N-jettiness Subtractions

[arXiv:1505.04794]

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In collaboration with
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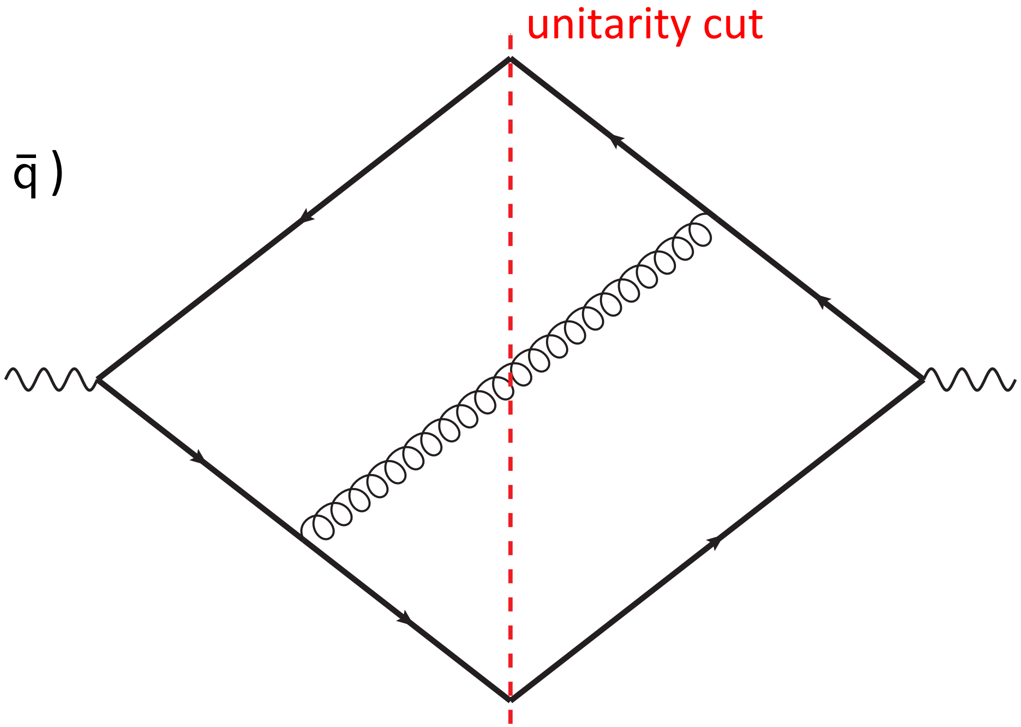
IR Singularities

Massless QCD:

(example: $e^+e^- \to q\,\bar{q}$)

NLO: $|\mathcal{M}|^2 \sim$  + ...

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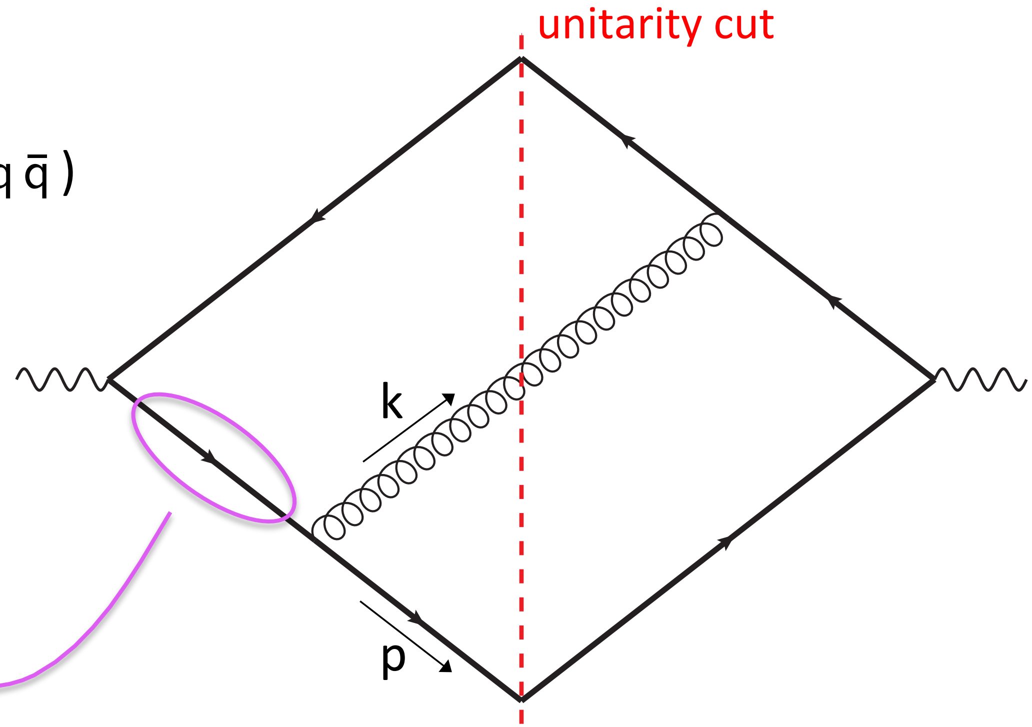


$+ \ldots$

$$\sim \frac{1}{(p+k)^2} = \frac{1}{2|\mathbf{p}||\mathbf{k}|(1-\cos\theta)}$$

$|\mathbf{k}| \longrightarrow 0$ soft singularity

$\theta \longrightarrow 0$ collinear singularity

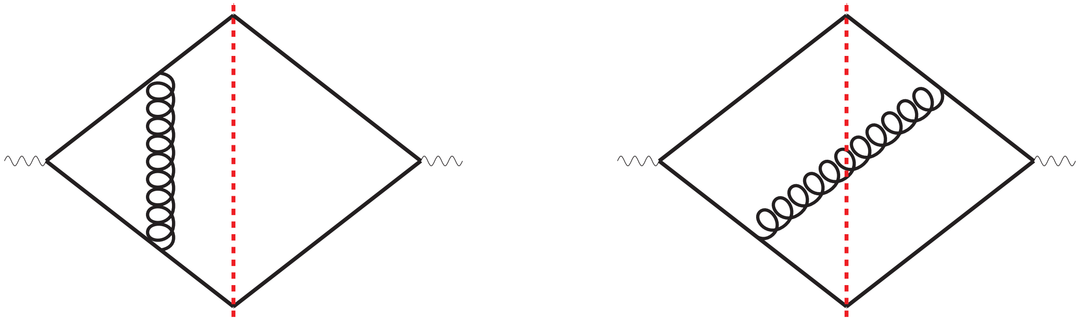
"IR singularities"

IR Singularities

Kinoshita-Lee-Nauenberg (KLN) theorem:

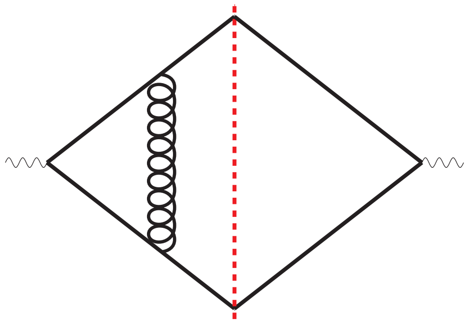
IR singularities cancel for “physical” (IR safe) observables.

- Implies sum over degenerate initial (→PDFs) and final states.
- Cancellation between virtual and real quantum corrections at every order in perturbation theory.



IR Singularities

Practical problem: regularization



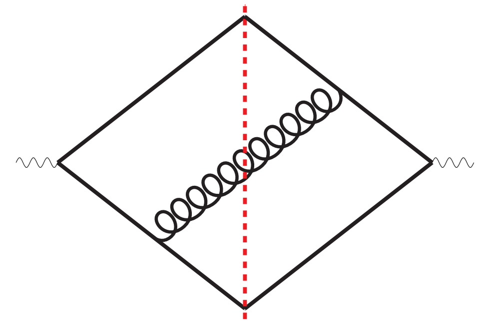
Virtual

Dimensional
Regularization

$\rightarrow \frac{1}{\epsilon^n}$ poles

(Must at higher orders)

Cancellation ⟷ fails



Real

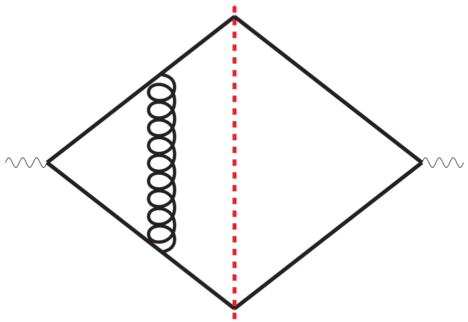
PS integration can be
(arbitrarily) complicated

→ Dim. Reg. impossible!

Aim: Numerical integration
→ Monte Carlo generator

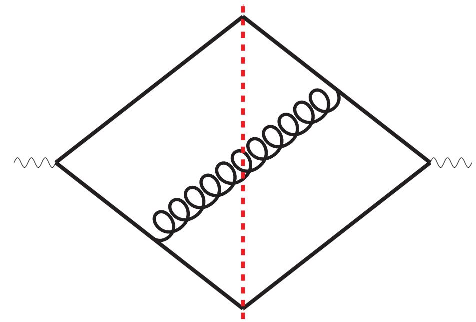
IR Singularities

Practical problem: regularization



Virtual

Dimensional Regularization



Real

PS integration can be (arbitrarily) complicated

Solution: “IR subtraction”

→ … impossible!

(Must at higher orders)

Aim: Numerical integration

→ Monte Carlo generator

Outline

- IR Subtraction/Slicing Methods
- The N-jettiness Event Shape
- NNLO Ingredients
- Applications/Results
- Possible Extensions
- Summary

IR Subtraction/Slicing Methods

N-jet cross section (i.e. N hard final state partons):

[X = set of (Born-level) kinematic variables the (differential) cross section depends on.]

$$\sigma_{\mathrm{LO}}(X) = \int_N d\sigma_{\mathrm{LO}}(X)$$

N-parton phase space

$$\sigma_{\mathrm{NLO}}(X) = \int_N d\sigma^{\mathrm{V}}_{\mathrm{NLO}}(X) + \int_{N+1} d\sigma^{\mathrm{R}}_{\mathrm{NLO}}(X)$$

$$\sigma_{\mathrm{NNLO}}(X) = \int_N d\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}(X) + \int_{N+1} d\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}(X) + \int_{N+2} d\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}(X)$$

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2-loop diagrams

1-loop diagrams

tree-level diagrams

IR Subtraction/Slicing Methods

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$$\int d\Phi_{\mathrm{N}}\left[\frac{\#}{\epsilon^4} + \frac{\#}{\epsilon^3} + \frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \#\right] \qquad \int d\Phi_{\mathrm{N+1}}\left[\frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \#\right] \qquad \int d\Phi_{\mathrm{N+2}}\left[\#\right]$$

Problem: Achieve cancellation of IR poles!

IR Subtraction/Slicing Methods

Idea: introduce subtraction (counter-) terms

$$\sigma_{\mathrm{NLO}}(X) = \int_N d\sigma^{V}_{\mathrm{NLO}}(X) + \int_{N+1}\left[d\sigma^{R}_{\mathrm{NLO}}(X) - d\sigma^{R}_{\mathrm{sing}}(X)\right] + \int_{N+1} d\sigma^{R}_{\mathrm{sing}}(X)$$

$$\sigma_{\mathrm{NNLO}}(X) = \int_N d\sigma^{VV}_{\mathrm{NNLO}}(X) + \int_{N+1}\left[d\sigma^{RV}_{\mathrm{NNLO}}(X) - d\sigma^{RV}_{\mathrm{sing}}(X)\right] + \int_{N+1} d\sigma^{RV}_{\mathrm{sing}}(X)$$
$$+ \int_{N+2}\left[d\sigma^{RR}_{\mathrm{NNLO}}(X) - d\sigma^{RR}_{\mathrm{sing}}(X)\right] + \int_{N+2} d\sigma^{RR}_{\mathrm{sing}}(X)$$

Local subtractions:

Construct $d\sigma^{R,RV,RR}_{\mathrm{sing}}$ such that they

- exactly match the $d\sigma^{R,RV,RR}_{(N)NLO}$ in the singular limit point by point in PS
- are simple enough to be integrable in D dimensions

IR Subtraction/Slicing Methods

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$$\sigma_{\mathrm{NLO}}(X) = \int_{N} d\sigma^{V}_{\mathrm{NLO}}(X) + \int_{N+1} \underbrace{\left[d\sigma^{R}_{\mathrm{NLO}}(X) - d\sigma^{R}_{\mathrm{sing}}(X) \right]}_{\text{integrable in 4D}} + \underbrace{\int_{N+1} d\sigma^{R}_{\mathrm{sing}}(X)}_{\text{integrable in D dim.}}$$

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$$+ \int_{N+2} \underbrace{\left[d\sigma^{RR}_{\mathrm{NNLO}}(X) - d\sigma^{RR}_{\mathrm{sing}}(X) \right]}_{\text{integrable in 4D}} + \underbrace{\int_{N+2} d\sigma^{RR}_{\mathrm{sing}}(X)}_{\text{integrable in D dim.}}$$

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IR Subtraction/Slicing Methods

Local subtraction schemes

NLO

- Catani-Seymour (Dipole) [Catani, Seymour, '96]
- FKS [Frixione, Kunszt, Signer, '96]
- Nagy-Soper [Nagy, Soper, '07; Chung, Krämer, Robens, '10]
- FDU [Sborlini, Driencourt-Magnin, Hernandez-Pinto, Rodrigo, '16]

✓ Well established

✓ Arbitrary N-jet processes

✓ Automated:

HERWIG++, SHERPA, MUNICH; MG5_aMC, POWHEG-BOX, WHIZARD; HELAC+DEDUCTOR; MCFM, ...

