ON-SHELL CONDITIONS IN THEORIES WITH FLAVOR MIXING

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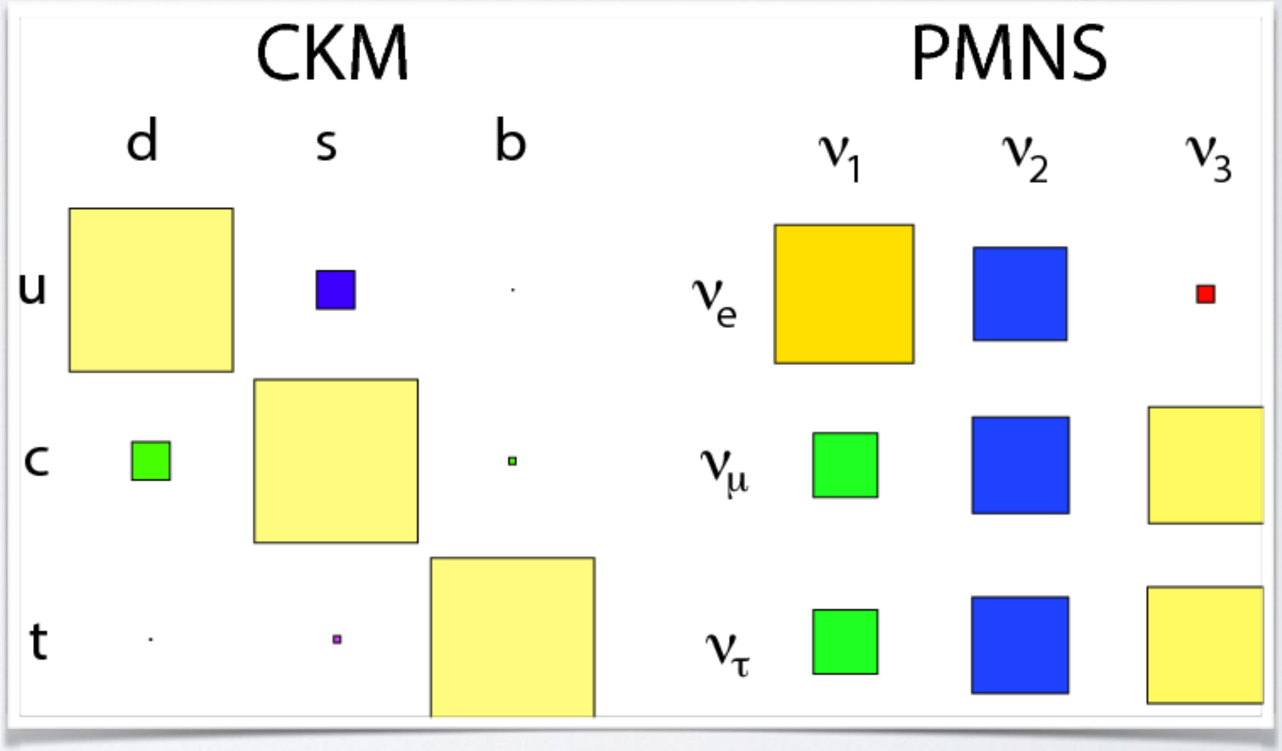
$\int dk \, \Pi$
Doktoratskolleg
Particles and Interactions

FWF
Der Wissenschaftsfonds.

INTRODUCTION

- On-shell conditions in theories with flavor mixing are already an **integral part of the Standard Model**

- In extensions of the SM, mixing for fermions and scalars are likely to occur
    - Want a proper foundation for the definition of these conditions



http://arxiv.org/abs/arXiv:1212.6374

http://media-cache-ak0.pinimg.com/736x/59/20/8c/59208cade2031154197d163b281a83d5.jpg

http://www.kitcheninnovationsinc.com/wp-content/uploads/2014/07/J218DISP-Ice-Cream.png

INTRODUCTION

- On-shell conditions already derived by
  Aoki et. al., Progr. Theor. Phys., No. 73 (1982)

- Renormalization of the quark mixing matrix by
  Denner & Sack, Nucl. Phys. B347 (1990)

  - **one-loop effects of the quark mixing matrix are practically negligible**

TABLE 2
Comparison of different approximations for the W-decay width (in GeV) with a top-quark mass $m_t = 150$ GeV and the corresponding W-mass $M_W = 80.199$ GeV

