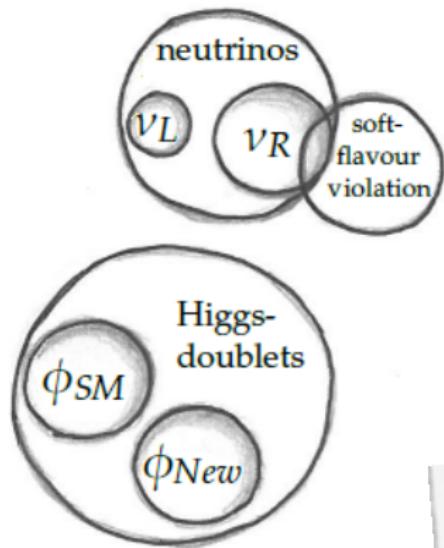
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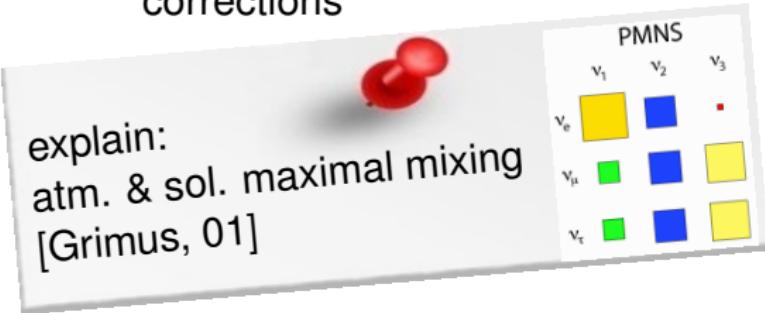
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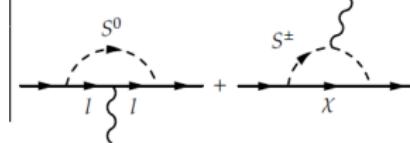
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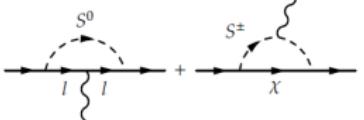
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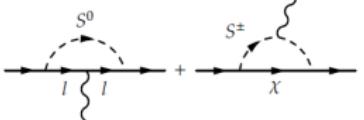
Charged lepton decays

exp. bounds	model contributions
$\text{BR}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13}$ $\text{BR}(\tau^- \rightarrow e^- \gamma) < 3.3 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 4.4 \times 10^{-8}$	$\mathcal{A}(\tau^- \rightarrow \mu^- \gamma) \propto \frac{1}{m_R^2}$
$\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$ $\text{BR}(\tau^- \rightarrow e^- e^+ e^-) < 2.7 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-) < 2.7 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 2.1 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-) < 1.8 \times 10^{-8}$	$\mathcal{A}(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-) \propto$ $\begin{cases} \frac{1}{m_R^2} & n_H = 1 \\ \text{const.} & n_H > 1 \end{cases}$
$(\text{exp/SM} = 1) \quad a_e^{err} = \pm 2.6 \times 10^{-13}$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 

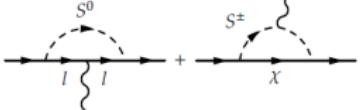
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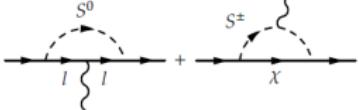
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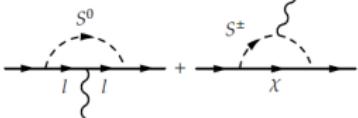
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a_e^{err} $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 	explain μ-MDM

Charged lepton decays

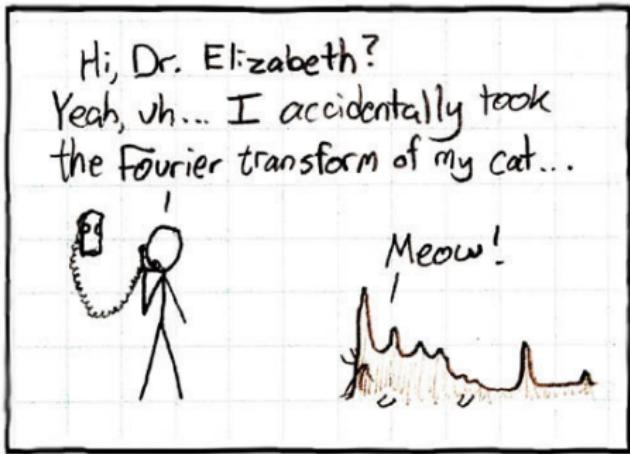
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Things might become a bit technical here...