IR Subtraction/Slicing Methods

Local subtraction schemes

NNLO — very complicated, hard to automate, but successfully applied to several LHC processes, e.g.:

- Sector decomposition [Anastasiou, Melnikov, Petriello, '03]
  - $pp \longrightarrow H, V$ [Anastasiou, Melnikov, Petriello, '03-04]
- Sector-improved subtraction schemes (STRIPPER) [Czakon, '10]
  - $pp \longrightarrow t\bar{t}$ [Czakon, Fiedler, Mitev, '13]
  - $pp \longrightarrow H + j$ [Boughezal, Caola, Melnikov, Petriello, Schulze, '13-15]
- Antenna subtraction [Gehrmann-De Ridder, Gehrmann, Glover, '05]
  - $e^+e^- \longrightarrow 3j$ [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07] [Weinzierl, '08]
  - $pp \longrightarrow jj$ (partial) [Gehrmann-De Ridder, Gehrmann, Glover, Pires '13]
  - $pp \longrightarrow H, Z + j$ [Chen, Gehrmann, Glover, Jaquier, 14-16] [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan, '15]
  - $pp \longrightarrow t\bar{t}$ (partial) [Abelof, Gehrmann-De Ridder, Maierhofer, Majer, Pozzorini, '11-15]
- Colorful NNLO [Del Duca, Somogyi, Trocsanyi, '05]
  - $e^+e^- \longrightarrow 3j$ [Del Duca, Duhr, Somogyi, Tramontano, Trocsanyi, '16]

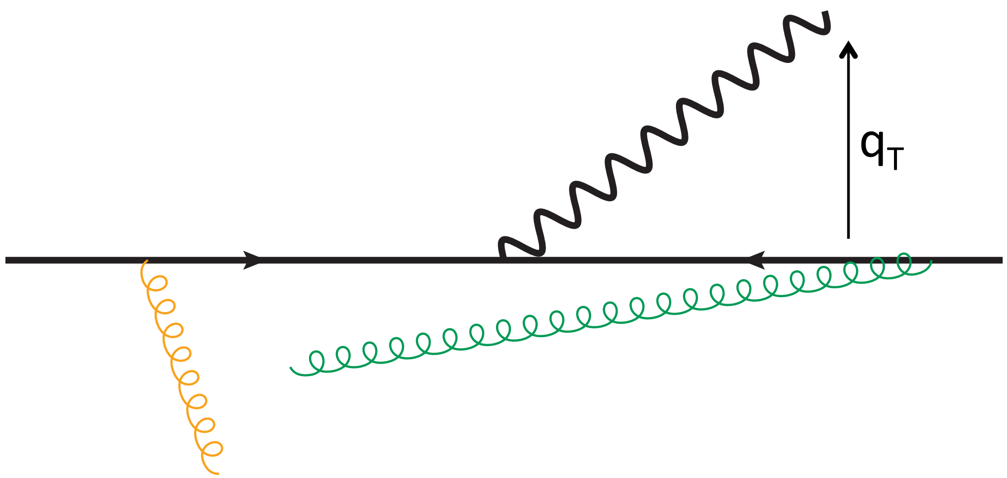
IR Subtraction/Slicing Methods

Alternative method:

Phase space Slicing (non-local subtraction)

prototype: $\boxed{q_T\text{ - subtraction}}$ [Catani, Grazzini, '07]

Transverse momentum q_T of color-neutral final state controls real emissions!



$q_T>0$
requires
at least one non-singular
real emission!

IR Subtraction/Slicing Methods

$\boxed{q_T \text{ - subtraction}}$ [Catani, Grazzini, '07]

$$\sigma_{\text{NNLO}}(X) = \int_0^{q_T^{\text{cut}}} dq_T \, \frac{d\sigma_{\text{NNLO}}(X)}{dq_T} + \int_{q_T^{\text{cut}}} dq_T \, \frac{d\sigma_{\text{NNLO}}(X)}{dq_T}$$

diff. measurement
(cuts) on Born-level
kinematics

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diff. measurement (cuts) on Born-level kinematics

$q_T>0$
1-jet NLO process computable with NLO tools ✔

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diff. measurement (cuts) on Born-level kinematics

expansion in small q_T

$$\frac{d\sigma}{dq_T} = H \times [B_a \otimes B_b \otimes S](q_T) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

[Collins, Soper, Sterman '84]

known analytically to NNLO ✔

$q_T>0$

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$q_T > 0$

1-jet NLO process computable with NLO tools ✔

- Exploits existing NLO technology
- Easy to implement
- Large numerical cancellations as $q_T^{\mathrm{cut}} \ll Q$ ($\rightarrow$ computer intensive)

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Very successful for LHC processes with colorless (N=0) final state:

- H [Catani, Grazzini, '07]
- W/Z [Catani, Cieri, Ferrera, de Florian, Grazzini, '09]
- γγ [Catani, Cieri, Ferrera, de Florian, Grazzini, '12]
- WH [Ferrera, Grazzini, Tramontano '11]
- ZH [Ferrera, Grazzini, Tramontano '15]
- ZZ [Cascioli et al., '14] [Grazzini, Kallweit, Rathlev '15]
- W^+W^- [Gehrmann et al., '14] [Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann, '16]
- Zγ/Wγ [Grazzini, Kallweit, Rathlev, Torre '14] [Grazzini, Kallweit, Rathlev '15]
- WZ [Grazzini, Kallweit, Rathlev, Wiesemann, '16]
- HH [De Florian et al., '16]

IR Subtraction/Slicing Methods

q_T - subtraction

[Catani, Grazzini, '07]

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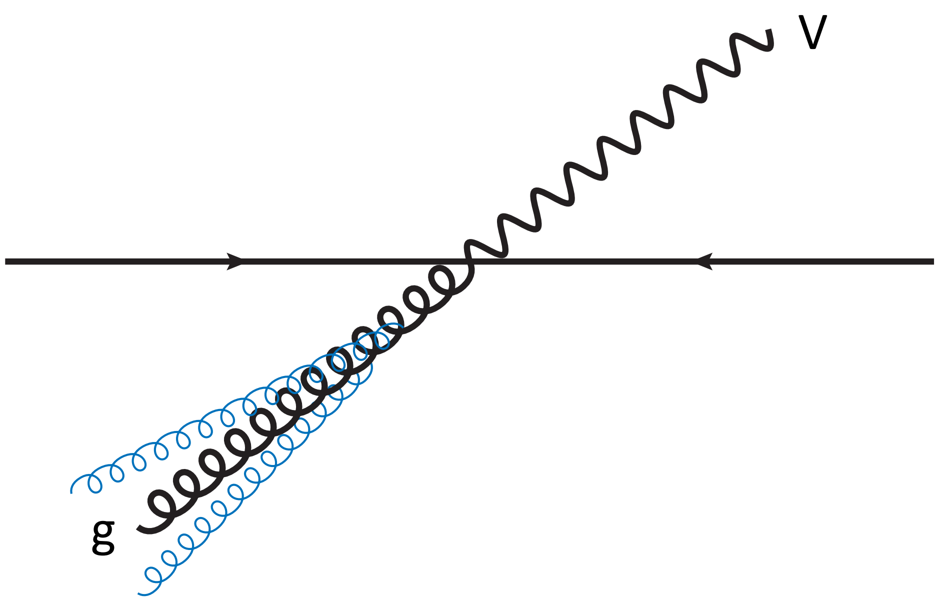
... but limited to colorless (N=0) final states!



transverse momentum of hard V+g final state (N=1): $q_T > 0$ even if, real emissions are in the collinear limit!

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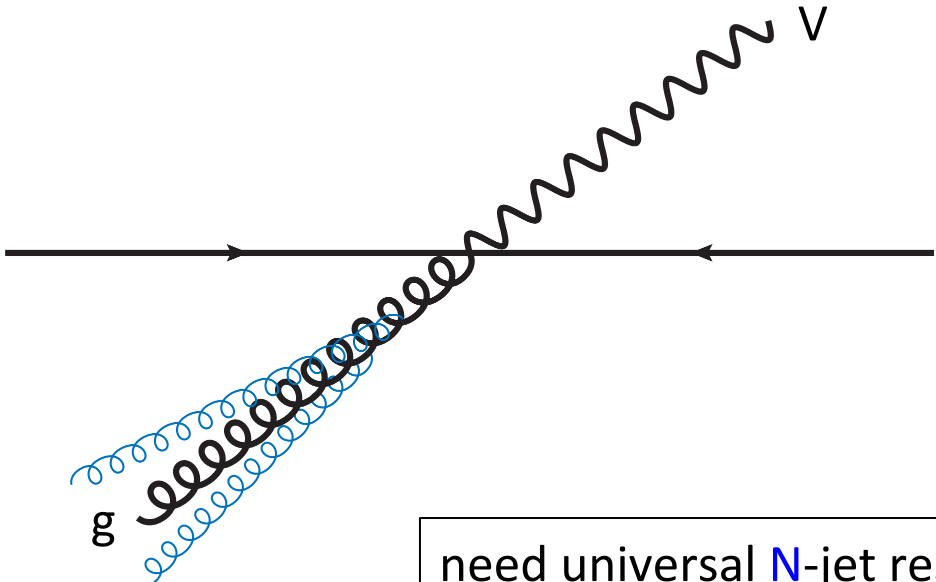

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need universal N-jet resolution variable → N-jettiness

The N-jettiness Event Shape

[Stewart, Tackmann, Waalewijn, '10]

Definition: $$\mathcal{T}_N = \sum_{k=1}^{M} \min_i \{2\, \hat{q}_i \cdot p_k\}$$

N final state jets

final state parton momenta

jet "directions": $\hat{q}_i = \frac{q_i}{Q_i}, \quad i = a, b, 1, \ldots, N$

$\hat{q}_{a,b}$ fixed to the beam directions!

(jet momenta q_i determined by any jet algorithm,
Q_i is arbitrary normalization, e.g. $Q_i = Q$ or $Q_i = 2E_i$)

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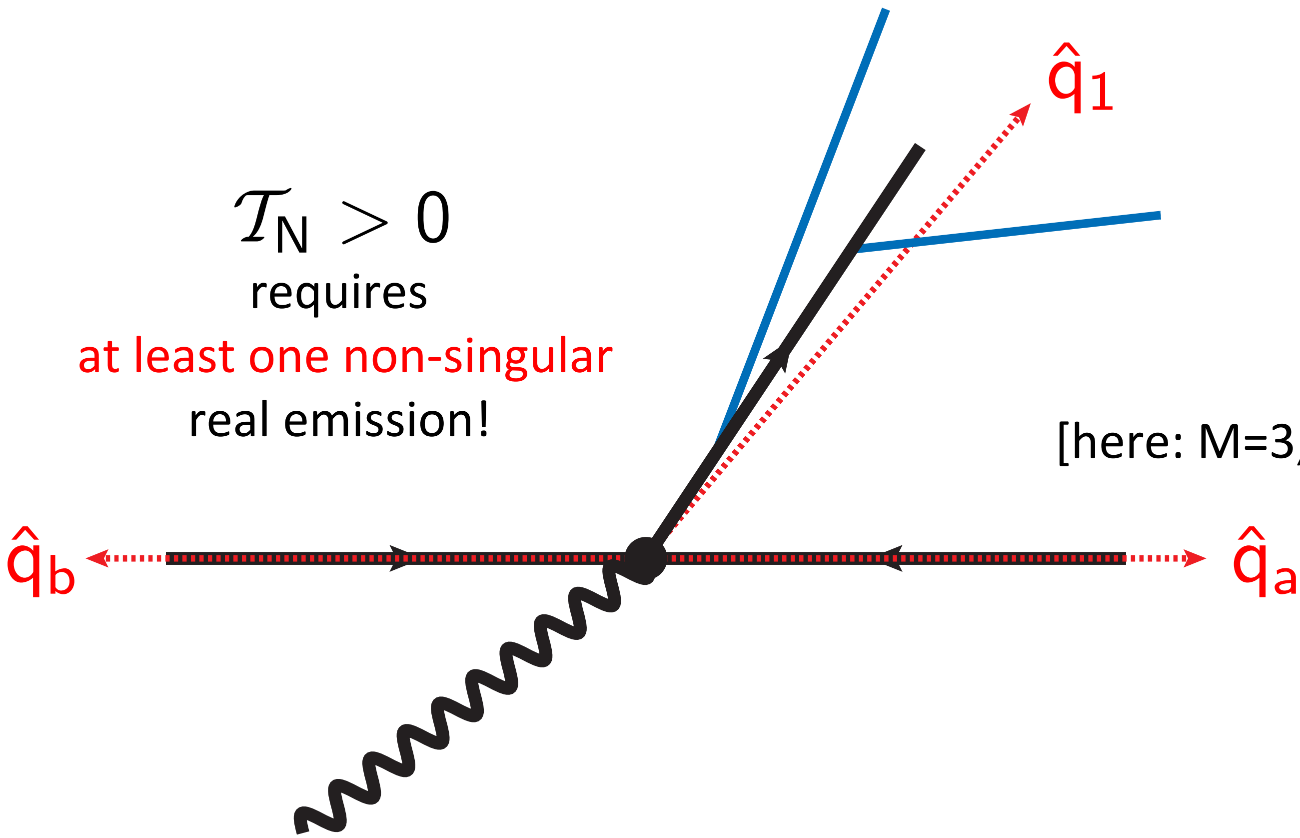
$\mathcal{T}_N > 0$

requires

at least one non-singular

real emission!