	Born decay width	Zero fermion masses	Degenerate quark masses	Exact result	Constituent quark masses
$\Gamma(W \to ud)$	0.644297	0.666486	0.666497	0.666497	0.666496
$\Gamma(W \to us)$	0.327699×10^{-1}	0.338985×10^{-1}	0.338992×10^{-1}	0.338992×10^{-1}	0.338989×10^{-1}
$\Gamma(W \to ub)$	0.330212×10^{-4}	0.341585×10^{-4}	0.342405×10^{-4}	0.342403×10^{-4}	0.342403×10^{-4}
$\Gamma(W \to cd)$	0.327787×10^{-1}	0.339076×10^{-1}	0.339201×10^{-1}	0.339201×10^{-1}	0.339201×10^{-1}
$\Gamma(W \to cs)$	0.642531	0.664660	0.664909	0.664909	0.664905
$\Gamma(W \to cb)$	0.142516×10^{-2}	0.147424×10^{-2}	0.147830×10^{-2}	0.147830×10^{-2}	0.147830×10^{-2}
$\Gamma(W \to \text{hadrons})$	$0.135384 \times 10^{+1}$	$0.140046 \times 10^{+1}$	$0.140074 \times 10^{+1}$	$0.140074 \times 10^{+1}$	$0.140073 \times 10^{+1}$
$\Gamma(W \to \text{leptons})$	0.676933	0.674700	0.674715	0.674715	0.674717
$\Gamma(W \to \text{all})$	$0.203077 \times 10^{+1}$	$0.207516 \times 10^{+1}$	$0.207545 \times 10^{+1}$	$0.207545 \times 10^{+1}$	$0.207545 \times 10^{+1}$

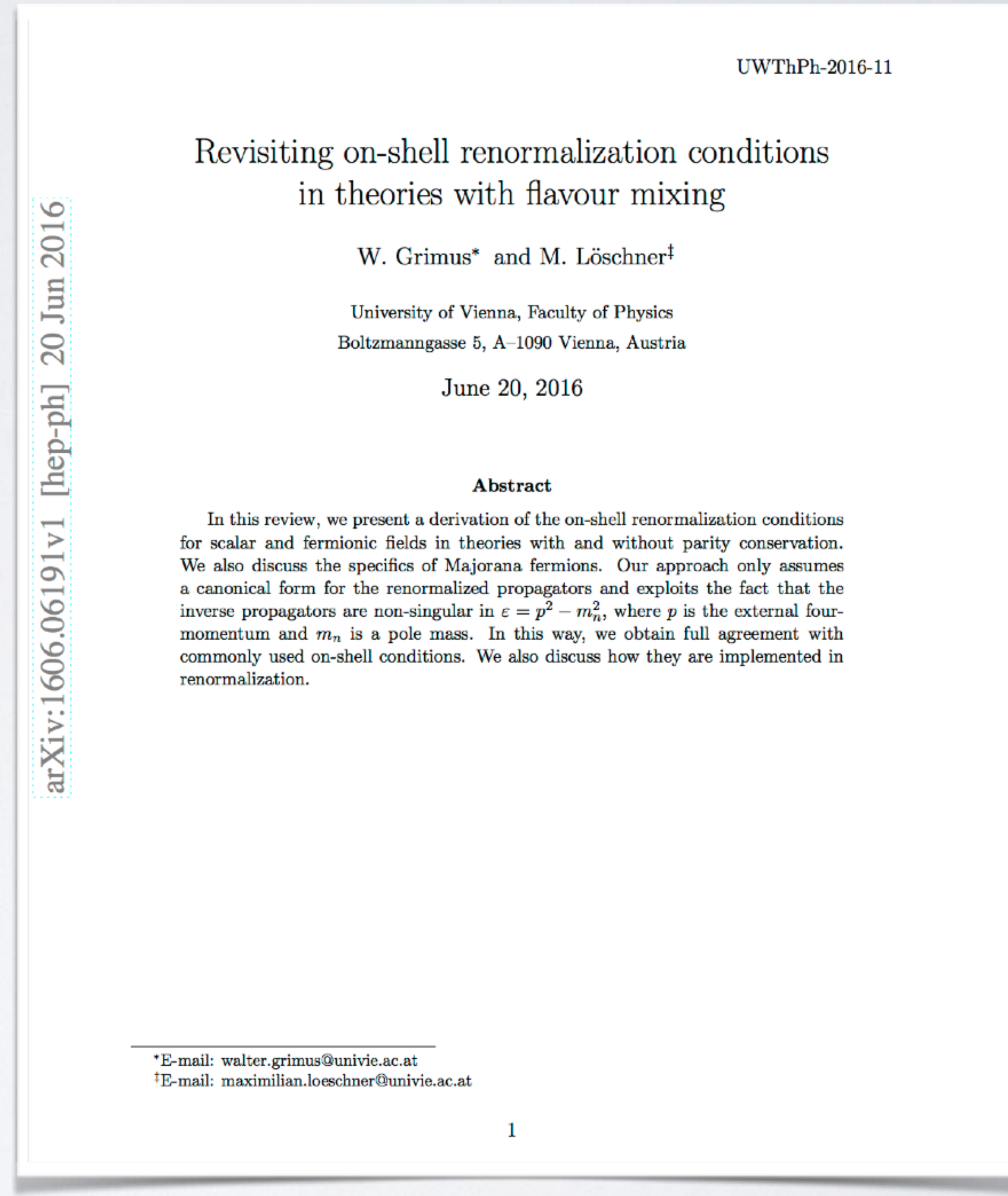
δV_{CKM}

INTRODUCTION

- Still, the derivation of on-shell conditions in theories with mixing remained a bit vague for the general reader of the relevant literature