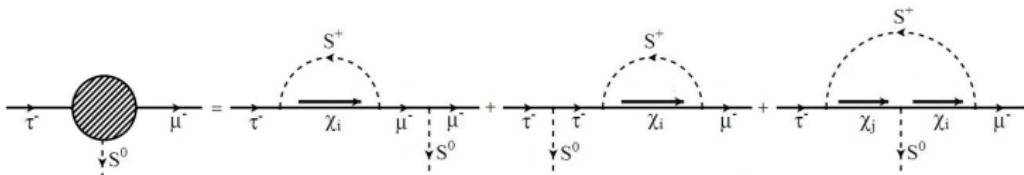
calculating problems:



Finding free parameters

Goal: $\text{BR}(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-)$ close to exp. bounds

→ constrain free parameters in non-suppressed contributions to BR:



BR calculation (one loop):

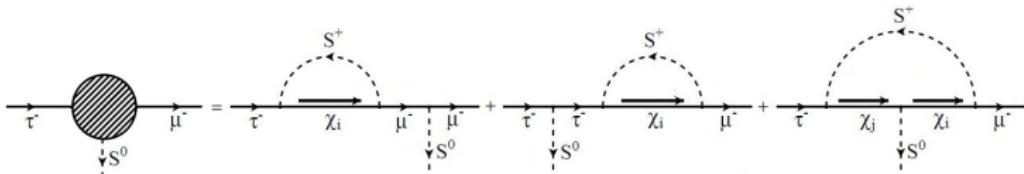
(S^0 : Higgs φ^0 in mass space)

$$\mathcal{L}_Y(S^0) = -\frac{1}{\sqrt{2}} \sum_{b\ell} S_b^0 \bar{\ell} \left[(\hat{\Gamma}_b)_{\ell\ell} \gamma_L + (\hat{\Gamma}_b^\dagger)_{\ell\ell} \gamma_R \right] \ell$$

Finding free parameters

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→ constrain **free parameters** in non-suppressed contributions to BR:



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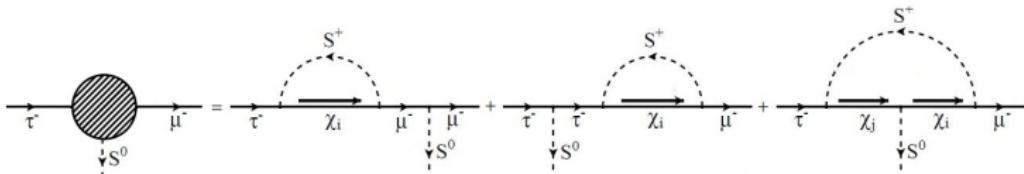
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$$\mathcal{L}_{Y\text{eff}}(S^0) = \sum_{b, \ell_1 \neq \ell_2} S^0 \bar{\ell}_1 \left[(A_L^b)_{\ell_1 \ell_2} \gamma_L + (A_R^b)_{\ell_1 \ell_2} \gamma_R \right] \ell_2 \text{ (one loop)}$$

Finding free parameters

Goal: $\text{BR}(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-)$ close to exp. bounds

→ constrain **free parameters** in non-suppressed contributions to BR:



BR calculation (one loop):

(S^0 : Higgs φ^0 in mass space)

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$$\rightarrow \Gamma(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-) = \frac{m_{\ell_1}^5}{6144\pi^3} |X_{\ell_2 \ell_1}|^2 |\gamma_{\ell_3}|^2 \frac{|A_{\ell_2 \ell_1}|^2 + |A_{\ell_1 \ell_2}|^2}{(m_{\ell_2}^2 - m_{\ell_1}^2)^2} \left(\frac{1}{M_3^4} + \frac{1}{M_4^4} \right)$$

$$\text{BR}(\ell) = \Gamma(\ell)/\Gamma_{\text{tot}}$$

Free parameters

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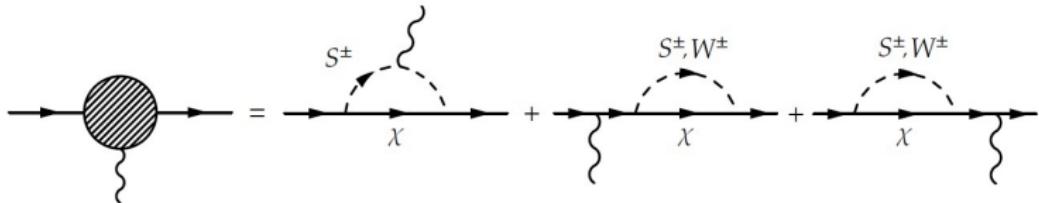
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Radiative corrections

A lower bound on the seesaw scale m_R is estimated from experimental limit of the branching ratio $\text{BR}(\ell_1^\pm \rightarrow \ell_2^\pm \gamma)$:

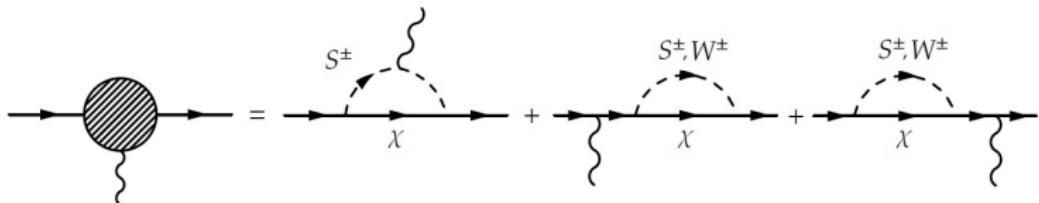


$$\Gamma(\ell_1^\pm \rightarrow \ell_2^\pm \gamma) = \frac{\alpha m_{\ell_1}^3}{4} (|A_L|^2 + |A_R|^2), \quad \text{with } A_{L,R} \sim \frac{1}{16\pi^2} \frac{m_{\ell_1}}{m_R^2}$$

$$\Rightarrow m_R \gtrsim 50 \text{ TeV } (\mu^+ \rightarrow e^+ \gamma), \quad m_R \gtrsim 2 \text{ TeV } (\tau^- \rightarrow e^- \gamma), (\tau^- \rightarrow \mu^- \gamma)$$

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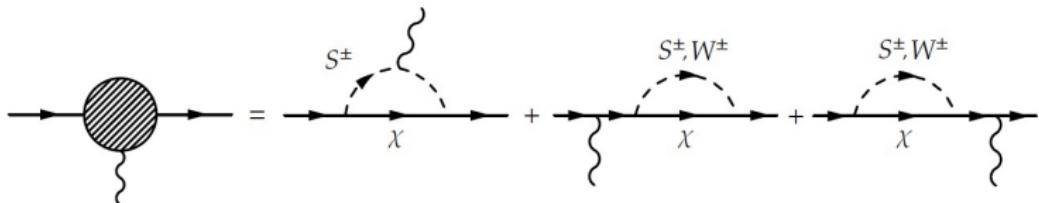
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With the choice for neutrino mass $m_1 = 0.05 \text{ eV}$,
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$$m_4 = 4.3 \times 10^{12} \text{ GeV}, \quad m_5 = 6.0 \times 10^{12} \text{ GeV}, \quad m_6 = 2.2 \times 10^{14} \text{ GeV}$$

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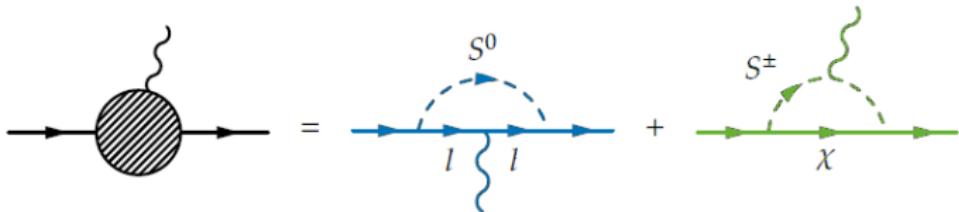
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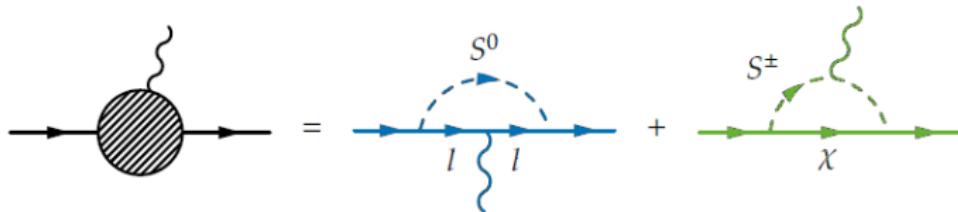
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Magnetic dipole moments



$$\begin{aligned}
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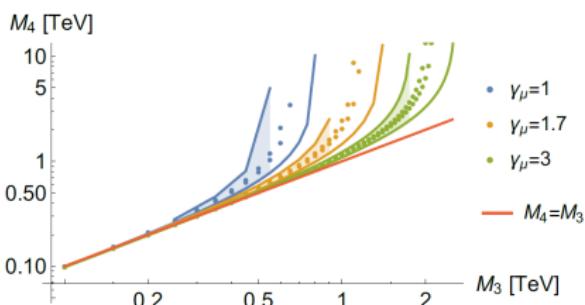
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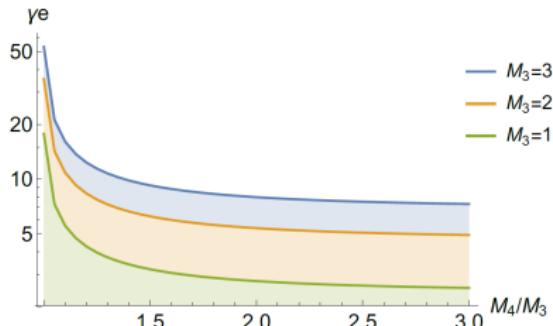
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Constrains on M_3 , M_4 , γ_μ , γ_e :