[here: M=3, N=1]



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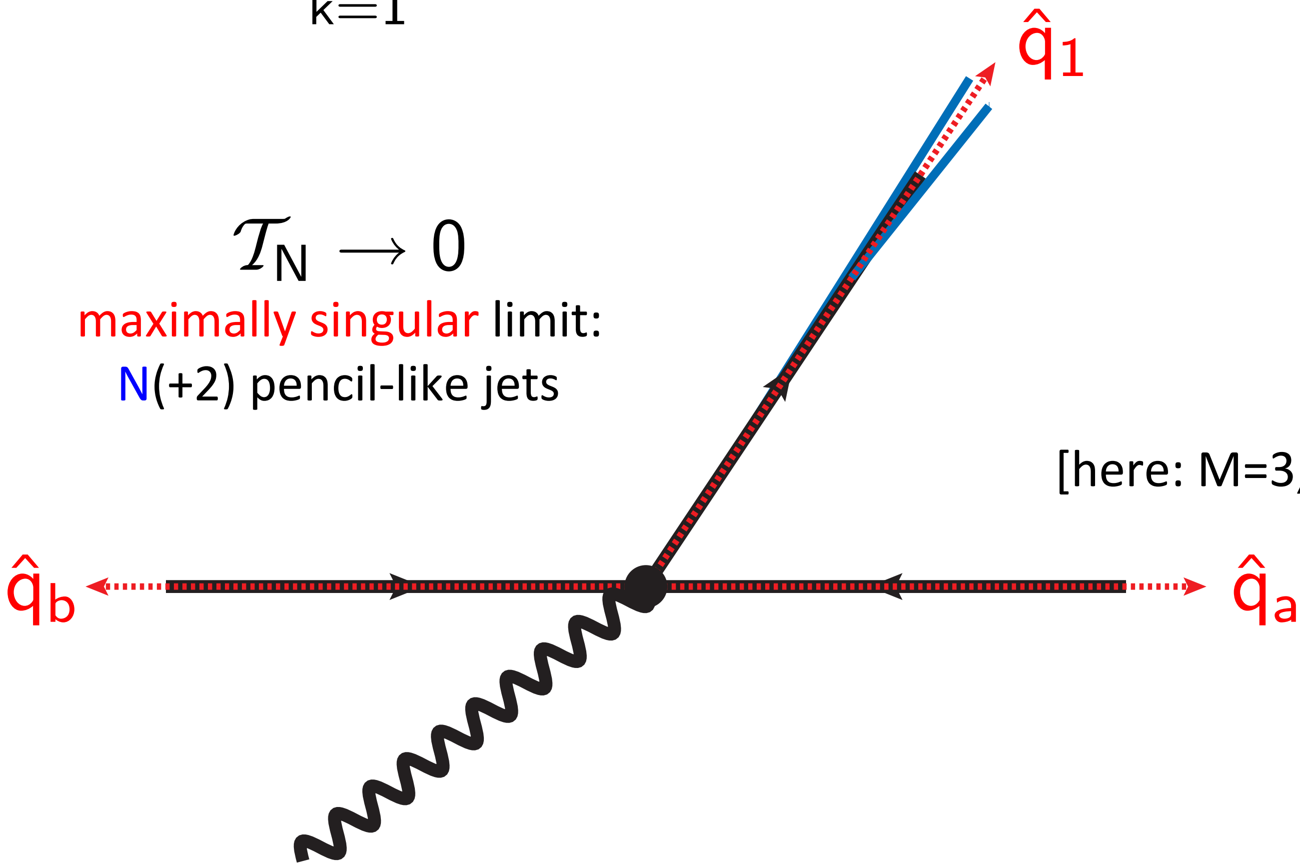
$\mathcal{T}_N \to 0$

maximally singular limit:

N(+2) pencil-like jets

[here: M=3, N=1]



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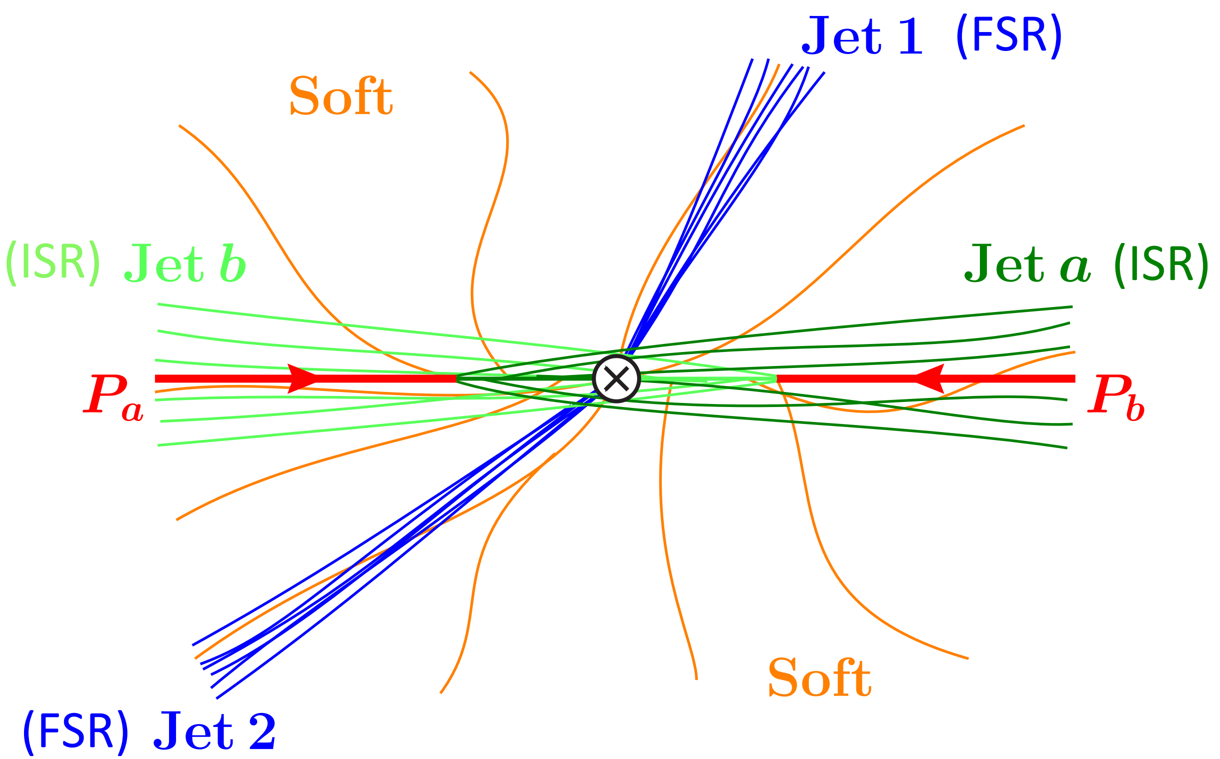
[Stewart, Tackmann, Waalewijn, '10]

Factorization formula for small $\mathcal{T}_N$:

$$\frac{d\sigma}{d\mathcal{T}_\mathcal{N}} = \mathrm{Tr}\left[\hat{H} \cdot \hat{S}_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i + \mathcal{O}\left(\frac{\mathcal{T}_N}{Q}\right)$$

$Q \sim E_{cm}$
"hard" scale



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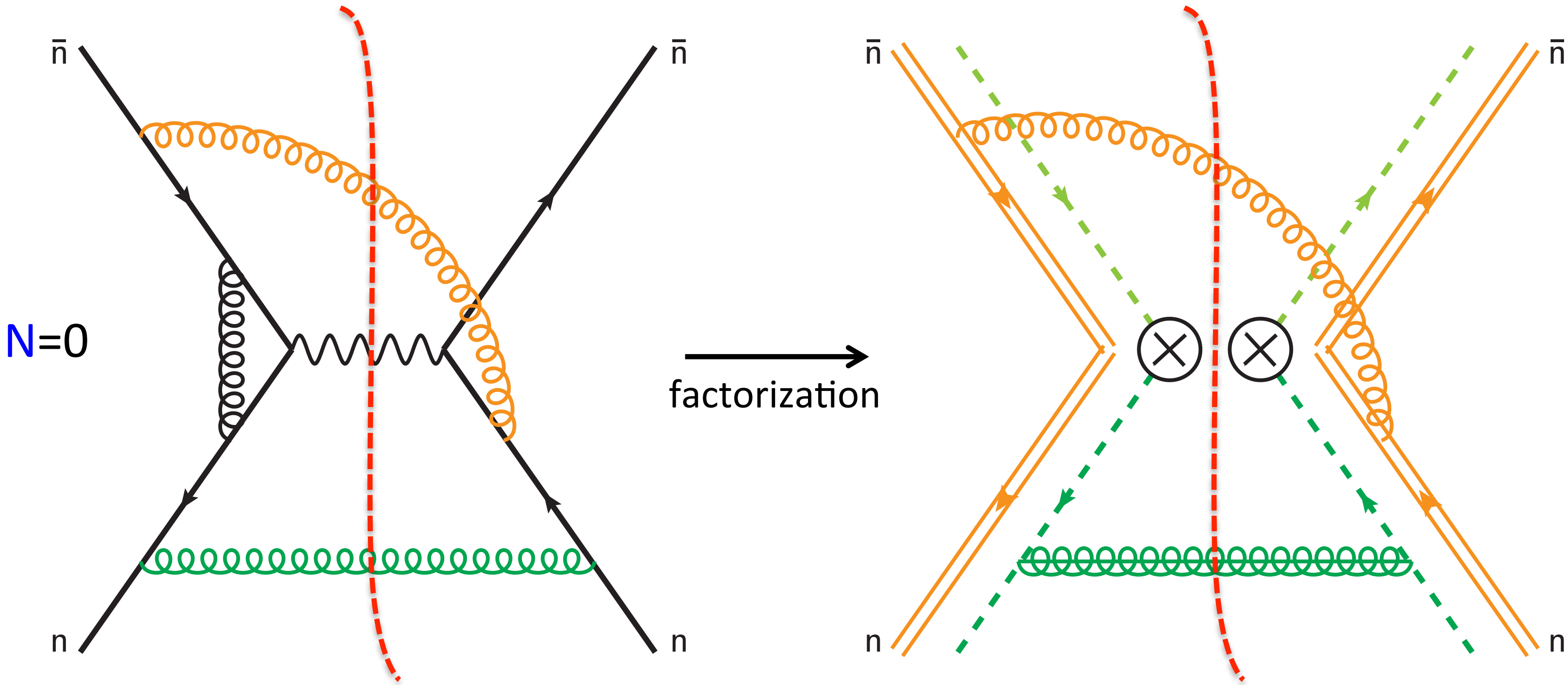

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$\hat{H}(\Phi_N, \mu)$

"hard function"

- depends on Born PS
- matrix in color space

$B_a(t_a, x_a, \mu)$

"beam function"

- depends on virtuality t_a
- and Bjorken variable x_a of incoming parton

$\hat{S}_N(k_a, \ldots, k_N, \{\hat{q}_i\}, \mu)$

"soft function"

- depends jet directions $\hat{q}_i$
- matrix in color space
- k_j : soft contributions to $\mathcal{T}_N$ from each jet

$J_i(s_i, \mu)$

"jet function"

- depends on virtuality s_i of outgoing parton

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Derivation in SCET

n-collinear vector $\quad p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu \rightarrow (p^-, p^+, p_\perp)$

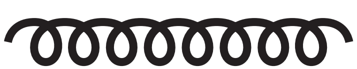
"lightcone coordinates"

$\left[n^2 = \bar{n}^2 = 0, \ n \cdot \bar{n} = 2\right]$

SCET$_\mathrm{I}$ degrees of freedom:

n-collinear quark  $p^\mu \sim (1, \lambda^2, \lambda)Q$

n-collinear gluon  $p^\mu \sim (1, \lambda^2, \lambda)Q$

(ultra)soft gluon  $p^\mu \sim (\lambda^2, \lambda^2, \lambda^2)Q$

power counting parameter

$\boxed{\lambda^2 = \mathcal{T}_N / Q}$

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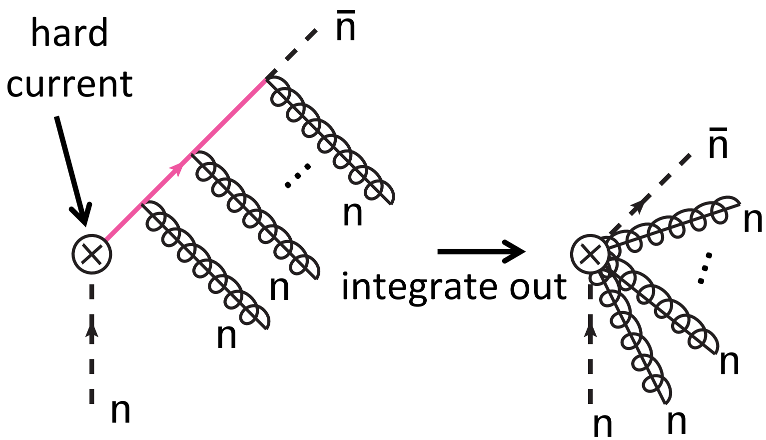
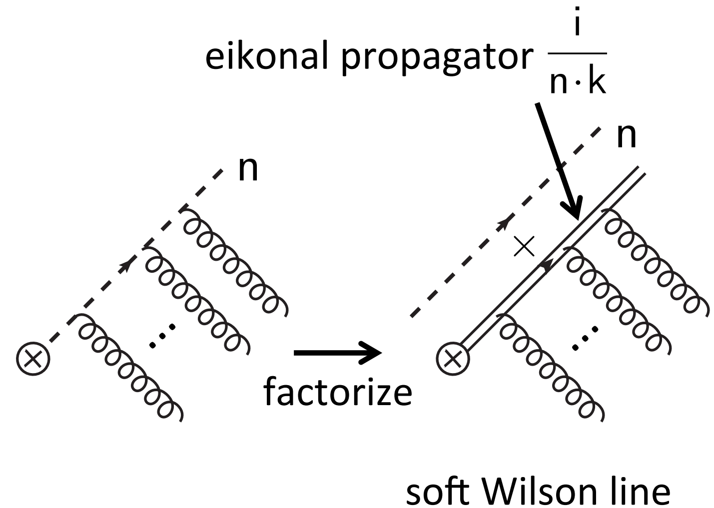
<u>Derivation in SCET</u>

Integrate out non-resonant dofs:



$$W_n = \sum_{\text{perms}} \exp\left(-\frac{g}{\bar{n}\cdot\mathcal{P}}\,\bar{n}\cdot A_n\right)$$

n-collinear Wilson line



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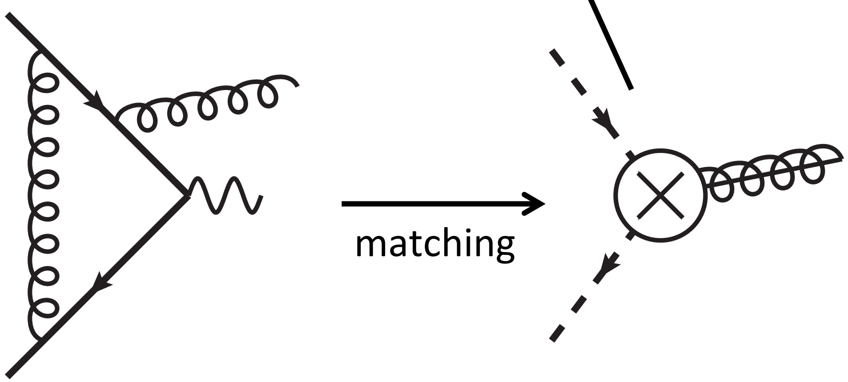
1-loop calculation