- An interesting theoretical problem in itself

  - **Review on the derivation** and use of on-shell conditions in theories with flavor mixing



arXiv:1606.06191v1 [hep-ph] 20 Jun 2016

UWThPh-2016-11

Revisiting on-shell renormalization conditions in theories with flavour mixing

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Abstract

In this review, we present a derivation of the on-shell renormalization conditions for scalar and fermionic fields in theories with and without parity conservation. We also discuss the specifics of Majorana fermions. Our approach only assumes a canonical form for the renormalized propagators and exploits the fact that the inverse propagators are non-singular in $\varepsilon = p^2 - m_n^2$, where p is the external four-momentum and m_n is a pole mass. In this way, we obtain full agreement with commonly used on-shell conditions. We also discuss how they are implemented in renormalization.

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[Int. J. Mod. Phys. A 31, 1630038 (2016)]

PREREQUISITES

- All masses are different

- Conditions only usable in regions where absorptive parts are negligible, otherwise only use dispersive part:

$$\frac{1}{p^2 - \mu^2 + i\epsilon} = \mathrm{P}\frac{1}{p^2 - \mu^2} - i\pi\delta(p^2 - \mu^2)$$

i.e. decompose propagator via principle value and delta function (origin: **Sokhotski-Plemelj theorem** for real line)

- Corresponds to commonly used definitions renormalization conditions only using the **real part of the self-energies**

- Hermitian counterterms in Lagrangian (alternatively e.g. *complex-mass scheme*)

REAL SCALAR PROPAGATOR

- Commonly used on-shell condition for real scalar propagator:

$$\Delta(p^2)\big|_{p^2\to m^2} = \frac{1}{p^2 - m^2}, \quad m = m_{\text{phys}}$$

- Inspires the form of the condition in multi-particle case

$$\Delta_{ij}(p^2)\big|_{p^2\to m_n^2} = \frac{\delta_{in}\delta_{nj}}{p^2 - m_n^2} + \Delta_{ij}^{(0)} + \mathcal{O}(p^2 - m_n^2), \quad \epsilon_n \equiv p^2 - m_n^2$$

  - On-shell condition for propagator

$$\Delta(p^2)\big|_{\epsilon\to 0} = \begin{pmatrix} \mathcal{O}(1) & & \dots & & \mathcal{O}(1) \\ & \ddots & & & \\ \vdots & & \frac{1}{\epsilon_n} + \mathcal{O}(1) & & \vdots \\ & & & \ddots & \\ \mathcal{O}(1) & & \dots & & \mathcal{O}(1) \end{pmatrix}$$

REAL SCALAR PROPAGATOR

- Problem: need conditions for the **inverse propagator**

- Reason: conditions should apply to the renormalized self energy (i.e. the counterterms therein)

$$\int \mathrm{d}(x-y)\, \langle\Omega|T\psi(x)\bar{\psi}(y)|\Omega\rangle e^{\mathrm{i}p\cdot(x-y)} =$$

$$+ \dots$$

$$= \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\left(-\mathrm{i}\Sigma(p^2)\right)\frac{\mathrm{i}}{p^2-m^2} + \dots$$

$$= \frac{\mathrm{i}}{p^2-m^2}\left(1 + \frac{\Sigma(p^2)}{p^2-m^2} + \left(\frac{\Sigma(p^2)}{p^2-m^2}\right)^2 + \dots\right)$$

$$= \frac{\mathrm{i}}{p^2-m^2-\Sigma(p^2)}.$$

  - Self-energy appears in the denominator of the two-point correlation function

REAL SCALAR PROPAGATOR

- Simple to translate on-shell conditions to the self-energy in the case without mixing:

$$\Delta(p^2) = \frac{1}{p^2 - m^2 - \Sigma(p^2)}$$

$$\Rightarrow \Sigma(p^2)\big|_{p^2=m^2} = 0, \qquad \frac{\mathrm{d}}{\mathrm{d}p^2}\Sigma(p^2)\bigg|_{p^2=m^2} = 0.$$