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} (\text{at } 3.6\sigma)^a \\ (261 \pm 78) \times 10^{-11} (\text{at } 3.6\sigma)^b \end{cases}, \quad a_e^{\text{err}} = \pm 2.6 \times 10^{-13}$$



For $M_3, M_4 \gtrsim 1 \text{ TeV}$
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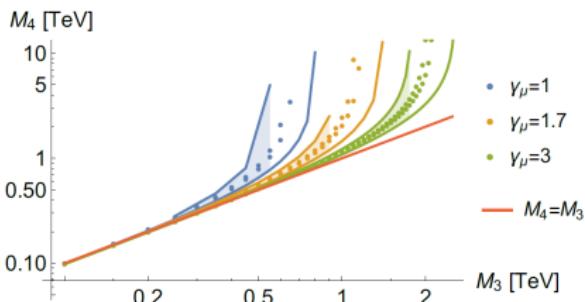
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^a[1010.4180/hep-ph], ^b[1105.3149/hep-ph]

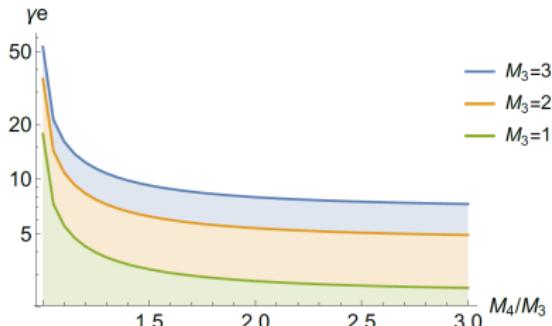
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$$\text{BR}(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-) = \frac{m_{\ell_1}^5}{6144\pi^3} |X_{\ell_2 \ell_1}|^2 |\gamma_{\ell_3}|^2 \frac{|A_{\ell_2 \ell_1}|^2 + |A_{\ell_1 \ell_2}|^2}{(m_{\ell_2}^2 - m_{\ell_1}^2)^2} \left(\frac{1}{M_3^4} + \frac{1}{M_4^4} \right) / \Gamma_{\text{tot}}$$