N=1

Wilson coefficient



purely virtual full QCD contributions (finite parts)

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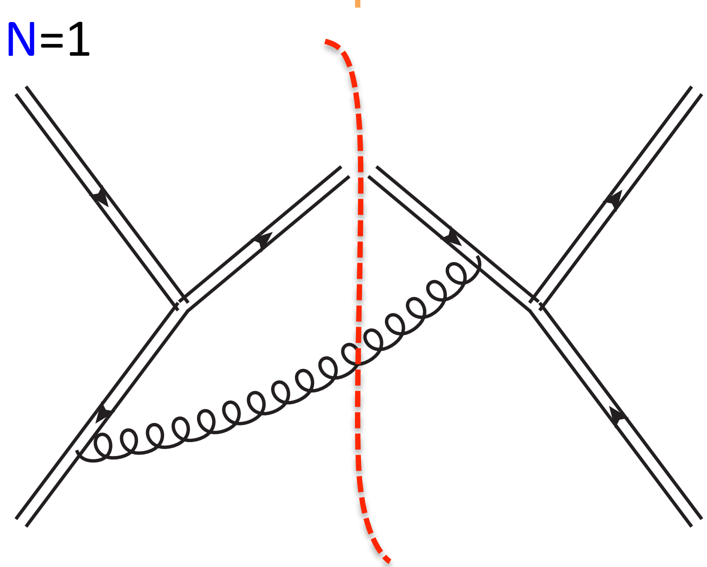
$$\frac{d\sigma}{d\mathcal{T}_{\mathcal{N}}} = \mathrm{Tr}\left[\hat{H} \cdot \hat{S}_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i + \mathcal{O}\left(\frac{\mathcal{T}_N}{Q}\right)$$

1-loop calculation



The N-jettiness Event Shape

[Stewart, Tackmann, Waalewijn, '10]

Factorization formula for small $\mathcal{T}_N$:

$$\frac{d\sigma}{d\mathcal{T}_\mathcal{N}} = \mathrm{Tr}\left[\hat{H} \cdot \hat{S}_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i + \mathcal{O}\left(\frac{\mathcal{T}_N}{Q}\right)$$

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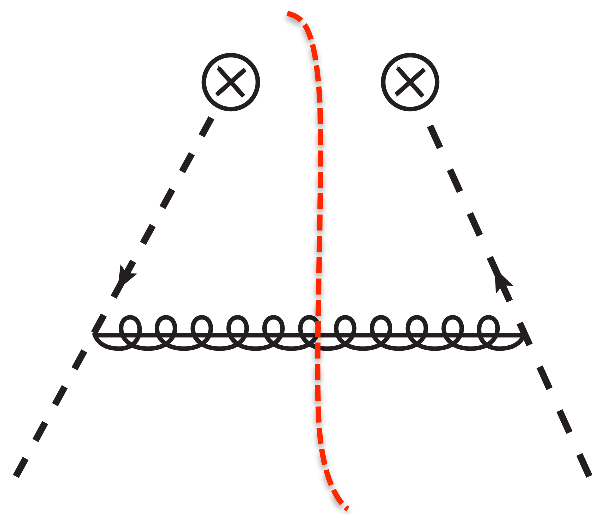

The N-jettiness Event Shape

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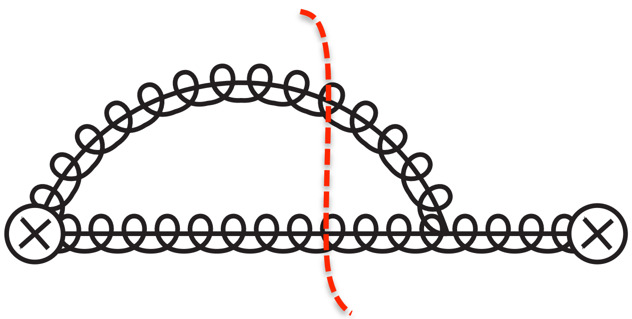


NNLO Ingredients

2-loop hard function:
- purely virtual QCD corrections
- process-dependent
- some known, many not

2-loop beam function:
✓ quark
✓ gluon [Gaunt, MS, Tackmann, '14]

2-loop jet function:
✓ quark [Becher, Neubert, '06]
✓ gluon [Becher, Bell, '10]

2-loop soft function:

✓ known analytically for N=0 (e.g. Drell-Yan, H-production, DIS, $e^+e^- \to jj$)
[Kelley, Schwartz, Schabinger, '11; Monni, Gehrmann, Luisoni, '11; Hornig, Lee, Stewart, Walsh, Zuberi, '11; Kang, Labun, Lee, '15]

✓ known numerically for N=1 (e.g. H/W/Z+j production)
[Boughezal, Liu, Petriello, '15]

✓ shown how to obtain numerically for arbitrary N [Boughezal, Liu, Petriello, '15]
from known 2-loop results + sector decomposition

NNLO Ingredients

n-collinear momentum: $p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu$

Generic quark beam function:

$\omega \equiv x\, p^-$

$$\boxed{B_q = \left\langle P_n(p^-) \middle| [\bar{q}_n W_n](0)\ \widehat{\mathcal{M}}\ \frac{\bar{n}\!\!\!/}{2} \left[\delta(\omega - \hat{p}^-)[W_n^\dagger q_n](0)\right] \middle| P_n(p^-) \right\rangle}$$

proton state — collinear Wilson line — collinear quark field

Gluon beam function analogous: $[W_n^\dagger q_n] \to \frac{1}{g}[W_n^\dagger\, iD_{n\perp} W_n]$

NNLO Ingredients

n-collinear momentum: $p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu$

Generic quark beam function:

$$\boxed{B_q = \Big\langle P_n(p^-) \Big| [\bar{q}_n W_n](0)\, \widehat{\mathcal{M}}\, \frac{\bar{\not{n}}}{2} \big[\delta(\omega - \hat{p}^-)[W_n^\dagger q_n](0)\big] \Big| P_n(p^-) \Big\rangle}$$

Typical measurement operators:

inclusive: $\widehat{\mathcal{M}} = \mathbb{1}$ (PDF)

N-jettiness: $\widehat{\mathcal{M}} = \delta(t - \omega \hat{p}^+)$

"inclusive" p_T: $\widehat{\mathcal{M}} = \delta(p_T - \hat{p}_\perp)$ ("TMDPDF")

fully differential: $\widehat{\mathcal{M}} = \delta(t - \omega \hat{p}^+)\delta(p_T - \hat{p}_\perp)$

NNLO Ingredients

n-collinear momentum: $p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu$

Generic quark beam function:

$$\boxed{B_q = \left\langle P_n(p^-) \middle| [\bar{q}_n W_n](0)\, \widehat{\mathcal{M}}\, \frac{\bar{n}\!\!\!/}{2} \left[\delta(\omega - \hat{p}^-)[W_n^\dagger q_n](0)\right] \middle| P_n(p^-) \right\rangle}$$

OPE: $$B_i(t, x, \mu) = \sum_j \int d\xi \; \mathcal{I}_{ij}\left(\frac{x}{\xi}, t, \mu\right) f_j(\xi, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{t}\right)$$

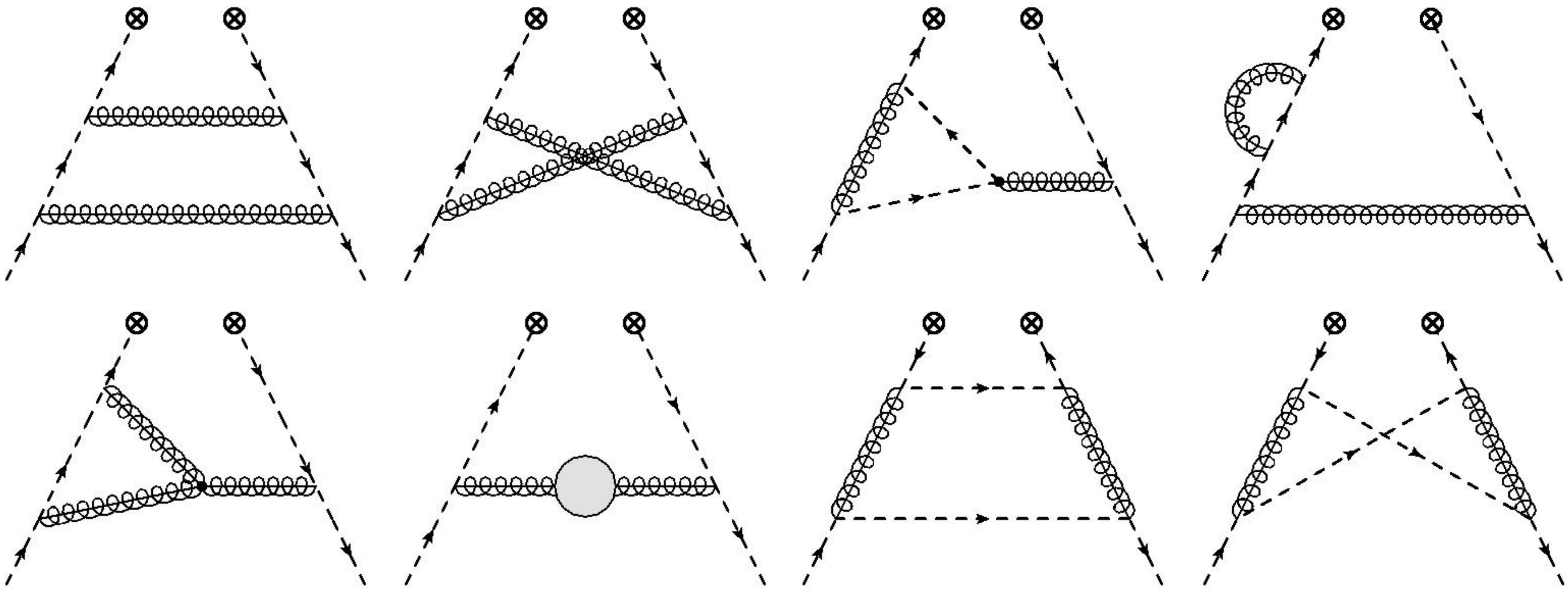
virtuality

Wilson coefficient

[Collins, Soper, Sterman; Stewart, Tackmann, Waalewijn]

NNLO Ingredients

2-loop qq-diagrams (axial gauge):



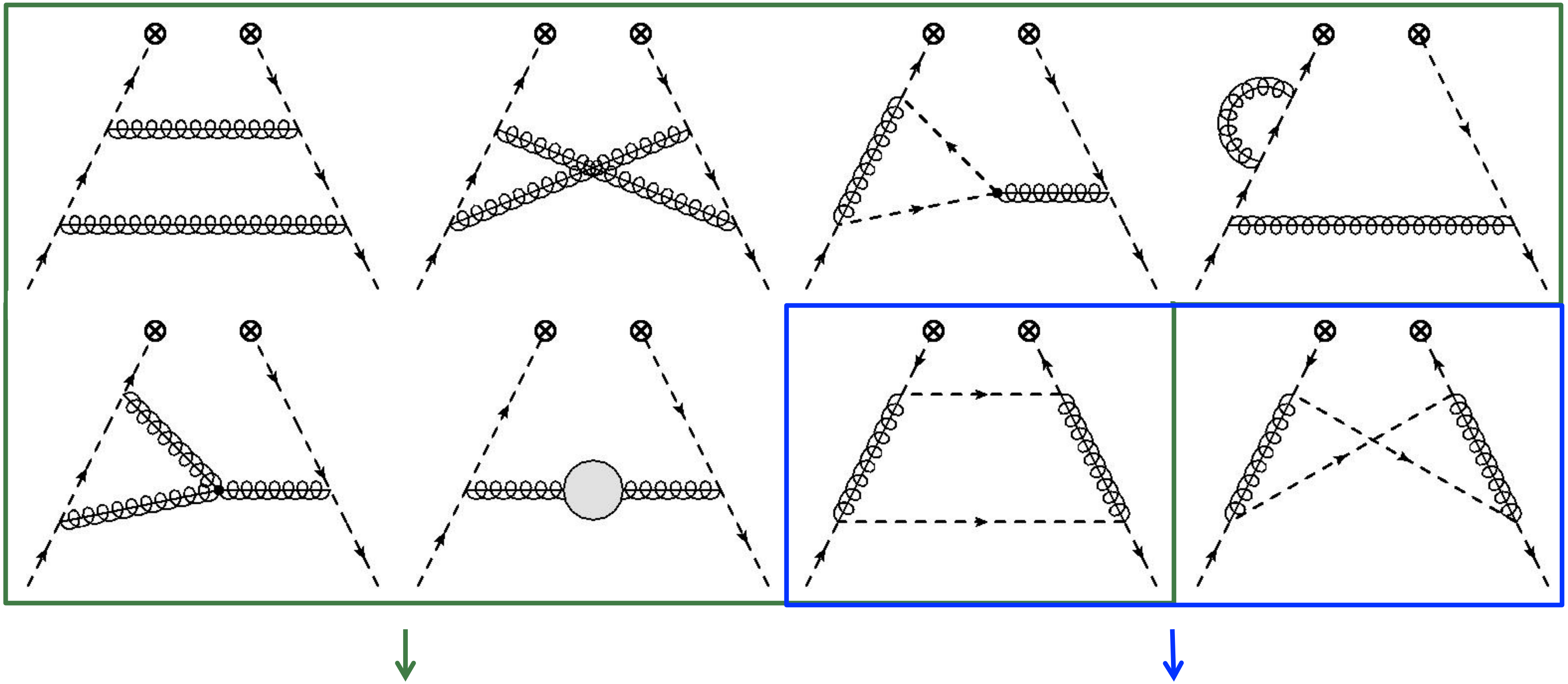
+ diagrams with gluons attached to Wilson lines (in Feynman gauge)

Consider all possible cuts!