- In order to define similar conditions in the multi-particle case:

$$\left(\Delta^{-1}\right)_{ij} =: A_{ij} = A_{ij}^{(0)} + \epsilon_n A_{ij}^{(1)} + \mathcal{O}(\epsilon_n^2), \quad \epsilon_n = p^2 - m_n^2$$

and use the inversion condition to the propagator:

$$\Delta_{ik} A_{kj} = A_{ik} \Delta_{kj} = \delta_{ij}$$

REAL SCALAR PROPAGATOR

- Yields **conditions for the inverse propagator**:

$$A_{in}^{(0)} = 0 \ \forall \, i = 1, \ldots, N \quad \text{and} \quad A_{nj}^{(0)} = 0 \ \forall \, j = 1, \ldots, N.$$

  and moreover for the entries on the diagonal:

$$A_{nn}^{(1)} = 1 \ \forall \, n$$

- Note that one can get even more conditions from the orthogonality, but these have nothing to do with the singularity structure ➤ not part of on-shell conditions

- Due to the choice of the inverse propagator, can equivalently use:

$$A_{in}(m_n^2) = A_{nj}(m_n^2) = 0 \ \forall \, i, j = 1, \ldots, N \quad \text{and} \quad \left. \frac{\mathrm{d}A_{nn}(p^2)}{\mathrm{d}p^2} \right|_{p^2 = m_n^2} = 1,$$

NUMBER OF CONDITIONS

- Rows and columns in principle get independent conditions:

$$A_{in}(m_n^2) = A_{nj}(m_n^2) = 0 \ \forall\, i,j = 1,\ldots,N \quad \text{and} \quad \left.\frac{\mathrm{d}A_{nn}(p^2)}{\mathrm{d}p^2}\right|_{p^2=m_n^2} = 1,$$

$$\begin{pmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & A_{ij} & \vdots \\ A_{N1} & \cdots & A_{NN} \end{pmatrix}$$

➡ total number of conditions: $2N^2 + N$

NUMBER OF CONDITIONS

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➡ total number of conditions: $2N^2 + N$

PARAMETER COUNTING

independent conditions vs. # counterterms

- First note that the propagator, as well as its inverse, are symmetric

$$\left(\Delta^{-1}(p^2)\right)^T_{\text{disp}} = \Delta^{-1}(p^2)_{\text{disp}}$$

  - Number of independent conditions reduced to $N^2 + N$

$$i \neq j: \; A_{ij}(m_j^2) = 0, \quad i = j: \; A_{ii}(m_i^2) = 0, \; \left.\frac{\mathrm{d}A_{ii}(p^2)}{\mathrm{d}p^2}\right|_{p^2=m_i^2} = 1.$$

  (note that this way of counting makes more sense for the fermions)

- Field strength renormalization constants form a general real matrix
  - $Z^{1/2}: \; N^2$ degrees of freedom
- Mass counterterms using a diagonal mass matrix:
  - $\delta\hat{m}: \; N$ degrees of freedom

    ➡ #renormalization condition coincides with #counterterms

PROPAGATOR SYMMETRY

$$\left(\Delta^{-1}(p^2)\right)^T_{\text{disp}} = \Delta^{-1}(p^2)_{\text{disp}}$$

- Use the Källén-Lehmann representation of the renormalized propagator to show that it is real and symmetric: (origin: $i\,(\Delta(x-y))_{ij} = \langle 0|\mathrm{T}\varphi_i(x)\varphi_j(y)|0\rangle$)

$$\Delta_{ij}(p^2) = \int_0^\infty \mathrm{d}\mu^2 \rho_{ij}(\mu^2)\frac{1}{p^2-\mu^2+i\epsilon}.$$

$$\rho_{ij}(q^2)\Theta(q^0) \equiv (2\pi)^3 \sum_n \delta^{(4)}(q-p_n)\langle 0|\varphi_i(0)|n\rangle\langle n|\varphi_j(0)|0\rangle$$

- Next invoke CPT invariance, which holds in any local, Lorentz-invariant theory:

$$\langle(\mathcal{CPT})x|(\mathcal{CPT})y\rangle = \langle x|y\rangle^* = \langle y|x\rangle$$
$$(\mathcal{CPT})\varphi_i(x)(\mathcal{CPT})^{-1} = \varphi_i(-x)$$

$$\begin{aligned}\Rightarrow \langle 0|\varphi_i(0)|n\rangle &= \langle(\mathcal{CPT})0|\,(\mathcal{CPT})\varphi_i(0)(\mathcal{CPT})^{-1}\,|(\mathcal{CPT})n\rangle^* \\ &= \langle 0|\,\varphi_i(0)\,|(\mathcal{CPT})n\rangle^* \\ &= \langle 0|\,\varphi_i(0)\,|n'\rangle^*\end{aligned}$$