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seesaw scale: m_R

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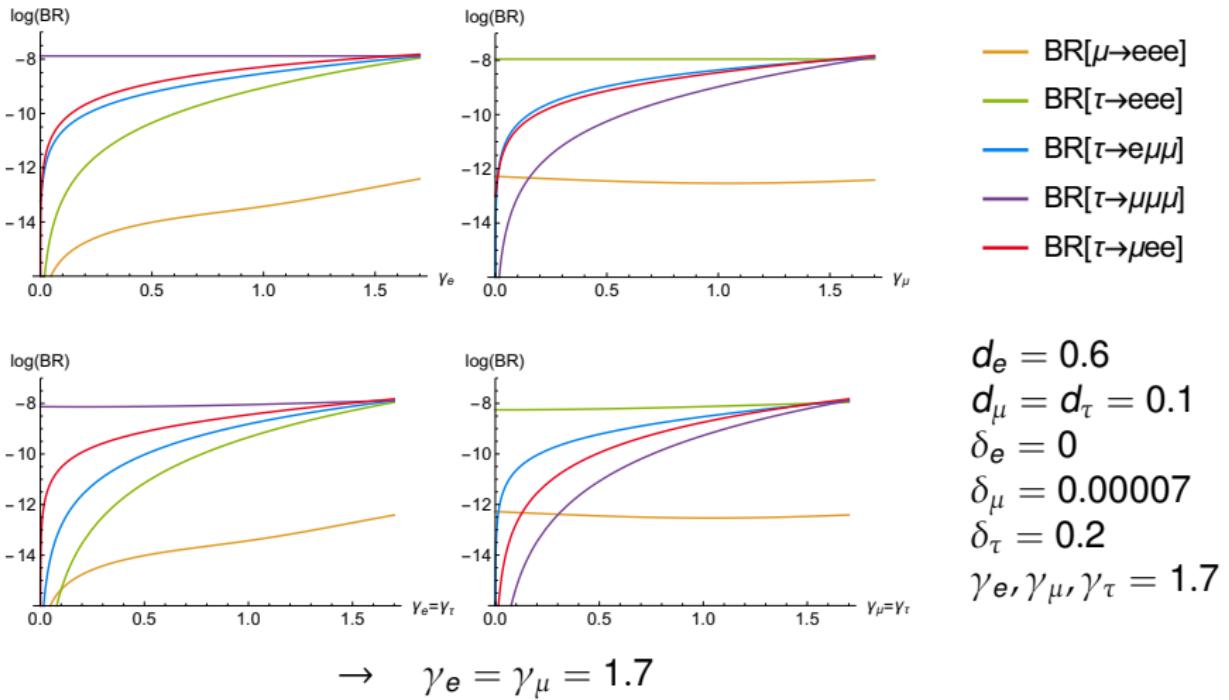
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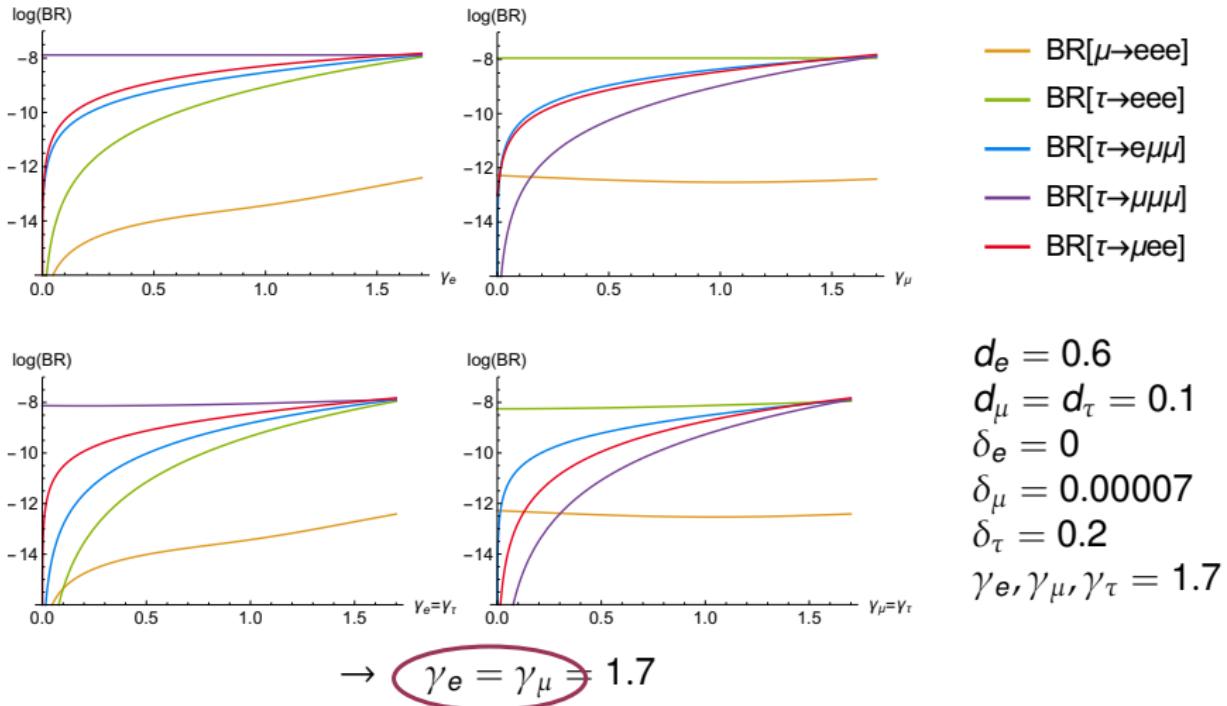
Constrain from 4ℓ -decays

We want $\text{BR} [\tau^- \rightarrow \ell^- e^+ e^-] (\gamma_e, \gamma_\mu, \gamma_\tau)$ and $\text{BR} [\tau^- \rightarrow \ell^- \mu^+ \mu^-] (\gamma_e, \gamma_\mu, \gamma_\tau)$ close to their experimental upper bounds ($< 2.7 \times 10^{-8}$):



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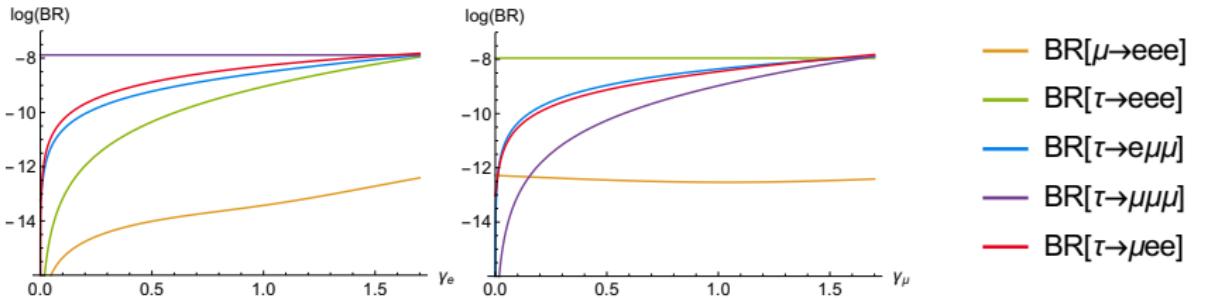
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$$\begin{aligned}
 d_e &= 0.6 \\
 d_\mu &= d_\tau = 0.1 \\
 \delta_e &= 0 \\
 \delta_\mu &= 0.00007 \\
 \delta_\tau &= 0.2 \\
 \gamma_e, \gamma_\mu, \gamma_\tau &= 1.7
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$$\rightarrow \gamma_e = \gamma_\mu = 1.7 \quad \checkmark \text{ MDM } (\gamma_\mu \geq 1.7, \gamma_e < 2.76 \text{ TeV})$$