NNLO Ingredients

2-loop qq-diagrams (axial gauge):

[Gaunt, MS, Tackmann, '14]



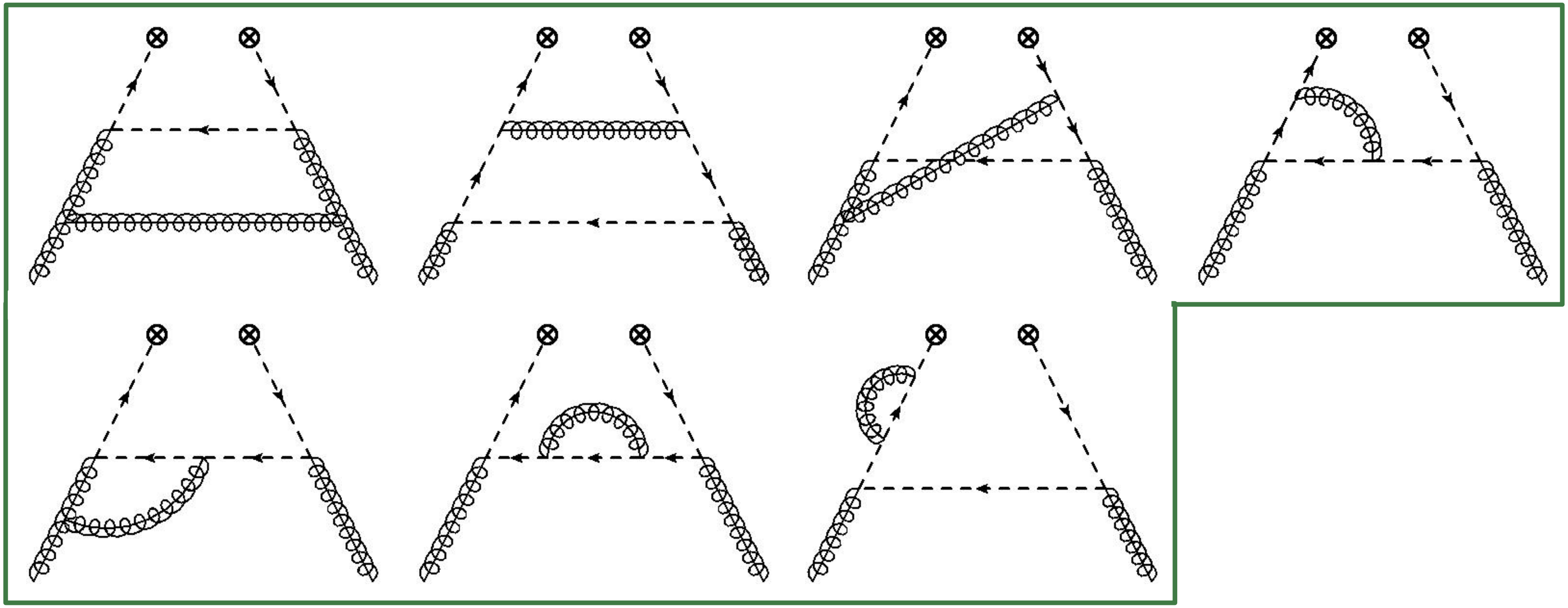
$\mathcal{I}^{(2)}_{\mathrm{qq}}$

$\mathcal{I}^{(2)}_{\bar{\mathrm{q}}\mathrm{q}}$

NNLO Ingredients

2-loop   qg-diagrams (axial gauge):

[Gaunt, MS, Tackmann, '14]



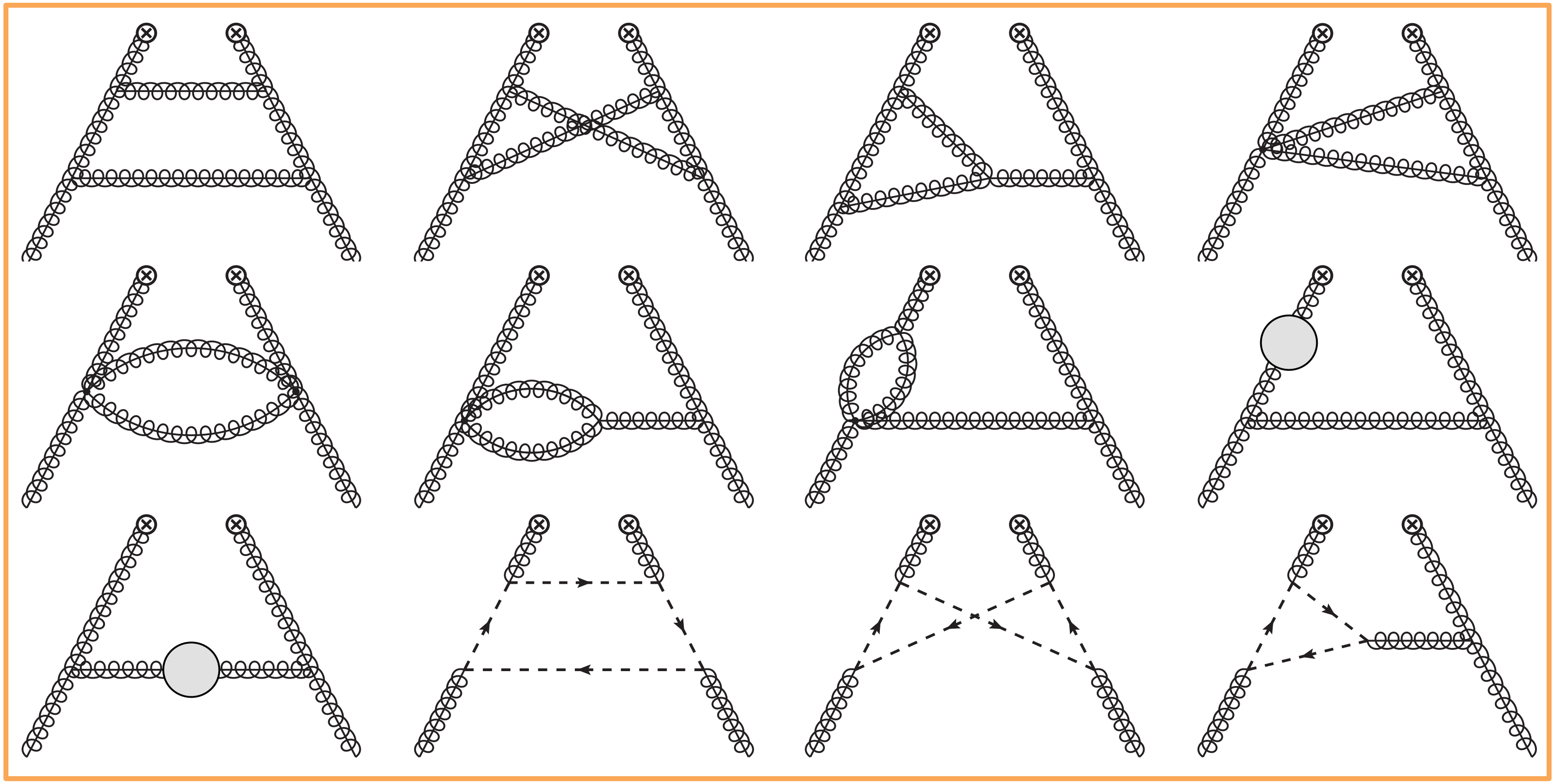
↓

$\mathcal{I}_{\mathrm{qg}}^{(2)}$

NNLO Ingredients

2-loop gg-diagrams (axial gauge):

[Gaunt, MS, Tackmann, ‘14]



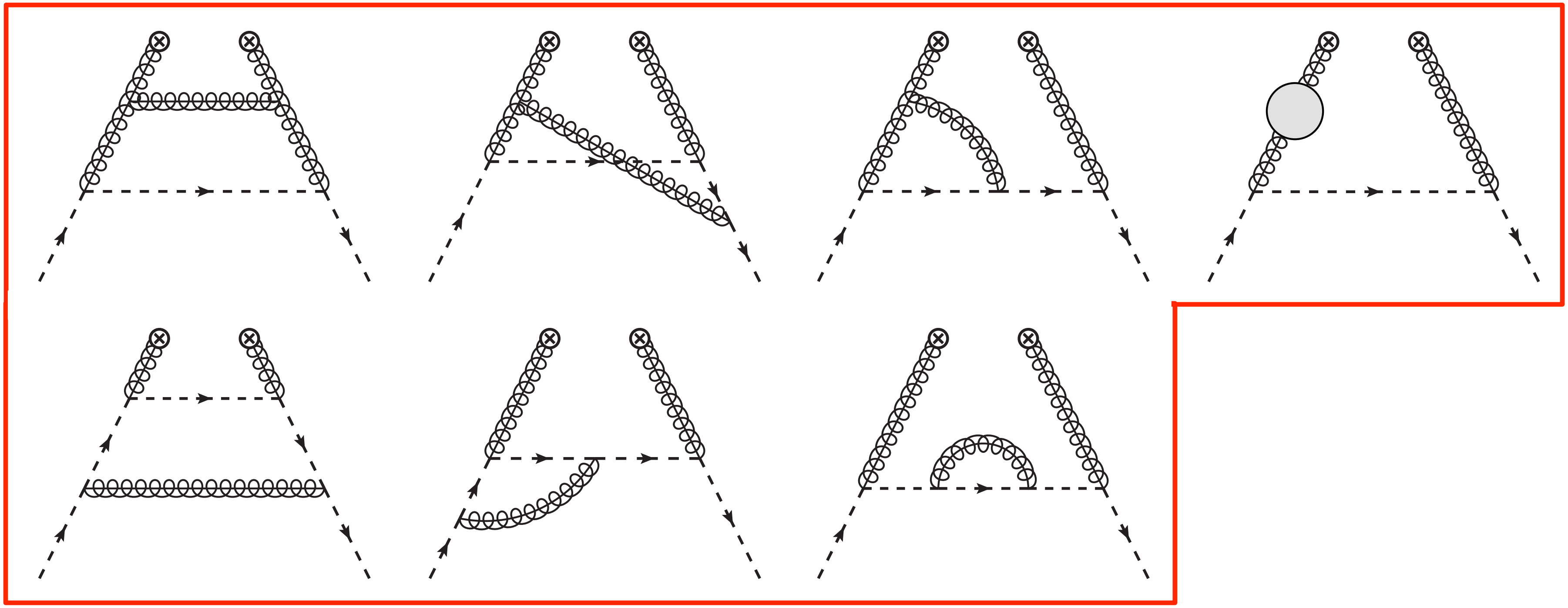
$\mathcal{I}_{gg}^{(2)}$

+Wilson line diagrams and ghost loops (in Feynman gauge)

NNLO Ingredients

$\boxed{\text{2-loop}}$ gq-diagrams (axial gauge):

[Gaunt, MS, Tackmann, '14]



$$\downarrow$$

$$\mathcal{I}_{\mathrm{gq}}^{(2)}$$

NNLO Ingredients

2-loop

$$\mathcal{L}_n(x) \equiv \left[\frac{\ln^n(x)}{x}\right]_+$$

$$\begin{aligned}
\mathcal{I}_{ij}^{(2)}(t,z,\mu) = {} & \frac{1}{\mu^2}\mathcal{L}_3\Big(\frac{t}{\mu^2}\Big)\frac{(\Gamma_0^i)^2}{2}\,\delta_{ij}\delta(1-z) \\
& + \frac{1}{\mu^2}\mathcal{L}_2\Big(\frac{t}{\mu^2}\Big)\Gamma_0^i\Big[-\Big(\frac{3}{4}\gamma_{B\,0}^i + \frac{\beta_0}{2}\Big)\delta_{ij}\delta(1-z) + 3P_{ij}^{(0)}(z)\Big] \\
& + \frac{1}{\mu^2}\mathcal{L}_1\Big(\frac{t}{\mu^2}\Big)\Big\{\Big[\Gamma_1^i - (\Gamma_0^i)^2\frac{\pi^2}{6} + \frac{(\gamma_{B\,0}^i)^2}{4} + \frac{\beta_0}{2}\gamma_{B\,0}^i\Big]\delta_{ij}\delta(1-z) \\
& \qquad + 2\Gamma_0^i\, I_{ij}^{(1)}(z) - 2(\gamma_{B\,0}^i + \beta_0)P_{ij}^{(0)}(z) + 4\sum_k P_{ik}^{(0)}(z)\otimes_z P_{kj}^{(0)}(z)\Big\} \\
& + \frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{t}{\mu^2}\Big)\Big\{\Big[(\Gamma_0^i)^2\zeta_3 + \Gamma_0^i\gamma_{B\,0}^i\frac{\pi^2}{12} - \frac{\gamma_{B\,1}^i}{2}\Big]\delta_{ij}\delta(1-z) - \Gamma_0^i\frac{\pi^2}{3}P_{ij}^{(0)}(z) \\
& \qquad - (\gamma_{B\,0}^i + 2\beta_0)I_{ij}^{(1)}(z) + 4\sum_k I_{ik}^{(1)}(z)\otimes_z P_{kj}^{(0)}(z) + 4P_{ij}^{(1)}(z)\Big\} \\
& \boxed{+\,\delta(t)\, 4I_{ij}^{(2)}(z)}
\end{aligned}$$

splitting function

New!