PROPAGATOR SYMMETRY

- Inserting this into the spectral density, we find:

$$\begin{aligned}\rho_{ij}(q^2)\Theta(q^0) &\equiv (2\pi)^3 \sum_n \delta^{(4)}(q-p_n)\langle 0|\varphi_i(0)|n\rangle\langle n|\varphi_j(0)|0\rangle \\ &= (2\pi)^3 \sum_{n'} \delta^{(4)}(q-p_n)\langle 0|\,\varphi_i(0)\,|n'\rangle^*\langle n'|\varphi_j(0)|0\rangle^* \\ &= \left(\rho_{ij}(q^2)\right)^* = \rho_{ji}(q^2)\end{aligned}$$

- With the spectral density being real and symmetric, we see that the same holds for the propagator (also the inverse):

$$\Delta_{ij}(p^2) = \Delta^*_{ij}(p^2) = \Delta_{ji}(p^2) \qquad \blacksquare$$

CONDITIONS FOR FERMIONS

- Choose condition for propagator similar to scalar case:

$$S_{ij} \overset{\varepsilon_n \to 0}{\longrightarrow} \frac{\delta_{in}\delta_{nj}}{\not{p} - m_n} + \tilde{S}_{ij}, \quad \varepsilon_n = p^2 - m_n^2$$

- Ansatz for propagator:

$$S_{ij}(p) = C_{ij}(p^2)\not{p} - D_{ij}(p^2)$$

- Find proper choice for expansion via:

$$\begin{aligned}\left(\not{p} - m_n\right) S_{ij} &= \delta_{in}\delta_{nj} + \left(\not{p} - m_n\right) \tilde{S}_{ij} \\ &= \varepsilon_n C_{ij} - \left(\not{p} - m_n\right) \left(D_{ij} + m_n C_{ij}\right)\end{aligned}$$

➡ Leads to general form of the propagator:

$$\begin{aligned}\Rightarrow C_{ij} &= \frac{\delta_{in}\delta_{nj}}{\varepsilon} + C_{ij}^{(0)} + \mathcal{O}(\varepsilon), \\ D_{ij} &= -\frac{m_n\delta_{in}\delta_{nj}}{\varepsilon} + D_{ij}^{(0)} + \mathcal{O}(\varepsilon).\end{aligned}$$

CONDITIONS FOR FERMIONS

- Choice for the inverse propagator: $(S^{-1})_{ij}(p) = A_{ij}(p^2)\not{p} - B_{ij}(p^2)$

- Choice for expansion of inverse propagator non-singular again:

$$A_{ij} = A_{ij}^{(0)} + \varepsilon_n A_{ij}^{(1)} + \mathcal{O}(\varepsilon_n^2), \quad B_{ij} = B_{ij}^{(0)} + \varepsilon_n B_{ij}^{(1)} + \mathcal{O}(\varepsilon_n^2)$$

- Use inversion relation to find:

$$\left(SS^{-1}\right)_{ij} = \delta_{ij} \Rightarrow \quad C_{ik}A_{kj}\,p^2 + D_{ik}B_{kj} = \delta_{ij}, \quad C_{ik}B_{kj} + D_{ik}A_{kj} = 0,$$
$$\left(S^{-1}S\right)_{ij} = \delta_{ij} \Rightarrow \quad A_{ik}C_{kj}\,p^2 + B_{ik}D_{kj} = \delta_{ij}, \quad B_{ik}C_{kj} + A_{ik}D_{kj} = 0.$$

- Inserting expansions for prop. and inverse prop. yields final conditions:

$$B_{in}(m_n^2) = m_n A_{in}(m_n^2) \ \forall\, i = 1, \dots, N;$$
$$B_{nj}(m_n^2) = m_n A_{nj}(m_n^2) \ \forall\, j = 1, \dots, N;$$
$$A_{nn}(m_n^2) + 2m_n^2 \left.\frac{\mathrm{d}A_{nn}(p^2)}{\mathrm{d}p^2}\right|_{p^2=m_n^2} - 2m_n \left.\frac{\mathrm{d}B_{nn}(p^2)}{\mathrm{d}p^2}\right|_{p^2=m_n^2} = 1.$$

INDEPENDENT CONDITIONS II

- Symmetry relation for the fermionic propagator:

$$\gamma_0 \left(S^{-1}(p)\right)^\dagger_{\text{disp}} \gamma_0 = S^{-1}(p)_{\text{disp}} \qquad \Rightarrow A^\dagger = A, \quad B^\dagger = B$$