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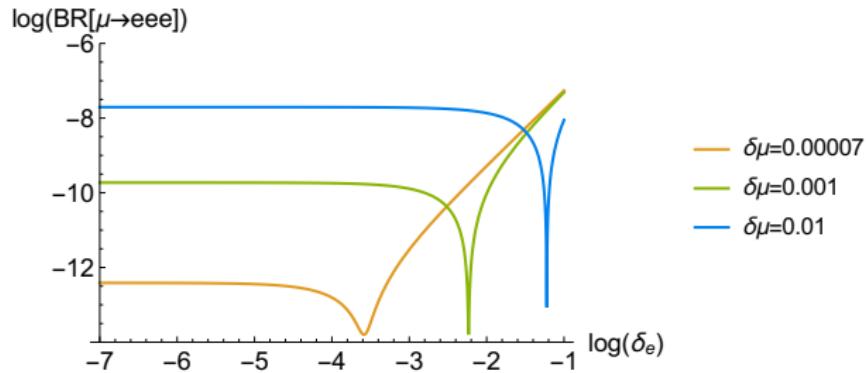
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Further constrains from 4ℓ -decays

$$\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$$



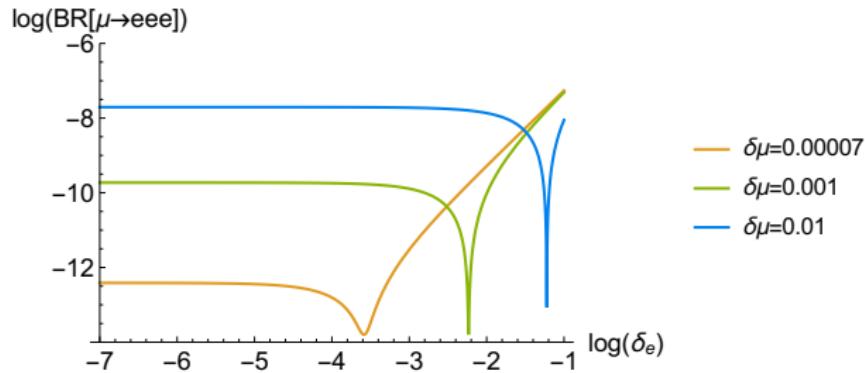
Large $\gamma_e = \gamma_\mu = 1.7$,
 $d_e = 0.6$, $d_\mu = 0.1$
give a too large
contribution to BR,
unless there is a
cancellation
between terms prop.
to δ_e and δ_μ .

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no finetuning with:

$$\delta_e = 0$$

$$\delta_\mu = 0.00007$$

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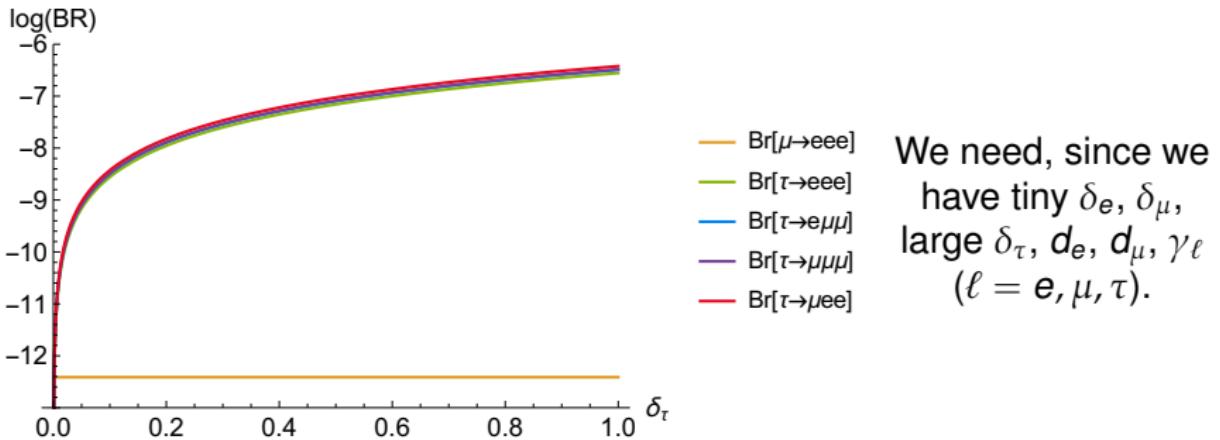
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Further constrain from 4ℓ -decays