[Gaunt, MS, Tackmann, '14]

NNLO Ingredients

2-loop hard function:
- purely virtual QCD corrections
- process-dependent
- some known, many not

2-loop beam function:
- ✓ quark
- ✓ gluon

[Gaunt, MS, Tackmann, '14]

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- ✓ known numerically for N=1 (e.g. H/W/Z+j production)
  [Boughezal, Liu, Petriello, '15]
- ✓ shown how to obtain numerically for arbitrary N [Boughezal, Liu, Petriello, '15]
  from known 2-loop results + sector decomposition

NNLO Ingredients

2-loop hard function:
- purely virtual QCD corrections
- process-dependent
-

2-loop beam function:
✓ quark
✓

[Gaunt, MS, Tackmann, ‘14]

2-
✓
✓

With these ingredients we can compute

$$\sigma_{\mathrm{NNLO}}^{\mathrm{sing}}(\mathrm{X}) = \int_0^{\mathcal{T}_\mathrm{N}^{\mathrm{cut}}} \mathrm{d}\mathcal{T}_\mathrm{N} \frac{\mathrm{d}\sigma_{\mathrm{NNLO}}(\mathrm{X})}{\mathrm{d}\mathcal{T}_\mathrm{N}} + \mathcal{O}\Big(\frac{\mathcal{T}_\mathrm{N}^{\mathrm{cut}}}{\mathrm{Q}}\Big)$$

...for a given 2-loop hard function

2-
✓ jj)

Kang, Labun, Lee, ‘15]

✓ known numerically for N=1 (e.g. H/W/Z+j production)
[Boughezal, Liu, Petriello, ‘15]

✓ shown how to obtain numerically for arbitrary N [Boughezal, Liu, Petriello, ‘15]
from known 2-loop results + sector decomposition

Applications/Results

N-jettiness slicing recipe

[Boughezal, Focke, Liu, Petriello, '15]
[Gaunt, MS, Tackmann, Walsh, '15]

① Compute NLO N+1 jet cross section with partonic event generator:

$$\int_{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma_{\rm NNLO}(X)}{d\mathcal{T}_N} = \int_{N+1} d\sigma_{\rm NLO}(X)\theta(\mathcal{T}_N > \mathcal{T}_N^{\rm cut}) \qquad (1)$$

- for each event determine N jet directions $\hat{q}_i$ by mapping parton- onto N jet-momenta
- differences in the mapping procedure (jet algorithm) $\sim \mathcal{T}_N^{\rm cut}/Q$ [Jouttenus, Stewart, Tackmann, Waalewijn, '11]
- compute $\mathcal{T}_N$ for each event and select events with $\mathcal{T}_N > \mathcal{T}_N^{\rm cut}$

② Compute singular cross section using SCET factorization formula

(input: 2-loop virtuals)

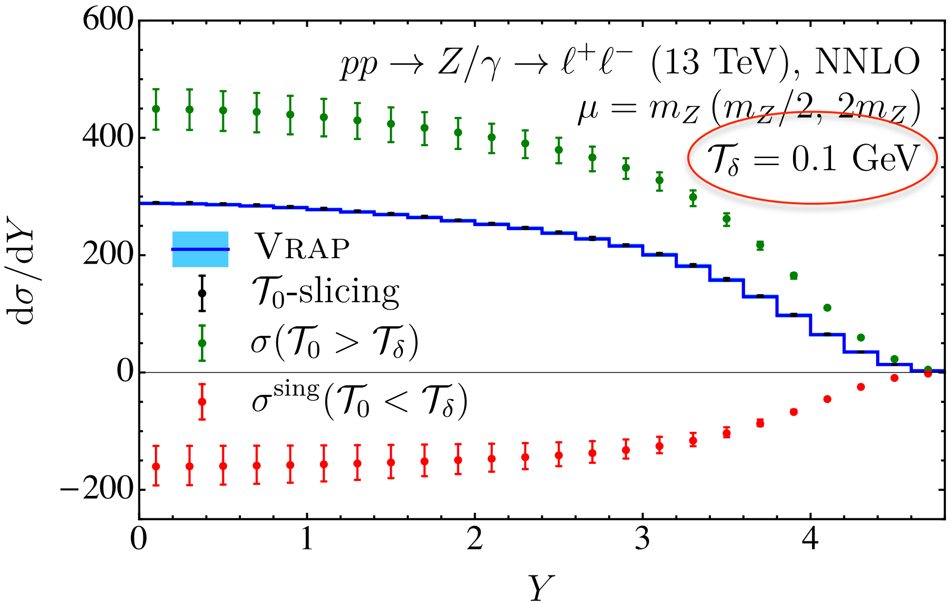
$$\sigma_{\rm NNLO}^{\rm sing}(X) = \int_0^{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma_{\rm NNLO}(X)}{d\mathcal{T}_N} \qquad (2)$$

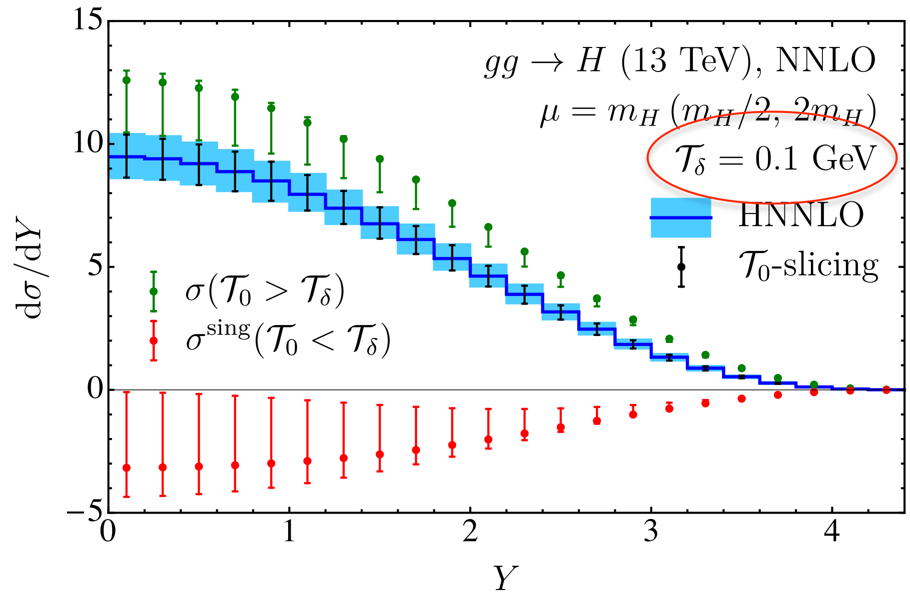
③ Add (1) + (2) and check $\mathcal{T}_N^{\rm cut}$ independence!

Applications/Results

NNLO Z and H rapidity distributions @LHC (as proof of principle):

- used $\mathcal{T}_0$ ("beam thrust") - slicing
- NLO Z/H+j computed with MCFM [Campbell, Ellis, '02, '10]





[Gaunt, MS, Tackmann, Walsh '15]

Good agreement with existing codes VRAP and HNNLO (per-mill level!)

[Anastasiou, Dixon, Melnikov, Petriello '03, '04] [Catani, Grazzini, '07, Grazzini '08]

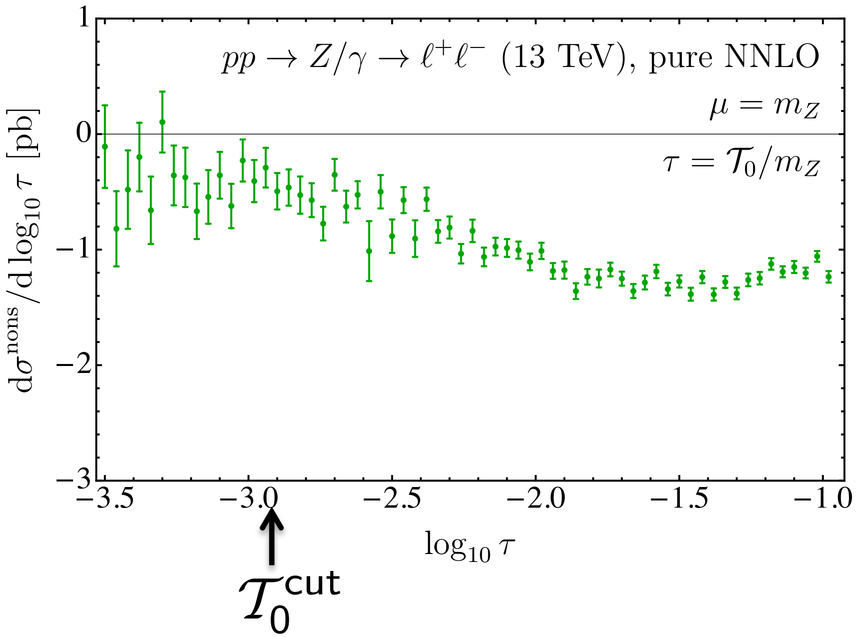
Applications/Results

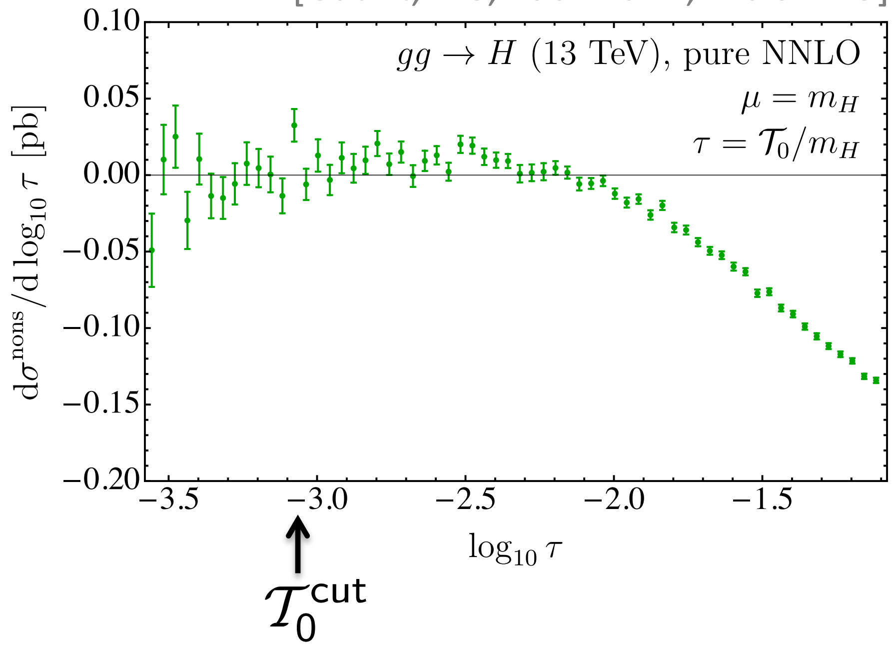
NNLO Z and H rapidity distributions @LHC (as proof of principle):

$$\frac{d\sigma^{\text{nons}}}{d\ln\tau} := \frac{d\sigma_{\text{NNLO}}}{d\ln\tau} - \frac{d\sigma^{\text{sing}}}{d\ln\tau} \sim \alpha_s^2 \tau \ln^3 \tau + \ldots\,, \qquad \tau = \mathcal{T}_N/Q$$

Check $\mathcal{T}_0^{\text{cut}}$ independence:

[Gaunt, MS, Tackmann, Walsh '15]



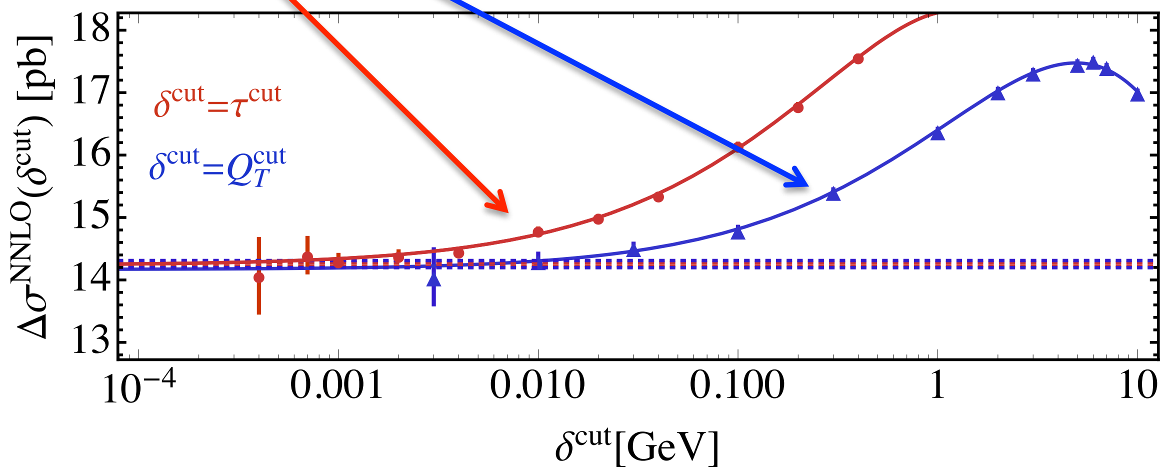


Observe convergence of nonsingular terms toward zero ✔

Applications/Results

Compare 0-jettiness to q_T slicing in pp $\to \gamma\gamma$:

[Campbell, Ellis, Li, Williams, '16]



$$\boxed{\Delta\sigma^{N^nLO}(\tau > \tau^{\rm cut})/\sigma^{LO} \sim \frac{1}{n!}\left(\frac{\alpha_s C_F}{\pi}\right)^n \log^{2n}\frac{\tau^{\rm cut}}{Q} + \ldots}$$

$$\boxed{\Delta\sigma^{N^nLO}(Q_T > Q_T^{\rm cut})/\sigma^{LO} \sim \frac{1}{n!}\left(\frac{2\alpha_s C_F}{\pi}\right)^n \log^{2n}\frac{Q_T^{\rm cut}}{Q} + \ldots}$$

$\Rightarrow$ For $\frac{\tau^{\rm cut}}{Q} \simeq \left(\frac{Q_T^{\rm cut}}{Q}\right)^{\sqrt{2}}$

log cancellations similarly large

→ similar computational effort!