  - Can again be derived from Källén-Lehmann representation

- Using this reduces the #independent renormalization conditions again:

$$i \neq j: B_{ij}(m_j^2) = m_j A_{ij}(m_j^2),$$
$$i = j: B_{ii}(m_i^2) = m_i A_{ii}(m_i^2),$$
$$A_{ii}(m_i^2) + 2m_i^2 \left.\frac{\mathrm{d}A_{ii}(p^2)}{\mathrm{d}p^2}\right|_{p^2=m_i^2} - 2m_i \left.\frac{\mathrm{d}B_{ii}(p^2)}{\mathrm{d}p^2}\right|_{p^2=m_i^2} = 1$$

- First relation is **complex** and **not symmetric in i,j**
  - **#independent conditions:** $2\left(N^2 - N\right) + 2N = 2N^2$

PARAMETER COUNTING II

- Renormalized self-energy can be written as:

$$\Sigma^{(r)}(p) = \Sigma(p) + \left(\mathbb{1} - \left(Z^{(1/2)}\right)^{\dagger} Z^{(1/2)}\right) \not{p} + \left(Z^{(1/2)}\right)^{\dagger} (\hat{m} + \delta\hat{m})\, Z^{(1/2)} - \hat{m}$$

  - Phase freedom in the complex field strength renormalization constants reduces #parameters to fulfill renorm. conditions:

$$Z^{(1/2)} \to e^{i\hat{\alpha}} Z^{(1/2)}, \quad e^{i\hat{\alpha}} = \mathrm{diag}(e^{i\alpha_1}, \ldots, e^{i\alpha_N})$$

- Then, #independent parameters is:

$$2N^2 - N + N = 2N^2$$

  - Coincides again with the #independent conditions

ALTERNATIVE FORMAL DERIVATION

- Can alternatively define: $\left(S^{-1}\right)_{ij}(\not{p}) =: T_{ij}(\not{p}) = A_{ij}(p^2)\not{p} - B_{ij}(p^2)$
- Note that $\dfrac{\mathrm{d}p^2}{\mathrm{d}\not{p}} = 2\not{p}$
- Then, one arrives at completely equivalent conditions:

$$T_{in}(\not{p} = m_n) = T_{nj}(\not{p} = m_n) = 0 \ \forall i, j$$
$$\left.\frac{\mathrm{d}T_{nn}(\not{p})}{\mathrm{d}\not{p}}\right|_{\not{p}=m_n} = 1.$$

FERMIONS W/O P-CONSERVATION

- Choices for propagator and inverse prop. similar to P-conserving case:

$$S = \not{p}\,(C_L\gamma_L + C_R\gamma_R) - (D_L\gamma_L + D_R\gamma_R)$$

$$S^{-1} = \not{p}\,(A_L\gamma_L + A_R\gamma_R) - (B_L\gamma_L + B_R\gamma_R)$$

- Expansions of propagator constituents works just as before

- Inversion relation looks similar too, but:
  left- and right-chiral parts mix:

$$\begin{aligned}
\left(SS^{-1}\right)_{ij} = \delta_{ij} \Rightarrow \quad & \left(C_R A_L\, p^2 + D_L B_L\right)_{ij} = \delta_{ij}, \quad (C_L B_L + D_R A_L)_{ij} = 0,\\
& \left(C_L A_R\, p^2 + D_R B_R\right)_{ij} = \delta_{ij}, \quad (C_R B_R + D_L A_R)_{ij} = 0,\\
\left(S^{-1}S\right)_{ij} = \delta_{ij} \Rightarrow \quad & \left(A_R C_L\, p^2 + B_L D_L\right)_{ij} = \delta_{ij}, \quad (B_R C_L + A_L D_L)_{ij} = 0,\\
& \left(A_L C_R\, p^2 + B_R D_R\right)_{ij} = \delta_{ij}, \quad (B_L C_R + A_R D_R)_{ij} = 0.
\end{aligned}$$

FERMIONS W/O CP-CONSERVATION

- Final conditions after invoking propagator symmetry:

$$(B_L)_{ij}(m_j^2) = m_j (A_R)_{ij}(m_j^2),$$
$$(B_R)_{ij}(m_j^2) = m_j (A_L)_{ij}(m_j^2),$$
$$(A_{L/R})_{ii}(m_i^2) + m_i^2 \, \frac{\mathrm{d}}{\mathrm{d}p^2} \left((A_L)_{ii}(p^2) + (A_R)_{ii}(p^2)\right)\Big|_{p^2=m_i^2}$$
$$-m_i \, \frac{\mathrm{d}}{\mathrm{d}p^2} \left((B_L)_{ii}(p^2) + (B_R)_{ii}(p^2)\right)\Big|_{p^2=m_i^2} = 1.$$