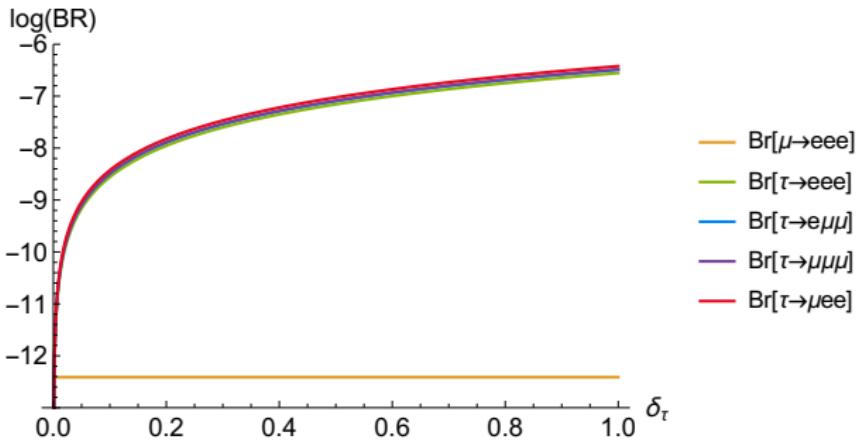


e.g.

$$\Gamma(\tau^- \rightarrow e^- e^+ e^-) \sim |\gamma_e|^2 (|A_{e\tau}|^2 + |A_{\tau e}|^2)$$

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Further constrain from 4ℓ -decays



We need, since we have tiny δ_e, δ_μ , large $\delta_\tau, d_e, d_\mu, \gamma_\ell$ ($\ell = e, \mu, \tau$).

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$$\begin{aligned} \delta_\tau &= 0.2 \\ d_e &= 0.6, d_\mu = 0.1 \\ \gamma_\ell &= 1.7 \end{aligned}$$

Results

seesaw scale: $m_R > 10^{12} \text{ GeV}$

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scalar masses: SM Higgs boson, Goldstone boson, 2 other scalars:

$$M_3 = 1 \text{ GeV}, M_4 = 2 \text{ GeV}, (M_{higgs} = 125 \text{ GeV})$$

Results

exp. bounds		model contributions
$\text{BR}(\mu^+ \rightarrow e^+ \gamma)$	< 4.2×10^{-13}	
$\text{BR}(\tau^- \rightarrow e^- \gamma)$	< 3.3×10^{-8}	$\sim \frac{1}{m_R^4}$
$\text{BR}(\tau^- \rightarrow \mu^- \gamma)$	< 4.4×10^{-8}	
$\text{BR}(\mu^- \rightarrow e^- e^+ e^-)$	< 1.0×10^{-12}	3.872×10^{-13}
$\text{BR}(\tau^- \rightarrow e^- e^+ e^-)$	< 2.7×10^{-8}	1.111×10^{-8}
$\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-)$	< 2.7×10^{-8}	1.280×10^{-8}
$\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)$	< 2.1×10^{-8}	1.307×10^{-8}
$\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-)$	< 1.8×10^{-8}	1.506×10^{-8}
$(\text{exp/SM} = 1) \quad a_e^{err}$	= $\pm 2.6 \times 10^{-13}$	$a_e^{mod} = 1.0 \times 10^{-13}$
$a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	= $\begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases}$	$a_\mu^{mod} = 258 \times 10^{-11}$

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Summary

Model:

- Right handed neutrinos, in seesaw formulation: expl. light m_μ
- 2 Higgs doublets: FCNI
- Soft flavour violation: violation only through neutrinos

ℓ -decays:

- Found upper bounds on flavour diagonal Yukawa couplings (Γ_ℓ , Δ_ν) at one loop.
- Found lower benchmarks on seesaw scale $m_R > 50 \text{ TeV}$
- Explain the discrepancy between experimental and theo. magnetic dipole moment of the myon
- Pointing out exp. signatures in ℓ -decays