NNLO 0-jet processes computed with N-jettiness slicing, automated in $\mathbb{MCFM}$:

pp $\to$ H, pp $\to$ Z, pp $\to$ W, pp $\to$ HZ, pp $\to$ HW, pp $\to \gamma\gamma$ (+decays)

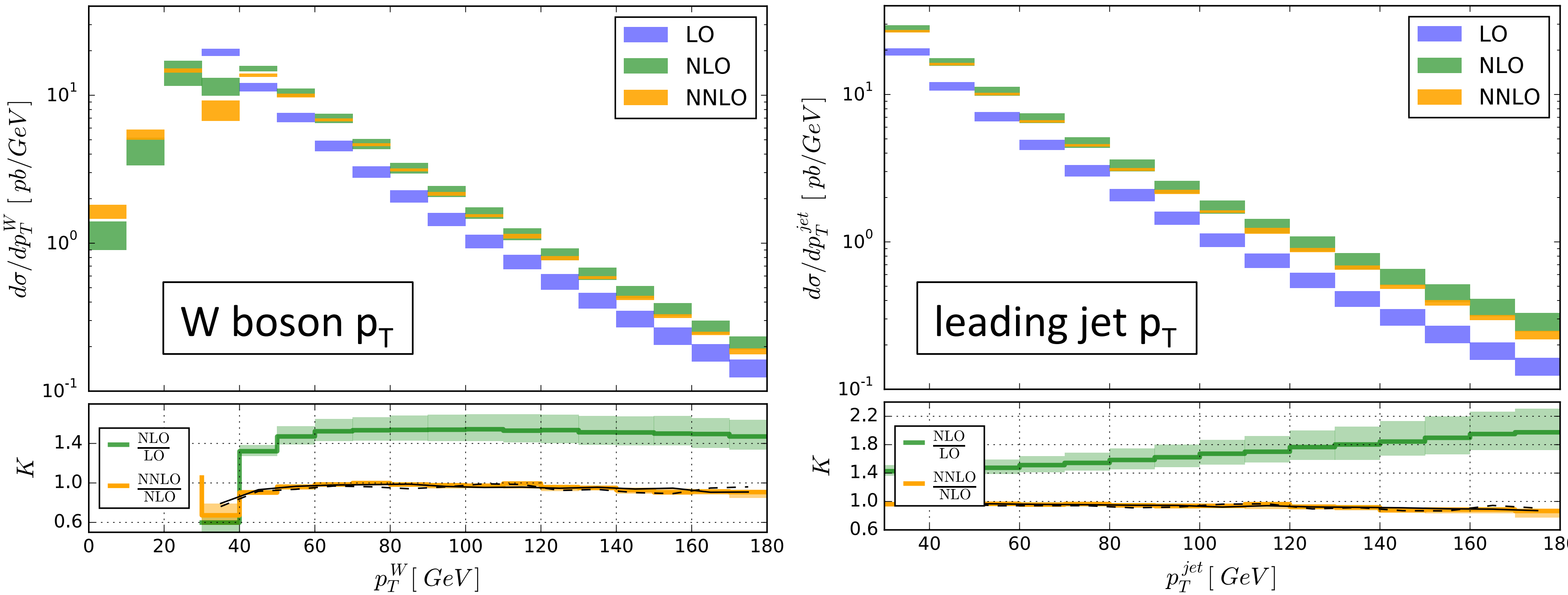
[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams, '16]

Applications/Results

More (impressive!) results for LHC: W+j@NNLO

- used $\mathcal{T}_1$ - slicing
- NLO W+jj computed with MCFM [Campbell, Ellis, '02, '10]

[Boughezal, Focke, Liu, Petriello, '15]



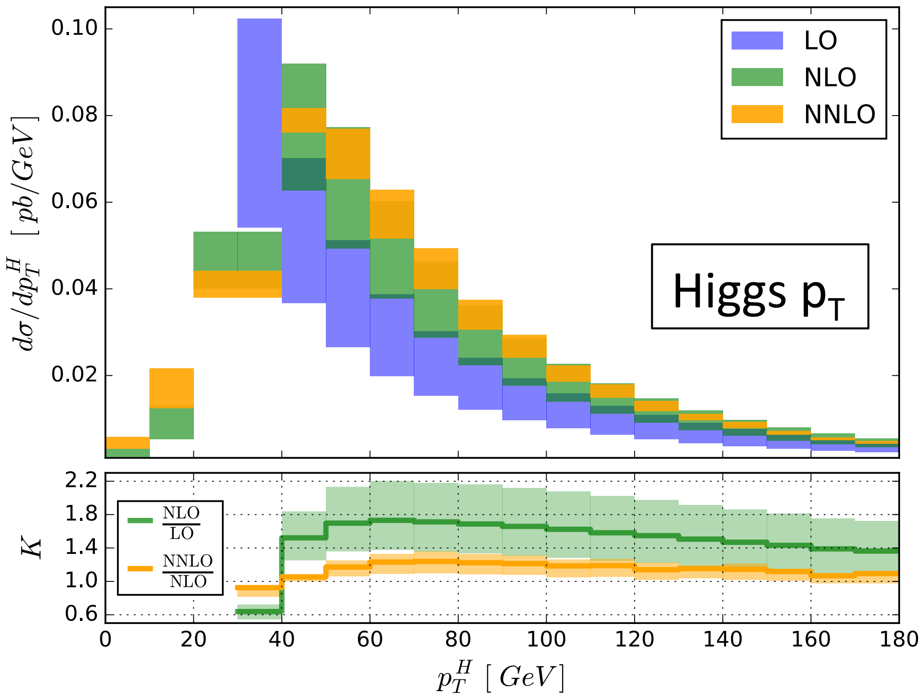
$\mathcal{T}_1^{\text{cut}}$ independence checked in the range $\mathcal{T}_1^{\text{cut}} \sim$ 0.05-0.1 GeV (per-mill level!) ✔

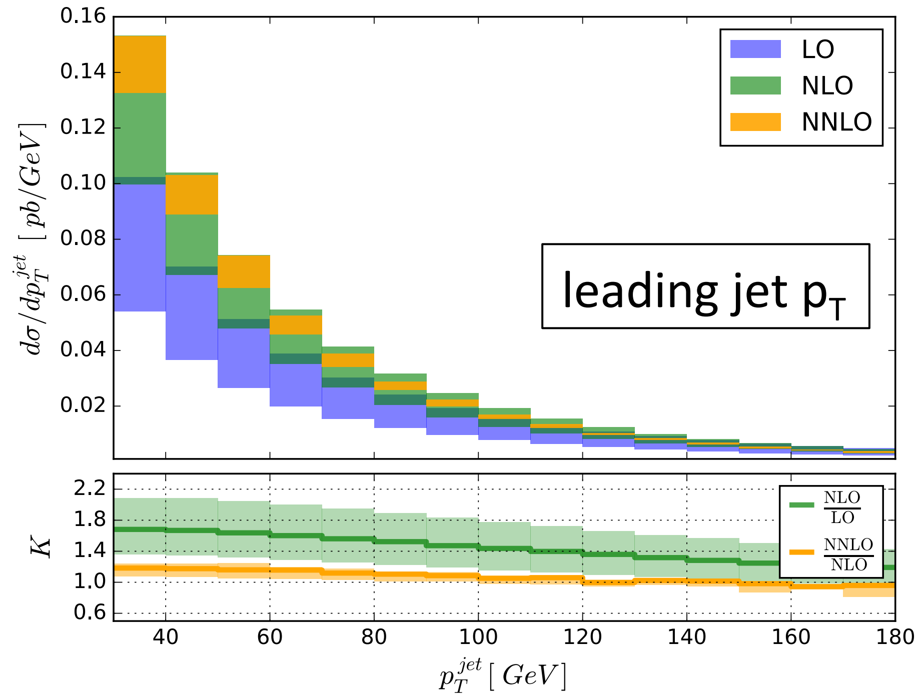
Applications/Results

More (impressive!) results for LHC: H+j@NNLO

- used $\mathcal{T}_1$ - slicing
- NLO H+jj computed with MCFM [Campbell, Ellis, '02, '10]

[Boughezal, Focke, Giele, Liu, Petriello, '15]





$\mathcal{T}_1^{\mathrm{cut}}$ independence checked in the range $\mathcal{T}_1^{\mathrm{cut}} \sim$ 0.05-0.1 GeV (per-mill level!) ✔

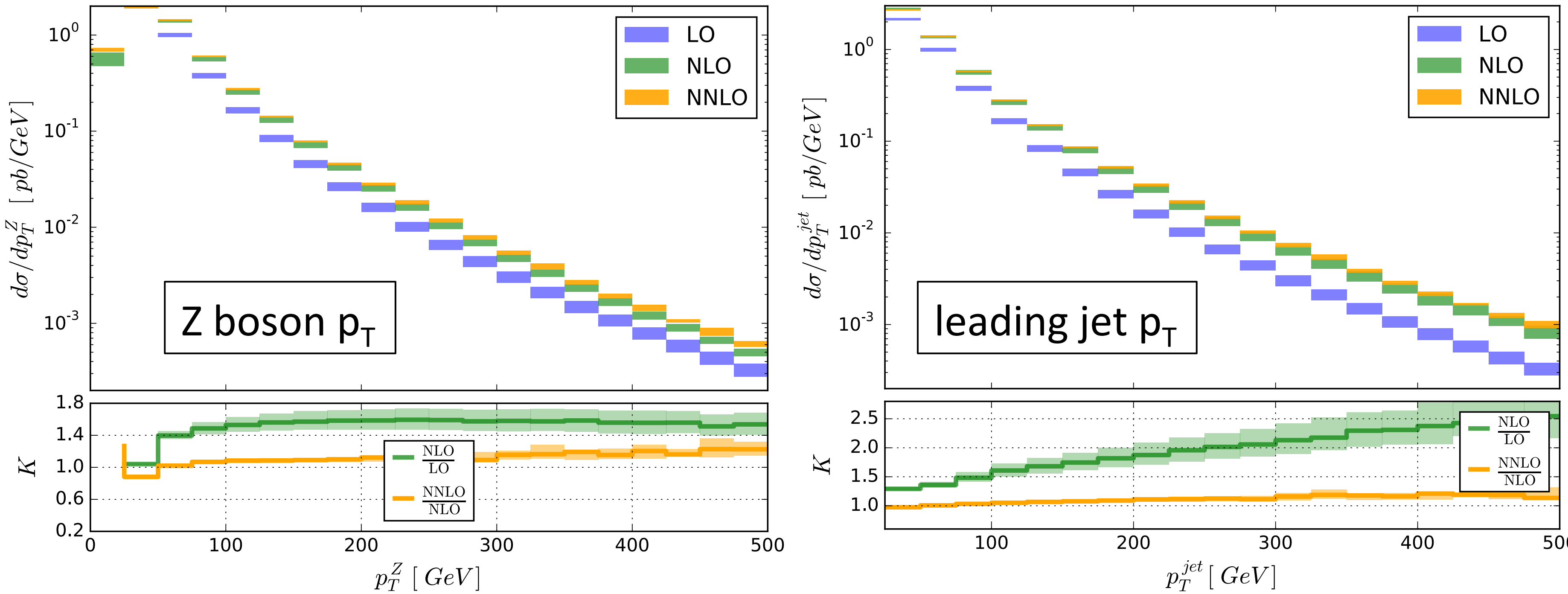
Agreement with results from sector-improved subtractions at per-mill level!

Applications/Results

More (impressive!) results for LHC: Z+j@NNLO

- used $\mathcal{T}_1$ - slicing
- NLO Z+jj computed with MCFM [Campbell, Ellis, '02, '10]

[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, '15]



$\mathcal{T}_1^{\text{cut}}$ independence checked in the range $\mathcal{T}_1^{\text{cut}} \sim$ 0.04-0.1 GeV (per-mill level!) ✔

Possible Extensions

- Single-differential N-jettiness subtraction:

$$\sigma(X) = \int_0^{\mathcal{T}_{\rm off}} d\mathcal{T}_N \frac{d\sigma^{\rm sing}(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_\delta} d\mathcal{T}_N \left[\frac{d\sigma(X)}{d\mathcal{T}_N} - \frac{d\sigma^{\rm sing}(X)}{d\mathcal{T}_N}\theta(\mathcal{T}_N < \mathcal{T}_{\rm off})\right] + \mathcal{O}\left(\frac{\mathcal{T}_\delta}{Q}\right)$$

  - point-by-point subtraction in $\mathcal{T}_N$
  - expect better numerical convergence → $\mathcal{T}_\delta \ll \mathcal{T}_N^{\rm cut}$ feasible (?)
  - requires PS map to disentangle $\mathcal{T}_N$ from rest of PS integration

- Multi-differential N-jettiness subtractions:
  - differential in $\mathcal{T}_N^i$ variable ($\sim$ invariant mass of each jet/beam sector)
  - differential in both, $\mathcal{T}_N$ and q_T [Jain, Procura, Waalewijn, '12] [Gaunt, MS '14] [Procura, Waalewijn, Zeune '15]
  - …

Possible Extensions

- Include sub-leading power corrections:

$$\sigma(X) = \int_0^{\mathcal{T}_{cut}} d\mathcal{T}_N \frac{d\sigma^{sing}(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_{cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \mathcal{O}\left(\frac{\mathcal{T}_{cut}}{Q}\right)$$

  - Could improve accuracy/performance of slicing method

Systematically calculable in SCET

- Include quark masses:
  - if $m_q \sim Q$, treat heavy partons similar to color-neutral final states, but $\longrightarrow$ more complicated soft function [Li, Wang '16]
  - if $m_q \ll Q$, use "massive SCET" [Pietrulewicz, Gritschacher, Hoang, Jemos, Mateu, '14] requires modified definition of $\mathcal{T}_N$!