- Counting more subtle here. Know from propagator symmetry:

$$A_L^\dagger = A_L, \quad A_R^\dagger = A_R, \quad B_L^\dagger = B_R.$$
$$\Rightarrow \mathrm{Im}\,(A_L)_{ii} = \mathrm{Im}\,(A_R)_{ii} = 0, \quad (B_R)_{ii}(m_i^2) = \left((B_L)_{ii}(m_i^2)\right)^*$$

- Inserting this into conditions leaves as independent ones (for i=j):

$$(A_L)_{ii}(m_i^2) = (A_R)_{ii}(m_i^2), \quad \mathrm{Im}\,(B_L)_{ii}(m_i^2) = 0, \quad \mathrm{Re}\,(B_L)_{ii}(m_i^2) = m_i (A_R)_{ii}(m_i^2)$$

  - #independent conditions: $4\left(N^2 - N\right) + 3N + N = 4N^2$

PARAMETER COUNTING III

- Self-energy for fermions w/o CP-conservation:

$$\Sigma^{(r)}(p) = \quad \Sigma(p) + \left(\mathbb{1} - \left(Z_L^{(1/2)}\right)^\dagger Z_L^{(1/2)}\right) \not{p}\gamma_L + \left(\mathbb{1} - \left(Z_R^{(1/2)}\right)^\dagger Z_R^{(1/2)}\right) \not{p}\gamma_R$$
$$+ \left(Z_R^{(1/2)}\right)^\dagger (\hat{m} + \delta\hat{m})\, Z_L^{(1/2)}\gamma_L + \left(Z_L^{(1/2)}\right)^\dagger (\hat{m} + \delta\hat{m})\, Z_R^{(1/2)}\gamma_R - \hat{m}$$

  - Again diagonal phase freedom, but the same for L/R:

$$Z_L^{(1/2)} \to e^{i\hat{\alpha}} Z_L^{(1/2)}, \quad Z_R^{(1/2)} \to e^{i\hat{\alpha}} Z_R^{(1/2)}, \quad e^{i\hat{\alpha}} = \mathrm{diag}(e^{i\alpha_1}, \ldots, e^{i\alpha_N})$$

  - Of course, #free parameters coincides with #conditions:

$$(4N^2 - N) + N = 4N^2$$

MAJORANAS

- Majorana fields are equal to their charge conjugate:

$$\psi_n(x) = C\gamma_0^T \psi_n^*(x)$$

$$\Rightarrow \psi_j^T = -\bar{\psi}_j C \text{ and } \psi_i = C\bar{\psi}_i^T$$

- Use these equalities in the identity

$$\langle 0|\mathrm{T}\psi_{ia}(x)\psi_{jb}(y)|0\rangle = -\langle 0|\mathrm{T}\psi_{jb}(y)\psi_{ia}(x)|0\rangle$$

- In the end this yields an additional propagator symmetry:

$$S(p) = CS^T(-p)C^{-1}$$

  - Inverse Majorana propagator has even less degrees of freedom:

$$\gamma_0 \left(S^{-1}(p)\right)^\dagger \gamma_0 = S^{-1}(p) \qquad \Rightarrow A_L^\dagger = A_L, \quad A_R^\dagger = A_R, \quad B_L^\dagger = B_R$$

$$S^{-1}(p) = C\left(S^{-1}(-p)\right)^T C^{-1} \qquad \Rightarrow A_L^T = A_R, \quad B_L^T = B_L, \quad B_R^T = B_R.$$

MAJORANAS

- Remaining independent renormalization conditions:

$$i \neq j : \quad (B_L)_{ij}(m_j^2) = m_j (A_R)_{ij}(m_j^2)$$

$$(A_L)_{ii}(m_i^2) = (A_R)_{ii}(m_i^2), \quad \mathrm{Im}\,(B_L)_{ii}(m_i^2) = 0, \quad \mathrm{Re}\,(B_L)_{ii}(m_i^2) = m_i (A_R)_{ii}(m_i^2)$$

(+ condition containing the derivatives)
which means we have as the #independent conditions:

$$2(N^2 - N) + 2N + N = 2N^2 + N$$

- Loose freedom of rephasing due to relation between L/R-parts

$$\psi_i^{(b)} = \psi_{Li}^{(b)} + \left(\psi_{Li}^{(b)}\right)^c \quad \Rightarrow Z_R^{(1/2)} = \left(Z_L^{(1/2)}\right)^*$$

$$Z_L^{(1/2)} \to e^{i\hat{\alpha}} Z_L^{(1/2)}, \quad Z_R^{(1/2)} \to e^{i\hat{\alpha}} Z_R^{(1/2)}, \quad e^{i\hat{\alpha}} = \mathrm{diag}(e^{i\alpha_1}, \ldots, e^{i\alpha_N})$$

(not to be confused with Majorana phases in mixing matrix)