Summary

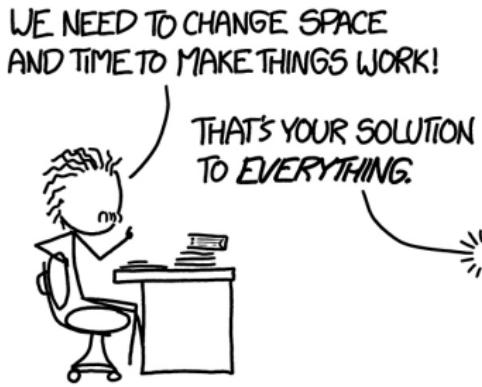
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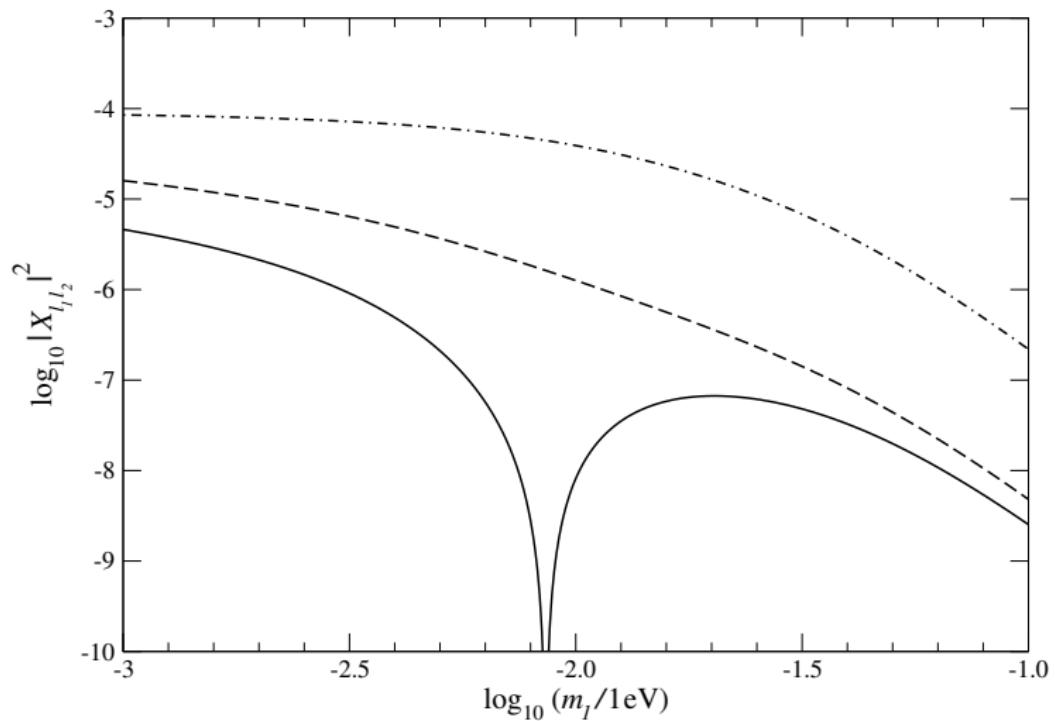
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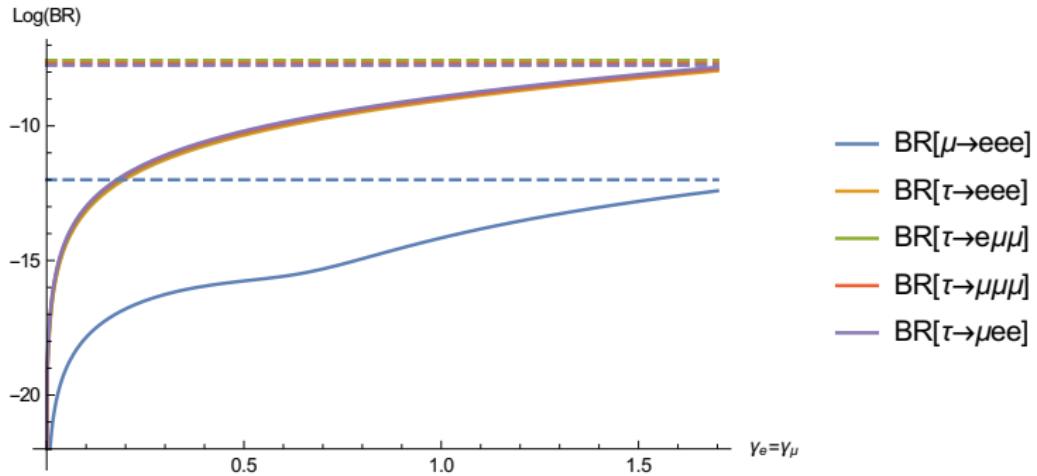
We found a different approach to make things work!



Thank you!

Backup slides





Advantage of right handed neutrinos

- **Explain mass hierarchy** in right handed neutrino mass models via the seesaw mechanism. [$m_{\nu_R} \gtrsim \text{TeV}$] (with additional Higgs doublets...)
- **Dark matter candidates** [$\text{keV} \lesssim m_{\nu_R} \lesssim \text{TeV}$]
- **Baryon asymmetry** via Leptogenesis in ν MSM models [$\text{keV} \lesssim m_{\nu_R} \lesssim \text{GeV}$]
- **Detected anomalies** at: LSND, MiniBooNE, gallium detectors: GALLEX, SAGE, reactor experiments... [$m_{\nu_R} \sim \text{eV}$] (a.o. also IceCube)

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