Summary

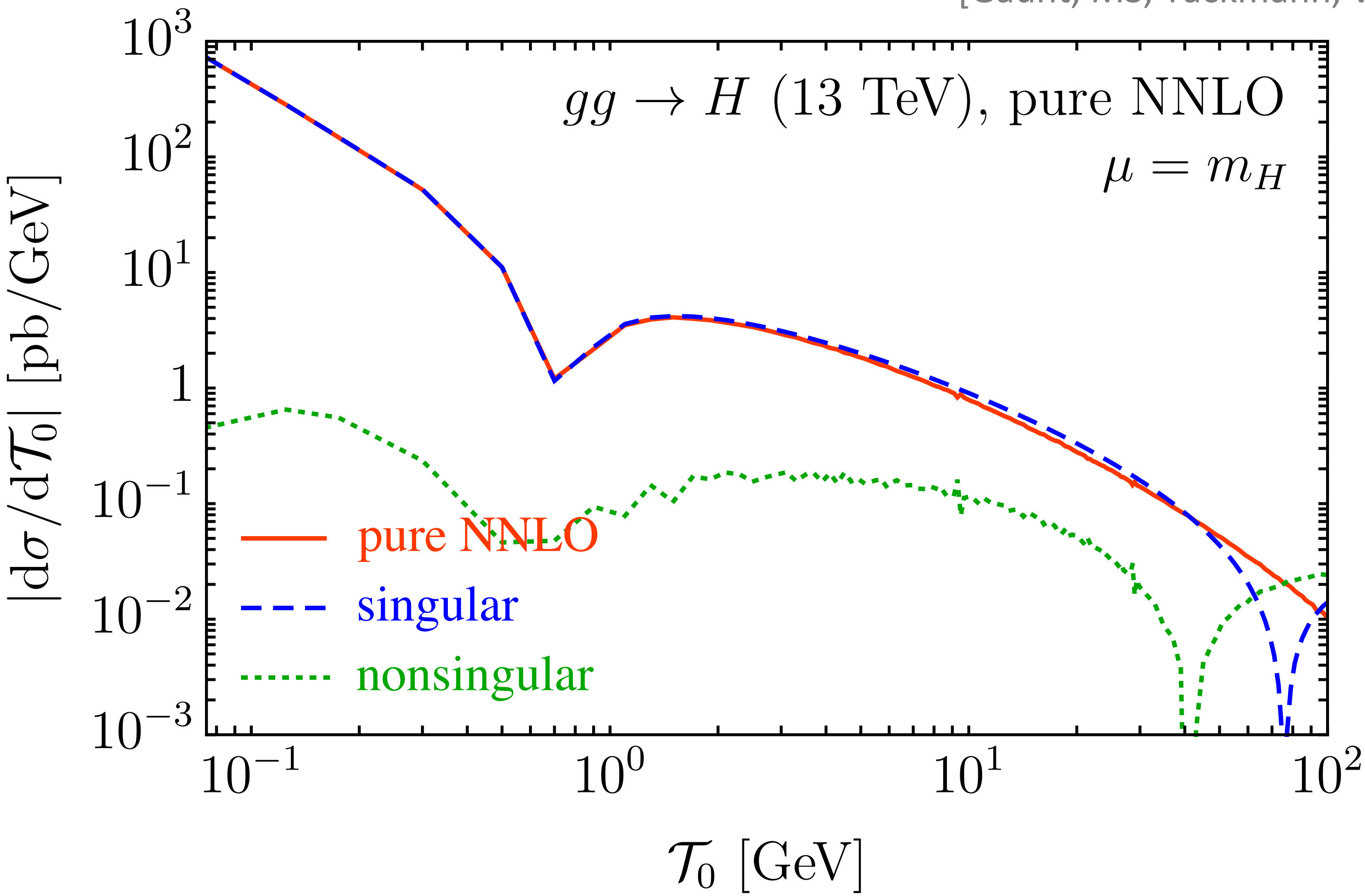
New NNLO subtraction method: N-jettiness slicing

$$\sigma(\mathrm{X}) = \int_0^{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_{\mathrm{N}} \frac{\mathrm{d}\sigma^{\mathrm{sing}}(\mathrm{X})}{\mathrm{d}\mathcal{T}_{\mathrm{N}}} + \int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_{\mathrm{N}} \frac{\mathrm{d}\sigma(\mathrm{X})}{\mathrm{d}\mathcal{T}_{\mathrm{N}}} + \mathcal{O}\left(\frac{\mathcal{T}_{\mathrm{cut}}}{\mathrm{Q}}\right)$$

- $\sigma(\mathcal{T}_{\mathrm{N}} > \mathcal{T}_{\mathrm{cut}})$ from NLO tools ✔
- $\sigma^{\mathrm{sing}}(\mathcal{T}_{\mathrm{N}} < \mathcal{T}_{\mathrm{cut}})$ from SCET, all universal ingredients known ✔
- Impressive NNLO results: $\mathrm{pp} \to \mathrm{W/Z/H} + \mathrm{j}$
- Possible extensions:
  more-differential subtraction, quark masses, power corrections ...

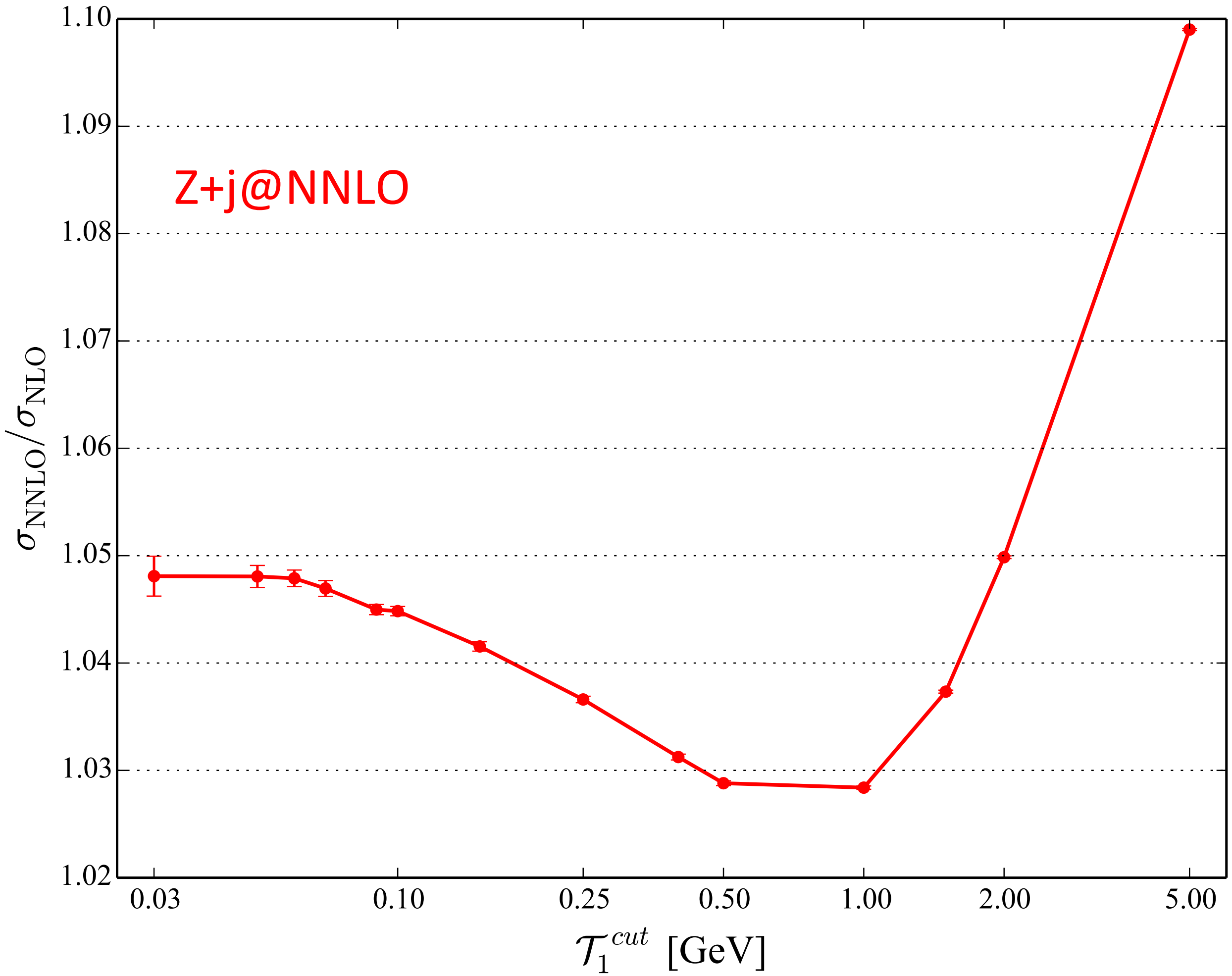
Back-Up

[Gaunt, MS, Tackmann, Walsh '15]



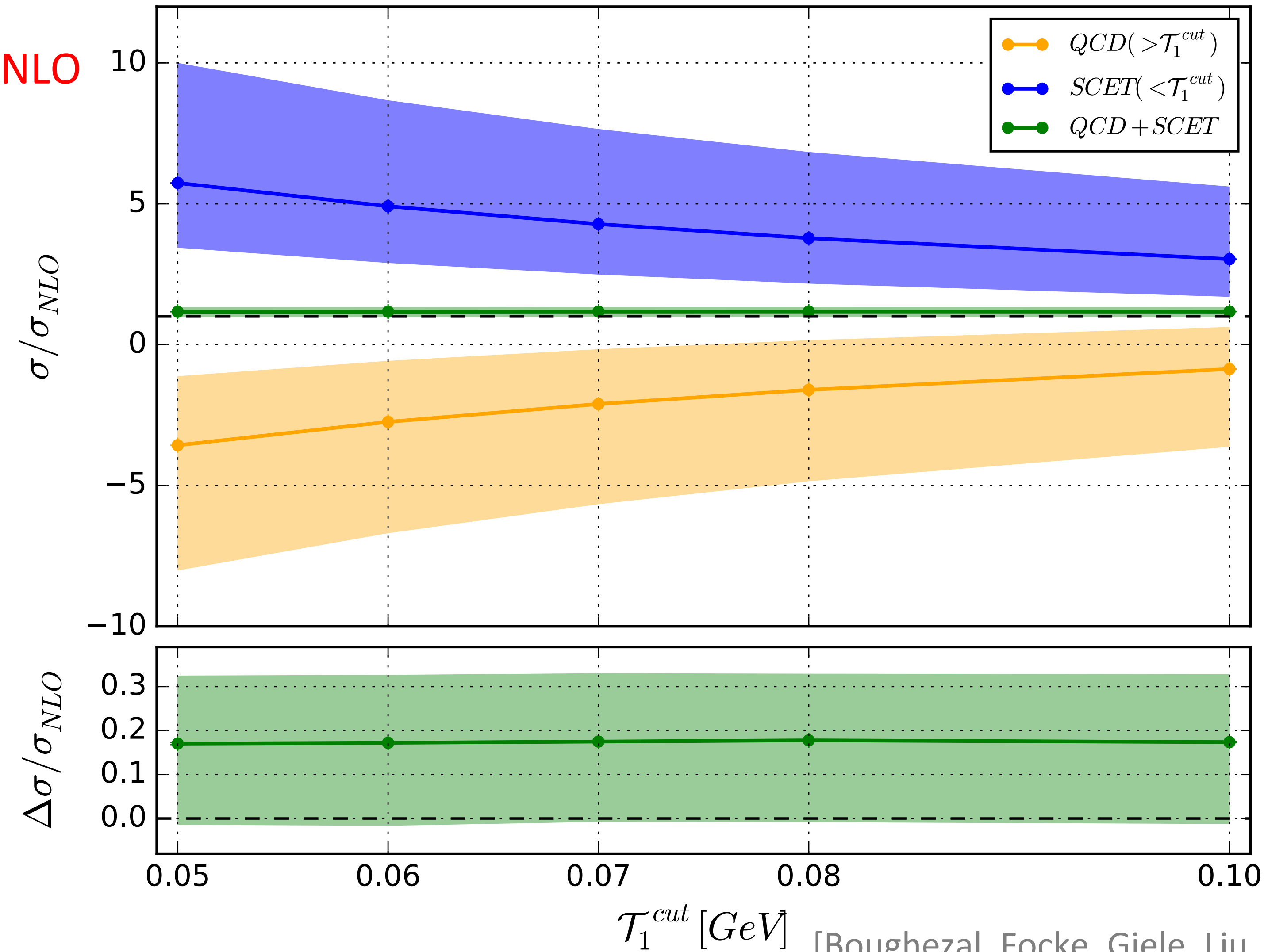
Back-Up

[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, '15]



Back-Up

H+j@NNLO



[Boughezal, Focke, Giele, Liu, Petriello, ‘15]

Back-Up

$$\mathcal{T}_N = \sum_{k=1}^{M} (E_k - |\vec{p}_k|) + \min\Big\{ \min_{j\in 1..M} \{|\vec{p}_j| - |p_j^z|\}, \min_{jk\in 1..M} \{|\vec{p}_j| + |\vec{p}_k| - |\vec{p}_j + \vec{p}_k|\} \Big\}. \quad (3.15)$$

The first term in the overall minimization corresponds to the first case above (extra emission clustered to the beam), whilst the second term corresponds to the second case (extra emission clustered to a jet).

When $M = N + 2$ there are two extra emissions. Now, $N - 2$ axes will always be aligned with $N-2$ of the p_k momenta, and there are four possible cases how the remaining two axes can be chosen based on the remaining four p_k. The appropriate expression for $\mathcal{T}_N$ for $M = N + 2$ is

$$\begin{aligned}
\mathcal{T}_N = \sum_{j=1}^{M} (E_j - |\vec{p}_j|) + \min\Big\{ & \min_{jk\in 1..M} \{|\vec{p}_j| + |\vec{p}_k| - |p_j^z| - |p_k^z|\}, \\
& \min_{jkl\in 1..M} \{|\vec{p}_j| + |\vec{p}_k| + |\vec{p}_l| - |\vec{p}_j + \vec{p}_k| - |p_l^z|\}, \\
& \min_{jkl\in 1..M} \{|\vec{p}_j| + |\vec{p}_k| + |\vec{p}_l| - |\vec{p}_j + \vec{p}_k + \vec{p}_l|\}, \\
& \min_{jklm\in 1..M} \{|\vec{p}_j| + |\vec{p}_k| + |\vec{p}_l| + |\vec{p}_m| - |\vec{p}_j + \vec{p}_k| - |\vec{p}_l + \vec{p}_m|\} \Big\}.
\end{aligned} \quad (3.16)$$

The first term in the overall minimization corresponds to both extra particles being clustered to a beam direction. The second term corresponds to one particle being clustered to a beam, and two particles being clustered together in a jet. The third term corresponds to clustering three particles together in a jet, and the final term corresponds to clustering two sets of two particles into two separate jets. In all cases the remaining jet directions are set by the remaining unclustered p_k momenta.

[Gaunt, MS, Tackmann, Walsh ‘15]

Back-Up

[Gaunt, MS, Tackmann, Walsh ‘15]