- **Again equals #counterterms** (one general complex FSRC + mass counterterms)

CONCLUSIONS

- On-shell renormalization conditions in theories with mixing already known, but still unclear in some specifics
- These conditions play an important role in extensions of the Standard Model in the fermion and scalar sector (with potentially strong mixing)
- Pains have been taken to dispel any unclear point in the derivation
- For more extras (e.g. explicit expressions for counterterms) and calculational details, see **Int. J. Mod. Phys. A 31, 1630038 (2016)**


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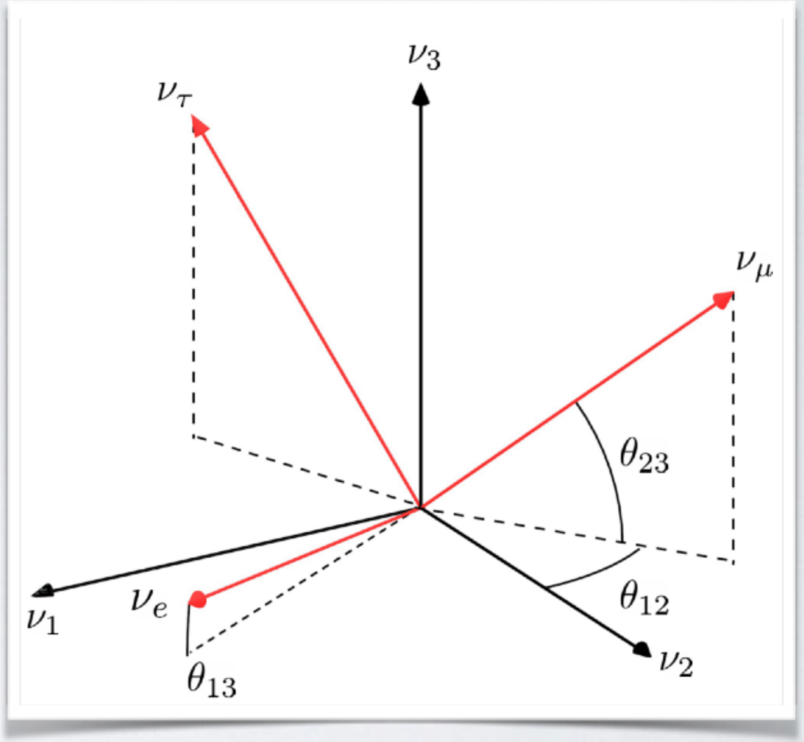
THANKS!

BACKUP SLIDES

FLAVOR SYMMETRIES

- Attempt to describe/explain structure of U_{PMNS} via symmetries of the mass matrix
- Use combination of discrete symmetries to approximate U_{PMNS}, e.g. μ-τ **symmetry**

[Phys. Lett. B 579 (2004), 113-122]

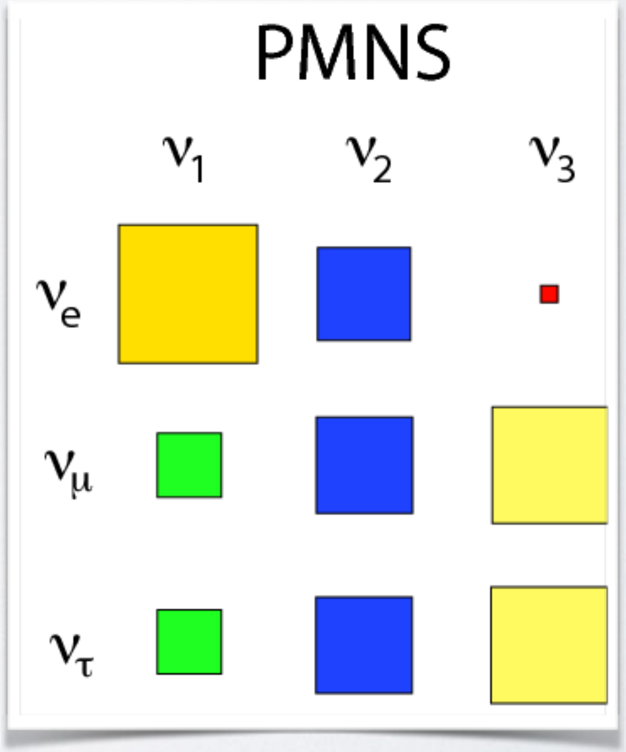


$$S = \begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S\mathcal{M}_\nu S = \mathcal{M}_\nu^*$$

$$\Rightarrow |U_{\mu i}| = |U_{\tau i}| \quad \forall i$$

$$\Rightarrow \theta_{23} = 45^\circ,\ \delta = \pm\frac{\pi}{2}.$$



http://arxiv.org/abs/arXiv:1212.6374